

# Triple prompt $J/\psi$ hadroproduction as a hard probe of multiple-parton scatterings

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Based on Hua-Sheng Shao, YJZ, 1902.04949

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## Outline

- 1 Introduction
- 2 The frame of Calculation
- 3 Numerical Result of  $J/\psi + J/\psi + J/\psi + X$
- 4 Summary

# Introduction

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- 3 PDF: collinear PDF, TMD PDF, ...

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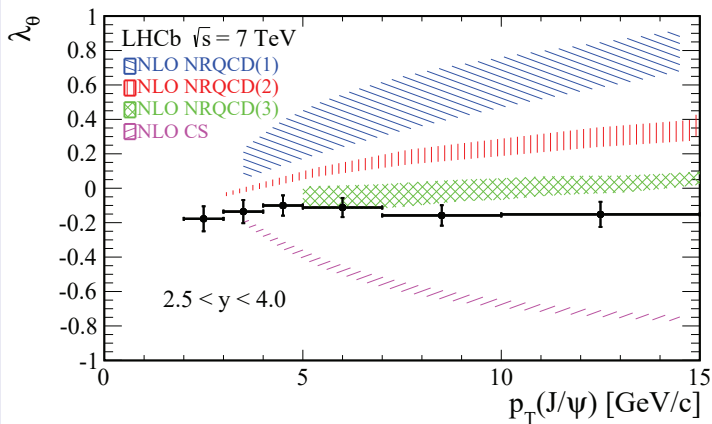
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- 5 Double-parton scattering (DPS), triple-parton scattering (TPS)



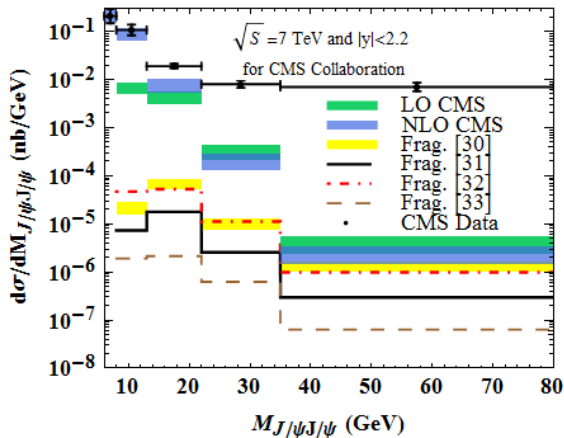
## Quarkonium productions

NLO  $J/\psi$  at LHCb, Chao/Wang/Kniehl, 1506.03981

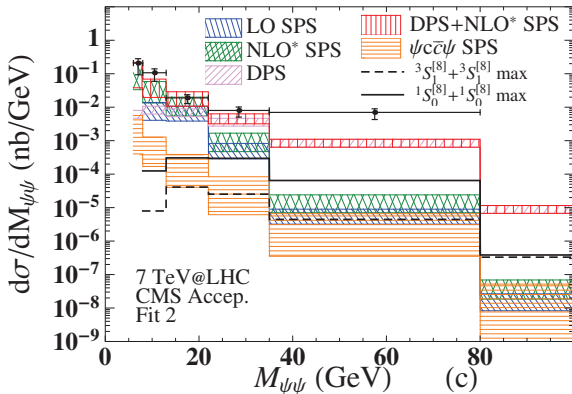
## CO LDMEs, 1212.2037

	Butenschoen, Kniehl <sup>18</sup>	Gong, Wang, Wan, Zhang <sup>53</sup>	Chao, Ma, Shao, Wang, Zhang <sup>52</sup>		
			default set	set 2	set 3
$\langle \mathcal{O}^{J/\psi} ({}^3S_1^{[1]}) \rangle$	1.32 GeV <sup>3</sup>	1.16 GeV <sup>3</sup>	1.16 GeV <sup>3</sup>	1.16 GeV <sup>3</sup>	1.16 GeV <sup>3</sup>
$\langle \mathcal{O}^{J/\psi} ({}^1S_0^{[8]}) \rangle$	0.0497 GeV <sup>3</sup>	0.097 GeV <sup>3</sup>	0.089 GeV <sup>3</sup>	0	0.11 GeV <sup>3</sup>
$\langle \mathcal{O}^{J/\psi} ({}^3S_1^{[8]}) \rangle$	0.0022 GeV <sup>3</sup>	-0.0046 GeV <sup>3</sup>	0.0030 GeV <sup>3</sup>	0.014 GeV <sup>3</sup>	0
$\langle \mathcal{O}^{J/\psi} ({}^3P_0^{[8]}) \rangle$	-0.0161 GeV <sup>5</sup>	-0.0214 GeV <sup>5</sup>	0.0126 GeV <sup>5</sup>	0.054 GeV <sup>5</sup>	0
$\langle \mathcal{O}^{\psi'} ({}^3S_1^{[1]}) \rangle$		0.758 GeV <sup>3</sup>			
$\langle \mathcal{O}^{\psi'} ({}^1S_0^{[8]}) \rangle$		-0.0001 GeV <sup>3</sup>			
$\langle \mathcal{O}^{\psi'} ({}^3S_1^{[8]}) \rangle$		0.0034 GeV <sup>3</sup>			
$\langle \mathcal{O}^{\psi'} ({}^3P_0^{[8]}) \rangle$		0.0095 GeV <sup>5</sup>			
$\langle \mathcal{O}^{\chi_0} ({}^3P_0^{[1]}) \rangle$		0.107 GeV <sup>5</sup>			
$\langle \mathcal{O}^{\chi_0} ({}^3S_1^{[8]}) \rangle$		0.0022 GeV <sup>3</sup>			

# Double $J/\psi$ at CMS, Sun, Han, Chao, 1404.4042



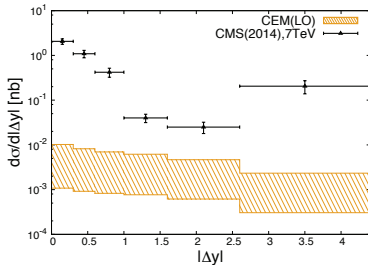
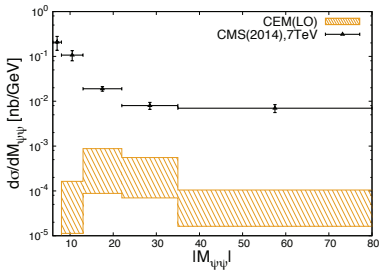
# Double $J/\psi$ at CMS, Lansberg, Shao, 1410.8822



# Double $J/\psi$ , Lansberg, Shao, 1410.8822

Energy and quarkonium cuts		$\sigma_{\text{exp.}}$	$\sigma_{\text{LO}}^{\text{SPS,prompt}}$	$\sigma_{\text{NLO}^{(*)}}^{\text{SPS,prompt}}$	$\sigma_{\text{DPS,prompt}}$	$\chi^2$
LHCb	$\sqrt{s} = 7 \text{ TeV}, P_T^{\psi_{1,2}} < 10 \text{ GeV},$ $2 < y_\psi < 5$ [34]	$18 \pm 5.3 \text{ pb}$	$41^{+51}_{-24} \text{ pb}$	$46^{+58}_{-27} \text{ pb}$	$31^{+11}_{-6.3} ({}^{+24}_{-15}) \text{ pb}$	$0.5 - 1.2$
D0	$\sqrt{s} = 1.96 \text{ TeV}, P_T^{\psi_{1,2}} > 4 \text{ GeV},$	SPS: $70 \pm 23 \text{ fb}$	$53^{+57}_{-27} \text{ fb}$	$170^{+340}_{-110} \text{ fb}$	–	–
	$ \eta_\psi  < 2.0$ [12] (+ $\mu$ cuts in caption)	DPS: $59 \pm 23 \text{ fb}$	–	–	$44^{+16}_{-9.1} ({}^{+7.5}_{-5.1}) \text{ fb}$	$0.06 - 0.5$
CMS	$\sqrt{s} = 7 \text{ TeV}, P_T^{\psi_{1,2}} > 6.5 \rightarrow 4.5 \text{ GeV}$ depending on $ y_{\psi_{1,2}}  \in [0, 2.2]$ (see the caption) [35]	$5.25 \pm 0.52 \text{ pb}$	$0.35^{+0.26}_{-0.17} \text{ pb}$	$1.5^{+2.2}_{-0.87} \text{ pb}$	$0.69^{+0.24(+0.039)}_{-0.14(-0.027)} \text{ pb}$	$1.09 - 1.14$
ATLAS	$\sqrt{s} = 7 \text{ TeV}, P_T^{\psi_{1,2}} > 5 \text{ GeV}$ and $ y_{\psi_{1,2}}  < 2.1$ (+ $\mu$ cuts in the caption) [48]	–	$6.4^{+4.3}_{-2.6} \text{ fb}$	$36^{+49}_{-20} \text{ fb}$	$19^{+6.8(+2.2)}_{-4.0(-1.6)} \text{ fb}$	N/A

# Double $J/\psi$ in the color evaporation model (CEM), Yamanaka, Lansberg, Shao, Zhang 1808.09884, 1811.07474



## Quarkonium production and double parton scattering

Many quarkonium associated production processes seems to be dominant by Double-Parton Scattering (DPS).

- 1  $J/\psi + W$  and  $J/\psi + Z$ , (ATLAS, arXiv:1401.2831, 1412.6428)

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- 3  $J/\psi + J/\psi$  (D0, arXiv:1406.2380; CMS, arXiv:1406.0484)

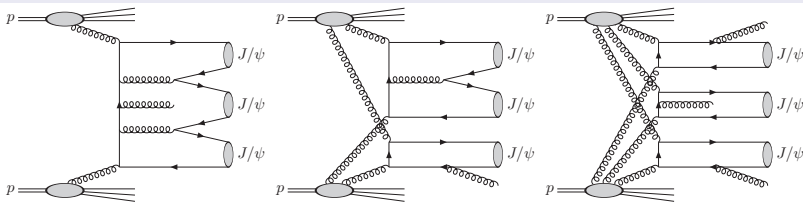
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- 3  $J/\psi + J/\psi$  (D0, arXiv:1406.2380; CMS, arXiv:1406.0484)
- 4  $\Upsilon + J/\psi$  (D0, arXiv:1511.02428)

# Single parton scattering (SPS), double parton scattering (DPS), and triple-parton scattering (TPS)

## SPS, DPS, TPS



**Figure:** SPS ( $\alpha_s^7$ ), DPS ( $\alpha_s^7$ ), and TPS ( $\alpha_s^9$ ) of  $pp \rightarrow 3J/\psi + X$ .

## DPS and TPS, 1708.07519

## DPS

$$\sigma_{pp \rightarrow a+b+X}^{DPS} = \frac{1}{1 + \delta_{ab}} \frac{\sigma(pp \rightarrow a + X) \times \sigma(pp \rightarrow b + X)}{\sigma_{eff,2}} \quad (1)$$

## TPS

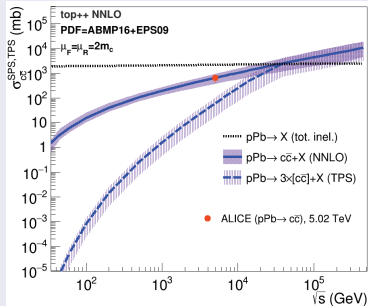
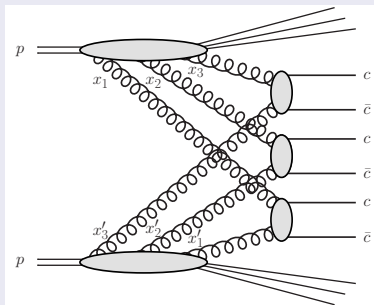
In particular, the TPS has never been observed yet by any experiment.

$$\sigma_{pp \rightarrow a+b+c+X}^{DPS} = \frac{1}{(1 + \delta_{ab} + \delta_{ac} + \delta_{bc} - \delta_{ab}\delta_{ac}\delta_{bc})!} \times \frac{\sigma(a + X) \times \sigma(b + X) \times \sigma(c + X)}{\sigma_{eff,3}^2} \quad (2)$$

$\sigma_{eff,2} = (0.82 + 0.11) \times \sigma_{eff,2}$  is drive after a global survey of

## Triple parton scattering

## TPS



**Figure:** TPS of  $pP_b \rightarrow c\bar{c} + c\bar{c} + c\bar{c}$  (PRL118, 122001).

## Double parton scattering Picture (Enterria, Snigirev, 1708.07519)

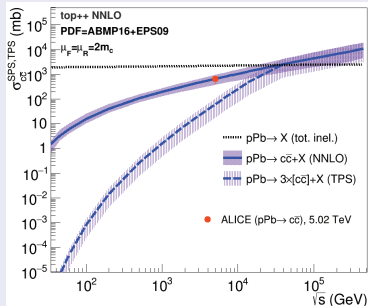
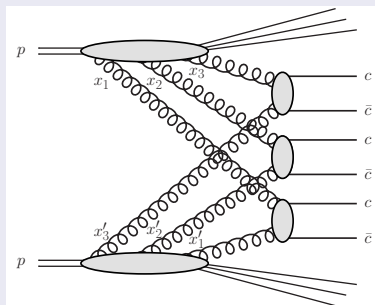
### DPS

$$\begin{aligned} P_{pp \rightarrow ab}^{DPS} &= P_{pp \rightarrow a}^{SPS} \times P_{pp \rightarrow b}^{SPS} \\ &= \frac{\sigma(pp \rightarrow a + X)}{\sigma^{inel}(pp)} \times \frac{\sigma(pp \rightarrow b + X)}{\sigma^{inel}(pp)} \end{aligned} \quad (3)$$

Then  $\sigma_{eff}^{nPS} \sim \sigma^{inel}(pp)$ . But  $\sigma^{inel}(pp) \sim 30 - 50\text{mb}$  and  $\sigma_{eff}^{nPS} \sim 10 - 15\text{mb}$ .

## Triple parton scattering

## TPS



**Figure:** TPS of  $pP_b \rightarrow c\bar{c} + c\bar{c} + c\bar{c}$  (PRL118, 122001).

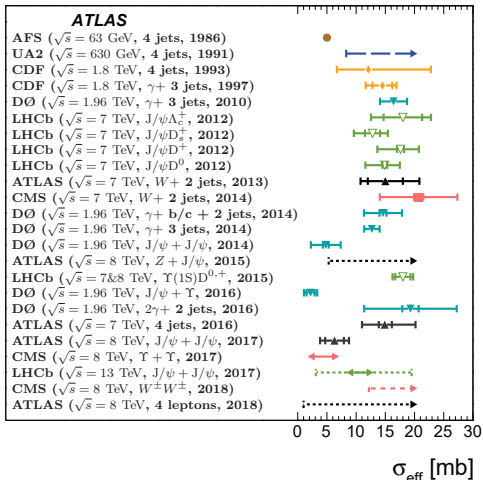


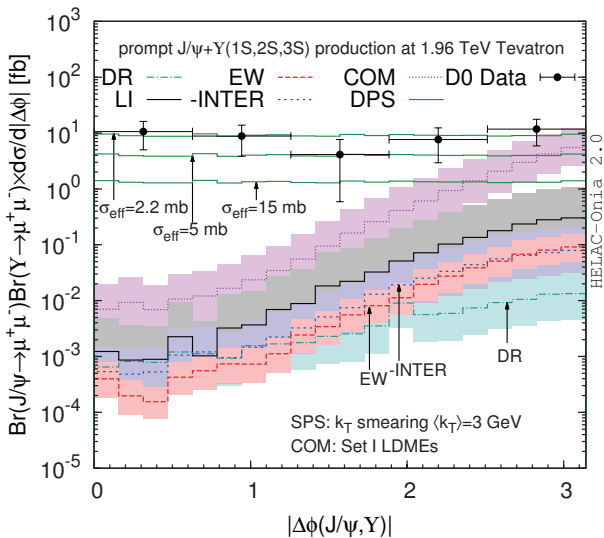


$\sigma_{\text{eff}}^{\text{DPS}}$ , Atlas, 1811.11094

## DPS and $W, Z$

Experiment (energy, final state, year)



$J/\psi + \Upsilon$  @ D0, Shao, Zhang, 1605.03061

## Quarkonium associated production

### Quarkonium associated production at hadron colliders

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- 5  $\sigma(\Upsilon + J/\psi)@ \alpha_s^6$ , Shao, Zhang, 1605.03061.
- 6  $\sigma(J/\psi + J/\psi + J/\psi + g)@ \alpha_s^7$ , Shao, Zhang, 1902.04949.



# The frame of Calculation

## Cross sections

### Hadron and Parton level cross sections

$$\sigma(h_1 h_2 \rightarrow \mathcal{J}\mathcal{J}\mathcal{J}) = \sum_{a,b} f_{a/h_1} \otimes f_{b/h_2} \otimes \hat{\sigma}(ab \rightarrow \mathcal{C} + \mathcal{J} + \mathcal{J} + X). \quad (4)$$

### Parton level cross section

$$d\hat{\sigma}(ab \rightarrow \mathcal{J}\mathcal{J}\mathcal{J}) = \sum_{n_1, n_2, n_3} \hat{\sigma}(ab \rightarrow c\bar{c}[n_1]c\bar{c}[n_2]c\bar{c}[n_3] + X) \langle O^{\mathcal{J}}(n_1) \rangle \langle O^{\mathcal{J}}(n_2) \rangle \langle O^{\mathcal{J}}(n_3) \rangle \quad (5)$$

## DPS and TPS cross sections of $3J/\psi$

$$\sigma^{\text{DPS}}(pp \rightarrow 3J/\psi + X) = \frac{\sigma^{\text{SPS}}(J/\psi J/\psi + X) \sigma^{\text{SPS}}(J/\psi + X)}{\sigma_{\text{eff},2}},$$

$$\sigma^{\text{TPS}}(pp \rightarrow 3J/\psi + X) = \frac{1}{6} \frac{(\sigma^{\text{SPS}}(pp \rightarrow J/\psi + X))^3}{(\sigma_{\text{eff},3})^2}. \quad (6)$$

Therefore, we have to calculate three different SPS cross sections for one, two and three  $J/\psi$  production in proton-proton collisions, Lansberg, Shao, 1504.06531.

## SPS cross sections for one and two $J/\psi$ production

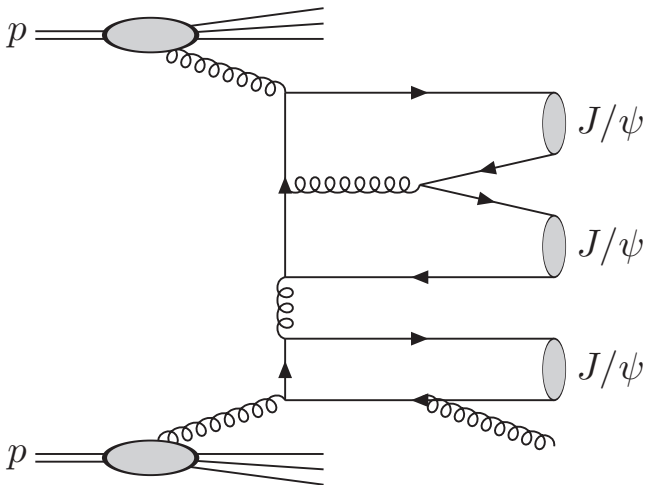
### SPS cross sections for $J/\psi$

We will use the data-driven approach to fit the matrix element of the single  $J/\psi$  production with the precise experimental data, Lansberg, Shao, 1410.8822.

### SPS cross sections for $J/\psi J/\psi$

We will use partial NLO result of double  $J/\psi$  SPS part by including infrared-safe real emission diagrams only (Lansberg, Shao, 1410.8822), which shows a reasonable agreement with the complete NLO calculation (Sun, Han, Chao, 1404.4042).

# One of 28774 Feynman Diagrams of SPS cross sections for $J/\psi J/\psi J/\psi$

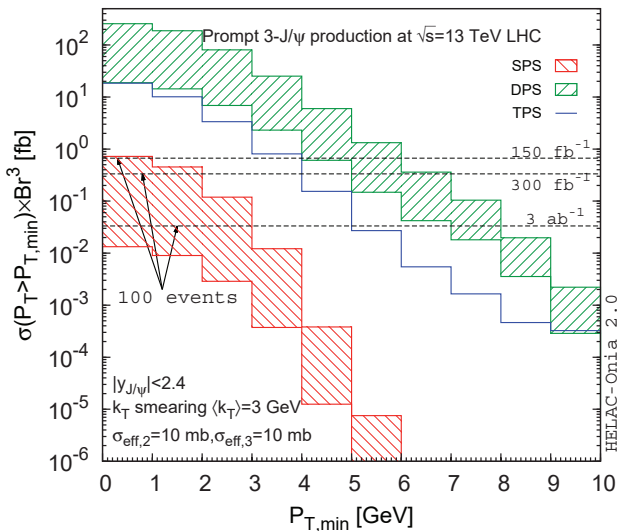


Numerical Result of  $J/\psi J/\psi J/\psi + X$

Cross sections of  $J/\psi J/\psi J/\psi$ 

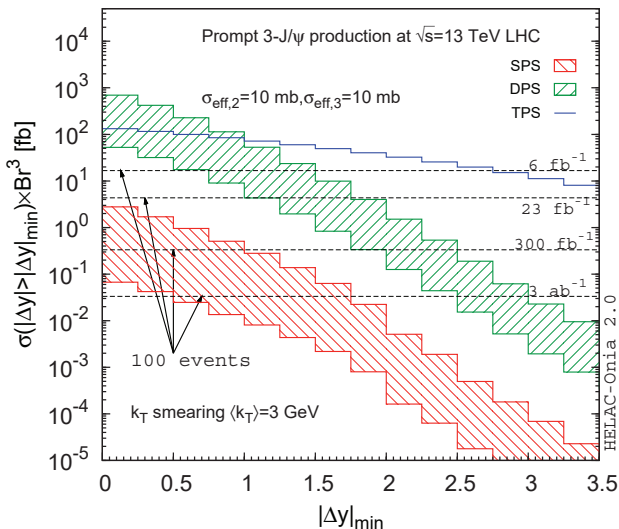
		inclusive	$2.0 < y_{J/\psi} < 4.5$	$ y_{J/\psi}  < 2.4$
13 TeV	SPS	$0.41_{-0.34}^{+2.4} \pm 0.0083$	$(1.8_{-1.5}^{+11} \pm 0.18) \times 10^{-2}$	$(8.7_{-7.5}^{+56} \pm 0.098) \times 10^{-2}$
	DPS	$(190_{-140}^{+501}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}^2}$	$(7.0_{-5.1}^{+18}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}^2}$	$(50_{-37}^{+140}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}^2}$
	TPS	$130 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)^2$	$1.3 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)^2$	$18 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)^2$
27 TeV	SPS	$0.46_{-0.39}^{+2.9} \pm 0.022$	$(3.2_{-2.8}^{+22} \pm 0.21) \times 10^{-2}$	$(5.8_{-5.1}^{+39} \pm 0.29) \times 10^{-2}$
	DPS	$(560_{-480}^{+2900}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}^2}$	$(19_{-16}^{+97}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}^2}$	$(120_{-100}^{+630}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}^2}$
	TPS	$570 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)^2$	$5.0 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)^2$	$57 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)^2$
75 TeV	SPS	$0.59_{-0.52}^{+4.4} \pm 0.016$	$(3.0_{-2.7}^{+25} \pm 0.23) \times 10^{-2}$	$(7.2_{-6.5}^{+63} \pm 0.38) \times 10^{-2}$
	DPS	$(1900_{-1600}^{+11000}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}^2}$	$(57_{-50}^{+340}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}^2}$	$(310_{-270}^{+2000}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}^2}$
	TPS	$3900 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)^2$	$27 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)^2$	$260 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)^2$
100 TeV	SPS	$1.1_{-1.0}^{+8.4} \pm 0.044$	$(4.5_{-4.0}^{+33} \pm 0.72) \times 10^{-2}$	$(36_{-32}^{+290} \pm 1.8) \times 10^{-2}$
	DPS	$(3400_{-2900}^{+19000}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}^2}$	$(100_{-86}^{+550}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}^2}$	$(490_{-430}^{+3000}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}^2}$
	TPS	$6500 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)^2$	$45 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)^2$	$380 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)^2$

# Cross sections of $J/\psi J/\psi J/\psi$





# Cross sections of $J/\psi J/\psi J/\psi$



## Summary

We performed a first complete theoretical study of  $3J/\psi$  by including SPS, DPS and TPS contributions.

The prompt  $3J/\psi$  is a very clean process to probe TPS and therefore the possible triple-parton correlations in a proton.

Although the process is rare, LHC Run2 is already more than enough to measure this process.

Finally, the minimal rapidity gap among  $3J/\psi$ s is a very useful observable to separate the TPS events from DPS and SPS.





## Multi parton scattering

The inclusive cross section to produce  $n$  hard particles in hadronic colliders is a convolution of generalized  $n$ -parton distribution functions (PDF) and elementary partonic cross sections summed over all involved partons,

$$\begin{aligned}
 \sigma_{hh' \rightarrow a_1 \dots a_n}^{\text{NPS}} = & \\
 & \left(\frac{m}{n!}\right) \sum_{i_1, \dots, i_n, i'_1, \dots, i'_n} \int \Gamma_h^{i_1 \dots i_n}(\mathbf{x}_1, \dots, \mathbf{x}_n; \mathbf{b}_1, \dots, \mathbf{b}_n; Q_1^2, \dots, Q_n^2) \\
 & \times \hat{\sigma}_{a_1}^{i_1 i'_1}(\mathbf{x}_1, \mathbf{x}'_1, Q_1^2) \cdots \hat{\sigma}_{a_n}^{i_n i'_n}(\mathbf{x}_n, \mathbf{x}'_n, Q_n^2) \quad (7) \\
 & \times \Gamma_{h'}^{i'_1 \dots i'_n}(\mathbf{x}'_1, \dots, \mathbf{x}'_n; \mathbf{b}_1 - \mathbf{b}, \dots, \mathbf{b}_n - \mathbf{b}; Q_1^2, \dots, Q_n^2) \\
 & \times d\mathbf{x}_1 \dots d\mathbf{x}_n d\mathbf{x}'_1, \dots, d\mathbf{x}'_n d^2b_1, \dots, d^2b_n d^2b.
 \end{aligned}$$

## The $n$ -parton distribution function (1708.07519)

**It encodes all the 3D structure information of the hadron.**

- 1 Assumption 1: the  $n$ -PDF are factored in terms of longitudinal and transverse components,

$$\Gamma_h^{i_1 \dots i_n} = D_h^{i_1 \dots i_n}(x_1, \dots, x_n; Q_1^2, \dots, Q_n^2) f(\mathbf{b}_1) \dots f(\mathbf{b}_n) \quad (8)$$

- 2 We can get hadron-hadron overlap function  
 $T(\mathbf{b}) = \int f(\mathbf{b}_1) f(\mathbf{b}_1 - \mathbf{b}) d^2 b_1$ , where  $1 = \int T(\mathbf{b}) d^2 b$ .
- 3 Assumption 2: the longitudinal components reduce to the product of independent single PDF

$$D_h^{i_1 \dots i_n}(x_1, \dots, x_n; Q_1^2, \dots, Q_n^2) = D_h^{i_1}(x_1; Q_1^2) \dots D_h^{i_n}(x_n; Q_n^2) \quad (9)$$

## The cross sections and $\sigma_{\text{eff}}^{nPS}$ (Enterria, Snigirev, 1708.07519)

### The cross sections of $n$ -particle associated production

Then we can get

$$\sigma_{hh' \rightarrow a_1 \dots a_n}^{nPS} = \left(\frac{m}{n!}\right) \frac{\sigma_{hh' \rightarrow a_1}^{SPS} \dots \sigma_{hh' \rightarrow a_n}^{SPS}}{(\sigma_{\text{eff}}^{nPS})^{n-1}}, \quad (10)$$

$\sigma_{\text{eff}}^{nPS}$

$$\left(\frac{1}{\sigma_{\text{eff}}^{nPS}}\right)^{n-1} = \int d^2b T^n(\mathbf{b}) \quad (11)$$