Probing the EW sphaleron with GWs

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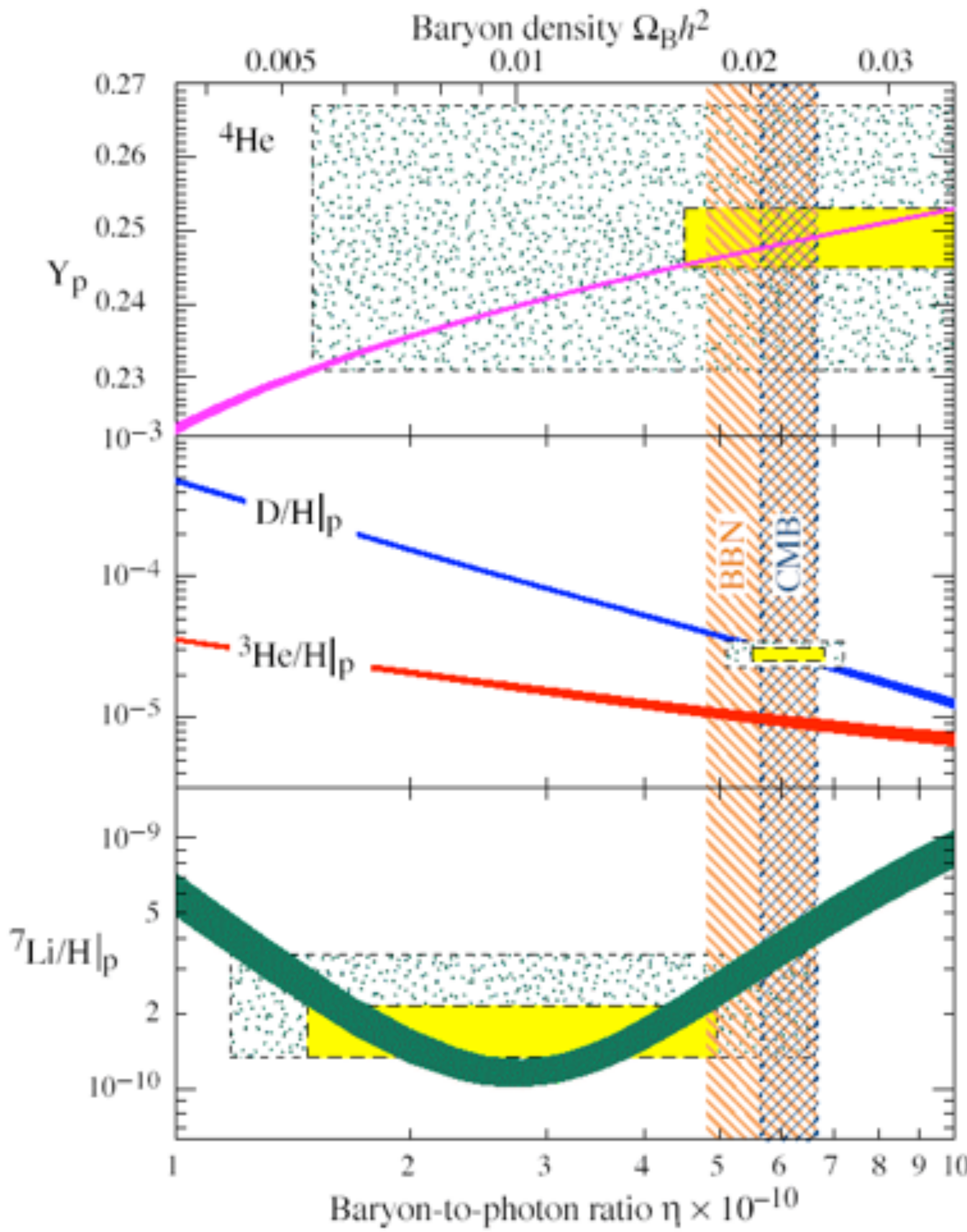
based on:RYZ, LGB,HKG, 1910.00234

Baryon asymmetry in the universe

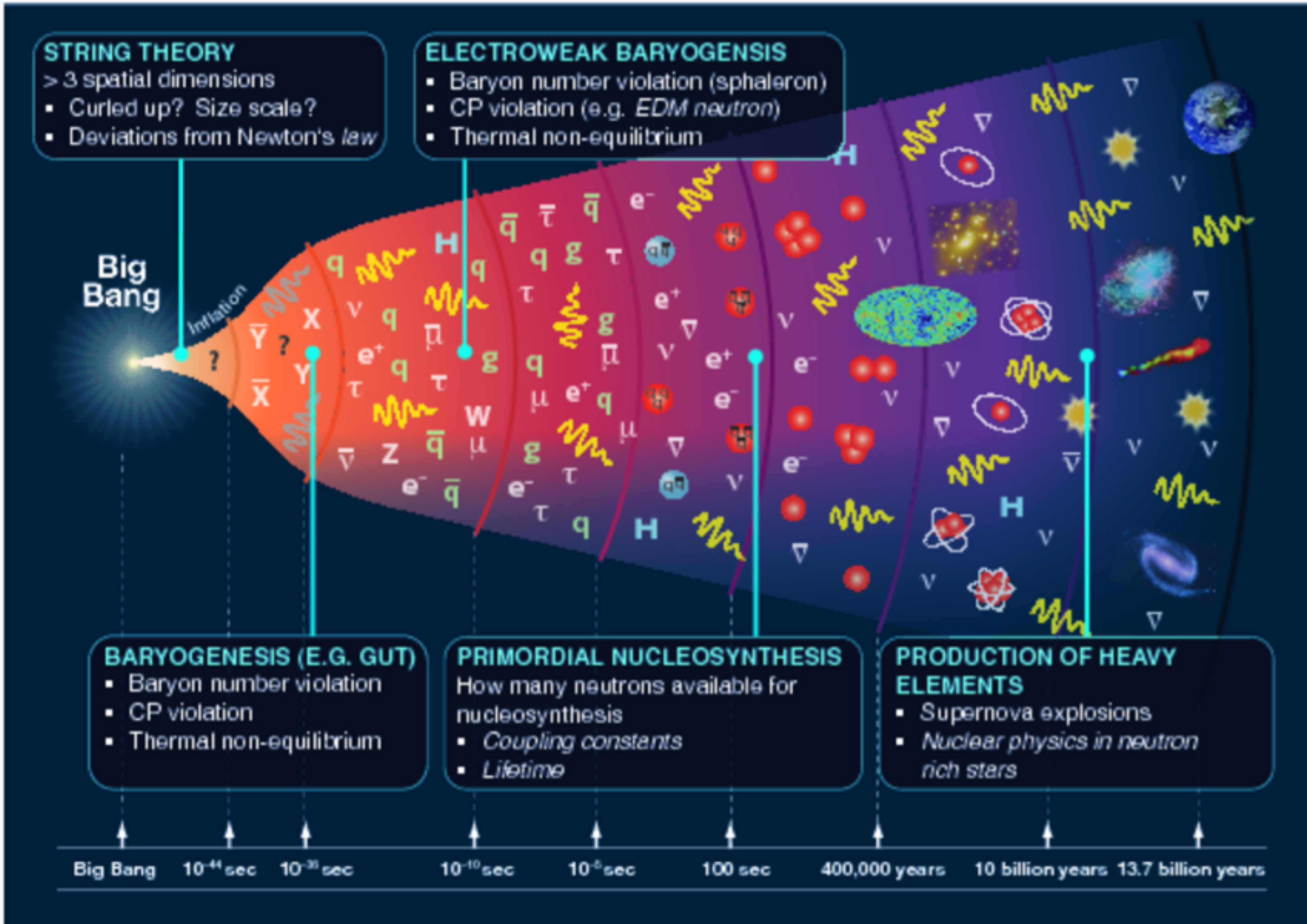


more baryons than anti-baryons (BBN & CMB, etc)

$$\frac{n_b}{s} \approx (0.7 - 0.9) \times 10^{-10} \neq 0$$

If the BAU is generated before T≃O(1) MeV(BBN), the light element abundances (D,^{3}He,^{4}He,^{7}Li) can be explained by the standard Big-Bang cosmology.

Baryogenesis = generate right nb/s

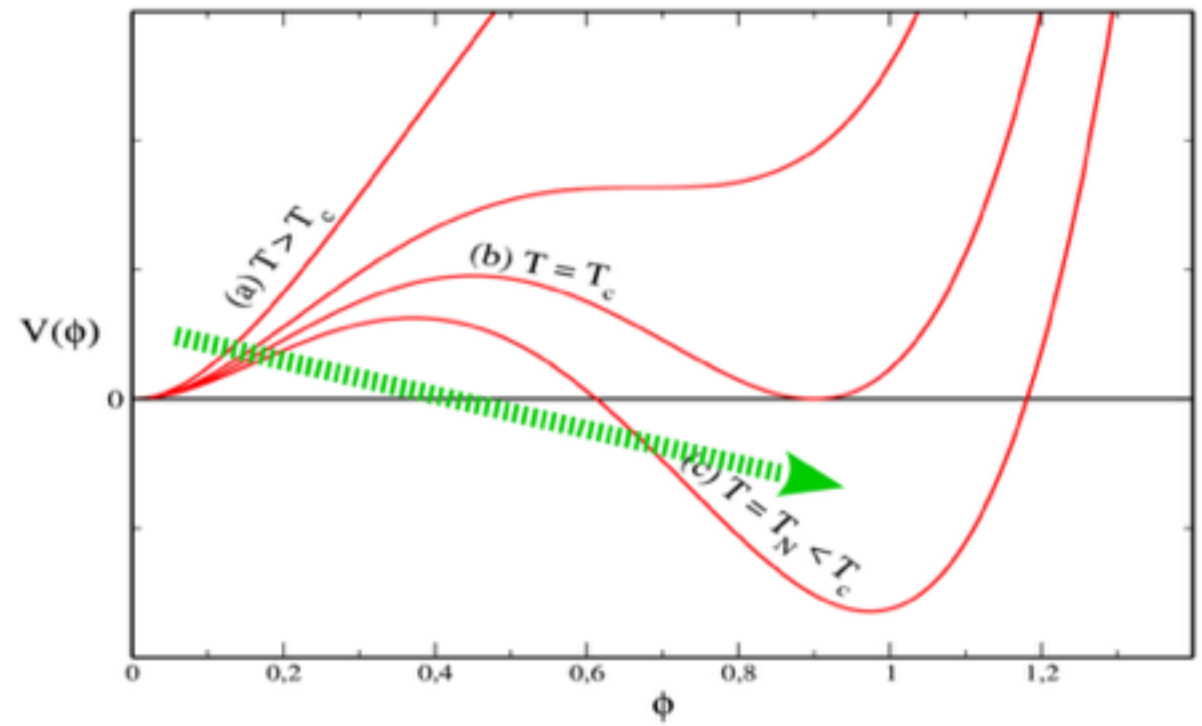


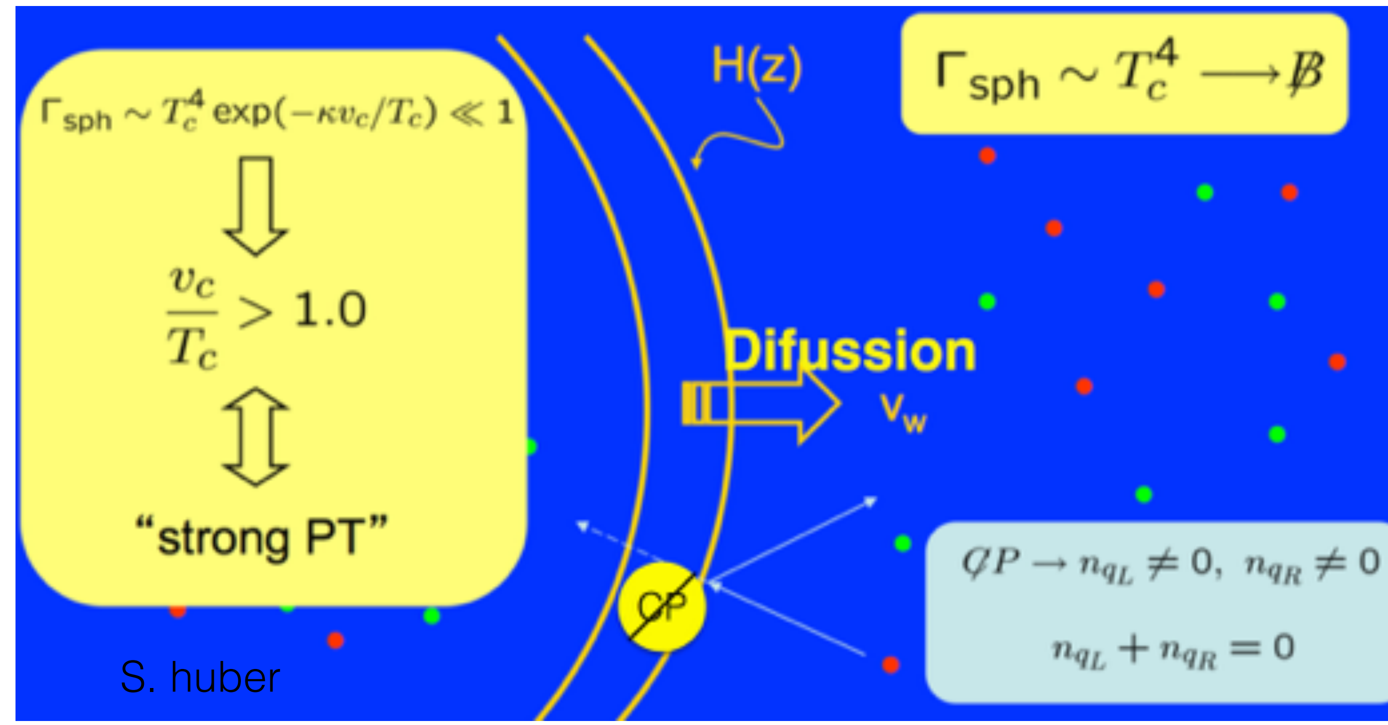
STRING THEORY
> 3 spatial dimensions
- Curled up? Size scale?
- Deviations from Newton's *law*
ELECTROWEAK BARYOGENSIS
- Baryon number violation (sphaleron)
- CP violation (e.g. *EDM neutron*)
- Thermal non-equilibrium
Big Bang
Inflation
BARYOGENESIS (E.G. GUT)
- Baryon number violation
- CP violation
- Thermal non-equilibrium
PRIMORDIAL NUCLEOSYNTHESIS
How many neutrons available for nucleosynthesis
- *Coupling constants*
- *Lifetime*
PRODUCTION OF HEAVY ELEMENTS
- Supernova explosions
- *Nuclear physics in neutron rich stars*
Big Bang
10^{-44} sec
10^{-35} sec
10^{-10} sec
10^{-5} sec
100 sec
400,000 years
10 billion years
13.7 billion years

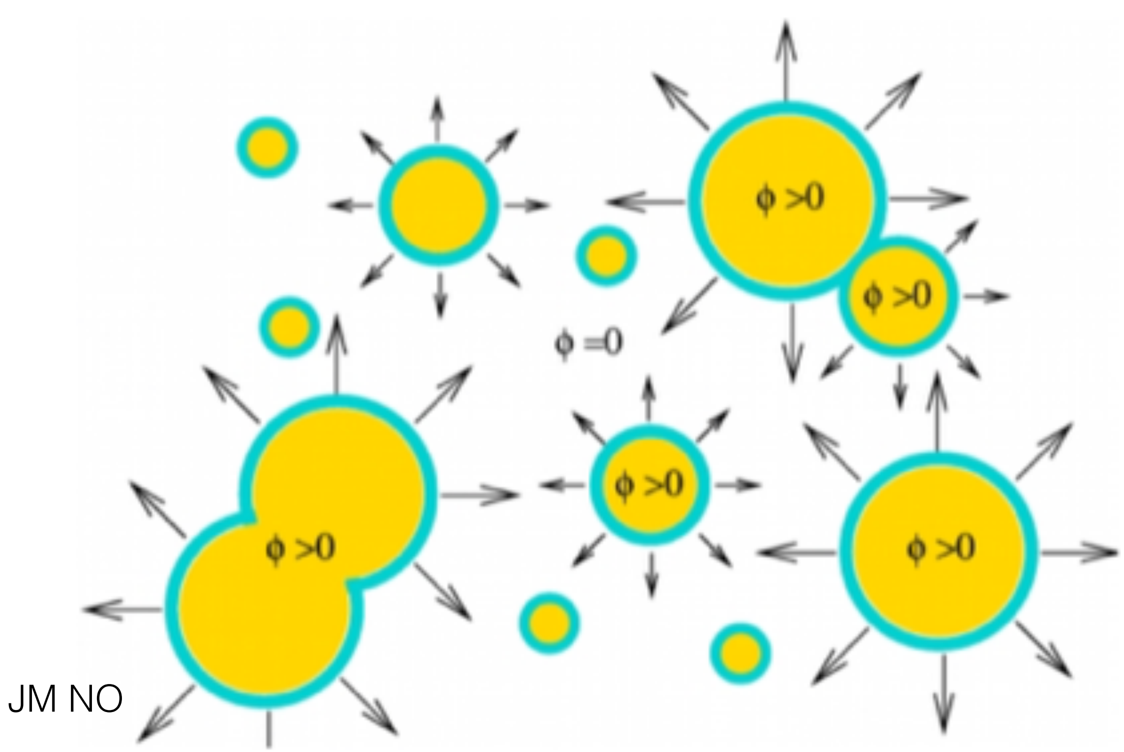
Tn~ 10^2 GeV

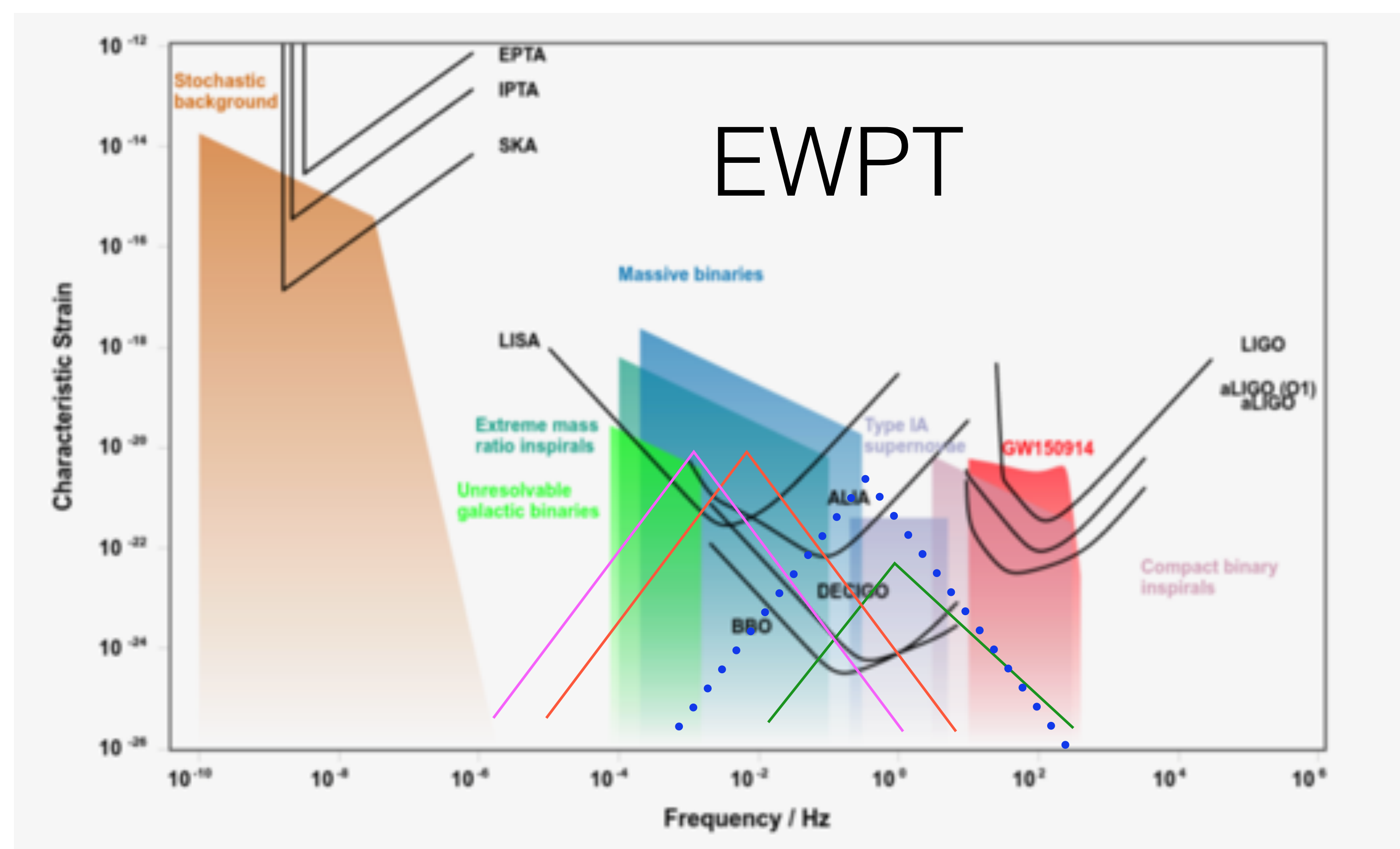
Why SFOEWPT

h



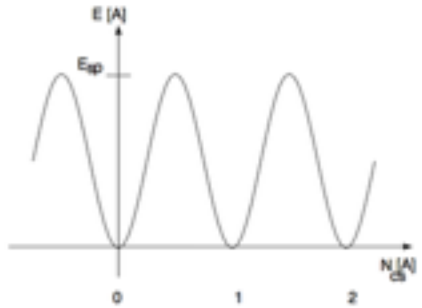






C J Moore et al. Class. Quantum Grav. 32 (2015) 015014.

BNPC, v/T and EW sphaleron



$$\partial_\mu J_B^\mu = i\frac{N_F}{32\pi^2}\left(-g_2^2 F^{a\mu\nu}\widetilde{F}^a_{\mu\nu} + g_1^2 f^{\mu\nu}\widetilde{f}_{\mu\nu}\right),$$

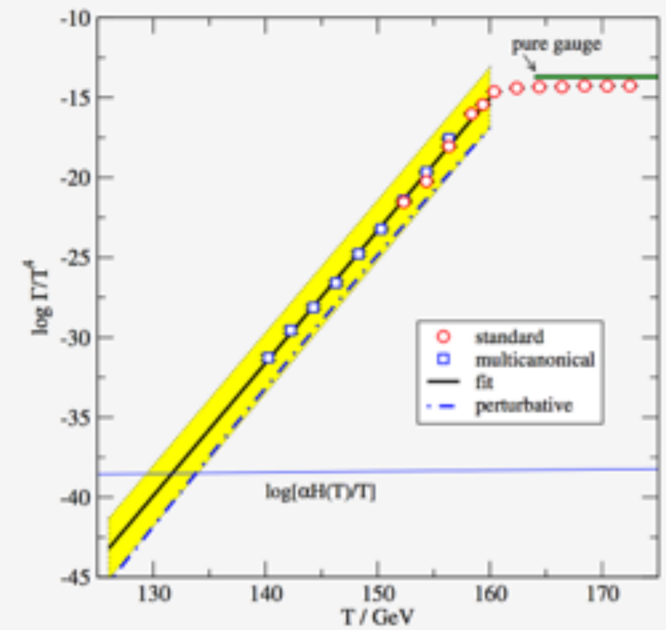
$$\Delta B = N_F(\Delta N_{\rm CS} - \Delta n_{\rm CS}),$$

$$N_{\rm CS} = -\frac{g_2^2}{16\pi^2}\int d^3x\, 2\epsilon^{ijk}\,{\rm Tr}\left[\partial_i A_j A_k + i\frac{2}{3}g_2 A_i A_j A_k\right],$$

$$n_{\rm CS} = -\frac{g_1^2}{16\pi^2}\int d^3x\, \epsilon^{ijk}\,\partial_i B_j B_k,$$

$$A_i \to U A_i U^{-1} + \frac{i}{g_2}(\partial_i U)U^{-1},$$

$$\delta N_{\rm CS} = \frac{1}{24\pi^2}\int d^3x\, {\rm Tr}\left[(\partial_i U)U^{-1}(\partial_j U)U^{-1}(\partial_k U)U^{-1}\right]\epsilon^{ijk}.$$



Lattice result, $\boxed{T_C = (159.5 \pm 1.5)\mathrm{GeV}}$, Phys.Rev.Lett,113, 141602 (2014

$$\boxed{\Gamma^{\rm sym} \approx 6\times(18\pm 3)\alpha_W^5 T^4, \qquad \Gamma^{\rm brok} \sim T^4\exp(-\frac{E_{\rm sph}}{T})}$$

Washout avoidance,BNPC

$$E_{{\rm sph},T} = \frac{4\pi v[T]}{g}\int_0^\infty d\xi\left[4\left(\frac{df}{d\xi}\right)^2 s_\mu^2 + \frac{8}{\xi^2}f^2(1-f)^2 s_\mu^4 + \frac{\xi^2}{2}\left(\frac{dh}{d\xi}\right)^2 s_\mu^2 + s_\mu^2((1-f)^2h^2 - 2fh(1-f)(1-h)c_\mu^2 + f^2(1-h)^2c_\mu^2) + \frac{\xi^2}{g^2 v[T]^4}(V_{eff}[\mu,h,T])\right]$$

$$\Gamma_{\rm sph} = A_{\rm sph}(T)\exp[-E_{\rm sph}(T)/T] < \mathrm{H(T)}$$

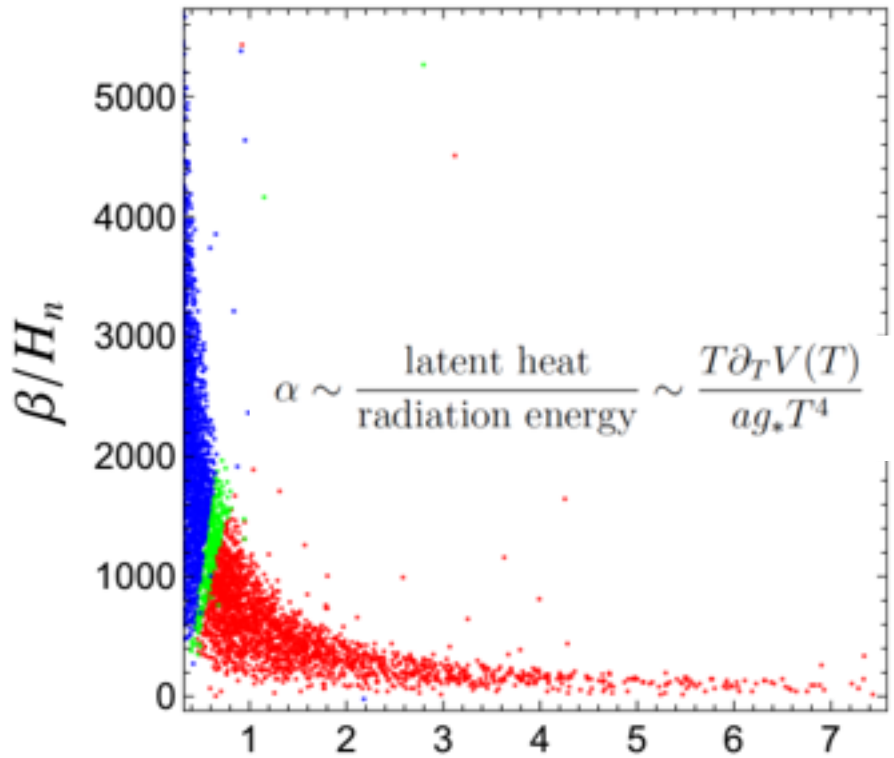
$$\frac{d^2f}{d\xi^2} = \frac{2}{\xi^2}f(1-f)(1-2f)s_\mu^2 - \frac{1}{8}(2h^2(1-f) - 2h(1-h)(1-2f)c_\mu^2 + 2f(1-h)^2c_\mu^2),$$

$$\frac{d}{d\xi}\left(\xi^2\frac{dh}{d\xi}\right) = 2h(1-f)^2 - 2f(1-f)(1-2h)c_\mu^2 - 2f^2(1-h)c_\mu^2 + \frac{\xi^2}{g_2^2}\frac{1}{v[T]^4}\frac{\partial V_{\rm eff}}{\partial h},$$

$$PT_{sph} \equiv \frac{E_{\rm sph}(T)}{T} - 7\ln\frac{v(T)}{T} + \ln\frac{T}{100\ {\rm GeV}} \qquad PT_{sph} > (35.9 - 42.8)$$

$$E_{\rm sph}(T) \approx E_{\rm sph,0}\frac{v(T)}{v} \qquad \frac{v(T)}{T} > (0.973 - 1.16)\left(\frac{E_{\rm sph,0}}{1.916\times 4\pi v/g}\right)^{-1}$$

GWs and EWPT



$$\frac{\beta}{H_*} = T_* \frac{d}{dT}\left(\frac{S_3}{T}\right)\Big|_{T_*}$$

β reflect the duration of the phase transition

$$v_b \simeq \frac{1/\sqrt{3} + \sqrt{\alpha^2 + 2\alpha/3}}{1+\alpha}\,, \qquad \kappa \simeq \frac{0.715\alpha + \frac{4}{27}\sqrt{3\alpha/2}}{1+0.715\alpha}$$

$$\Omega_{\rm col}h^2 = 1.67\times 10^{-5}\left(\frac{H_*}{\beta}\right)^2\left(\frac{\kappa\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{1/3}\left(\frac{0.11 v_b^3}{0.42+v_b^2}\right)\frac{3.8(f/f_{\rm env})^{2.8}}{1+2.8(f/f_{\rm env})^{3.8}}$$

envelop approximation

$$f_{\rm env} = 16.5\times 10^{-6}\left(\frac{f_*}{H_*}\right)\left(\frac{T_*}{100{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{1/6}{\rm Hz}$$

$$\Omega_{\rm sw}h^2 = 2.65\times 10^{-6}\left(\frac{H_*}{\beta}\right)\left(\frac{\kappa_v\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{1/3} v_b\left(\frac{f}{f_{\rm sw}}\right)^3\left(\frac{7}{4+3(f/f_{\rm sw})^2}\right)^{7/2} \quad (5.6)$$

$$\Omega_{\rm turb}h^2 = 3.35\times 10^{-4}\left(\frac{H_*}{\beta}\right)\left(\frac{\kappa_{\rm turb}\alpha}{1+\alpha}\right)^{3/2}\left(\frac{100}{g_*}\right)^{1/3} v_b\frac{(f/f_{\rm turb})^3}{[1+(f/f_{\rm turb})]^{11/3}(1+8\pi f/h_*)} \quad (5.7)$$

$\kappa_v \approx \alpha(0.73 + 0.083\sqrt{\alpha} + \alpha)^{-1}$ and $\kappa_{\rm turb} \approx 0.1\kappa_v$

$$f_{\rm sw} = 1.9\times 10^{-5}\frac{1}{v_b}\left(\frac{\beta}{H_*}\right)\left(\frac{T_*}{100{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{1/6}{\rm Hz}\,,$$

$$f_{\rm turb} = 2.7\times 10^{-5}\frac{1}{v_b}\left(\frac{\beta}{H_*}\right)\left(\frac{T_*}{100{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{1/6}{\rm Hz}$$

kv, kturb: the fraction of latent heat transformed into the bulk motion of the fluid for sound waves and MHD

GW sources

$$\Omega_{\rm GW}(f) = \begin{cases} \Omega_{\rm GW*}\left(\frac{f}{f_*}\right)^{n_{\rm GW1}} & \text{for } f < f_*, \\ \Omega_{\rm GW*}\left(\frac{f}{f_*}\right)^{n_{\rm GW2}} & \text{for } f > f_*, \end{cases}$$

1807.00786

Table 1. Cosmological GW sources

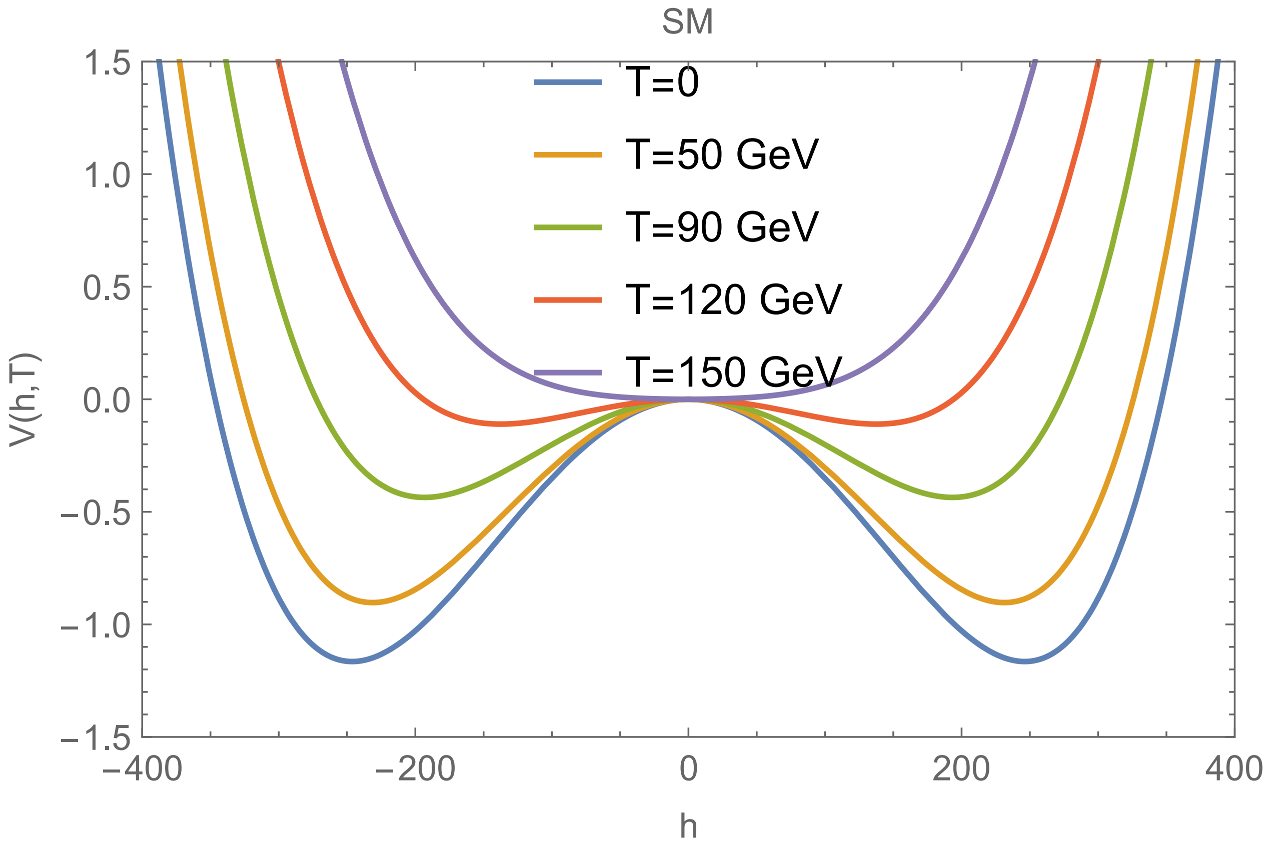
source	$n_{\rm GW1}$	$n_{\rm GW2}$	f_* [Hz]	$\Omega_{\rm GW}$
Phase transition (bubble collision)	2.8	-2	$\sim 10^{-5}\left(\frac{f_{\rm PT}}{\beta}\right)\left(\frac{\beta}{H_{\rm PT}}\right)\left(\frac{T_{\rm PT}}{100\ {\rm GeV}}\right)$	$\sim 10^{-5}\left(\frac{H_{\rm PT}}{\beta}\right)^2\left(\frac{\kappa_\phi\alpha}{1+\alpha}\right)^2\left(\frac{0.11v_w^3}{0.42+v_w^2}\right)$
Phase transition (turbulence)	3	$-5/3$	$\sim 3\times 10^{-5}\left(\frac{1}{v_w}\right)\left(\frac{\beta}{H_{\rm PT}}\right)\left(\frac{T_{\rm PT}}{100\ {\rm GeV}}\right)$	$\sim 3\times 10^{-4}\left(\frac{H_{\rm PT}}{\beta}\right)\left(\frac{\kappa_{\rm turb}\alpha}{1+\alpha}\right)^{3/2} v_w$
Phase transition (sound waves)	3	-4	$\sim 2\times 10^{-5}\left(\frac{1}{v_w}\right)\left(\frac{\beta}{H_{\rm PT}}\right)\left(\frac{T_{\rm PT}}{100\ {\rm GeV}}\right)$	$\sim 3\times 10^{-6}\left(\frac{H_{\rm PT}}{\beta}\right)\left(\frac{\kappa_v\alpha}{1+\alpha}\right)^2 v_w$
Preheating ($\lambda\phi^4$)	3	cutoff	$\sim 10^7$	$\sim 10^{-11}\left(\frac{g^2/\lambda}{100}\right)^{-0.5}$
Preheating (hybrid)	2	cutoff	$\sim \frac{g}{\sqrt{\lambda}}\lambda^{1/4}10^{10.25}$	$\sim 10^{-5}\left(\frac{\lambda}{g^2}\right)^{1.16}\left(\frac{v}{M_{\rm pl}}\right)^2$
Cosmic strings (loops 1)	$[1,2]$	$[-1,-0.1]$	$\sim 3\times 10^{-8}\left(\frac{G\mu}{10^{-11}}\right)^{-1}$	$\sim 10^{-9}\left(\frac{G\mu}{10^{-12}}\right)\left(\frac{\alpha_{\rm loop}}{10^{-1}}\right)^{-1/2}$ (for $\alpha_{\rm loop} \gg \Gamma G\mu$)
Cosmic strings (loops 2)	$[-1,-0.1]$	0	$\sim 3\times 10^{-8}\left(\frac{G\mu}{10^{-11}}\right)^{-1}$	$\sim 10^{-9.5}\left(\frac{G\mu}{10^{-12}}\right)\left(\frac{\alpha_{\rm loop}}{10^{-1}}\right)^{-1/2}$ (for $\alpha_{\rm loop} \gg \Gamma G\mu$)
Cosmic strings (infinite strings)	$[0, 0.2]$	$[0, 0.2]$	—	$\sim 10^{-[11,13]}\left(\frac{G\mu}{10^{-8}}\right)$
Domain walls	3	-1	$\sim 10^{-9}\left(\frac{T_{\rm ann}}{10^{-2}{\rm GeV}}\right)$	$\sim 10^{-17}\left(\frac{\sigma}{1{\rm TeV}^3}\right)^2\left(\frac{T_{\rm ann}}{10^{-2}{\rm GeV}}\right)^{-4}$
Self-ordering scalar fields	0	0	—	$\sim \frac{511}{N}\Omega_{\rm rad}\left(\frac{v}{M_{\rm pl}}\right)^4$
Self-ordering scalar + reheating	0	-2	$\sim 0.4\left(\frac{T_R}{10^7\ {\rm GeV}}\right)$	$\sim \frac{511}{N}\Omega_{\rm rad}\left(\frac{v}{M_{\rm pl}}\right)^4$
Magnetic fields	3	α_B+1	$\sim 10^{-6}\left(\frac{T_*}{10^2{\rm GeV}}\right)$	$\sim 10^{-16}\left(\frac{B}{10^{-10}{\rm G}}\right)$
Inflation+reheating	~ 0	-2	$\sim 0.3\left(\frac{T_R}{10^7\ {\rm GeV}}\right)$	$\sim 2\times 10^{-17}\left(\frac{r}{0.01}\right)$
Inflation+kination	~ 0	1	$\sim 0.3\left(\frac{T_R}{10^7\ {\rm GeV}}\right)$	$\sim 2\times 10^{-17}\left(\frac{r}{0.01}\right)$
Particle prod. during inf.	-2ϵ	$-4\epsilon(4\pi\xi-6)(\epsilon-\eta)$	—	$\sim 2\times 10^{-17}\left(\frac{r}{0.01}\right)$
2nd-order (inflation)	1	drop-off	$\sim 7\times 10^5\left(\frac{T_{\rm reh}}{10^9\ {\rm GeV}}\right)^{1/3}\left(\frac{M_{\rm inf}}{10^{16}\ {\rm GeV}}\right)^{2/3}$	$\sim 10^{-12}\left(\frac{T_{\rm reh}}{10^9\ {\rm GeV}}\right)^{-4/3}\left(\frac{M_{\rm inf}}{10^{16}\ {\rm GeV}}\right)^{4/3}$
2nd-order (PBHs)	2	drop-off	$\sim 4\times 10^{-2}\left(\frac{M_{\rm PBH}}{10^{20}\ {\rm g}}\right)^{-1/2}$	$\sim 7\times 10^{-9}\left(\frac{\mathcal{A}^2}{10^{-3}}\right)^2$
Pre-Big-Bang	3	$3-2\mu$	—	$\sim 1.4\times 10^{-6}\left(\frac{H_s}{0.15M_{\rm pl}}\right)^4$

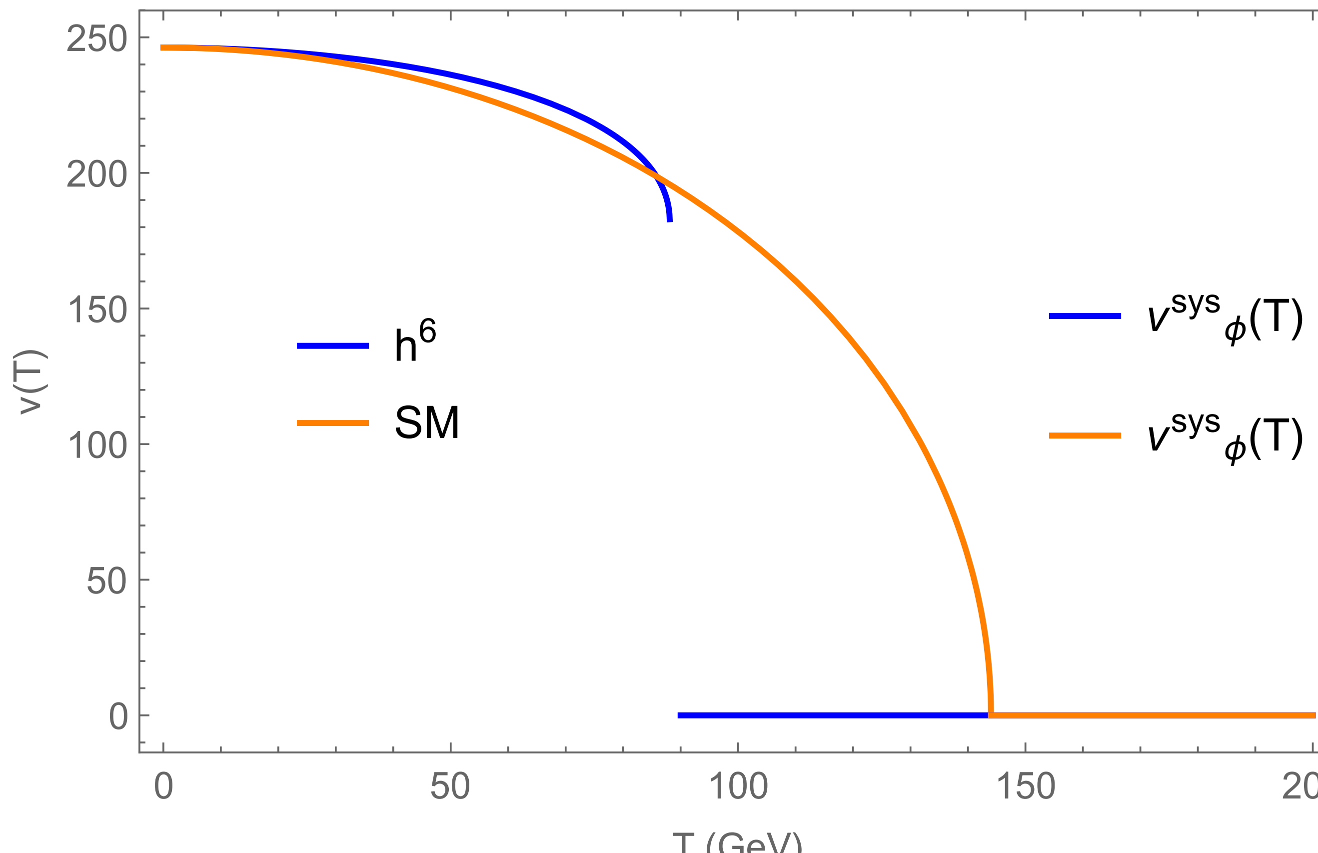
BSM for EWPT

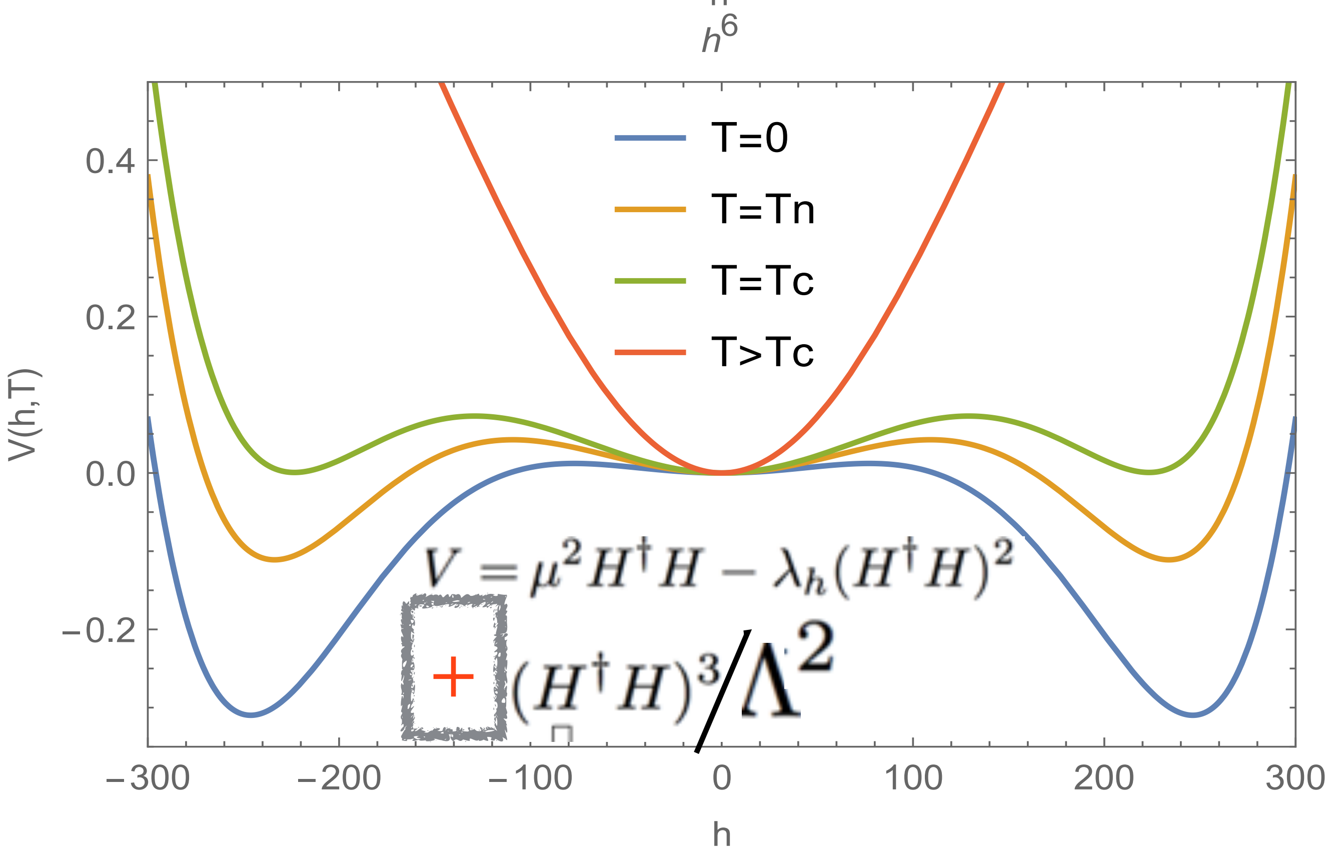
SM+Scalar Singlet	Espinosa, Quiros 93, Benson 93, Choi, Volkas 93, Vergara 96, Branco, Delepine, Emmanuel- Costa, Gonzalez 98, Ham, Jeong, Oh 04, Ahriche 07, Espinosa, Quiros 07, Profumo, Ramsey-Musolf, Shaughnessy 07, Noble, Perelstein 07, Espinosa, Konstandin, No, Quiros 08, Barger, Langacker, McCaskey, Ramsey-Musolf, Shaughnessy 09, Ashoorioon, Konstandin 09, Das, Fox, Kumar, Weiner 09, Espinosa, Konstandin, Riva 11, Chung, Long 11, Barger, Chung, Long, Wang 12, Huang, Shu, Zhang 12, Fairbairn, Hogan 13, Katz, Perelstein 14, Profumo, Ramsey-Musolf, Wainwright, Winslow 14, Jiang, Bian, Huang, Shu 15, Kozaczuk 15, Cline, Kainulainen, Tucker-Smith 17, Kurup, Perelstein 17, Chen, Kozaczuk, Lewis 17,Cheng, Bian 17, Bian, Tang 18,…
SM+Scalar Doublet	Turok, Zadrozny 92, Davies, Froggatt, Jenkins, Moorhouse 94, Cline, Lemieux 97, Huber 06, Froome, Huber, Seniuch 06, Cline, Kainulainen, Trott 11, Dorsch, Huber, No 13, Dorsch, Huber, Mimasu, No 14, Basler, Krause, Muhlleitner, Wittbrodt, Wlotzka 16, Dorsch, Huber, Mimasu, No 17, Bernon, Bian, Jiang 17, Bian, Liu 18,…
SM + Scalar Triplet	Profumo, Ramsey-Musolf 12, Chiang 14, Zhou, Cheng, Deng, Bian, Wu 18,…
NMSSM	Pietroni 93, Davies, Froggatt, Moorhouse 95, Huber, Schmidt 01, Ham, Oh, Kim, Yoo, Son 04, Menon, Morrissey, Wagner 04, Funakubo, Tao, Yokoda 05, Huber, Konstandin, Prokopec, Schmidt 07, Chung, Long 10, Kozaczuk, Profumo, Stephenson Haskins, Wainwright 15, Bi, Bian, Huang, Shu, Yin 15, Bian, Guo, Shu 17,…
Composite Higgs	Espinosa, Gripaios, Konstandin, Riva 11, Bruggisser, Von Harling, Matsedonskyi, Servant 18, Bian,Wu,Xie 19, De Curtis, Delle Rose, Panico 19
EFT Approach (h^6 and ??)	Grojean, Servant, Wells 05, Bodeker, Froome, Huber, Seniuch 05, Huang, Joglekar, Li, Wagner 15,Cai,Sasaki ,Wang17,Zhou, Bian, Guo 19, …

Higgs Potential Shape??? EFT or ???

First or second order



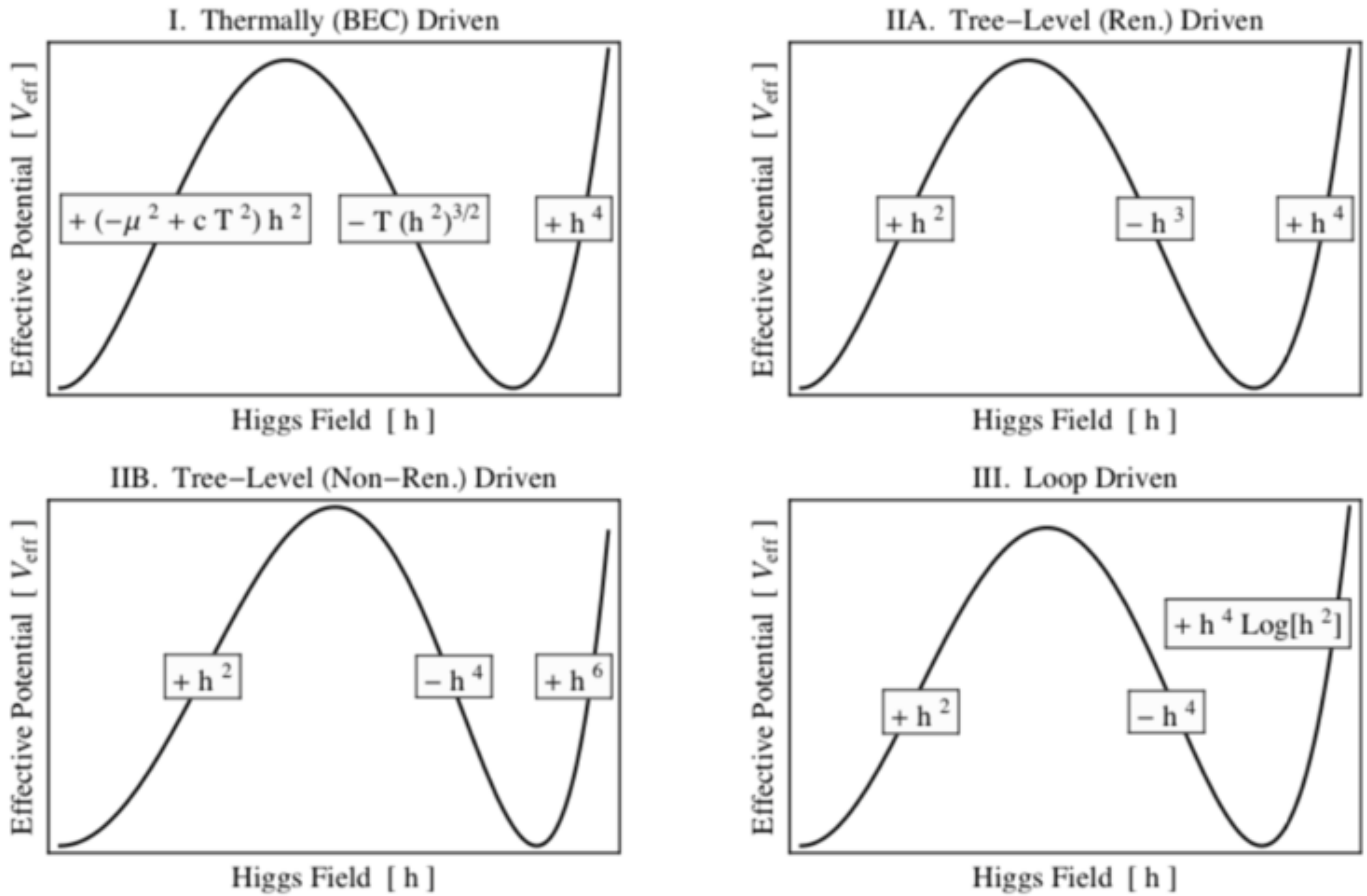




Grojean, Servant, Wells 05, P. Huang, Jokelar, Li, Wagner (2015)
F.P. Huang, Gu, Yin, Yu, Zhang (2015) F.P. Huang, Wan, Wang, Cai, Zhang (2016) Cao, F.P. Huang, Xie, & Zhang (2017)

LHC say the quantum fluctuation (quadratic oscillation) around h=v with mh=126 GeV, not sensitive to the specifically potential shape

Model classes for catalyzing a strongly first order electroweak phase transition



Daniel J H. Chung, Andrew J. Long, and Lian-Tao Wang PRD87, 023509 (2013)

Dim. six operator, SMEFT

Higgs potential $$V(H) = -m^2(H^\dagger H) + \lambda(H^\dagger H)^2 + \frac{(H^\dagger H)^3}{\Lambda^2}$$

Finite temperature potential $$V_T(h,T) = V(h) + \frac{1}{2}c_{hT}h^2$$

Thermal correction $$c_{hT} = (4y_t^2 + 3g^2 + g'^2 + 8\lambda)T^2/16$$

Electroweak minimum being the global one $$\Lambda \geq v^2/m_h$$

Potential barrier requirement $$\Lambda < \sqrt{3}v^2/m_h$$

EWPT in the SM + Real Singlet: ‘xSM’ model

For the “xSM” model, the gauge invariant finite temperature effective potential is found to be:

$$V(h,s,T) = -\frac{1}{2}[\mu^2 - \Pi_h(T)]h^2 - \frac{1}{2}[-b_2 - \Pi_s(T)]s^2 + \frac{1}{4}\lambda h^4 + \frac{1}{4}a_1 h^2 s + \frac{1}{4}a_2 h^2 s^2 + \frac{b_3}{3}s^3 + \frac{b_4}{4}s^4, \quad \text{(C1)}$$

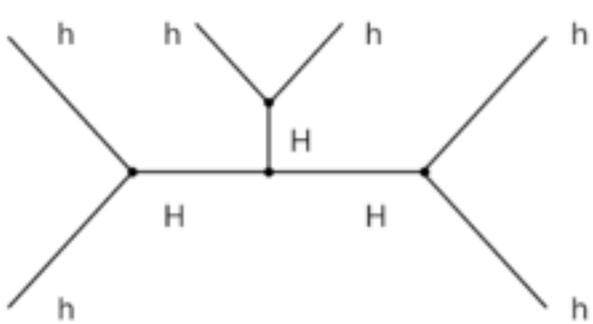
with the thermal masses given by

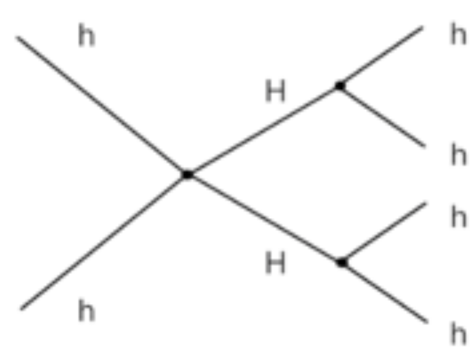
$$\Pi_h(T) = \left(\frac{2m_W^2 + m_Z^2 + 2m_t^2}{4v^2} + \frac{\lambda}{2} + \frac{a_2}{24}\right)T^2,$$
$$\Pi_s(T) = \left(\frac{a_2}{6} + \frac{b_4}{4}\right)T^2, \quad \text{(C2)}$$

$$v^{\mathrm{xSM}}/T \equiv \frac{v_h(T)}{T} = \frac{\sqrt{v_h^2(T) + v_s^2(T)}\cos\theta(T)}{T},$$
$$\cos\theta(T) \equiv \frac{v_h(T)}{\sqrt{v_h^2(T) + v_s^2(T)}},$$

For small mixing limit between the extra Higgs and the SM Higgs, one have

$$c_4^{xSM} = -\frac{a_1^2 - 8b_2\lambda}{32b_2} + \frac{\theta^2(a_1^2(6b_2 - \mu^2) - 8a_1b_2b_3 + 8b_2^2(a_2 - 2\lambda))}{32b_2^2} + O(\theta^3)$$
$$c_6^{xSM} = -\frac{a_1^2(a_1b_3 - 3a_2b_2)}{192b_2^3} - \frac{\theta^2 a_1}{256b_2^4}(a_1^3b_2 + 4a_1^2b_3(\mu^2 - 3b_2)$$
$$+4a_1b_2(a2(11b_2 - 2\mu^2) - 6b_2(b4 + \lambda) + 4b_3^2) - 32a_2b_2^2b_3) + O(\theta^3)$$
$$c_8^{xSM} = \frac{a_1^4b_4}{1024b_2^4} + \frac{a_1^3\theta^2}{1024b_2^5}(a_1(a_2b_2 + 4b_4(\mu^2 - 3b_2)) + 16b_2b_3b_4) + O(\theta^3)$$





Phase transition strength and the BNPC

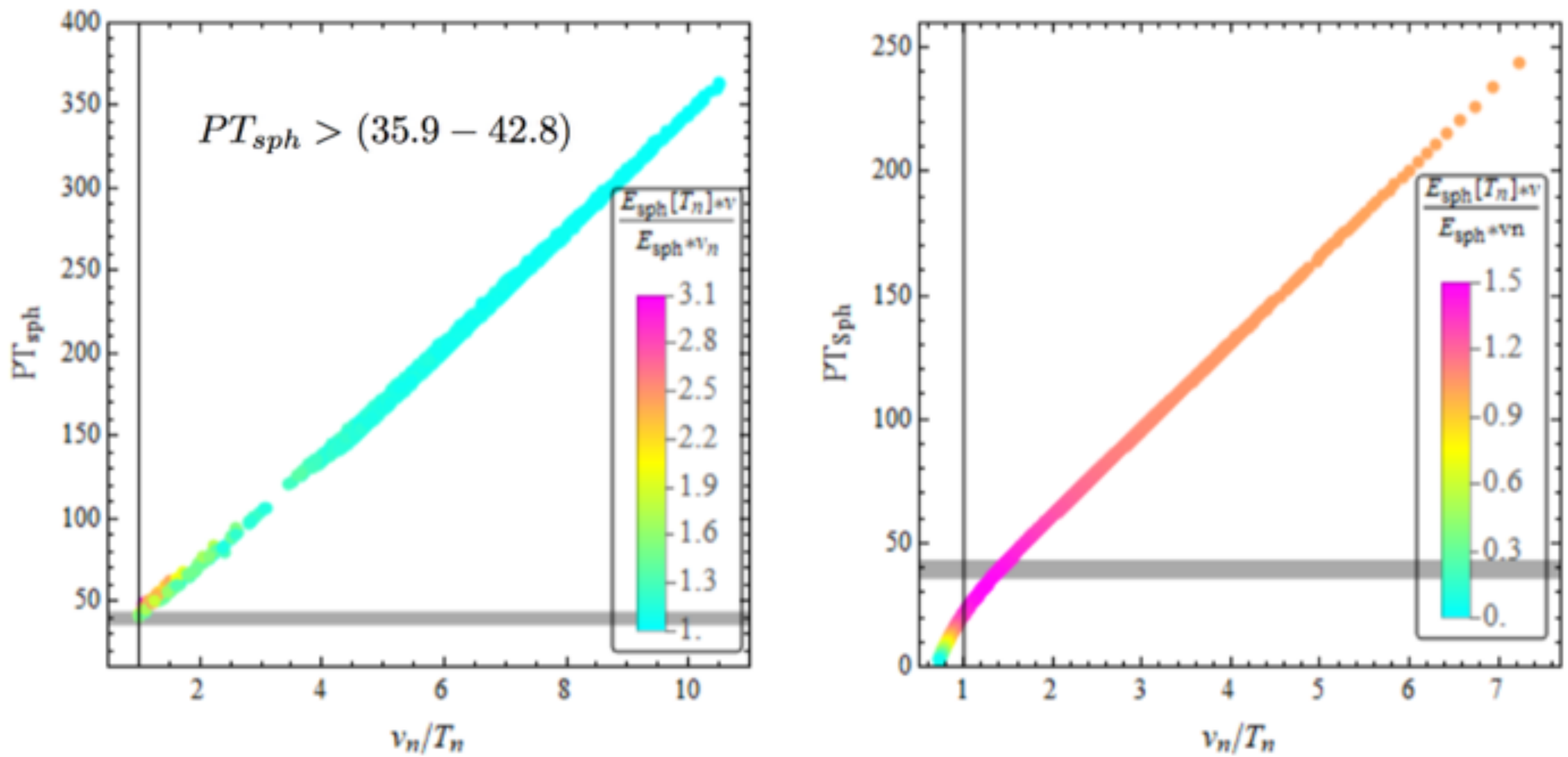
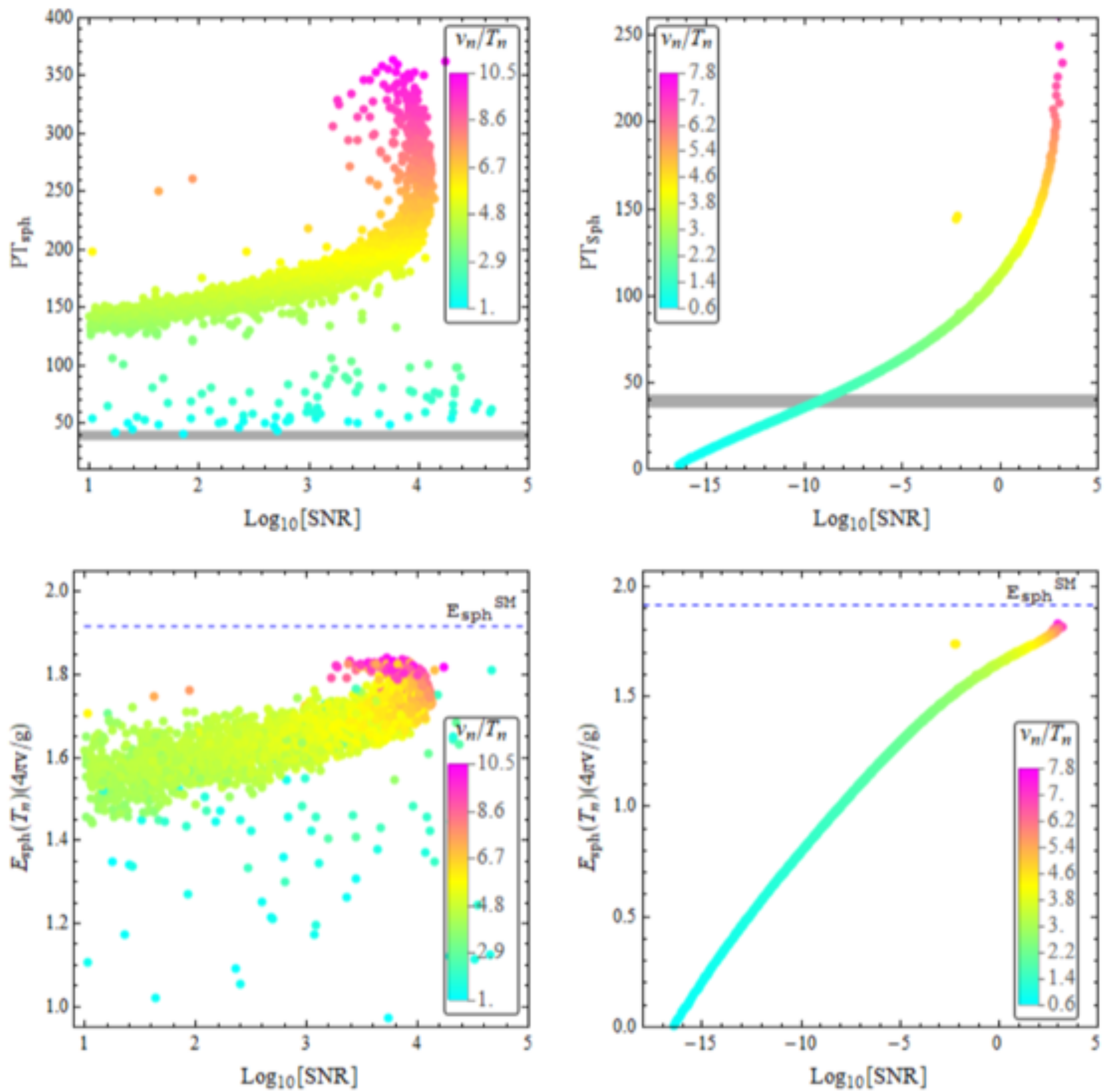


FIG. 1: PT_{sph} versus v_n/T_n for SMEFT (left) and xSM (right) by by color-coding $E_{sph}(T_n)v/(E_{sph}v_n)$.

Search for sphaleron with GW

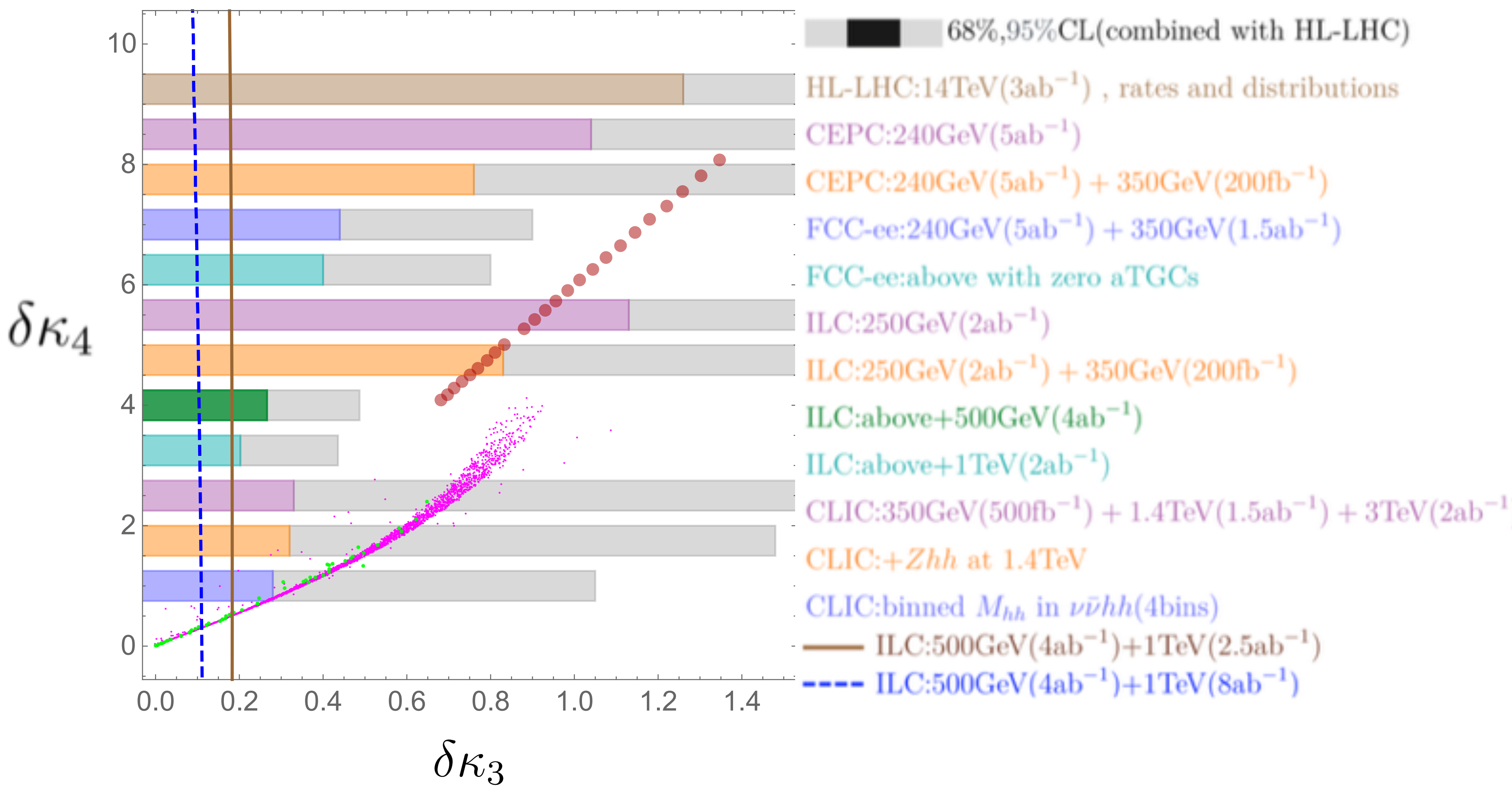


Gravitational waves can be searched for by cross-correlating outputs from two or more detectors, with the resulting signal-to- noise ratio(SNR)

$$\mathrm{SNR} = \sqrt{\mathcal{T} \int df \left[\frac{h^2 \Omega_{\mathrm{GW}}(f)}{h^2 \Omega_{\mathrm{exp}}(f)} \right]^2}$$

where T is the duration of the data in years and Ω_{exp} the power spectral density of the detector.

Triple and quartic Higgs coupling deviation, GW



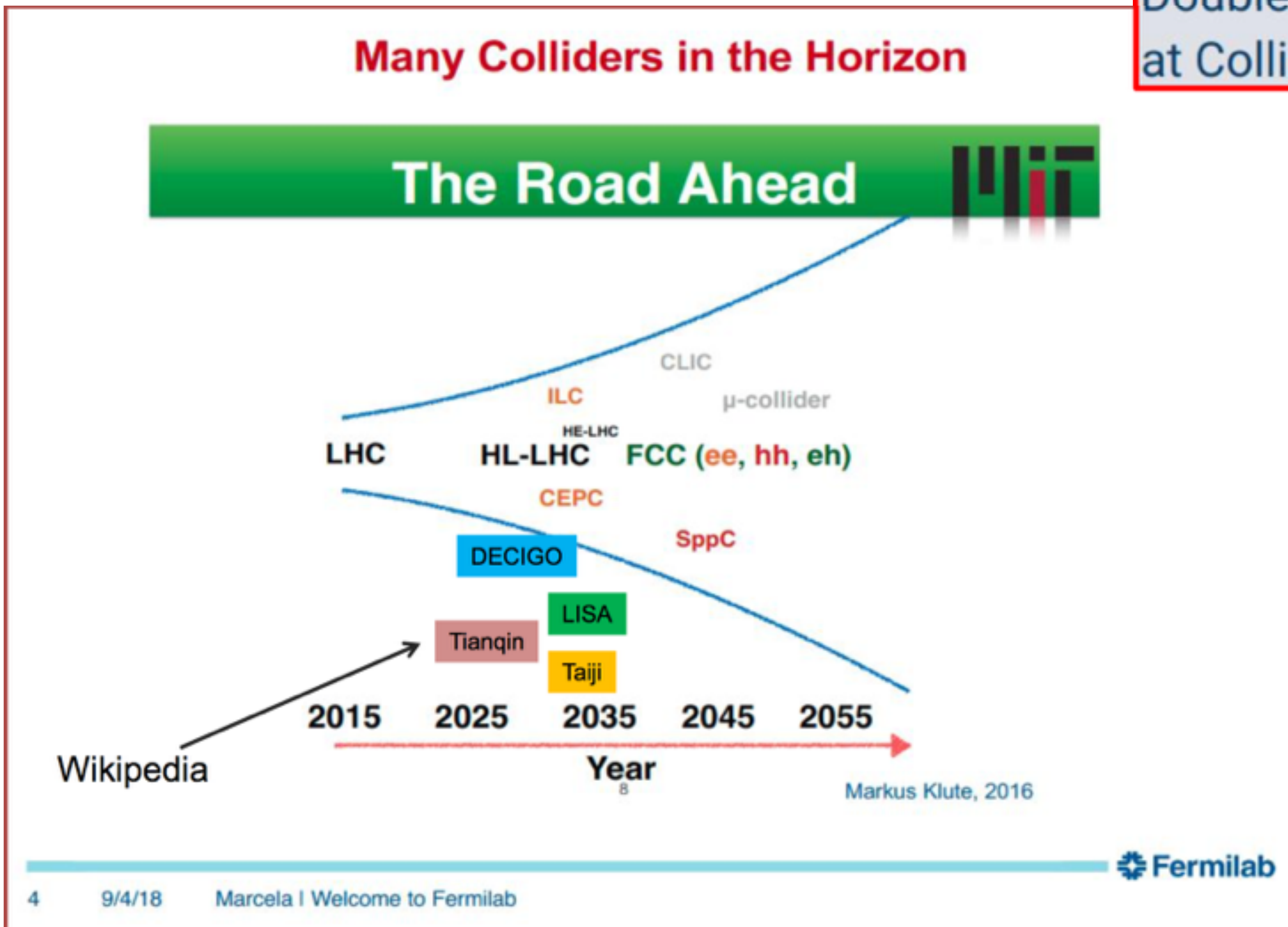
$$\Delta\mathcal{L} = -\frac{1}{2}\frac{m_h^2}{v}(1+\delta\kappa_3)h^3 - \frac{1}{8}\frac{m_h^2}{v^2}(1+\delta\kappa_4)h^4$$

$$\delta\kappa_3^{h^6} = \frac{2v^4}{\Lambda^2 m_h^2}\,,\ \delta\kappa_4^{h^6} = \frac{12v^4}{\Lambda^2 m_h^2}$$

$$\delta\kappa_3^{\rm xSM} = \alpha^2\left[-\frac{3}{2} + \frac{2m_H^2 - 2b_3 v_s - 4b_4 v_s^2}{m_h^2}\right]$$

$$\delta\kappa_4^{\rm xSM} = \alpha^2\left[-3 + \frac{5m_H^2 - 4b_3 v_s - 8b_4 v_s^2}{m_h^2}\right]$$

Double Higgs Production at Colliders Workshop



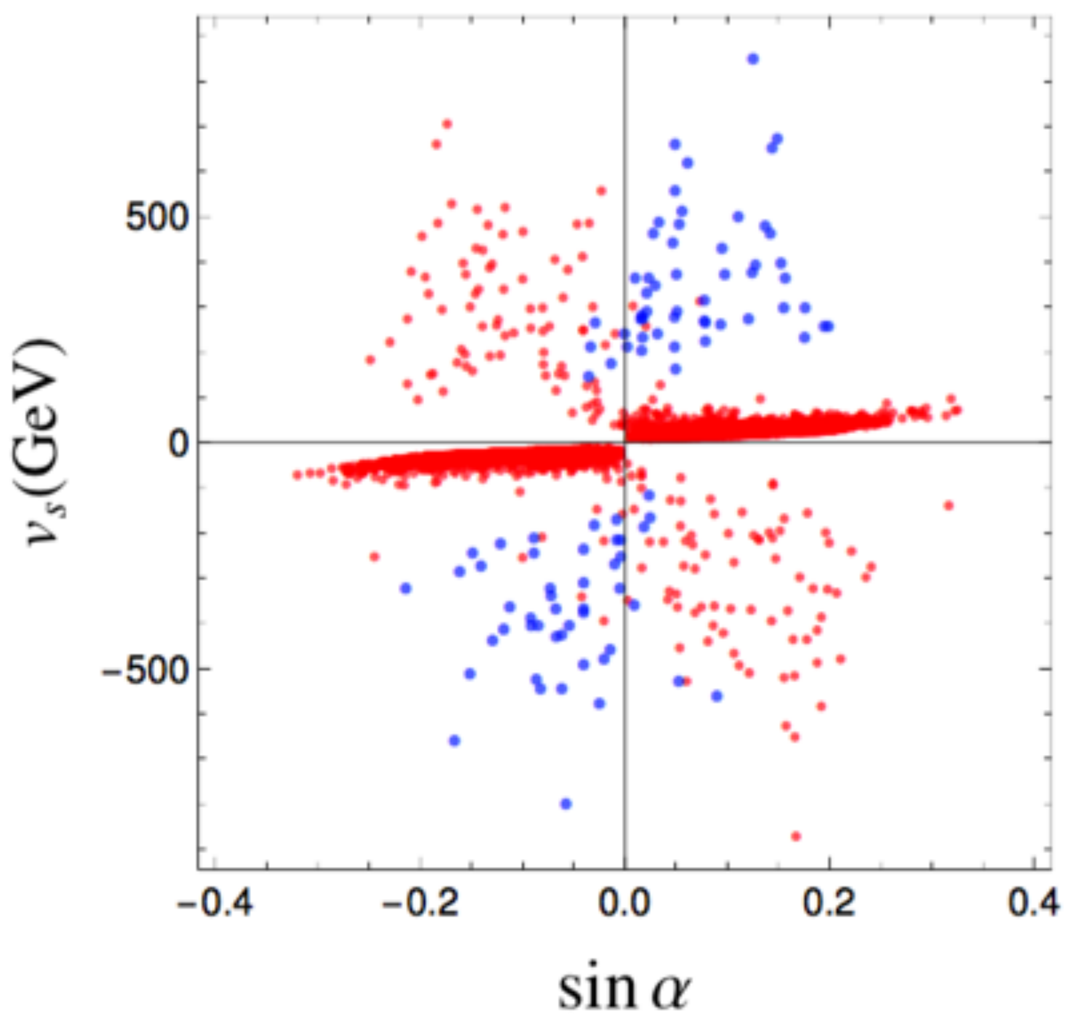
Future ....

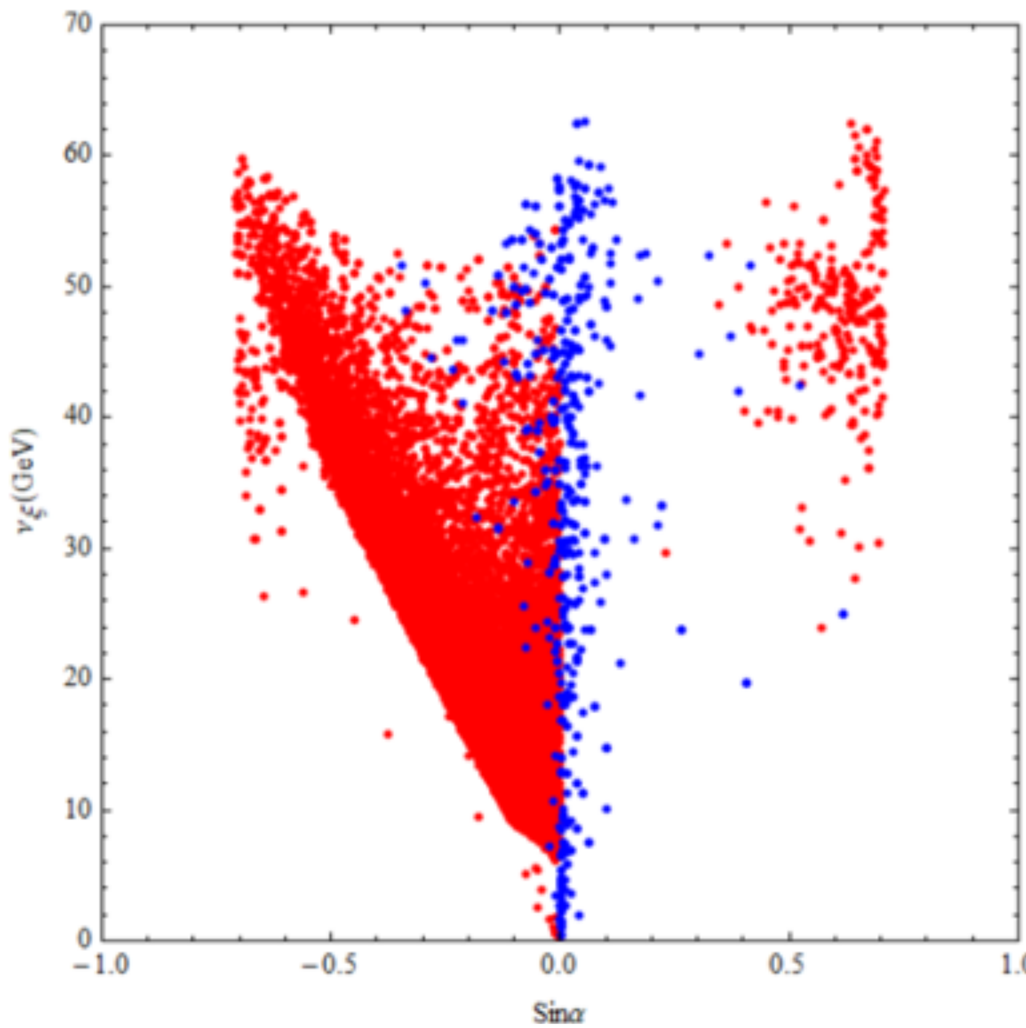
- Nonperturbative evaluation of EWPT and GW

- Sphaleron rate simulation and collider search for B+L Violation process

谢谢！

xSM: without extra EWSB GM: with extra EWSB



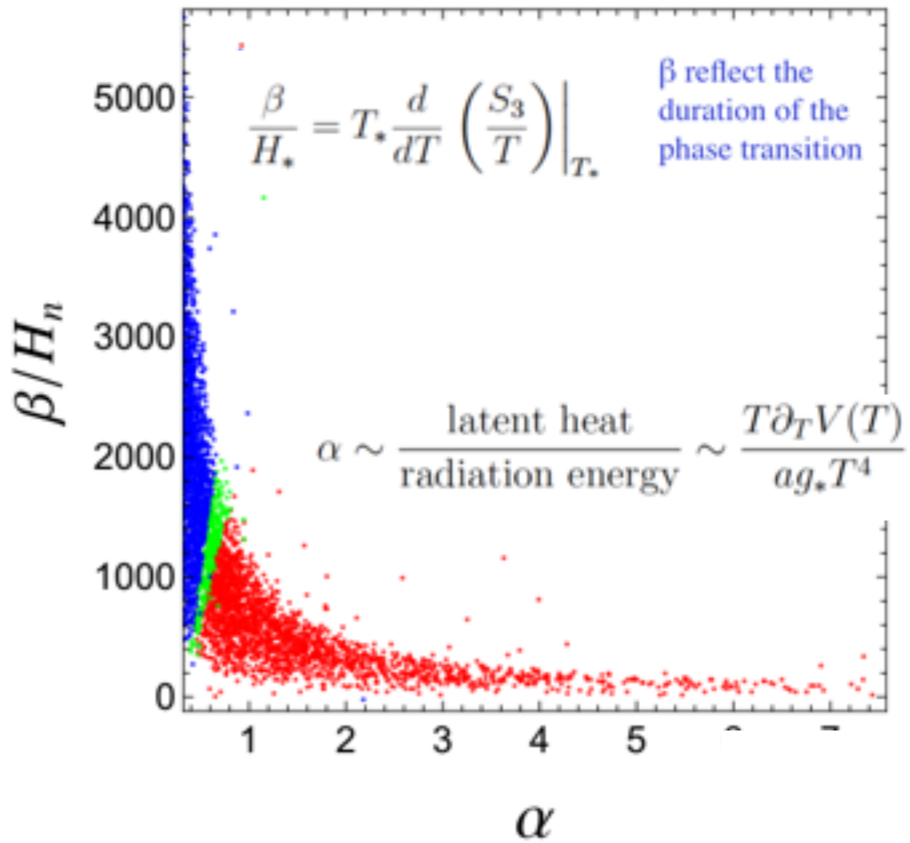


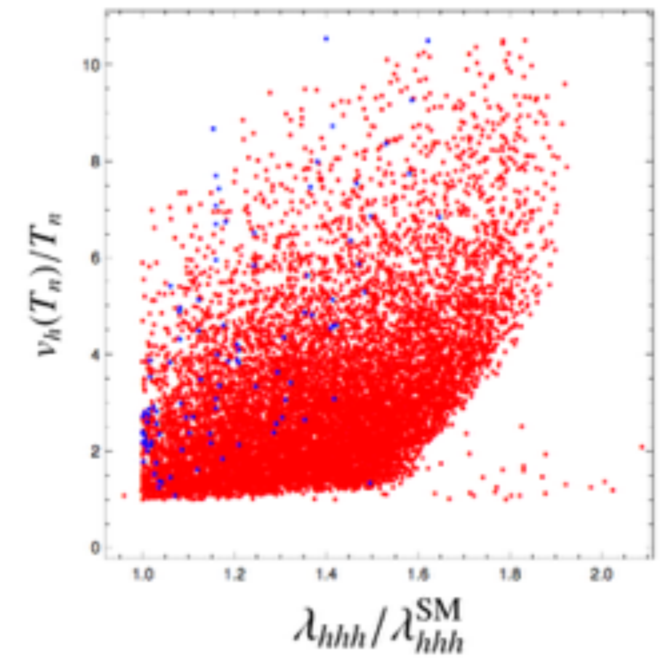
$$g_{hxx} = \cos\alpha\, g_{hxx}^{SM}$$

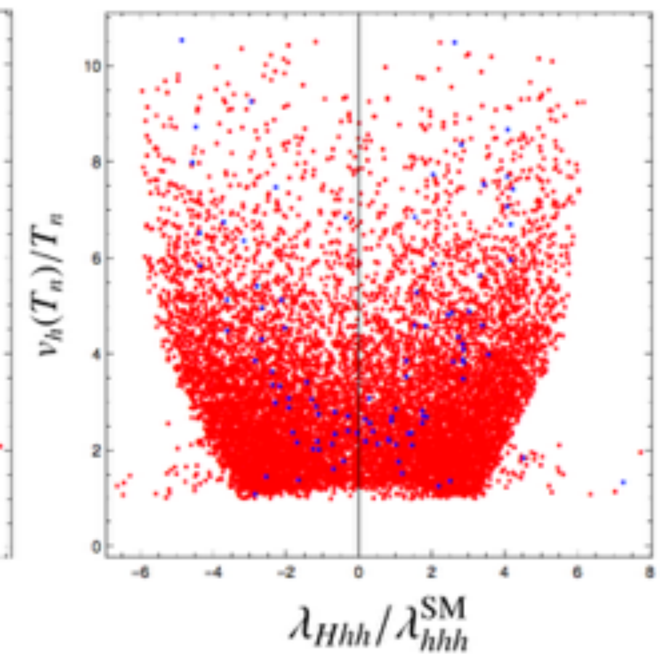
$$g_{hf\bar{f}} = \cos\alpha / \cos\theta_H g_{hf\bar{f}}^{SM},\ g_{hVV} = (\cos\alpha\cos\theta_H - \sqrt{\frac{8}{3}}\sin\alpha\sin\theta_H) g_{hf\bar{f}}^{SM},$$

$$g_{Hf\bar{f}} = \sin\alpha / \cos\theta_H g_{hf\bar{f}}^{SM},\ g_{HVV} = (\sin\alpha\cos\theta_H + \sqrt{\frac{8}{3}}\cos\alpha\sin\theta_H) g_{hVV}^{SM}$$

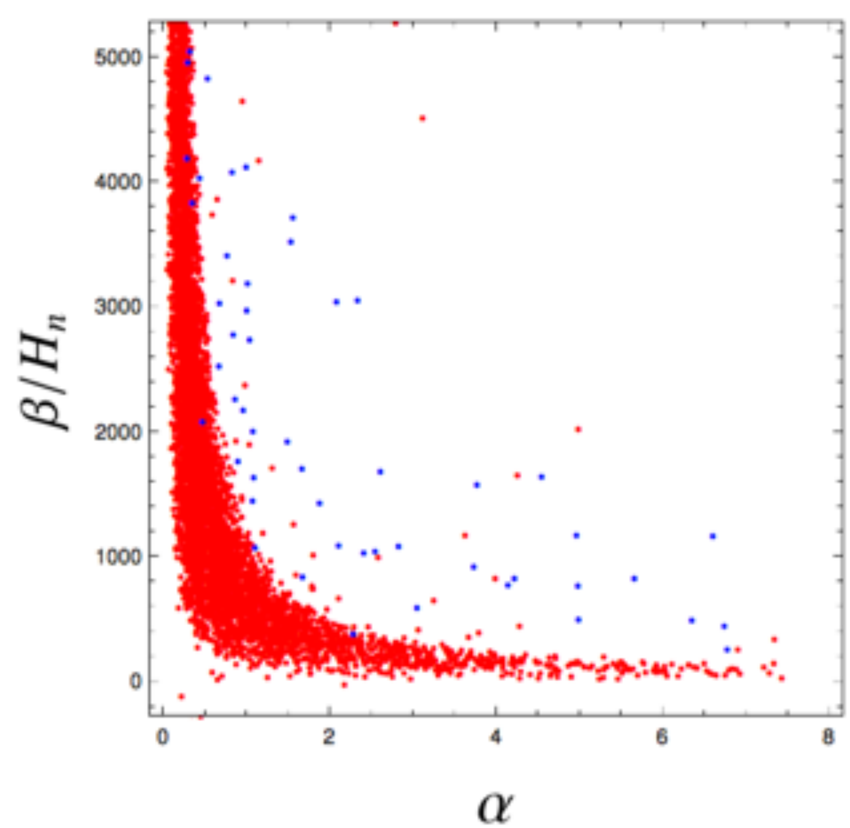
GW

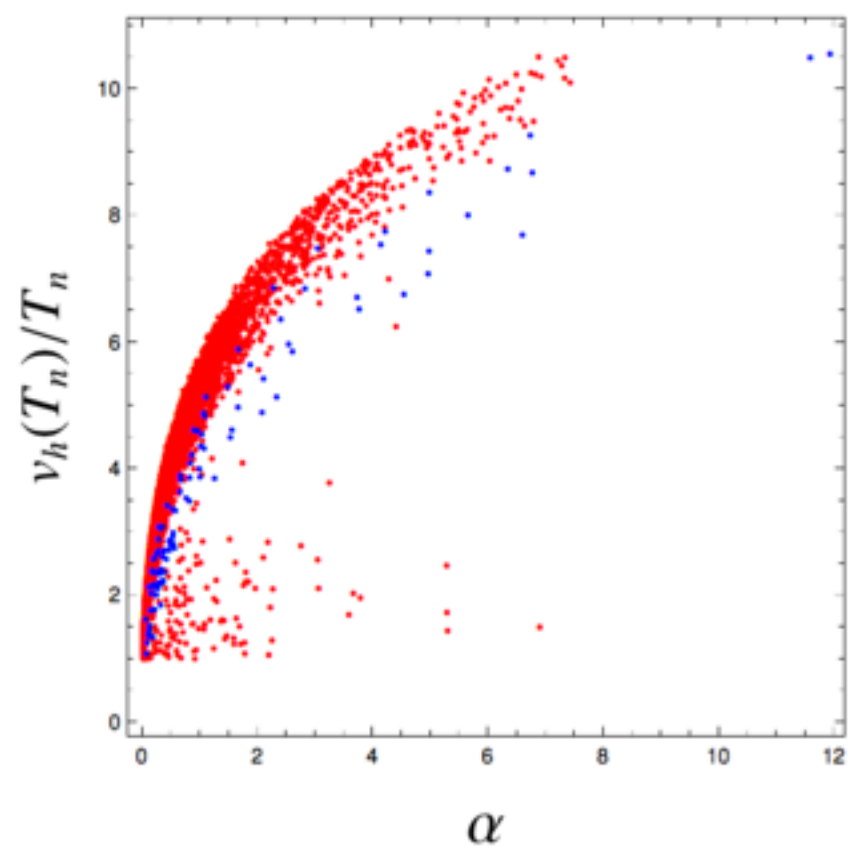

$\frac{\beta}{H_*} = T_* \frac{d}{dT}\left(\frac{S_3}{T}\right)\Big|_{T_*}$
β reflect the duration of the phase transition
$\alpha \sim \frac{\text{latent heat}}{\text{radiation energy}} \sim \frac{T\partial_T V(T)}{a g_* T^4}$
β/H_n
α

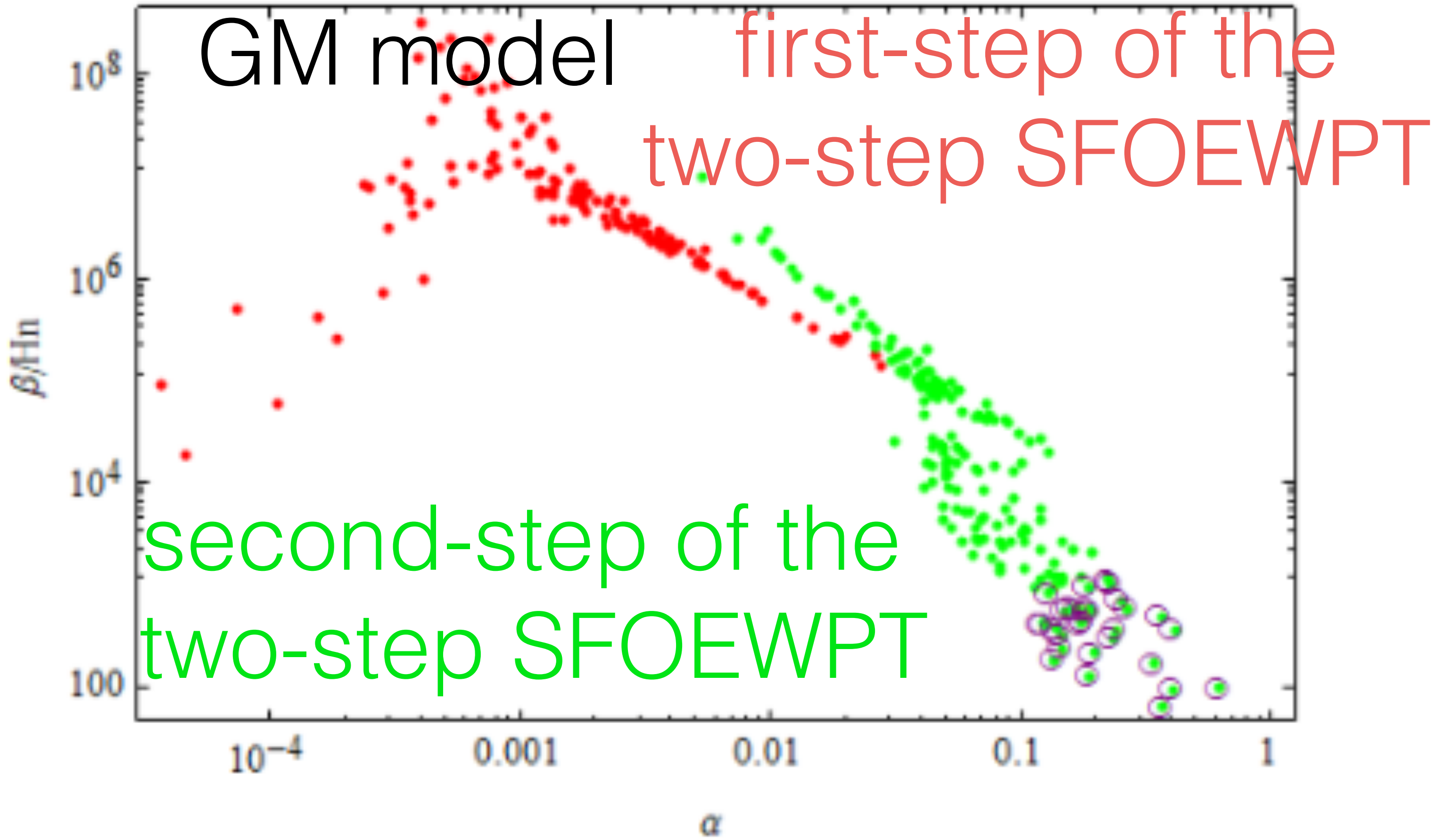

$v_h(T_n)/T_n$
$\lambda_{hhh}/\lambda_{hhh}^{SM}$

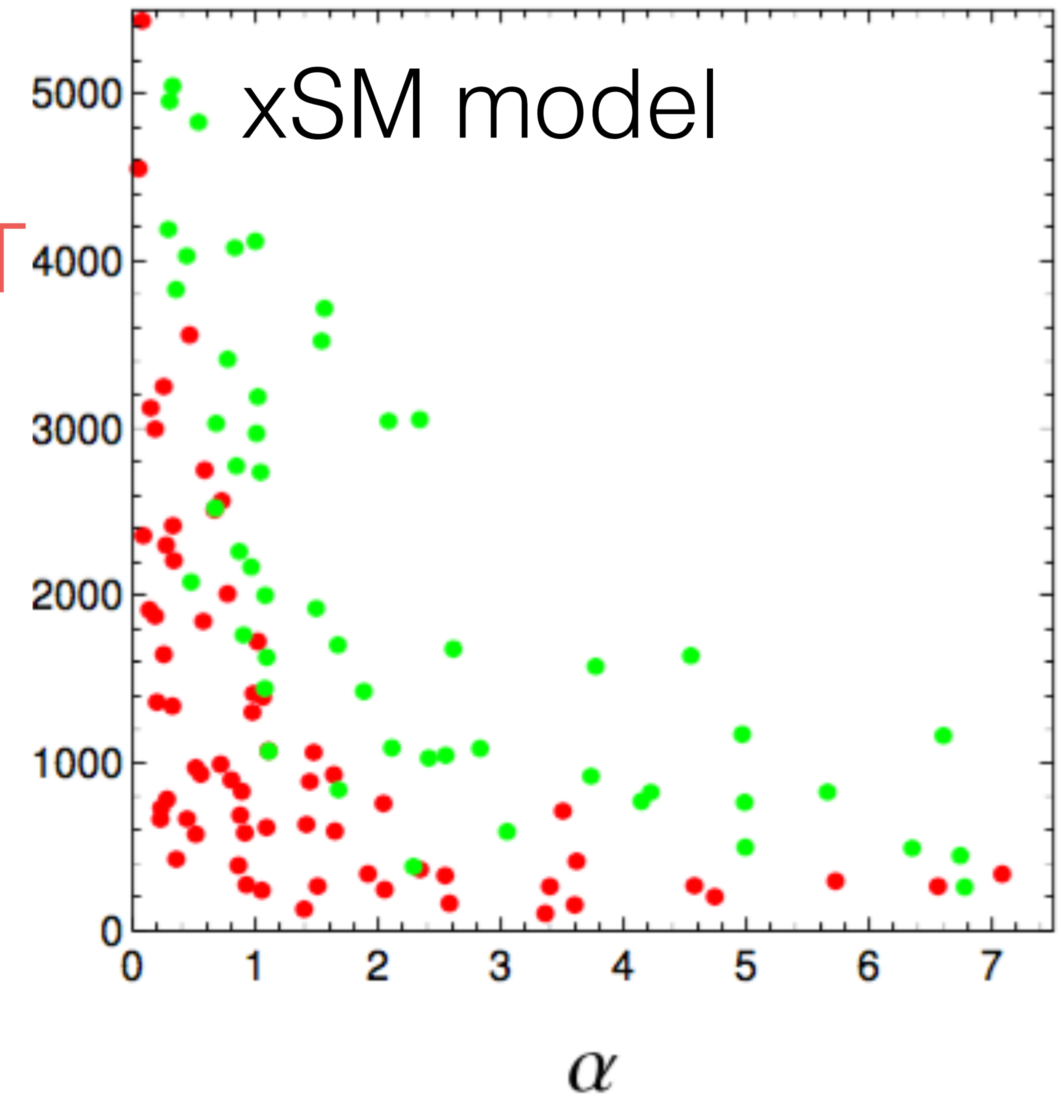

$v_h(T_n)/T_n$
$\lambda_{Hhh}/\lambda_{hhh}^{SM}$

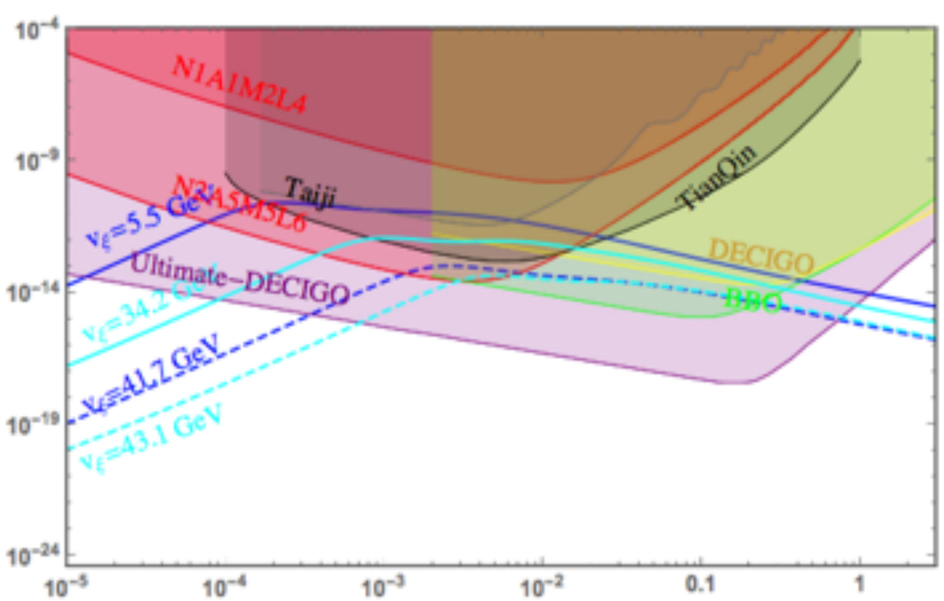
xSM

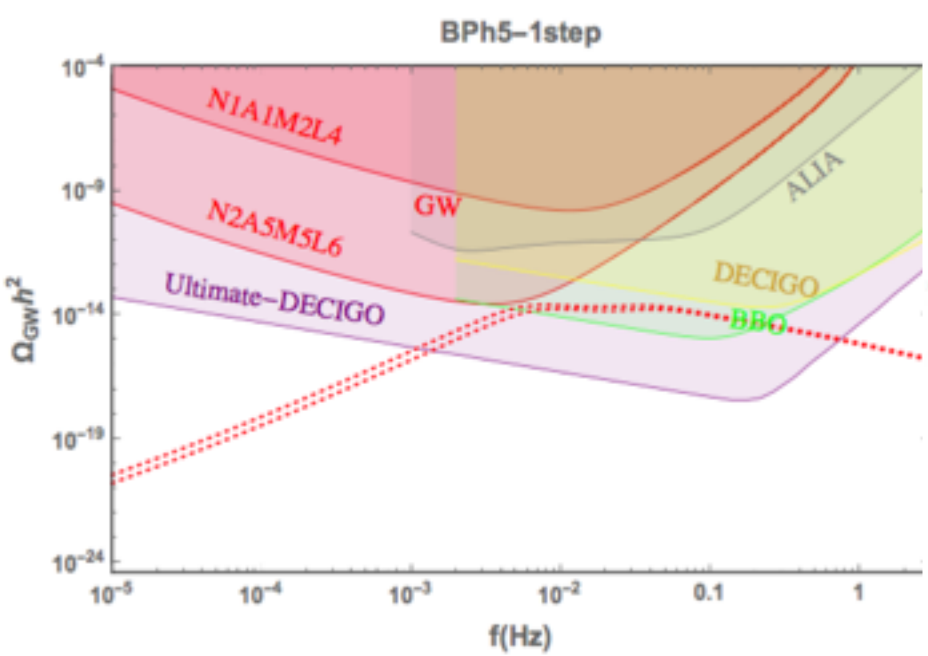

β/H_n
α

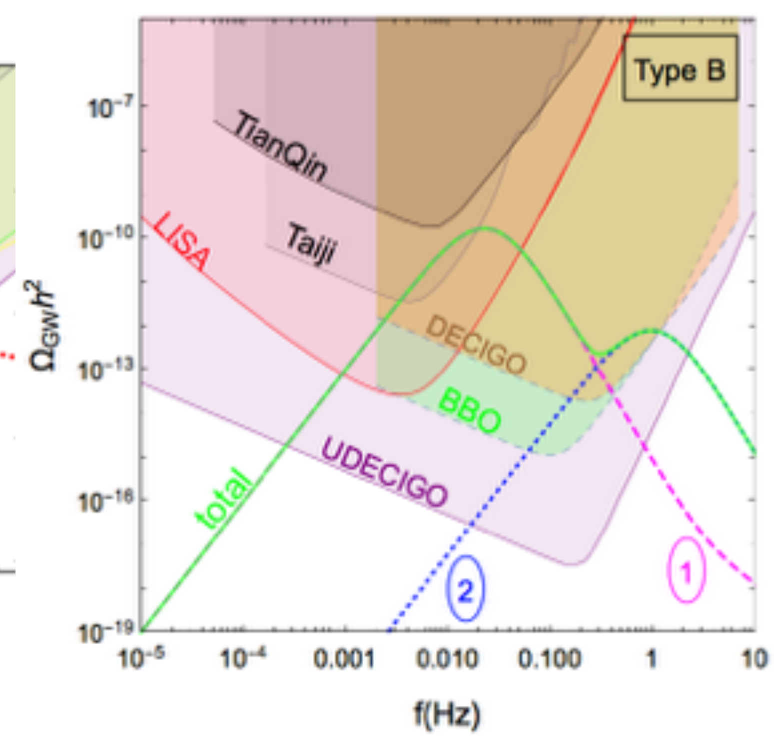

$v_h(T_n)/T_n$
α


GM model
first-step of the
two-step SFOEWPT
second-step of the
two-step SFOEWPT
β/Hn
α

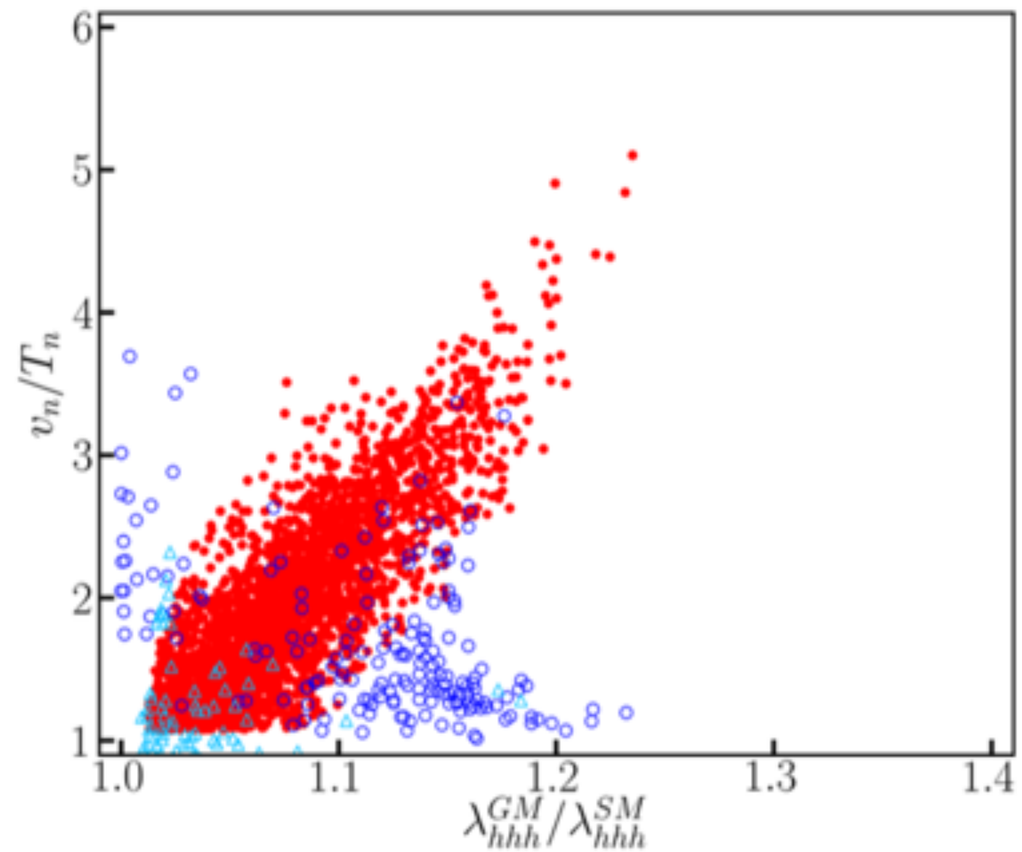

xSM model
α

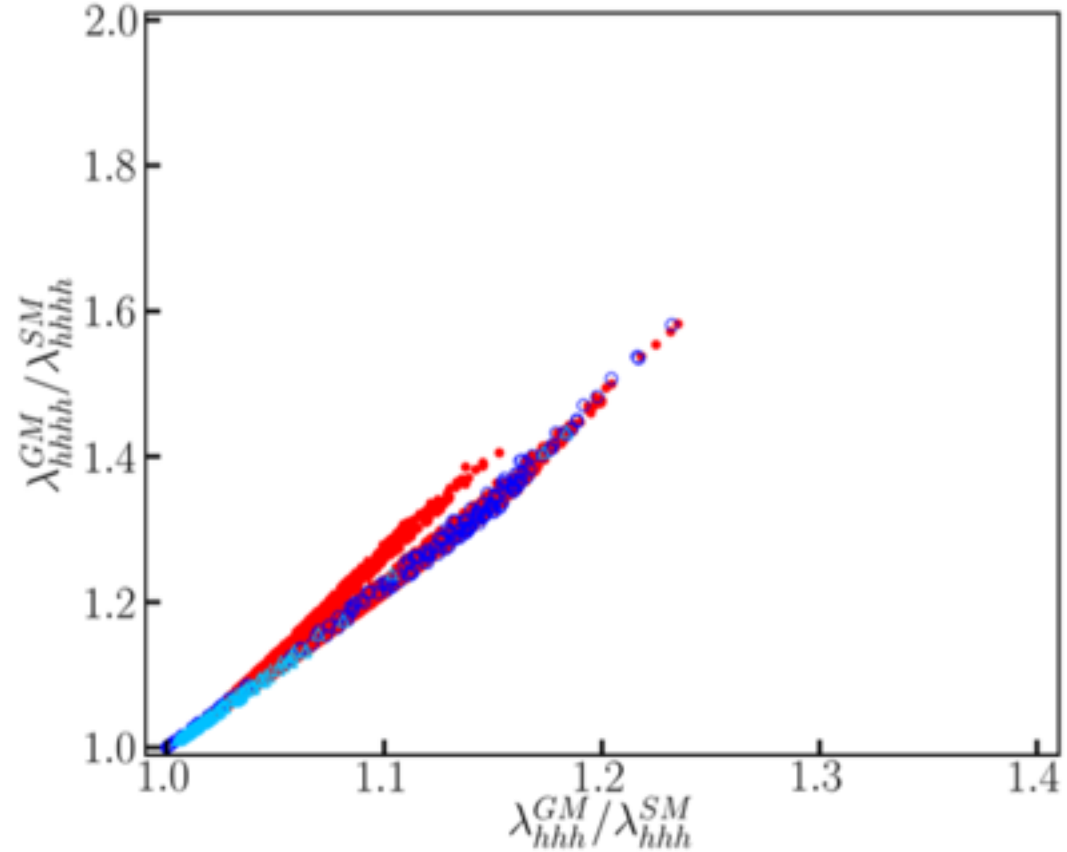

N1A1M2L4
N2A5M5L6
Taiji
TianQin
DECIGO
BBO
Ultimate-DECIGO

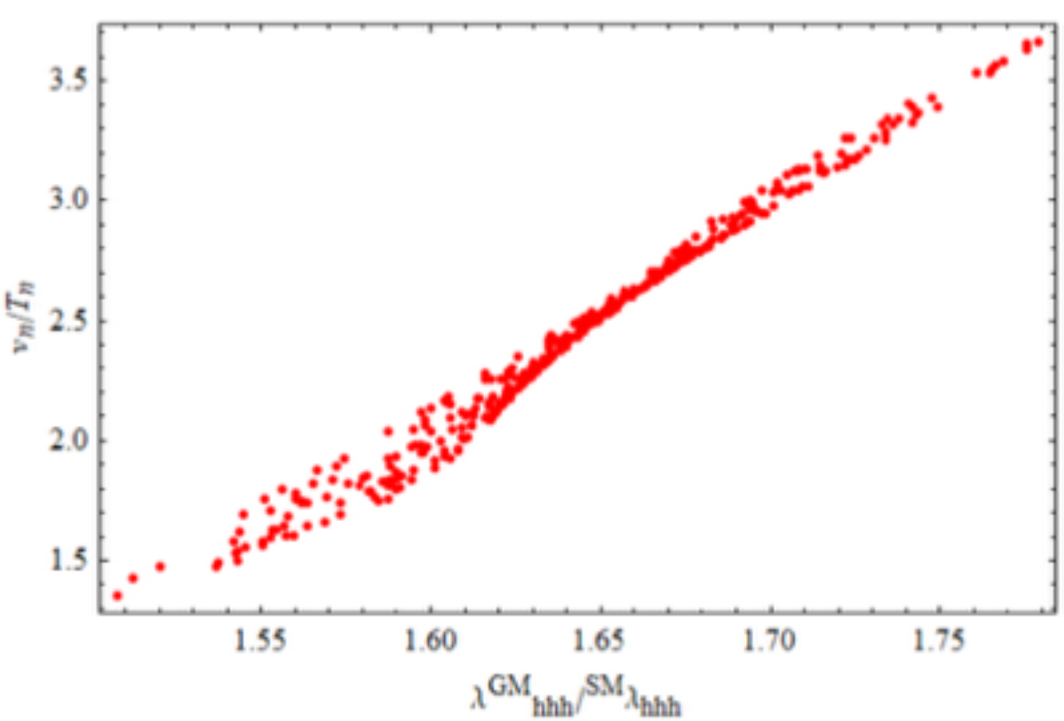

BPh5-1step
N1A1M2L4
N2A5M5L6
GW
ALIA
DECIGO
BBO
Ultimate-DECIGO
$\Omega_{GW}h^2$
f(Hz)

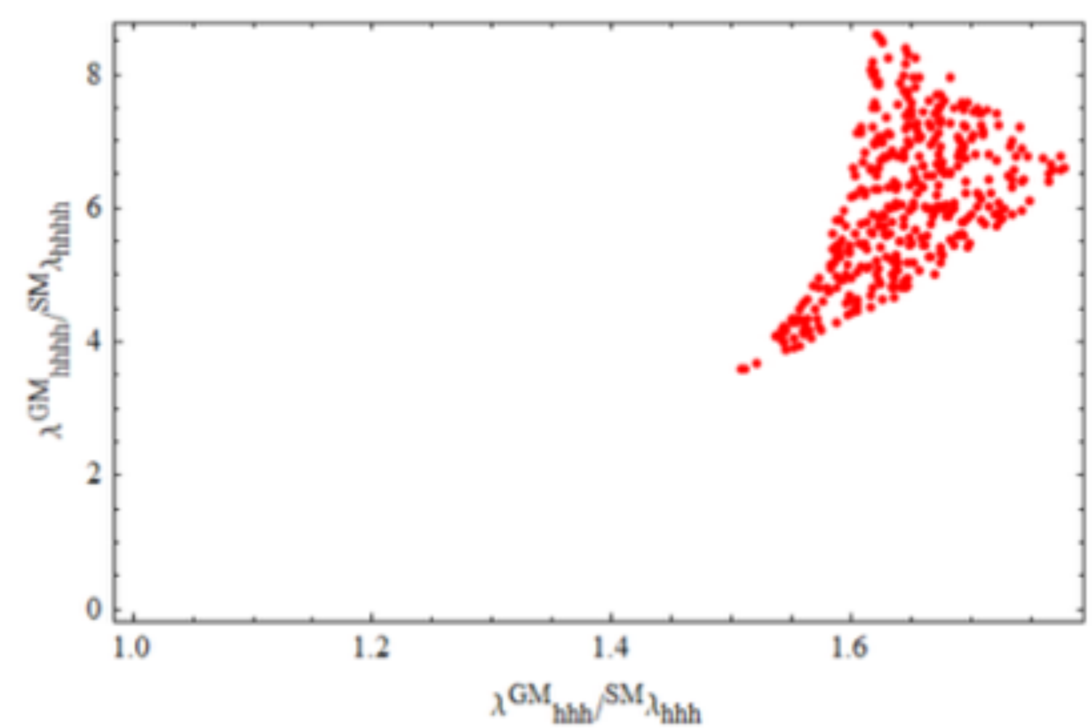

Type B
TianQin
Taiji
LISA
DECIGO
BBO
UDECIGO
total
$\Omega_{GW}h^2$
f(Hz)

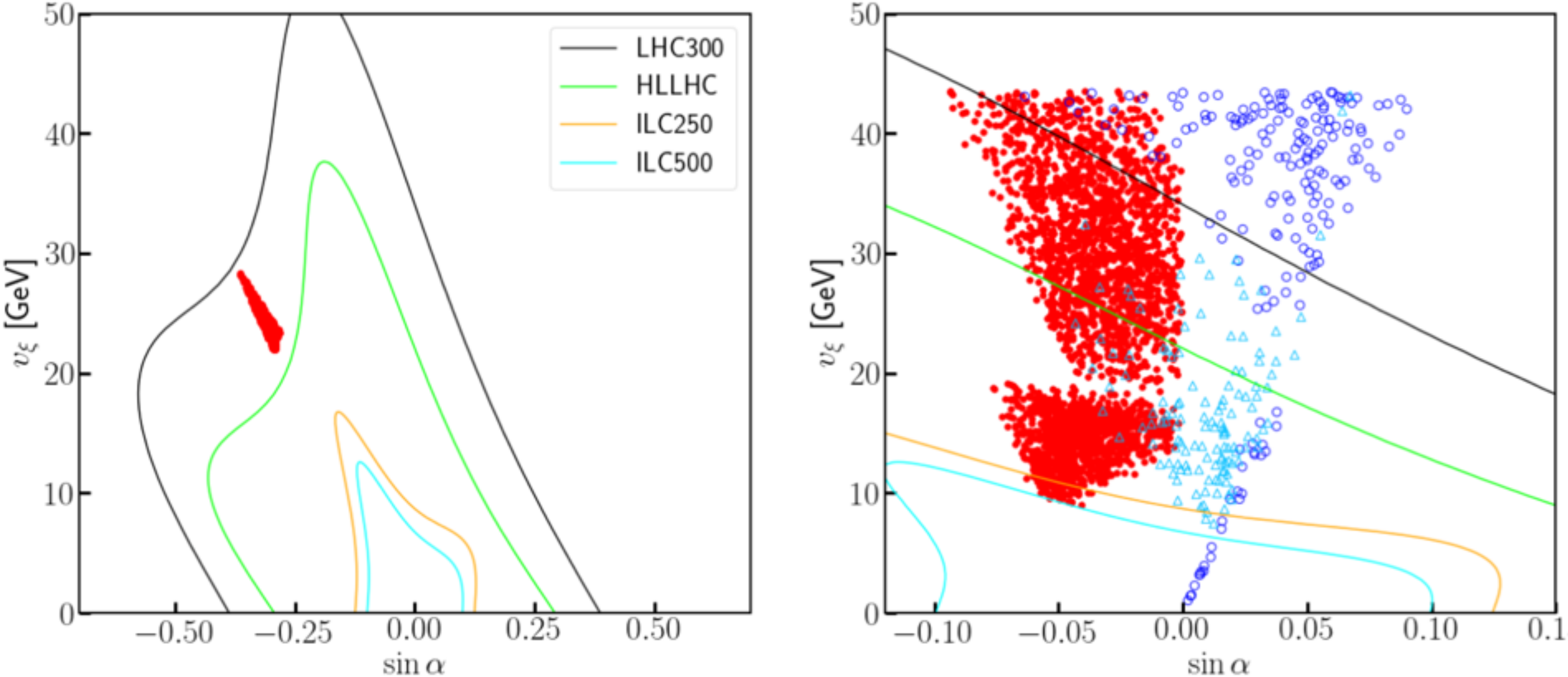
PT strength and Higgs triple and quartic couplings





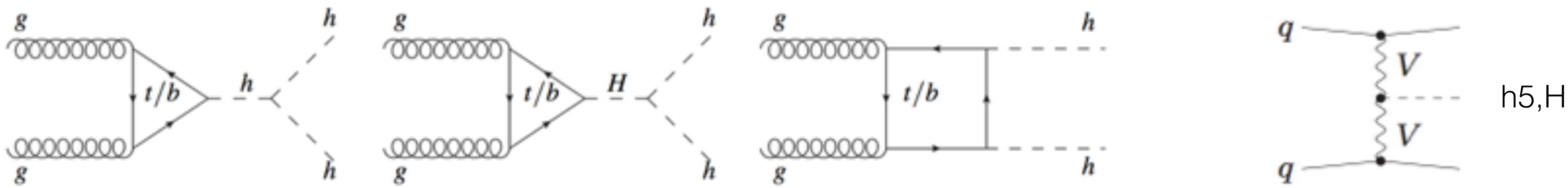


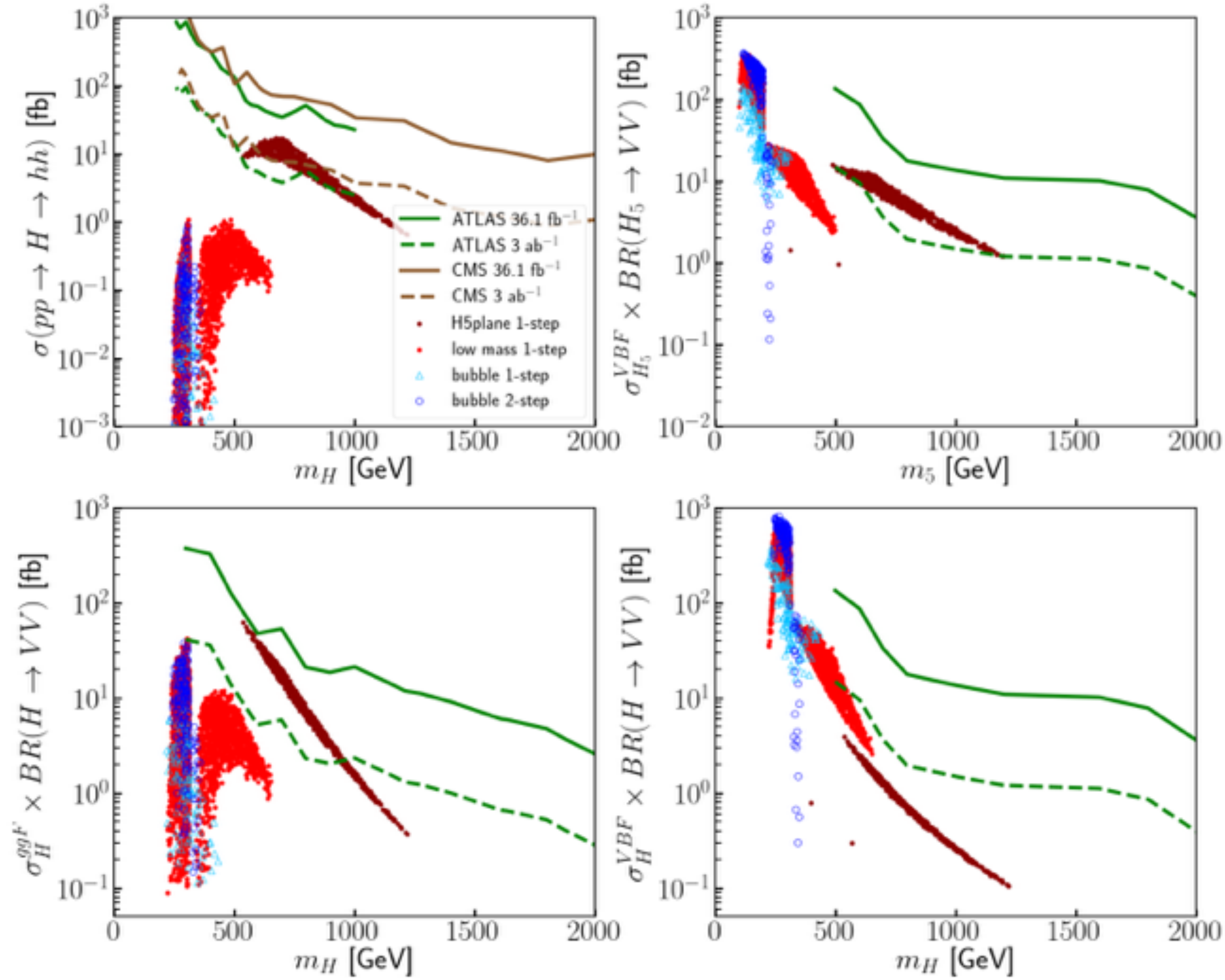




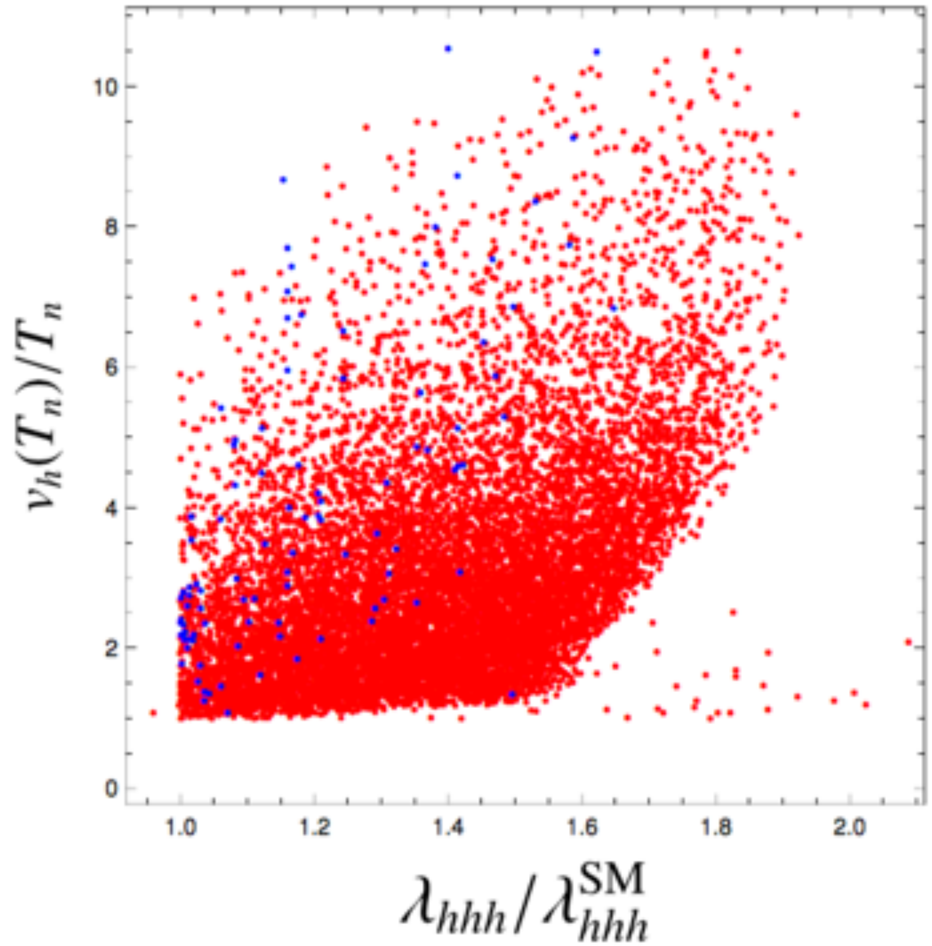
LHC300
HLLHC
ILC250
ILC500
v_ξ [GeV]
$\sin\alpha$
v_ξ [GeV]
$\sin\alpha$

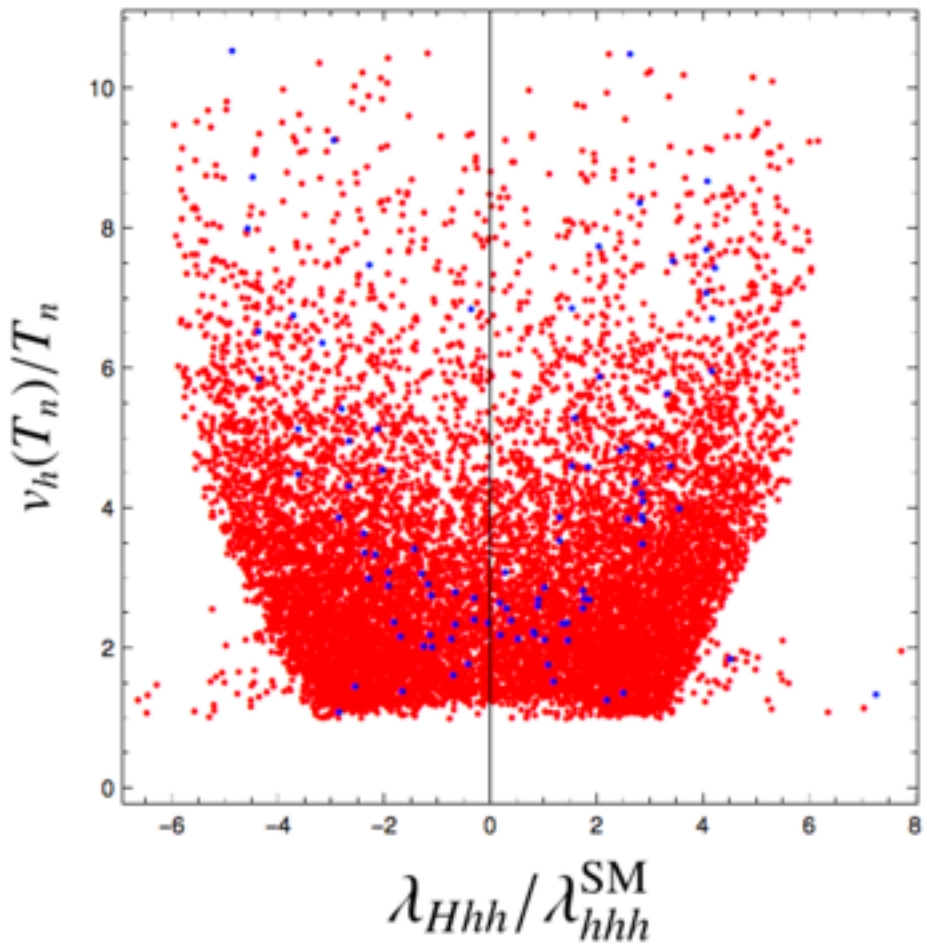
hadron collider

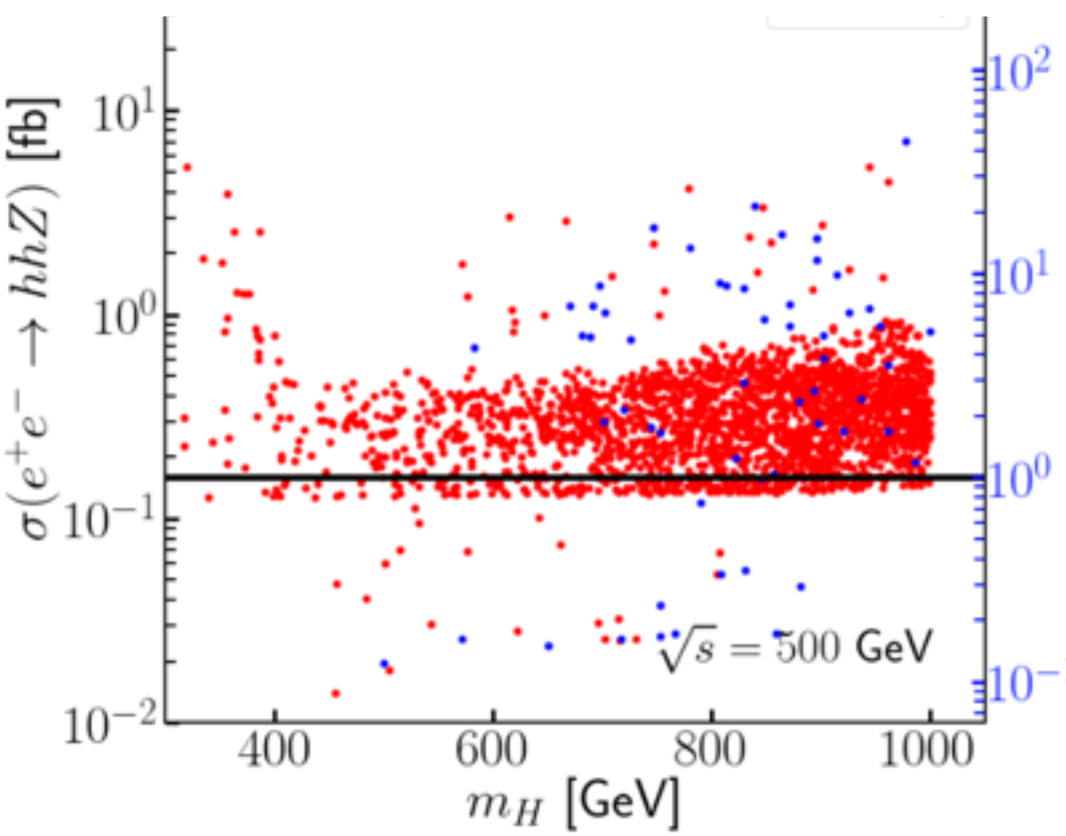


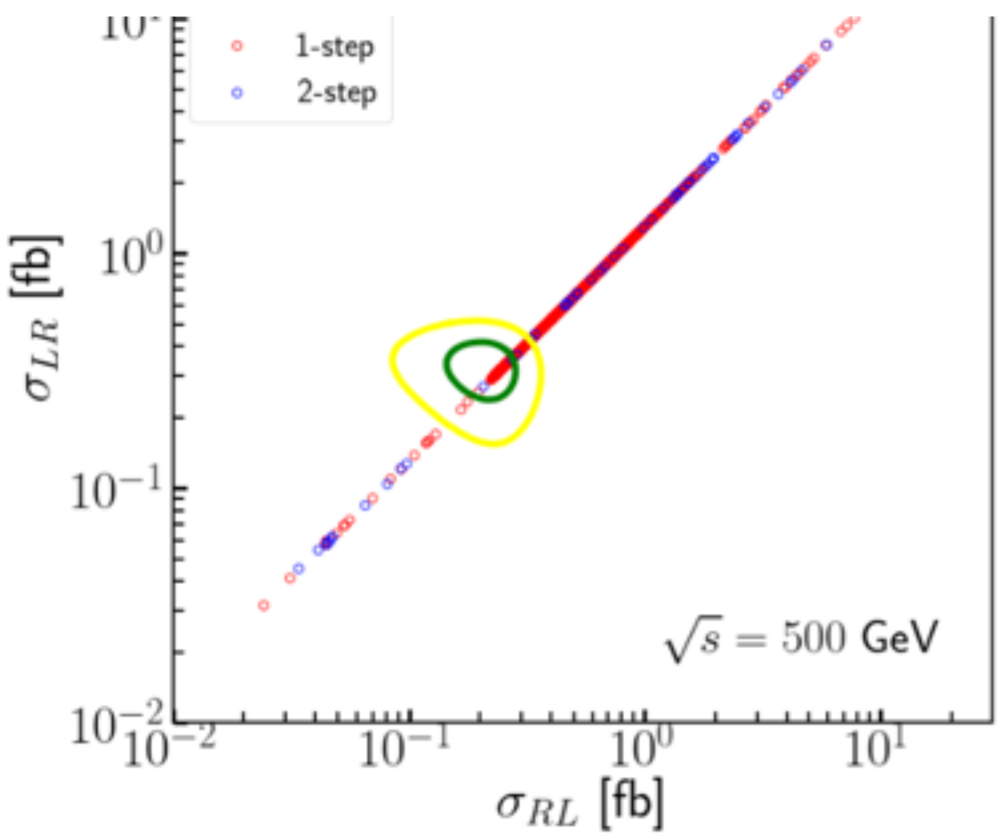


xSM


$v_h(T_n)/T_n$
$\lambda_{hhh}/\lambda_{hhh}^{SM}$

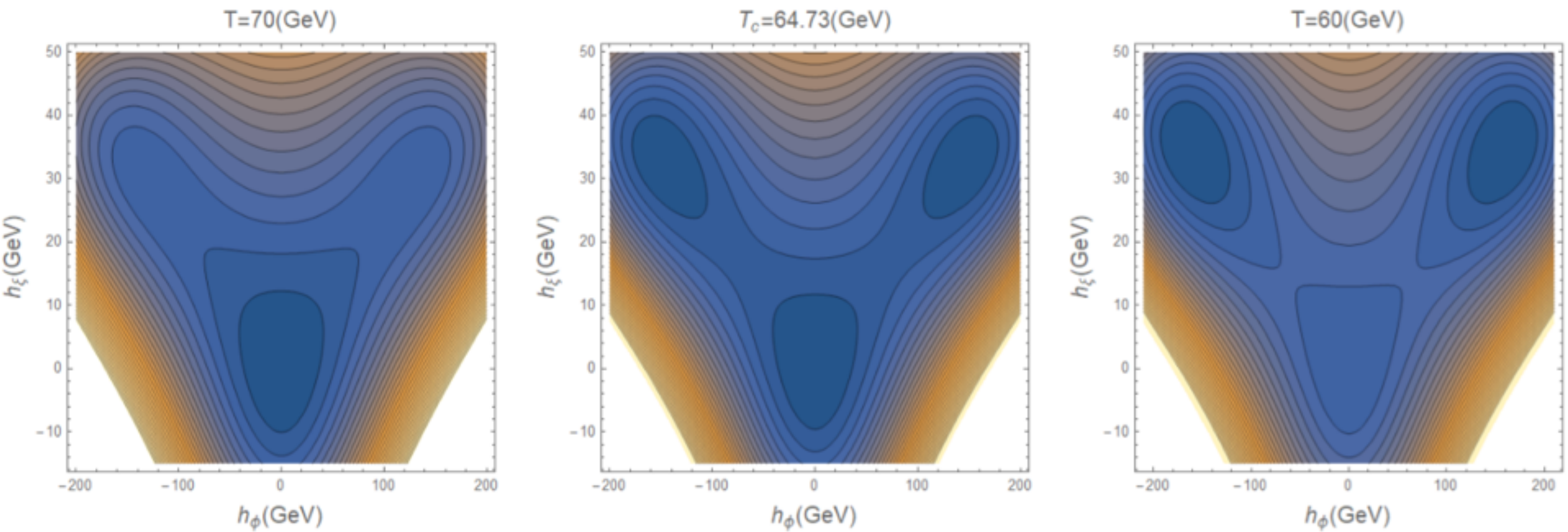

$v_h(T_n)/T_n$
$\lambda_{Hhh}/\lambda_{hhh}^{SM}$


$\sigma(e^+e^- \to hhZ)$ [fb]
κ
$\sqrt{s} = 500$ GeV
m_H [GeV]


1-step
2-step
σ_{LR} [fb]
$\sqrt{s} = 500$ GeV
σ_{RL} [fb]

SFOEWPT

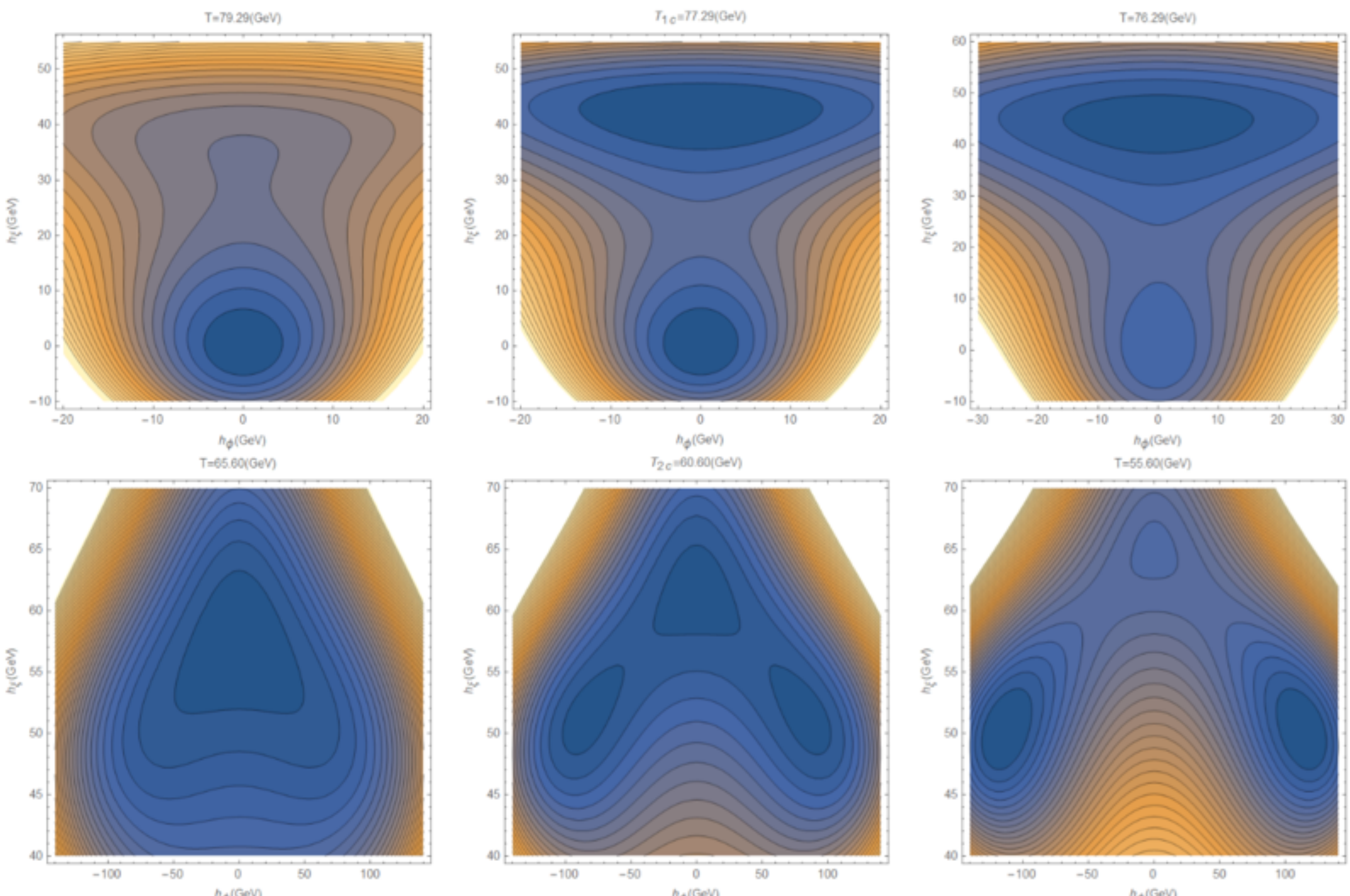
one-step



O → SU(2)V EWSB

SFOEWPT

multi-step. general with two fields



0->(0,<hxi>) ->(<hphi>,<hxi>)

Bubble, Sphaleron and BAU

Instanton

$$\frac{\Gamma}{V} = A(T)e^{-S_3/T}.$$

$$\frac{S_3(T_N)}{T_N} - \frac{3}{2}\ln\left(\frac{S_3(T_N)}{T_N}\right)$$
$$= 152.59 - 2\ln g_*(T_N) - 4\ln\left(\frac{T_N}{100\ \text{GeV}}\right)$$

Bubble nucleation

S3 (TN)/TN ~140-150

Washout avoid

$$\Gamma_{\text{sph}} = A_{\text{sph}}(T)\exp[-E_{\text{sph}}(T)/T] < H(T)$$

$$E_{\text{sph}}(T) \approx E_{\text{sph},0}\frac{v(T)}{v} \qquad \frac{v(T)}{T} > (0.973 - 1.16)\left(\frac{E_{\text{sph},0}}{1.916 \times 4\pi v/g}\right)^{-1}$$

1708.03061

SM+S

$$E_{\text{sph}}[f,h,k] = \frac{4\pi v}{g_2}\int_0^\infty d\xi\left[4\left(\frac{df}{d\xi}\right)^2 + \frac{8}{\xi^2}(f-f^2)^2 + \frac{\xi^2}{2}\left(\frac{dh}{d\xi}\right)^2 + h^2(1-f)^2 + \frac{\xi^2}{2}\frac{v_S^2}{v^2}\left(\frac{dk}{d\xi}\right)^2 + \frac{\xi^2}{g_2^2 v^4}V_{\text{eff}}(h,k,T)\right].$$

$$V_{1\ell} = V_{\rm tree} + \Delta V_{1\ell}$$

$$\Delta V_{1\ell} = \Delta V_{1\ell,T=0} + V_{1\ell,T\neq 0} \ ,$$

$$\Delta V_{1\ell,T=0} = \sum_{i=h,\chi,W,Z,t} \frac{n_i\, m_i^2(h_c)}{64\pi^2} \left(\log \frac{m_i^4(h_c)}{v^2} - C_i \right) \ ,$$

$$V_{1\ell,T\neq 0} = \frac{n_t\, T^4}{2\pi^2} J_f\left(m_t^2(h_c)/T^2\right) + \sum_{i=h,\chi,W,Z} \frac{n_i\, T^4}{2\pi^2} J_b\left(m_i^2(h_c)/T^2\right)$$

the high-temperature expansion of J_b and J_f leading terms,

$J_b(x) \to \pi^2 x/12$ and $J_f(x) \to -\pi^2 x/24$

GM model

The most general scalar potential $V(\Phi,\Delta)$ invariant under $SU(2)_L \times SU(2)_R \times U(1)_Y$ is given by

$$\begin{aligned} V(\Phi,\Delta) =& \frac{1}{2}m_1^2 \mathrm{tr}[\Phi^\dagger\Phi] + \frac{1}{2}m_2^2\mathrm{tr}[\Delta^\dagger\Delta] + \lambda_1\left(\mathrm{tr}[\Phi^\dagger\Phi]\right)^2 \\ &+ \lambda_2\left(\mathrm{tr}[\Delta^\dagger\Delta]\right)^2 + \lambda_3\mathrm{tr}\left[\left(\Delta^\dagger\Delta\right)^2\right] + \lambda_4\mathrm{tr}[\Phi^\dagger\Phi]\mathrm{tr}[\Delta^\dagger\Delta] \\ &+ \lambda_5\mathrm{tr}\left[\Phi^\dagger\frac{\sigma^a}{2}\Phi\frac{\sigma^b}{2}\right]\mathrm{tr}[\Delta^\dagger T^a\Delta T^b] \\ &+ \mu_1\mathrm{tr}\left[\Phi^\dagger\frac{\sigma^a}{2}\Phi\frac{\sigma^b}{2}\right](P^\dagger\Delta P)_{ab} + \mu_2\mathrm{tr}[\Delta^\dagger T^a\Delta T^b](P^\dagger\Delta P)_{ab}\,, \end{aligned} \quad (3)$$

$$v_\phi^2 + 8v_\xi^2 \equiv v^2 \approx (246\mathrm{GeV})^2$$

$$v_\chi = \sqrt{2}v_\xi$$

$$\Phi \equiv (\epsilon_2\phi^*,\phi) = \begin{pmatrix} \phi^{0*} & \phi^+ \\ -\phi^{+*} & \phi^0 \end{pmatrix}, \quad \Delta \equiv (\epsilon_3\chi^*,\xi,\chi) = \begin{pmatrix} \chi^{0*} & \xi^+ & \chi^{++} \\ -\chi^{+*} & \xi^0 & \chi^+ \\ \chi^{++*} & -\xi^{+*} & \chi^0 \end{pmatrix}, \quad (1)$$

with

$$\epsilon_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \epsilon_3 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad (2)$$

where the phase convention for the scalar field components is: $\chi^{--} = \chi^{++*}$, $\chi^- = \chi^{+*}$, $\xi^- = \xi^{+*}$, $\phi^- = \phi^{+*}$. Φ and Δ are transformed under $SU(2)_L \times SU(2)_R$ as $\Phi \to U_{2,L}\Phi U_{2,R}^\dagger$ and $\Delta \to U_{3,L}\Delta U_{3,R}^\dagger$ with $U_{L,R} = exp(i\theta^a_{L,R}T^a)$ and T^a being the $SU(2)$ generators.

where summations over $a,b = 1,2,3$ are understood, σ's and T's are the 2×2 (Pauli matrices) and 3×3 matrix representations of the $SU(2)$ generators, respectively

$$T_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, T_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, T_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad (4)$$

The P matrix, which is the similarity transformation relating the generators in the triplet and the adjoint representations, is given by

$$P = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 & i & 0 \\ 0 & 0 & \sqrt{2} \\ 1 & i & 0 \end{pmatrix}. \quad (5)$$

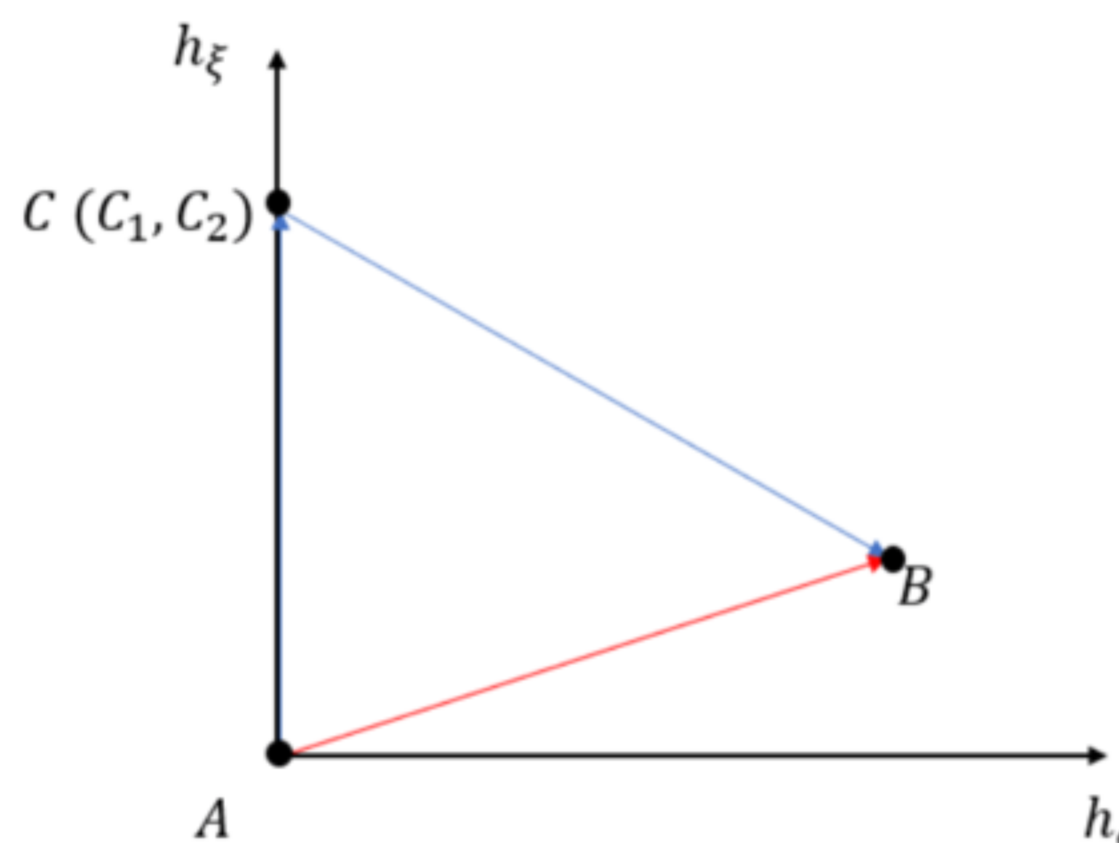
The finite-T potential

$$V_T = V_0 + \frac{1}{2}c_\phi T^2 h_\phi^2 + \frac{1}{2}c_\xi T^2 h_\xi^2 + \frac{1}{2}c_\chi T^2 h_\chi^2$$

$$V_0 = \frac{1}{4}(4h_\phi^4\lambda_1 + 2(h_\xi^2 + h_\chi^2)(m_2^2 + 2\lambda_2(h_\xi^2 + h_\chi^2)) + 2\lambda_3(2h_\xi^4 + h_\chi^4)$$
$$+ h_\phi^2(2m_1^2 + 4\lambda_4 h_\xi^2 + h_\xi(2\sqrt{2}\lambda_5 h_\chi + \mu_1) + h_\chi(4\lambda_4 h_\chi + \lambda_5 h_\chi + \sqrt{2}\mu_1)) + 12\mu_2 h_\xi h_\chi^2)$$

$$c_\phi = \frac{3g^2}{16} + \frac{g'^2}{16} + 2\lambda_1 + \frac{3\lambda_4}{2} + \frac{1}{4}y_t^2 \sec^2\theta_H$$
$$c_\xi = \frac{g^2}{2} + \frac{11\lambda_2}{3} + \frac{7\lambda_3}{3} + \frac{2\lambda_4}{3},$$
$$c_\chi = \frac{g^2}{2} + \frac{g'^2}{4} + \frac{11\lambda_2}{3} + \frac{7\lambda_3}{3} + \frac{2\lambda_4}{3}.$$

$$v^{GM}/T \equiv \frac{\sqrt{v_\phi^2(T) + 8v_\xi^2(T)}}{T} = \frac{v_\phi(T)\cos\theta_H(T)^{-1}}{T},$$
$$\cos\theta_H(T) \equiv \frac{v_\phi(T)}{\sqrt{v_\phi^2(T) + 8v_\xi^2(T)}},$$



$h_\chi = \sqrt{2}h_\xi$ as required by the custodial symmetry