Transport model study of chiral magnetic effects

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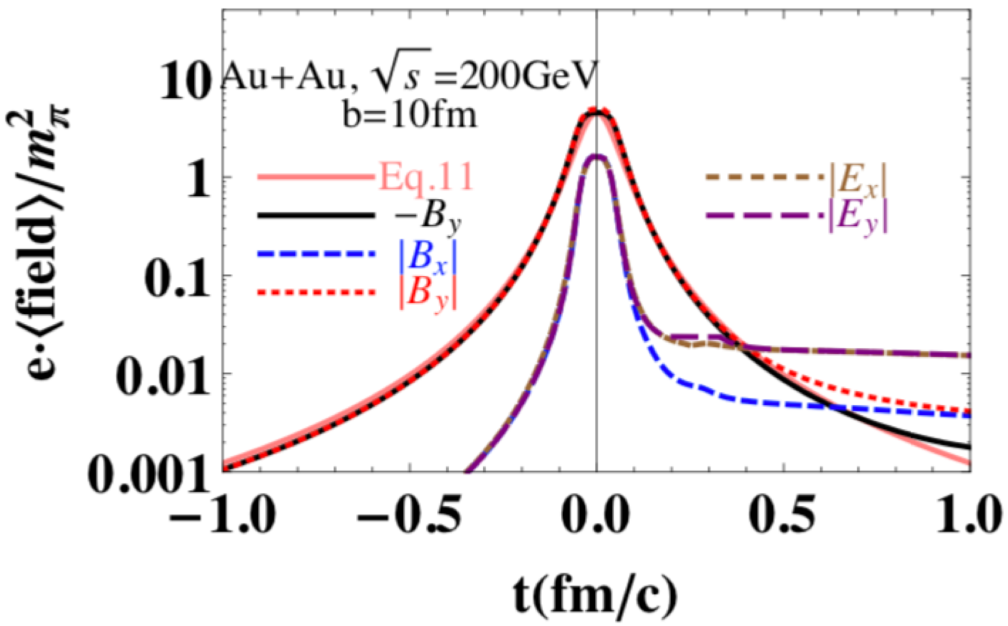
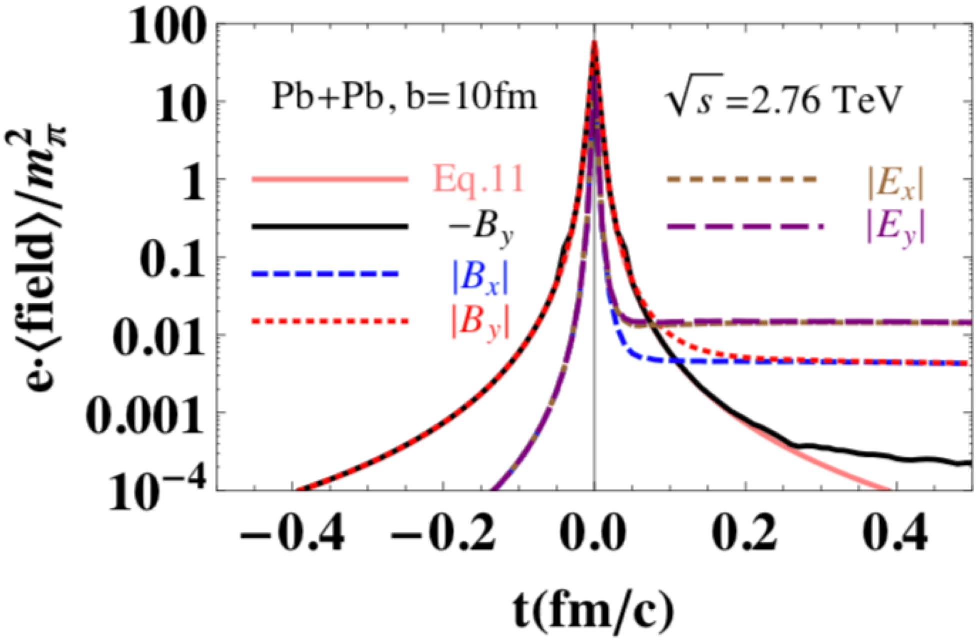
- ☐ Chiral transport model
- ☐ Chiral magnetic wave
- ☐ Chiral magnetic effect
- ☐ Chiral vortical effect
- ☐ Summary

Introduction: Electromagnetic field in relativistic HIC

W. T. Deng and X. G. Huang, PRC 85, 044907 (2012)





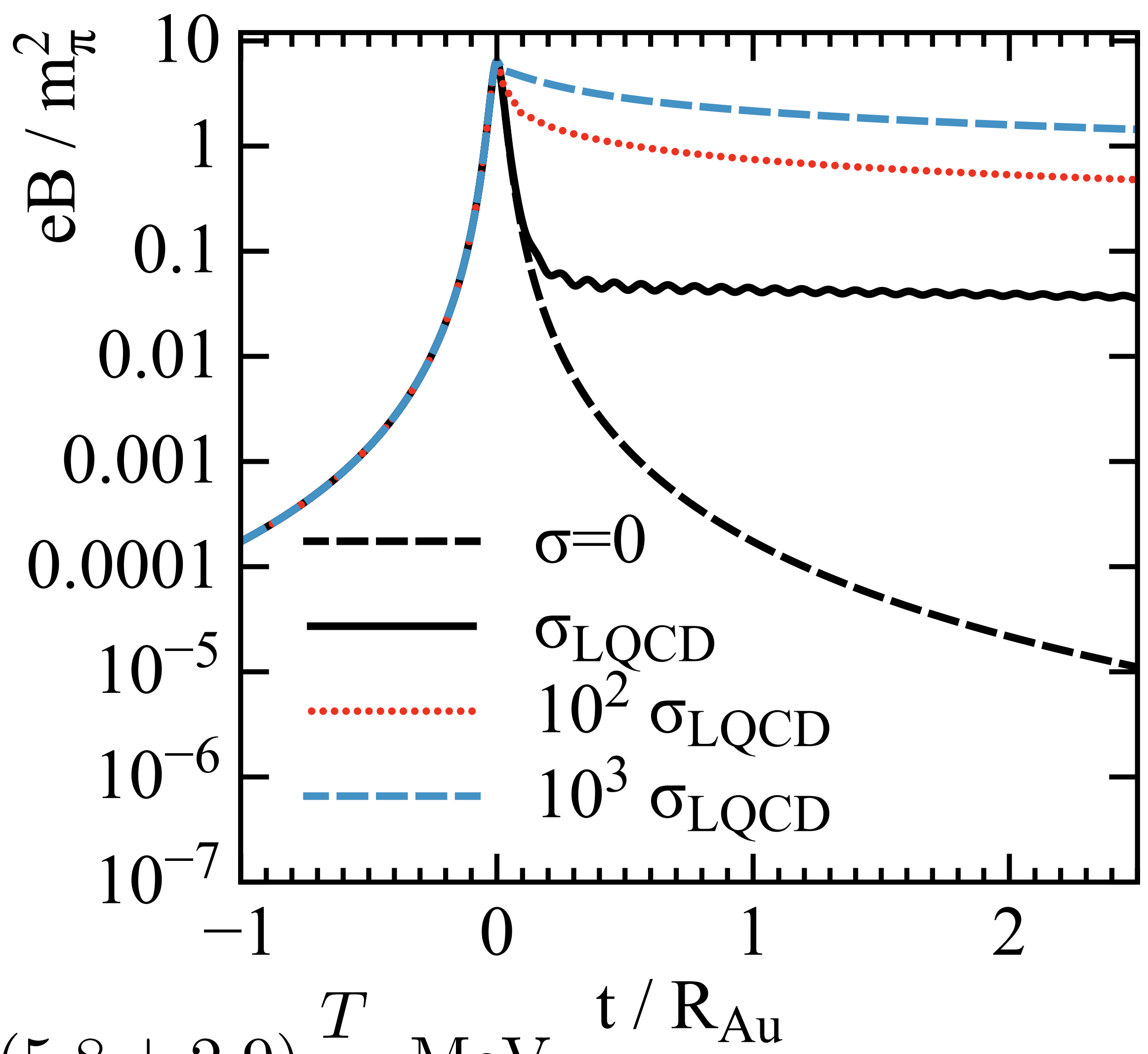
$$\langle eB_y(t)\rangle \approx \frac{\langle eB_y(0)\rangle}{(1+t^2/t_B^2)^{3/2}}, \quad t_B \approx \frac{R_A}{\gamma v_z} \approx \frac{2m_N}{\sqrt{s}} R_A \approx 0.065 \text{ fm}$$

at RHIC

- Calculated from the Lienard-Wiechert potential using the spatial and momentum information of the protons from HIJING.
- Magnitude of electromagnetic field is large but the lifetime is short.

Effect of QGP conductivity on magnetic field

L. Mclerran & V. Skokov, NPA 929, 184 (2014)



$$\sigma_{\mathrm{Ohm}}^{\mathrm{LQCD}} = (5.8 \pm 2.9)\frac{T}{T_0}\ \mathrm{MeV}$$

- Lifetime of magnetic field is long only if QGP is a perfect conductor.

Anomalous chiral effects

Vector current $J^{\mu} = \langle \bar{\Psi}\gamma^{\mu}\Psi \rangle = J^{\mu}_{R} + J^{\mu}_{L}$

Axial vector current $J^{\mu}_{5} = \langle \bar{\Psi}\gamma^{\mu}\gamma_{5}\Psi \rangle = J^{\mu}_{R} - J^{\mu}_{L}$

Axial anomaly $\partial_{\mu} J^{\mu}_{5} = \frac{Q^2}{2\pi^2}\mathbf{E}\cdot\mathbf{B} + \frac{N_f g^2}{16\pi^2} G^{a}_{\mu\nu}\tilde{G}^{\mu\nu}_{a}$

$$\begin{pmatrix} \mathbf{J} \\ \mathbf{J_5} \end{pmatrix} = \begin{pmatrix} \sigma & \sigma_5 \\ \sigma_{\chi_e} & \sigma_S \end{pmatrix} \begin{pmatrix} \mathbf{E} \\ \mathbf{B} \end{pmatrix}$$

Ohm′s law $\mathbf{J} = \sigma\mathbf{E}$

Chiral magnetic effect $\mathbf{J} = \sigma_{\mathbf{5}}\mathbf{B}$, $\sigma_{\mathbf{5}} = \frac{\mathbf{Q^2}\mu_{\mathbf{5}}}{\mathbf{2}\pi^{\mathbf{2}}}$

Chiral electric separation effect $\mathbf{J_5} = \sigma_{\chi_{\mathbf{e}}}\mathbf{E}$, $\sigma_{\chi_{\mathbf{e}}} = \chi_{\mathbf{e}}\mu\mu_{\mathbf{5}}$

Chiral separation effect $\mathbf{J_5} = \sigma_{\mathbf{S}}\mathbf{B}$ $\sigma_{\mathbf{S}} = \frac{\mathbf{Q^2}\mu}{\mathbf{2}\pi^{\mathbf{2}}}$

Chiral magnetic wave

Kharzeev & Yee, PRD 83, 085007 (2011)
Burnier, Kharzeev, Liao & Yee, PRL 107, 052303 (2011)

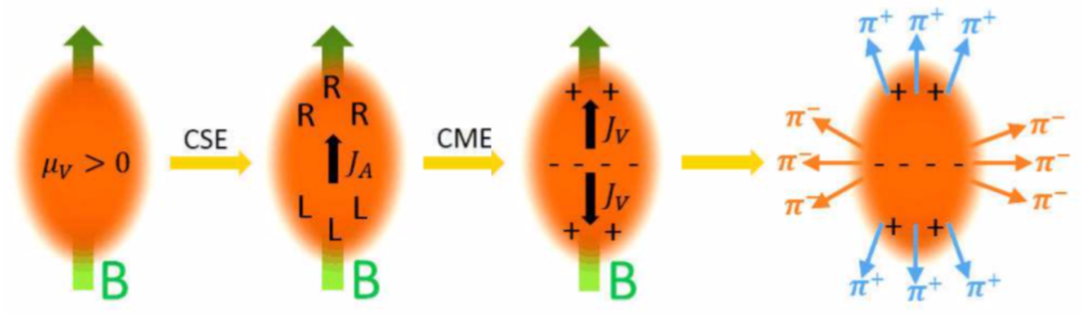
From the chiral magnetic and separation effects, $\mathbf{J}_{R,L} = \pm \frac{Q^2 \mu_{R,L}}{4\pi^2}\mathbf{B},$

Separate conservation of vector charges of particles of right and left chiralities

$$\frac{\partial \rho_{R,L}}{\partial t} \pm \boldsymbol{\nabla} \cdot \left(\frac{Q^2 \mu_{R,L}}{4\pi^2}\mathbf{B} \right) = 0$$

Small deviation from equilibrium leads to chiral magnetic waves propagating along and opposite to the direction of magnetic field

$$\frac{\partial \delta\rho_{R,L}}{\partial t} \pm \left(\frac{Q^2}{4\pi^2 \chi_{R,L}} \right) \mathbf{B} \cdot \boldsymbol{\nabla} \delta\rho_{R,L} = 0, \quad \chi_{R,L} = \frac{\partial \rho_{R,L}}{\partial \mu_{R,L}}$$

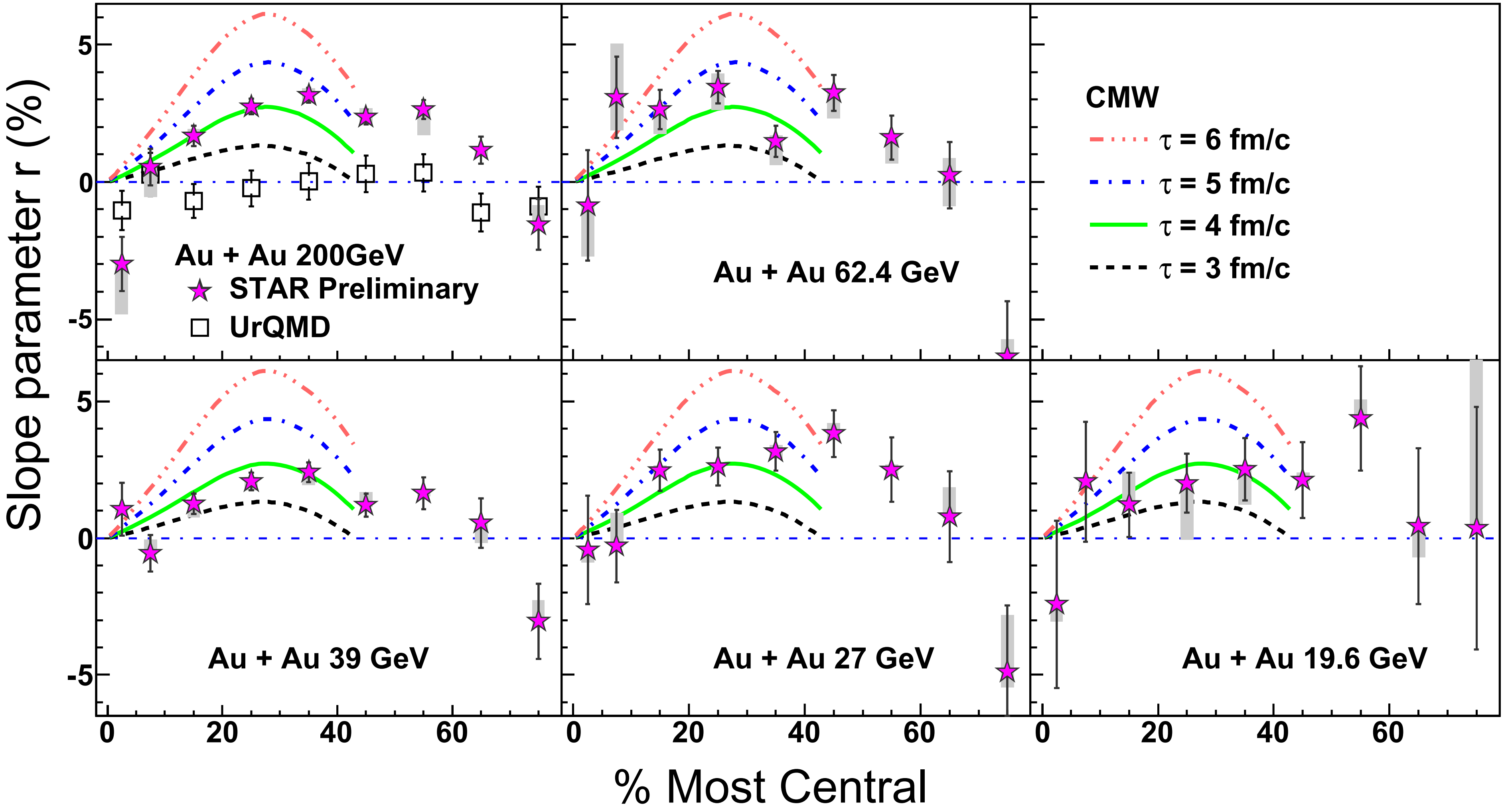


→ Larger elliptic flow for negatively charged than positively charged particles

Chiral magnetic wave and elliptic flow splitting

G. Wang et al., NPA 904-905, 248c (2013)

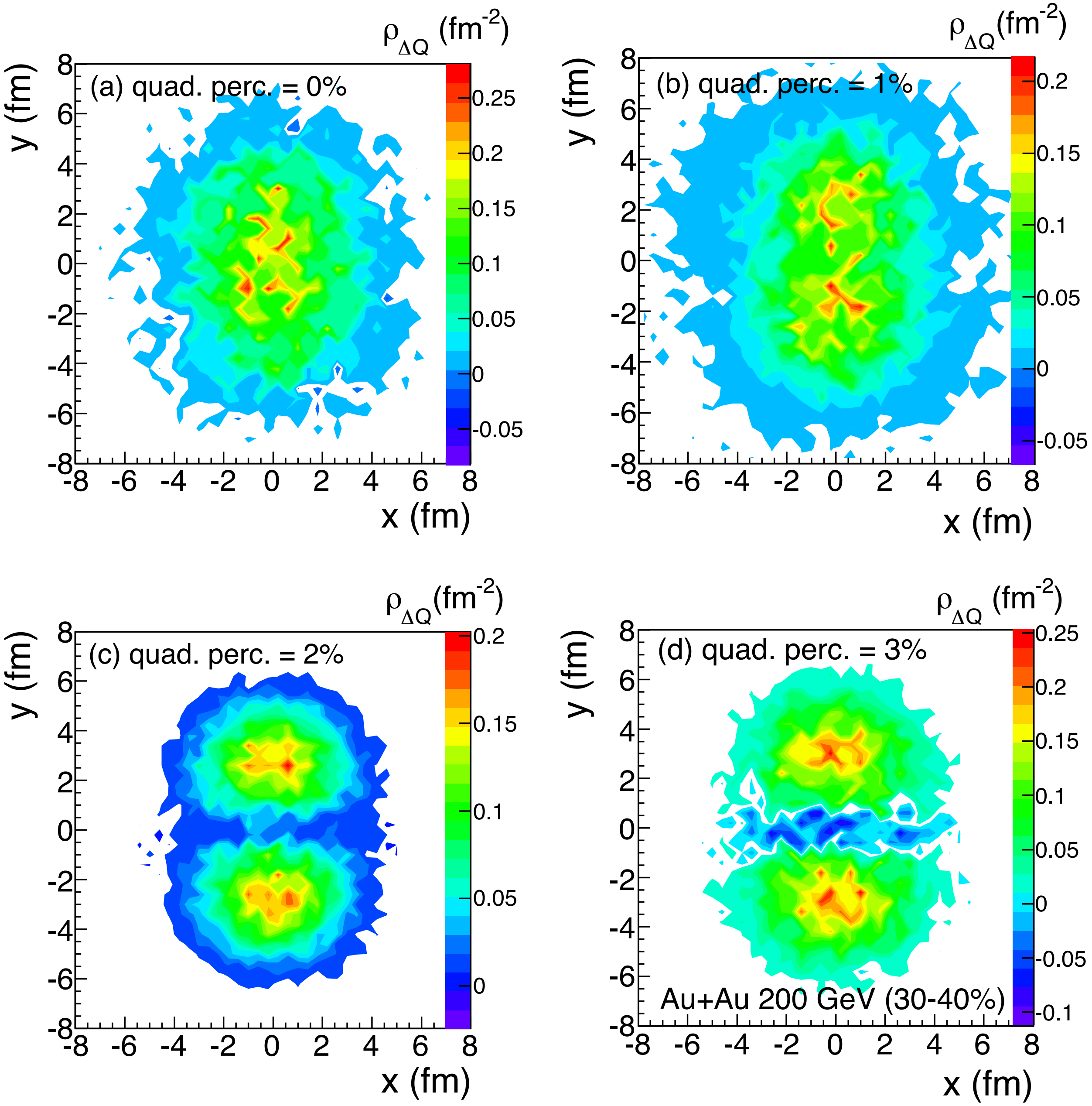
Require magnetic field of lifetime of 4 fm/c



$$\Delta v_2 = v_2(-) - v_2(+), \quad A_{\pm} = \frac{N_+ - N_-}{N_+ + N_-}, \quad \text{slope parameter} = \frac{\Delta v_2}{A_{\pm}}$$

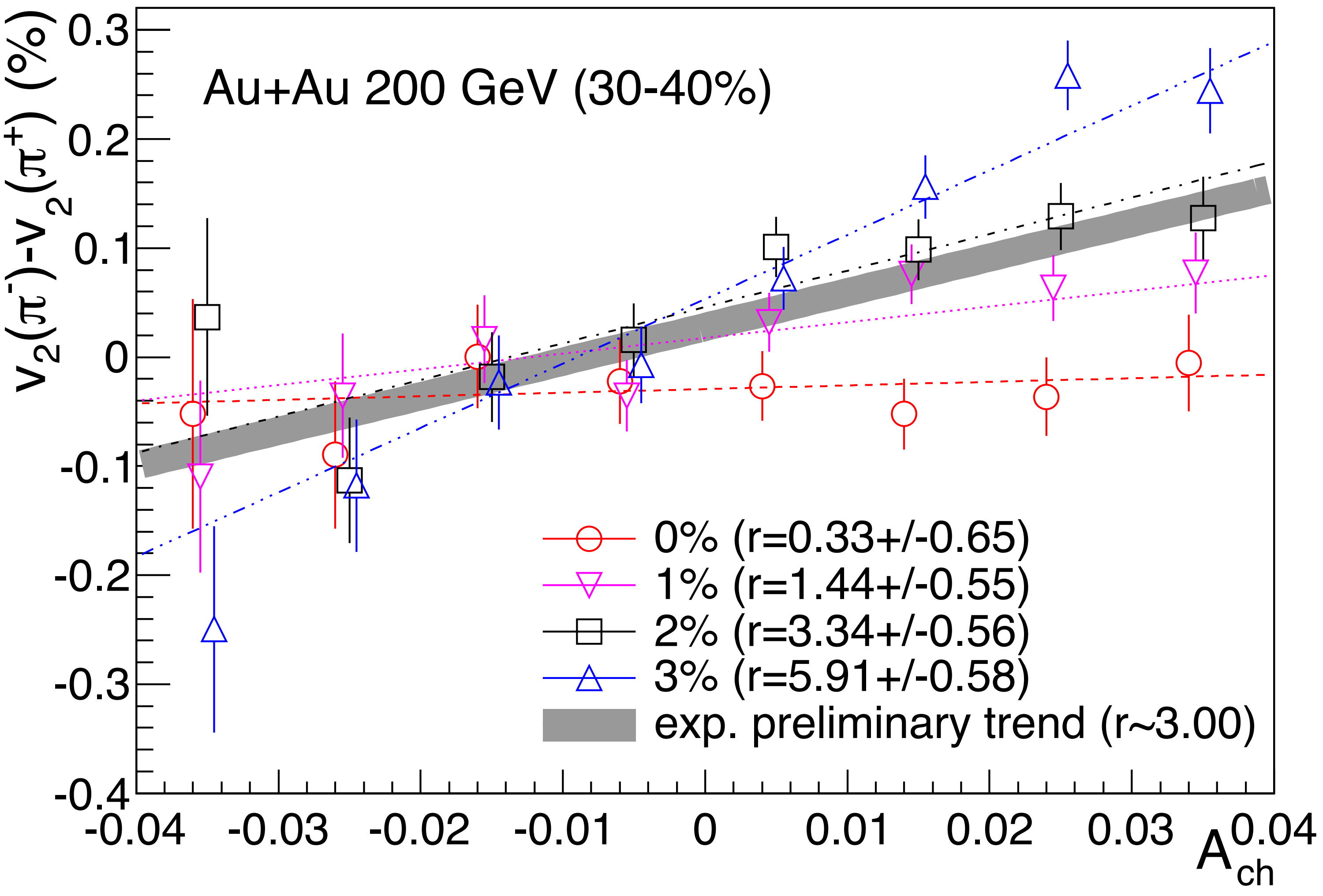
Final-state effect on charge asymmetry of pion elliptic flow

G. L. Ma, PLB 735, 383 (2014)



Modified initial distributions in the transverse plane of a collision described by AMPT

Charge asymmetry of pion elliptic flow



- Intersection at A_{ch} = 0 is sensitive to initial quadrupole moment in transverse plane.

Chiral kinetic equation with electromagnetic field

- Path integral: Stephanov & Yin, PRL 109, 162001 (2012)
- Poisson brackets: Son & Yamamoto, PRD 87, 085016 (2013)
- Covariant Wigner function: Chen, Pu, Wang & Wang, PRL 110, 262301 (2013)

$$\frac{dt}{d\tau}\partial_t f_{R/L} + \frac{d\mathbf{x}}{d\tau}\cdot\nabla_{\mathbf{x}} f_{R/L} + \frac{d\mathbf{p}}{d\tau}\cdot\nabla_{\mathbf{p}} f_{R/L} = 0$$

$$\frac{dt}{d\tau} = 1 \pm Q\mathbf{b}\cdot\mathbf{B}$$

Plus: positive helicity
Minus: negative helicity

$$\frac{d\mathbf{x}}{d\tau} = \hat{\mathbf{p}} \pm Q(\hat{\mathbf{p}}\cdot\mathbf{b})\mathbf{B} \pm Q(\mathbf{E}\times\mathbf{b})$$

$$\frac{d\mathbf{p}}{d\tau} = Q(\mathbf{E}+\hat{\mathbf{p}}\times\mathbf{B}) \pm Q^2(\mathbf{E}\cdot\mathbf{B})\mathbf{b} \mp Q|\mathbf{p}|(\mathbf{E}\cdot\mathbf{b})\mathbf{b}$$

$$\text{Three}-\text{dimensional Berry curvature} \quad \mathbf{b} = \frac{\mathbf{p}}{2|\mathbf{p}|^3}$$

Modified quark scattering

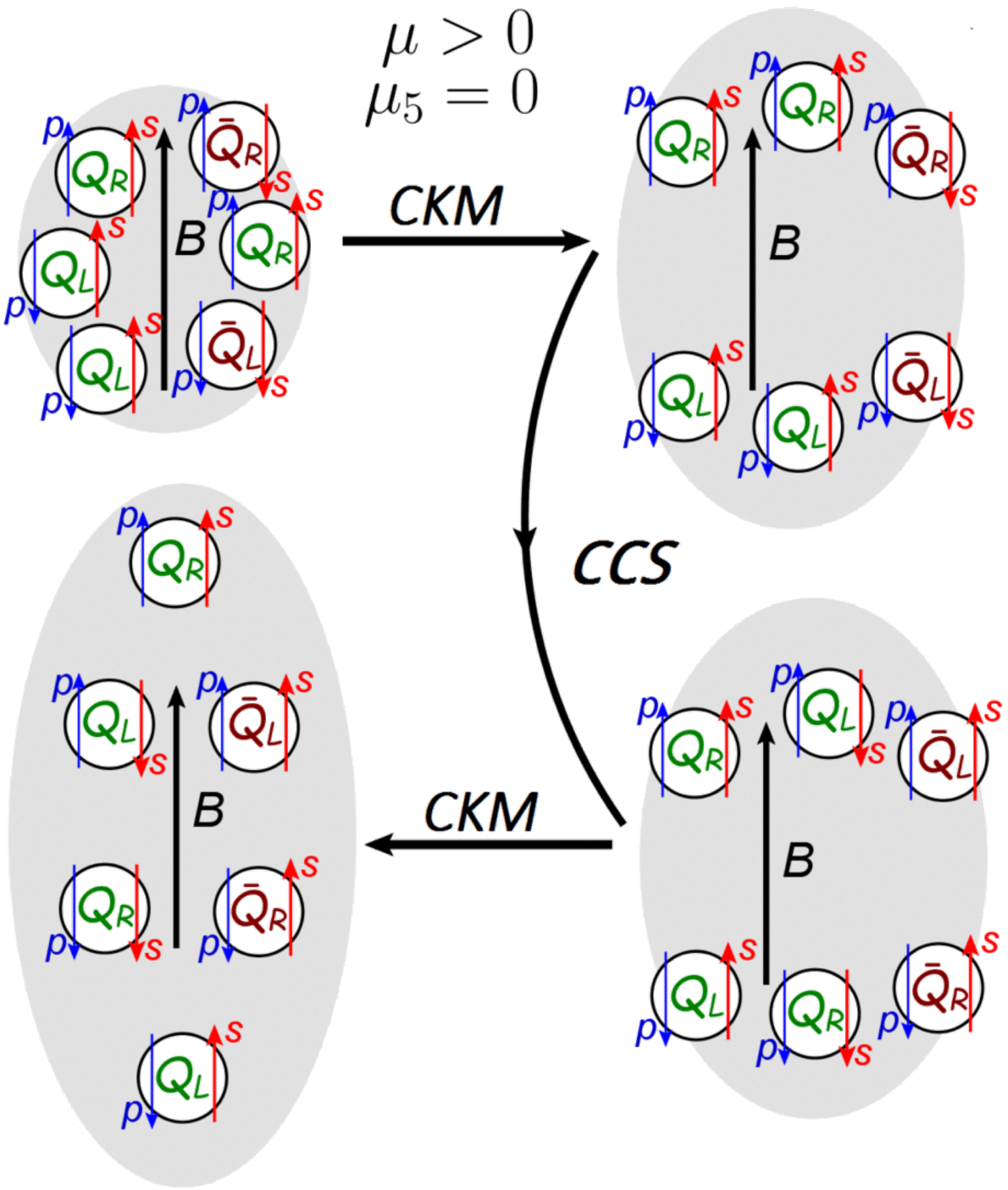
- To ensure massless fermions to reach the equilibrium distribution

$$\sqrt{G} f\left(\frac{p-\mu}{T}\right), \quad \sqrt{G}=1+\lambda Q(\mathbf{b}\cdot\mathbf{B})$$

the momenta $\mathbf{p}_3$ and $\mathbf{p}_4$ of two colliding fermions after a collision are determined by momentum conservation with the probability proportional to $\sqrt{G(\boldsymbol{p}_3)G(\boldsymbol{p}_4)}$, which is set to zero if negative.

- For collision between a fermion and its antiparticle that have opposite helicities, their helicities can be flipped after the collision [Sun, Ko & Li, PRC 94, 045204 (2016); 95, 034909 (2017)].

Chirality changing scattering (CCS)



- CKE leads to the separation of particles of right chirality and left chirality.

- CCS ($R\overline{R} \leftrightharpoons L\overline{L}$) resulting in more positively charged particles moving in y-direction.

$$\dot{\mathbf{r}} = \frac{\hat{\mathbf{p}} + Qh(\hat{\mathbf{p}} \cdot \mathbf{b})\mathbf{B}}{1 + Qh\mathbf{B} \cdot \mathbf{b}} \qquad \mathbf{b} = \frac{\mathbf{p}}{2p^3}$$

Chiral transport model

$$\frac{dt}{d\tau}\partial_t f_{R/L} + \frac{d\mathbf{x}}{d\tau}\cdot\nabla_{\mathbf{x}} f_{R/L} + \frac{d\mathbf{p}}{d\tau}\cdot\nabla_{\mathbf{p}} f_{R/L} = C(f_{R/L})$$

Application to non-central HIC with initial conditions

$$T(x,y) = \frac{T_0}{\left(1+e^{\frac{\sqrt{x^2+y^2/c^2}-R}{a}}\right)^{1/3}}$$

$$eB_y = \frac{eB_0}{1+(t/\tau)^2}$$

Longitudinal distribution

$z = \tau_0 \sinh y$, $p_z = m_T \cosh y$

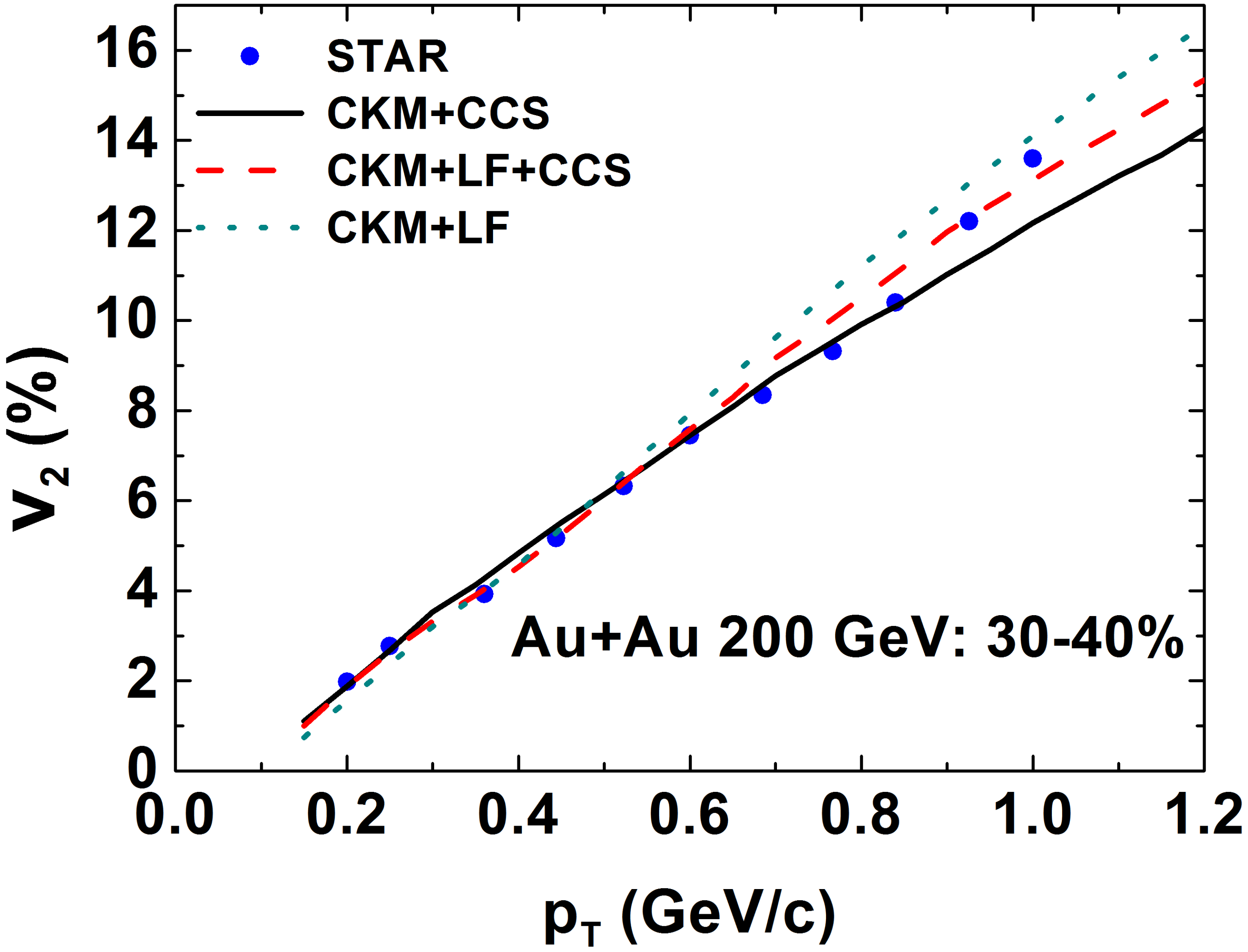
$\tau_0 = 0.4$ fm/c

$$T_0 = 300\ \text{MeV},\ R = 3.5\ \text{fm},\ a = 0.5\ \text{fm},\ c = 1.5\ \text{fm}$$

$$eB_0 = 7\ m_\pi^2,\ \tau = 6\ \text{fm}/c$$

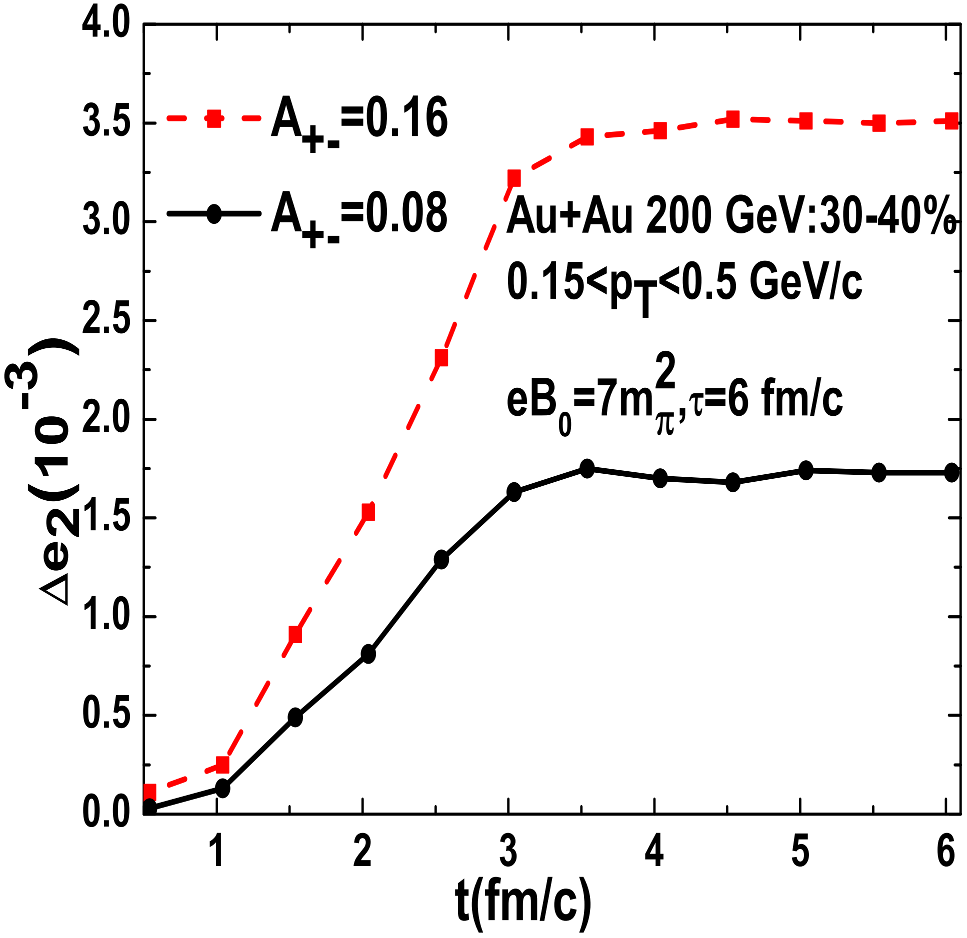
$$\sigma = \sigma_0 (T_0/T)^3 \text{ with } \sigma_0 = 13 \sim 15 \text{ mb by fitting to measured } v_2$$

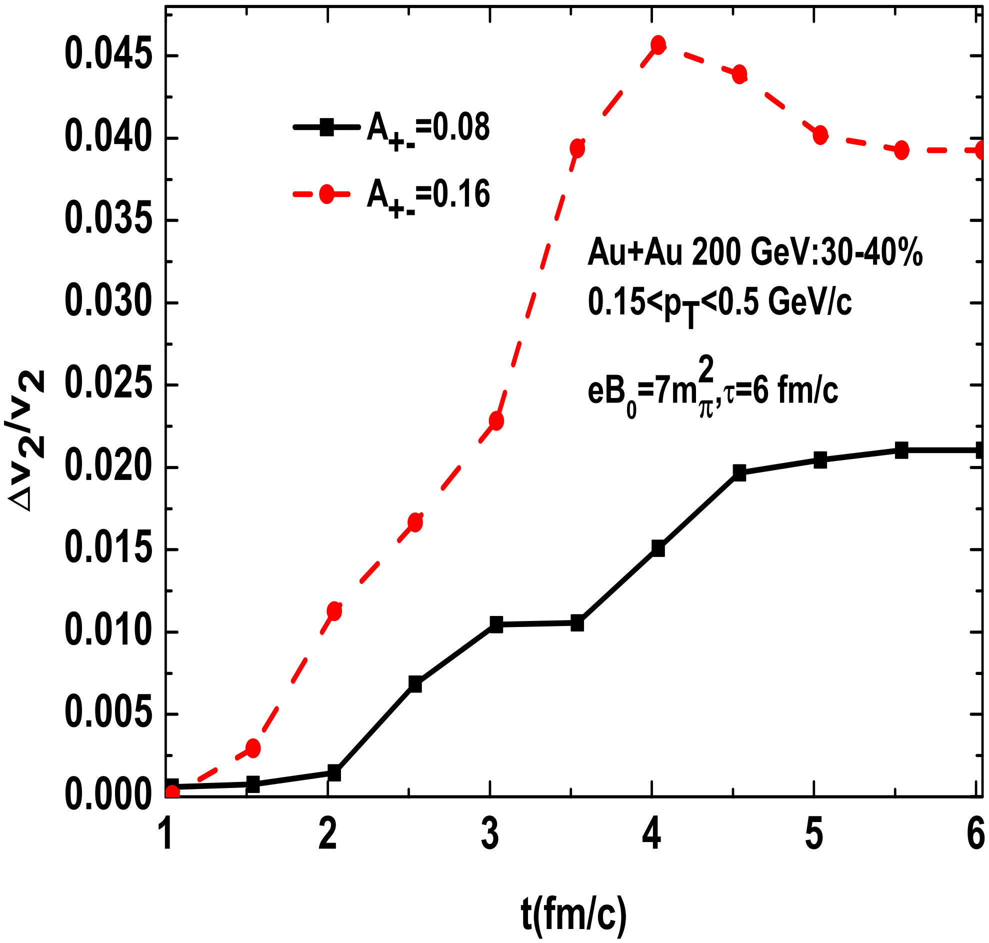
Differential elliptic flow



Data from Adams *et al.* (STAR Collaboration), PRC 72, 014904 (2005)

Time evolution of eccentricity and v_2 splittings

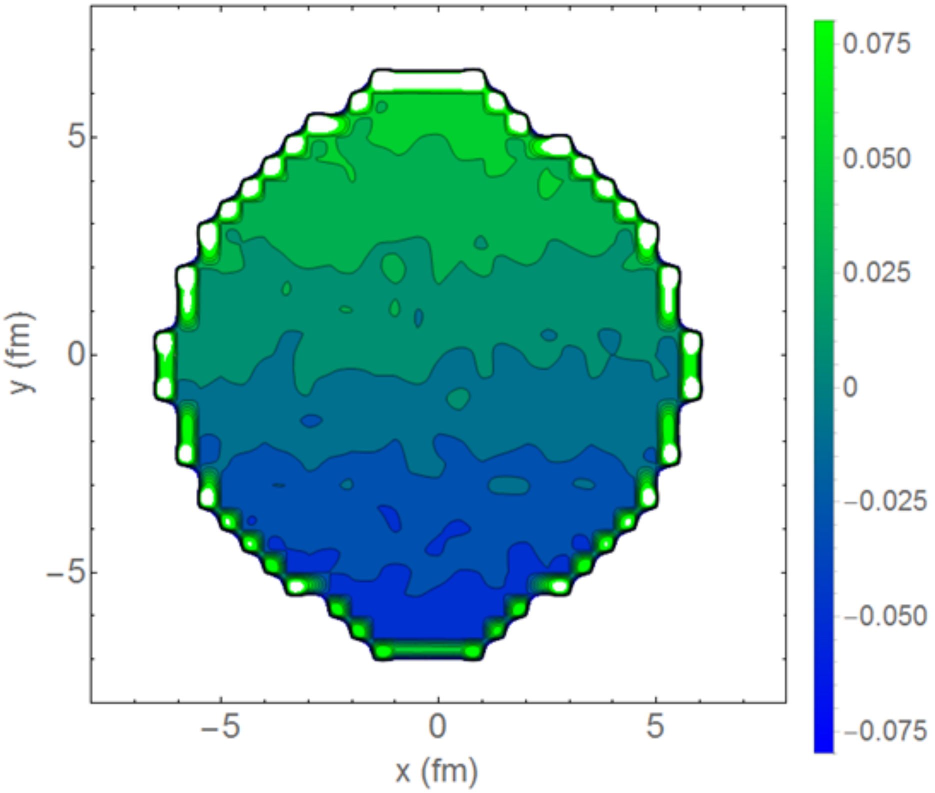




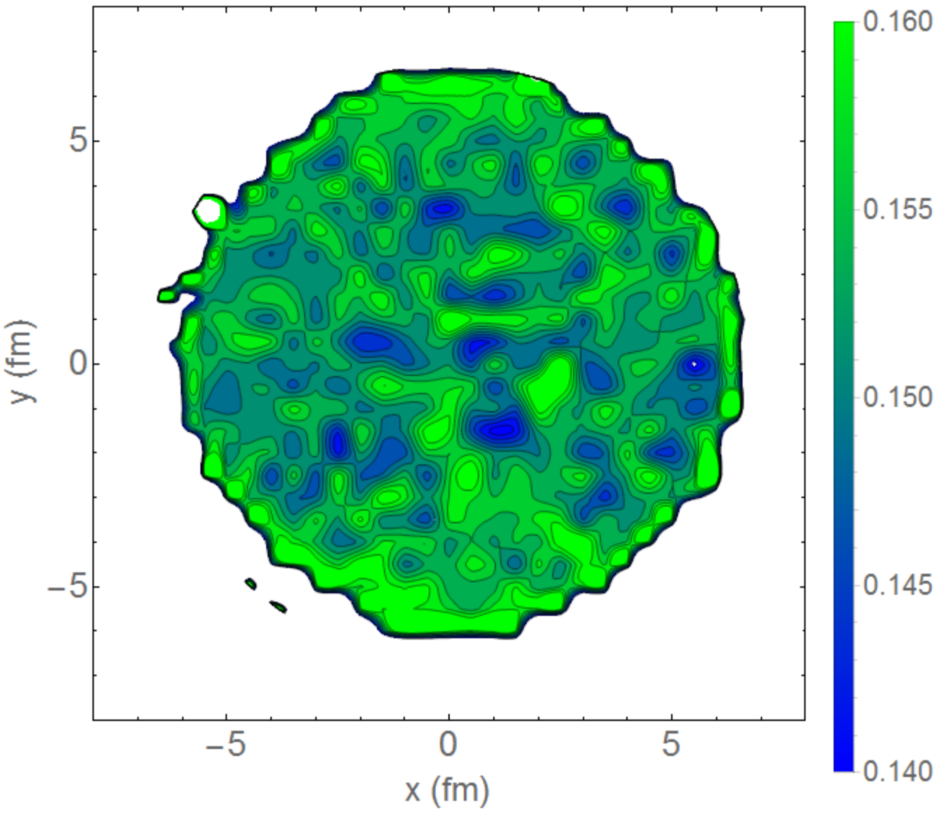
- Including only chiral kinetic motion (CKM) and chirality changing quark-antiquark scattering (CCS) and neglecting the Lorentz force.

Vector and axial vector charge distributions

@ z = 0 & $A_{\pm} = 0.16$

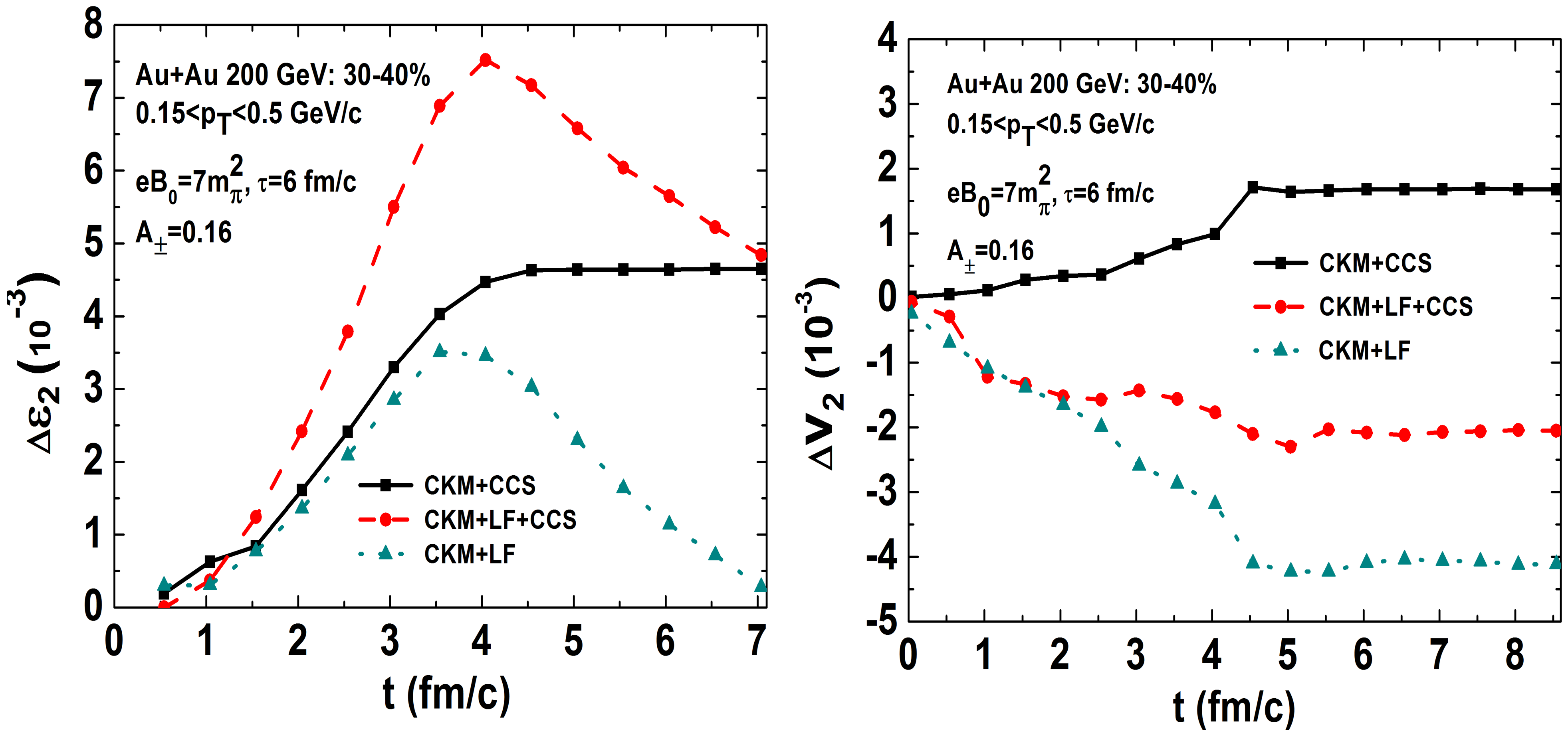


Axil charge distribution
(dipole moment)



Charge distribution
(quadrupole moment)

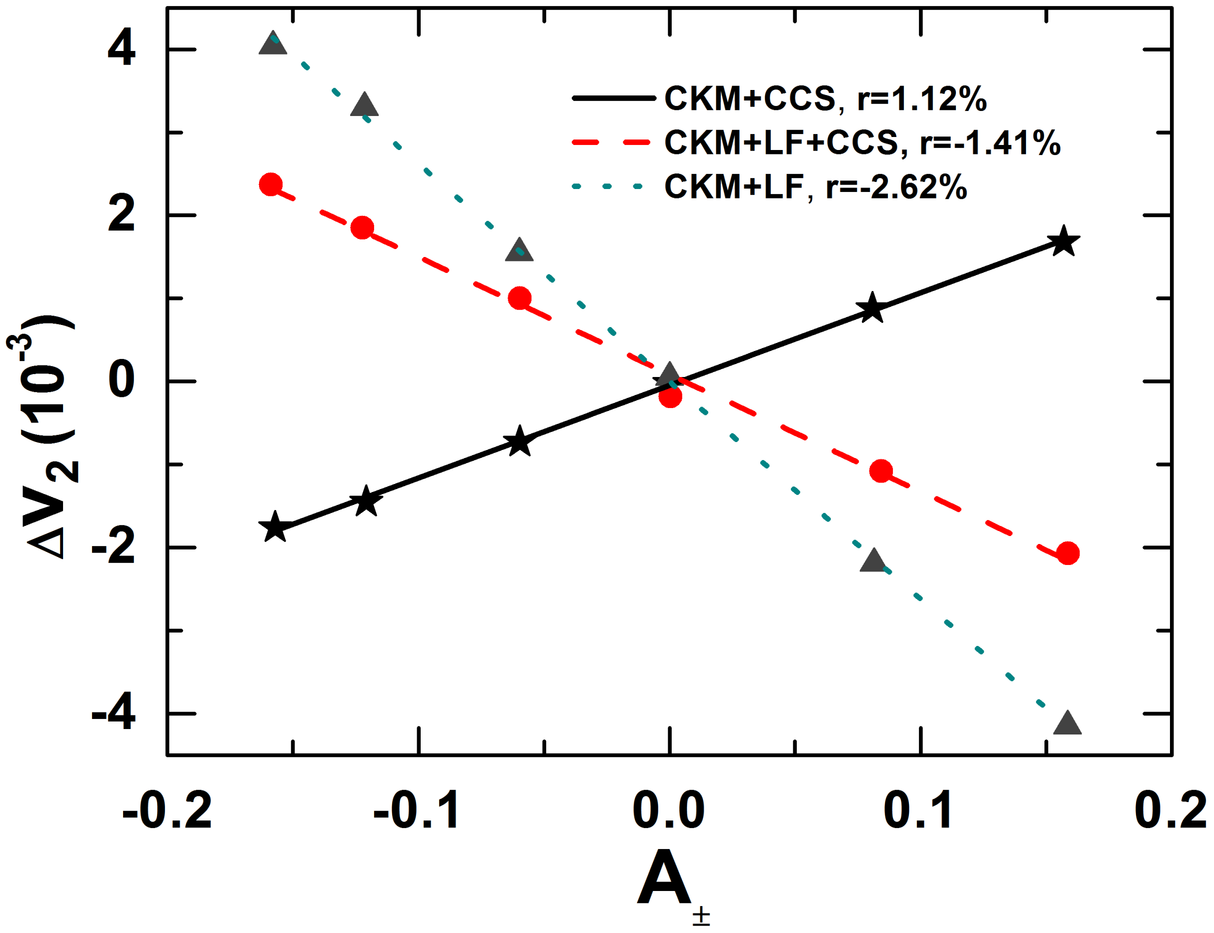
Effect of Lorentz force



- Not included before [Y. Burnier et al., PRL 107 (2011); M. Hongo *et al.*, arXiv 1309.2823 (2013); Yee & Yin, PRC 89 (2014)].
- Larger elliptic flow for positively charged than for negatively charged particles, leading to negative v_2 splitting.

Charge asymmetry dependence of v_2 splitting

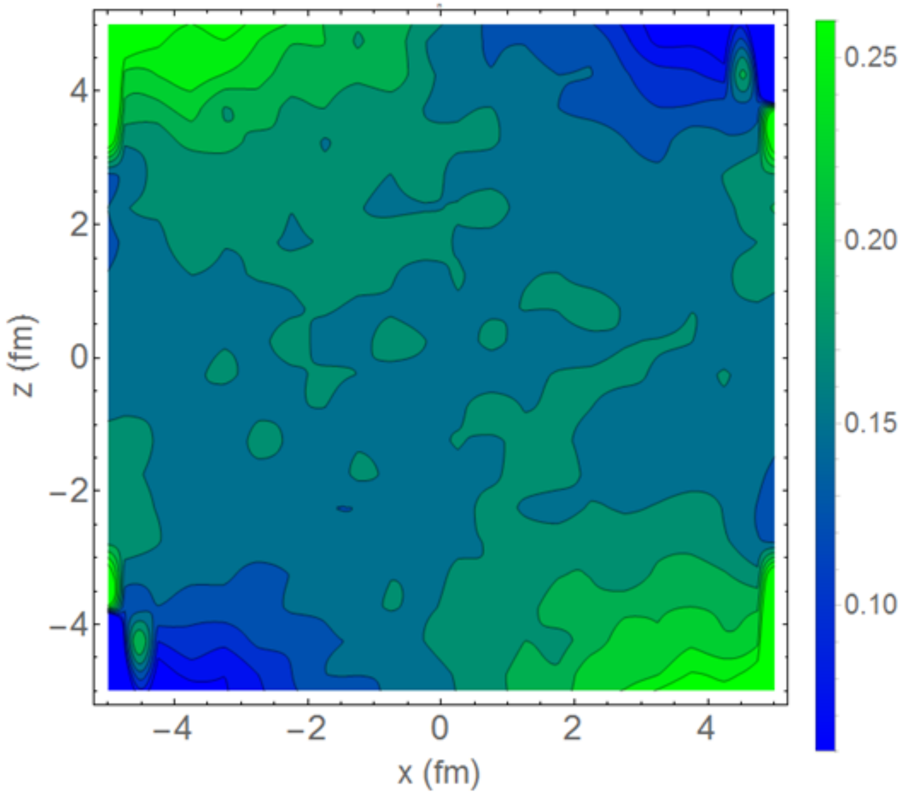
Sun & Ko, PRC 94, 045204 (2016); 95, 034909 (2017)

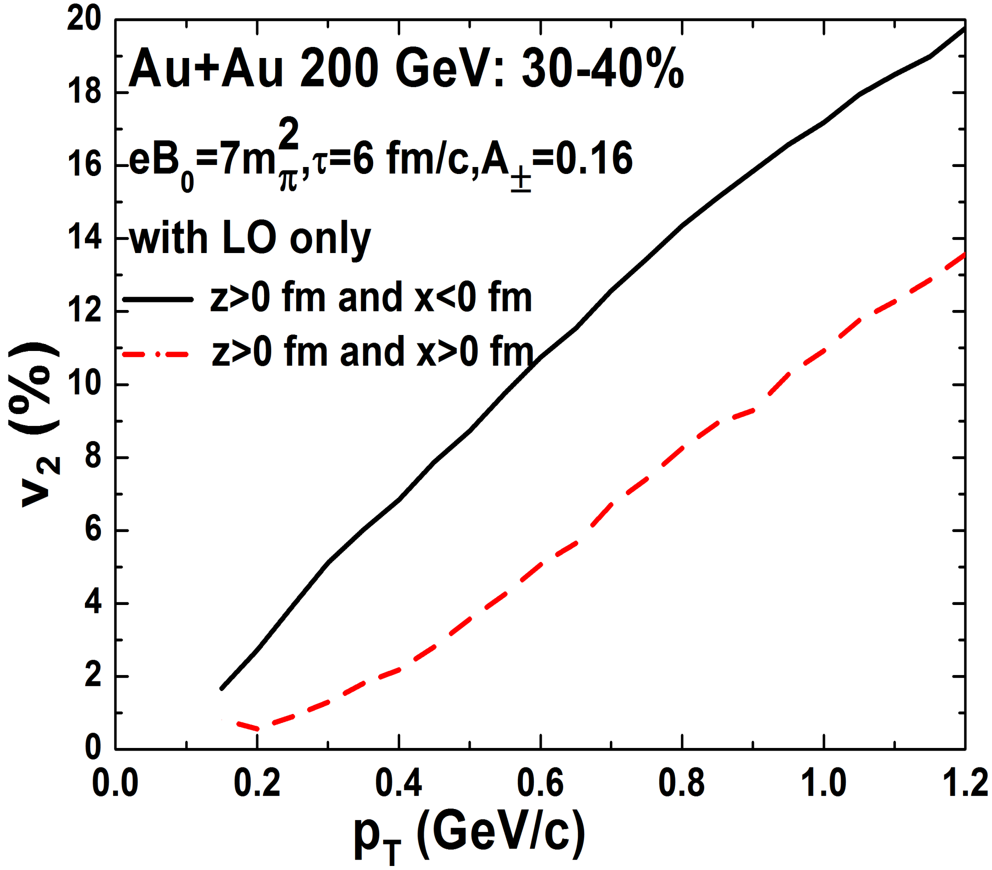


- Lorentz force leads to negative slope parameter.
- The positive slope parameter (r = 1%) without LF is smaller than experiment data (r = 3%).

Effect of Lorentz force on charge distribution

$$\frac{\mu}{T} = \frac{N_R - N_{\bar{R}} + N_L - N_{\bar{L}}}{N_R + N_{\bar{R}} + N_L + N_{\bar{L}}}$$





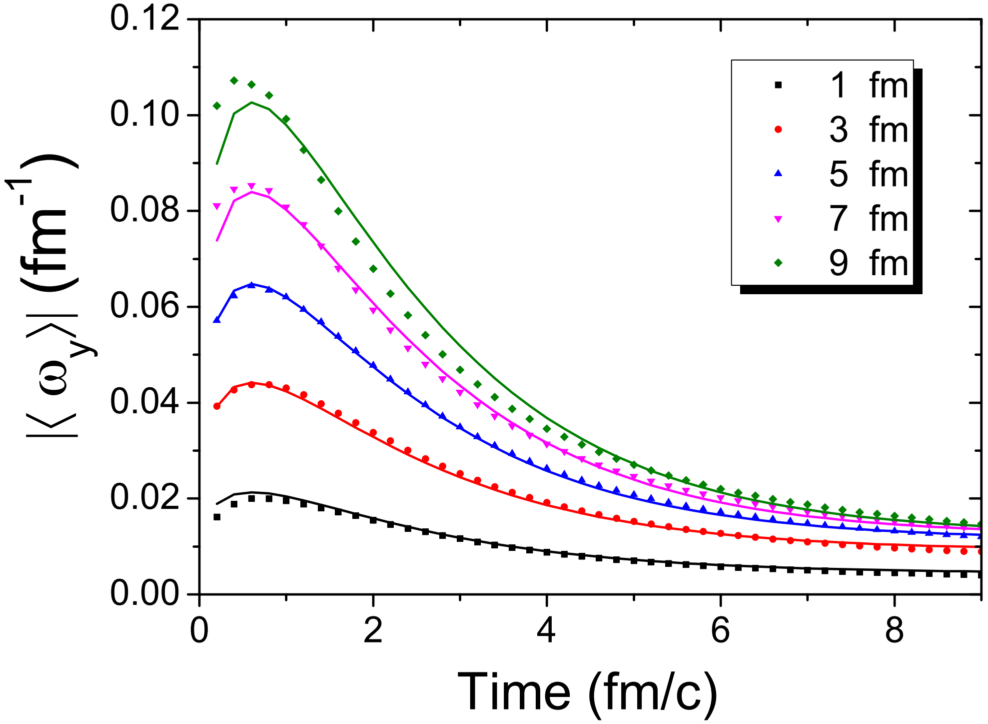
- Flow is larger in z-direction because of initial narrow size in z-direction.
- Lorentz force leads to different v_1 for positively and negatively charged particles.
- Elliptic flow is larger for particles in upper left and lower right quadrants.

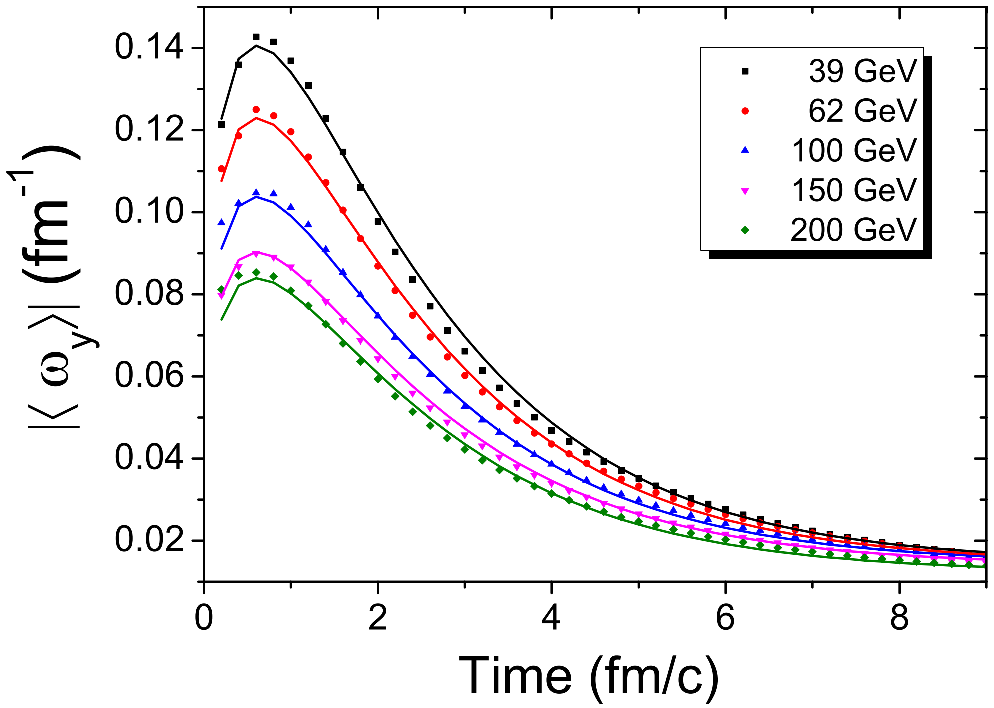
Vorticity in relativistic heavy ion collisions

Jiang, Lin & Liao,
PRC 94, 044910 (2016)

Based on AMPT

$$\vec{\omega} = \frac{1}{2}\nabla\times\vec{v}, \quad \langle\omega_y\rangle = \frac{\int d^3\vec{r}\,[\mathcal{W}(\vec{r})]\,\omega_y(\vec{r})}{\int d^3\vec{r}\,[\mathcal{W}(\vec{r})]}, \quad \mathcal{W}(\vec{r}) = \rho\epsilon(\vec{r})$$





- Average vorticity decreases with time, decreasing impact parameter, and increasing collision energy.

Chiral kinetic equation with both electromagnetic and vorticity fields

- Coriolis force: Stephanov & Yin, PRL 109, 162001 (2012)
- Covariant Wigner function: Chen, Pu, Wang & Wang, PRL 110, 262301 (2013)

$$\frac{dt}{d\tau}\partial_t f_{R/L} + \frac{d\mathbf{x}}{d\tau}\cdot\nabla_{\mathbf{x}} f_{R/L} + \frac{d\mathbf{p}}{d\tau}\cdot\nabla_{\mathbf{p}} f_{R/L} = 0$$

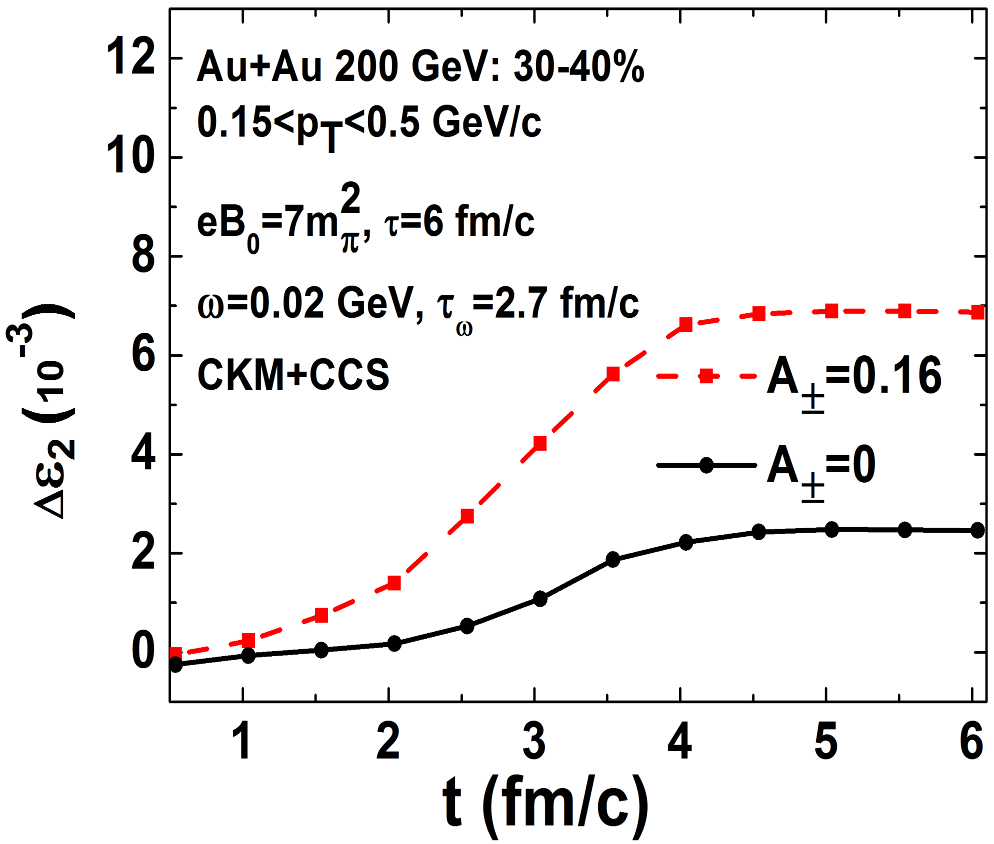
$$\frac{dt}{d\tau} = 1 \pm Q\mathbf{b}\cdot\mathbf{B} \pm 4|\mathbf{p}|(\mathbf{b}\cdot\omega)$$

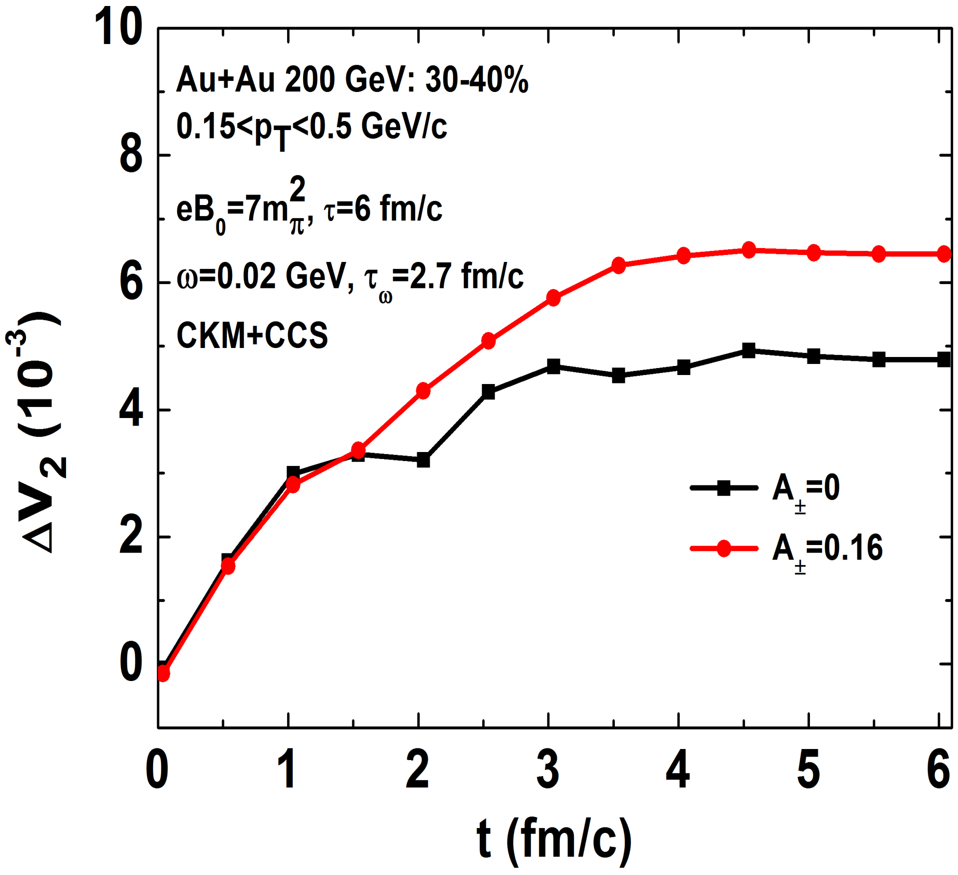
$$\frac{d\mathbf{x}}{d\tau} = \hat{\mathbf{p}} \pm Q(\hat{\mathbf{p}}\cdot\mathbf{b})\mathbf{B} \pm Q(\mathbf{E}\times\mathbf{b}) \pm \frac{1}{|\mathbf{p}|}\omega$$

$$\frac{d\mathbf{p}}{d\tau} = Q(\mathbf{E}+\hat{\mathbf{p}}\times\mathbf{B}) \pm Q^2(\mathbf{E}\cdot\mathbf{B})\mathbf{b} \mp Q|\mathbf{p}|(\mathbf{E}\cdot\mathbf{b})\mathbf{b}$$
$$\pm 3Q(\mathbf{b}\cdot\omega)(\mathbf{p}\cdot\mathbf{E})\hat{\mathbf{p}}$$

$$\mathrm{Three-dimensional\ Berry\ curvature} \quad \mathbf{b} = \frac{\mathbf{p}}{2|\mathbf{p}|^3}$$

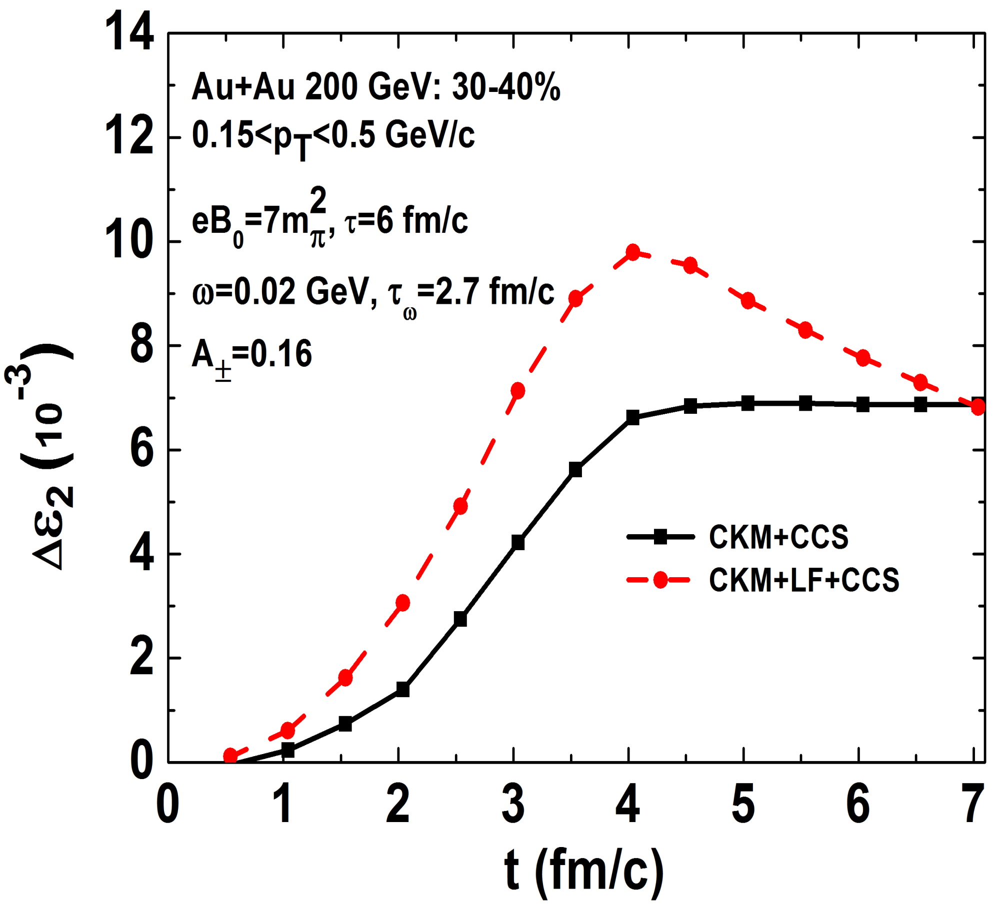
Time evolution of eccentricity and v_2 splittings with both vorticity and magetic field effects

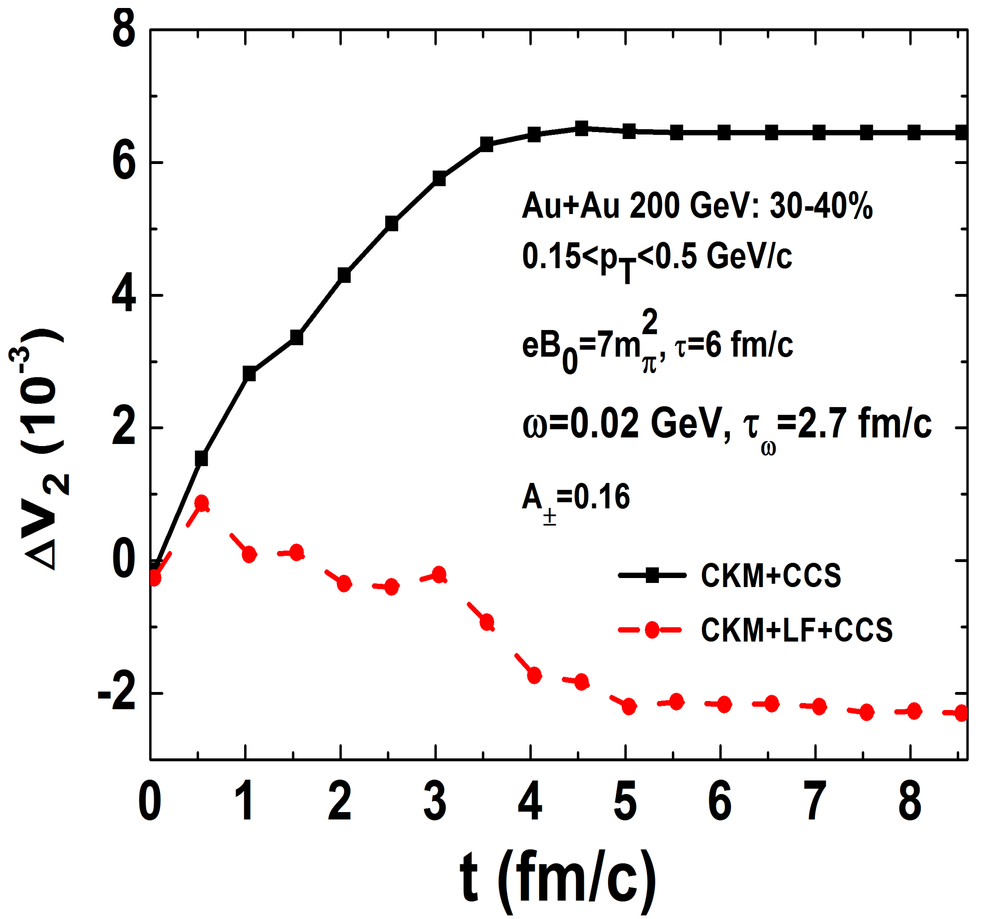




- Finite eccentricity and elliptic flow splittings even when charge asymmetry is zero.
- Elliptic flow splitting develops faster than in the presence of magnetic field only.

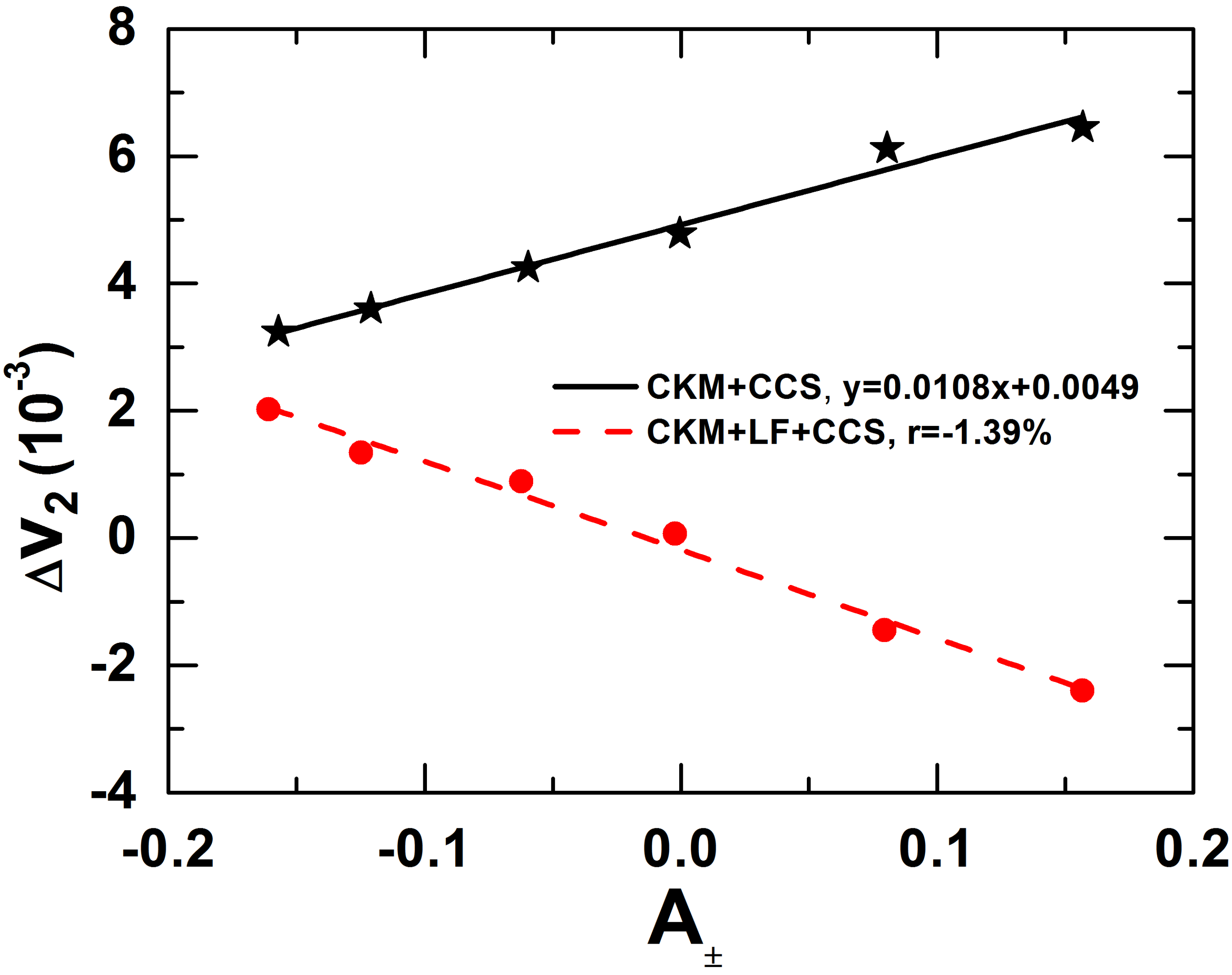
Effect of Lorentz force on time evolution of eccentricity and v_2 splittings





- Lorentz force leads to larger elliptic flow for positively charged than negatively charged particles.

Charge asymmetry dependence of v_2 splitting



- Large elliptic flow splitting when charge asymmetry is zero.
- Lorentz force destroys chiral effects.

Chiral magnetic effect in isobaric (Zr+Zr and Ru+Ru) collisions

Shi, Jiang, Lilleskov & Liao, Ann. Phys. 394, 50 (2018)

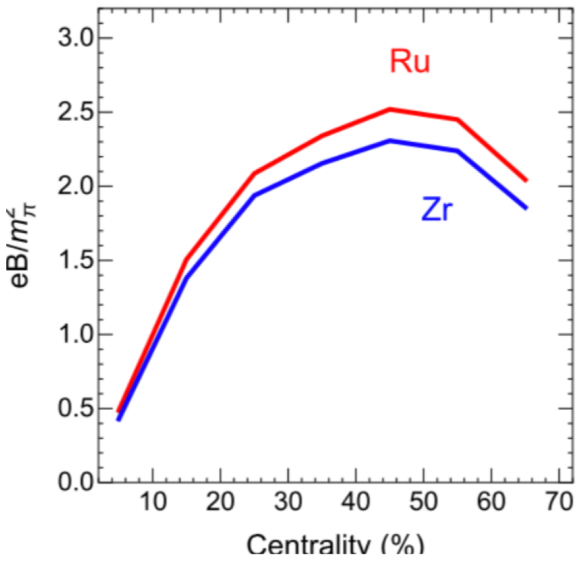
- Magnetic field can lead to electric current in the presence of non- vanishing axial charges → charge separation

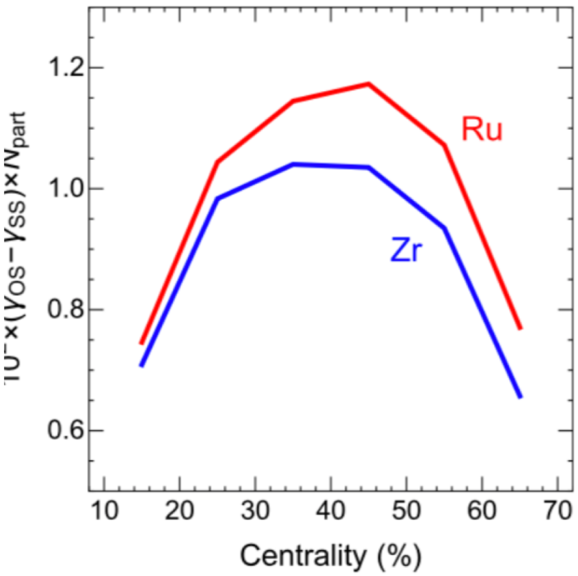
$$\mathbf{J} = \frac{N_c \mu_5 Q}{2\pi^2}\mathbf{B}$$

$$\boldsymbol{B} = \frac{B_0}{1+\tau^2/\tau_B^2}\hat{\boldsymbol{y}}$$

- Based on Anomalous-Viscous Fluid Dynamics (AVFD) with magnetic field parametrized by a Lorentzian function and having strength B_0 determined by spectator protons and $\tau_B = 0.6$ fm/*c*.

- Initial axial charges are estimated using the chirality imbalance arising from gluonic topological charge fluctuations in the early-stage glasma → $\frac{n_5}{s} \approx 0.12$ at 40-50% centrality.





$\boldsymbol{\gamma^{OS} - \gamma^{SS}}$ **correlator** (Voloshin)

$$\gamma^{\mathrm{OS}} = \langle \cos(\phi^{+(-)} + \phi^{-(+)} - 2\Psi)\rangle$$

$$\gamma^{\mathrm{SS}} = \langle \cos(\phi^{+(-)} + \phi^{+(-)} - 2\Psi)\rangle$$

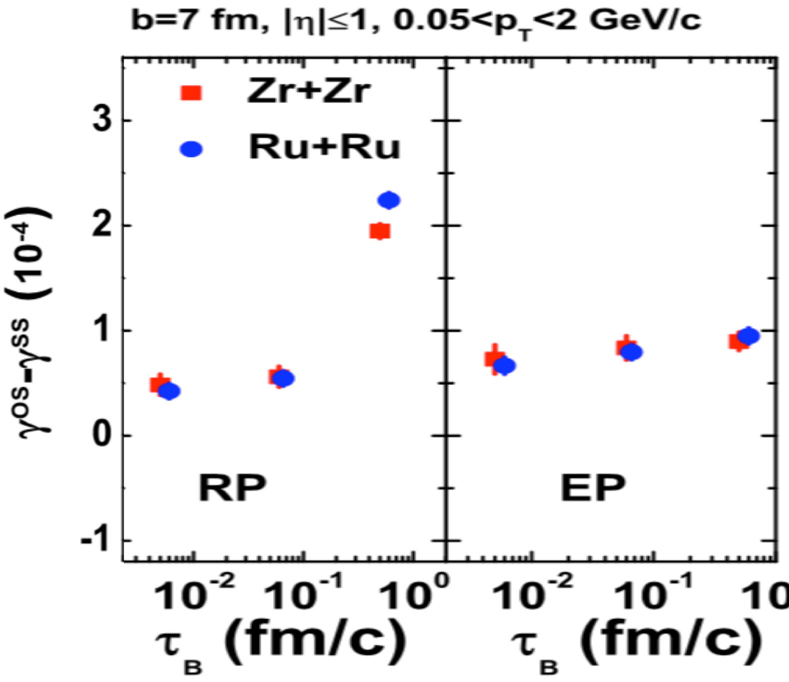
Chiral magnetic effect from chiral kinetic approach

Sun & Ko, PRC 98, 014911 (2018)

Probability for quark helicity $P_\lambda = \frac{1+\lambda p}{2}$

$$p = \frac{d\sqrt{\langle N_5^2\rangle}/d\eta}{dN/d\eta} \approx 0.2$$

$\gamma^{OS} - \gamma^{SS}$ correlator



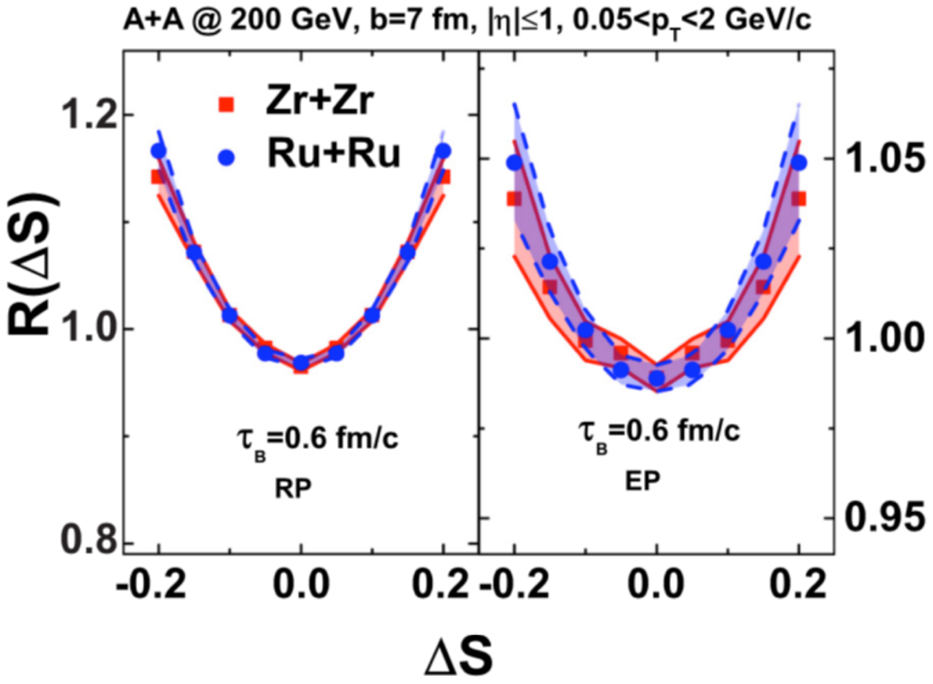
- Signal small with event plane

$R(\Delta S)$ correlator (Lacey)

$$R(\Delta S) = \frac{N_{\text{real}}(\Delta S)/N_{\text{shuffle}}(\Delta S)}{N^{\perp}_{\text{real}}(\Delta S)/N^{\perp}_{\text{shuffle}}(\Delta S)}$$

$$\Delta S = \frac{\sum_i \sin(\phi_i^+ - \Psi)}{N_p} - \frac{\sum_i \sin(\phi_i^- - \Psi)}{N_n}$$

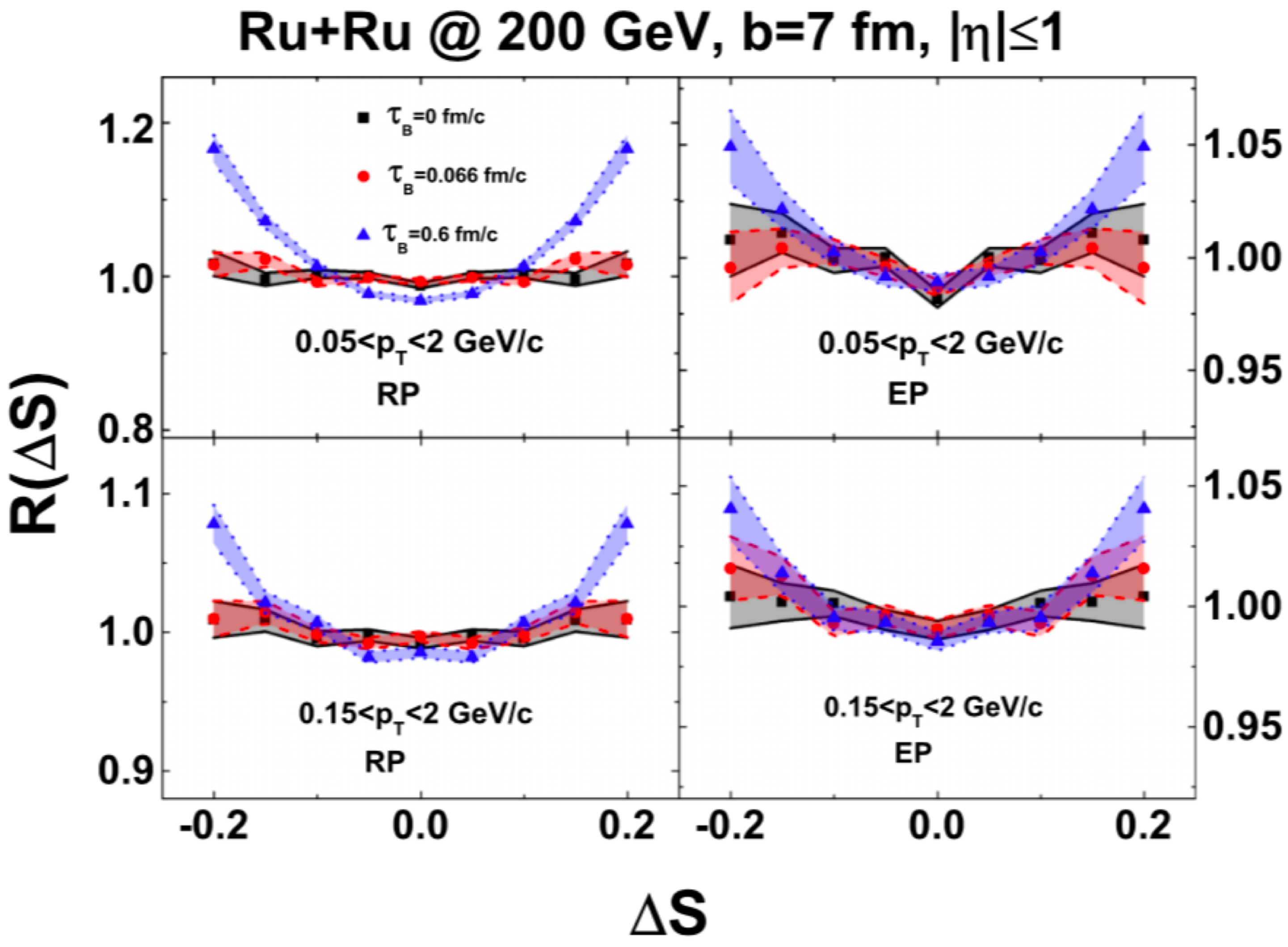
$$\perp\!: \Psi \to \Psi + \pi/2$$



- Effect plausible for long-lived B

- Similar to that based on Anomalous-Viscous Fluid Dynamics (AVFD) [Shi et al., Ann. Phys. 394, 50 (2018); Magdy et al., PRC 98, 061902 (2018)].

Magnetic lifetime dependence of chiral magnetic effect



- Chiral magnetic effect is sensitive to the lifetime of magnetic field even in the presence of appreciable initial axial charge fluctuations.

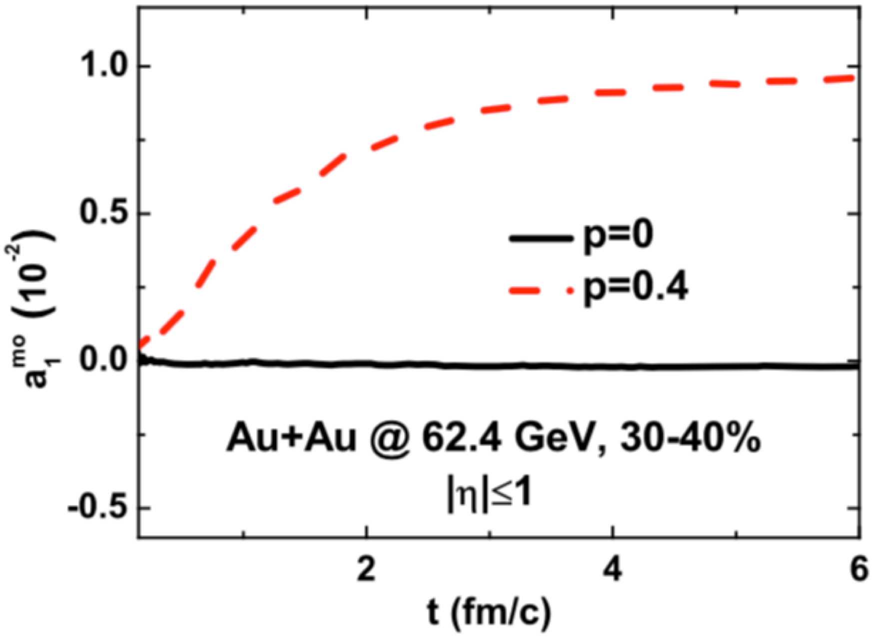
Multiplicity up-down asymmetry

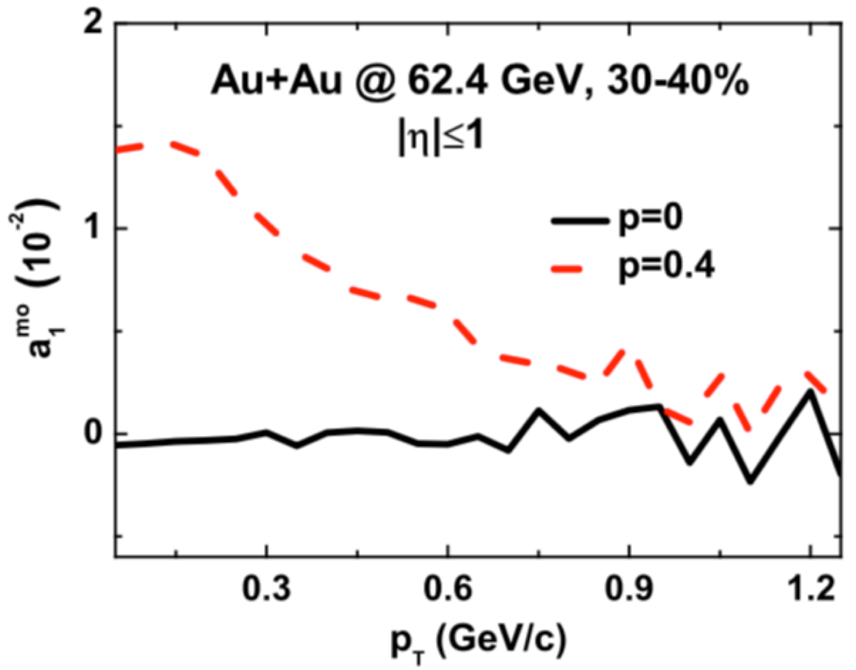
Sun & Ko, PLB 789, 228 (2019)

In the presence of both magnetic and vorticity fields and with net axial charge density, azimuthal angle distribution of charged particles

$$\frac{dN_\pm}{d\phi} \propto 1 + 2v_2 \cos(2\phi - 2\Psi_{\mathrm{RP}}) + 2(a_{\mathrm{CVE}} \pm a_{\mathrm{CME}}) \sin(\phi - \Psi_{\mathrm{RP}})$$

Considering both positively and negatively charged particles leads to a multiplicity up-down asymmetry $a_1^{mo} = \langle \sin\phi \rangle = a_{CVE}$

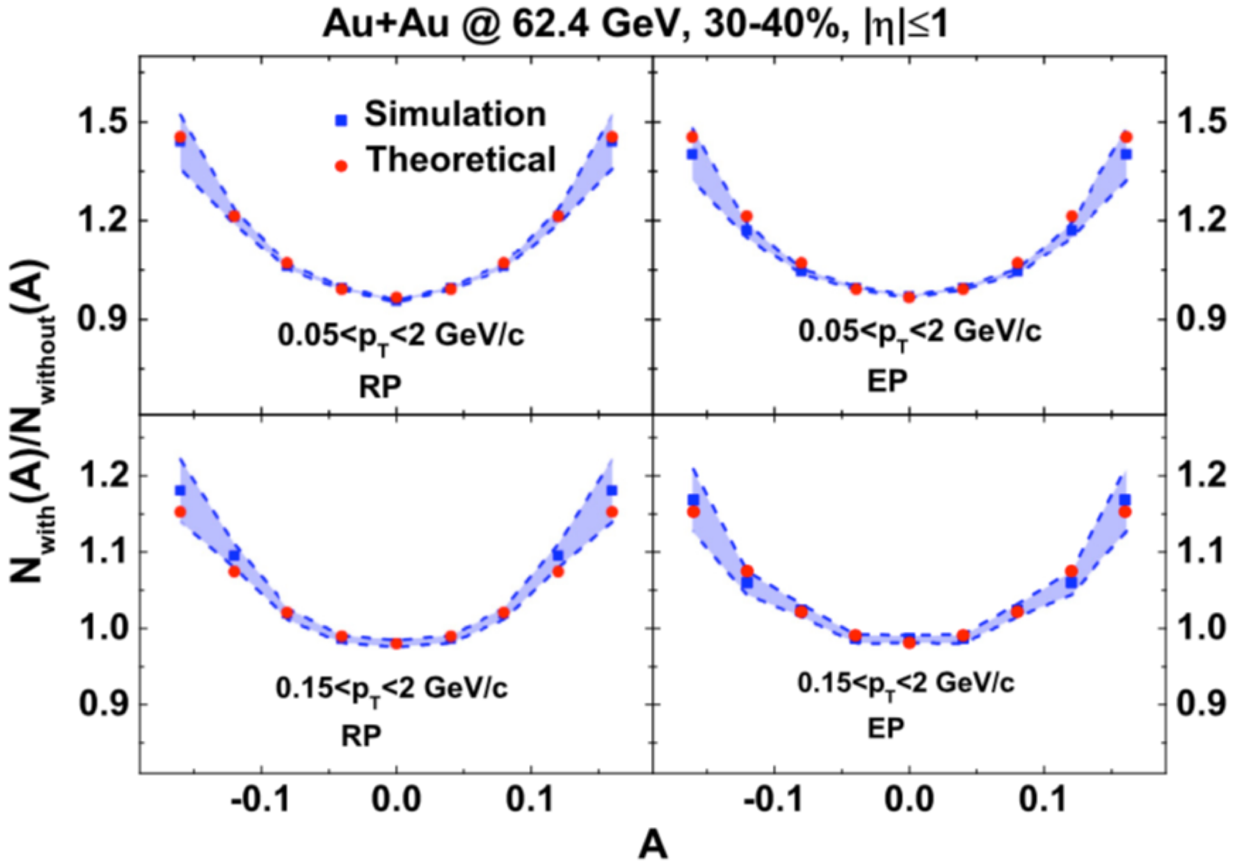




- Multiplicity up-down asymmetry event distribution

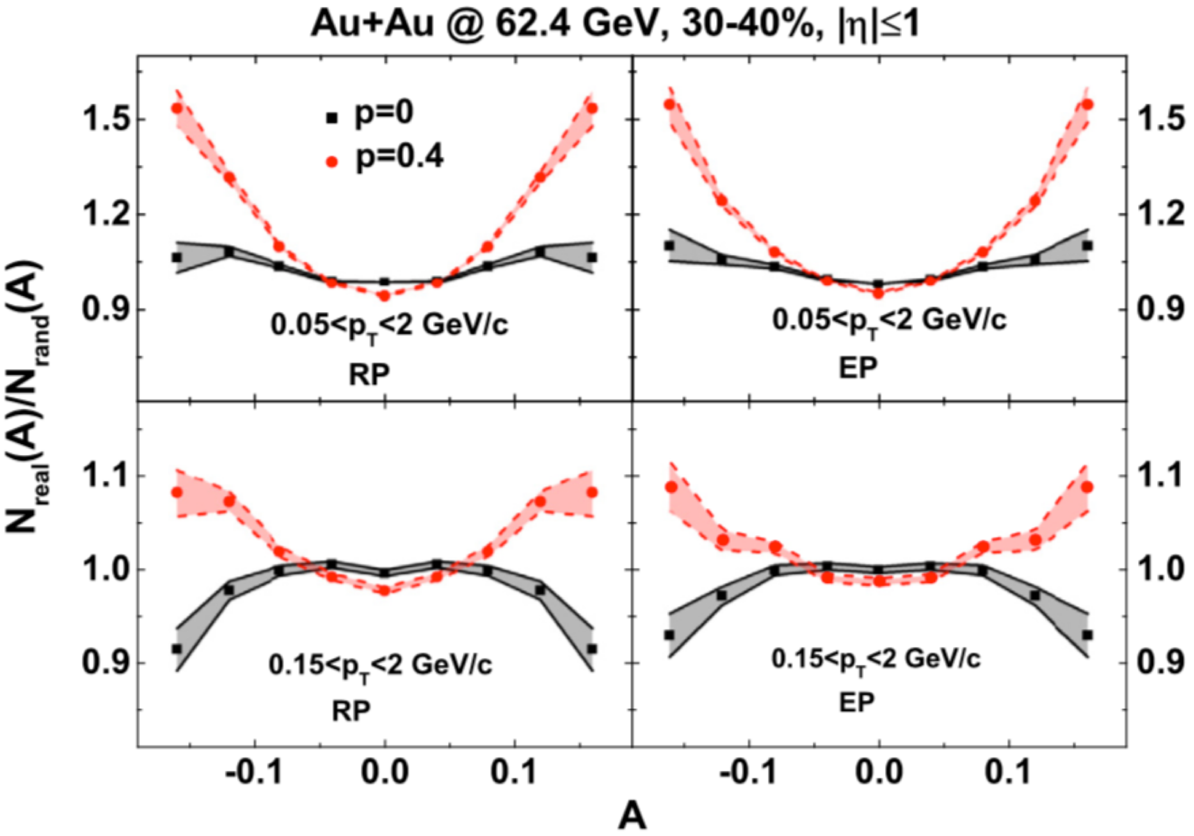
$$A = \frac{N_U - N_D}{N_U + N_D}$$

N_U and N_D are numbers of particles with momenta pointing towards upper and lower hemispheres of a collision



It has a concave shape that vanishes at A=0 and has a width equal to $\frac{4a_{CVE}}{\pi}$ as expected for a large number of uncorrelated particles in an event.

- Ratio of multiplicity up-down asymmetry distribution to that of random distribution



$$p = \frac{d\sqrt{\langle N_5^2 \rangle}/d\eta}{dN/d\eta}$$

- Width of the distribution is sensitive to the value of axial charge fluctuations in quark matter.
- Chiral separation effect due to the triangular anomaly [Son & Surowka, PRL 103, 191601 (2009)]?

Summary

- Magnetic field generated in non-central relativistic heavy ion collisions is large but short-lived.

- In the presence of long-lived magnetic field chiral transport study shows that
  - Chirality changing scattering is essential for generating different eccentricities and v_2 splittings between particles and antiparticles.
  - CMW enhances v_2 of negatively charged particles and leads to a positive slope parameter in $\Delta v_2/A\pm$. Including also CVW leads to nonzero v_2 splitting at zero charge asymmetry.
  - Lorentz force enhances v_2 of positively charged particles and leads to a negative slope parameter or destroys the effects due to CMW and CVW
  - there is a difference in the chiral magnetic effect (charge separation) in collisions of isobaric nuclei Zr+Zr and Ru+Ru.

- Large and long lived vorticity field generated in noncentral HIC can lead to an **up-down asymmetry** of produced particles if **axial charge fluctuations** are present in the produced quark matter.