FCPPL-Satellite

# About Glueball Hunting


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April 23, 2019, Tsinghua

# Contents:


- A Brief Review of Glueball Studies
- Gluon-pair Vacuum Coupling Vertex
- Oddballs via QCDSR
- Hunting for Oddballs
- Concluding remarks

# Contents:

- A Brief Review of Glueball Studies
- $0^{--}$ Oddballs via QCDSR
- Hunting for Oddballs
- Concluding remarks

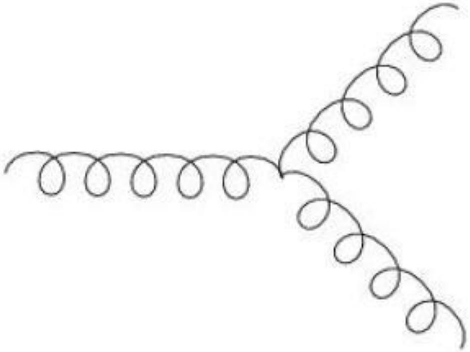
# I. A Brief Review of Glueball Studies

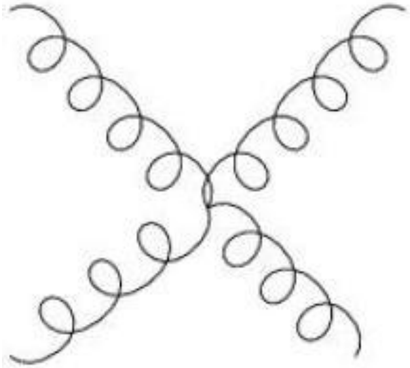
**The QCD Lagrangian:**

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu} + \sum_q \bar{\psi}_q (i\gamma^\mu D_\mu - m_q)\psi_q$$

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f^{abc} A^b_\mu A^c_\nu$$

**There exist gluon self-interactions**





# I. A Brief Review of Glueball Studies

- **Color structure**
  - **Quark= fundamental representation 3**
  - **Gluon= Adjoint representation 8**
  - **Observable particles=color singlet 1**
    - **Mesons** $3 \bigotimes 3 = 1 \bigoplus 8$
    - **Baryons** $3 \bigotimes 3 \bigotimes 3 = 1 \bigoplus 8 \bigotimes 8 \bigoplus 10$
    - **Glueballs** $\begin{cases} 8 \bigotimes 8 = 1 \bigoplus 8 \bigoplus 8 \bigoplus 10 \bigoplus \bar{10} \bigoplus 27 \\ 8 \bigotimes \cdots \bigotimes 8 = 1 \bigoplus 8 \bigoplus \cdots \end{cases}$

# I. A Brief Review of Glueball Studies

- Glueballs are allowed by QCD
- No definite observation in experiment up to now

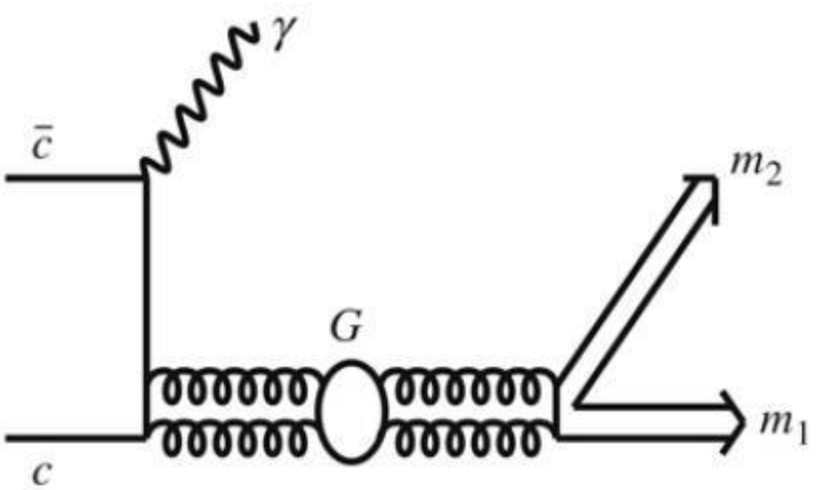
**The main difficulties in observing glueballs lie in**

- lack of the knowledge about their production & decay properties
- mixing with quark states adds difficulty to isolate them.

# I. A Brief Review of Glueball Studies

- **Gluon-rich processes (Taking gg as an example)**

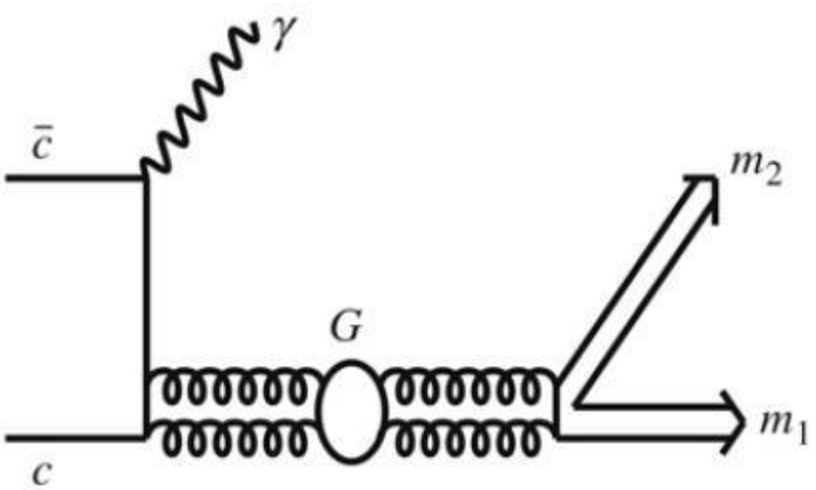
**The most prominent example in $e^+e^-$ colliders.**



# I. A Brief Review of Glueball Studies

➢ **Gluon-rich processes (Taking gg as an example)**

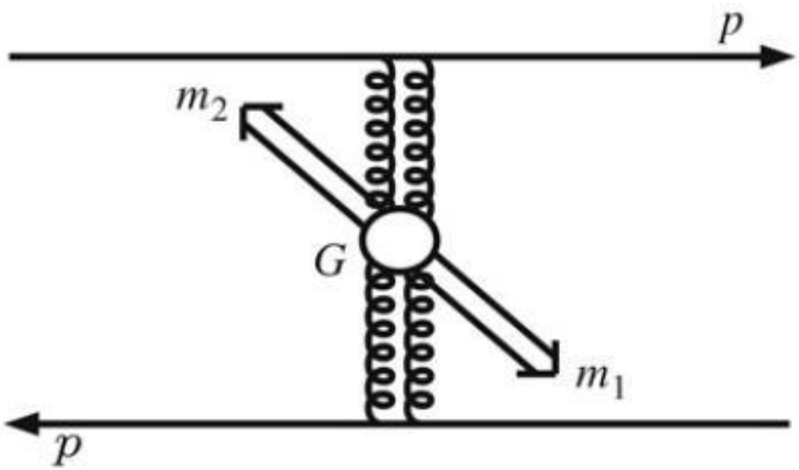
**The most prominent example in $e^+e^-$ colliders.**

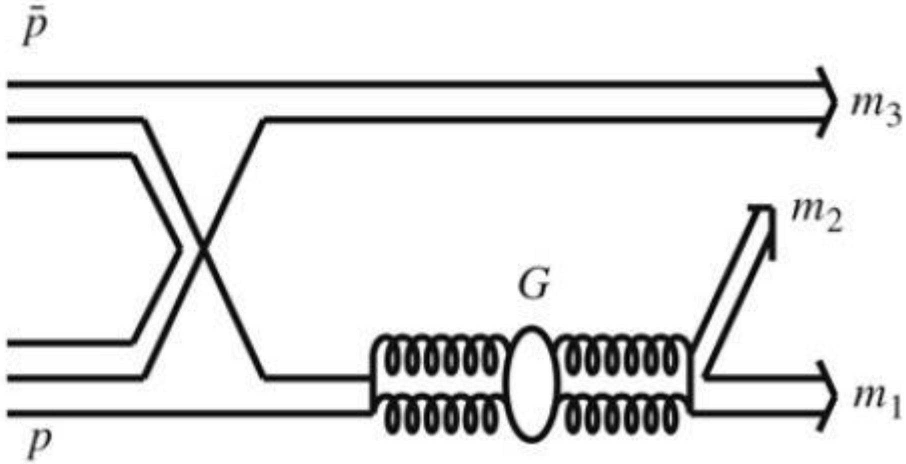


# I. A Brief Review of Glueball Studies

- **Gluon-rich processes (Taking gg as an example)**

**Very promising examples in hadron colliders.**



# I. Glueballs and Glueball Studies

- **Good evidence exists for the lightest scalar guleball $0^{++}$ , which however mixes with nearby mesons. There are several candidates, e.g. $f_0(980)$, $f_0(1500)$, $f_0(1710)$, but no definitive conclusions can be drawn concerning the nature of these states.**
- **Evidence for tensor $2^{++}$ and pseudoscalar $0^{-+}$ glueballs are weak**
- **The study of the oddballs in experiment is still in absence**

**To pin down a glueball in experiment is a challenging task**

V. Crede and C.A. Meyer, Prog. Part. Nucl. Phys. 63(2009) 74-116, and refs. therein

# I. A Brief Review of Glueball Studies

- **Theoretically:**
  - Lattice QCD
  - Flux tube model
  - MIT bag model
  - Coulomb gauge model
  - QCD Sum Rules (QCDSR)

Flux tube model, MIT bag model, Coulomb gauge model } **Constituent Models**

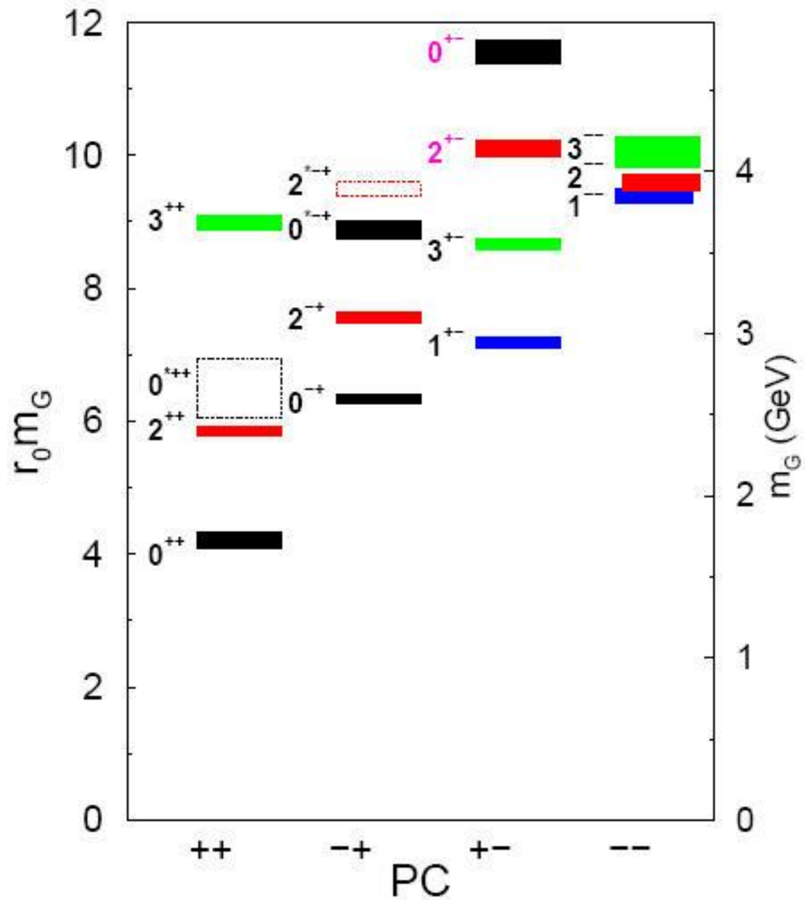
V.Mathieu, N.Kochelev&V.Vento, Int.J.Mod.Phys. E18,1(2009)

**Morningstar & Peardon, PRD60 (1999) 034509**

# ● Results from Lattice QCD

$$r_0^{-1} = 410 \pm 20 \text{ MeV}$$

| $J^{PC}$ | Other $J$ | $r_0 m_G$ | $m_G$ (MeV) |
|---|---|---|---|
| $0^{++}$ | | 4.21 (11)(4) | 1730 (50)(80) |
| $2^{++}$ | | 5.85 (2)(6) | 2400 (25)(120) |
| $0^{-+}$ | | 6.33 (7)(6) | 2590 (40)(130) |
| $0^{*++}$ | | 6.50 (44)(7)$^\dagger$ | 2670 (180)(130) |
| $1^{+-}$ | | 7.18 (4)(7) | 2940 (30)(140) |
| $2^{-+}$ | | 7.55 (3)(8) | 3100 (30)(150) |
| $3^{+-}$ | | 8.66 (4)(9) | 3550 (40)(170) |
| $0^{*-+}$ | | 8.88 (11)(9) | 3640 (60)(180) |
| $3^{++}$ | 6, 7, 9, ... | 8.99 (4)(9) | 3690 (40)(180) |
| $1^{--}$ | 3, 5, 7, ... | 9.40 (6)(9) | 3850 (50)(190) |
| $2^{*-+}$ | 4, 5, 8, ... | 9.50 (4)(9)$^\dagger$ | 3890 (40)(190) |
| $2^{--}$ | 3, 5, 7, ... | 9.59 (4)(10) | 3930 (40)(190) |
| $3^{--}$ | 6, 7, 9, ... | 10.06 (21)(10) | 4130 (90)(200) |
| $2^{+-}$ | 5, 7, 11, ... | 10.10 (7)(10) | 4140 (50)(200) |
| $0^{+-}$ | 4, 6, 8, ... | 11.57 (12)(12) | 4740 (70)(230) |



# I. A Brief Review of Glueball Studies

- **Results of Lattice QCD**

| $R^{PC}$ | Possible $J^{PC}$ | $r_0 M_G$ | $r_0 M_G$ |
|---|---|---|---|
| $A_1^{++}$ | $0^{++}$ | 4.16(11) | 4.21(11) |
| $E^{++}$ | $2^{++}$ | 5.82(5) | 5.85(2) |
| $T_2^{++}$ | $2^{++}$ | 5.83(4) | 5.85(2) |
| $A_2^{++}$ | $3^{++}$ | 9.00(8) | 8.99(4) |
| $T_1^{++}$ | $3^{++}$ | 8.87(8) | 8.99(4) |
| $A_1^{-+}$ | $0^{-+}$ | 6.25(6) | 6.33(7) |
| $T_1^{+-}$ | $1^{+-}$ | 7.27(4) | 7.18(3) |
| $E^{-+}$ | $2^{-+}$ | 7.49(7) | 7.55(3) |
| $T_2^{-+}$ | $2^{-+}$ | 7.34(11) | 7.55(3) |
| $T_2^{+-}$ | $3^{+-}$ | 8.80(3) | 8.66(4) |
| $A_2^{+-}$ | $3^{+-}$ | 8.78(5) | 8.66(3) |
| $T_1^{--}$ | $1^{--}$ | 9.34(4) | 9.50(4) |
| $E^{--}$ | $2^{--}$ | 9.71(3) | 9.59(4) |
| $T_2^{--}$ | $2^{--}$ | 9.83(8) | 9.59(4) |
| $A_2^{--}$ | $3^{--}$ | 10.25(4) | 10.06(21) |
| $E^{+-}$ | $2^{+-}$ | 10.32(7) | 10.10(7) |
| $A_1^{+-}$ | $0^{+-}$ | 11.66(7) | 11.57(12) |

Chen *et al.*, PRD73(2006) 014516

Morningstar & Peardon, PRD60 (1999) 034509

**Mass($0^{--}$) = (5166±1000) MeV (Unquenched)**

Gregory, *et al.*, JHEP1210 (2012) 170.

# I. A Brief Review of Glueball Studies

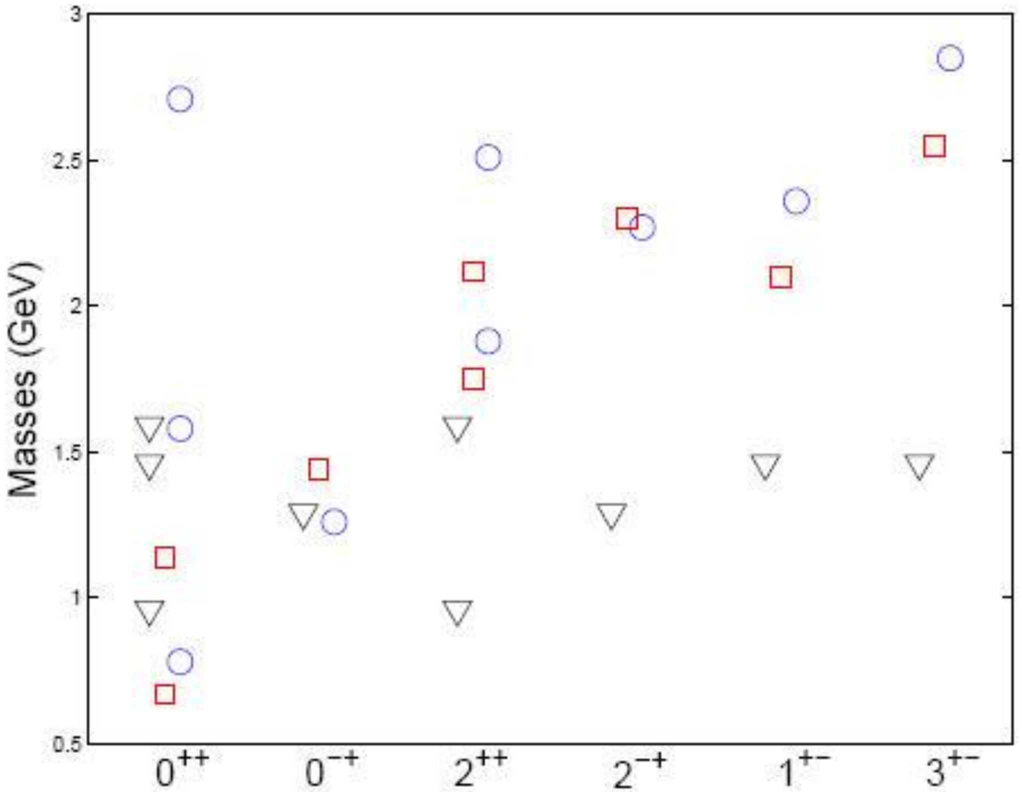
- **Flux tube model**

| $J^{PC}$ | Mass (GeV) |
|---|---|
| $0^{++}$ | 1.52 |
| $1^{+-}$ | 2.25 |
| $0^{++}$ | 2.75 |
| $0^{++}, 0^{+-}, 0^{-+}, 0^{--}$ | 2.79 |
| $2^{++}$ | 2.84 |
| $2^{++}, 2^{++}, 2^{++}, 2^{++}$ | 2.84 |
| $1^{+-}$ | 3.25 |
| $3^{+-}$ | 3.35 |

Isgur & Parton, PRD31(1985)2910.

# I. A Brief Review of Glueball Studies

- **MIT bag model**




▽ =Jaffe &Johnson, PLB60,201(1976).

○ =Carlson *et al.*, PRD27 (1983)1556.

□ =Chanowitz &Sharpe, NPB222(1983)211.

# I. A Brief Review of Glueball Studies

- **Coulomb Gauge model**

| Model | $J^{PC}$ | $0^{-+}$ | $1^{--}$ | $2^{--}$ | $3^{--}$ | $5^{--}$ | $7^{--}$ |
|---|---|---|---|---|---|---|---|
| | color | $f$ | $d$ | $d$ | $d$ | $d$ | $d$ |
| | $S$ | 0 | 1 | 2 | 3 | 3 | 3 |
| | $L$ | 0 | 0 | 0 | 0 | 2 | 4 |
| $H^g_{\rm eff}$ (this work) | | 3900 | 3950 | 4150 | 4150 | 5050 | 5900 |
| $H_M$ (this work) | | 3400 | 3490 | 3660 | 3920 | 5150 | 6140 |

Llances-Estrada, Bicudo &Cotanch, PRL96 (2006) 081601

# I. A Brief Review of Glueball Studies

- **QCD Sum Rules**

Two-gluon glueballs in QCDSR

| | Novikov *et.al.* | Forkel | Bagan *et.al.* | Huang *et.al* |
|---|---|---|---|---|
| $0^{++}$ | 0.7-0.9 GeV | 1.25 GeV | 1.7 GeV | 1.66 GeV |
| $0^{-+}$ | - | 2.2 GeV | - | - |

Novikov et al., NPB165 (1980) 67.

Bagan&Steele, PLB243 (1990) 43.

Forkel, PRD64 (2001) 034015.

Huang, Jin&Zhang, PRD59 (1999) 034026.

# I. A Brief Review of Glueball Studies

- **QCD Sum Rules**

Tri-gluon glueballs in QCDSR

| | $0^{++}$ | $0^{-+}$ | $1^{-+}$ | $1^{--}$ | $2^{++}$ |
|---|---|---|---|---|---|
| Latorre *et. al.* | 3.1 GeV | - | - | - | - |
| Liu *et. al.* | 1.45 GeV | - | 1.87 GeV | 2.4 GeV | 2.0 GeV |
| Hao *et. al.* | - | 1.9-2.7 GeV | - | - | - |

**Latorre *et al*., PLB191 (1987) 437.**

**Liu, CPL15 (1998) 784.**

**G. Hao, CFQ, A.L. Zhang, PLB642 (2006) 53.**

# I. A Brief Review of Glueball Studies

- **Production studies of glueballs via Lattice QCD**

  **For example:**

- ***Scalar glueball in radiative J/ψ decay on lattice,***
  **Long-Cheng Gui, *et al*., (CLQCD Collaboration),**
  **Phys. Rev. Lett. 110 (2013) 021601**

- ***Lattice study of radiative J/ψ decay to a tensor glueball,***
  **Yi-Bong Yang, *et al*., (CLQCD Collaboration),**
  **Phys. Rev. Lett. 111 (2013) 091601.**

# I. A Brief Review of Glueball Studies

- **Decay analysis of glueballs**

  **For example:**

- ***Comment on "Chiral Suppression of Scalar-Glueball Decay",***
  **Kuang-Ta Chao, Xiao-Gang He, and Jian-Ping Ma,**
  **Phys. Rev. Lett. 98, 149103 (2007).**

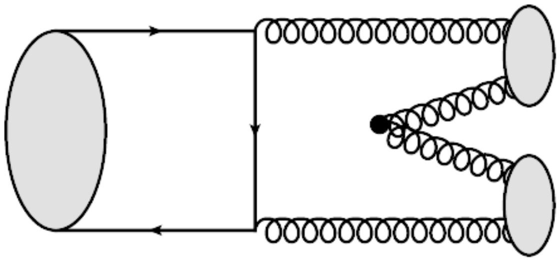
- ***On Two-Body Decays of A Scalar Glueball,***
  **Kuang-Ta Chao, Xiao-Gang He, and Jian-Ping Ma,**
  **Eur. Phys. J. C55: 417-421(2008).**

# Contents

- Glueballs and Glueball Studies
- **Gluon-pair Vacuum Coupling Vertex**
- Hunting for Oddballs
- Concluding remarks

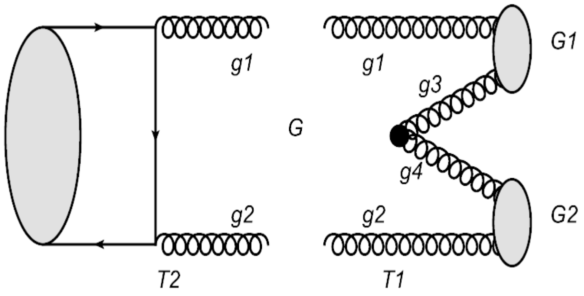
# II. Gluon-pair Vacuum Coupling Vertex

## $\eta_c/\eta_b$ decays to glueballs exclusively via $0^{++}$ model



**The transition amplitude writes:**

$$\langle G_1 G_2|T|\eta_c\rangle = g_s^2 \gamma_g \langle G_1 G_2|\bar{q}_i t^a_{ij} \gamma_\mu q_j A^\mu_a \bar{q}_m \times t^b_{mn} \gamma_\nu q_n A^\nu_b \delta_{cd} \eta_{\rho\sigma} A^\rho_c A^\sigma_d|\eta_c\rangle \ ,$$

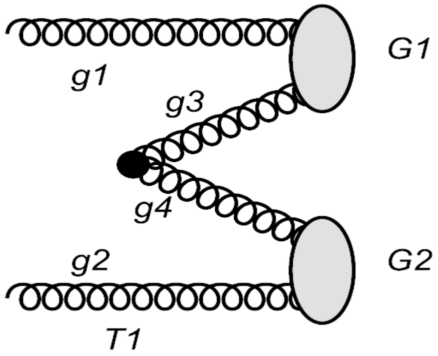



**Inserting the completeness**

$$\langle G_1 G_2|T|\eta_c\rangle = \frac{1}{2E_G}\sum_G \gamma_g \langle G_1 G_2|\delta_{cd}\eta_{\rho\sigma} A^\rho_c A^\sigma_d|G\rangle \times g_s^2 \langle G|\bar{q}_i t^a_{ij} \gamma_\mu q_j A^\mu_a \bar{q}_m t^b_{mn} \gamma_\nu q_n A^\nu_b|\eta_c\rangle \equiv \frac{1}{2E_G}\sum_G \gamma_g \langle G_1 G_2|T_1|G\rangle g_s^2 \langle G|T_2|\eta_b\rangle.$$

# II. Gluon-pair Vacuum Coupling Vertex

## The $0^{++}$ model




$$\hat{T}_1 = I_1 \otimes I_2 \otimes \hat{T}_{vac}$$

$I_i \rightarrow$ Identity matrices for $g_i$

$\hat{T}_{vac} \rightarrow$ Gluon-pair vacuum vertex

$$J^{PC}_{vac} = 0^{++} \Rightarrow \begin{cases} L = 0 \\ S = 0 \end{cases}$$

$$\Rightarrow \langle LM_L; SM_S | J_{vac} M_{vac} \rangle = 1$$

$$\hat{T}_{vac} = \gamma_g \int \mathrm{d}^3\mathbf{k}_3 \, \mathrm{d}^3\mathbf{k}_4 \delta^3(\mathbf{k}_3 + \mathbf{k}_4) \mathcal{Y}_{00}\left(\frac{\mathbf{k}_3 - \mathbf{k}_4}{2}\right) \chi^{34}_{0,0} \, \delta_{cd} a^\dagger_{3c}(\mathbf{k}_3) \, a^\dagger_{4d}(\mathbf{k}_4).$$

# II. Gluon-pair Vacuum Coupling Vertex

## The $0^{++}$ model

$$J_G^{PC} = J_{\eta_c}^{PC} = 0^{+-}$$

$|G\rangle$ **can be expressed as**

$$|G\rangle = \sqrt{2E_G}\int \mathrm{d}^3\mathbf{k}_1\mathrm{d}^3\mathbf{k}_2\delta^3\left(\mathbf{K}_\mathrm{G}-\mathbf{k}_1-\mathbf{k}_2\right)$$
$$\times \sum_{M_{L_G},M_{S_G}} \langle L_G M_{L_G} S_G M_{S_G}|J_G M_{J_G}\rangle\, \psi_{n_G L_G M_{L_G}}\left(\mathbf{k}_1,\mathbf{k}_2\right)\chi^{12}_{S_G M_{S_G}}\delta_{ab}\left|g_1^a g_2^b\right\rangle .$$

**The normalization conditions**

$$\langle G(\mathbf{K}_G)|G(\mathbf{K}'_G)\rangle = 2E_G\delta^3(\mathbf{K}_G-\mathbf{K}'_G),$$
$$\langle g_i^a(\mathbf{k}_i)|g_j^b(\mathbf{k}_j)\rangle = \delta_{ij}\delta^{ab}\delta^3(\mathbf{k}_i-\mathbf{k}_j),$$
$$\int \mathrm{d}^3\mathbf{k}_1\mathrm{d}^3\mathbf{k}_2\delta^3(\mathbf{K}_\mathrm{G}-\mathbf{k}_1-\mathbf{k}_2)\psi_\mathrm{G}(\mathbf{k}_1,\mathbf{k}_2)\psi_{\mathrm{G}'}(\mathbf{k}_1,\mathbf{k}_2) = \delta_{\mathrm{G}'\mathrm{G}}.$$

# II. Gluon-pair Vacuum Coupling Vertex

## The $0^{++}$ model

$|G_1\rangle$ **and** $|G_2\rangle$ **can be constructed similarly, then**

$$\begin{aligned}\langle G_1G_2|T_1|G\rangle &= \sqrt{8E_GE_{G_1}E_{G_2}}\,\gamma_g \sum_{M_{L_G},M_{S_G}M_{L_{G_1}},M_{S_{G_1}}M_{L_{G_2}},M_{S_{G_2}}} \\ &\times \langle L_GM_{L_G}S_GM_{S_G}|J_GM_{J_G}\rangle\langle L_{G_1}M_{L_{G_1}}S_{G_1}M_{S_{G_1}}|J_{G_1}M_{J_{G_1}}\rangle\langle L_{G_2}M_{L_{G_2}}S_{G_2}M_{S_{G_2}}|J_{G_2}M_{J_{G_2}}\rangle \\ &\times \langle\chi^{13}_{S_{G_1}M_{S_{G_1}}}\chi^{24}_{S_{G_2}M_{S_{G_2}}}|\chi^{12}_{S_GM_{S_G}}\chi^{34}_{00}\rangle I_{M_{L_G},M_{L_{G_1}},M_{L_{G_2}}}(\mathbf{K})(\delta_{ab}\delta_{cd}\delta_{ac}\delta_{bd})_{color-octet}\,.\end{aligned}$$

$$\begin{aligned}I_{M_{L_G},M_{L_{G_1}},M_{L_{G_2}}}(\mathbf{K}) &= \int d^3\mathbf{k}_1d^3\mathbf{k}_2d^3\mathbf{k}_3d^3\mathbf{k}_4\,\delta^3(\mathbf{k}_1+\mathbf{k}_2)\delta^3(\mathbf{k}_3+\mathbf{k}_4)\delta^3(\mathbf{K}_{G_1}-\mathbf{k}_1-\mathbf{k}_3)\delta^3(\mathbf{K}_{G_2}-\mathbf{k}_2-\mathbf{k}_4) \\ &\times \psi^*_{n_{G_1}L_{G_1}M_{L_{G_1}}}(\mathbf{k}_1,\mathbf{k}_3)\psi^*_{n_{G_2}L_{G_2}M_{L_{G_2}}}(\mathbf{k}_2,\mathbf{k}_4)\psi_{n_GL_GM_{L_G}}(\mathbf{k}_1,\mathbf{k}_2)\mathcal{Y}_{00}\left(\frac{\mathbf{k}_3-\mathbf{k}_4}{2}\right).\end{aligned}$$

# II. Gluon-pair Vacuum Coupling Vertex

## The $0^{++}$ model

In momentum space, the wave function in form of harmonic oscillator

$$\psi_{nLM}(\mathbf{k}) = \mathcal{N}_{nL} \exp\left(-\frac{R^2\mathbf{k}^2}{2}\right) \mathcal{Y}_{LM}(\mathbf{k})\, \mathcal{P}(\mathbf{k}^2).$$

The spin coupling can be expressed by winger's 9j symbol

$$\langle \chi^{13}_{S_{G_1} M_{S_{G_1}}} \chi^{24}_{S_{G_2} M_{S_{G_2}}} | \chi^{12}_{S_G M_{S_G}} \chi^{34}_{00} \rangle = (-1)^{S_{G_2}+1} \Big[ (2S_{G_1}+1)(2S_{G_2}+1)(2S_G+1) \Big]^{1/2}$$
$$\times \sum_{S, M_s} \langle S_{G_1} M_{S_{G_1}}; S_{G_2} M_{S_{G_2}} | S M_s \rangle \langle S M_s | S_G M_{S_G}; 00 \rangle \begin{Bmatrix} s_1 & s_3 & S_{G_1} \\ s_2 & s_4 & S_{G_2} \\ S_G & 0 & S \end{Bmatrix}.$$

**Z. G. Luo, X. L. Chen, and X. Liu, Phys. Rev. D 79, 074020 (2009).**
**A. Le Yaouanc, L. Oliver, O. Pene and J. Raynal, Hadron Transitions in the Quark Model**

# II. Gluon-pair Vacuum Coupling Vertex

## The $0^{++}$ model

**Extract helicity amplitude from**

$$\langle G_2|T_1|G\rangle = \delta^3(\mathbf{K}_{G_1}+\mathbf{K}_{G_2}-\mathbf{K}_G) \times \mathcal{M}_1^{M_{J_G}M_{J_{G_1}}M_{J_{G_2}}},$$

**$T_2$ can be calculated in pQCD**

$$|\mathcal{M}_2|^2 = \frac{8g_s^4|R(0)_{\eta_c}|^2}{3\pi m_c}.$$

$$\mathcal{M}^{JL} = \frac{\mathcal{M}_1^{JL}\mathcal{M}_2}{2E_G}$$

$$\mathcal{M}_1^{JL} = \frac{\sqrt{2L+1}}{2J_G+1}\sum_{M_{G_1},M_{G_2}}\langle L0JM_{J_G}|J_G M_{J_G}\rangle \times\langle J_{G_1}M_{J_{G_1}}J_{G_2}M_{J_{G_2}}|JM_{J_G}\rangle\mathcal{M}_1^{M_{J_G}M_{J_{G_1}}M_{J_{G_2}}}$$

**M. Jacob and G. C. Wick, Ann. Phys.7, 404 (1959)**

**The decay width then**

$$\Gamma = \pi^2\frac{|\mathbf{K}|}{M_{\eta_c}^2}\sum_{JL}\left|\mathcal{M}^{JL}\right|^2,$$

# II. Gluon-pair Vacuum Coupling Vertex

**Glueballs production in $\eta_c/\eta_b$ decays via the $0^{++}$ model**

$f_0(1500)$ **and** $\eta(1405)$ **as the glueball candidates**

$$\langle G_1G_2|T_1|G\rangle = \sum_{M_G,M_{G_2}} 8\gamma_g\sqrt{8E_GE_{G_1}E_{G_2}} \times \langle 1M_0;1-M_0|00\rangle \times \langle 1M_2;1-M_2|00\rangle \times \langle \chi^{13}_{00}\chi^{24}_{1-M_2}|\chi^{12}_{1-M_0}\chi^{34}_{00}\rangle I_{M_0,0,M_2}(\mathbf{K}) .$$

| | $J^{PC}$ | $L$ | $M$ | $S$ | $M_S$ |
|---|---|---|---|---|---|
| $\eta_c$ | $0^{-+}$ | 1 | $M_0$ | 1 | $-M_0$ |
| $f_0(1500)$ | $0^{++}$ | 0 | 0 | 0 | 0 |
| $\eta(1405)$ | $0^{-+}$ | 1 | $M_2$ | 1 | $-M_2$ |

# II. Gluon-pair Vacuum Coupling Vertex

In simple harmonic oscillator :

$$E_{in} = (2n + L + 3/2)\hbar\omega, \quad \alpha = \sqrt{\mu\omega/\hbar}, \quad R = 1/\alpha.$$

R and other parameters

| | $E$ (GeV) | $E_{in}$(GeV) | $\omega$ (GeV) | $\alpha$ (GeV) | $R$ (GeV)$^{-1}$ |
|---|---|---|---|---|---|
| $G$ | 2.98 | 2.98 | 0.66 | 0.45 | 2.24 |
| $G_1$ | 1.53 | 1.50 | 0.43 | 0.36 | 2.79 |
| $G_2$ | 1.45 | 1.41 | 0.31 | 0.31 | 3.26 |

$$I_{0,0,0} = -0.41\delta^3(\mathrm{K}_G - \mathrm{K}_{G_1} - \mathrm{K}_{G_2}),$$

$$\mathcal{M}_1^{JL} = \mathcal{M}_1^{00} = 0.11\gamma_g.$$

# II. Gluon-pair Vacuum Coupling Vertex

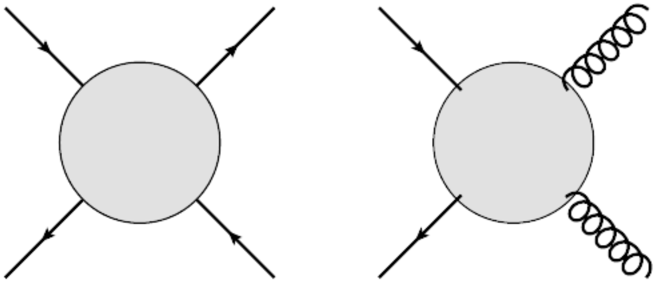
**$\gamma_g$ estimated by comparing to the $^3P_0$ model.**

$\gamma = g/2m$ **quark-pair creation strength**

**Z. G. Luo, X. L. Chen, and X. Liu, Phys. Rev. D 79, 074020 (2009), J. Segovia, D.R. Entem, and F. Fernandez Grupo, Phys. Lett. B 715, 322 (2012).**

$$\gamma_g^2/g_s^2 \sim \Gamma_{q\bar{q}\to gg}/\Gamma_{q\bar{q}} \sim 0.4,$$

$$\gamma_g^2 = 3.32^{+2.05}_{-1.10}\ \mathrm{GeV}^2.$$



$$\Gamma_{q\bar{q}\to q\bar{q}} \sim \frac{4}{9}\left(\frac{s^2+u^2}{t^2}+\frac{t^2+u^2}{s^2}-\frac{2u^2}{3st}\right),$$

$$\Gamma_{q\bar{q}\to gg} \sim \frac{32}{27}\left(-\frac{9\,(t^2+u^2)}{4s^2}+\frac{t}{u}+\frac{u}{t}\right).$$

# II. Gluon-pair Vacuum Coupling Vertex

- The decay width

$$\Gamma = \pi^2 \frac{|\mathbf{K}|}{M_{\eta_c}^2} \sum_{JL} \left| \mathcal{M}^{JL} \right|^2 = \pi^2 \frac{|\mathbf{K}|}{4M_{\eta_c}^4} |\mathcal{M}_1^{00}|^2 |\mathcal{M}_2|^2$$

$$= \frac{16\pi^2 g_s^4 |R(0)_{\eta_c}|^2 \gamma_g^2 |\mathbf{K}| E_G E_{G_1} E_{G_2} I^2}{3^7 \times \pi m_c M_{\eta_c}^4} = 5.17^{+3.52}_{-1.84}\ \mathrm{KeV}\ ,$$

- The branching ratio

$$Br_{\eta_c \to f_0(1500)\eta(1405)} = \frac{\Gamma_{\eta_c \to f_0(1500)\eta(1405)}}{\Gamma_{total}} = 1.63^{+1.18}_{-0.61} \times 10^{-4}.$$

# II. Oddballs via QCDSR

- Glueballs and Glueball Studies
- **Oddballs via QCDSR**
- Hunting for Oddballs
- Concluding remarks

# II. Oddballs via QCDSR

- **Oddballs**

**Oddballs: glueballs with exotic quantum numbers**

$$J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, 3^{-+} \text{ and so on}$$

Trigluon glueballs

**Physics at BESIII, Editors Kuang-Ta Chao & Yifang Wang, Int. JMPA24,1,(2009).**

**V.Mathieu, N.Kochelev&V.Vento, Int.J.Mod.Phys. E18,1(2009).**

- $C=-1 \rightarrow$ **Trigluon glueballs.**
- **Exotic quantum numbers ➡ Do not mix with** $q\bar{q}$
- $0^{--}$ **oddball may be the lowest lying one. Besides, it has the simplest Lorentz structure.**

# II. Oddballs via QCDSR

- **It can be produced in the decay of heavy vector quarkonium or quarkoniumlike states.**

**= exotic state (not easy to be detected, only $\pi_1(1400)$ with $1^{-+}$ in PDG )**

**= unfavorable production channel**

**Like in football game + =**

$$1^{--} \to G_{0^{--}} + 1^{++}$$

$$1^{--} \to G_{0^{+-}} + 1^{-+}$$

$$1^{--} \to G_{1^{-+}} + 0^{--}$$

$$1^{--} \to G_{2^{+-}} + 1^{-+}/2^{-+}/3^{-+}$$

$$1^{--} \to G_{3^{-+}} + 2^{+-}$$

$$1^{++} = f_1(1285)/\chi_{c1}(3511)$$

# II. Oddballs via QCDSR

- **QCDSR**

CFQ & Liang Tang, PRL113 (2014) 221601

CFQ & Liang Tang, NPB904 (2016) 282

- Field strength tensor

$$G^a_{\mu\nu}(x) = G^a_{0\mu\nu}(x) + g_s f^{abc} A^b_\mu(x) A^c_\nu(x)$$

- In coordinate gauge

$$A^a_\mu(x) \simeq \frac{1}{2} x^\nu G^a_{\nu\mu}(0); \qquad A^a_\mu(0) \simeq 0$$

$$G^a_{\mu\nu}(x) = G^a_{0\mu\nu}(0) + \frac{1}{4} g_s f^{abc} x^\rho x^\sigma G^b_{\rho\mu}(0) G^c_{\sigma\nu}(0)$$

$$G^a_{0\mu\nu}(x) = \partial_\mu A^a_\nu(x) - \partial_\nu A^a_\mu(x)$$

$$G^a_{\mu\nu}(0) = G^a_{0\mu\nu}(0) = G^a_{0\mu\nu}(x)$$

# II. Oddballs via QCDSR

- Some contractions

$$\underbrace{G^a_{0\mu\nu}(x)G^i_{0\alpha\beta}(y)} = \int \frac{d^4p}{(2\pi)^4}\frac{-i\delta^{ai}}{p^2}\Gamma_{\mu\nu\alpha\beta}(p)e^{-ip\cdot(x-y)}$$

$$\Gamma_{\mu\nu\alpha\beta}(p) = p_\mu p_\alpha g_{\nu\beta} + p_\nu p_\beta g_{\mu\alpha} - p_\mu p_\beta g_{\nu\alpha} - p_\nu p_\alpha g_{\mu\beta}$$

$$\underbrace{A^m_\mu(x)G^j_{0\beta\gamma}(y)} = \int \frac{d^4p}{(2\pi)^4}\frac{\delta^{mj}}{p_1^2}(p_\beta g_{\mu\gamma} - p_\gamma g_{\mu\beta})e^{-ip\cdot(x-y)}$$

$$\underbrace{G^j_{0\beta\gamma}(x)A^m_\mu(y)} = \int \frac{d^4p}{(2\pi)^4}\frac{-\delta^{jm}}{p^2}(p_\beta g_{\mu\gamma} - p_\gamma g_{\mu\beta})e^{-ip\cdot(x-y)}$$

$$\underbrace{\widetilde{G}^a_{0\mu\nu}(x)\widetilde{G}^i_{0\rho\sigma}(y)} = \int \frac{d^4p}{(2\pi)^4}\frac{-i\delta^{ai}}{p^2}\widetilde{\Gamma}_{\mu\nu\rho\sigma}(p)e^{-ip\cdot(x-y)}$$

$$\widetilde{\Gamma}_{\mu\nu\rho\sigma}(p) = p_\mu p_\rho g_{\nu\sigma} + p_\nu p_\sigma g_{\mu\rho} - p_\mu p_\sigma g_{\nu\rho} - p_\nu p_\rho g_{\mu\sigma} + p^2(g_{\mu\sigma}g_{\nu\rho} - g_{\mu\rho}g_{\nu\sigma})$$

# II. Oddballs via QCDSR

- Gluon condensates

$$\delta^{ab}\langle 0|G^a_{\mu\nu}(0)G^b_{\rho\sigma}(0)|0\rangle = \frac{1}{D(D-1)}(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho})\langle GG\rangle$$

$$\delta^{ab}\langle 0|\widetilde{G}^a_{\mu\nu}(0)\widetilde{G}^b_{\rho\sigma}(0)|0\rangle = \frac{2-D}{2D(D-1)}(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho})\langle GG\rangle$$

$$f^{abc}\langle 0|G^a_{\mu\nu}(0)G^b_{\rho\sigma}(0)G^c_{\alpha\beta}(0)|0\rangle = \frac{1}{D(D-1)(D-2)}T^3_{\mu\nu\rho\sigma\alpha\beta}\langle GGG\rangle$$

$$\begin{aligned} T^3_{\mu\nu\rho\sigma\alpha\beta} \quad = \quad & g_{\mu\rho}g_{\nu\alpha}g_{\sigma\beta} - g_{\mu\rho}g_{\nu\beta}g_{\sigma\alpha} - g_{\mu\sigma}g_{\nu\alpha}g_{\rho\beta} + g_{\mu\sigma}g_{\nu\beta}g_{\rho\alpha} \\ & - g_{\mu\alpha}g_{\nu\rho}g_{\sigma\beta} + g_{\mu\alpha}g_{\nu\sigma}g_{\rho\beta} + g_{\mu\beta}g_{\nu\rho}g_{\sigma\alpha} - g_{\mu\beta}g_{\nu\sigma}g_{\rho\alpha} \end{aligned}$$

# Gluon condensates

$$
\begin{aligned}
& f^{abe} f^{cde} \langle 0 | G^a_{\mu\nu}(0) G^b_{\rho\sigma}(0) G^c_{\alpha\beta}(0) G^d_{\gamma\delta}(0) | 0 \rangle \\
= \quad & A\{ g_{\mu\rho}g_{\nu\alpha}g_{\sigma\gamma}g_{\beta\delta} - g_{\mu\rho}g_{\nu\alpha}g_{\sigma\delta}g_{\beta\gamma} - g_{\mu\rho}g_{\nu\beta}g_{\sigma\gamma}g_{\alpha\delta} + g_{\mu\rho}g_{\nu\beta}g_{\sigma\delta}g_{\alpha\gamma} \\
& -g_{\mu\rho}g_{\nu\gamma}g_{\sigma\alpha}g_{\beta\delta} + g_{\mu\rho}g_{\nu\gamma}g_{\sigma\beta}g_{\alpha\delta} + g_{\mu\rho}g_{\nu\delta}g_{\sigma\alpha}g_{\beta\gamma} - g_{\mu\rho}g_{\nu\delta}g_{\sigma\beta}g_{\alpha\gamma} \\
& -g_{\mu\sigma}g_{\nu\alpha}g_{\rho\gamma}g_{\beta\delta} + g_{\mu\sigma}g_{\nu\alpha}g_{\rho\delta}g_{\beta\gamma} + g_{\mu\sigma}g_{\nu\beta}g_{\rho\gamma}g_{\alpha\delta} - g_{\mu\sigma}g_{\nu\beta}g_{\rho\delta}g_{\alpha\gamma} \\
& +g_{\mu\sigma}g_{\nu\gamma}g_{\rho\alpha}g_{\beta\delta} - g_{\mu\sigma}g_{\nu\gamma}g_{\rho\beta}g_{\alpha\delta} - g_{\mu\sigma}g_{\nu\delta}g_{\rho\alpha}g_{\beta\gamma} + g_{\mu\sigma}g_{\nu\delta}g_{\rho\beta}g_{\alpha\gamma} \\
& -g_{\mu\alpha}g_{\nu\rho}g_{\sigma\gamma}g_{\beta\delta} + g_{\mu\alpha}g_{\nu\rho}g_{\sigma\delta}g_{\beta\gamma} + g_{\mu\alpha}g_{\nu\sigma}g_{\rho\gamma}g_{\beta\delta} - g_{\mu\alpha}g_{\nu\sigma}g_{\rho\delta}g_{\beta\gamma} \\
& +g_{\mu\beta}g_{\nu\rho}g_{\sigma\gamma}g_{\alpha\delta} - g_{\mu\beta}g_{\nu\rho}g_{\sigma\delta}g_{\alpha\gamma} - g_{\mu\beta}g_{\nu\sigma}g_{\rho\gamma}g_{\alpha\delta} + g_{\mu\beta}g_{\nu\sigma}g_{\rho\delta}g_{\alpha\gamma} \\
& +g_{\mu\gamma}g_{\nu\rho}g_{\sigma\alpha}g_{\beta\delta} - g_{\mu\gamma}g_{\nu\rho}g_{\sigma\beta}g_{\alpha\delta} - g_{\mu\gamma}g_{\nu\sigma}g_{\rho\alpha}g_{\beta\delta} + g_{\mu\gamma}g_{\nu\sigma}g_{\rho\beta}g_{\alpha\delta} \\
& -g_{\mu\delta}g_{\nu\rho}g_{\sigma\alpha}g_{\beta\gamma} + g_{\mu\delta}g_{\nu\rho}g_{\sigma\beta}g_{\alpha\gamma} + g_{\mu\delta}g_{\nu\sigma}g_{\rho\alpha}g_{\beta\gamma} - g_{\mu\delta}g_{\nu\sigma}g_{\rho\beta}g_{\alpha\gamma}\} \\
& +B[(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha})(g_{\rho\gamma}g_{\sigma\delta} - g_{\rho\delta}g_{\sigma\gamma}) - (g_{\mu\gamma}g_{\nu\delta} - g_{\mu\delta}g_{\nu\gamma})(g_{\rho\alpha}g_{\sigma\beta} - g_{\rho\beta}g_{\sigma\alpha})]
\end{aligned}
$$

$$
\begin{aligned}
A &= \frac{(D+1)\langle (f^{abc}G^a_{\mu\nu}G^b_{\nu\sigma})^2\rangle - \langle (f^{abc}G^a_{\mu\nu}G^b_{\rho\sigma})^2\rangle}{(D+2)D(D-1)(D-2)(D-3)} \\
B &= \frac{-4\langle (f^{abc}G^a_{\mu\nu}G^b_{\nu\sigma})^2\rangle + (D-2)\langle (f^{abc}G^a_{\mu\nu}G^b_{\rho\sigma})^2\rangle}{(D+2)D(D-1)(D-2)(D-3)}
\end{aligned}
$$

$$
f^{abe} f^{cde} \langle 0 | \widetilde{G}^a_{\mu\nu}(0) \widetilde{G}^b_{\rho\sigma}(0) \widetilde{G}^c_{\alpha\beta}(0) \widetilde{G}^d_{\gamma\delta}(0) | 0 \rangle = f^{abe} f^{cde} \langle 0 | G^a_{\mu\nu}(0) G^b_{\rho\sigma}(0) G^c_{\alpha\beta}(0) G^d_{\gamma\delta}(0) | 0 \rangle
$$

# II. Oddballs via QCDSR

## ➢ QCDSR

- **The two-point correlation function**

$$\Pi(q^2) = i \int d^4x e^{iq\cdot x} \langle 0|T\Big\{ j_{0^{--}}(x), j_{0^{--}}(0) \Big\}|0\rangle \,,$$

- **The QCD side of the correlation function**

$$\Pi^{\rm QCD}(Q^2) = a_0 Q^{12} \ln\frac{Q^2}{\mu^2} + b_0 Q^8 \langle \alpha_s G^2\rangle + \left(c_0 + c_1 \ln\frac{Q^2}{\mu^2}\right) Q^6 \langle g_s G^3\rangle + d_0 Q^4 \langle \alpha_s G^2\rangle^2 \,.$$

- **The phenomenological side of the correlation function**

$$\frac{1}{\pi}{\rm Im}\Pi^{\rm phe}(s) = f_G^2 M_{0^{--}}^{12} \delta(s - M_{0^{--}}^2) + \rho(s)\theta(s - s_0) \,.$$

# II. Oddballs via QCDSR

- **The dispersion relation**

$$\Pi(Q^2) = \frac{1}{\pi}\int_0^\infty ds \frac{\mathrm{Im}\Pi(s)}{s+Q^2} + \Big(\Pi(0) - Q^2\Pi'(0) + \frac{1}{2}Q^4\Pi''(0) - \frac{1}{6}Q^6\Pi'''(0)\Big)\,,$$

- **The Borel transformation**

$$\hat{B}_\tau \equiv \lim_{\substack{Q^2\to\infty,n\to\infty \\ \frac{Q^2}{n}=\frac{1}{\tau}}} \frac{(-Q^2)^n}{(n-1)!}\left(\frac{d}{dQ^2}\right)^n \,,$$

- **The quark-hadron duality approximation**

$$\frac{1}{\pi}\int_{s_0}^\infty e^{-s\tau}\mathrm{Im}\Pi^{\mathrm{QCD}}(s)ds \simeq \int_{s_0}^\infty \rho(s)e^{-s\tau}ds\,,$$

# II. Oddballs via QCDSR

- **The moments**

$$L_0(\tau, s_0) = \frac{1}{\pi}\int_0^{s_0} e^{-s\tau}\mathrm{Im}\Pi^{\mathrm{QCD}}(s)ds\ ,$$

$$L_1(\tau, s_0) = \frac{1}{\pi}\int_0^{s_0} s e^{-s\tau}\mathrm{Im}\Pi^{\mathrm{QCD}}(s)ds\ ,$$

- **The mass function**

$$M^i_{0^{--}}(\tau, s_0) = \sqrt{\frac{L_1(\tau, s_0)}{L_0(\tau, s_0)}}$$

- **Ratios to constrain the windows of**

$$R_i^{\mathrm{OPE}} = \frac{\int_0^{s_0} e^{-s\tau}\mathrm{Im}\Pi^{\langle \mathrm{g_s G^3}\rangle}(s)ds}{\int_0^{s_0} e^{-s\tau}\mathrm{Im}\Pi^{\mathrm{QCD}}(s)ds}$$

$$R_i^{\mathrm{PC}} = \frac{L_0(\tau, s_0)}{L_0(\tau, \infty)}\ .$$

# II. Oddballs via QCDSR

## ➢ Interpolating currents of $0^{--}$ oddballs

- **Constraints: quantum number, gauge invariance, Lorentz invariance and $SU_c(3)$ symmetry**

$$j^A_{0^{--}}(x) = g_s^3 d^{abc}[g^t_{\alpha\beta}\tilde{G}^a_{\mu\nu}(x)][\partial_\alpha\partial_\beta G^b_{\nu\rho}(x)][G^c_{\rho\mu}(x)]\,,$$
$$j^B_{0^{--}}(x) = g_s^3 d^{abc}[g^t_{\alpha\beta}G^a_{\mu\nu}(x)][\partial_\alpha\partial_\beta \tilde{G}^b_{\nu\rho}(x)][G^c_{\rho\mu}(x)]\,,$$
$$j^C_{0^{--}}(x) = g_s^3 d^{abc}[g^t_{\alpha\beta}G^a_{\mu\nu}(x)][\partial_\alpha\partial_\beta G^b_{\nu\rho}(x)][\tilde{G}^c_{\rho\mu}(x)]\,,$$
$$j^D_{0^{--}}(x) = g_s^3 d^{abc}[g^t_{\alpha\beta}\tilde{G}^a_{\mu\nu}(x)][\partial_\alpha\partial_\beta \tilde{G}^b_{\nu\rho}(x)][\tilde{G}^c_{\rho\mu}(x)]\,,$$

**where** $g^t_{\alpha\beta} = g_{\alpha\beta} - \partial_\alpha\partial_\beta/\partial^2$ $\quad \tilde{G}^a_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\kappa\tau}G^a_{\kappa\tau}$

# II. Oddballs via QCDSR

- **Interpolating currents of $0^{+-}$ oddballs**

$$
\begin{aligned}
J_A^{0^{+-}}(x) &= g_s^3 d^{abc}[g^t_{\alpha\beta} G^a_{\mu\nu}(x)][\partial_\alpha\partial_\beta G^b_{\nu\rho}(x)][G^c_{\rho\mu}(x)]\,, \\
J_B^{0^{+-}}(x) &= g_s^3 d^{abc}[g^t_{\alpha\beta} G^a_{\mu\nu}(x)][\partial_\alpha\partial_\beta \tilde{G}^b_{\nu\rho}(x)][\tilde{G}^c_{\rho\mu}(x)]\,, \\
J_C^{0^{+-}}(x) &= g_s^3 d^{abc}[g^t_{\alpha\beta} \tilde{G}^a_{\mu\nu}(x)][\partial_\alpha\partial_\beta G^b_{\nu\rho}(x)][\tilde{G}^c_{\rho\mu}(x)]\,, \\
J_D^{0^{+-}}(x) &= g_s^3 d^{abc}[g^t_{\alpha\beta} \tilde{G}^a_{\mu\nu}(x)][\partial_\alpha\partial_\beta \tilde{G}^b_{\nu\rho}(x)][G^c_{\rho\mu}(x)]\,;
\end{aligned}
$$

- **Interpolating currents of $1^{-+}$ oddballs**

$$
\begin{aligned}
J_{A,\alpha}^{1^{-+}}(x) &= g_s^3 f^{abc}\partial_\mu[G^a_{\mu\nu}(x)][G^b_{\nu\rho}(x)][G^c_{\rho\alpha}(x)]\,, \\
J_{B,\alpha}^{1^{-+}}(x) &= g_s^3 f^{abc}\partial_\mu[G^a_{\mu\nu}(x)][\tilde{G}^b_{\nu\rho}(x)][\tilde{G}^c_{\rho\alpha}(x)]\,, \\
J_{C,\alpha}^{1^{-+}}(x) &= g_s^3 f^{abc}\partial_\mu[\tilde{G}^a_{\mu\nu}(x)][G^b_{\nu\rho}(x)][\tilde{G}^c_{\rho\alpha}(x)]\,, \\
J_{D,\alpha}^{1^{-+}}(x) &= g_s^3 f^{abc}\partial_\mu[\tilde{G}^a_{\mu\nu}(x)][\tilde{G}^b_{\nu\rho}(x)][G^c_{\rho\alpha}(x)]\,,
\end{aligned}
$$

# II. Oddballs via QCDSR

- **Interpolating currents of $2^{+-}$ oddballs**

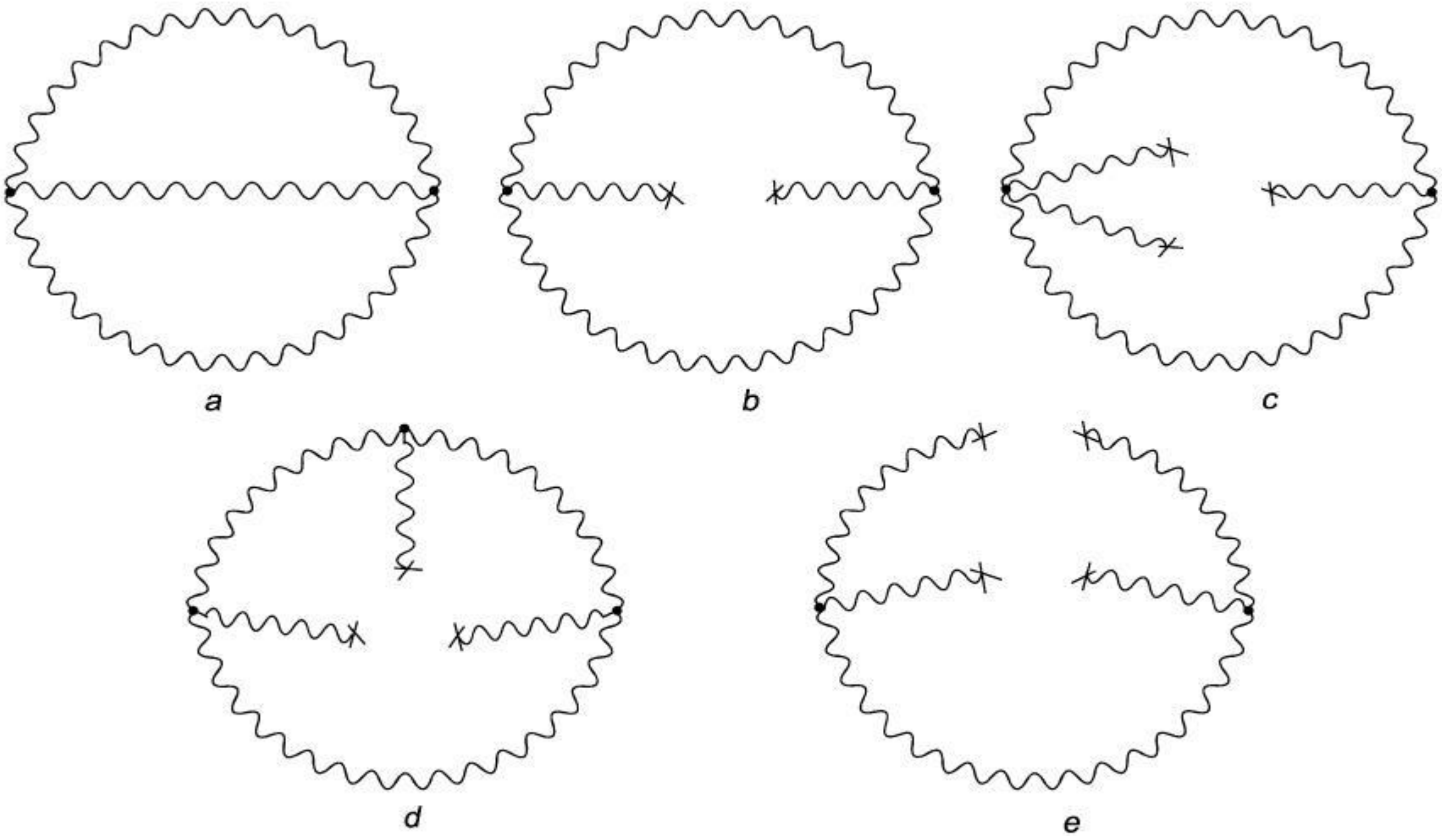
$$
\begin{aligned}
J^{2^{+-}}_{A,\mu\alpha}(x) &= g_s^3 d^{abc}[G^a_{\mu\nu}(x)G^b_{\nu\rho}(x)G^c_{\rho\alpha}(x)] \, , \\
J^{2^{+-}}_{B,\mu\alpha}(x) &= g_s^3 d^{abc}[G^a_{\mu\nu}(x)\tilde{G}^b_{\nu\rho}(x)\tilde{G}^c_{\rho\alpha}(x)] \, , \\
J^{2^{+-}}_{C,\mu\alpha}(x) &= g_s^3 d^{abc}[\tilde{G}^a_{\mu\nu}(x)G^b_{\nu\rho}(x)\tilde{G}^c_{\rho\alpha}(x)] \, , \\
J^{2^{+-}}_{D,\mu\alpha}(x) &= g_s^3 d^{abc}[\tilde{G}^a_{\mu\nu}(x)\tilde{G}^b_{\nu\rho}(x)G^c_{\rho\alpha}(x)] \, .
\end{aligned}
$$

- The two-point correlation function

$$
\Pi^{J^{PC},\,k}_{\alpha_1\cdots\alpha_j,\,\beta_1\cdots\beta_j}(q^2)=i\int d^4x e^{iq\cdot x}\langle 0|T\left\{j^{J^{PC},\,k}_{\alpha_1\cdots\alpha_j}(x), j^{J^{PC},\,k}_{\beta_1\cdots\beta_j}(0)\right\}|0\rangle \, ,
$$

# II. Oddballs via QCDSR

➢ **Typical Feynman diagrams of trigluon glueballs**



# II. Oddballs via QCDSR

- The QCD side of the correlation function

$$\begin{aligned}\Pi^{\rm k,\,QCD}_{\rm JPC}(q^2) &= a_0(q^2)^n\ln\frac{-q^2}{\mu^2}+\left(b_0+b_1\ln\frac{-q^2}{\mu^2}\right)(q^2)^{n-2}\langle\alpha_s G^2\rangle \\ &+ \left(c_0+c_1\ln\frac{-q^2}{\mu^2}\right)(q^2)^{n-3}\langle g_s G^3\rangle \\ &+ d_0(q^2)^{n-4}\langle\alpha_s G^2\rangle^2\ ,\end{aligned}$$

where n represents the corresponding power of $q^2$ for each oddballs.

- The phenomenological side of the correlation function

$$\begin{aligned}\frac{1}{\pi}{\rm Im}\Pi^{\rm k,\,phe}_{\rm JPC}(s) &= (f^k_{JPC})^2(M^k_{JPC})^{2n}\delta\left(s-(M^k_{JPC})^2\right) \\ &+ \rho^k_{JPC}(s)\theta(s-s_0)\ .\end{aligned}$$

- The dispersion relation

$$\begin{aligned}\Pi^k_{JPC}(q^2) &= \frac{1}{\pi}\int_0^\infty ds\frac{{\rm Im}\Pi^k_{JPC}(s)}{s-q^2}+\left(\Pi^k_{JPC}(0)+q^2\Pi^{k\,\prime}_{JPC}(0)\right. \\ &+ \left.\frac{1}{2}q^4\Pi^{k\,\prime\prime}_{JPC}(0)+\frac{1}{6}q^6\Pi^{k\,\prime\prime\prime}_{JPC}(0)\right)\ ,\end{aligned}$$

# II. Oddballs via QCDSR

- The main function

$$\frac{1}{\pi}\int_0^\infty \frac{\mathrm{Im}\Pi^{\mathrm{k,\,QCD}}_{\mathrm{JPC}}(s)}{s-q^2}ds$$

$$= \frac{(f^k_{JPC})^2 (M^k_{JPC})^{2n}}{(M^k_{JPC})^2 - q^2} + \int_{s_0}^\infty \frac{\rho^k_{JPC}(s)\theta(s-s_0)}{s-q^2}ds \ .$$

- The Borel transformation

$$\hat{B}_\tau \equiv \lim_{\substack{-q^2\to\infty,n\to\infty \\ \frac{-q^2}{n}=\frac{1}{\tau}}} \frac{(q^2)^n}{(n-1)!}\left(-\frac{d}{dq^2}\right)^n \ ,$$

- The quark-hadron duality approximation

$$\frac{1}{\pi}\int_{s_0}^\infty e^{-s\tau}\mathrm{Im}\Pi^{\mathrm{k,\,QCD}}_{\mathrm{JPC}}(s)ds \simeq \int_{s_0}^\infty \rho^k_{JPC}(s)e^{-s\tau}ds \ ,$$

# II. Oddballs via QCDSR

- The moments

$$L^k_{J^{PC},\,0}(\tau, s_0) = \frac{1}{\pi}\int_0^{s_0} e^{-s\tau}\mathrm{Im}\Pi^{\mathrm{k,\,QCD}}_{\mathrm{J^{PC}}}(s)ds\ ,$$

$$L^k_{J^{PC},\,1}(\tau, s_0) = \frac{1}{\pi}\int_0^{s_0} s e^{-s\tau}\mathrm{Im}\Pi^{\mathrm{k,\,QCD}}_{\mathrm{J^{PC}}}(s)ds\ ,$$

- The mass function

$$M^k_{J^{PC}}(\tau, s_0) = \sqrt{\frac{L^k_{J^{PC},\,1}(\tau, s_0)}{L^k_{J^{PC},\,0}(\tau, s_0)}}\ ,$$

- Ratio to constrain $\tau$ & $s_0$ by the pole contribution (PC)

$$R^{k,\ \mathrm{PC}}_J = \frac{L^k_{J^{PC},\,0}(\tau, s_0)}{L^k_{J^{PC},\,0}(\tau, \infty)}\ .$$

# II. Oddballs via QCDSR

- Ratio to constrain τ and $s_0$ by convergence of the OPE

$$R_J^{k,\,\mathrm{G}^2} = \frac{\int_0^{s_0} e^{-s\tau}\mathrm{Im}\Pi_{\mathrm{JPC}}^{\mathrm{k},\,\langle\alpha_s\mathrm{G}^2\rangle}(s)ds}{\int_0^{s_0} e^{-s\tau}\mathrm{Im}\Pi_{\mathrm{JPC}}^{\mathrm{k},\,\mathrm{QCD}}(s)ds}\,,$$

$$R_J^{k,\,\mathrm{G}^3} = \frac{\int_0^{s_0} e^{-s\tau}\mathrm{Im}\Pi_{\mathrm{JPC}}^{\mathrm{k},\,\langle g_s\mathrm{G}^3\rangle}(s)ds}{\int_0^{s_0} e^{-s\tau}\mathrm{Im}\Pi_{\mathrm{JPC}}^{\mathrm{k},\,\mathrm{QCD}}(s)ds}\,.$$

- Input parameters

$$\langle\alpha_s G^2\rangle = 0.06\,\mathrm{GeV}^4\ ,\ \langle g_s G^3\rangle = (0.27\,\mathrm{GeV}^2)\langle\alpha_s G^2\rangle\ ,$$

$$\Lambda_{\overline{\mathrm{MS}}} = 300\,\mathrm{MeV}\ ,\ \alpha_s = \frac{-4\pi}{11\ln(\tau\Lambda^2_{\overline{\mathrm{MS}}})}\ ,$$

# II. Oddballs via QCDSR

- Wilson coefficients of $0^{--}$ in the QCD-side

$$a_0^i = \frac{487\alpha_s^3}{143\times 2^6\times 3^3\pi}\ ,\ b_0^i = -\frac{5\pi}{36}\alpha_s^2\ , c_0^A = -\frac{205}{12}\pi\alpha_s^3\ ,$$
$$c_1^A = -\frac{775}{144}\pi\alpha_s^3\ ,\ c_0^B = -\frac{2065}{48}\pi\alpha_s^3\ , c_1^B = -\frac{1075}{96}\pi\alpha_s^3\ ,$$
$$c_0^C = \frac{2275}{72}\pi\alpha_s^3\ ,\ c_1^C = \frac{2125}{144}\pi\alpha_s^3\ ,\ c_0^D = -\frac{1045}{144}\pi\alpha_s^3\ ,$$
$$c_1^D = -\frac{25}{32}\pi\alpha_s^3\ ,\ d_0^j = 0\ ,\ d_0^D = -\frac{5}{9}\pi^3\alpha_s\ ,$$

**where, i=A, B, C, D; j=A, B, C; with A, B, C and D corresponding to the above four currents.**

**There are symmetries within Wilson coefficients $a_0^i$, $b_0^i$ and $d_0^j$. The position and number of $\tilde{G}$ do not influence the perturbative and $\langle\alpha_s G^2\rangle$ contributions, whereas they influence $\langle g_s G^3\rangle$ term. Since $\langle\alpha_s G^2\rangle^2$ involves no loop contribution, $d_0^j$ are governed by the number of $\tilde{G}$.**

# ➢Wilson coefficients of $0^{+-}$ in the QCD-side

$$a_0^A=\frac{487}{143\times 2^6\times 3^3}\frac{\alpha_s^3}{\pi}\,,\quad b_0^A=\frac{5}{36}\pi\alpha_s^2\,,\quad b_1^A=0\,,$$
$$c_0^A=-\frac{325}{72}\pi\alpha_s^3\,,\quad c_1^A=-\frac{2125}{144}\pi\alpha_s^3\,,\quad d_0^A=0\,;$$
$$a_0^B=\frac{487}{143\times 2^6\times 3^3}\frac{\alpha_s^3}{\pi}\,,\quad b_0^B=\frac{5}{36}\pi\alpha_s^2\,,\quad b_1^B=0\,,$$
$$c_0^B=\frac{7445}{144}\pi\alpha_s^3\,,\quad c_1^B=\frac{1075}{96}\pi\alpha_s^3\,,\quad d_0^B=0\,;$$
$$a_0^C=\frac{487}{143\times 2^6\times 3^3}\frac{\alpha_s^3}{\pi}\,,\quad b_0^C=\frac{5}{36}\pi\alpha_s^2\,,\quad b_1^C=0\,,$$
$$c_0^C=\frac{1955}{72}\pi\alpha_s^3\,,\quad c_1^C=\frac{775}{144}\pi\alpha_s^3\,,\quad d_0^C=0\,;$$
$$a_0^D=\frac{487}{143\times 2^6\times 3^3}\frac{\alpha_s^3}{\pi}\,,\quad b_0^D=\frac{5}{36}\pi\alpha_s^2\,,\quad b_1^D=0\,,$$
$$c_0^D=\frac{235}{72}\pi\alpha_s^3\,,\quad c_1^D=\frac{25}{32}\pi\alpha_s^3\,,\quad d_0^D=0\,,$$

where we notice that except for $c_0^k$ and $c_1^k$, $a_0^k$, $\mathrm{b}_0^k$ , $b_1^k$, and $d_0^k$ are equal for case $A$ to $D$. This situation is similar to the $0^{--}$ oddballs.

# ➢Wilson coefficients of $1^{-+}$ in the QCD-side

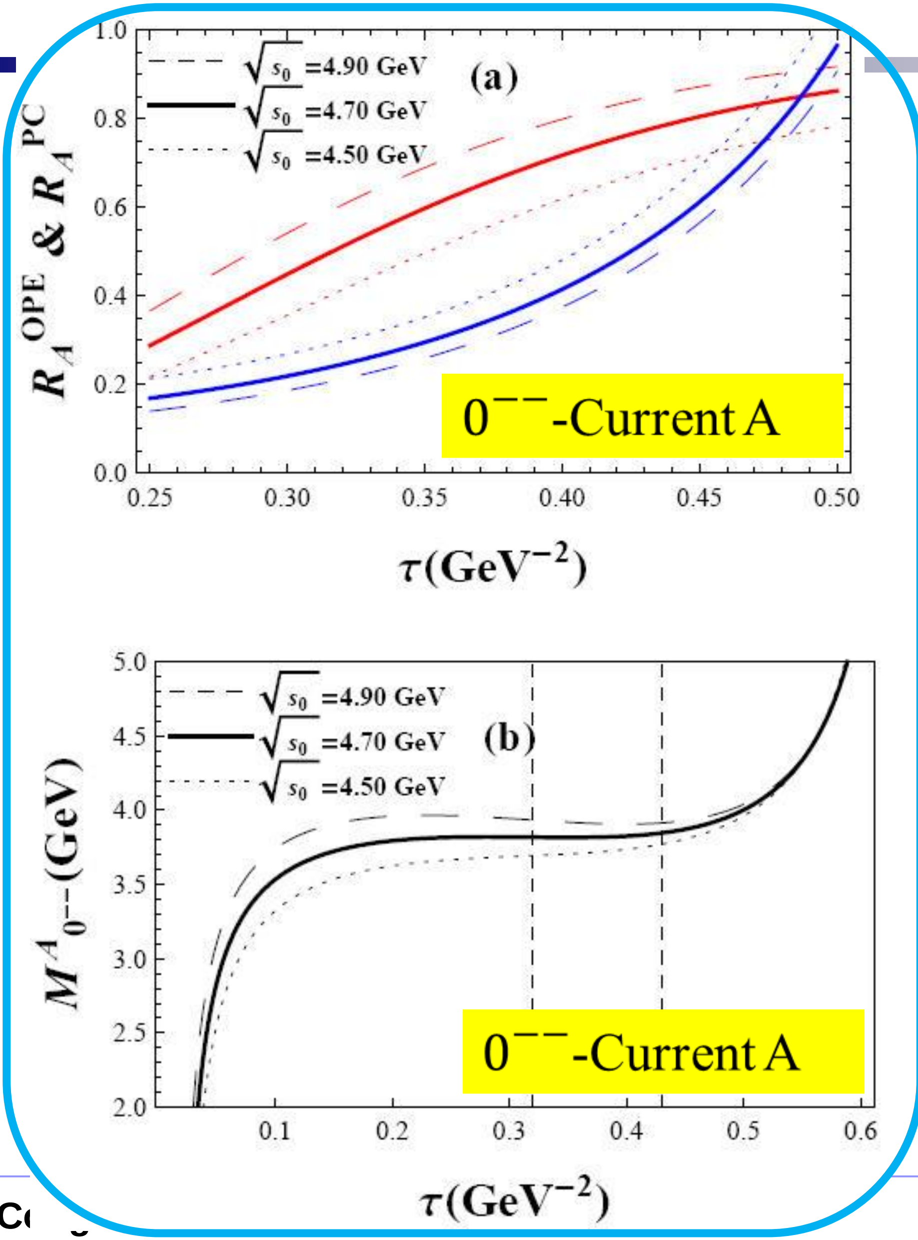
$$a_0^A=\frac{1}{1008}\frac{\alpha_s^3}{\pi}\,,\qquad b_0^A=-\frac{1}{72}\pi\alpha_s^2\,,\qquad b_1^A=\frac{1}{12}\pi\alpha_s^2\,,$$
$$c_0^A=\frac{71}{96}\pi\alpha_s^3\,,\qquad c_1^A=\frac{23}{48}\pi\alpha_s^3\,,\qquad d_0^A=\frac{1}{3}\pi^3\alpha_s\,;$$
$$a_0^B=\frac{1}{1008\pi}\frac{\alpha_s^3}{\pi}\,,\qquad b_0^B=\frac{23}{72}\pi\alpha_s^2\,,\qquad b_1^B=\frac{1}{12}\pi\alpha_s^2\,,$$
$$c_0^B=\frac{89}{64}\pi\alpha_s^3\,,\qquad c_1^B=\frac{27}{128}\pi\alpha_s^3\,,\qquad d_0^B=\frac{1}{3}\pi^3\alpha_s\,;$$
$$a_0^C=\frac{1}{112}\frac{\alpha_s^3}{\pi}\,,\qquad b_0^C=-\frac{1}{8}\pi\alpha_s^2\,,\qquad b_1^C=\frac{3}{4}\pi\alpha_s^2\,,$$
$$c_0^C=\frac{79}{48}\pi\alpha_s^3\,,\qquad c_1^C=\frac{845}{384}\pi\alpha_s^3\,,\qquad d_0^C=3\pi^3\alpha_s\,;$$
$$a_0^D=\frac{1}{1008}\frac{\alpha_s^3}{\pi}\,,\qquad b_0^D=\frac{23}{72}\pi\alpha_s^2\,,\qquad b_1^D=\frac{1}{12}\pi\alpha_s^2\,,$$
$$c_0^D=-\frac{47}{64}\pi\alpha_s^3\,,\qquad c_1^D=-\frac{1}{64}\pi\alpha_s^3\,,\qquad d_0^D=\frac{1}{3}\pi^3\alpha_s\,,$$

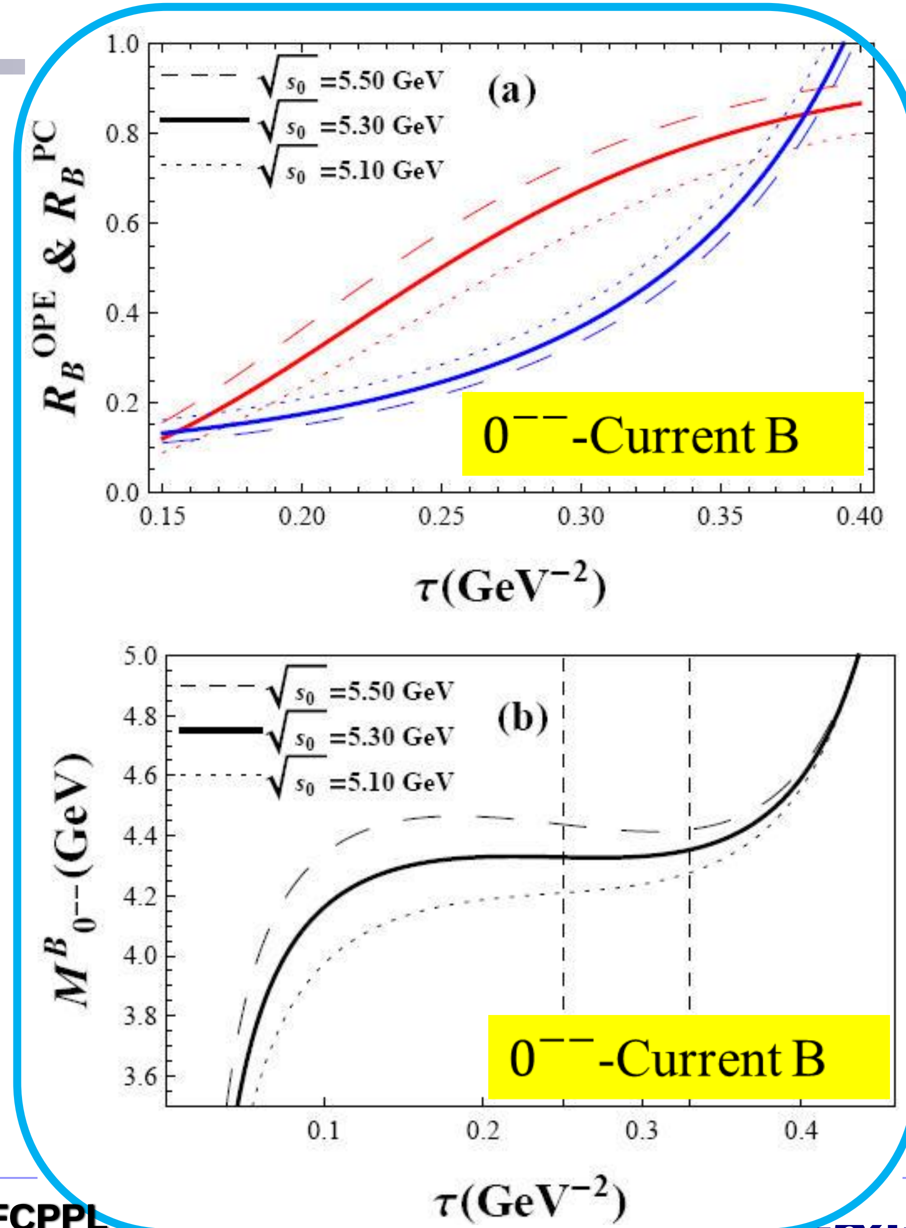
where the ratios of $a_0^k$ to $b_1^k$ are equal for case $A$ to $D$. This implies that the mass curves of case $A$ to $D$ will be very similar, since if we neglect the $\langle g_sG^3\rangle$ term which is much smaller than the $\langle\alpha_sG^2\rangle$ term in mass Eq., the mass of the oddball only depends on the ratio of $a_0^k$ to $b_1^k$.

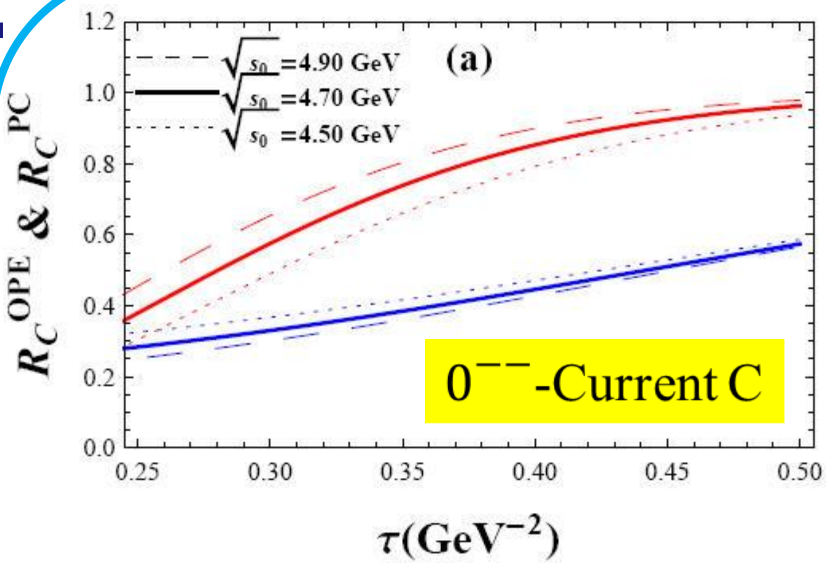
# ➢Wilson coefficients of $2^{+-}$ in the QCD-side

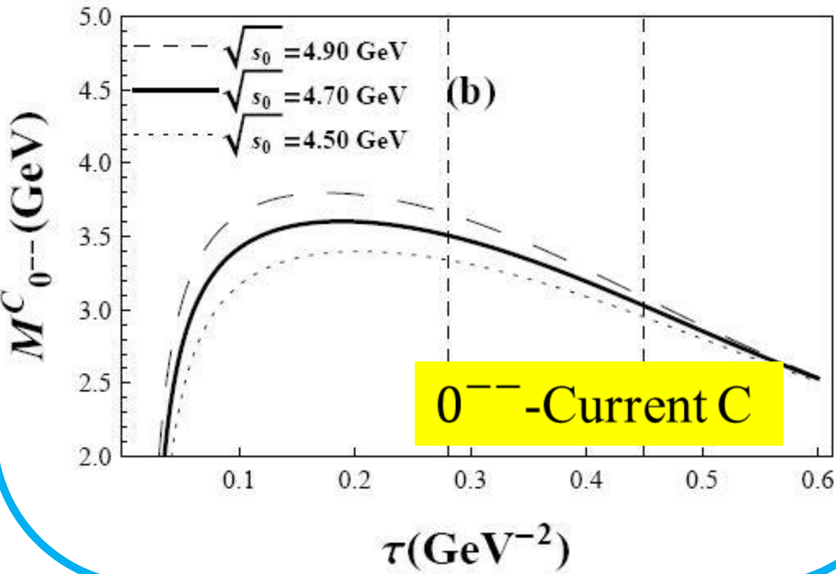
$$
\begin{aligned}
&a_0^A = -\frac{2}{81}\frac{\alpha_s^3}{\pi}\,, && b_0^A = \frac{20}{3}\pi\alpha_s^2\,, && b_1^A = -\frac{20}{9}\pi\alpha_s^2\,,\\
&c_0^A = \frac{205}{54}\pi\alpha_s^3\,, && c_1^A = -\frac{40}{9}\pi\alpha_s^3\,, && d_0^A = \frac{20}{9}\pi^3\alpha_s\,;\\
&a_0^B = -\frac{1}{324}\frac{\alpha_s^3}{\pi}\,, && b_0^B = \frac{5}{81}\pi\alpha_s^2\,, && b_1^B = \frac{10}{27}\pi\alpha_s^2\,,\\
&c_0^B = \frac{415}{162}\pi\alpha_s^3\,, && c_1^B = \frac{20}{27}\pi\alpha_s^3\,, && d_0^B = \frac{10}{9}\pi^3\alpha_s\,;\\
&a_0^C = -\frac{1}{324}\frac{\alpha_s^3}{\pi}\,, && b_0^C = -\frac{115}{81}\pi\alpha_s^2\,, && b_1^C = \frac{10}{27}\pi\alpha_s^2\,,\\
&c_0^C = -\frac{65}{162}\pi\alpha_s^3\,, && c_1^C = \frac{20}{27}\pi\alpha_s^3\,, && d_0^C = \frac{10}{9}\pi^3\alpha_s\,;\\
&a_0^D = -\frac{1}{324}\frac{\alpha_s^3}{\pi}\,, && b_0^D = \frac{5}{81}\pi\alpha_s^2\,, && b_1^D = \frac{10}{27}\pi\alpha_s^2\,,\\
&c_0^D = \frac{415}{162}\pi\alpha_s^3\,, && c_1^D = \frac{20}{27}\pi\alpha_s^3\,, && d_0^D = \frac{10}{9}\pi^3\alpha_s\,,
\end{aligned}
$$

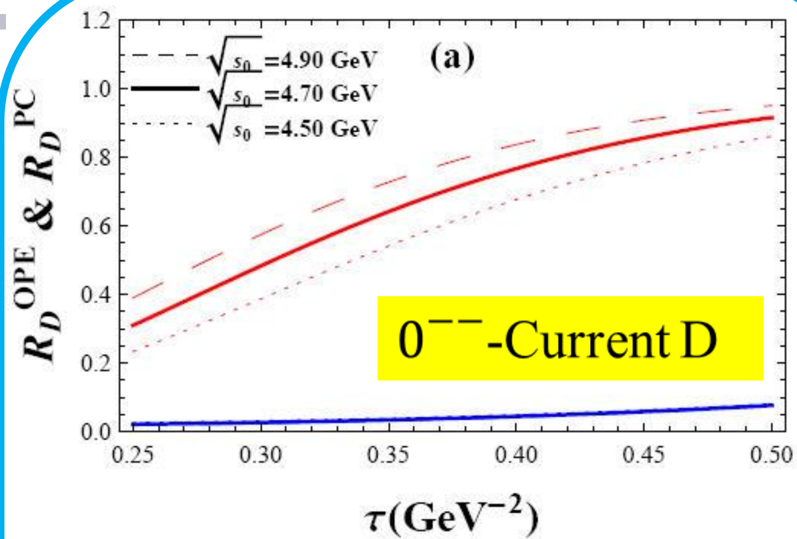
where $a_0^k$, $b_1^k$, and $c_1^k$ are equal for case $B$ to $D$. This implies that the mass curves of case $B$ to $D$ will be exactly equal, because they are determined by the Wilson coefficients $a_0^k$, $b_1^k$, and $c_1^k$.

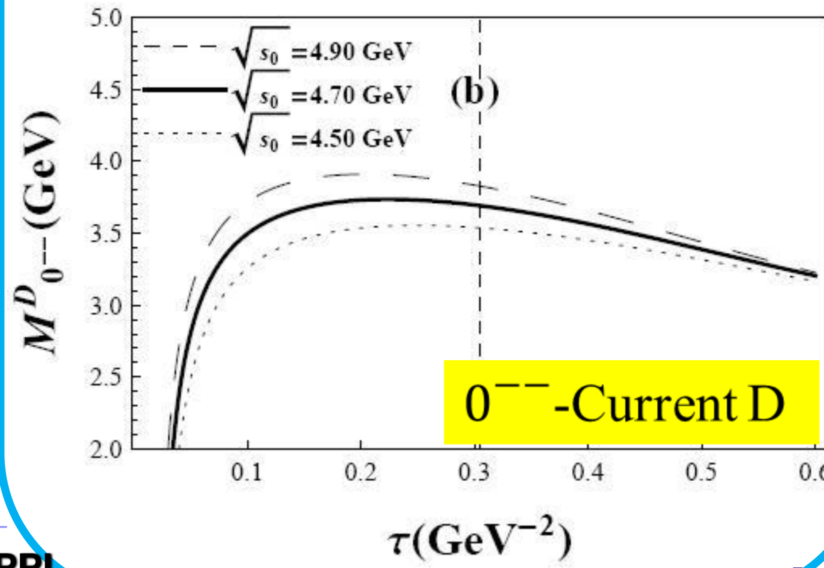

(a)
$\sqrt{s_0}$ =4.90 GeV
$\sqrt{s_0}$ =4.70 GeV
$\sqrt{s_0}$ =4.50 GeV
$R_A^{OPE}$ & $R_A^{PC}$
$0^{--}$-Current A
$\tau(GeV^{-2})$
(b)
$\sqrt{s_0}$ =4.90 GeV
$\sqrt{s_0}$ =4.70 GeV
$\sqrt{s_0}$ =4.50 GeV
$M^A_{0^{--}}$(GeV)
$0^{--}$-Current A
$\tau(GeV^{-2})$

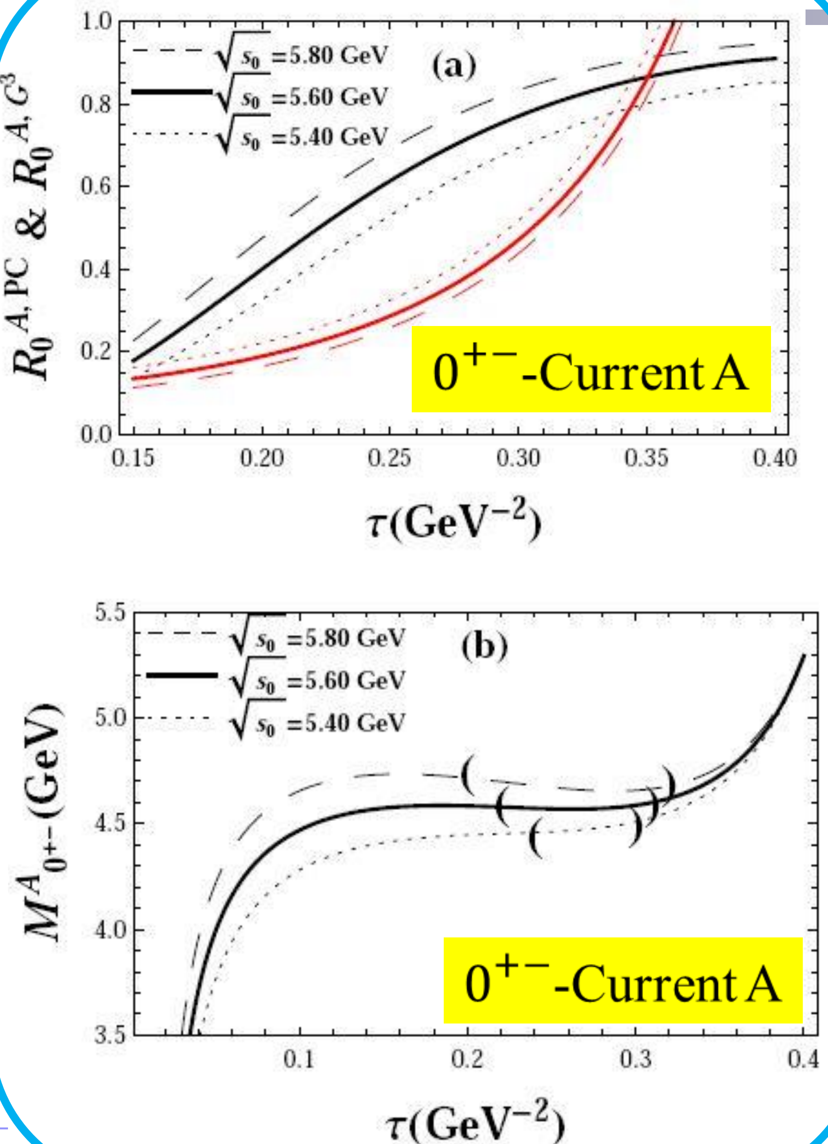

(a)
$\sqrt{s_0}$ =5.50 GeV
$\sqrt{s_0}$ =5.30 GeV
$\sqrt{s_0}$ =5.10 GeV
$R_B^{OPE}$ & $R_B^{PC}$
$0^{--}$-Current B
$\tau(GeV^{-2})$
(b)
$\sqrt{s_0}$ =5.50 GeV
$\sqrt{s_0}$ =5.30 GeV
$\sqrt{s_0}$ =5.10 GeV
$M^B_{0^{--}}$(GeV)
$0^{--}$-Current B
$\tau(GeV^{-2})$

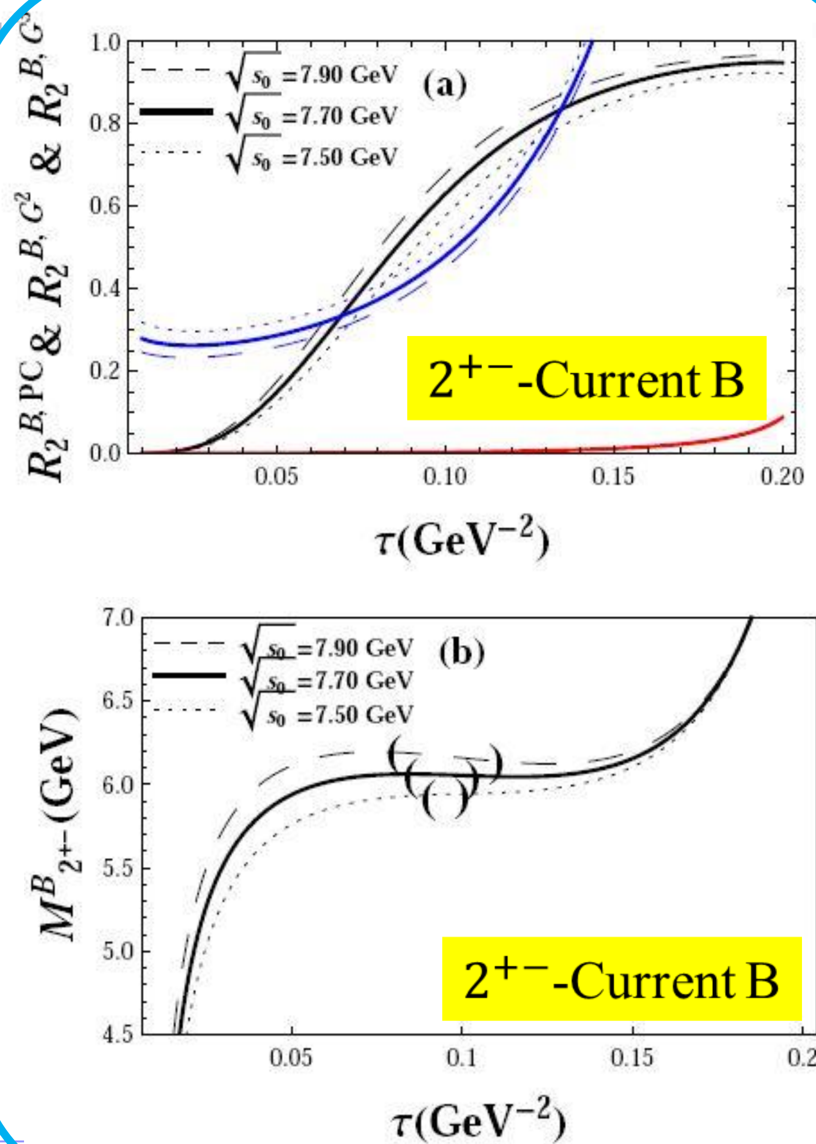

$\sqrt{s_0}$ =4.90 GeV
$\sqrt{s_0}$ =4.70 GeV
$\sqrt{s_0}$ =4.50 GeV
(a)
$0^{--}$-Current C
$R_C^{OPE}$ & $R_C^{PC}$
$\tau(GeV^{-2})$

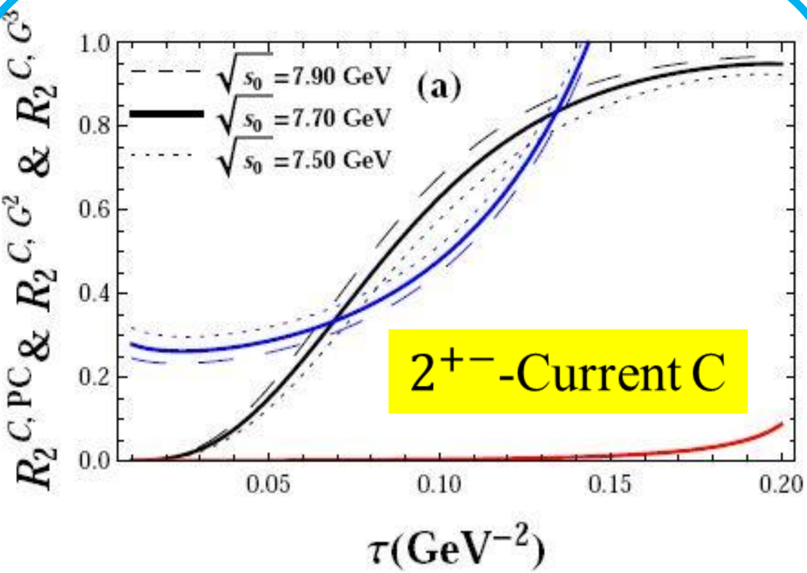

$\sqrt{s_0}$ =4.90 GeV
$\sqrt{s_0}$ =4.70 GeV
$\sqrt{s_0}$ =4.50 GeV
(b)
$0^{--}$-Current C
$M^C_{0^{--}}$(GeV)
$\tau(GeV^{-2})$

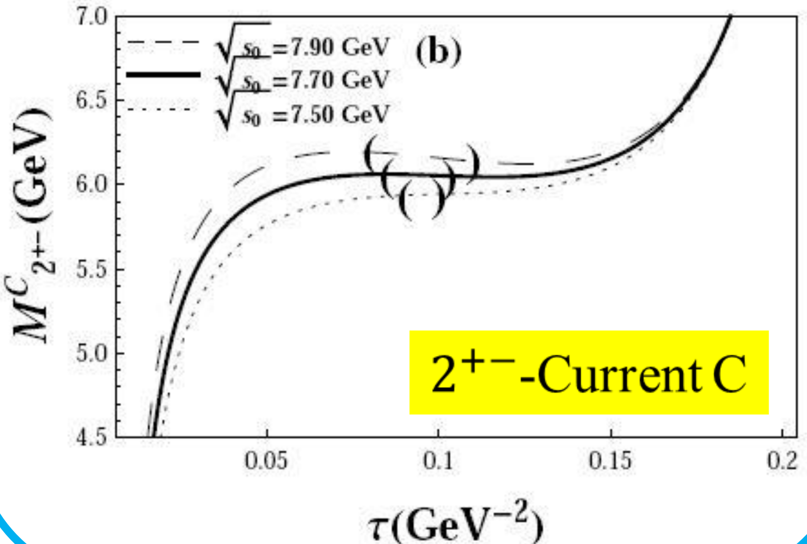

$\sqrt{s_0}$ =4.90 GeV
$\sqrt{s_0}$ =4.70 GeV
$\sqrt{s_0}$ =4.50 GeV
(a)
$0^{--}$-Current D
$R_D^{OPE}$ & $R_D^{PC}$
$\tau(GeV^{-2})$

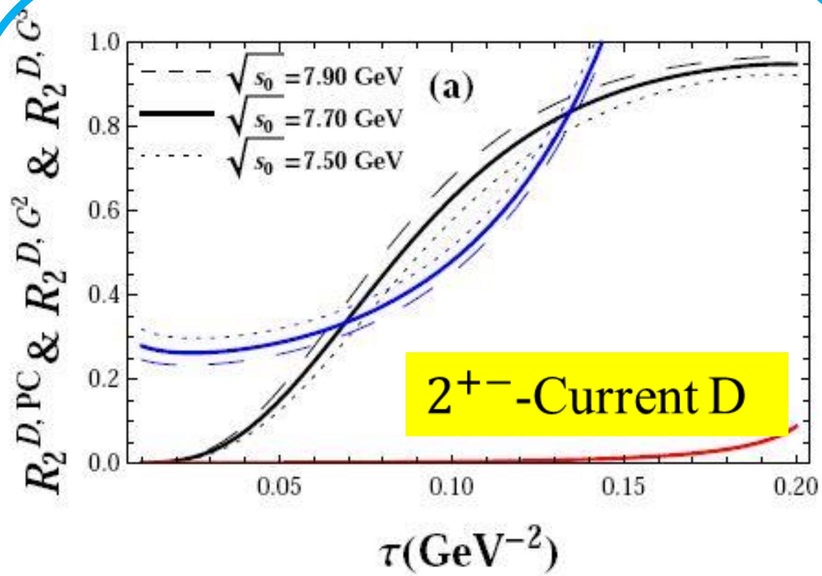

$\sqrt{s_0}$ =4.90 GeV
$\sqrt{s_0}$ =4.70 GeV
$\sqrt{s_0}$ =4.50 GeV
(b)
$0^{--}$-Current D
$M^D_{0^{--}}$(GeV)
$\tau(GeV^{-2})$

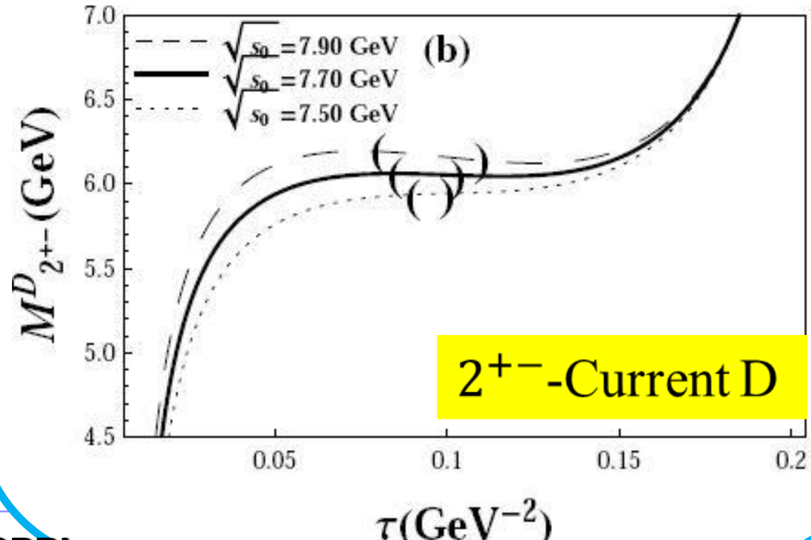

(a)
$\sqrt{s_0}$ =5.80 GeV
$\sqrt{s_0}$ =5.60 GeV
$\sqrt{s_0}$ =5.40 GeV
$R_0^{A,PC}$ & $R_0^{A,G^3}$
$\tau(\mathrm{GeV}^{-2})$
$0^{+-}$-Current A
(b)
$\sqrt{s_0}$ =5.80 GeV
$\sqrt{s_0}$ =5.60 GeV
$\sqrt{s_0}$ =5.40 GeV
$M^A_{0^{+-}}$(GeV)
$\tau(\mathrm{GeV}^{-2})$
$0^{+-}$-Current A


(a)
$\sqrt{s_0}$ =7.90 GeV
$\sqrt{s_0}$ =7.70 GeV
$\sqrt{s_0}$ =7.50 GeV
$R_2^{B,PC}$ & $R_2^{B,G^2}$ & $R_2^{B,G^3}$
$\tau(\mathrm{GeV}^{-2})$
$2^{+-}$-Current B
(b)
$\sqrt{s_0}$ =7.90 GeV
$\sqrt{s_0}$ =7.70 GeV
$\sqrt{s_0}$ =7.50 GeV
$M^B_{2^{+-}}$(GeV)
$\tau(\mathrm{GeV}^{-2})$
$2^{+-}$-Current B


$R_2^{C,PC}$ & $R_2^{C,G^2}$ & $R_2^{C,G^3}$
$\sqrt{s_0}$ =7.90 GeV
$\sqrt{s_0}$ =7.70 GeV
$\sqrt{s_0}$ =7.50 GeV
(a)
$2^{+-}$-Current C
$\tau(\mathrm{GeV}^{-2})$


$M^C_{2+-}$(GeV)
$\sqrt{s_0}$ =7.90 GeV
$\sqrt{s_0}$ =7.70 GeV
$\sqrt{s_0}$ =7.50 GeV
(b)
$2^{+-}$-Current C
$\tau(\mathrm{GeV}^{-2})$


$R_2^{D,PC}$ & $R_2^{D,G^2}$ & $R_2^{D,G^3}$
$\sqrt{s_0}$ =7.90 GeV
$\sqrt{s_0}$ =7.70 GeV
$\sqrt{s_0}$ =7.50 GeV
(a)
$2^{+-}$-Current D
$\tau(\mathrm{GeV}^{-2})$


$M^D_{2+-}$(GeV)
$\sqrt{s_0}$ =7.90 GeV
$\sqrt{s_0}$ =7.70 GeV
$\sqrt{s_0}$ =7.50 GeV
(b)
$2^{+-}$-Current D
$\tau(\mathrm{GeV}^{-2})$

- Comparison with other methods, in unit of GeV.

| $J^{PC}$ | Flux tube model | Lattice QCD | Holography QCD | Our results (QCDSR) |
|---|---|---|---|---|
| $0^{--}$ | 2.79 [1] | 5.166 [2] | 2.8 [6], 3.817 [7] | 3.81, 4.33 |
| $0^{+-}$ | 2.79 [1] | 5.45 [2], 4.74 [3], 4.78 [4] | X | 4.57 |
| $1^{-+}$ | X | 1.68 [5] | X | X |
| $2^{+-}$ | X | 4.14 [3], 4.23 [4] | 2.786 [7] | 6.06 |

[1] N.Isgur and J.E.Paton, PRD 31, 2910 (1985).
[2] E.Gregory, et al., JHEP1210, 170 (2012).
[3] C.J.Morningstar and M.J.Peardon,PRD60, 034509 (1999).
[4] Y.Chen et al.,PRD73, 014516 (2006).
[5] K.Ishikawa, et al., PLB120, 387 (1983).
[6] L. Bellantuono, et al., JHEP 1510 (2015) 137.
[7] Y.-D Chen and Mei Huang, arXiv:1511.07018.

# III. Hunting for Oddballs

- Glueballs and Glueball Studies
- $0^{--}$ Oddballs via QCDSR
- **Hunting for Oddballs**
- Concluding remarks

# III. Hunting for Oddballs

- Proposed production channels of $0^{--}$ oddball G(3810)

$$X(3872) \to \gamma + G_{0^{--}}(3810), \qquad \Upsilon(1S) \to f_1(1285) + G_{0^{--}}(3810),$$

$$\Upsilon(1S) \to \chi_{c_1} + G_{0^{--}}(3810), \qquad \chi_{b_1} \to J/\psi + G_{0^{--}}(3810),$$

$$\chi_{b_1} \to \omega + G_{0^{--}}(3810).$$

- Proposed decay channels of $0^{--}$oddball

$$G_{0^{--}}(3810) \to \gamma + f_1(1285),$$

$$G_{0^{--}}(3810) \to \omega + f_1(1285).$$

$$G_{0^{--}}(3810) \to \gamma + \chi_{c_1},$$

# III. Hunting for Oddballs

➢ Proposed production channels of $0^{+-}$ and $2^{+-}$ oddballs

| $J^{PC}$ | S-wave | P-wave |
|---|---|---|
| $0^{+-}$ | $h_b \to \{f_1(1285),\ \chi_{c1},\ X(3872)\} + G_{0^{+-}}(4570)$ | $\Upsilon(1S) \to \{f_1(1285),\ \chi_{c1},\ X(3872)\} + G_{0^{+-}}(4570)$<br>$\chi_{bJ} \to \{\gamma,\ \omega,\ \phi,\ J/\psi,\ \psi(2S)\} + G_{0^{+-}}(4570)$<br>$h_b \to \{\eta,\ \eta',\ \eta_c\} + G_{0^{+-}}(4570)$ |
| $2^{+-}$ | $\Upsilon(1S) \to \eta_2(1645) + G_{2^{+-}}(6060)$<br>$\chi_{b1,2} \to \{h_1(1170),\ h_c\} + G_{2^{+-}}(6060)$<br>$h_b \to \{f_1(1285),\ f_2(1270),\ \chi_{c1,2},\} + G_{2^{+-}}(6060)$ | $\Upsilon(1S) \to f_1(1285) + G_{2^{+-}}(6060)$ |

➢ Proposed decay channels of $0^{+-}$ and $2^{+-}$ oddballs

| $J^{PC}$ | S-wave | P-wave |
|---|---|---|
| $0^{+-}$ | $G_{0^{+-}}(4570) \to h_1(1170) + f_1(1285)$ | $G_{0^{+-}}(4570) \to \{\gamma,\ \omega,\ \phi,\ J/\psi\} + f_0(980)$<br>$G_{0^{+-}}(4570) \to h_1(1170) + \{\eta, \eta',\ \eta_c\}$<br>$G_{0^{+-}} \to h_c + \{\eta, \eta'\}$ |
| $2^{+-}$ | $G_{2^{+-}}(6060) \to \{h_1(1170),\ h_c\} + f_1(1285)$ | $G_{2^{+-}}(6060) \to \{\gamma,\ \omega,\ \phi, J/\psi,\ \psi(2S)\} + f_1(1285)$ |

# Note: for $0^{--}$ oddballs

- **Compare the lighter one with Flux tube model:**

$$G_{0^{--}}(3810) > 2.79\,\mathrm{GeV}$$

**Isgur & Parton, PRD31 (1985) 2910.**

- **Compare the heavier one with Lattice QCD:**

$$G_{0^{--}}(4330) < (5166 \pm 1000)\,\mathrm{MeV}$$

**Gregory, *et al.*, JHEP1210( 2012) 170.**

# III. Hunting for Oddballs

- **BESIII Collaboration, by …**
- **Belle Collaboration, by C.P. Shen, …**
- **Fermilab, Mike Albrow**
- **LHCb Collaboration, Paolo Gandini.**
- **phys.org, "*Long-searched-for glueball could soon be detected*", by Lisa Zyga.**

# IV. Concluding remarks

- Glueballs and Glueball Studies
- $0^{--}$ Oddballs via QCDSR
- Hunting for Oddballs
- **Concluding remarks**

# IV. Concluding remarks

- We obtain two stable $0^{--}$ oddballs with masses about 3.81 GeV and 4.33 GeV

- We find there might be also $0^{+-}$ and $2^{+-}$ oddballs with masses of 4.57 GeV and 6.06 GeV respectively

- Oddballs can in principle mix with hybrids and tetraquark states, though naively the OZI suppression may hinder the mixing in certain degree.

# IV. Concluding remarks

- We briefly analyzed the oddballs optimal production and decay mechanism. They are expected to be measured in BESIII, BELLEII, Super-B, PANDA, and LHCb experiments.

- Oddballs are looming somewhere ahead

- glueball production and decay properties are important..

- It is time for we people to take it more seriously to pin down the glueballs

# IV. Concluding remarks

- Glueball may be scanned by diff. scattering
- …

# Thank you for your attention!