Precision QCD Calculations for Heavy Quark Decays

Yu-Ming Wang

Nankai University

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Why precision calculations?

- Understanding the general properties of power expansion in EFTs (HQET, SCET, NRQCD).

- Interesting to understand the strong interaction dynamics of heavy quark decays.
  - Factorization properties of the subleading-power amplitudes.
  - Renormalization and asymptotic properties of the higher-twist B-meson DAs.
  - Interplay of different QCD techniques.

- Precision determinations of the CKM matrix elements $|V_{ub}|$ and $|V_{cb}|$.
  Power corrections, QED corrections, BSM physics.

- Crucial to understand the CP violation in B-meson decays.
  Strong phase of $\mathscr{A}(B \to M_1 M_2)$ @ m_b scale in the leading power.

- Indispensable for understanding the flavour puzzles (see EPS-HEP 2019 for updates).
  - P_5' and $R_{K^{(*)}}$ anomalies in $B \to K^{(*)} \ell^+ \ell^-$.
  - $R_{D^{(*)}}$ anomalies in $B \to D^{(*)} \ell \bar{\nu}_\ell$.
  - Color suppressed hadronic B-meson decays.
  - Polarization fractions of penguin dominated $B_{(s)} \to VV$ decays.

Theory tools for precision flavor physics

New Physics: $\mathscr{L}_{NP}$

$\downarrow$

EW scale (m_W): $\mathscr{L}_{SM} + \mathscr{L}_{D>4}$

$\downarrow$

Heavy-quark scale (m_b): $\mathscr{L}_{eff} = -\frac{G_F}{\sqrt{2}} \sum_i C_i Q_i + \mathscr{L}_{eff,D>6}$

$\downarrow$

QCD scale ($\Lambda_{\rm QCD}$)

- Aim: $\langle f | Q_i | \bar{B} \rangle = ?$
- QCD factorization [Diagrammatic approach].
- SCET factorization [Operator formalism].
- TMD factorization.
- (Light-cone) QCD sum rules.
- Lattice QCD.

- Key concepts: Factorization, Resummation, Evolution.

Factorization in Classical Physics

- Galileo's Leaning Tower of Pisa Experiment:

$$\mathscr{L}_{\text{eff}} = \frac{1}{2} m \dot{h}^2 - m g h \,.$$

Symmetry of the effective Lagrangian: $h \to h + a$.
Dynamical interpretation: The force acting on the ball, $F = m g$, independent of h.

- Newton's Gravity Theory:

$$V_{\text{full}}(h) = -G \frac{M m}{r} = -G \frac{M m}{R + h} \,.$$

  - Power expansion of the full potential energy:

$$V_{\text{eff}}(h) = C_1(R)\, m\, (h/R) + C_2(R)\, m\, (h/R)^2 + \dots \,.$$

    The general form of the effective potential can be written without knowing V_{full}.
  - Matching the full theory and the effective theory:

$$C_1(R) = -C_2(R) = \frac{G M}{R} \,, \qquad V_{\text{eff}}(h) = m g h - \frac{m g}{R} h^2 + \dots \,.$$

- Symmetry of the effective Lagrangian broken in the full theory.

$$g(r) = \frac{G M}{r^2} \,, \qquad r \frac{\partial}{\partial r} g(r) = \gamma_g \, g(r) \,.$$

This differential equation is actually a renormalization group equation.

Factorization in Quantum Physics

- Wigner-Eckart theorem:

$$\langle \tau' j' m' | T_q^k | \tau j m \rangle = \langle j k j' | m q m' \rangle \, \langle \tau' j' || T^k || \tau j \rangle \, .$$

  Separation of geometry and dynamics.

- Generalized Wigner-Eckart theorem in Lie algebra.
  An example from SU(3): u, v and W are all 8s.

$$\langle u | W | v \rangle = \lambda_1 \, \mathrm{Tr}[\bar{u} \, W \, v] + \lambda_2 \, \mathrm{Tr}[\bar{u} \, v \, W] \, .$$

  Notice that $8^3 = 512$ matrix elements expressed in terms of only two parameters.

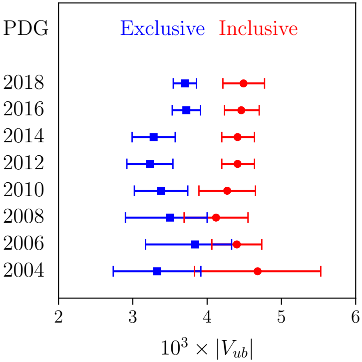
- Factorization for strong interaction physics.
  An example from $B \to \gamma \ell \nu_\ell$:

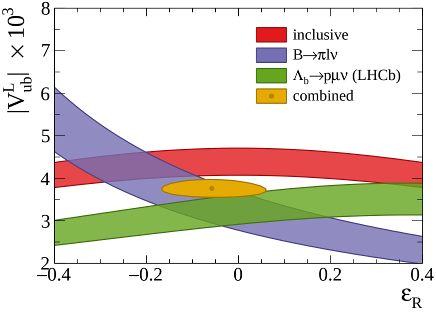
$$F_{V,\mathrm{LP}}(n \cdot p) = \frac{Q_u \, m_B}{n \cdot p} \tilde{f}_B(\mu) \, C_\perp(n \cdot p, \mu) \int_0^\infty d\omega \, \frac{\phi_B^+(\omega, \mu)}{\omega} \, J_\perp(n \cdot p, \omega, \mu) \, .$$

  - Separation of hard, hard-collinear and soft fluctuations.
  - Key input: B-meson light-cone distribution amplitude $\phi_B^+(\omega, \mu)$.

Heavy-to-light form factors in QCD

- Long-standing tension between the exclusive and inclusive $|V_{ub}|$.





- Right handed current, underestimate of QCD uncertainties?

Factorization for heavy-to-light form factors

- Factorization formulae for semeleptonic B-meson decays [BBNS, BPRS, and many others].

$$F_i^{B\to M}(E) = C_i^{(\mathrm{A0})}(E)\,\xi_a(E) + \int_0^\infty \frac{d\omega}{\omega} \int_0^1 dv\, \underbrace{T_i(E;\ln\omega, v)}_{C_i^{(\mathrm{B1})} * J_i}\, \phi_B^+(\omega)\phi_M(v)\,.$$

Need the hadronic matrix elements of both the LP and NLP SCET currents!

- Diagrammatic factorization for heavy-to-light form factors at one loop [Beneke, Feldmann, 2001].

- Perturbative calculations of the hard matching coefficients $C_i^{(\mathrm{A0})}(E)$:
  - One-loop SCET computations in [Bauer, Fleming, Pirjol, Stewart, 2001; Beneke, Kiyo, Yang, 2004].
  - Two-loop SCET computations in [Bonciani, Ferroglia, 2008; Asatrian, Greub, Pecjak, 2008; Beneke, Huber, Li, 2009; Bell, 2009; Bell, Beneke, Huber, Li, 2011].

- Perturbative calculations of the hard matching coefficients $C_i^{(\mathrm{B1})}$:
  - Infrared subtractions complicated by the appearance of evanescent operators and the D-dimensional Fierz transformation.
  - One-loop SCET computations in [Becher, Hill, 2004; Hill, Becher, Lee, Neubert, 2004; Beneke, Yang, 2006].

- Factorization of $\xi_P(E)$ in $\mathrm{SCET_{II}}$ is not yet established at present.
  $\Leftrightarrow$ $\xi_P(E)$ taken from the LCSR/LQCD calculations.

Different versions of light-cone sum rules

- Light-cone QCD sum rules with the light-meson LCDA [Ball, Braun, Khodjamirian, etc]:
  - Interpolating the heavy B-meson by a local QCD current.
  - Diagrammatical factorization for the vacuum-to-light-meson correction functions.
  - Disadvantage: different non-perturbative inputs for different decay observables.

- Light-cone QCD sum rules with the B-meson LCDA [Khodjamirian, Lü, Shen, Wang, etc]:
  - Interpolating the light energetic meson by a local QCD current.
  - Diagrammatical factorization for the vacuum-to-B-meson correction functions.
  - Advantage: universal non-perturbative inputs for different decay observables.

- Light-cone SCET sum rules with the B-meson LCDA [Feldmann, Lü, Shen, Wang, etc]:
  - Interpolating the light energetic meson by a local SCET current.
  - SCET factorization for the vacuum-to-B-meson correction functions.
  - Advantage I: Computation of the short-distance functions much easier.
  - Advantage II: Systematic resummation of enhanced logarithms beyond the LL accuracy.

- Light-cone QCD sum rules with the chiral current for the light meson [Huang, Li, Wu, etc]:
  - Advantage: Twist-three light-meson LCDAs do not contribute (at least) at NLO.
  - Heavy hadrons of both positive and negative parities enter the hadronic dispersion relation.

- Yet more versions have been constructed with different assumptions/approximations.

Light-cone QCD sum rules with light-hadron LCDA

- Semileptonic $B \to P$ form factors:
  - LO QCD calculations of $B \to P$ form factors [Belyaev, Khodjamirian, Rückl, 1993].
  - NLO QCD calculations of $B \to P$ form factors at twist-2 accuracy [Khodjamirian, Rückl, Weinzierl, Yakovlev, 1997].
  - NLO QCD calculations of $B \to P$ form factors at twist-3 accuracy [Ball, Zwicky, 2005; Duplancic, Khodjamirian, Mannel, Melic, Offen, 2008].
  - Improved NLO QCD calculations of $B \to P$ form factors at twist-3 accuracy [Khodjamirian, Mannel, Offen, Wang, 2011].
  - (Partial) NNLO QCD calculations of $B \to P$ form factors at twist-2 accuracy [Bharucha, 2012].

- Semileptonic $B \to V(\to P_1 P_2)$ form factors:
  - NLO QCD calculations of $B \to V$ form factors at twist-3 accuracy [Ball, Zwicky, 2005].
  - Updated NLO QCD calculations of $B \to V$ form factors at twist-3 accuracy [Bharucha, Straub, Zwicky, 2015].
  - LO QCD calculations of $B \to \pi K$ form factors [Meißner, Wei Wang, 2014].
  - LO QCD calculations of $B \to \pi\pi$ form factors [Hambrock, Khodjamirian, 2016].

- Semileptonic Λ_b-baryon form factors:
  - LO QCD calculation of $\Lambda_b \to \Lambda$ form factors [Wang, Li, Lü, 2008].
  - LO QCD calculation of $\Lambda_b \to p$ form factors [Khodjamirian, Mannel, Klein, Wang, 2011].

Light-cone QCD sum rules with heavy-hadron LCDA

- LO QCD calculations of $B \to M$ form factors [Khodjamirian, Offen, Mannel, 2006].
- NLO QCD calculations of $B \to \pi$ form factors at leading-twist accuracy [Wang, Shen, 2015].
- NLO QCD calculations of $B \to D$ form factors at leading-twist accuracy and LO QCD calculations of higher-twsit corrections up to the twist-six accuracy [Wang, Wei, Shen, Lü, 2017].
  - NLO leading-twist jet function complicated by two distinct hard-collinear variables.
  - Power-enhanced charm-quark mass effect.
- LO QCD calculations of $B \to P$ and $B \to V$ form factors at the twist-four accuracy [Gubernari, Kokulu, van Dyk, 2018].
  - No definite power counting scheme for QCD calculations of the correlation functions
  - Violation of the QCD equations of motion at tree level.
  - Sizable theory uncertainties for phenomenological applications.
- NLO QCD calculations of $B \to \pi, K$ form factors at leading-twist accuracy and LO QCD calculations of higher-twsit corrections up to the twist-six accuracy [Lü, Shen, Wang, Wei, 2019].
- NLO QCD calculations of $\Lambda_b \to \Lambda$ form factors at twist-four accuracy [Wang, Shen, 2016].
  - Non-trivial demonstration of the factorization-scale independence.
  - Immediately confirmed by the lattice QCD calculations [Detmold, Meinel, 2016].

Light-cone SCET sum rules with B-meson LCDA

- NLO QCD calculations of $B \to M$ form factors at leading-twist accuracy [De Fazio, Feldmann, Hurth, 2006; 2008].

- NLO QCD calculations of $B \to V$ form factors at leading-twist accuracy and LO QCD calculation of higher-twsit corrections up to the twist-six accuracy [Gao, Lü, Shen, Wang, Wei, 2019].
  - Rigorous perturbative matching with the evanescent-operator approach.
  - First SCET computation of the SU(3)-symmetry breaking effects.
  - Three-particle higher-twist corrections compatible with the EOM constraints.
  - Identify mechanisms responsible for the discrepancies between QCDF and LCSR.

- Key task: construct the sum rules for the $\mathrm{SCET_I}$ matrix elements ξ_a and Ξ_a.

$$F_i^{B\to V}(n\cdot p) = C_i^{(\mathrm{A0})}(n\cdot p)\,\xi_a(n\cdot p) + \int d\tau\, C_i^{(\mathrm{B1})}(\tau, n\cdot p)\,\Xi_a(\tau, n\cdot p)\,, \quad (a = \parallel, \perp)\,.$$

The operator-level definitions for longitudinal form factors:

$$\langle V(p,\varepsilon^*)|\left(\bar{\xi}\, W_c\right)\gamma_5\, h_v\, |\bar{B}_v\rangle = -n\cdot p\,(\varepsilon^*\cdot v)\,\xi_{\parallel}(n\cdot p)\,,$$

$$\langle V(p,\varepsilon^*)|\left(\bar{\xi}\, W_c\right)\gamma_5\left(W_c^\dagger\, i\not{D}_{c\perp}\, W_c\right)(rn)\, h_v\, |\bar{B}_v\rangle = -n\cdot p\, m_b\, \varepsilon^*\cdot v \int_0^1 d\tau\, e^{i\tau n\cdot p r}\,\Xi_{\parallel}(\tau, n\cdot p)\,.$$

Light-cone SCET sum rules for $\xi_{\|}(n \cdot p)$

- Constructing the vacuum-to-B-meson correlation function [Gao, Lü, Shen, Wang, Wei, 2019]:

$$\Pi_{\nu,\|}(p,q) = \int d^4x\, e^{ip\cdot x} \langle 0|\mathrm{T}\left\{\bar{q}'(x)\,\gamma_\nu\, q(x),\, \left(\bar{\xi}\, W_c\right)(0)\,\gamma_5\, h_v(0)\right\}|\bar{B}_v\rangle\,.$$

Interpolating the collinear vector meson by a local vector current.

- Identify the leading power contributions to $\Pi_{\nu,\|}(p,q)$ in $\mathrm{SCET_I}$:

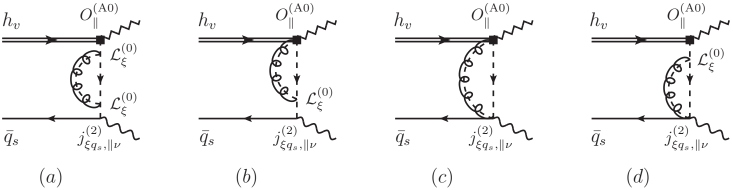
$$\begin{aligned}
\Pi_{\nu,\|}(p,q) &= \int d^4x\, e^{ip\cdot x} \langle 0|\mathrm{T}\left\{j^{(2)}_{\xi q_s,\|\,\nu}(x),\, \left(\bar{\xi}\, W_c\right)(0)\,\gamma_5\, h_v(0)\right\}|\bar{B}_v\rangle \\
&\quad + \int d^4x\, e^{ip\cdot x} \int d^4y\, \langle 0|\mathrm{T}\left\{j^{(0)}_{\xi\xi,\nu}(x),\, i\mathcal{L}^{(2)}_{\xi q_s}(y),\, \left(\bar{\xi}\, W_c\right)(0)\,\gamma_5\, h_v(0)\right\}|\bar{B}_v\rangle \\
&\quad + \int d^4x\, e^{ip\cdot x} \int d^4y \int d^4z\, \langle 0|\mathrm{T}\left\{j^{(0)}_{\xi\xi,\nu}(x),\, i\mathcal{L}^{(1)}_{\xi q_s}(y),\, i\mathcal{L}^{(1)}_{\xi m}(z),\, \left(\bar{\xi}\, W_c\right)(0)\,\gamma_5\, h_v(0)\right\}|\bar{B}_v\rangle\,, \\
&\equiv \Pi^{A}_{\nu,\|}(p,q) + \Pi^{B}_{\nu,\|}(p,q) + \Pi^{C}_{\nu,\|}(p,q)\,.
\end{aligned}$$

Need the power-suppressed currents or SCET Lagrangians to project out the soft quark state.

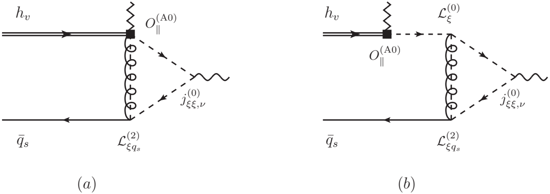
- Establish soft-collinear factorization formulae for the three different pieces respectively.

Light-cone SCET sum rules for $\xi_{\|}(n \cdot p)$

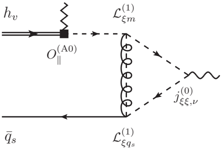
- Diagrammatic representation for $\Pi^{A}_{\nu,\|}(p,q)$ at one loop:



- Diagrammatic representation for $\Pi^{B}_{\nu,\|}(p,q)$ at one loop:



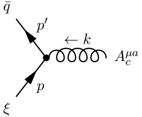
- Diagrammatic representation for $\Pi^{C}_{\nu,\|}(p,q)$ at one loop:



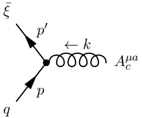
- No evanescent SCET operators for the longitudinal correlation functions.

Light-cone SCET sum rules for $\xi_{\|}(n \cdot p)$

- Subleading power SCET Feynman rules for soft-collinear interactions [Beneke, Garny, Szafron, Wang, 2018]:



$$ig_s t^a \begin{cases} 0 & \mathcal{O}(\lambda^0) \\ \Gamma_\mu(p) & \mathcal{O}(\lambda) \\ \left[n_{-\mu} + \gamma_{\perp\mu} \frac{\not{p}_\perp}{n_+ p} + \frac{n_{+\mu}}{n_+ k} \frac{p^2}{n_+ p}\right] \frac{\not{n}_+}{2} - (p' X_\perp)\Gamma_\mu(p) & \mathcal{O}(\lambda^2) \end{cases} \tag{A.35}$$



$$ig_s t^a \begin{cases} 0 & \mathcal{O}(\lambda^0) \\ \Gamma_\mu(p') & \mathcal{O}(\lambda) \\ \frac{\not{n}_+}{2} \left[n_{-\mu} + \frac{\not{p}'_\perp}{n_+ p'} \gamma_{\perp\mu} - \frac{n_{+\mu}}{n_+ k} \frac{(p')^2}{n_+ p'}\right] + (p X_\perp)\Gamma_\mu(p') & \mathcal{O}(\lambda^2) \end{cases}$$

Here we introduce the short-hand notation

$$X_\mu = \partial_\mu \left[(2\pi)^4 \, \delta^{(4)} \left(\sum p_{in} - \sum p_{out}\right)\right].$$

- Soft-collinear factorization formula for $\Pi_{v,\|}(p,q)$ [Gao, Lü, Shen, Wang, Wei, 2019]:

$$\Pi_{v,\|}(p,q) = \frac{\tilde{f}_B(\mu)\, m_B}{2} \int_0^{+\infty} \frac{d\omega}{\bar{n}\cdot p - \omega + i0} \left\{ \left[1 + \frac{\alpha_s C_F}{4\pi} \hat{J}^{(\mathrm{A0})}_{\|,-}\left(\frac{\mu^2}{n\cdot p\,\omega}, \frac{\omega}{\bar{n}\cdot p}\right)\right] \phi_B^-(\omega,\mu) + \left[\frac{\alpha_s C_F}{4\pi} \hat{J}^{(m)}_{\|,+}\left(\frac{\mu^2}{n\cdot p\,\omega}, \frac{\omega}{\bar{n}\cdot p}\right)\right] \phi_B^+(\omega,\mu) \right\} \bar{n}_v \,.$$

Light-cone SCET sum rules for $\Xi_{\parallel}(n \cdot p)$

- Constructing the vacuum-to-B-meson correlation function [Gao, Lü, Shen, Wang, Wei, 2019]:

$$\widetilde{\Pi}_{\nu,\parallel}(p,q,\tau) = \frac{n \cdot p}{2\pi} \int d^4x \, e^{ip\cdot x} \int d^4y \int dr \, e^{-in\cdot p \, \tau \, r} \langle 0 | \mathrm{T} \left\{ j^{(0)}_{\xi \xi, \nu}(x), \, i \mathscr{L}^{(1)}_{\xi q_s}(y), \, \left(\bar{\xi} \, W_c \right)(0) \, \gamma_5 \, (W_c^{\dagger} \, i \not{D}_{c\perp} \, W_c)(rn) \, h_v(0) \right\} | \bar{B}_v \rangle .$$

- SCET factorization formula for the correlation function at tree level:

$$\widetilde{\Pi}_{\nu,\parallel}(p,q,\tau) = \frac{\alpha_s \, C_F}{4\pi} \tilde{f}_B(\mu) \, m_B \, [(1-\tau)\,\theta(\tau)\,\theta(1-\tau)] \times \int_0^{\infty} \frac{d\omega'}{\omega' - \bar{n}\cdot p - i0} \left[\int_{\omega'}^{\infty} d\omega \, \frac{n\cdot p}{\omega} \, \phi_B^+(\omega,\mu) \right] \bar{n}_{\nu} \, .$$

Non-trivial τ-dependence beyond the tree level requires two-loop computations.

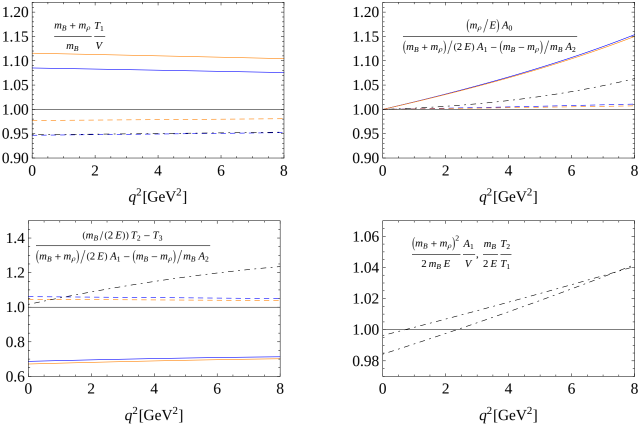
- SCET sum rules for $\Xi_{\parallel}(n \cdot p)$ at LO in α_s:

$$\Xi_{\parallel}(\tau, n\cdot p) = -\frac{\alpha_s \, C_F}{\pi} \, \frac{U_2(\mu_{h2},\mu) \, \tilde{f}_B(\mu_{h2})}{f_{V,\parallel}} \, \frac{m_B \, m_V}{n\cdot p \, m_b} \, [(1-\tau)\,\theta(\tau)\,\theta(1-\tau)] \times \int_0^{\omega_s} d\omega' \exp\left[-\frac{n\cdot p \, \omega' - m_V^2}{n\cdot p \, \omega_M} \right] \int_{\omega'}^{\infty} d\omega \, \frac{\phi_B^+(\omega,\mu)}{\omega} + \mathscr{O}(\alpha_s^2) \, .$$

In agreement with the SCET factorization formula for $\Xi_{\parallel}(n \cdot p)$ obtained in [Beneke, Yang, 2006].

Light-cone SCET sum rules for $B \to V$ form factors

- Long-standing puzzles for 18 years [Bell, Beneke, Huber, Li, 2011]:

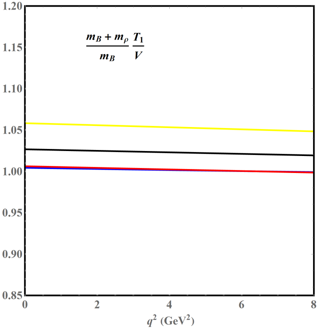


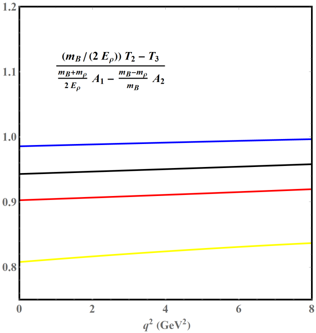
Discrepancies between the SCET predictions (blue solid) and the LCSR results with light vector meson DAs (black dashed-dotted) [Ball, Zwicky, 2005].

- Subleading power correction in heavy quark expansion, systematic uncertainties?

Light-cone SCET sum rules for $B \to V$ form factors

- Comparison between the SCET calculations and the B-meson LCSR predictions:





- The underlying mechanism responsible for the discrepancies:

$$\mathscr{R}_{1,\mathrm{LCSR}} = 1 + (-0.049)\big|_{C_i^{(\mathrm{A0})}} + (+0.054)\big|_{C_i^{(\mathrm{B1})}} + (-3.5 \times 10^{-5})\big|_{\mathrm{3PHT}},$$

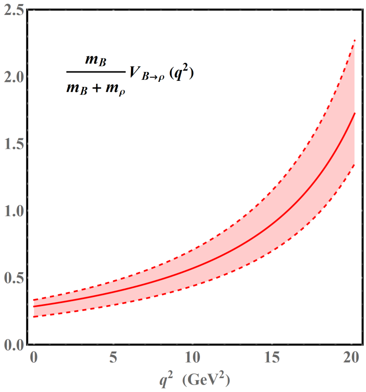
$$\mathscr{R}_{1,\mathrm{QCDF}} = 1 + (-0.023)\big|_{C_i^{(\mathrm{A0})}} + (+0.086)\,[1 + \mathscr{O}(\alpha_s)]\big|_{C_i^{(\mathrm{B1})}}.$$

(a) No perturbative expansion for the A0-type $\mathrm{SCET_I}$ form factor in QCDF.

(b) Almost a factor of two smaller prediction of the B1-type $\mathrm{SCET_I}$ form factor in LCSR.

Light-cone SCET sum rules for $B \to V$ form factors

- The predicted vector $B \to \rho$ form factor:
  - Dominant uncertainty from the model dependence of the B-meson DAs at a reference scale.
  - $V_{B\to\rho}(0) = 0.327 \pm 0.031$ from the light-meson LCSR as an input to determine λ_B.
  - Extrapolation toward the large q^2 region with the z-series expansion.



- Exclusive $|V_{ub}|$ from $B \to \rho \ell \nu_\ell$ [Gao, Lü, Shen, Wang, Wei, 2019]:

$$|V_{ub}| = \left(3.05\,^{+0.67}_{-0.52}\Big|_{\rm th.}\,^{+0.19}_{-0.20}\Big|_{\rm exp.}\right) \times 10^{-3}\,.$$

Confronted with $|V_{ub}|_{\rm PDG} = \left(3.70 \pm 0.12\big|_{\rm th.} \pm 0.10\big|_{\rm exp.}\right)$ from $B \to \pi \ell \nu_\ell$.

Theoretical wishlist

- Systematic understanding of the (high-twist) B-meson distribution amplitudes.
  - Renormalization properties beyond the one-loop approximation [conformal symmetry].
  - Perturbative constraints at large ω_i [OPE technique].
  - Renormalon analysis and the renormalization-scheme dependence.
  - Precision determinations of the inverse moment λ_B.

- QCD factorization for the subleading power corrections.
  - SCET analysis for the pion-photon form factor as the first step [operator structures, symmetry constraints, etc].
  - General treatment of the rapidity divergences in the (naïve)-factorization formulae.
  - Rigorous factorization proof taking into account the Glauber gluons.
  - Novel resummation techniques for enhanced logarithms [symmetry, geometry].

- Technical issues for future improvements.
  - Factorization techniques for electromagnetic corrections.
  - NNLO QCD computations for $B \to V\ell\ell$ and $B \to V\gamma$.
  - QCD factorization for the radiative and electroweak penguin decays of the Λ_b-baryon.
  - Improved understanding of the parton-hadron duality violation.

- Very promising future for QCD aspects of heavy-quark physics!