Jet-medium interaction in heavy-ion collisions

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and Related Physics

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Outline

- **Introduction**

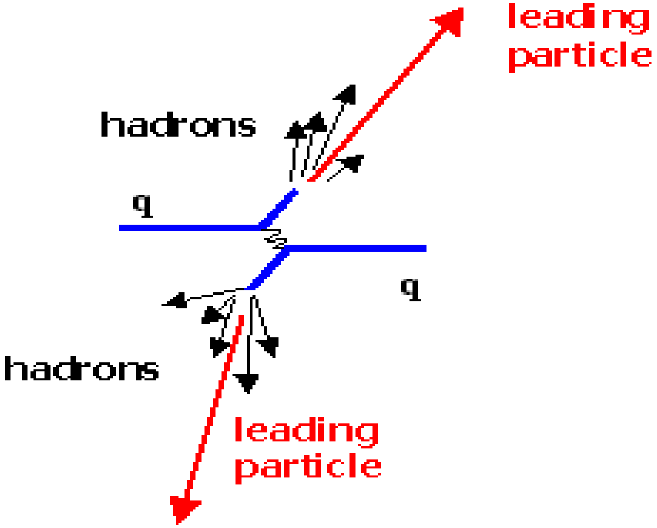
- **Medium-induced gluon emission**
  - Beyond collinear expansion and soft gluon emission approximation for massive/massless quarks with transverse and longitudinal scatterings

- **Full jets in a coupled jet-fluid model**
  - Jet evolution in quark-gluon plasma with medium response
  - Nuclear modification of full jet yield and jet shape

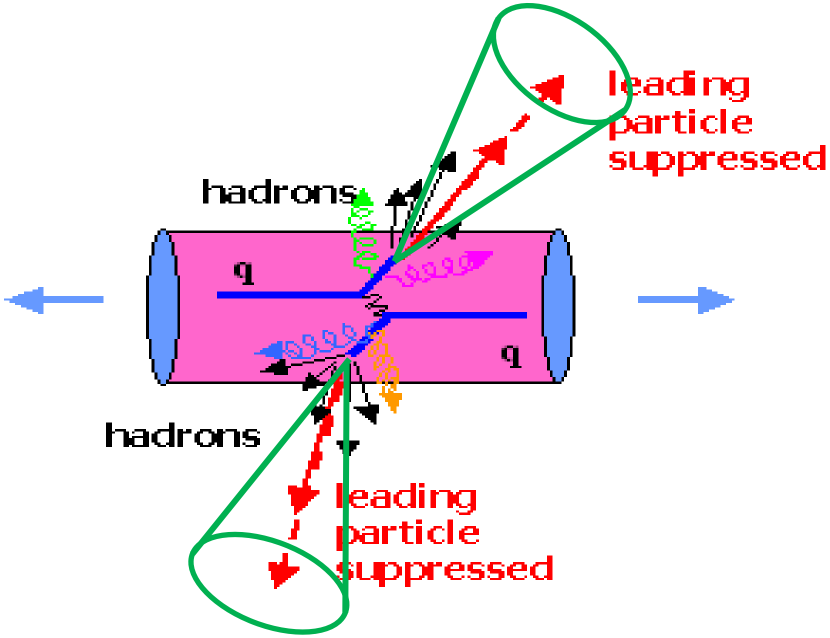
- **Jet quenching at forward rapidity**

- **Summary**

Jets are hard probes of QGP



N+N

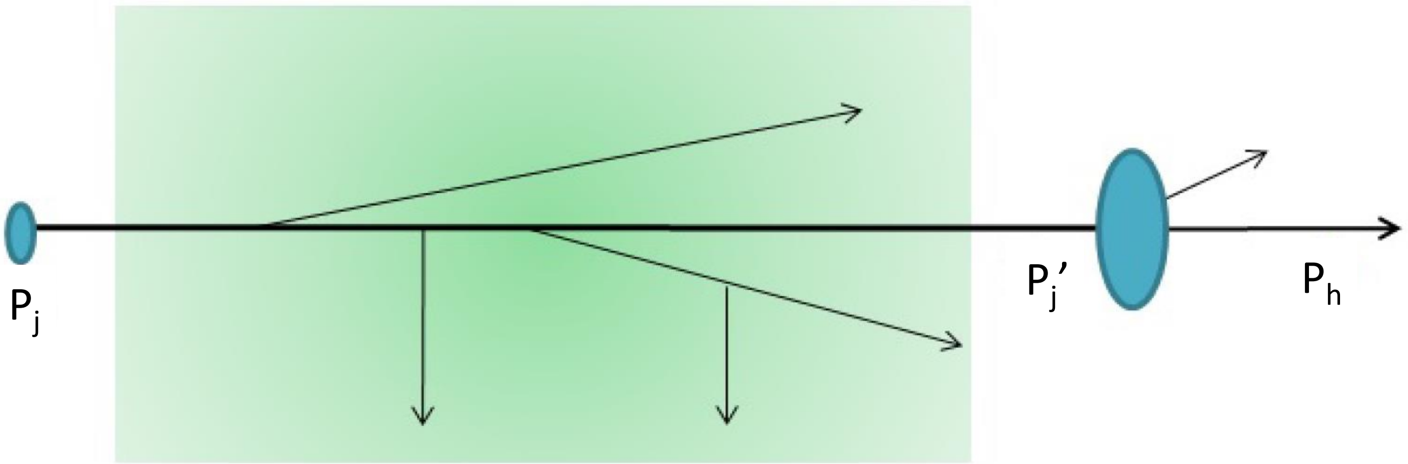


A+A: jet quenching

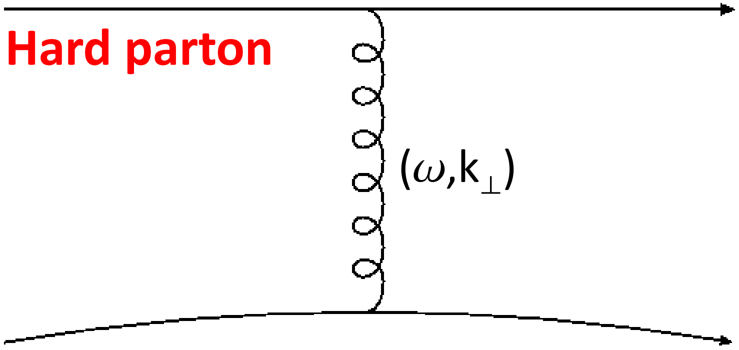
Jets (and **jet-medium interaction, jet quenching**) provide valuable tools to probe hot & dense QGP in relativistic heavy-ion collisions (at RHIC & LHC):

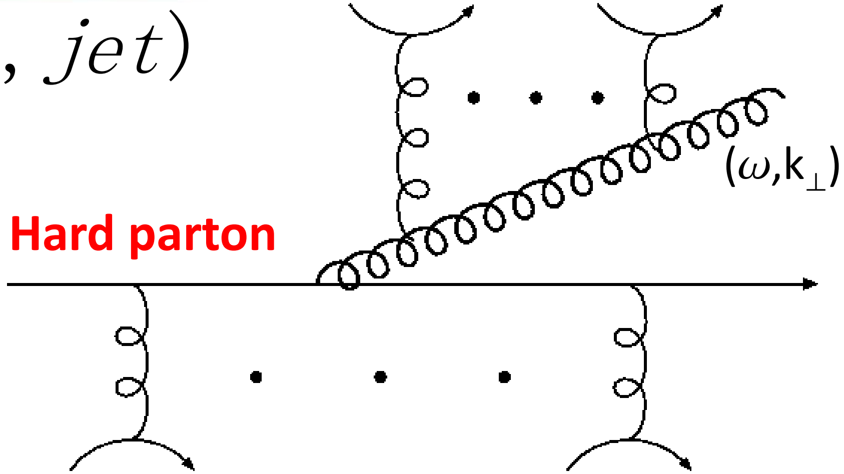
(1) parton energy loss (2) deflection and broadening (3) modification of jet substructure (4) jet-induced medium excitation

Elastic and inelastic interactions



$$P_{j\to j'}(medium,\ jet)$$





Elastic (collisional)

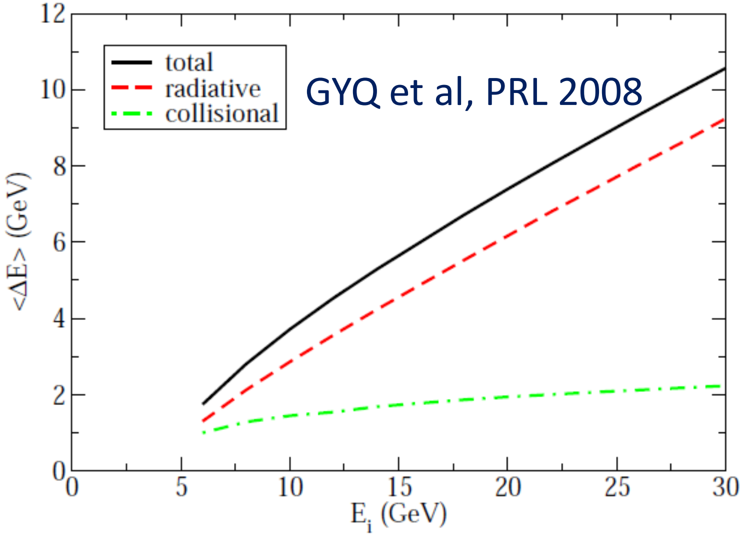
$$\frac{d\Gamma_{coll}}{d\omega dk_\perp^2 dt}(T,\ E,\ \ldots)\ =\ ?$$

Inelastic (radiative)

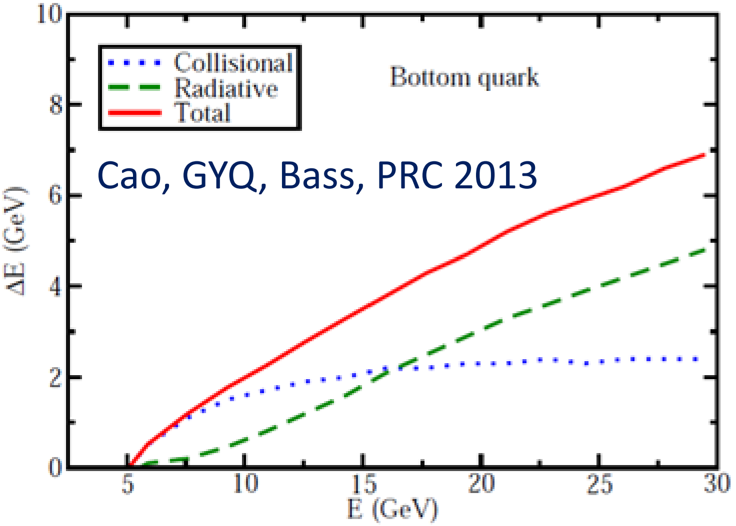
$$\frac{d\Gamma_{rad}}{d\omega dk_\perp^2 dt}(T,\ E,\ \ldots)\ =\ ?$$

Elastic (collisional) energy loss

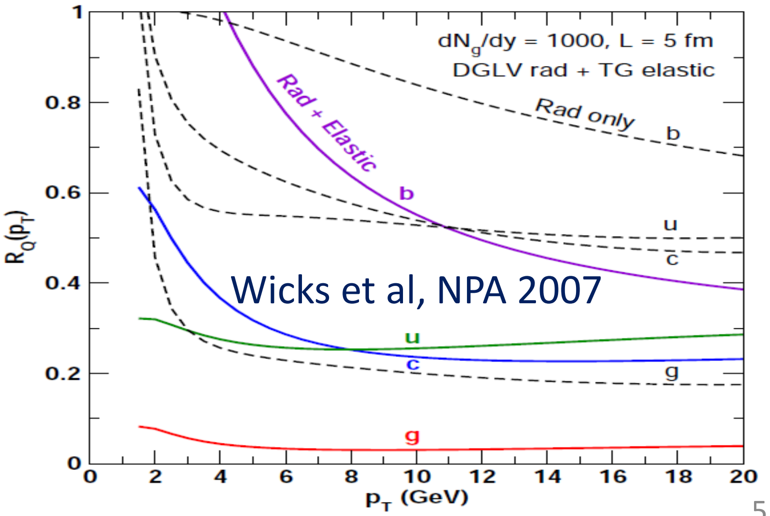
- **First studied by Bjorken:**
  - Bjorken 1982; Bratten, Thoma 1991; Thoma, Gyulassy, 1991; Mustafa, Thoma 2005; Peigne, Peshier, 2006; Djordjevic (GLV), 2006; Wicks et al (DGLV), 2007; GYQ et al (AMY), 2008...
- **Main findings:**
  - **dE/E small compared to rad. for large E**
  - **But non-negligible in R_{AA} calculation (especially for heavy flavors)**
  - **Important when studying full jet energy loss and medium response**



GYQ et al, PRL 2008



Cao, GYQ, Bass, PRC 2013

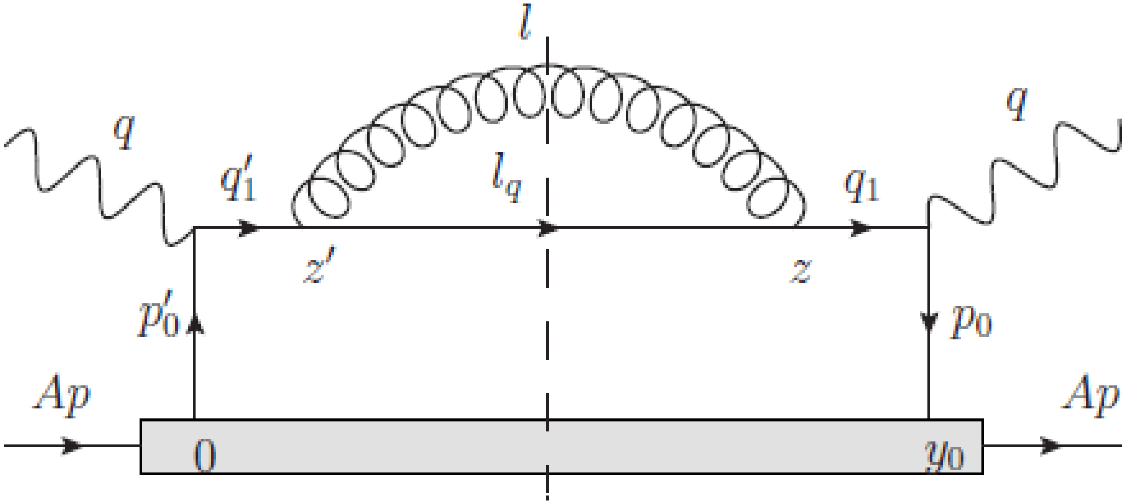


Wicks et al, NPA 2007

Medium-induced inelastic (radiative) process

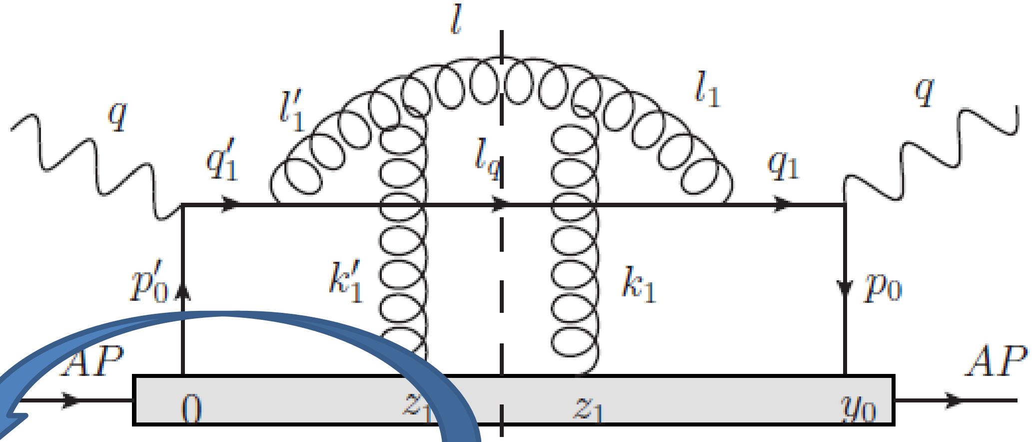
- **pQCD-based formalisms**
  - **BDMPS-Z**: Baier-Dokshitzer-Mueller-Peigne-Schiff-Zakharov
  - **ASW**: Amesto-Salgado-Wiedemann
  - **AMY**: Arnold-Moore-Yaffe (& Caron-Huot, Gale)
  - **GLV**: Gyulassy-Levai-Vitev (& Djordjevic, Heinz)
  - **HT**: Wang-Guo (& Zhang, Wang, Majumder)
- **Various approximations:**
  - High energy & eikonal approximations
  - Soft gluon emission approximation (ASW, GLV)
  - Collinear expansion (BDMPS-Z, HT)
  - Gluon emission induced by transverse scatterings
- **Recent improvements:**
  - Include non-eikonal corrections within the path integral formalism (Apolinrio, Armesto, Milhano, Salgado, JHEP (2015), arXiv:1407.0599)
  - Reinvestigate GLV/DGLV formalism by relaxing the soft gluon emission approximation (Blagojevic, Djordjevic, Djordjevic, arXiv:1804.07593; Sievert, Vitev, arXiv:1807.03799)
  - Generalize HT formalism by going **beyond the collinear expansion and soft gluon emission approximation**, including **both transverse and longitudinal scatterings**, for **massless and massive quarks** (Zhang, Hou, GYQ, PRC 2018, arXiv:1804.00470 & arXiv:1812.11048; Zhang, GYQ, Wang, in prep.)

Gluon emission in vacuum



$$\frac{dN_g^{\mathrm{vac}}}{dy\, d^2 \mathbf{l}_\perp} = C_F \frac{\alpha_s}{2\pi^2} P(y) \frac{l_\perp^2 + \frac{y^4}{1+(1-y)^2} M^2}{(l_\perp^2 + y^2 M^2)^2}.$$

Medium-induced inelastic (radiative) process



Zhang, Hou, GYQ, PRC 2018
& arXiv:1812.11048;
Zhang, GYQ, Wang, in prep.

+ other 20 diagrams

$$\frac{dN_g^{med}}{dy\,d^2\mathbf{l}_\perp}=\frac{\alpha_s}{2\pi^2}P(y)\int dZ_1^-\int\frac{dk^-d^2\mathbf{k}_{1\perp}}{(2\pi)^3}\mathcal{D}(k_1^-,\mathbf{k}_{1\perp})$$

$$\times\Bigg\{\left[2-2\cos\left(\frac{y(1-y)}{(y-\lambda_1^-)\,(1+\lambda_1^- -y)}\frac{(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+(y-\lambda_1^-)^2M^2}{l_\perp^2+y^2M^2}\frac{Z_1^-}{\tilde{\tau}^-_{\text{form}}}\right)\right]$$

$$C_A\left[\frac{1+(1+\lambda_1^- -y)^2}{1+(1-y)^2}\left(\frac{y-\frac{\lambda_1^-}{2}}{y-\lambda_1^-}\right)^2\frac{(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+\frac{(y-\lambda_1^-)^4M^2}{1+(1+\lambda_1^- -y)^2}}{\left[(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+(y-\lambda_1^-)^2M^2\right]^2}\right.$$

$$-\frac{1+(1+\lambda_1^- -y)(1-y)}{2[1+(1-y)^2]}\left(\frac{y-\frac{\lambda_1^-}{2}}{y-\lambda_1^-}\right)\frac{\mathbf{l}_\perp\cdot(\mathbf{l}_\perp-\mathbf{k}_{1\perp})+\frac{y^2(y-\lambda_1^-)^2}{1+(1+\lambda_1^- -y)(1-y)}M^2}{[l_\perp^2+y^2M^2]\left[(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+(y-\lambda_1^-)^2M^2\right]}$$

$$\left.-\frac{1+(1+\lambda_1^- -y)(1-\frac{y}{1+\lambda_1^-})}{2[1+(1-y)^2]}\left(\frac{y-\frac{\lambda_1^-}{2}}{y-\lambda_1^-}\right)\frac{(\mathbf{l}_\perp-\mathbf{k}_{1\perp})\cdot\left(\mathbf{l}_\perp-\frac{y}{1+\lambda_1^-}\mathbf{k}_{1\perp}\right)+\frac{\left(\frac{y}{1+\lambda_1^-}\right)^2(y-\lambda_1^-)^2}{1+(1+\lambda_1^- -y)(1-\frac{y}{1+\lambda_1^-})}M^2}{\left[\left(\mathbf{l}_\perp-\frac{y}{1+\lambda_1^-}\mathbf{k}_{1\perp}\right)^2+(\frac{y}{1+\lambda_1^-})^2M^2\right]\left[(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+(y-\lambda_1^-)^2M^2\right]}\right]+\ldots\ldots\Bigg\}$$

$$\mathcal{D}(k_1^-,\mathbf{k}_{1\perp})=(2\pi)^3\frac{dP_{\text{el}}}{dk_1^-d^2\mathbf{k}_{1\perp}dZ_1^-}$$

Medium-induced gluon emission beyond collinear expansion & soft emission limit with transverse & longitudinal scatterings for massive quarks

Only transverse scatterings

- Model the traversed nuclear medium by heavy static scattering centers (only transverse scatterings)

$$
\begin{aligned}
\frac{dN_g^{\rm med}}{dy\,d^2\mathbf{l}_\perp} &= \frac{\alpha_s}{2\pi^2}P(y)\int dZ_1^- \int \frac{d^2\mathbf{k}_{1\perp}}{(2\pi)^2}\mathcal{D}_\perp(\mathbf{k}_{1\perp}) \\
&\times \Bigg\{ C_A\left[2-2\cos\left(\frac{(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+y^2M^2}{l_\perp^2+y^2M^2}\,\frac{Z_1^-}{\tilde{\tau}^-_{\rm form}}\right)\right] \times \Bigg[\frac{(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+\frac{y^4}{1+(1-y)^2}M^2}{\left[(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+y^2M^2\right]^2} \\
&\quad -\frac{1}{2}\,\frac{\mathbf{l}_\perp\cdot(\mathbf{l}_\perp-\mathbf{k}_{1\perp})+\frac{y^4}{1+(1-y)^2}M^2}{[l_\perp^2+y^2M^2]\left[(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+y^2M^2\right]} - \frac{1}{2}\,\frac{(\mathbf{l}_\perp-\mathbf{k}_{1\perp})\cdot(\mathbf{l}_\perp-y\mathbf{k}_{1\perp})+\frac{y^4}{1+(1-y)^2}M^2}{\left[(\mathbf{l}_\perp-y\mathbf{k}_{1\perp})^2+y^2M^2\right]\left[(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+y^2M^2\right]}\Bigg] \\
&\quad +\left(\frac{C_A}{2}-C_F\right)\left[2-2\cos\left(\frac{Z_1^-}{\tilde{\tau}^-_{\rm form}}\right)\right]\left[\frac{\mathbf{l}_\perp\cdot(\mathbf{l}_\perp-y\mathbf{k}_{1\perp})+\frac{y^4}{1+(1-y)^2}M^2}{[l_\perp^2+y^2M^2]\left[(\mathbf{l}_\perp-y\mathbf{k}_{1\perp})^2+y^2M^2\right]} - \frac{l_\perp^2+\frac{y^4}{1+(1-y)^2}M^2}{[l_\perp^2+y^2M^2]^2}\right] \\
&\quad +C_F\left[\frac{(\mathbf{l}_\perp-y\mathbf{k}_{1\perp})^2+\frac{y^4}{1+(1-y)^2}M^2}{\left[(\mathbf{l}_\perp-y\mathbf{k}_{1\perp})^2+y^2M^2\right]^2} - \frac{l_\perp^2+\frac{y^4}{1+(1-y)^2}M^2}{[l_\perp^2+y^2M^2]^2}\right]\Bigg\}.
\end{aligned}
$$

Soft gluon emission approximation

- Further taking soft gluon emission approximation $y^2 M \ll yM \sim l_\perp \sim k_{1\perp}$:

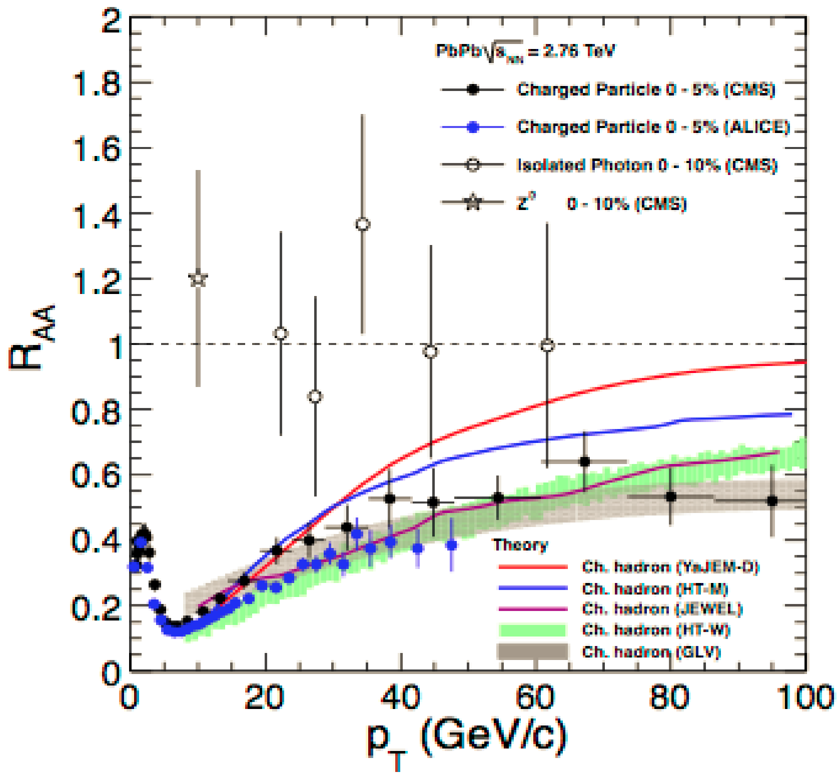
$$\frac{dN_g^{\mathrm{med}}}{dy\,d^2\mathbf{l}_\perp} = \frac{\alpha_s}{2\pi^2} P(y) \int dZ_1^- \int d^2\mathbf{k}_{1\perp} \frac{dP_{\mathrm{el}}}{d^2\mathbf{k}_{1\perp} dZ_1^-} \times C_A \left[2 - 2\cos\left(\frac{(\mathbf{l}_\perp - \mathbf{k}_{1\perp})^2 + y^2 M^2}{l_\perp^2 + y^2 M^2} \frac{Z_1^-}{\tilde{\tau}_{\mathrm{form}}^-} \right) \right] \\ \times \left[\frac{(\mathbf{l}_\perp - \mathbf{k}_{1\perp})^2}{\left[(\mathbf{l}_\perp - \mathbf{k}_{1\perp})^2 + y^2 M^2 \right]^2} - \frac{\mathbf{l}_\perp \cdot (\mathbf{l}_\perp - \mathbf{k}_{1\perp})}{[l_\perp^2 + y^2 M^2] \left[(\mathbf{l}_\perp - \mathbf{k}_{1\perp})^2 + y^2 M^2 \right]} \right].$$

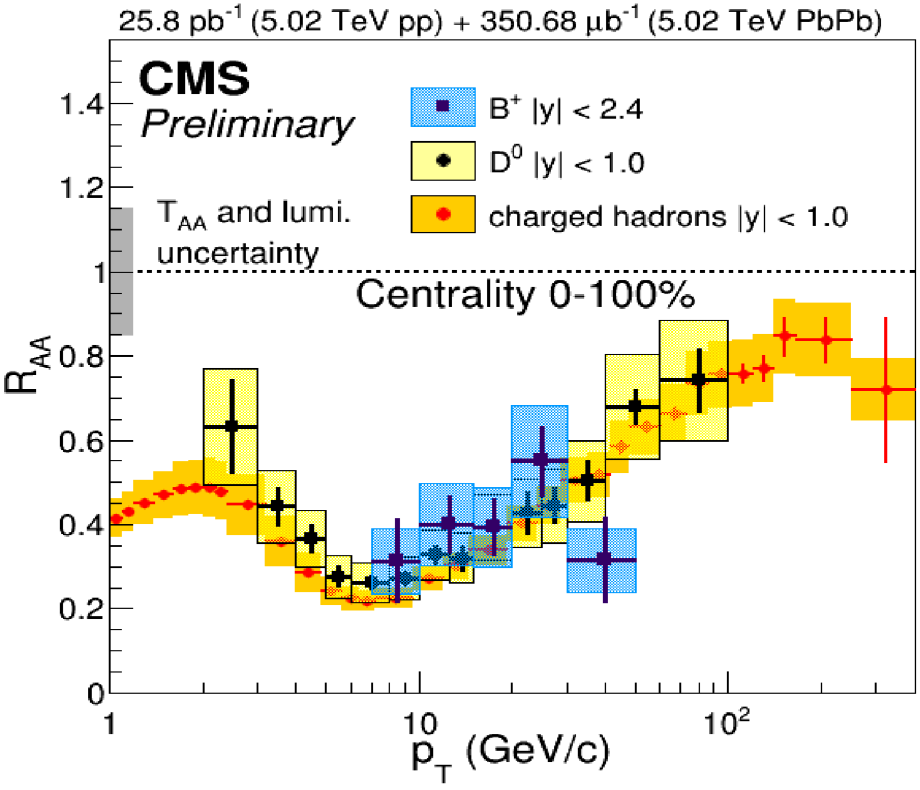
- This agrees with the GLV/DGLV first-order-in-opacity formula.

- Jet transport parameter is related to the differential elastic scattering rate as follows:

$$\hat{q}_{lc} = \frac{d\langle k_{1\perp}^2 \rangle}{dL^-} = \int \frac{dk_1^- d^2\mathbf{k}_{1\perp}}{(2\pi)^3} \mathbf{k}_{1\perp}^2 \mathcal{D}(k_1^-, \mathbf{k}_{1\perp}) = \int \frac{d^2\mathbf{k}_{1\perp}}{(2\pi)^2} \mathbf{k}_{1\perp}^2 \mathcal{D}_\perp(\mathbf{k}_{1\perp}) = \int d^2\mathbf{k}_{1\perp} \mathbf{k}_{1\perp}^2 \rho^- \frac{d\sigma_{\mathrm{el}}}{d^2\mathbf{k}_{1\perp}}.$$

Nuclear modifications of large p_T hadrons

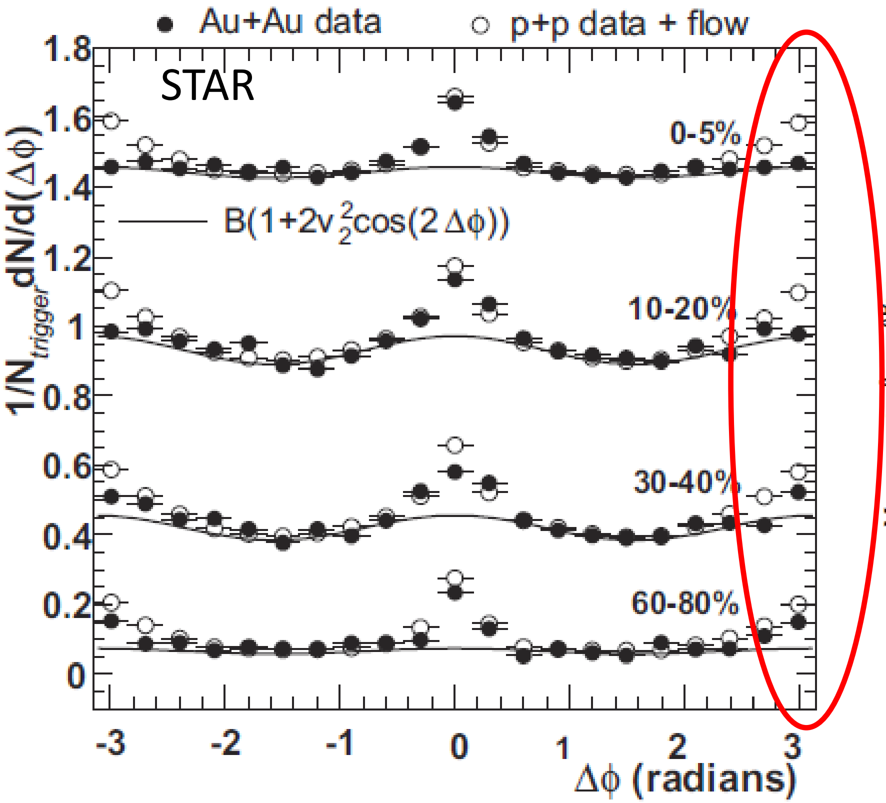


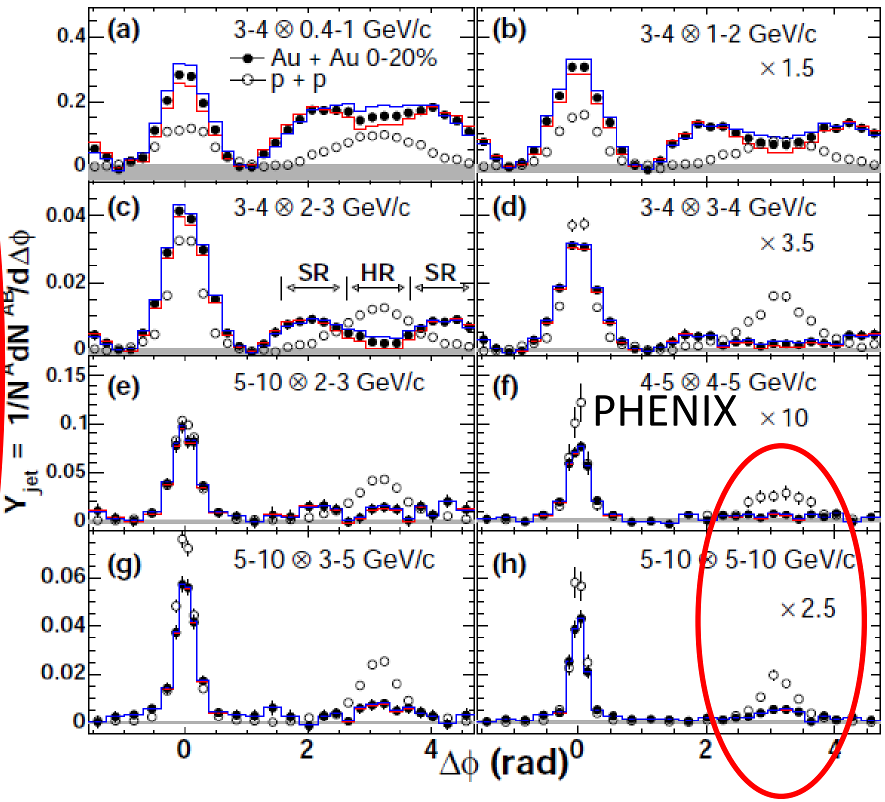


$$R_{AA} = \frac{1}{N_{\mathrm{coll}}} \frac{dN^{AA} / d^2 p_T dy}{dN^{pp} / d^2 p_T dy}$$

Color & flavor dependences of parton energy loss: $\Delta E_g > \Delta E_{uds} > \Delta E_c > \Delta E_b$?

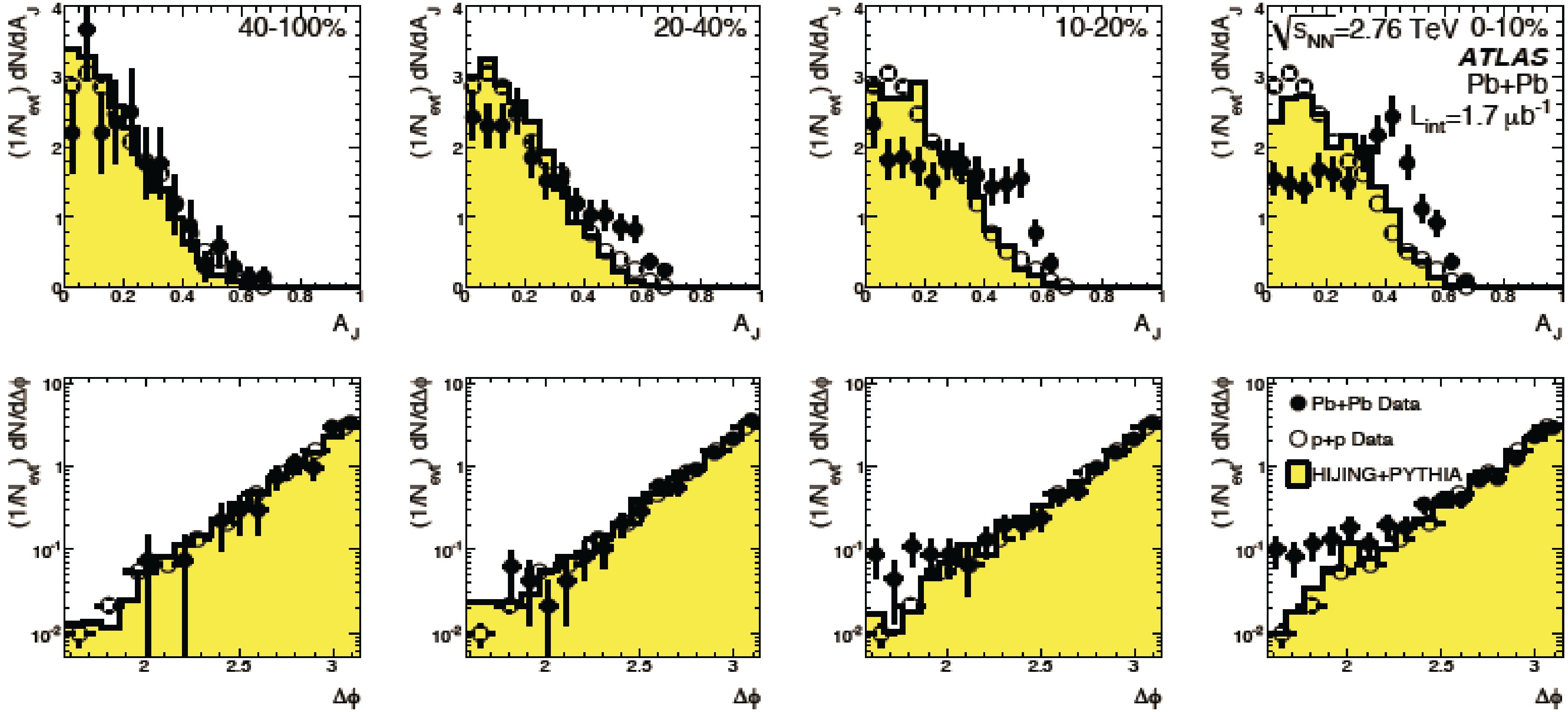
Jet-related correlations





Both per-trigger yield and the shape of the angular distribution are modified by QGP. Can probe parton energy loss and angular deflection (broadening) effects.

Dijet (γ-jet) correlations



$$A_J = \frac{p_{T,1} - p_{T,2}}{p_{T,1} + p_{T,2}}$$

$$\Delta\phi = \left|\phi_1 - \phi_2\right|$$

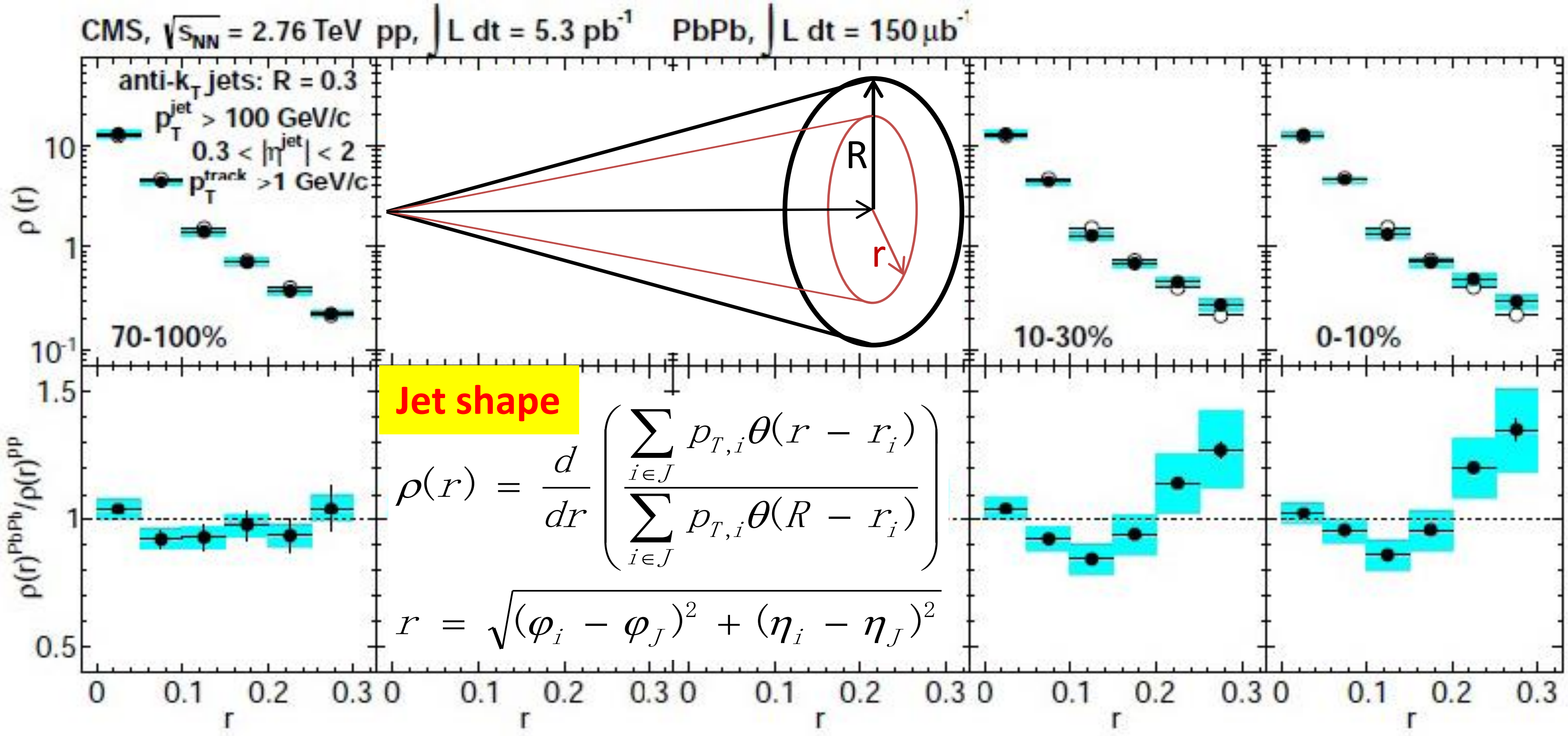
Strong modification of momentum imbalance distribution

=> Significant energy loss experienced by the subleading jets

Small change in angular distribution

=> medium-induced broadening effect is quite modest (here)

Jet structure/substructure



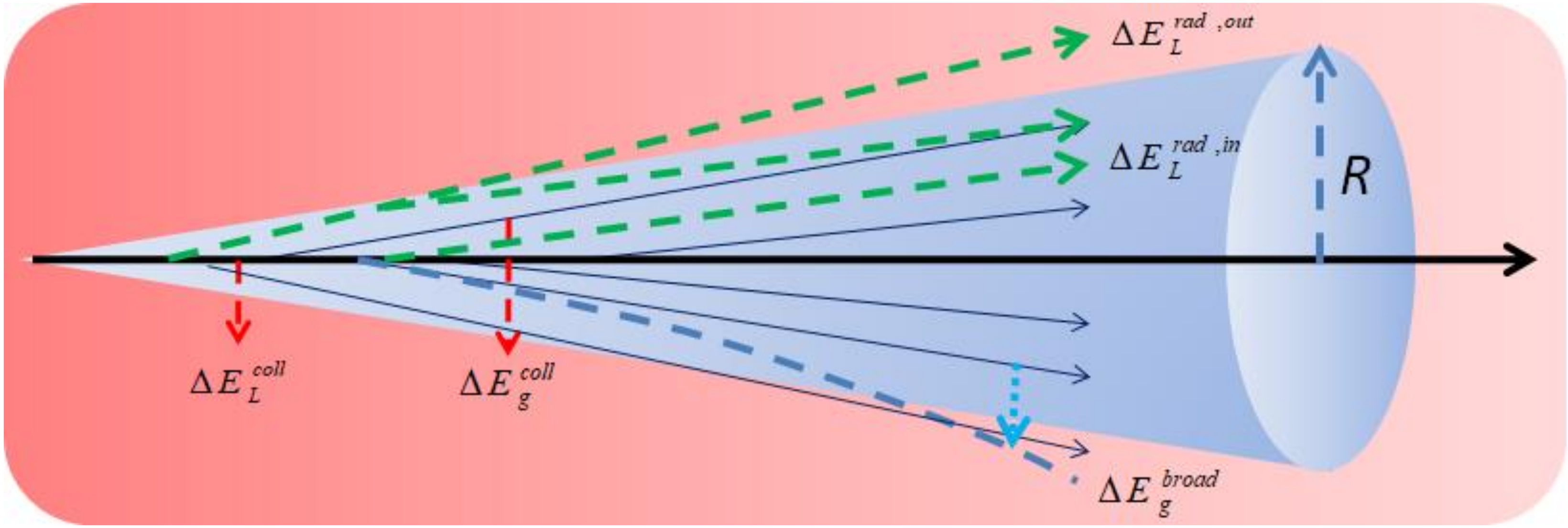
Jet shape

$$\rho(r) = \frac{d}{dr}\left(\frac{\sum_{i \in J} p_{T,i}\,\theta(r - r_i)}{\sum_{i \in J} p_{T,i}\,\theta(R - r_i)}\right)$$

$$r = \sqrt{(\varphi_i - \varphi_J)^2 + (\eta_i - \eta_J)^2}$$

- **The observed enhancement at large r** is consistent with jet broadening (& medium-induced radiation)
- The soft outer part of the jet is easier to modify, while changing the inner hard cone is more difficult

Full jet evolution & energy loss in medium



$$E_{jet} = E_{in} + E_{lost} = E_{in} + E_{rad,out} + E_{kick,out} + (E_{th} - E_{th,in})$$

GYQ, Muller, PRL, 2011; Casalderrey-Solana, Milhano, Wiedemann, JPG 2011; Young, Schenke, Jeon, Gale, PRC, 2011; Dai, Vitev, Zhang, PRL 2013; Wang, Zhu, PRL 2013; Blaizot, Iancu, Mehtar-Tani, PRL 2013; etc.

A model for full jet evolution in medium

- **Solve the 3D (energy & transverse momentum) evolution for shower partons inside the full jet**
- **Include both collisional (the longitudinal drag and transverse diffusion) and all radiative/splitting processes**

$$\frac{d}{dt} f_j(\omega_j, k_{j\perp}^2, t) = \left(\hat{e}_j \frac{\partial}{\partial \omega_j} + \frac{1}{4} \hat{q}_j \nabla_{k_\perp}^2 \right) f_j(\omega_j, k_{j\perp}^2, t)$$

Drag & transverse broadening

$$+ \sum_i \int d\omega_i dk_{i\perp}^2 \frac{d\tilde{\Gamma}_{i\to j}(\omega_j, k_{j\perp}^2 | \omega_i, k_{i\perp}^2)}{d\omega_j d^2 k_{j\perp} dt} f_i(\omega_i, k_{i\perp}^2, t)$$

Gain terms

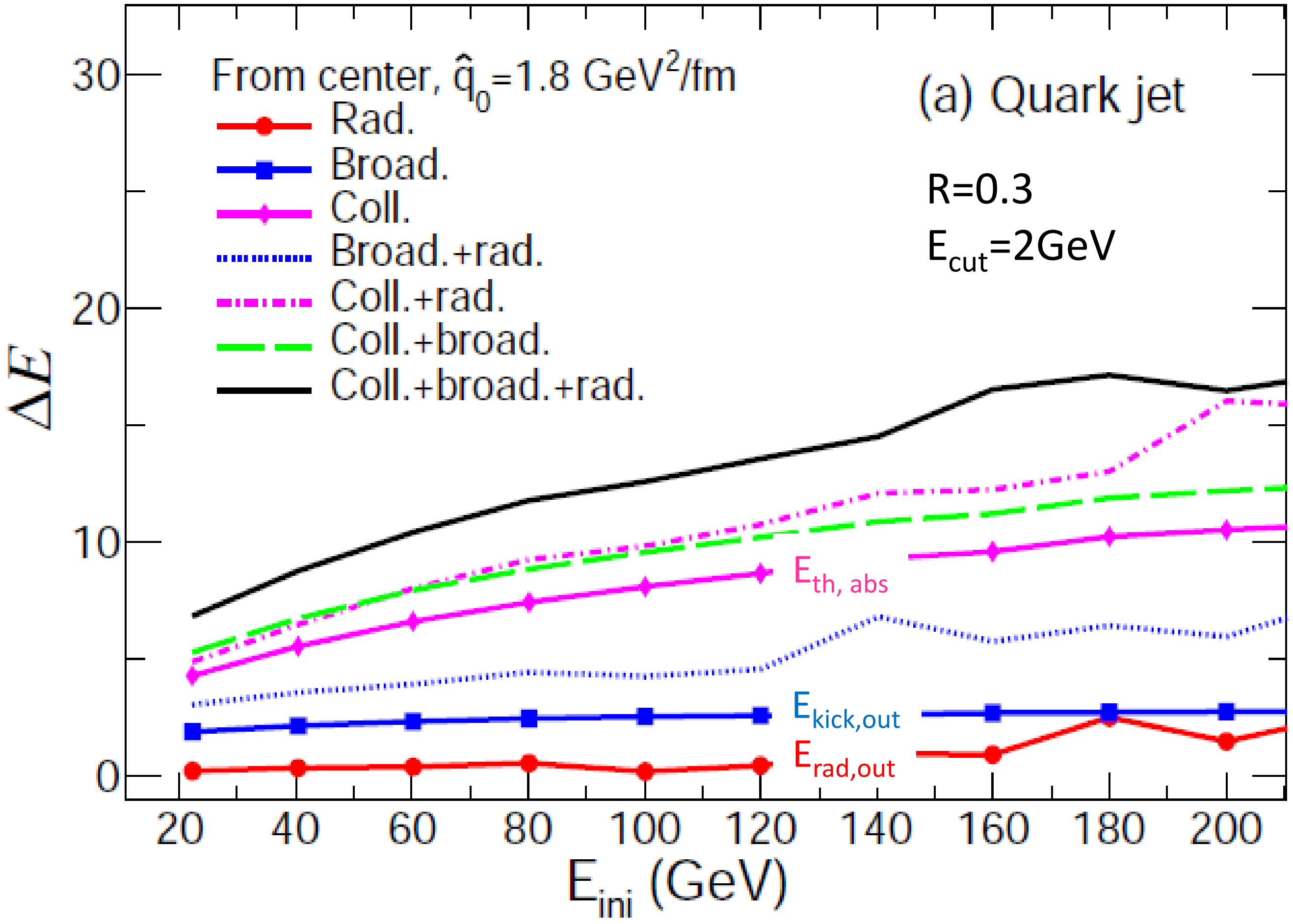
$$- \sum_i \int d\omega_i dk_{i\perp}^2 \frac{d\tilde{\Gamma}_{j\to i}(\omega_i, k_{i\perp}^2 | \omega_j, k_{j\perp}^2)}{d\omega_i d^2 k_{i\perp} dt} f_j(\omega_j, k_{j\perp}^2, t)$$

Loss terms

$$E_{jet}(R) = \sum_i \int_R \omega_i f_i(\omega_i, k_{i\perp}^2) d\omega_i dk_{i\perp}^2$$

Ningbo Chang, GYQ, PRC 2016

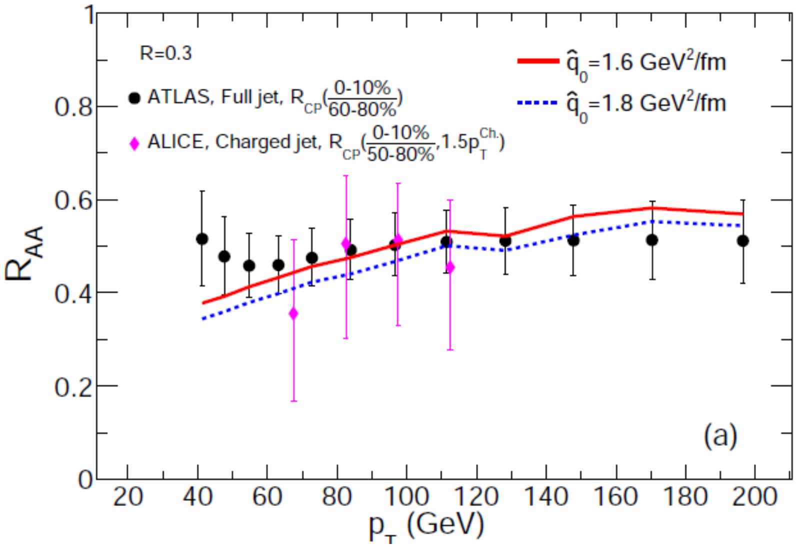
Full jet energy loss (radiative, collisional, broadening)

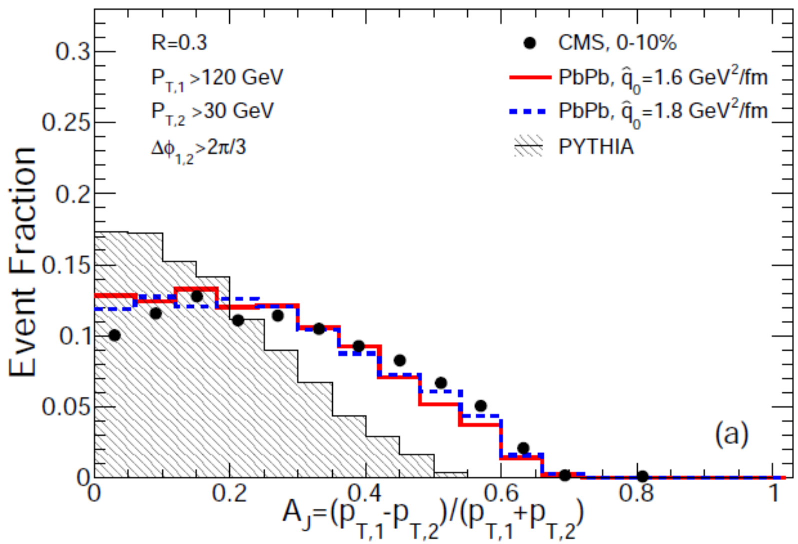


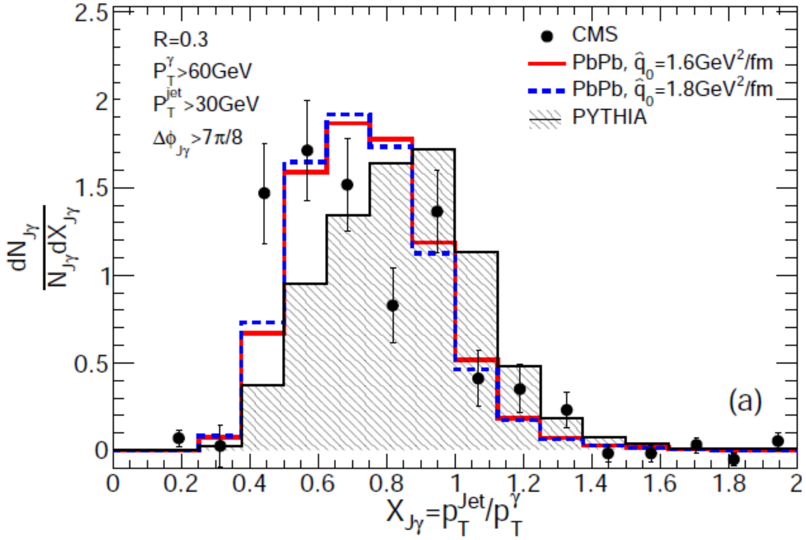
Chang, GYQ, PRC 2016

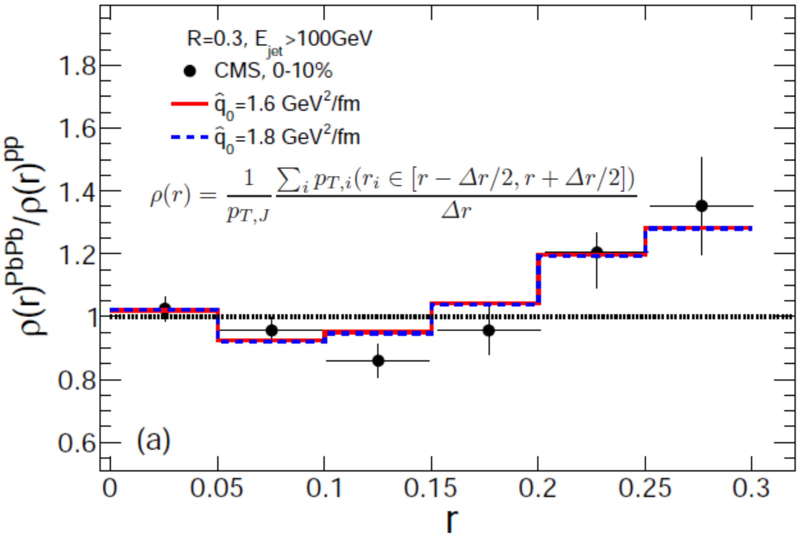
$$\frac{df(\vec{p},t)}{dt} = C_{coll.E.loss}\left[f\right] + C_{coll.broad}\left[f\right] + C_{rad}\left[f\right]$$

Various full jet observables



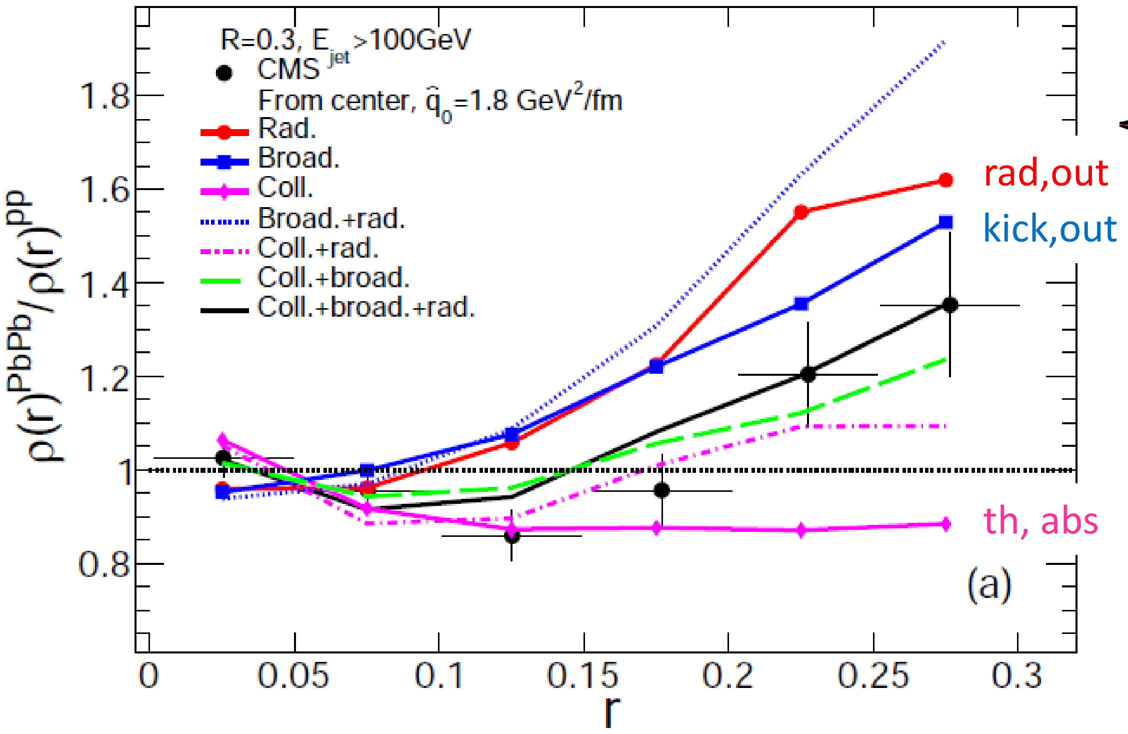


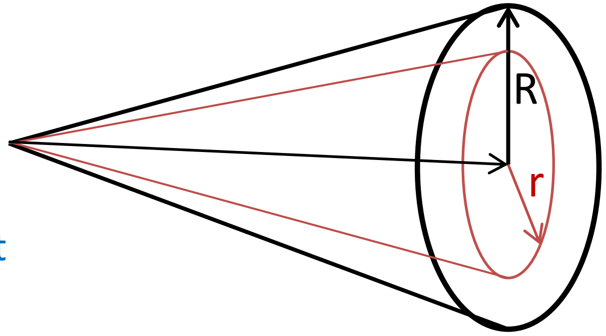




N. B. Chang, GYQ, PRC 2016

Nuclear modification of jet shape function





$$\rho(r) = \frac{d}{dr}\left(\frac{\sum_{i\in J} p_{T,i}\theta(r-r_i)}{\sum_{i\in J} p_{T,i}\theta(R-r_i)}\right)$$

$$r_i = \sqrt{(\varphi_i-\varphi_J)^2+(\eta_i-\eta_J)^2}$$

The enhancement at large r is consistent with jet broadening (& medium-induced radiation)

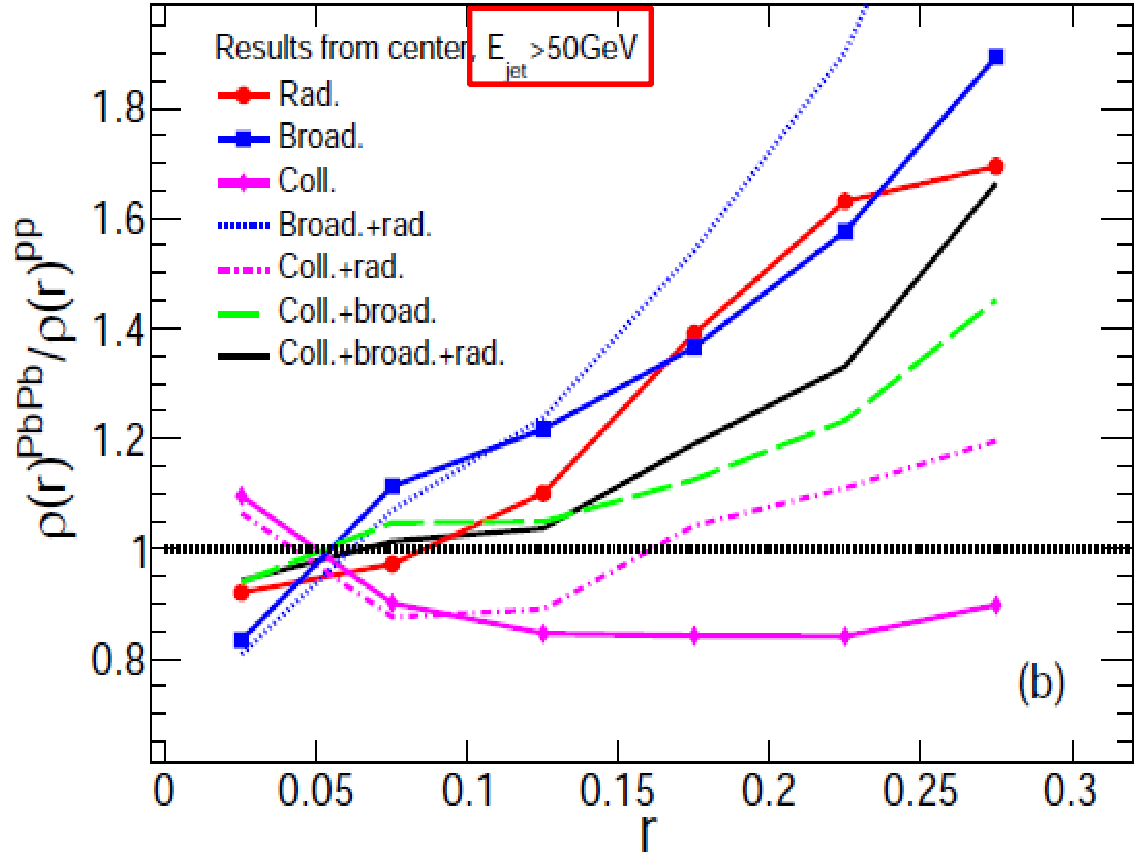
The soft outer part is easier to modify, while changing the inner hard cone is more difficult

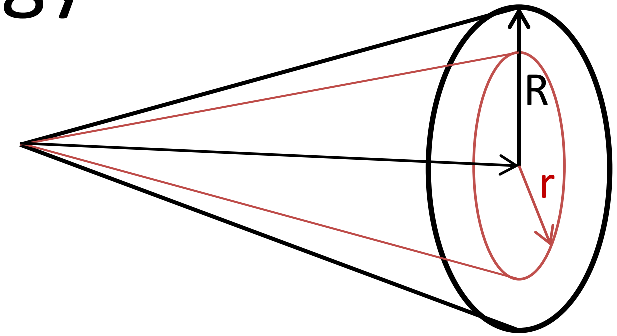
The final jet shape is the interplay of different jet-medium interaction mechanisms

N. B. Chang, GYQ, PRC 2016

$$\frac{df(\vec{p},t)}{dt} = C_{coll.E.loss}\left[f\right] + C_{coll.broad}\left[f\right] + C_{rad}\left[f\right]$$

Nuclear modification of jet shape function: lower jet energy



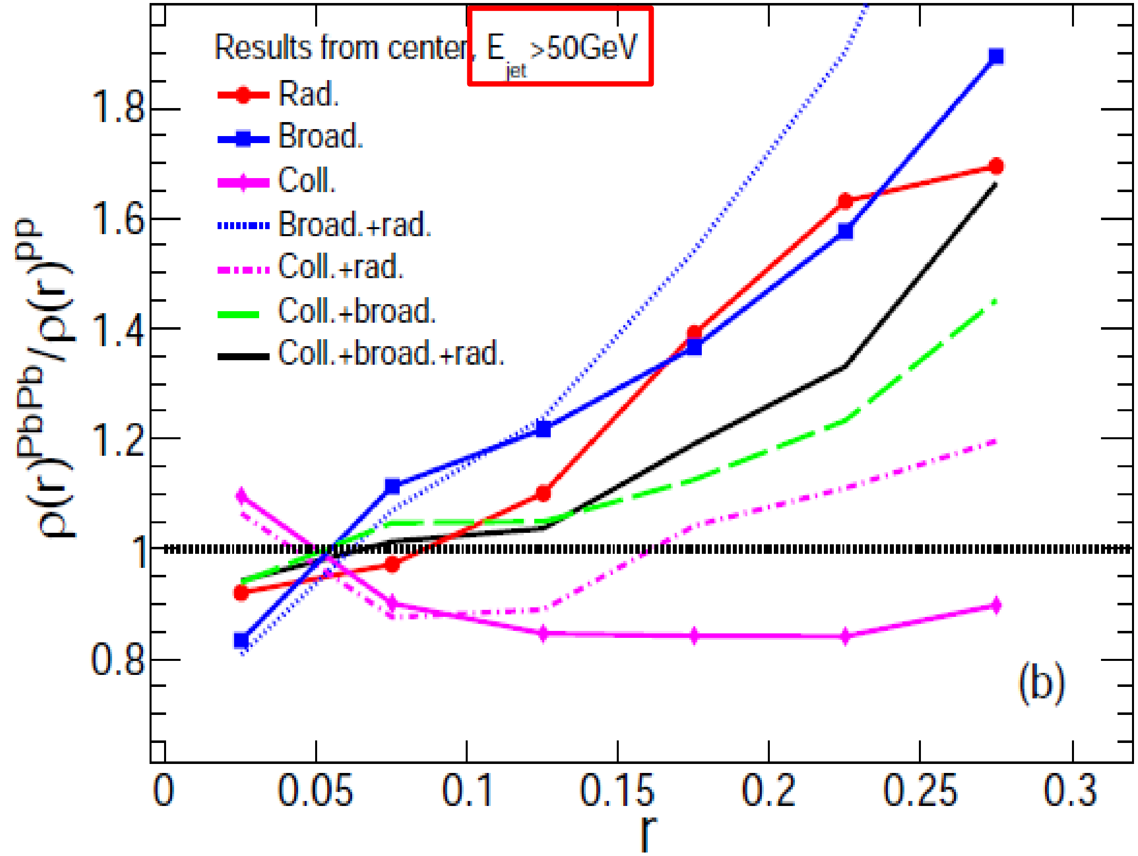


There is a chance to see the modification of jet core for lower energy jets (at RHIC) since the jet core is not too hard to be modified

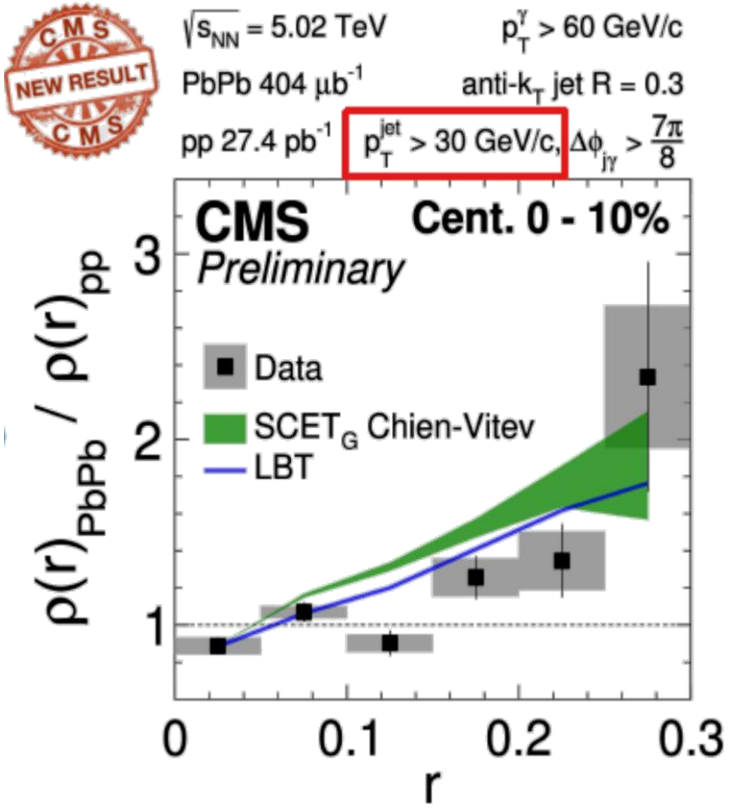
Ningbo Chang, GYQ, PRC 2016

$$\frac{df(\vec{p},t)}{dt} = C_{coll.E.loss}\left[f\right] + C_{coll.broad}\left[f\right] + C_{rad}\left[f\right]$$

Nuclear modification of jet shape function: lower jet energy



CMS-PAS HIN-18-006



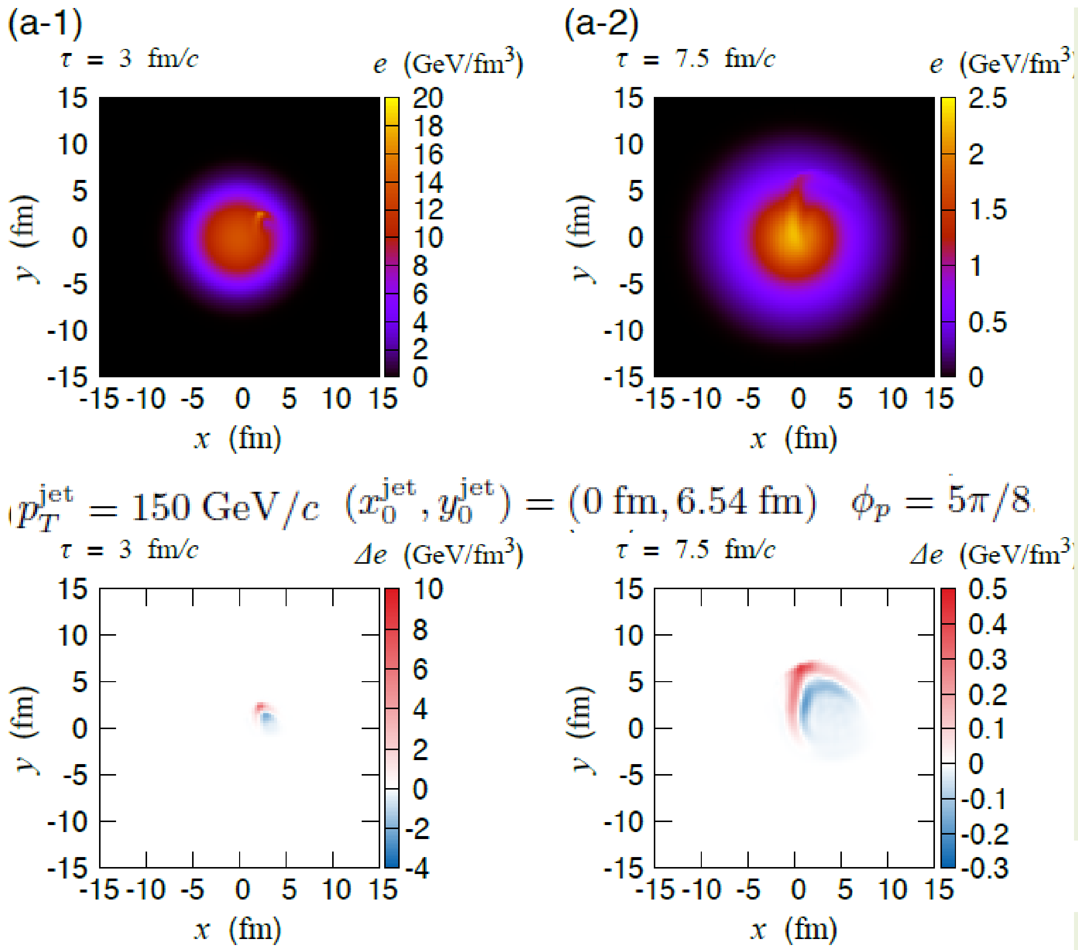
Ningbo Chang, GYQ, PRC 2016

$$\frac{df(\vec{p},t)}{dt} = C_{coll.E.loss}[f] + C_{coll.broad}[f] + C_{rad}[f]$$

A coupled jet-fluid model: jet evolution & medium response

$$\frac{df(\vec{p},t)}{dt} = C_{coll.E.loss}[f] + C_{coll.broad}[f] + C_{rad}[f]$$

$$\partial_\mu T^{\mu\nu}_{\mathrm{QGP}}(x) = J^\nu(x) = -\partial_\mu T^{\mu\nu}_{\mathrm{jet}}(x) = -\frac{dP^\nu_{\mathrm{jet}}}{dt d^3x} = -\sum_j \int \frac{d^3k_j}{\omega_j} k_j^\nu k_j^\mu \partial_\mu f_j(\boldsymbol{k}_j, \boldsymbol{x}, t)$$

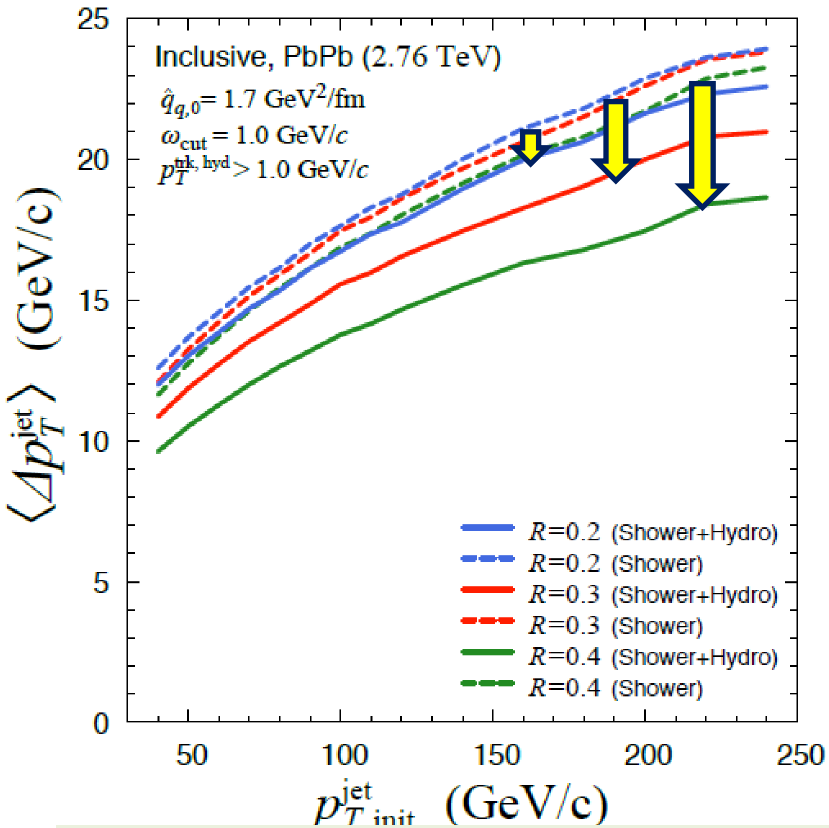


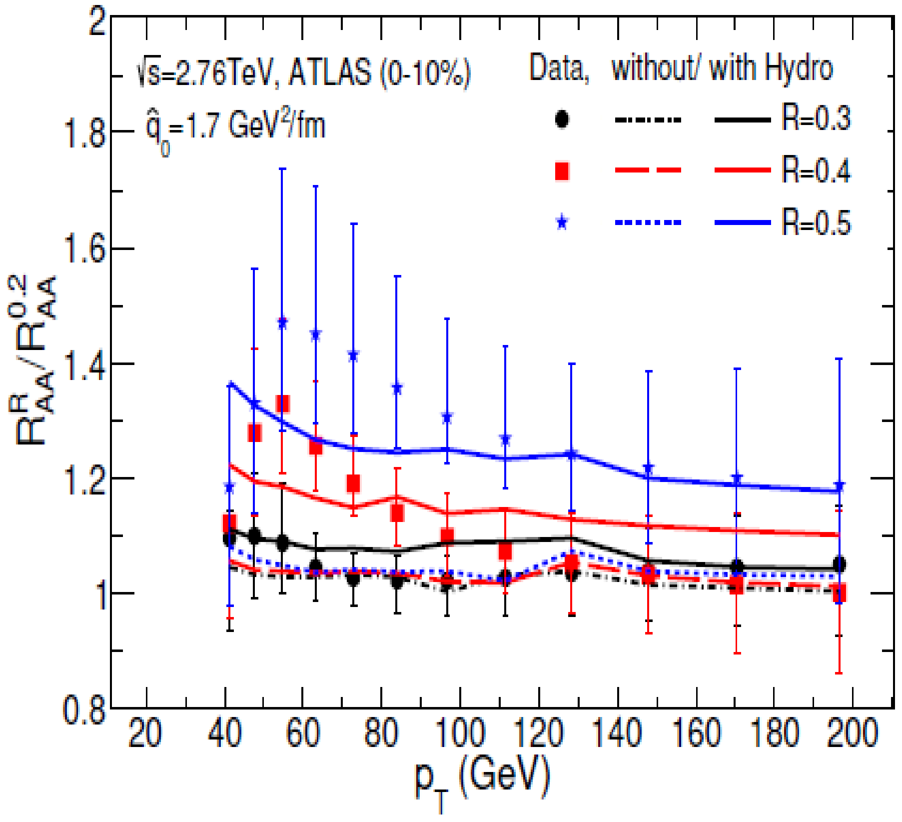
$p_T^{\mathrm{jet}} = 150\ \mathrm{GeV}/c \quad (x_0^{\mathrm{jet}}, y_0^{\mathrm{jet}}) = (0\ \mathrm{fm}, 6.54\ \mathrm{fm}) \quad \phi_p = 5\pi/8$

- V-shaped wave fronts are induced by the jet, and develop with time
- The wave fronts carry the energy & momentum, propagates outward & lowers energy density behind the jet
- Jet-induced flow and the radial flow of the medium are pushed and distorted by each other

Tachibana, Chang, GYQ, PRC 2017

Effect of jet-induced flow on jet energy loss & suppression

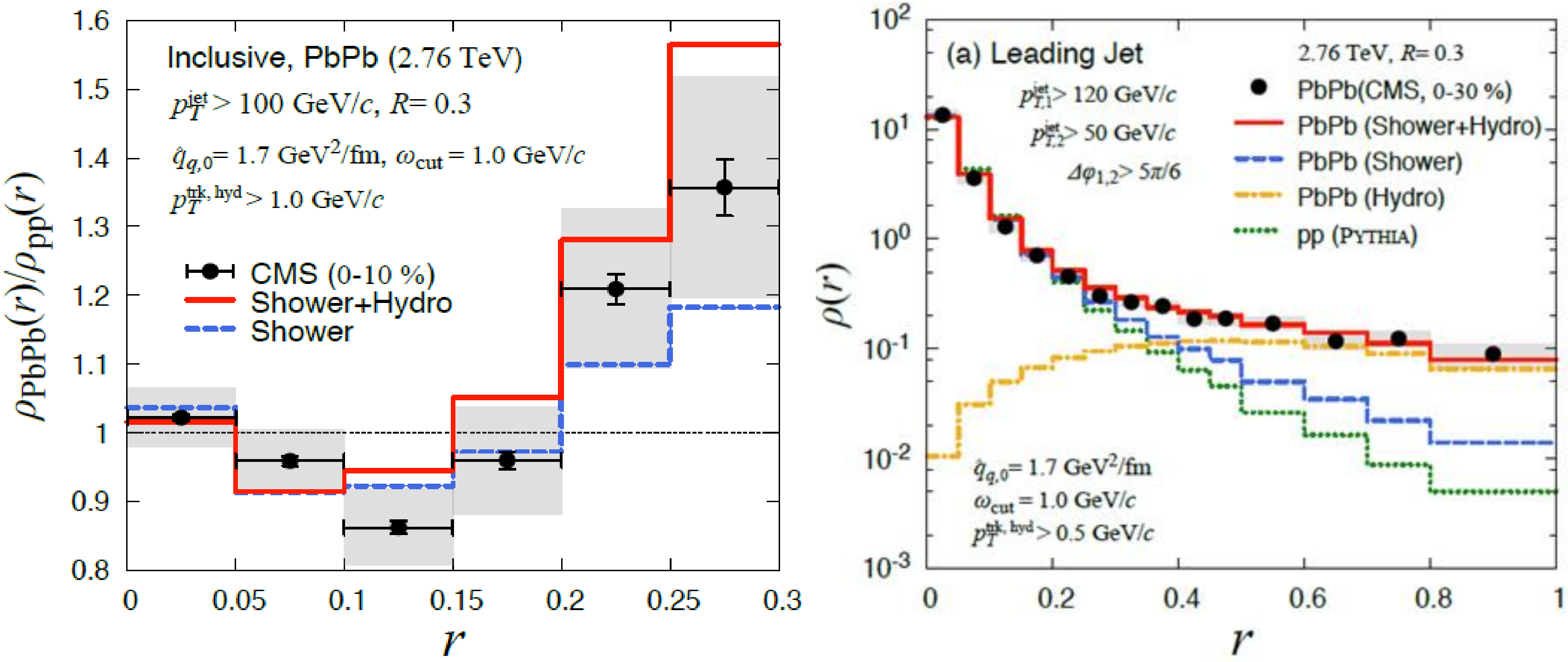




- **Hydro part** (the lost energy from shower part to medium still inside the jet cone) **partially compensates the energy loss experienced by jet shower part.**
- **Jet-induced flow evolves with medium, diffuses, and spreads widely around jet axis, leading to stronger jet cone size dependence.**

Tachibana, Chang, GYQ, PRC 2017; in preparation

Effect of jet-induced flow on jet shape



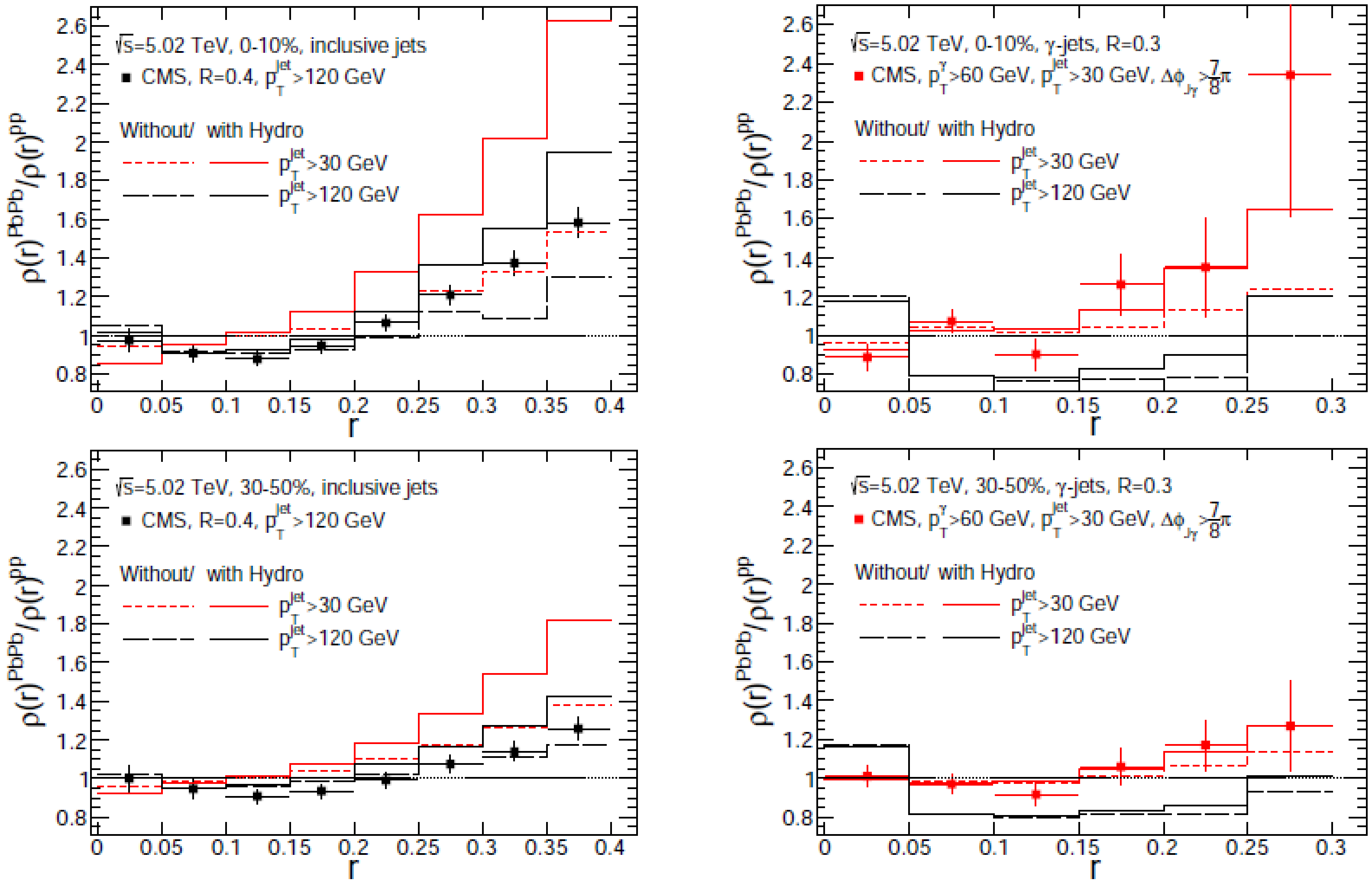
Tachibana, Chang, GYQ, PRC 2017

The inclusion of jet-induced medium flow does not modify jet shape at small r, but significantly enhance jet broadening effect at large r (r > 0.2-0.25).

The contribution from the hydro part is quite flat and finally dominates over the shower part in the region from r = 0.4-0.5.

Signal of jet-induced medium excitation in full jet shape at large r.

Jet shape function for inclusive jets and γ-jets

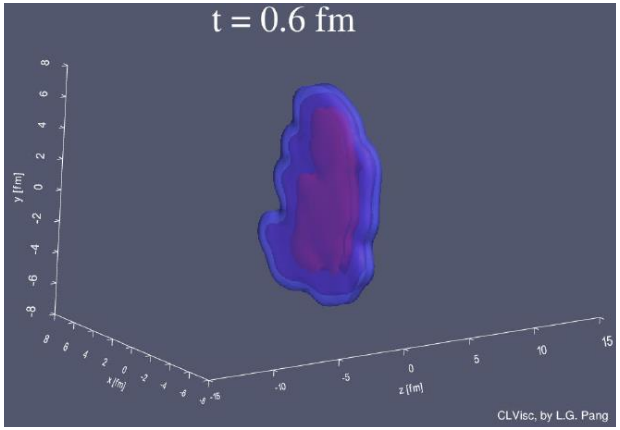


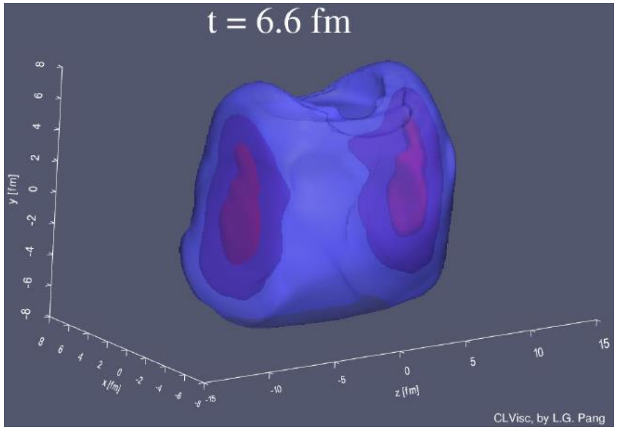
Tachibana, Chang, GYQ, in preparation

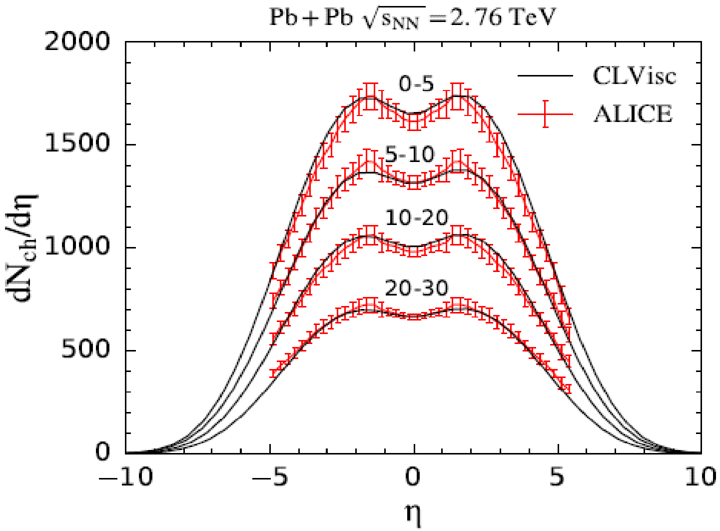
Jet quenching at forward rapidity

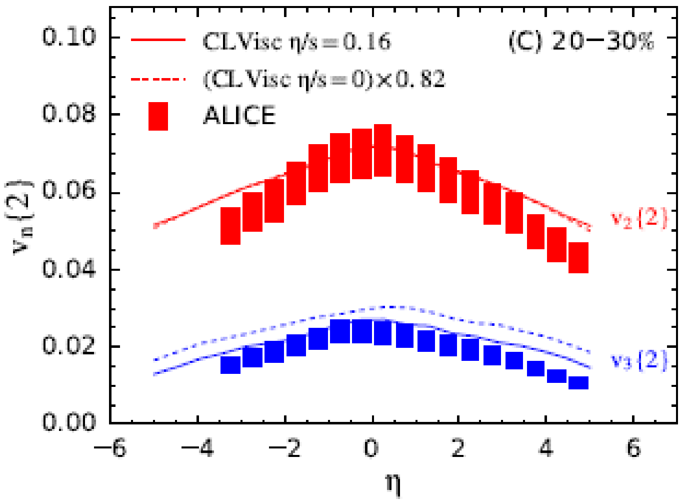
- Forward rapidity provides versatile tools for studying jet quenching
  - Bulk medium profiles
  - Jet spectra
  - Jet structures
  - Jet quenching effects

Bulk medium profiles
(longitudinal dependence, fluctuations & decorrelations)



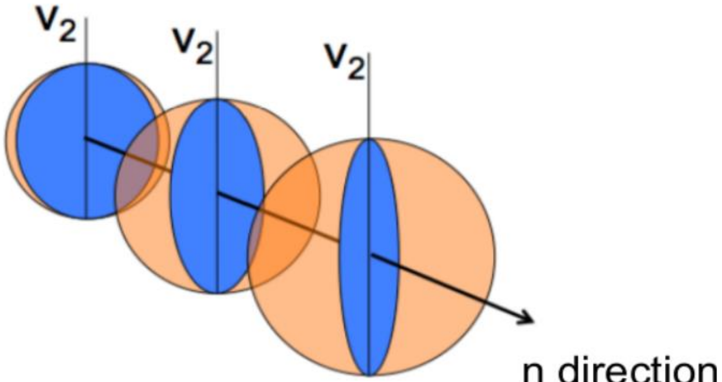


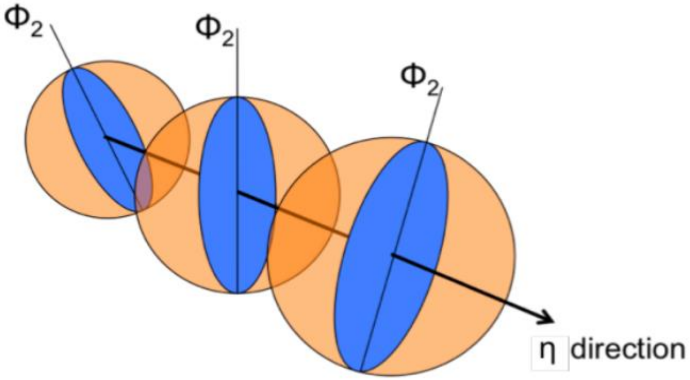




Pang, Wang, Wang, PRC 2012; Pang, Petersen, Wang, PRC 2018

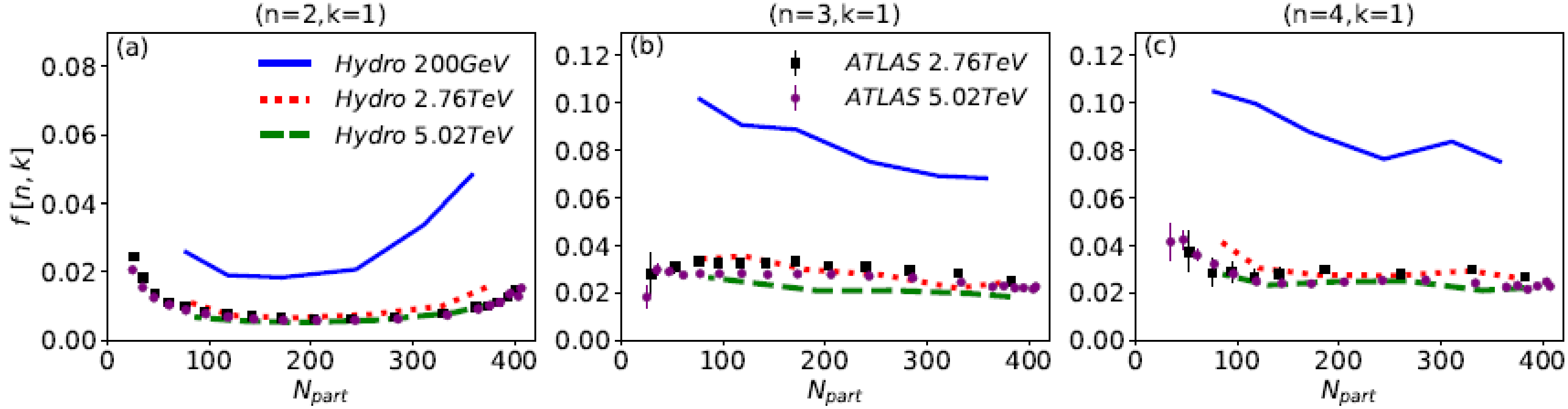
Bulk medium profiles
(longitudinal dependence, fluctuations & decorrelations)





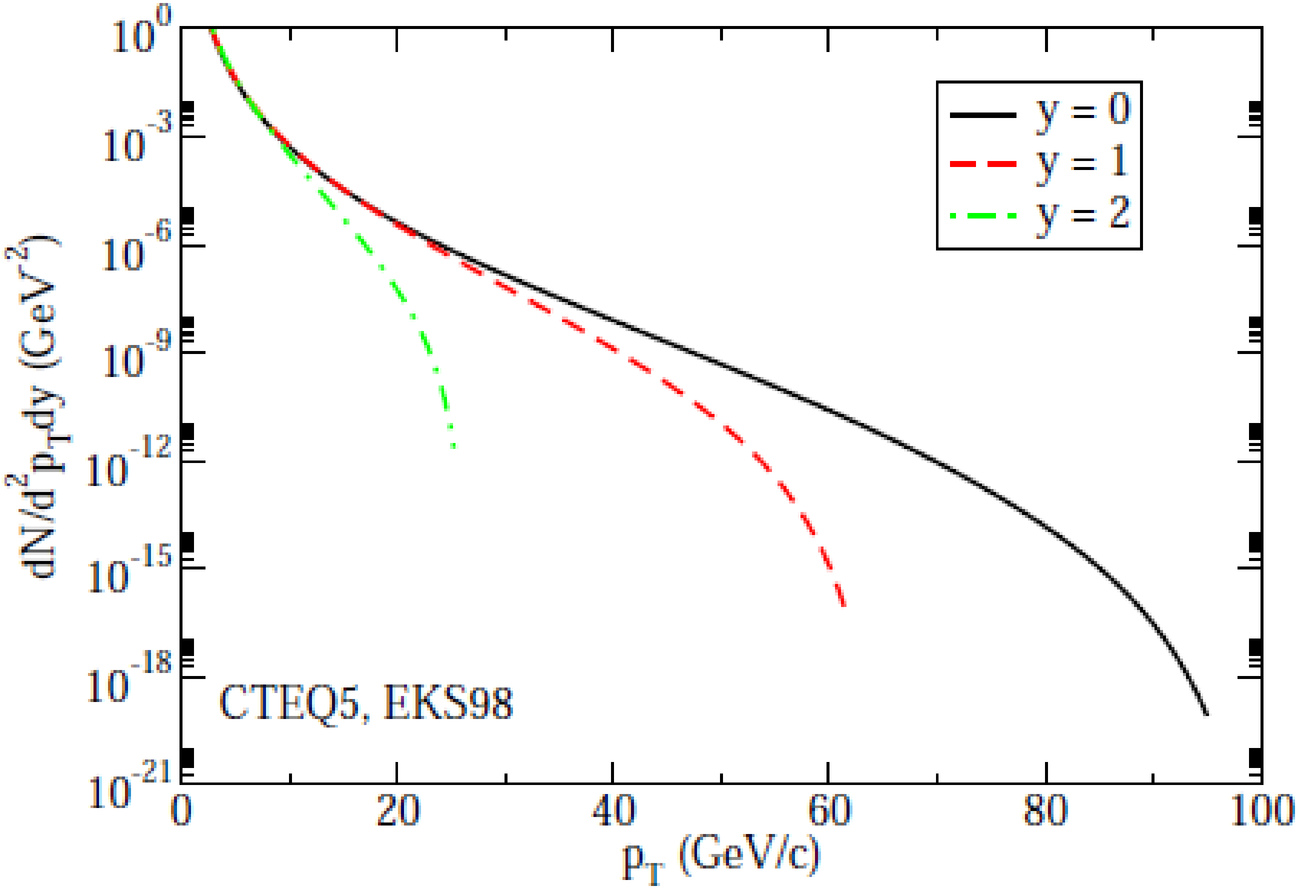
$$r[n,k](\eta) = \frac{\langle Q_n^k(-\eta) Q_n^{*k}(\eta_r)\rangle}{\langle Q_n^k(\eta) Q_n^{*k}(\eta_r)\rangle}$$

$$r[n,k](\eta) \approx 1 - 2f[n,k]\eta$$



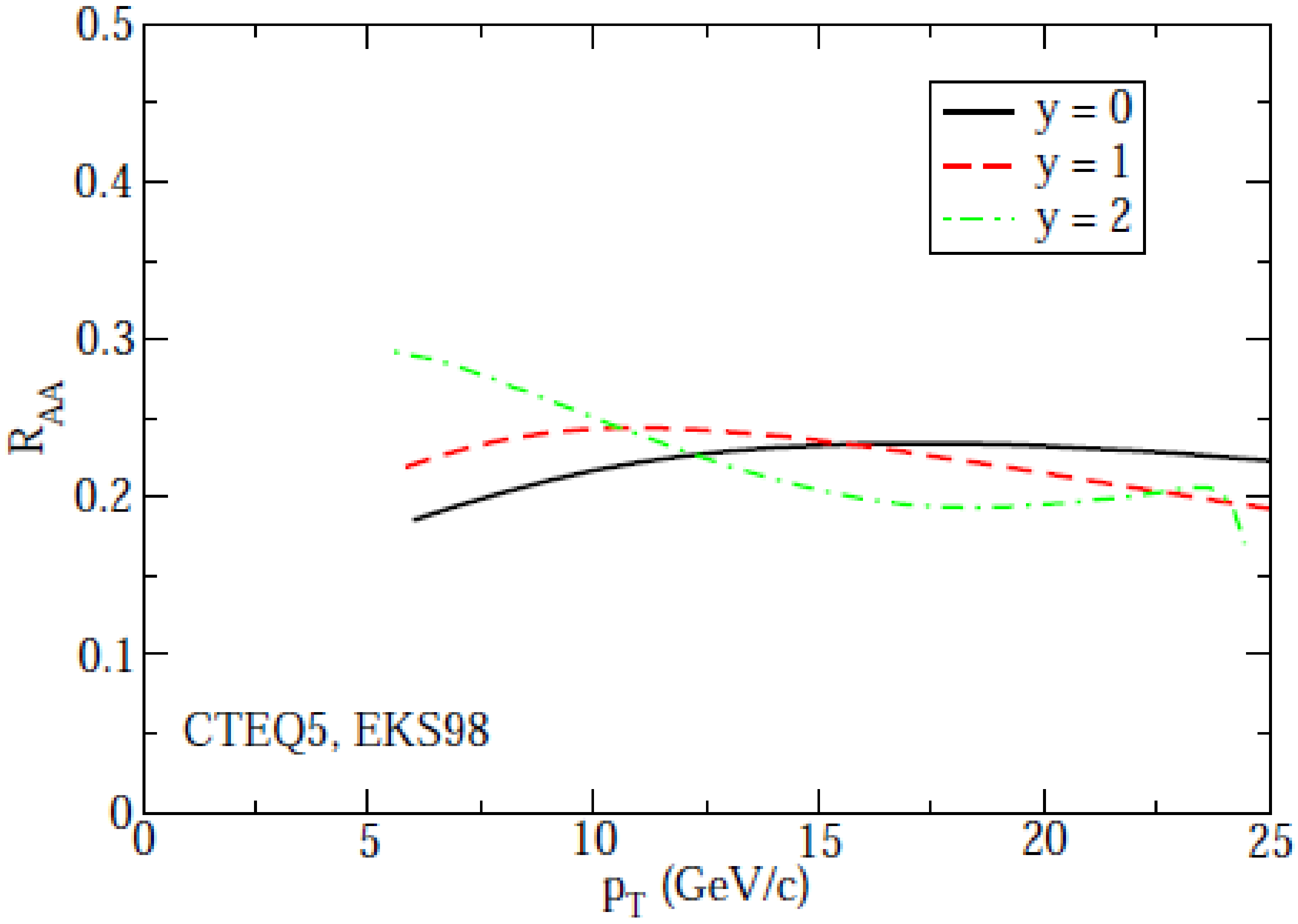
Xiang-Yu Wu, Pang, GYQ, Wang, PRC 2018

Jet spectra



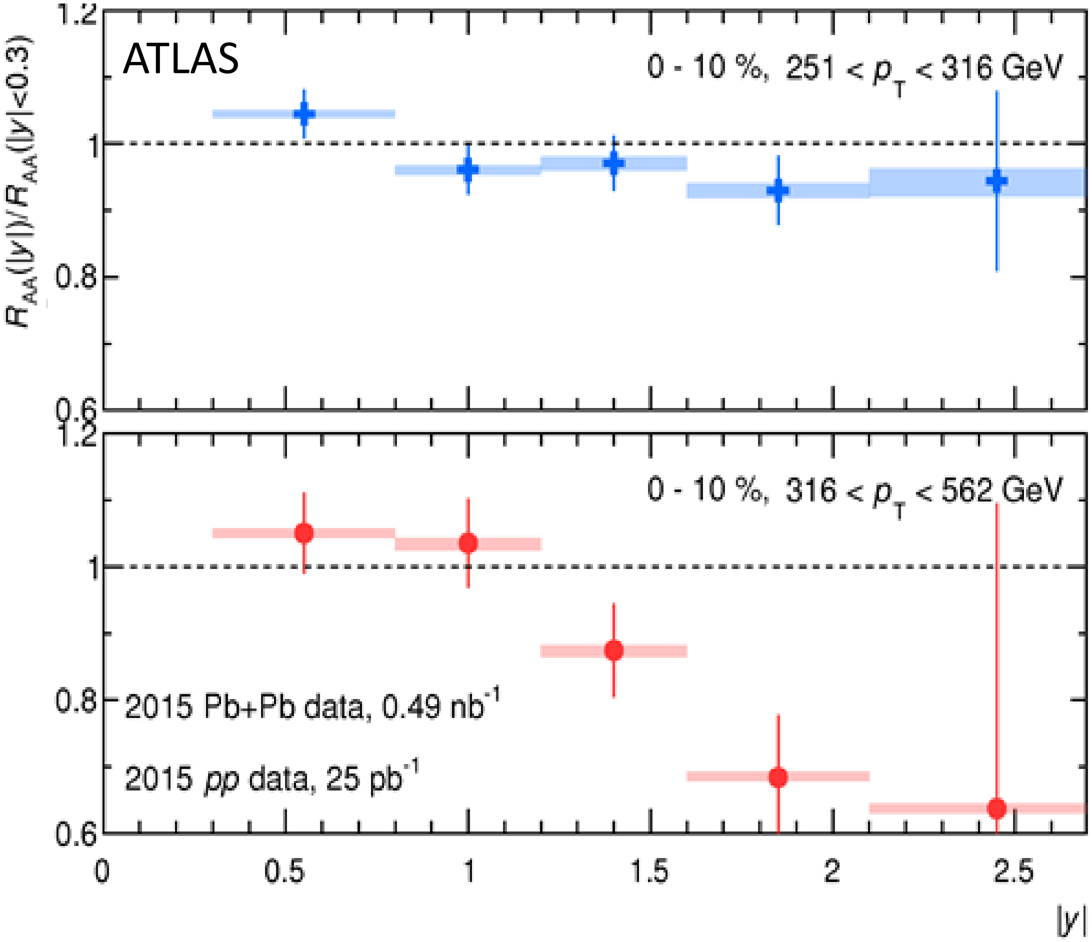
GYQ, et al, PRC 2007

Hadron R_{AA}

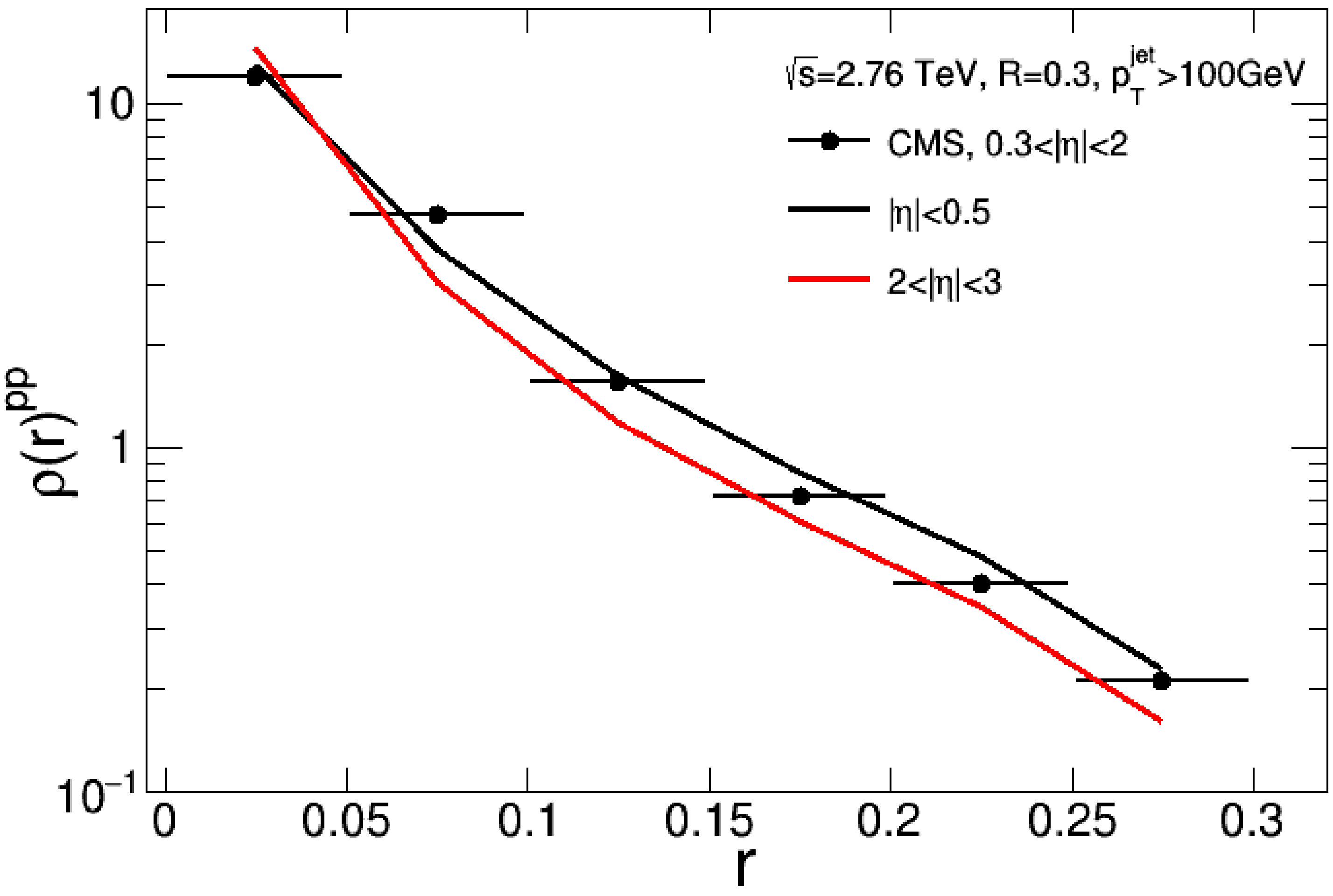


GYQ, et al, PRC 2007

Jet R_{AA}


ATLAS
0 - 10 %, 251 < p_T < 316 GeV
0 - 10 %, 316 < p_T < 562 GeV
2015 Pb+Pb data, 0.49 nb^{-1}
2015 pp data, 25 pb^{-1}
$R_{AA}(|y|)/R_{AA}(|y|<0.3)$
1.2
1
0.8
0.6
0
0.5
1
1.5
2
2.5
$|y|$

Jet structure

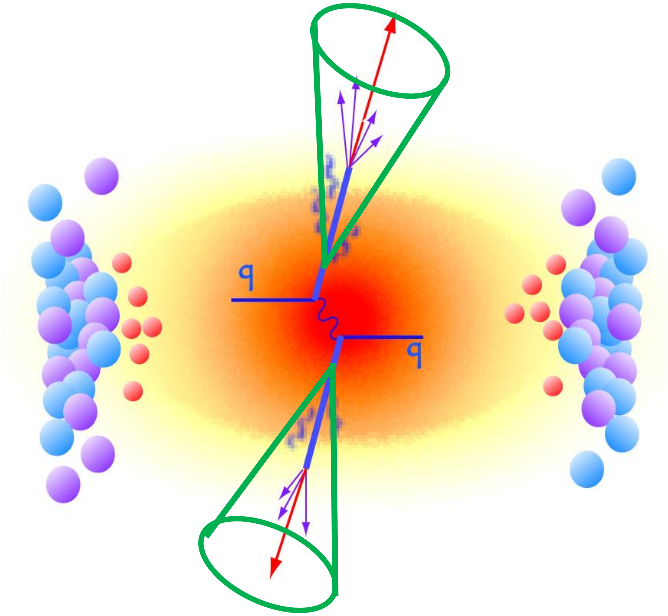


Summary

- Medium-induced gluon emission beyond collinear expansion and soft emission limit for massive quarks including transverse and longitudinal scatterings
- A coupled jet-fluid model with full jet evolution and medium response
- Interplay of different mechanisms in full jet evolution, jet energy loss and nuclear modification of jet substructure
- Signal of jet-induced medium excitation in jet shape at large r
- Forward rapidity provides versatile tools for studying jet quenching

Full jets in heavy-ion collisions

- Jets are spray of charged (and neutral) particles originating from fragmentation of hard-scattered partons
- Jet reconstruction: recombine hadron (or parton) fragments, approximate the original parton energy and momentum
- Parameters: e.g., jet size R
- With the inclusion of sub-leading fragments, fully reconstructed jets are expected to provide more detailed information (about jet-medium interaction) than leading hadron observables



$$R = \sqrt{(\phi - \phi_J)^2 + (\eta - \eta_J)^2}$$

Generalized k_T family of jet reconstruction algorithms

- (1) Consider all particles in the list, and compute all distances d_{iB} and d_{ij}
- (2) For particle i, find $\min(d_{ij}, d_{iB})$
- (3) If $\min(d_{iB}, d_{ij}) = d_{iB}$, declare particle i to be a jet, and remove it from the list of particles. Then return to (1)
- (4) If $\min(d_{iB}, d_{ij})=d_{ij}$, recombine i & j into a single new particle. Then return to (1)
- (5) Stop when no particles are left

$$d_{iB} = p_{T,i}^{2p}$$

$$d_{ij} = \min(p_{T,i}^{2p}, p_{T,j}^{2p}) \frac{\Delta R_{ij}^2}{R^2}$$

$$\Delta R_{ij}^2 = (\phi_i - \phi_j)^2 + (\eta_i - \eta_j)^2$$

p=1: k_T algorithm
p=0: Cambridge/Aachen algorithm
p=-1: anti-k_T algorithm

Jet substructure observables

• **Jet shape**	$\rho(r) = \left\langle \frac{1}{p_{T,J}} \sum_{i \in J} p_{T,i} \delta(r - r_i) \right\rangle_{jets}$	**Transverse profile**
• **Jet fragmentation function**	$D(z) = \left\langle \sum_{i \in J} \delta(z - \frac{p_{T,i}}{p_{T,J}}) \right\rangle_{jets}$	**Longitudinal profile**
• **Girth**	$g = \frac{1}{p_{T,J}} \sum_{i \in J} p_{T,i} r_i$	**Transverse size**
• **Jet mass**	$m_J^2 = \left(\sum_{i \in J} p_i^{\mu} \right)^2$	**Energy & size**
• **Groomed jet**	$z_g = \frac{\min(p_{T,1}, p_{T,2})}{p_{T,1} + p_{T,2}} > z_{cut} \theta^{\beta} = z_{cut} \left(\frac{\Delta R_{12}}{R} \right)^{\beta}$	**momentum sharing (splitting function)**

Medium response to jet-deposited energy/momentum

$$\partial_\mu T^{\mu\nu}_{\mathrm{QGP}}(x) = J^\nu(x) = -\partial_\mu T^{\mu\nu}_{\mathrm{jet}}(x) = -\frac{dP^\nu_{\mathrm{jet}}}{dt d^3x} = -\sum_j \int \frac{d^3k_j}{\omega_j} k_j^\nu k_j^\mu \partial_\mu f_j(\boldsymbol{k}_j, \boldsymbol{x}, t)$$

$$= -\sum_j \int \frac{d^3k_j}{\omega_j} k_j^\nu k_j^\mu \left[\partial_\mu f_j(\boldsymbol{k}_j, \boldsymbol{x}, t)\big|_{\hat{e},\hat{q}}\right] + \sum_j \int \frac{d^3k_j}{\omega_j} k_j^\nu k_j^\mu \left[\partial_\mu f_j(\boldsymbol{k}_j, \boldsymbol{x}, t)\big|_{\mathrm{rad.}}\right]$$

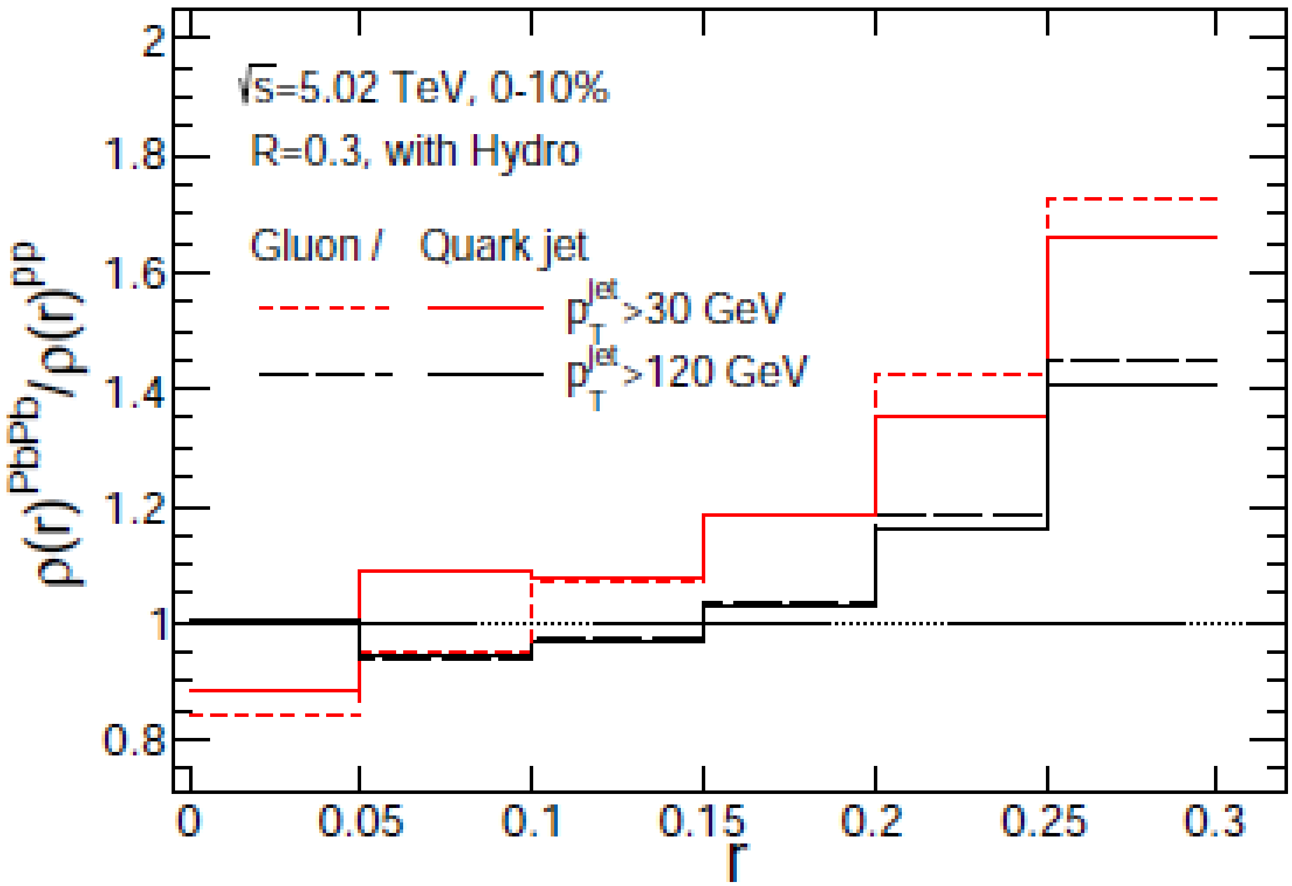
$$= -\sum_j \int d^3k_j k_j^\nu \left.\frac{df_j(\boldsymbol{k}_j, t)}{dt}\right|_{\mathrm{col.}} \delta^{(3)}\left(\boldsymbol{x} - \boldsymbol{x}_0^{\mathrm{jet}} - \frac{\boldsymbol{k}_j}{\omega_j} t\right)$$

$$J^\nu(x) \approx -\frac{1}{2\pi r t^3}(x^\nu - x^\nu_{\mathrm{jet},0}) \left.\frac{dE^{\mathrm{jet}}}{dt dr}\right|_{\mathrm{col.}} \delta\left(|\boldsymbol{x} - \boldsymbol{x}_0^{\mathrm{jet}}| - t\right)$$

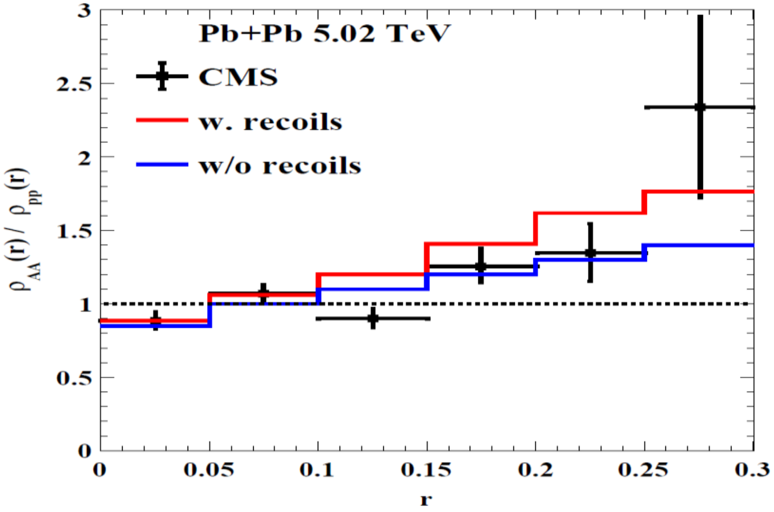
$$\left.\frac{dE^{\mathrm{jet}}}{dt dr}\right|_{\mathrm{col.}} = \sum_j \int d\omega dk_{j\perp}^2 \omega_j \left.\frac{df_j\left(\omega_j, k_{j\perp}^2, t\right)}{dt}\right|_{\mathrm{col.}} \delta\left(r - \frac{k_{j\perp}}{\omega_j}\right)$$

$$J^{\bar{\nu}}(\tau, x, y, \eta_s) = -\frac{dP^{\bar{\nu}}_{\mathrm{jet}}}{\tau d\tau dx dy d\eta_s} = \Lambda^{\bar{\nu}}_\mu J^\mu(x) = -\Lambda^{\bar{\nu}}_\mu \frac{dP^\mu_{\mathrm{jet}}}{dt d^3x}$$

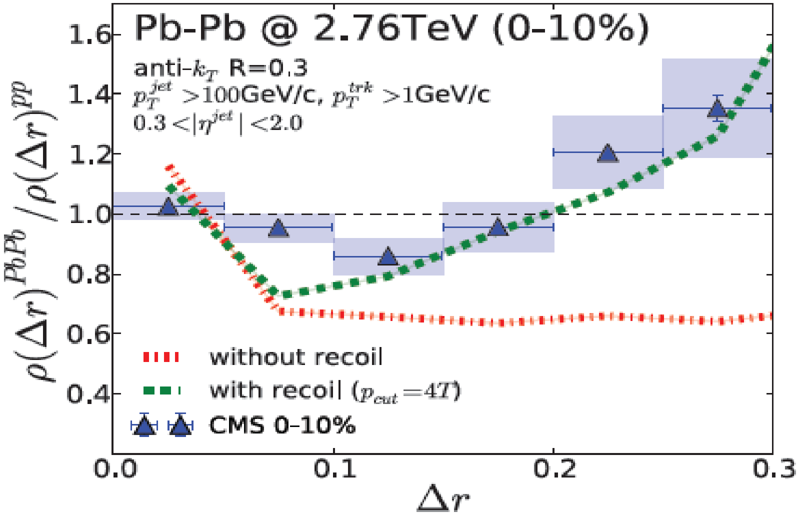
Jet shape function for quark & gluon jets



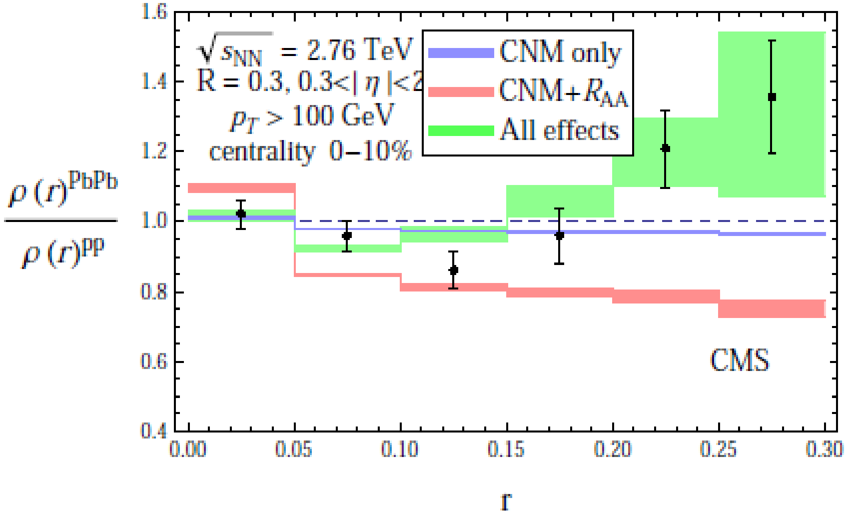
Effect of jet-induced flow on jet shape



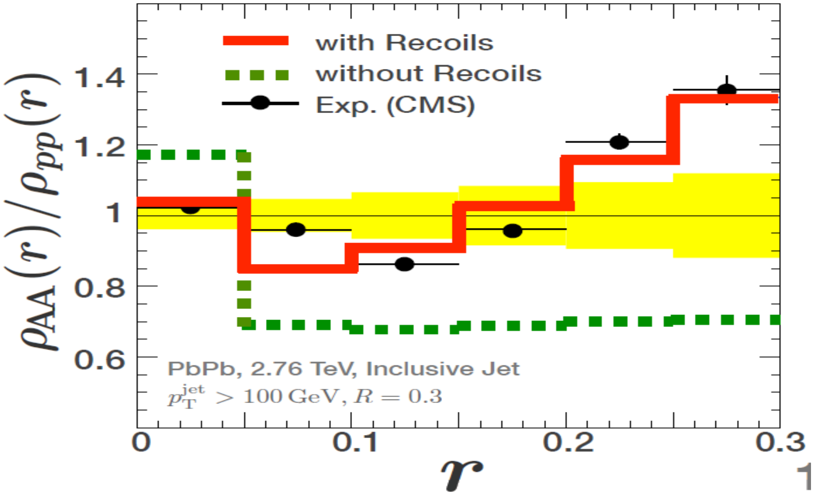
Luo, Cao, He, Wang, arXiv:1803.06785



C. Park, S. Jeon, C. Gale, 2018



Chien, Vitev, JHEP 2016



Elayavalli, Zapp, JHEP 2017