QCD Phase Structure within the FRG Approach

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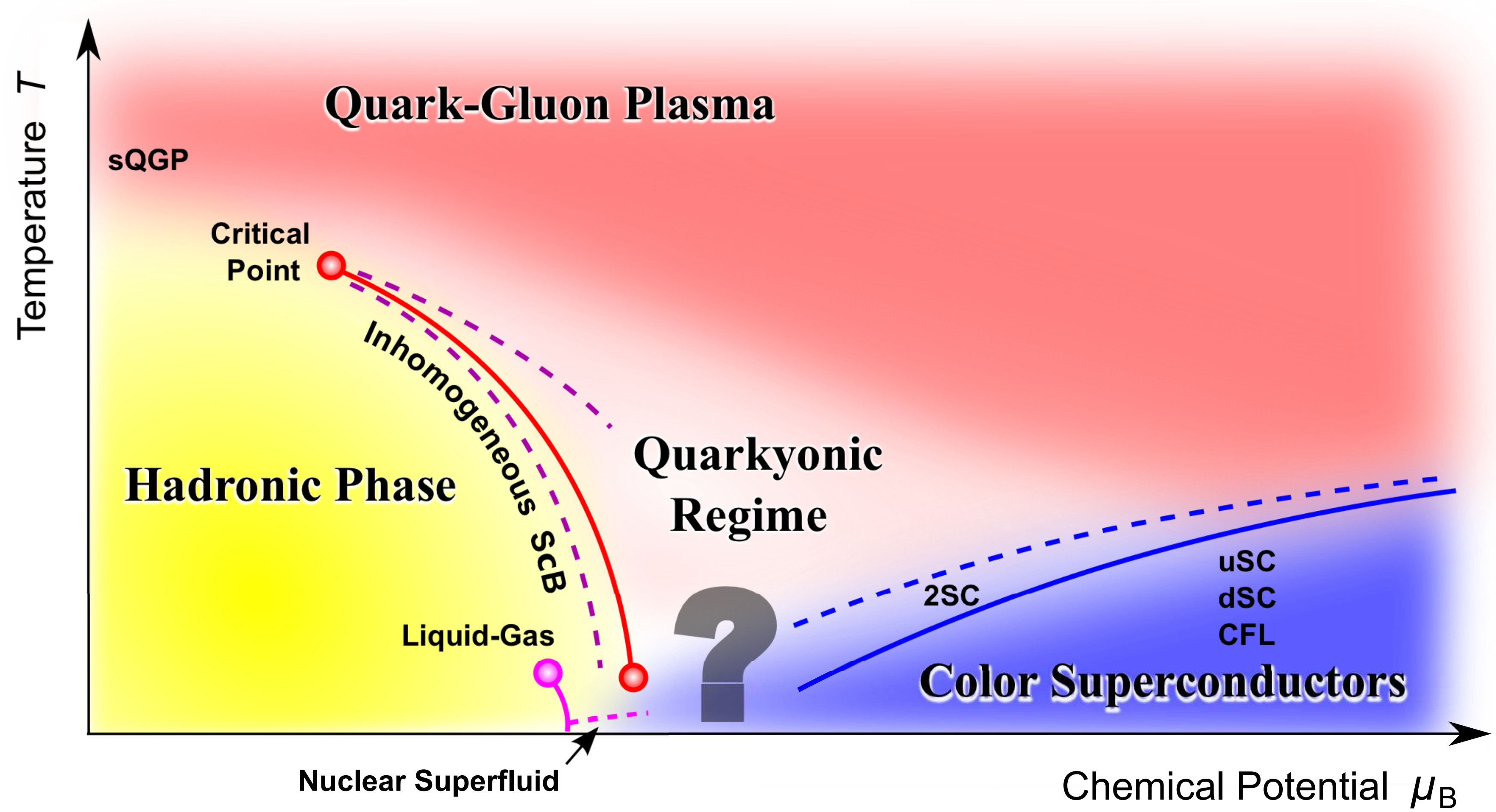
Outline

- Introduction
- QCD within the functional renormalization group approach
- Phase structure in QCD
- Thermodynamics in effective models
- Summary and outlook

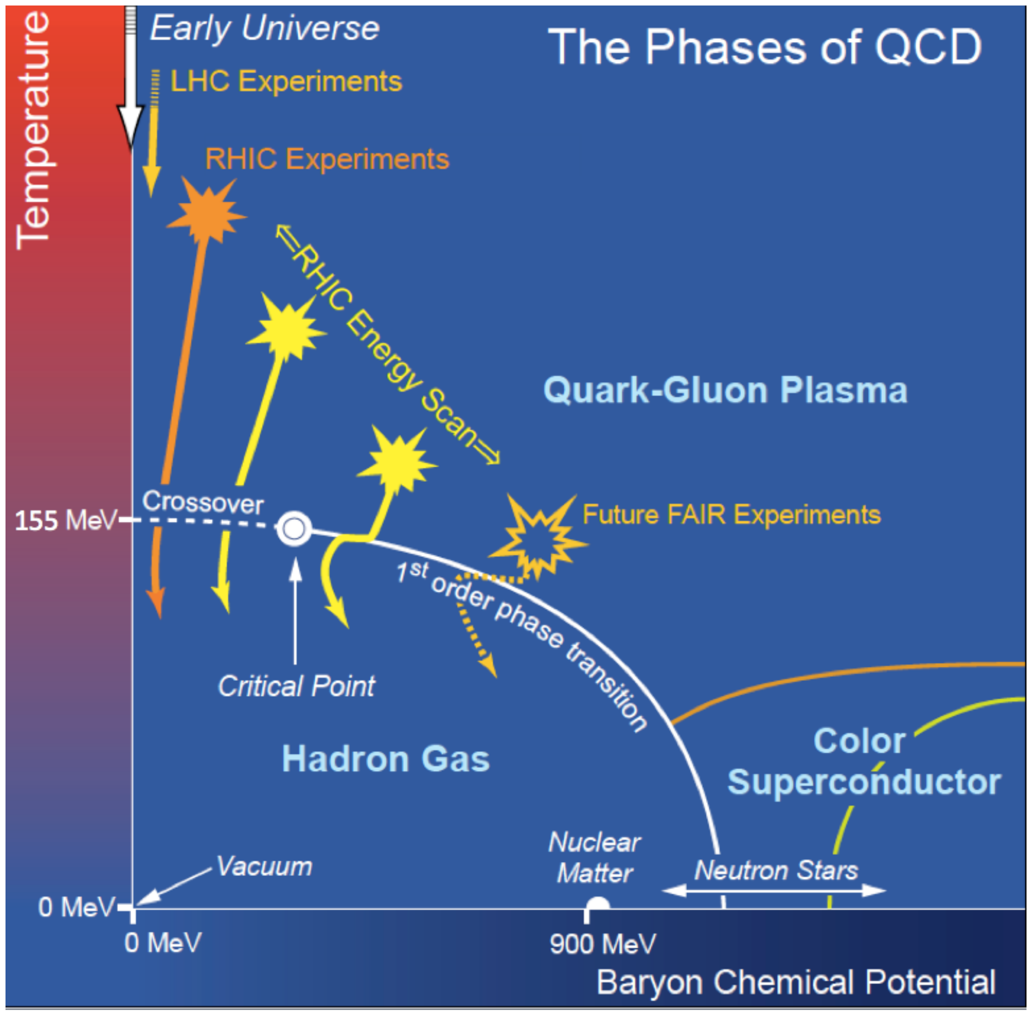
QCD phase diagram



K. Fukushima, C. Sasaki, arXiv:1301.6377.

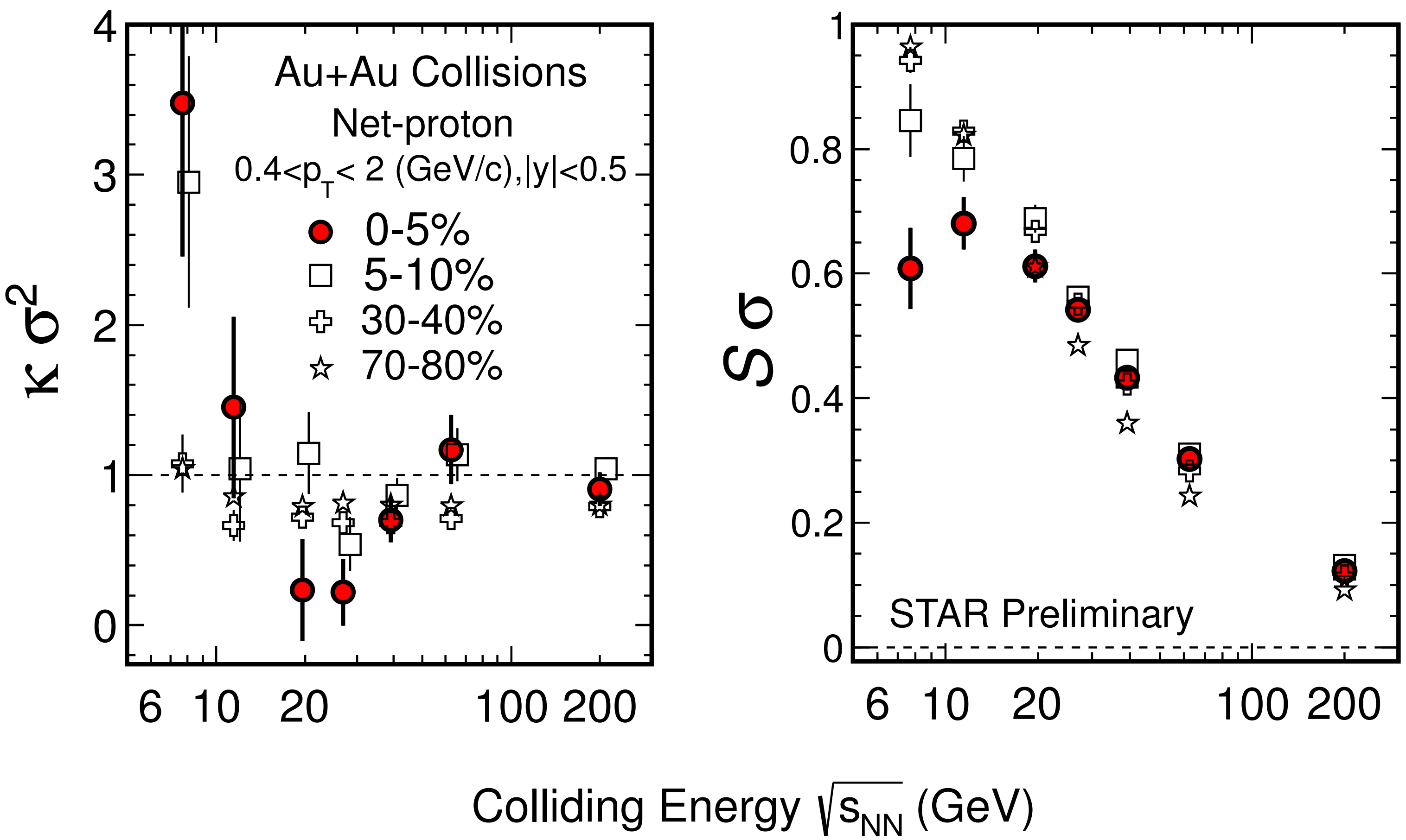
- QCD phase diagram in the T-muB plane.
- Critical end point (or critical point) is a key feature of QCD phase structure.
- Experimental programs: RHIC-BES, FAIR, NICA, HIAF.
- Some hints from RHIC-BES experiment: net-baryon (proton) cumulants, directed flow, HBT radii, light nuclei……

High-order net-proton cumulants



The Hot QCD White Paper (2015)

RHIC:



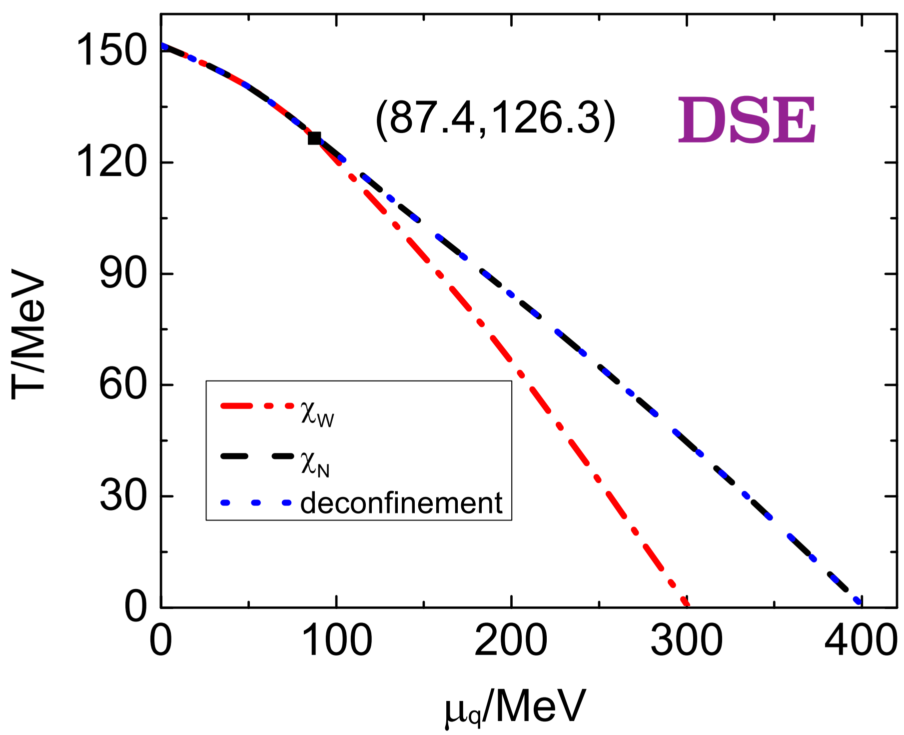
X.Luo (STAR), PoS CPOD2014, 019 (2014)

cf. Xiaofeng's talk

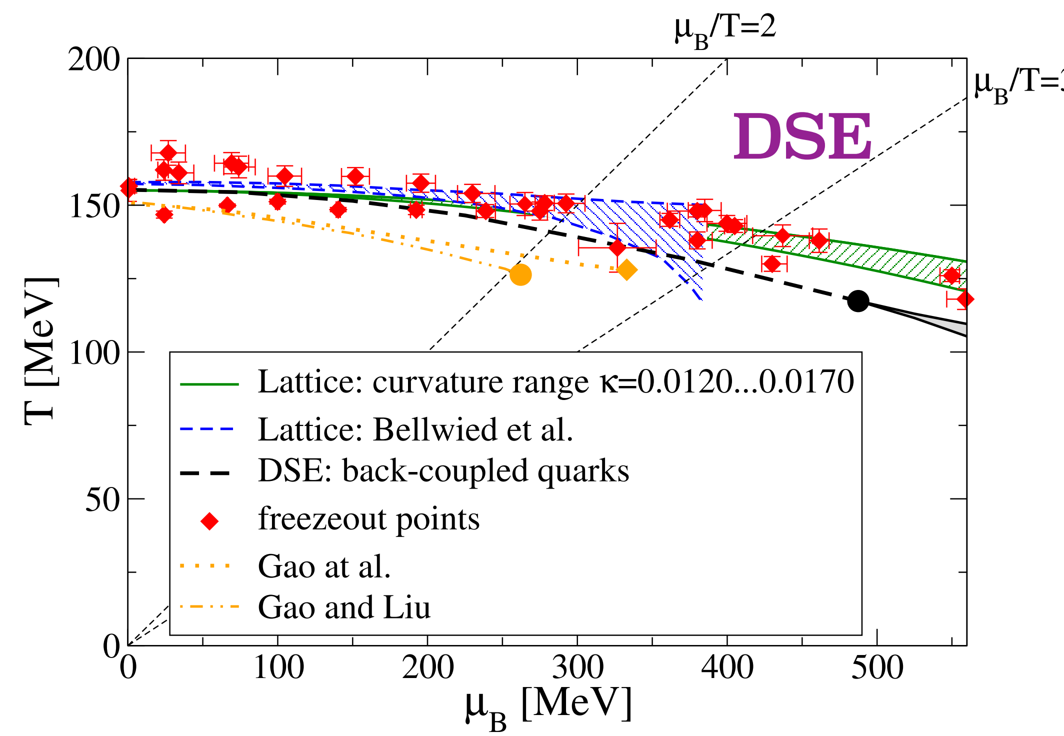
- Non-monotonic energy dependence of the kurtosis => hint of entering critical region.

QCD Phase Structure: Lattice, DSE and FRG

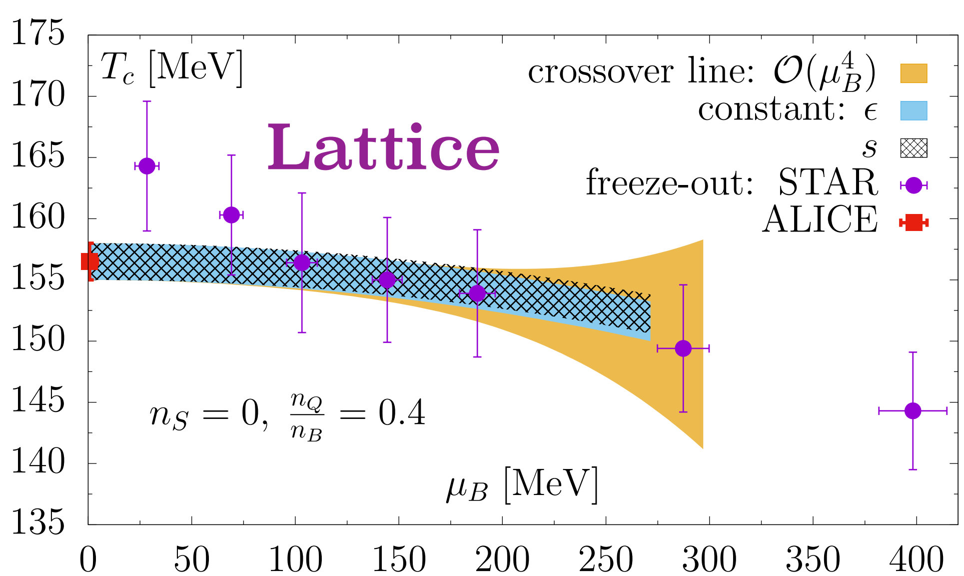
cf. Yu-xin's talk



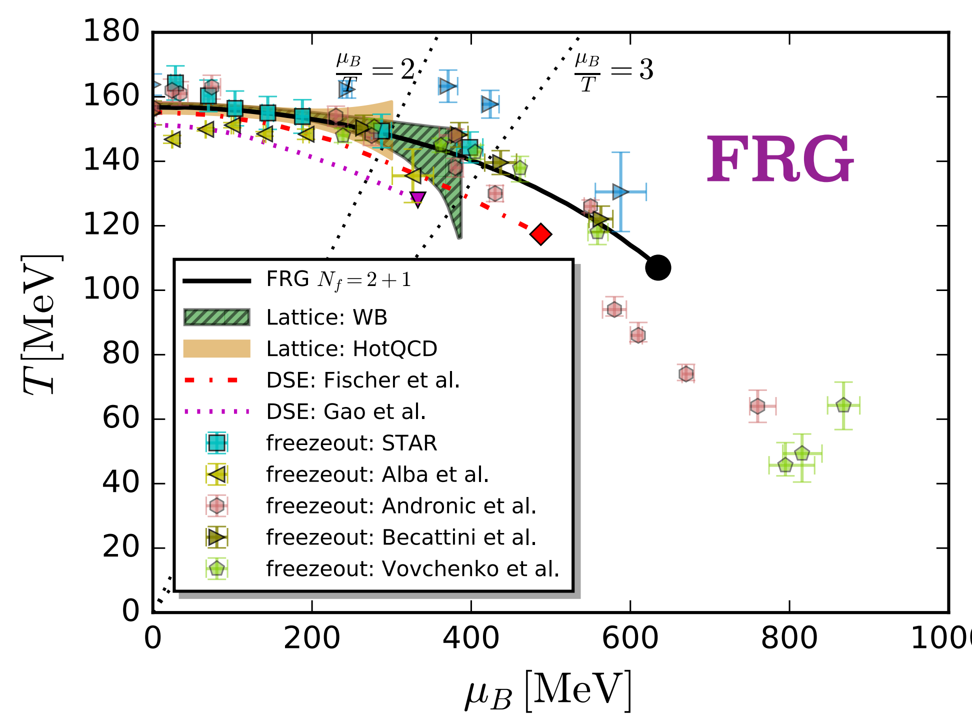
F. Gao and Y.-x. Liu , PRD 94 (2016) 076009



C. S. Fischer, (2018), arXiv:1810.12938.



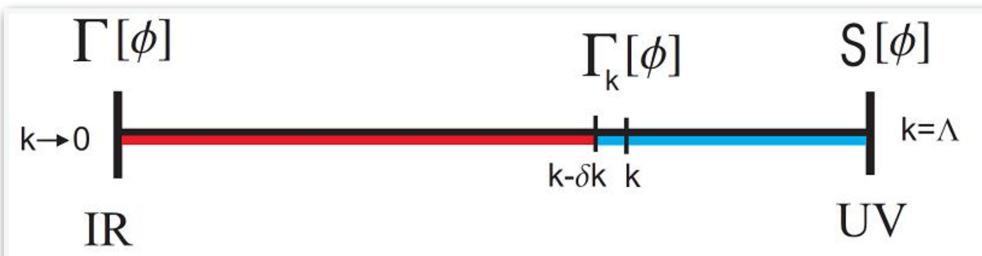
A. Bazavov *et al.* (HotQCD), arXiv: 1812.08235. cf. Heng-tong's talk



WF, J.M. Pawlowski, F. Rennecke, arXiv:1907.xxxxx.

Quantum fluctuations with FRG

FRG



Rebosonized QCD Effective action:

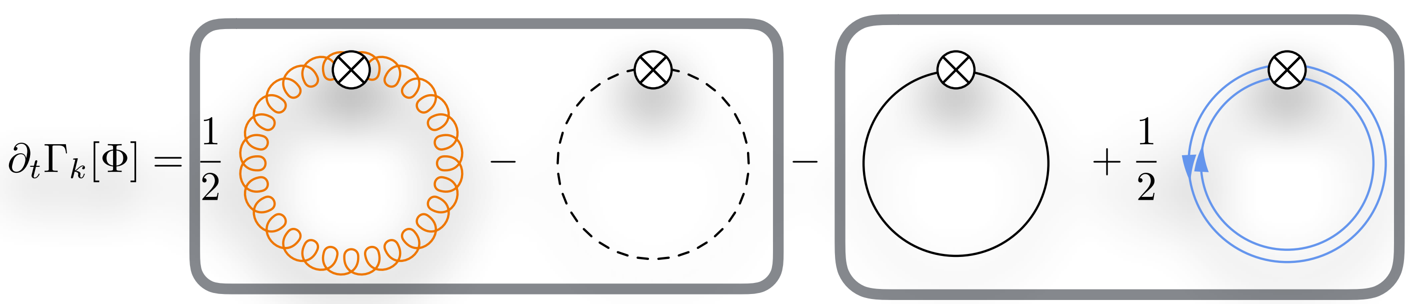
$$\Gamma_k = \int_x \Big\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + Z_c (\partial_\mu \bar{c}^a) D^{ab}_\mu c^b + \frac{1}{2\xi} (\partial_\mu A^a_\mu)^2$$

$$+ Z_q \bar{q} \left(\gamma_\mu D_\mu + m^s_0 \right) q$$

$$- \lambda_q \Big[(\bar{q}\, \tau^0 q)^2 + (\bar{q}\, \vec{\tau} q)^2 \Big] + h_k\, \bar{q} \left(\tau^0 \sigma + \vec{\tau} \cdot \vec{\pi} \right) q$$

$$+ \frac{1}{2} Z_\phi (\partial_\mu \phi)^2 + V_k(\rho) - c_\sigma\, \sigma \Big\} ,$$

Flow equation:



glue sector

matter part

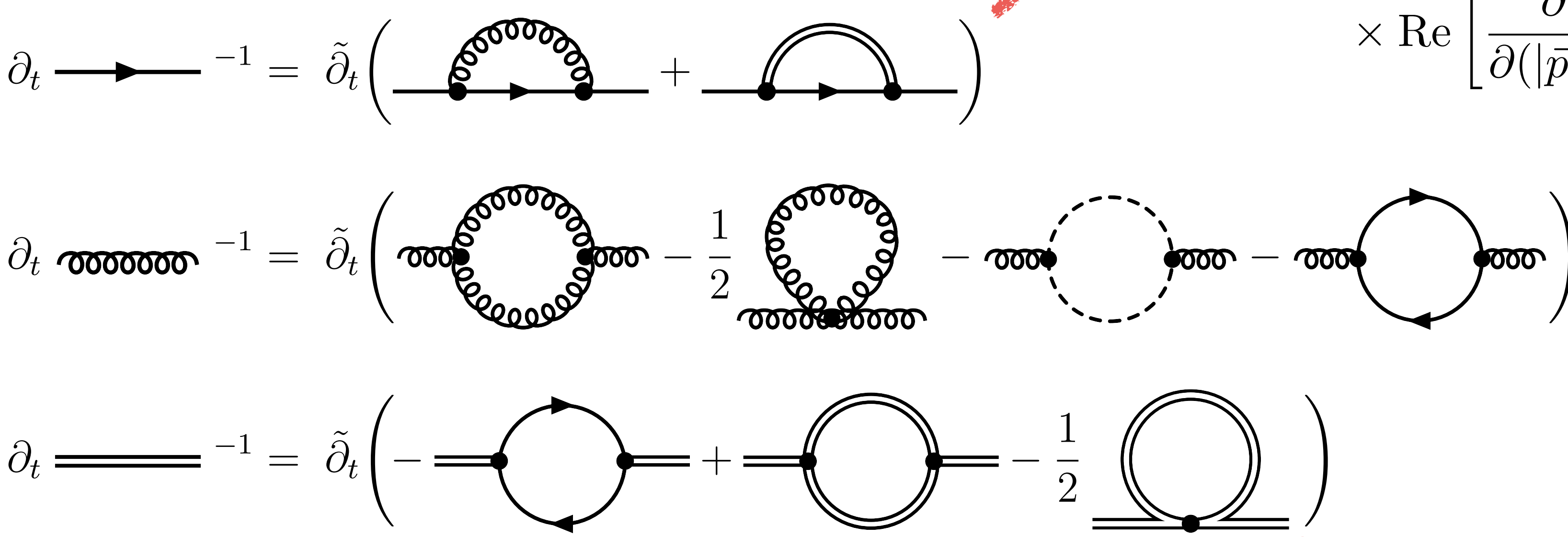
$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} \mathrm{STr} \Big[\partial_t R_k \big(\Gamma^{(2)}_k[\Phi] + R_k \big)^{-1} \Big]$$

$$= \frac{1}{2} \mathrm{Tr} \big(G^{AA}_k[\Phi] \partial_t R^A_k \big) - \mathrm{Tr} \big(G^{c\bar{c}}_k[\Phi] \partial_t R^c_k \big)$$

$$- \mathrm{Tr} \big(G^{q\bar{q}}_k[\Phi] \partial_t R^q_k \big) + \frac{1}{2} \mathrm{Tr} \big(G^{\phi\phi}_k[\Phi] \partial_t R^\phi_k \big) ,$$

Propagators and anomalous dimensions

Flow equation:



Quark anomalous dimension:

$$\eta_{q,k}(p_0,\vec{p}) = \frac{1}{4Z_{q,k}(p_0,\vec{p})}$$

$$\times\, \mathrm{Re}\left[\frac{\partial}{\partial(|\vec{p}|^2)}\mathrm{tr}\left(i\vec{\gamma}\cdot\vec{p}\left(-\frac{\delta^2}{\delta\bar{q}(p)\delta q(p)}\partial_t\Gamma_k\right)\right)\right],$$

Glue sector:

$$\eta_A = \eta^{\mathrm{QCD}}_{A,T=0} + \Delta\eta^{\mathrm{glue}}_{A,T} + \Delta\eta^{q}_{A,T}\,,$$

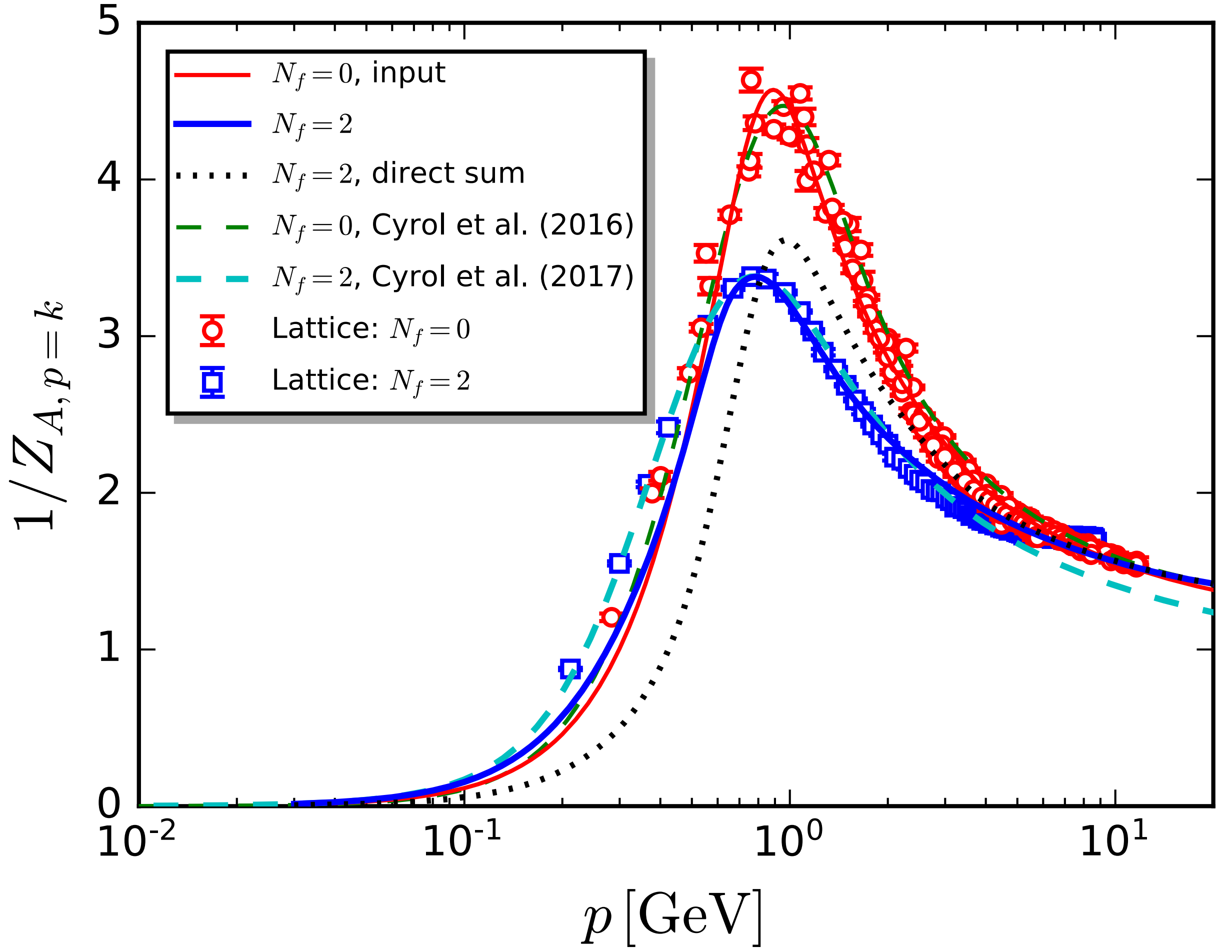
with

$$\eta^{\mathrm{QCD}}_{A,k;T=0} = -\frac{\partial_t Z^{\mathrm{QCD}}_{A,k=0}(p=k)}{Z^{\mathrm{QCD}}_{A,k=0}}\,.$$

Meson anomalous dimension:

$$\eta_{\phi,k}(p_0,\vec{p}) = -\frac{1}{Z_{\phi,k}}\frac{1}{\delta_{ij}}\frac{\partial}{\partial(|\vec{p}|^2)}\frac{\delta^2\partial_t\Gamma_k}{\delta\pi_i(-p)\delta\pi_j(p)}\,,$$

Direct sum versus full back coupling for the gluon



WF, J.M. Pawlowski, F. Rennecke, arXiv:1907.xxxxx.

Full back coupling:

$$\eta_{A,k}^{\mathrm{QCD}}=\eta_A^{\mathrm{glue}}(\alpha_s,\bar m_A^2)+\eta_A^{\mathrm{quark}}(\alpha_{\bar q A q},\bar m_q^2)\,,$$

Direct sum:

$$\eta_{A,k}^{\mathrm{QCD}}=\eta_{A,k}^{\mathrm{YM}}+\Delta\eta_{A,k}(\alpha_{\bar q A q},\bar m_q^2)\,.$$

In this work we adopt:

$$\eta_A=\eta_{A,T=0}^{\mathrm{QCD}}+\Delta\eta_{A,T}^{\mathrm{glue}}+\Delta\eta_{A,T}^{q}\,,$$

Thermal quark loop:

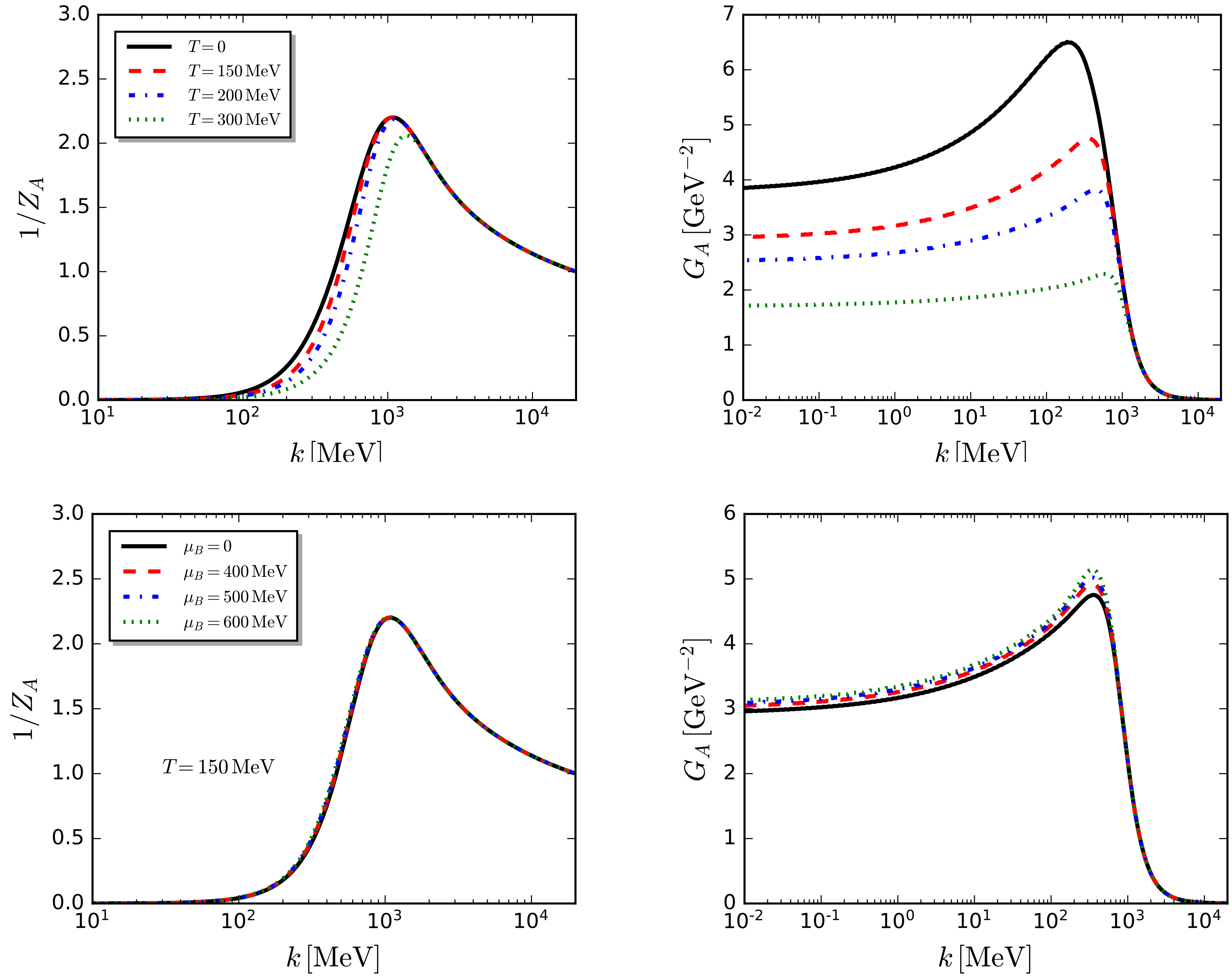
$$\Delta\eta_{A,T}^{q}=-\frac{1}{2(N_c^2-1)}\delta_{ab}\Pi_{\mu\nu}^{\mathrm{M}}(p)\left(\left[\overline{\mathrm{Flow}}_{AA\,\mu\nu}^{(2)\,ab}(p)\right]_T^{(q)}-\left[\overline{\mathrm{Flow}}_{AA\,\mu\nu}^{(2)\,ab}(p)\right]_{T=0}^{(q)}\right)_{\substack{p_0=0\\|\vec p|=k}}\,,$$

Debye screening mass for the gluon:

$$\bar G_A^{-1}(k)=\bar Z_{A,k}k^2=Z_{A,k}k^2+\Delta m_s^2(k,T)\,.$$

$$\Delta m_s^2(k,T)=(c\,T)^2\exp\left[-\left(\frac{k}{\pi T}\right)^n\right]\,.$$

Gluon propagator at finite T and muB



WF, J.M. Pawlowski, F. Rennecke, arXiv:1907.xxxxx.

Dynamical hadronization

Introduce a scale-dependent meson field:

$$\langle \partial_t \hat{\phi}_k \rangle = \dot{A}_k \, \bar{q}\tau q + \dot{B}_k \, \phi + \dot{C}_k \, \hat{e}_\sigma \,,$$

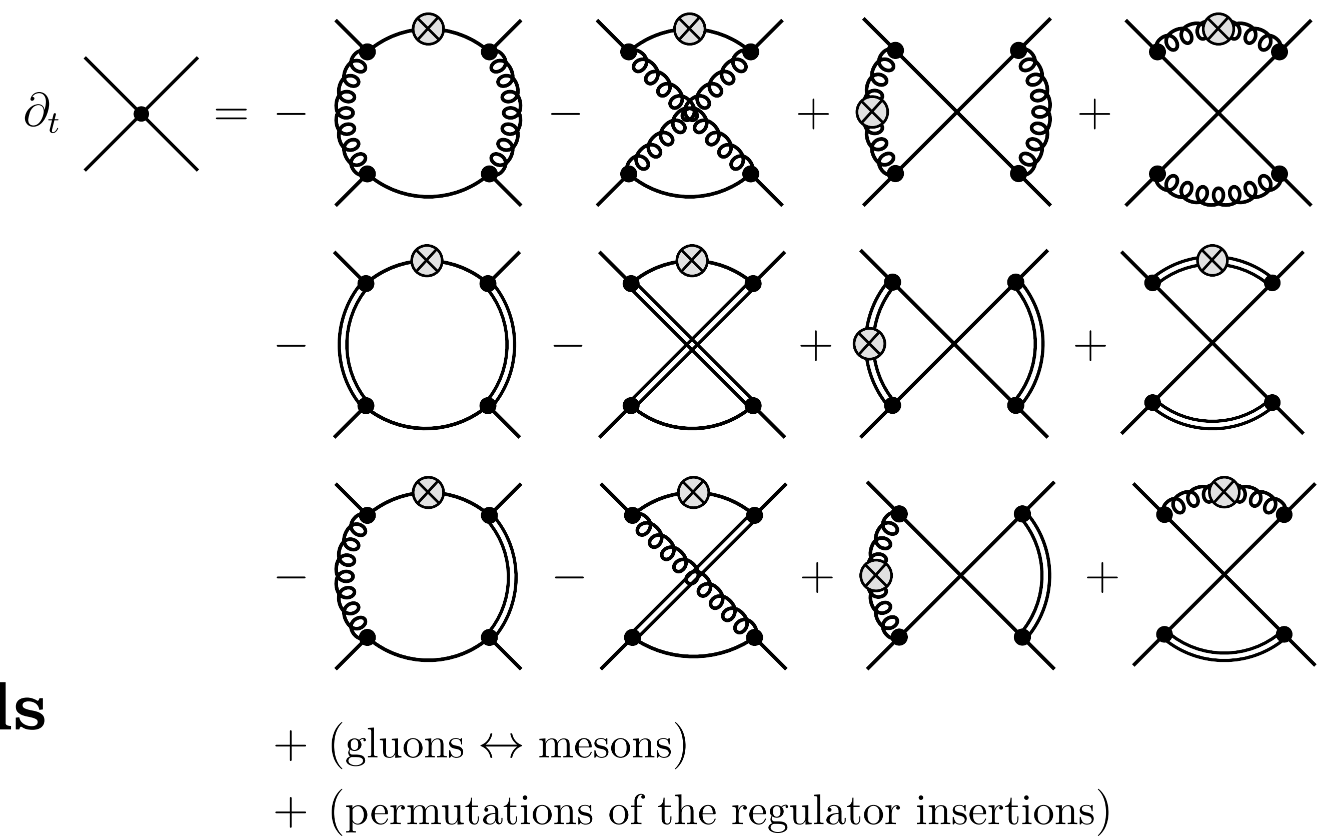
The Wetterich equation is modified:

$$\partial_t \Gamma_k[\Phi] + \int \langle \partial_t \hat{\phi}_{k,i} \rangle \frac{\delta \Gamma_k[\Phi]}{\delta \phi_i}$$

$$= \frac{1}{2} \mathrm{Tr}\, G_k[\phi] \, \partial_t R_k + \frac{1}{2} \mathrm{Tr}\, G_{\phi\varphi_j}[\Phi] \frac{\delta \langle \partial_t \hat{\phi}_k \rangle}{\delta \varphi_j} \, R_\phi \,,$$

The four-fermion couplings:

$$\partial_t \bar{\lambda}_q - 2\,(1+\eta_q)\,\bar{\lambda}_q - \bar{h}\,\dot{\bar{A}} = \frac{1}{4} \overline{\mathrm{Flow}}^{(4)}_{(\bar{q}q)(\bar{q}q)} \,,$$



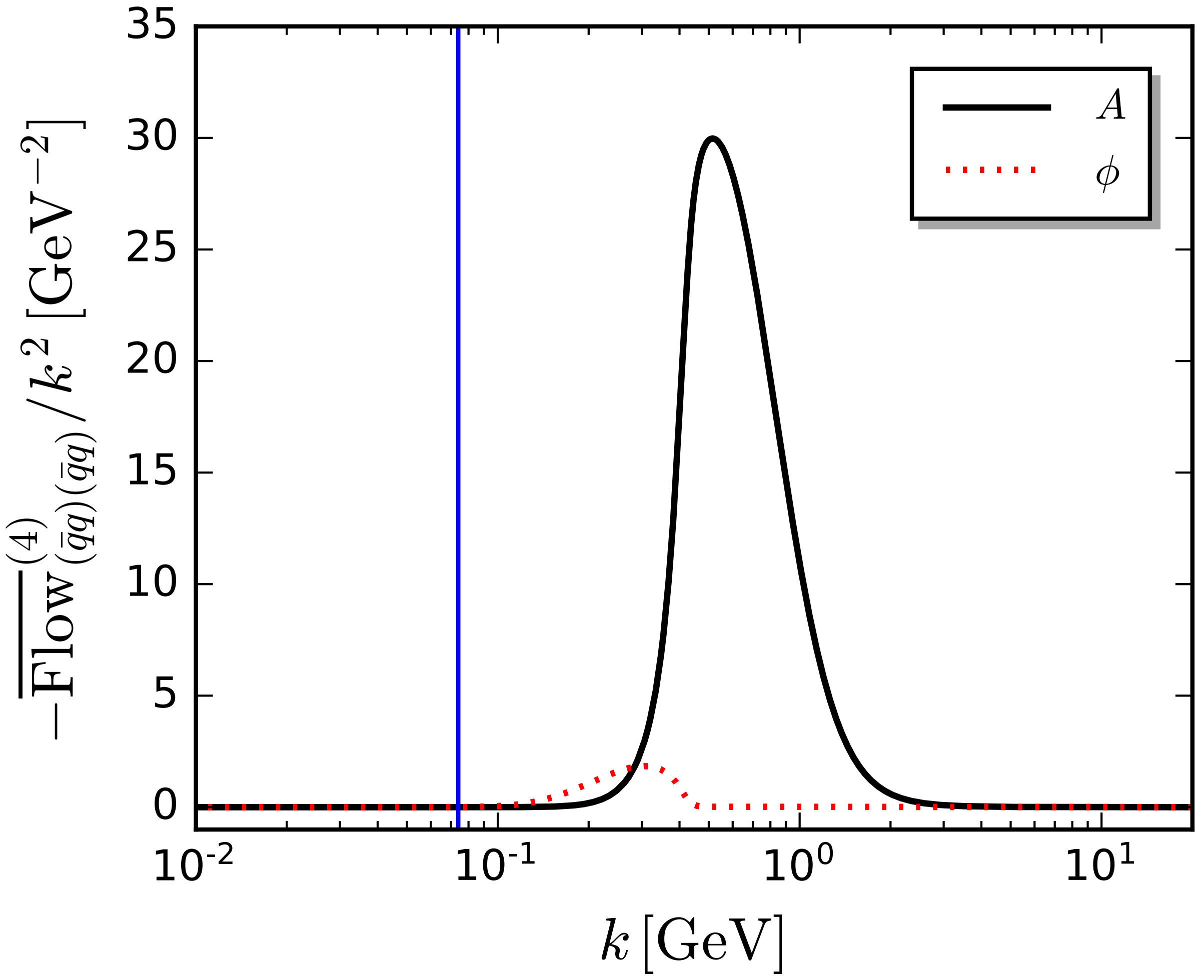
Demanding

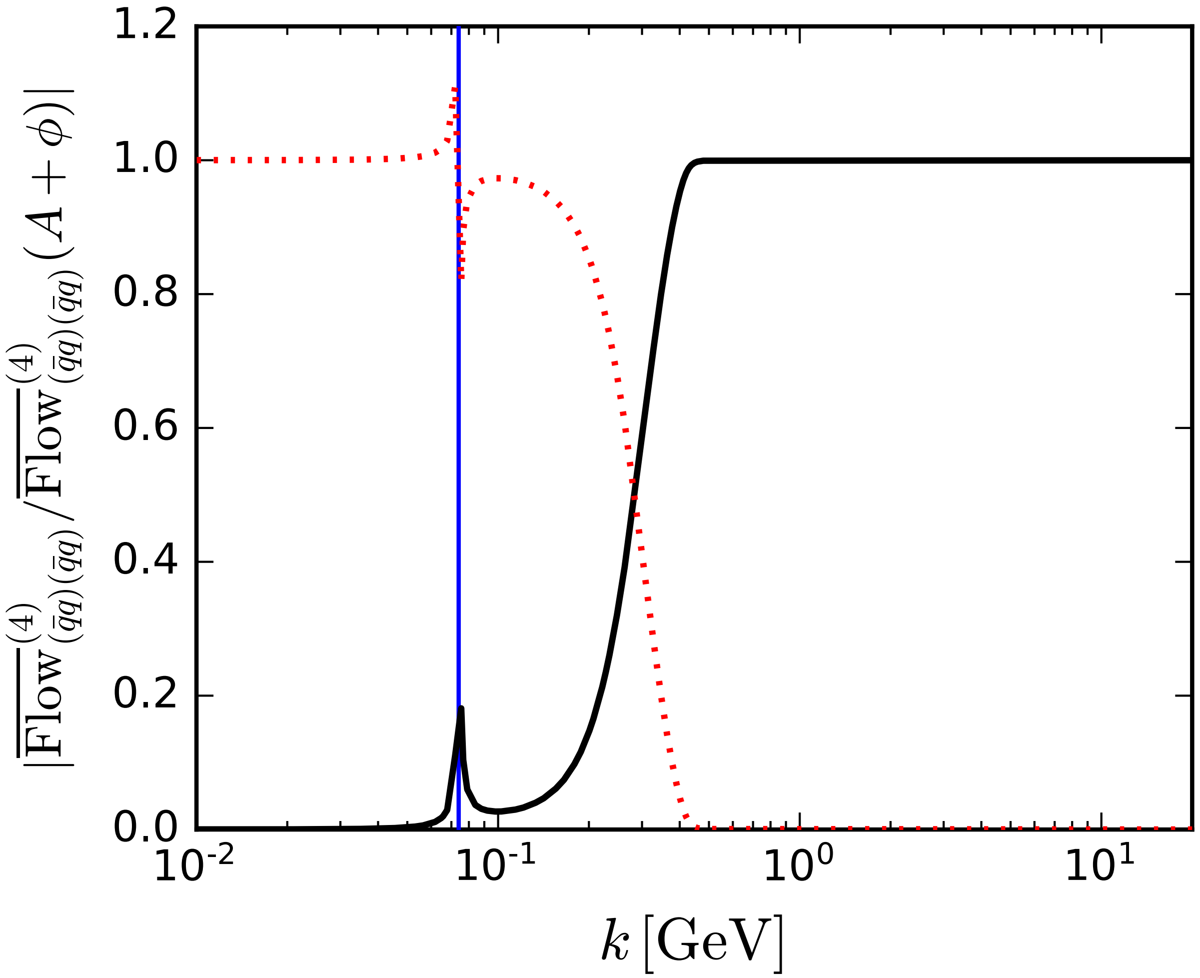
$$\bar{\lambda}_q \equiv 0 \,, \qquad \forall k \,.$$

The hadronization function reads

$$\dot{\bar{A}} = -\frac{1}{\bar{h}} \overline{\mathrm{Flow}}^{(4)}_{(\bar{q}q)(\bar{q}q)/4} \,.$$

Flow of 4-quark coupling—gluon versus meson





WF, J.M. Pawlowski, F. Rennecke, arXiv:1907.xxxxx.

Yukawa coupling

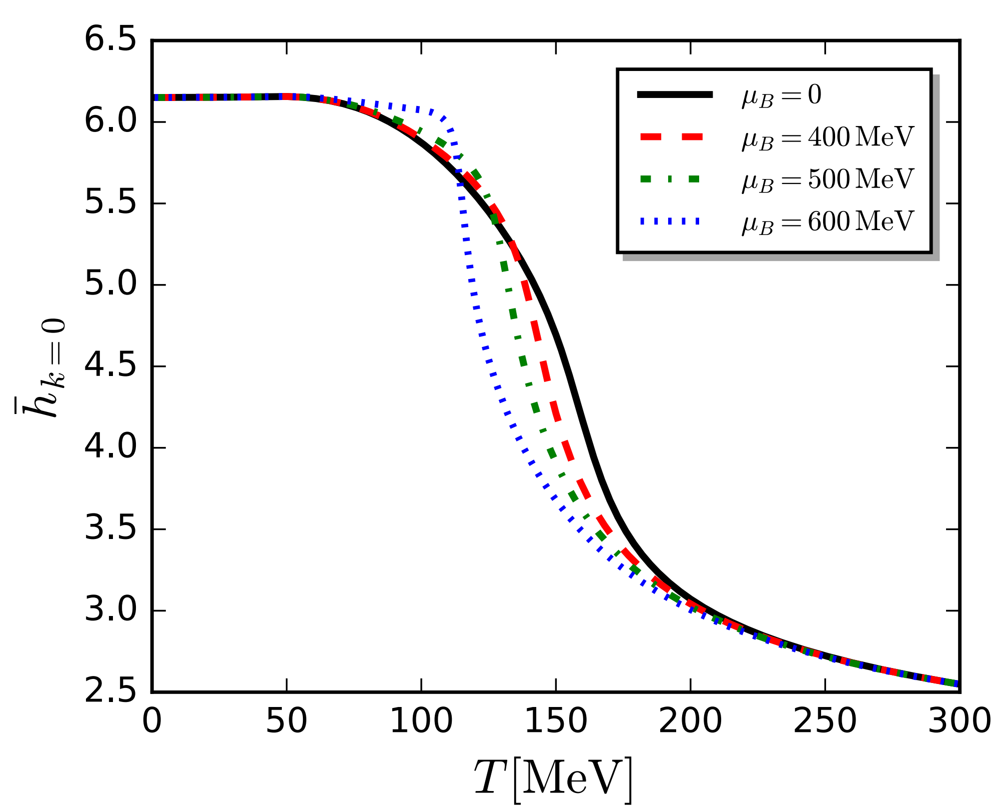
Two different Yukawa coupling:

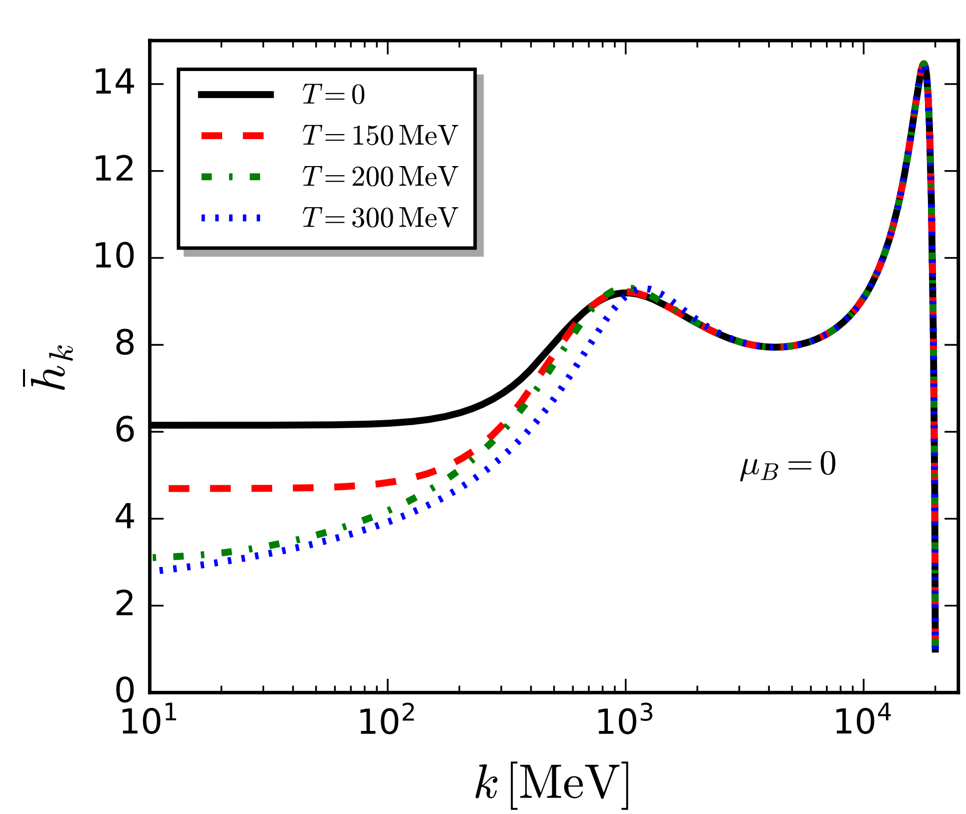
$$h_\pi = h(\rho_0) = \Gamma^{(3)}_{(\bar{q}\vec{\tau}q)\vec{\pi}}[\Phi_0]\,,$$

$$h_\sigma = h(\rho_0) + \rho\, h'(\rho_0) = \Gamma^{(3)}_{(\bar{q}\tau^0 q)\sigma}[\Phi_0]\,.$$

The flow equation:

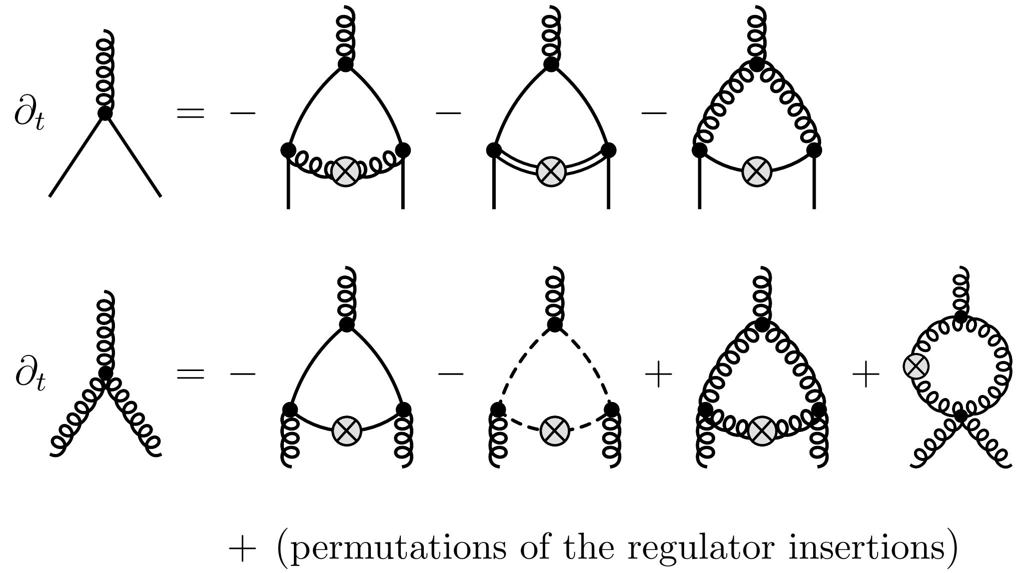
$$\partial_t \bar{h} = \left(\frac{1}{2}\eta_{\phi,k} + \eta_{q,k}\right)\bar{h} - \bar{m}_\pi^2\,\dot{\bar{A}} + \frac{1}{\bar{\sigma}}\mathrm{Re}\,\overline{\mathrm{Flow}}^{(2)}_{(\bar{q}\tau^0 q)}\,,$$

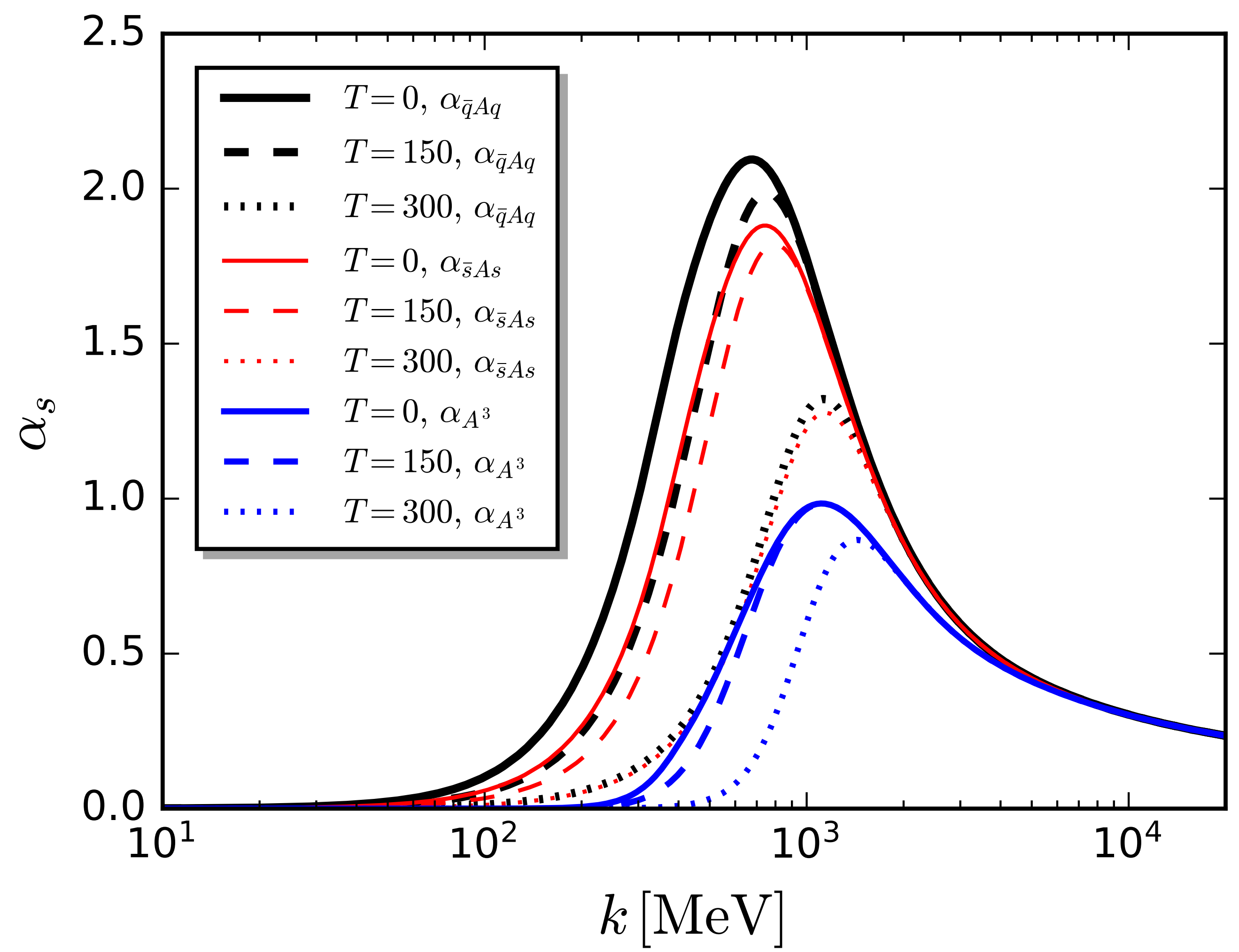




WF, J.M. Pawlowski, F. Rennecke, arXiv:1907.xxxxx.

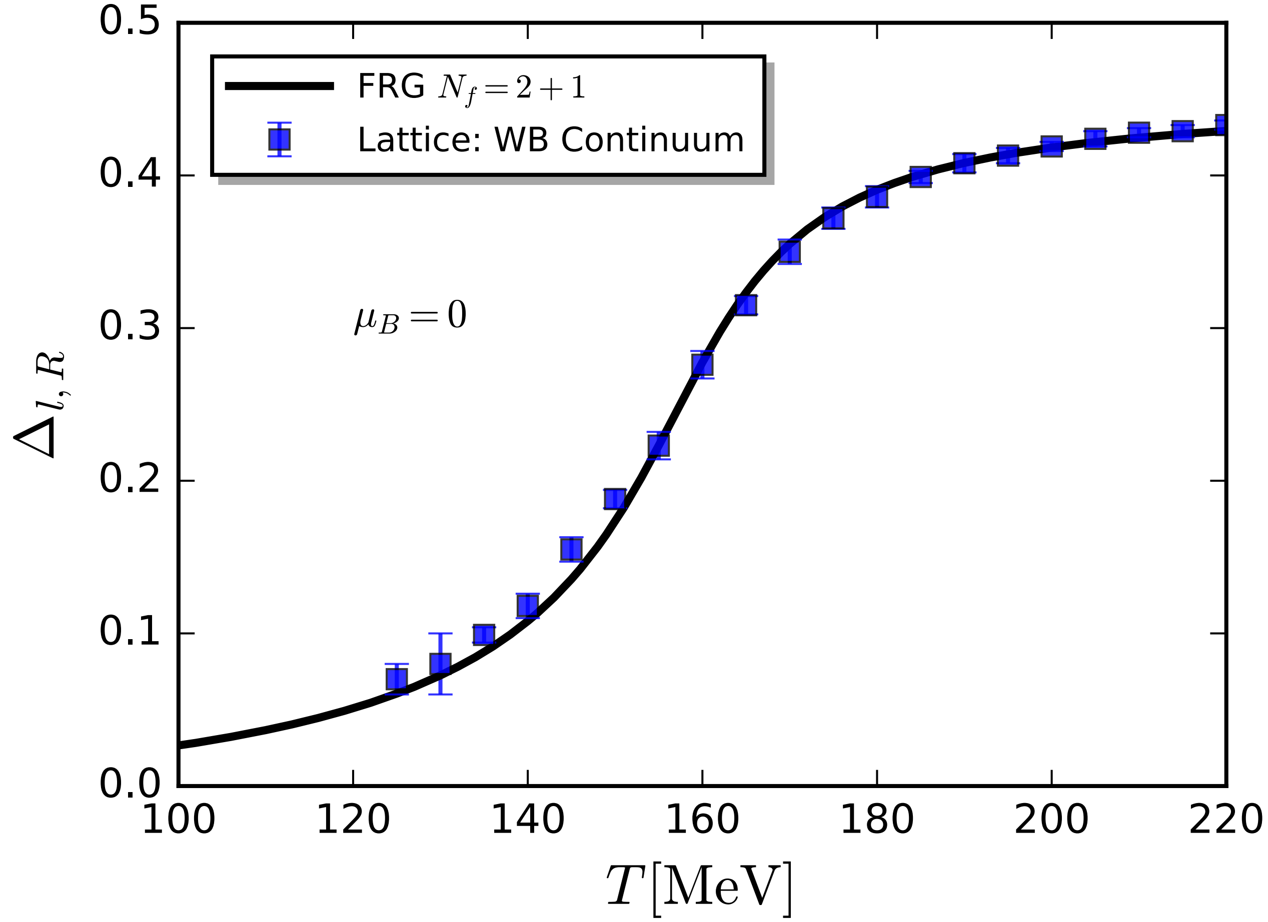
QCD strong couplings among quarks and gluons





WF, J.M. Pawlowski, F. Rennecke, arXiv:1907.xxxxx.

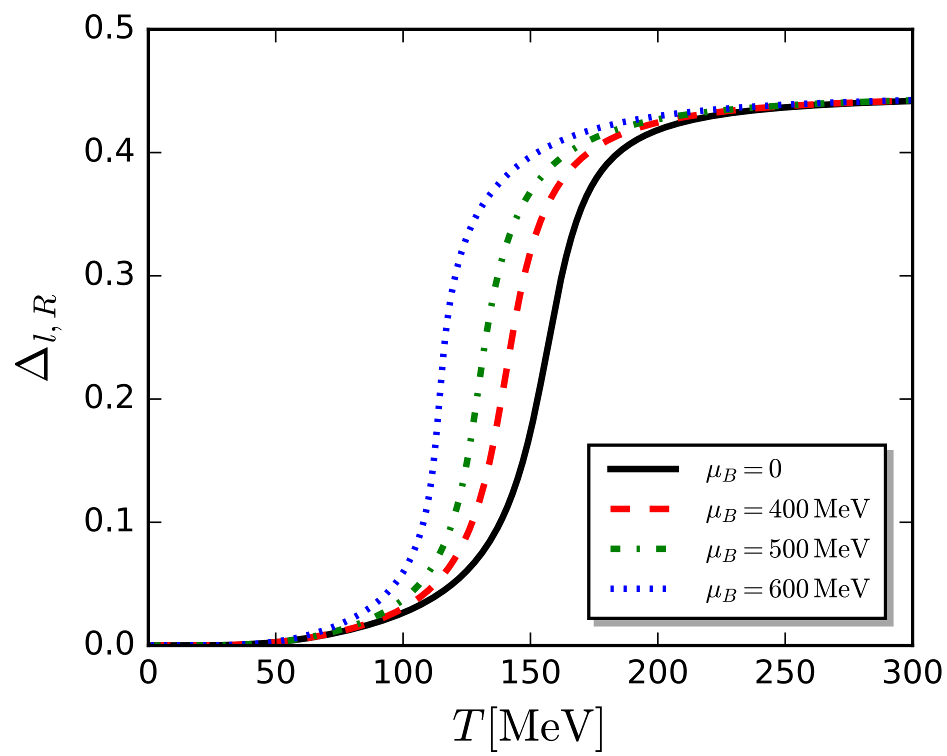
Renormalized light quark condensate

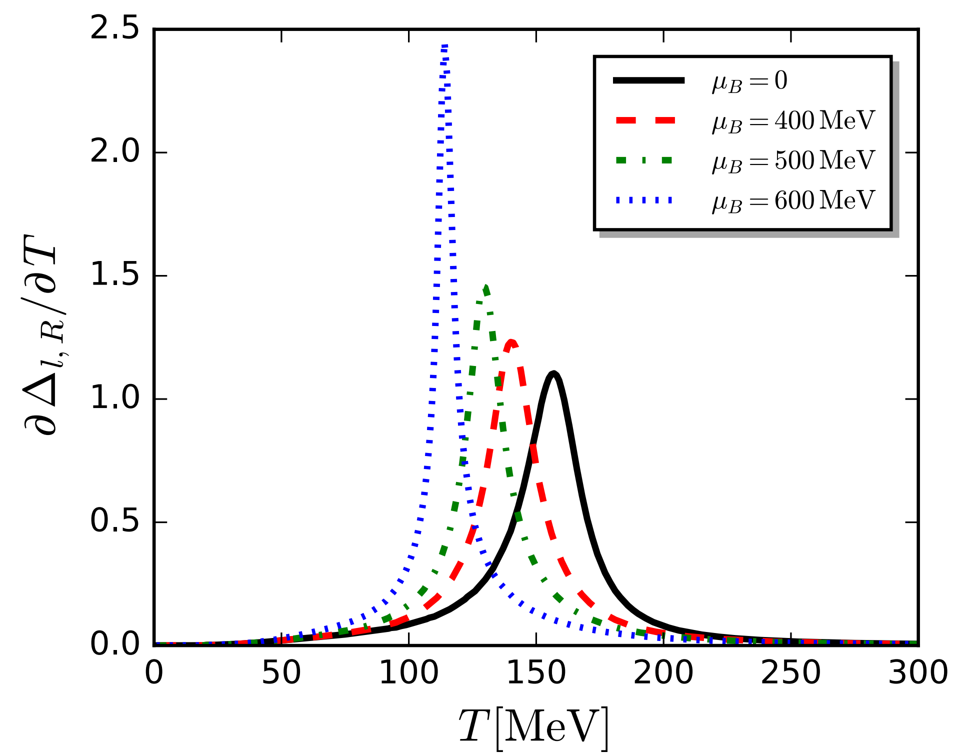


$$\Delta_{q_i} \simeq -m_{q_i}^0 T \sum_{n\in\mathbb{Z}} \int \frac{d^3q}{(2\pi)^3} \mathrm{tr}\, G_{q_i\bar{q}_i}(q)\,.$$

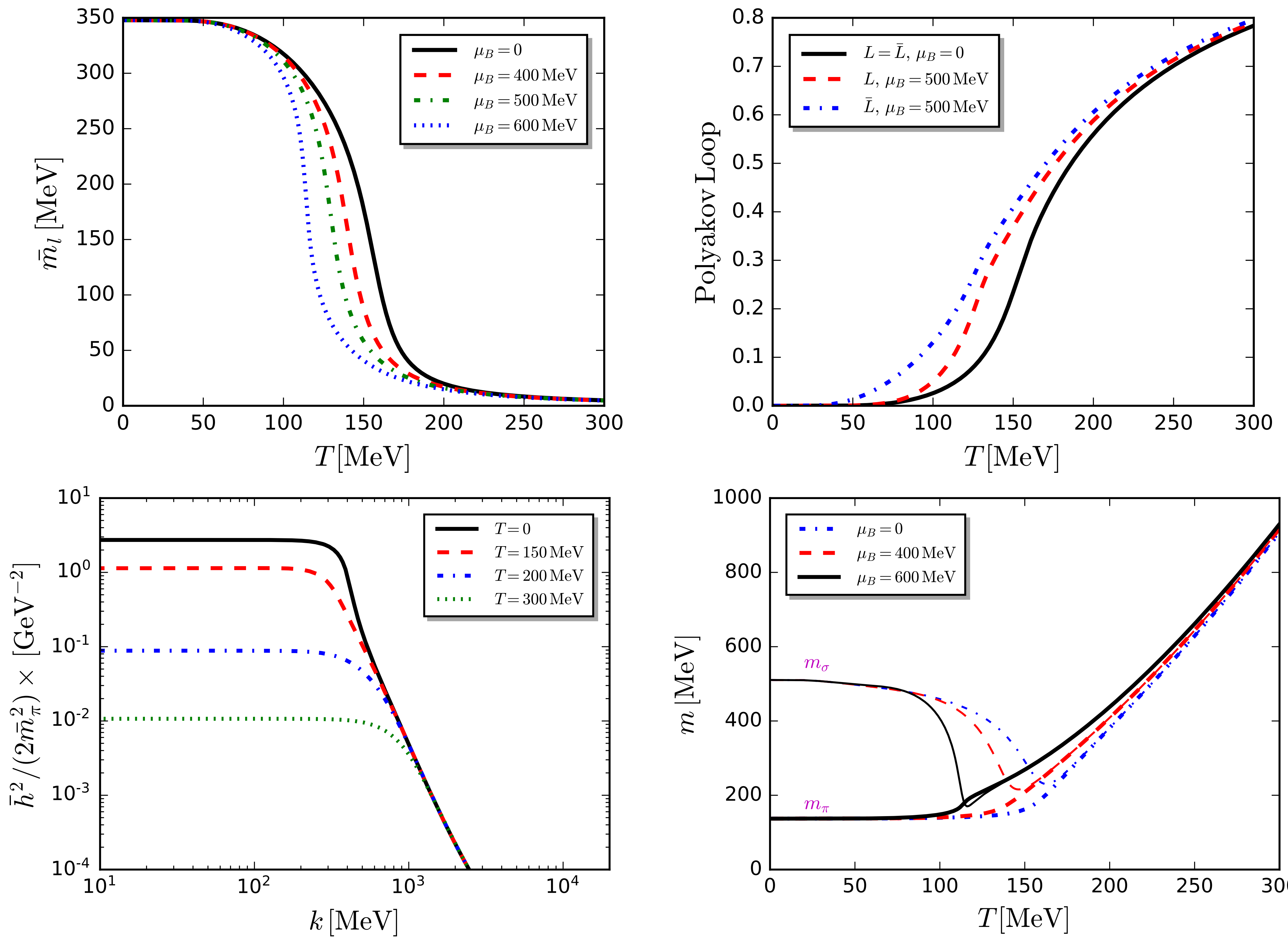
$$\Delta_{q_i,R} = \frac{1}{\mathcal{N}_R}\left[\Delta_{q_i}(T,\mu_q) - \Delta_{q_i}(0,0)\right]\,.$$

WF, J.M. Pawlowski, F. Rennecke, arXiv:1907.xxxxx.



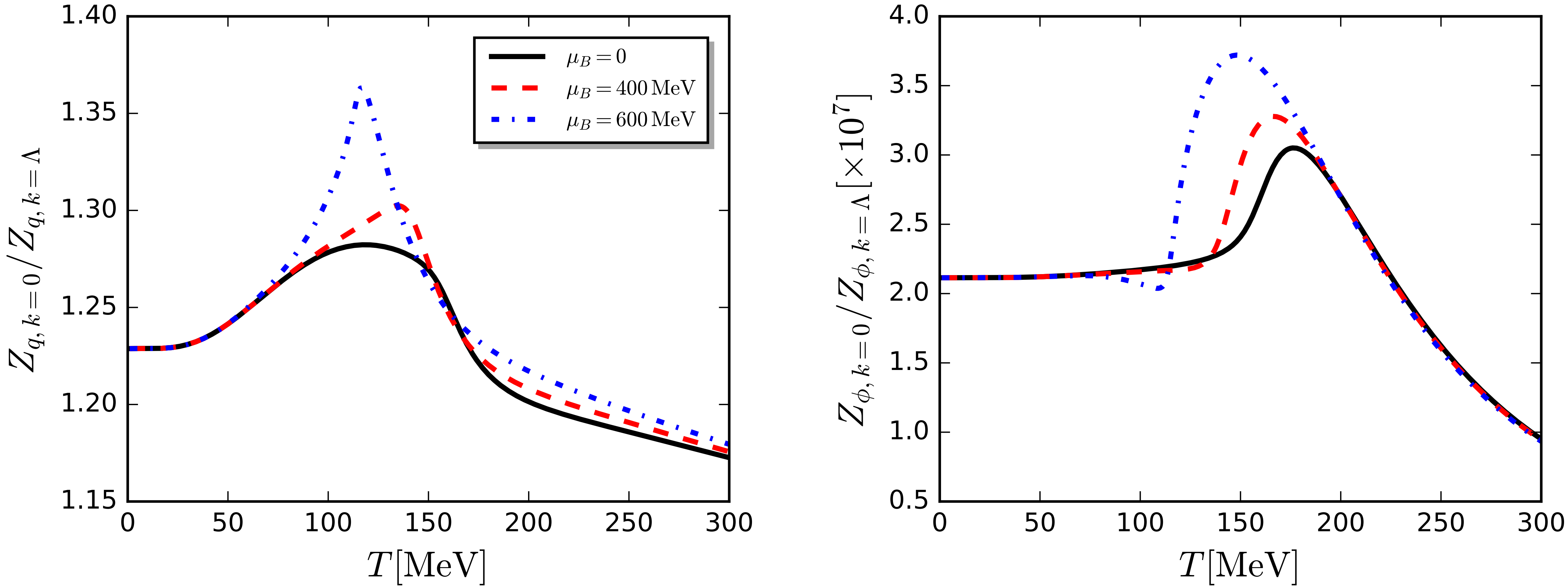


Quark mass, Polyakov loop and effective 4-fermion coupling



WF, J.M. Pawlowski, F. Rennecke, arXiv:1907.xxxxx.

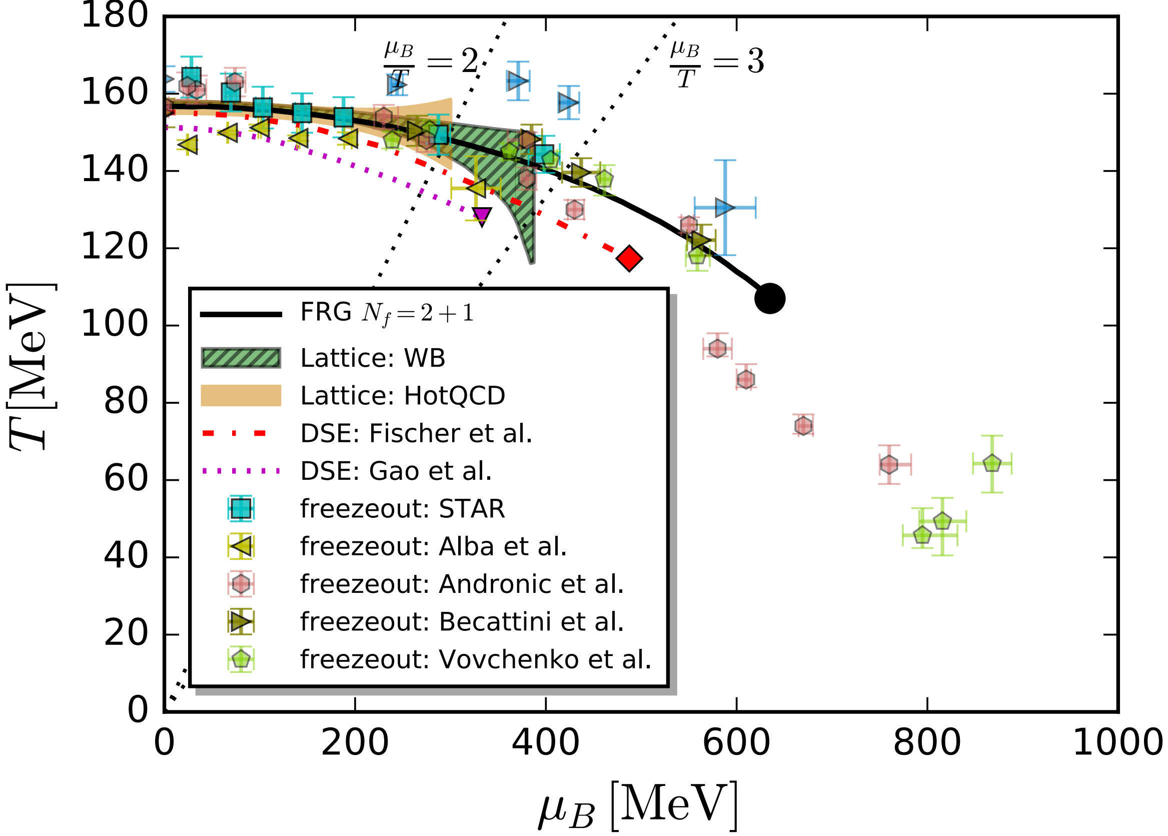
Quark and meson wave function renormalization

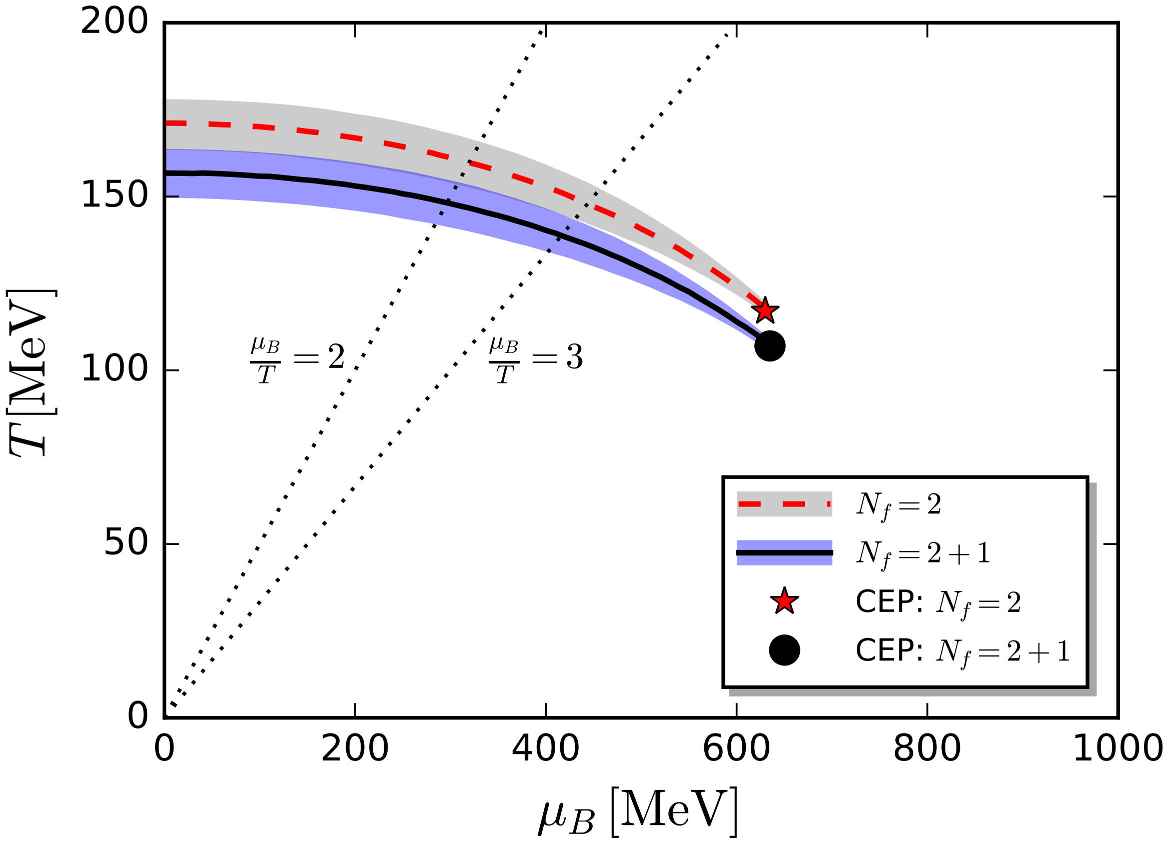


WF, J.M. Pawlowski, F. Rennecke, arXiv:1907.xxxxx.

Phase diagram and curvature

WF, J.M. Pawlowski, F. Rennecke, arXiv:1907.xxxxx.





CEP:

$$(T_{\mathrm{CEP}}, \mu_{B_{\mathrm{CEP}}})_{N_f=2+1} = (107\,\mathrm{MeV}, 635\,\mathrm{MeV})\,,$$

$$(T_{\mathrm{CEP}}, \mu_{B_{\mathrm{CEP}}})_{N_f=2} = (117\,\mathrm{MeV}, 630\,\mathrm{MeV})\,,$$

FRG curvature of the phase boundary:

$$\frac{T_c(\mu_B)}{T_c} = 1 - \kappa\left(\frac{\mu_B}{T_c}\right)^2 + \lambda\left(\frac{\mu_B}{T_c}\right)^4 + \cdots\,,$$

$$\kappa_{N_f=2+1} = 0.0142(2)$$

$$\kappa_{N_f=2} = 0.0176(1)$$

Lattice result:

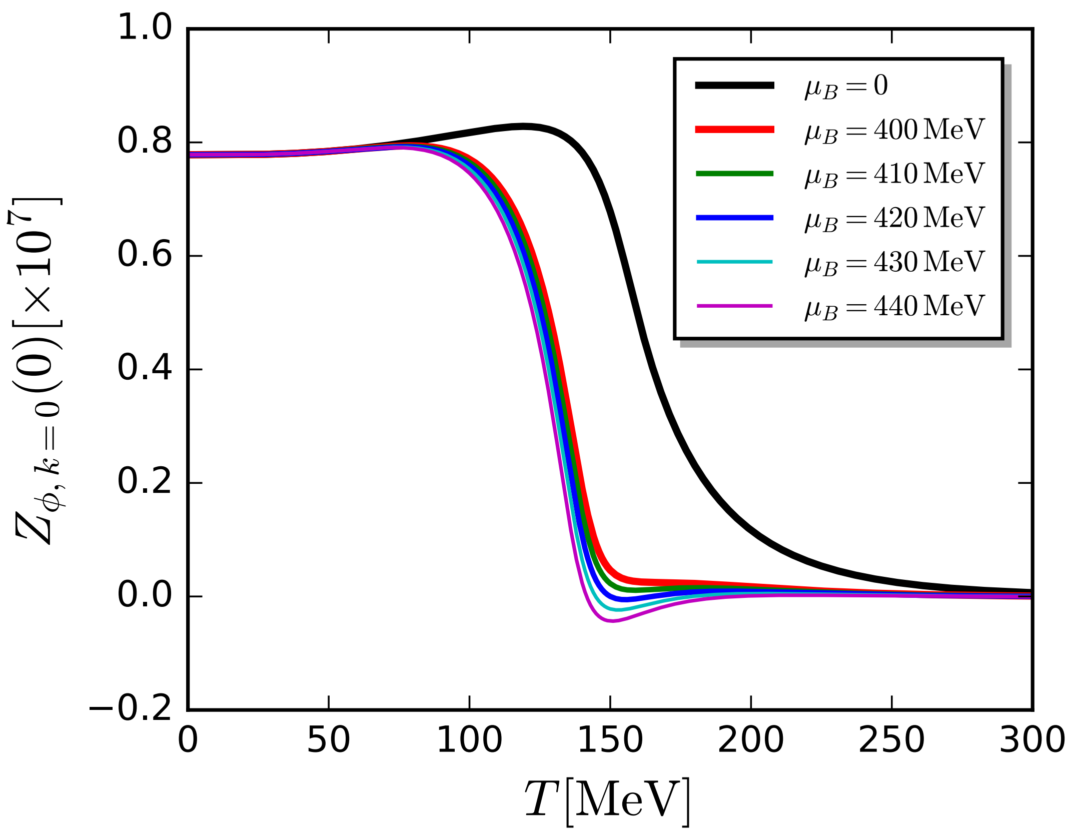
$$\kappa = 0.0149 \pm 0.0021$$

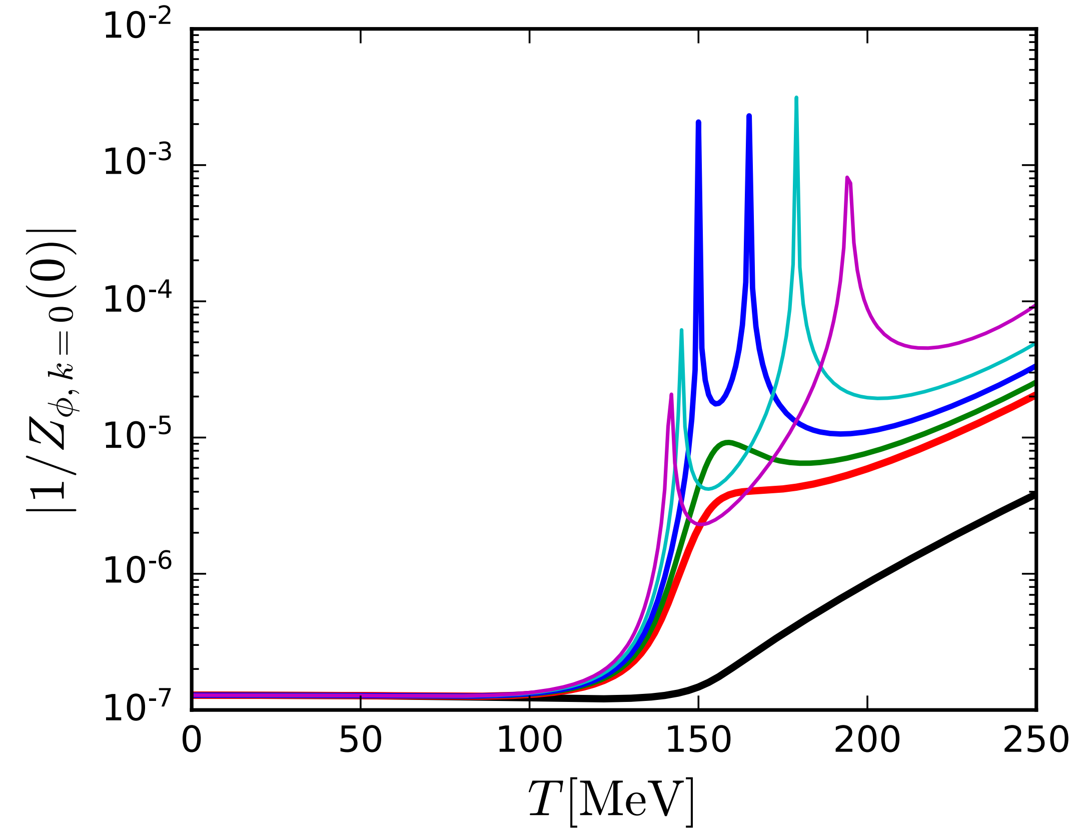
R. Bellwied *et al.* (WB), arXiv:1507.07510.

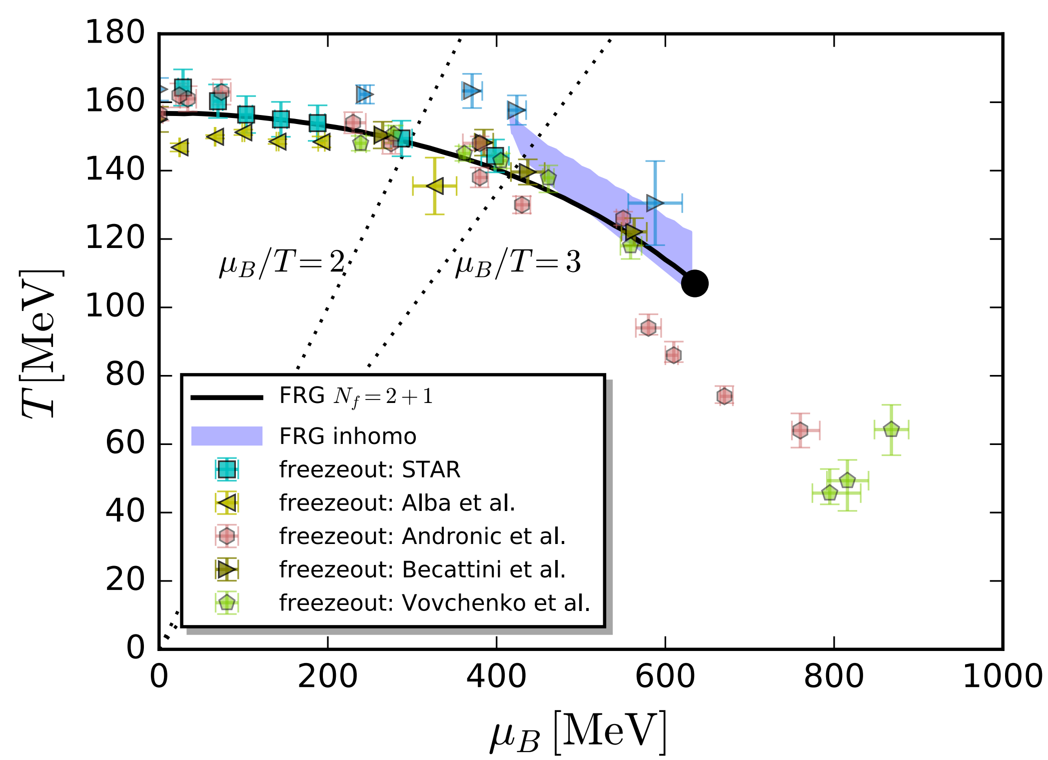
$$\kappa = 0.015 \pm 0.004$$

A. Bazavov *et al.* (HotQCD), arXiv:1812.08235.

Inhomogeneous phase?





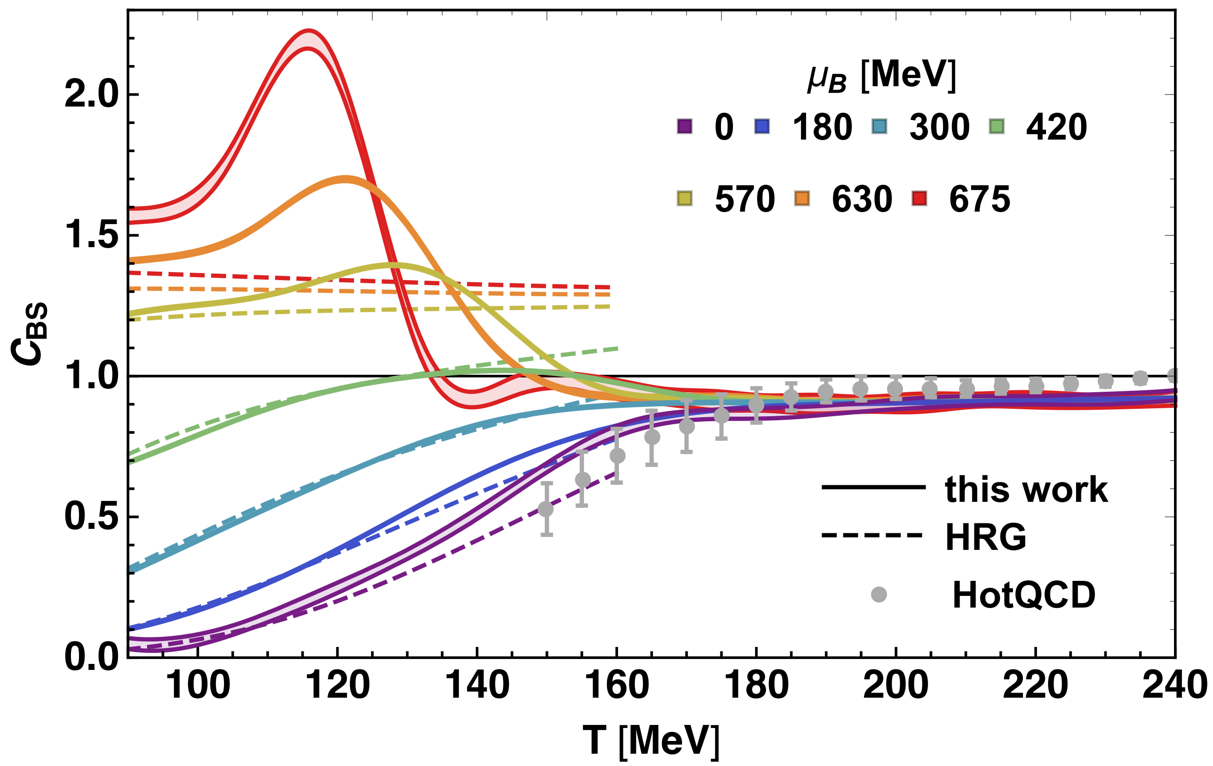


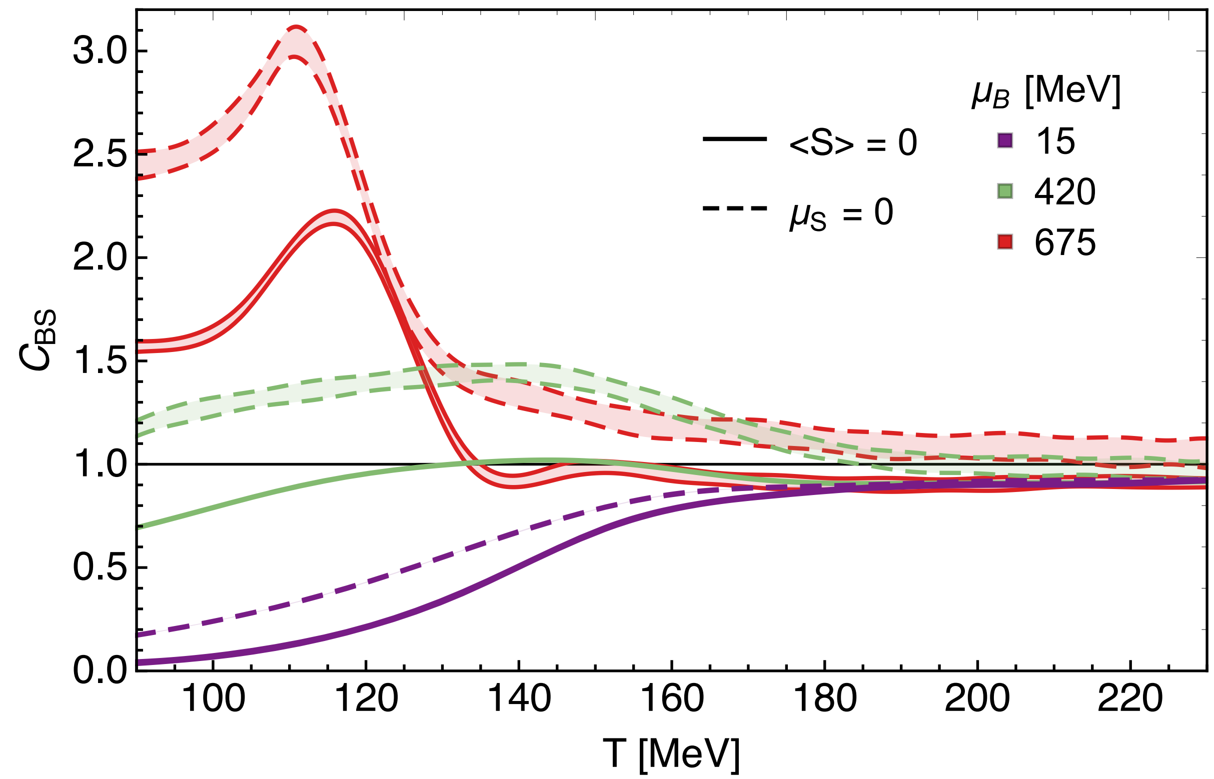
Two point function for the meson:

$$\Gamma^{(2)}_{\phi\phi}(p) = Z_\phi(p^2)\, p^2 + m_\phi^2\,,$$

WF, J.M. Pawlowski, F. Rennecke, arXiv:1907.xxxxx.

Strangeness neutrality and baryon-strangeness correlations





Baryon-strangeness correlation:

$$C_{BS}(T, \mu_B, \mu_{S0}) = -3\frac{\langle BS\rangle}{\langle S^2\rangle} = -3\frac{\chi_{11}^{BS}(T, \mu_B, \mu_{S0})}{\chi_2^S(T, \mu_B, \mu_{S0})}$$

Strangeness neutrality:

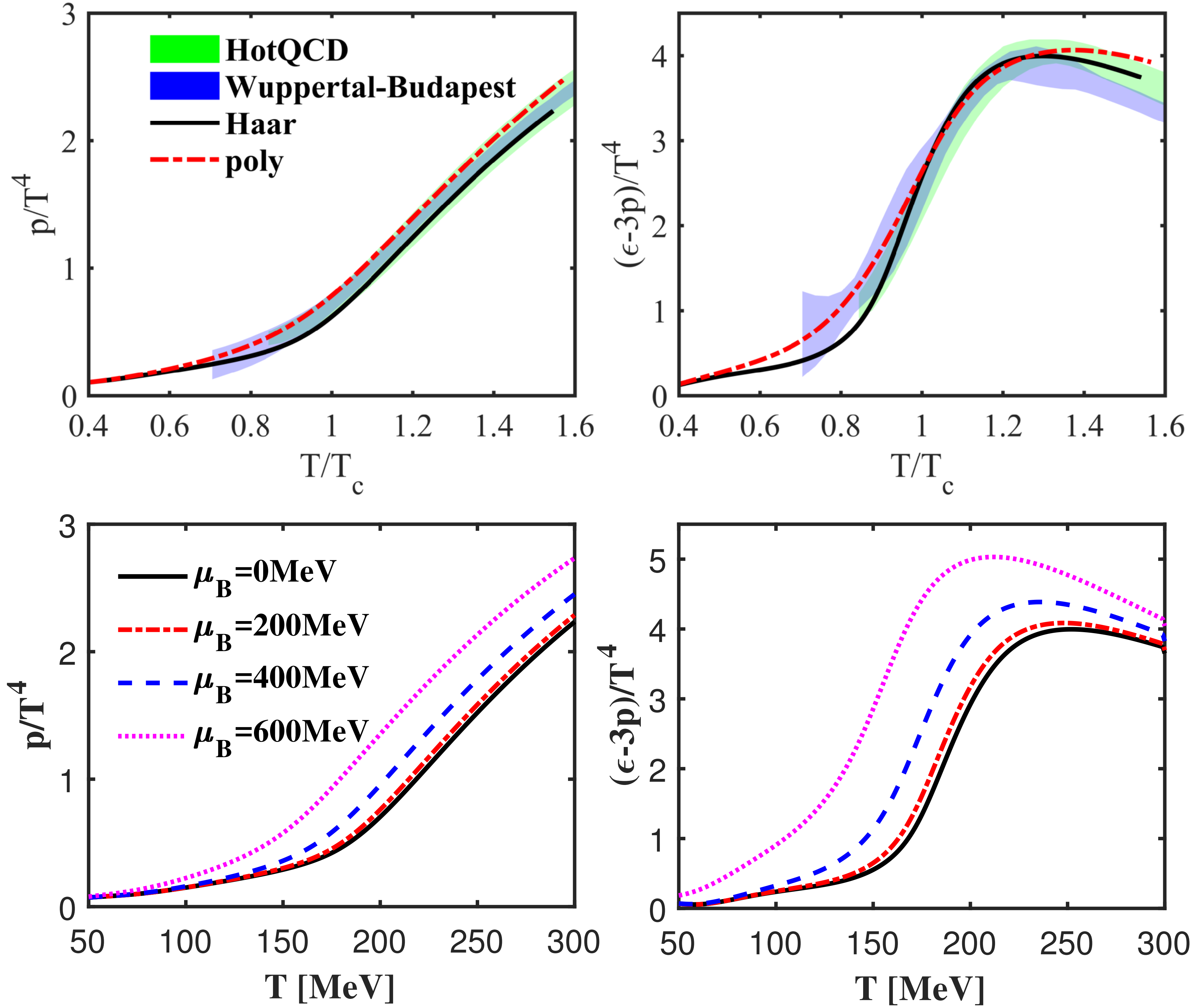
$$0 = \frac{d}{d\hat{\mu}_B}\chi_1^S(T, \mu_B, \mu_{S0})$$

$$= \chi_{11}^{BS}(T, \mu_B, \mu_{S0}) + \chi_2^S(T, \mu_B, \mu_{S0})\frac{\partial\hat{\mu}_{S0}}{\partial\hat{\mu}_B}$$

$$\frac{\partial\mu_{S0}(T, \mu_B)}{\partial\mu_B} = \frac{1}{3}C_{BS}(T, \mu_B, \mu_{S0})\,.$$

WF, J.M. Pawlowski, F. Rennecke, arXiv:1808.00410, 1809.01594

QCD equation of state



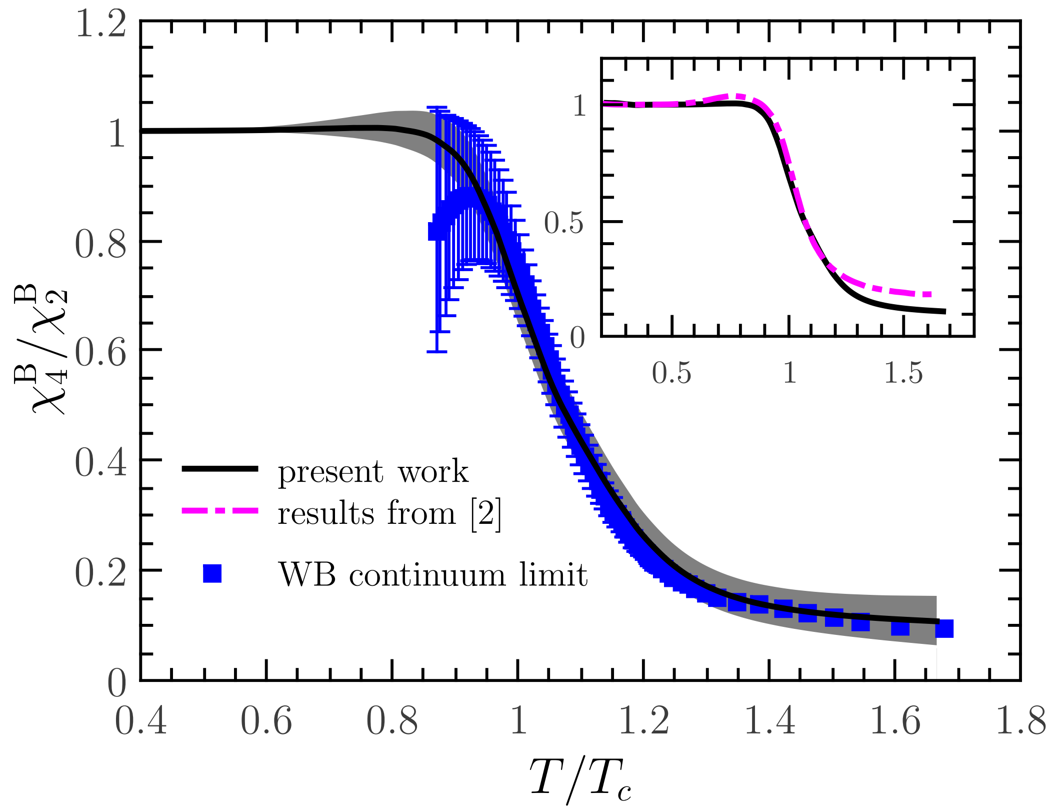
FRG results at muB=0

FRG results at high muB

R.Wen, C. Huang, WF, PRD 99 (2019) 094019

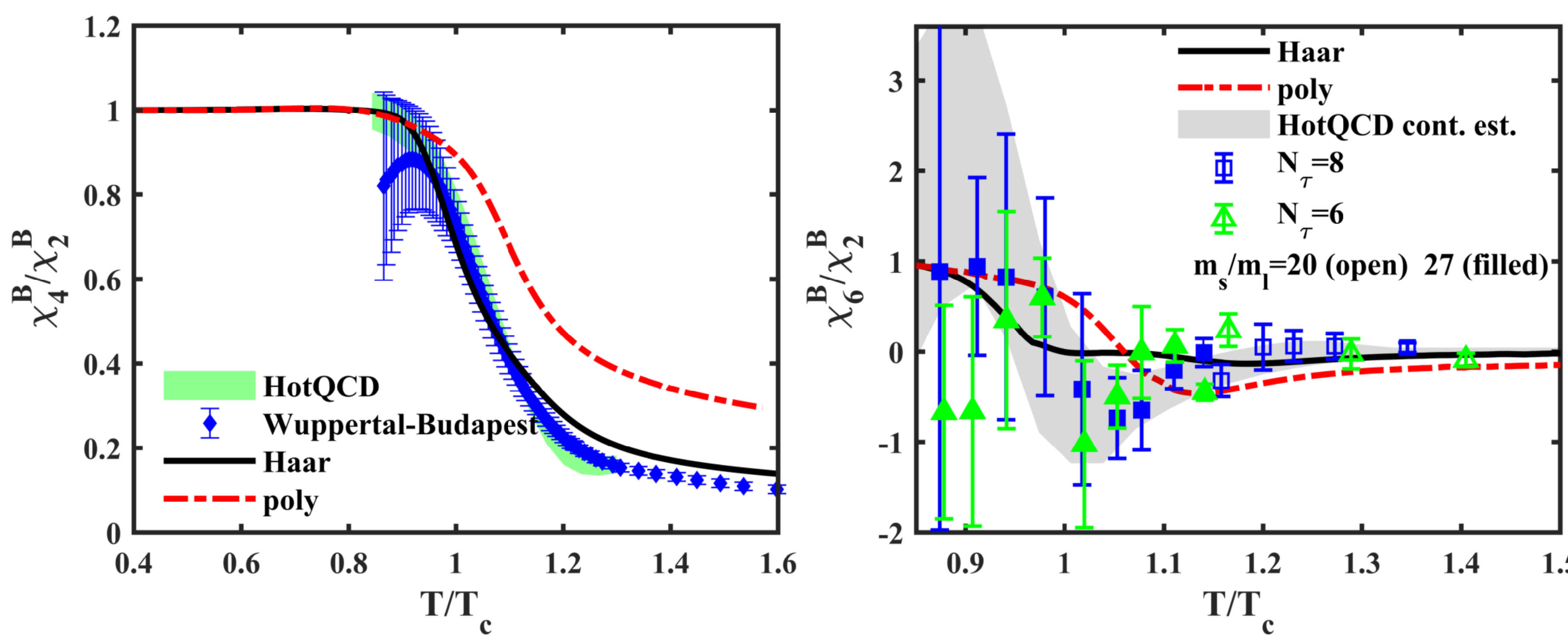
Baryon number fluctuations from FRG

$N_f = 2$



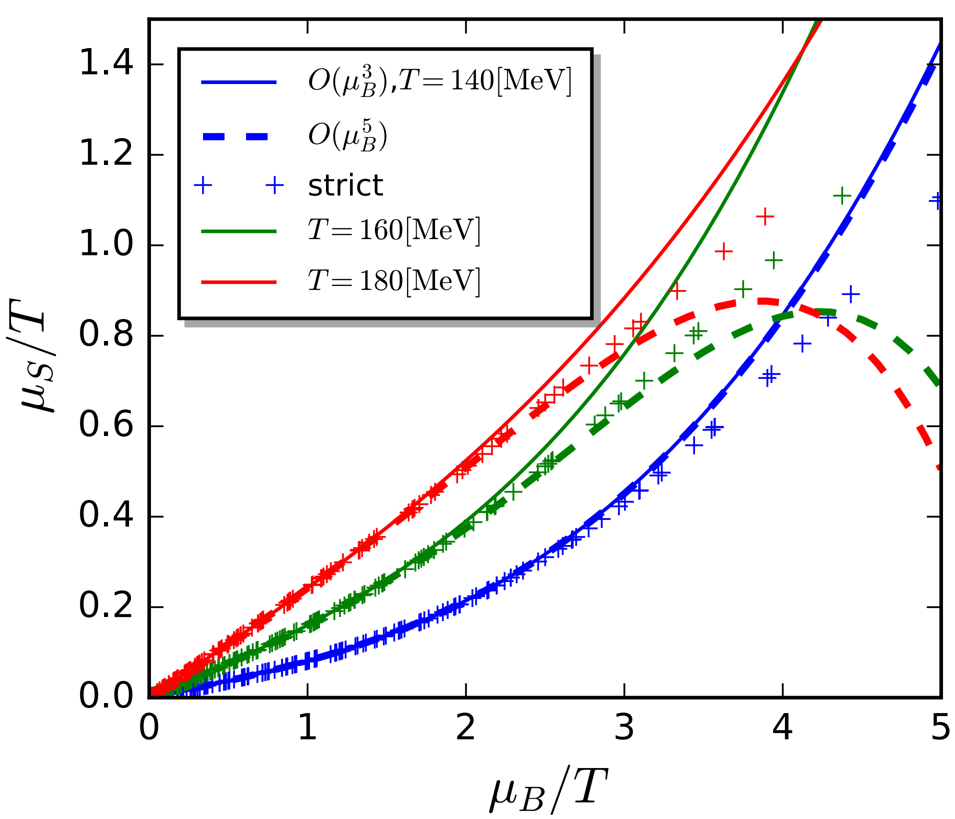
WF, J.M. Pawlowski, F. Rennecke, Bernd-Jochen Schaefer, PRD 94 (2016) 116020

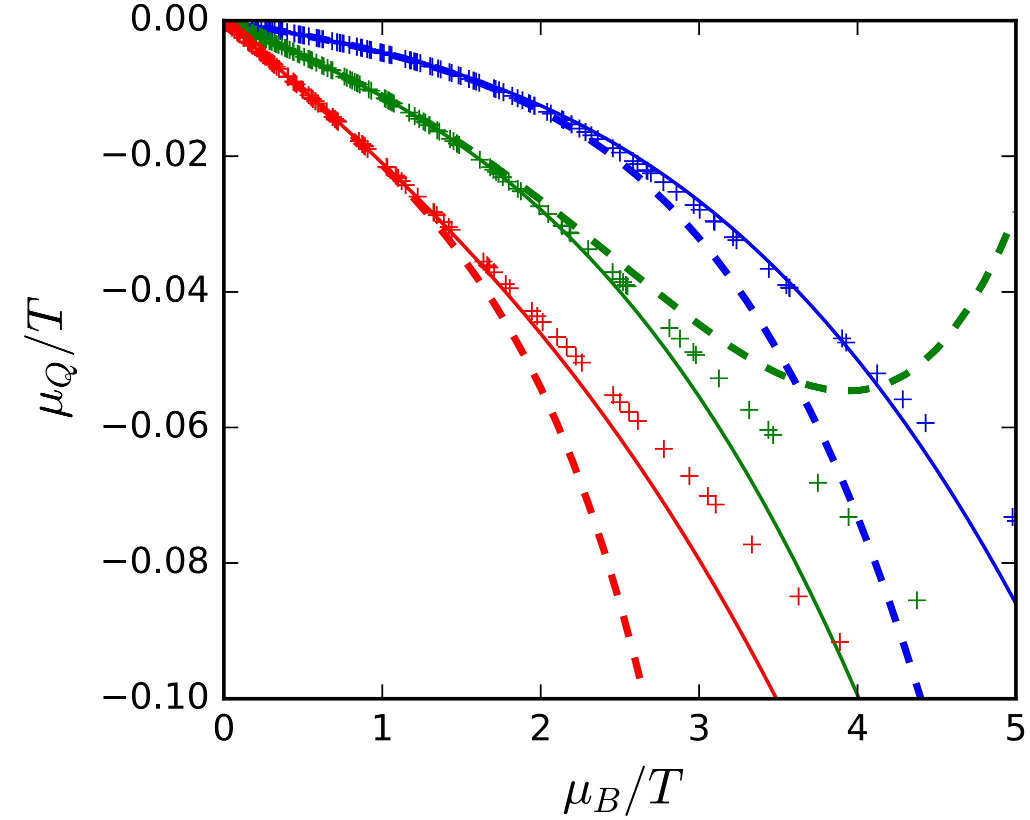
$N_f = 2+1$



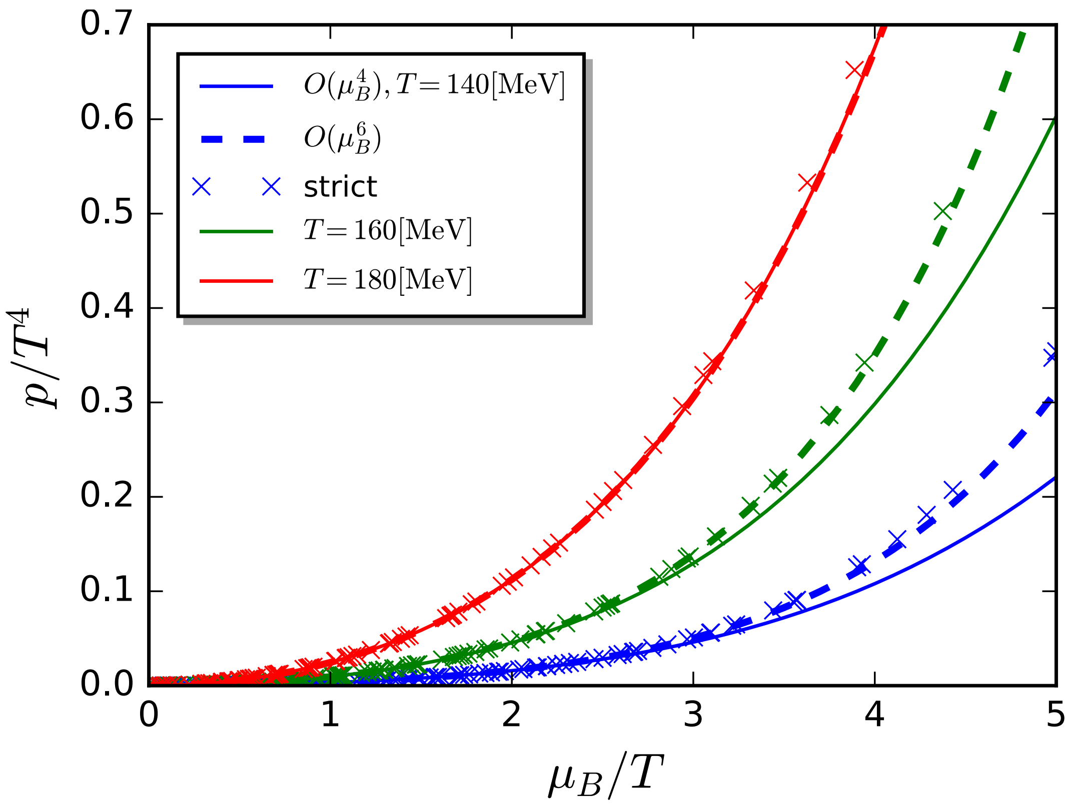
R.Wen, C. Huang, WF, PRD 99 (2019) 094019

Convergency of the Taylor expansion for muB



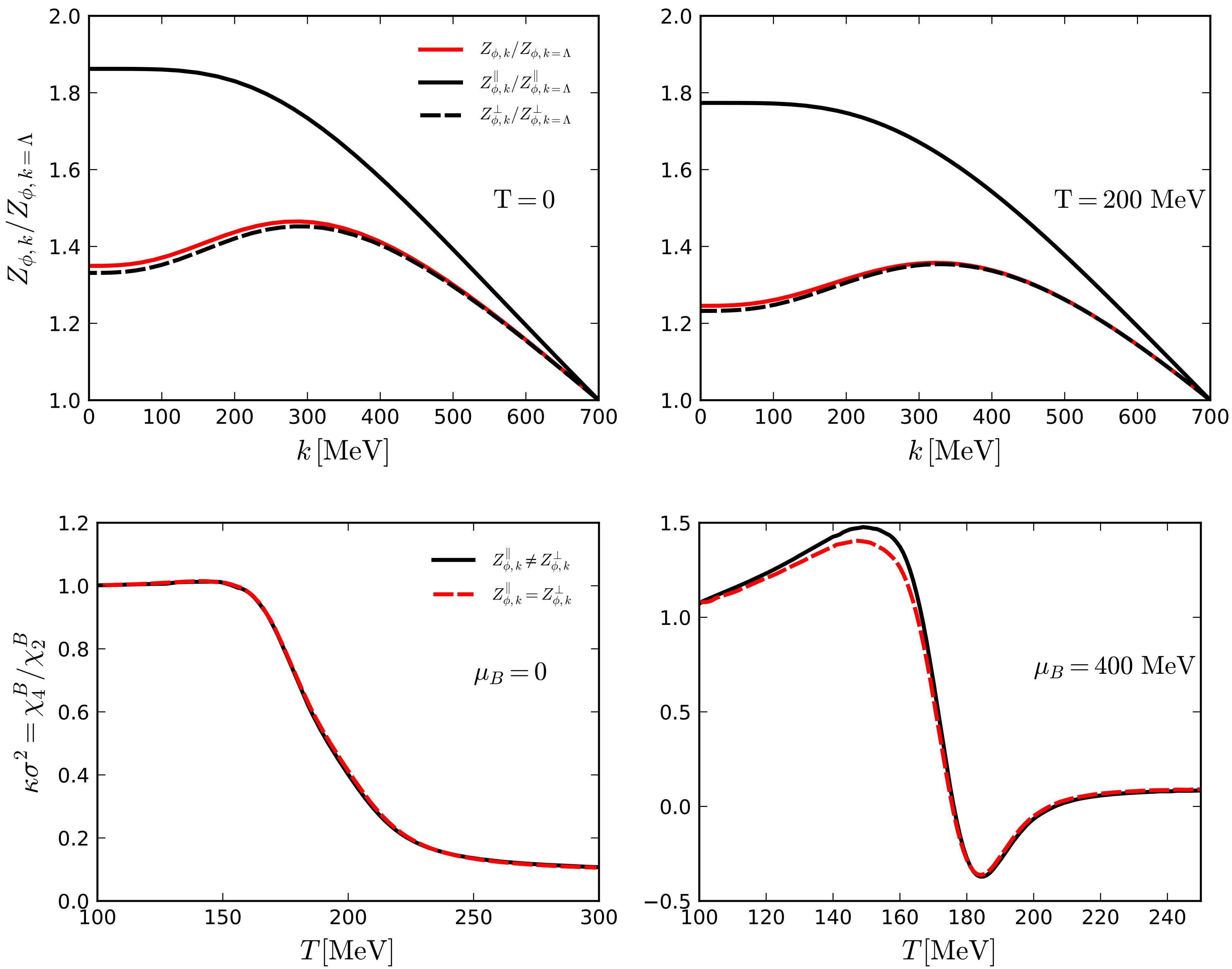


Pressure:



R.Wen, WF, in preparation.

Thermal splitting of the meson wave function renormalization



Zphi

Kurtosis

S. Yin, R.Wen, WF, in preparation.

Summary and outlook

★ We have investigated the QCD phase structure within FRG.

★ A CEP at (T, muB)=(107,635) MeV, as well as indications for an inhomogeneous phase for muB >=420 MeV is founded.

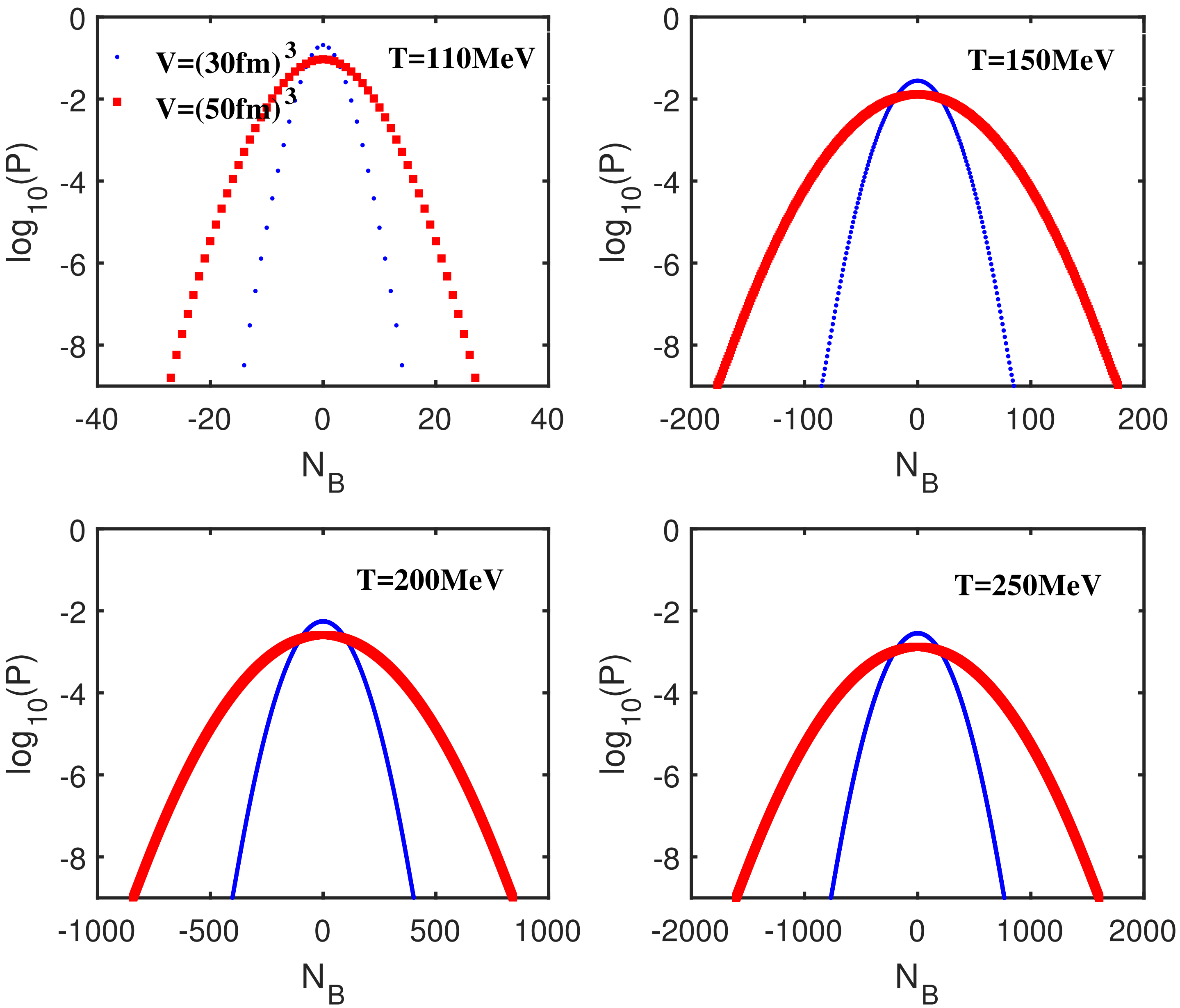
★ The curvature of the phase boundary at small chemical potential is kappa=0.0142(2), in agreement with lattice results.

★ Improvement of our calculation and its applications on other observables, e.g., EoS, fluctuations, etc. are highly required.

Thank you very much for your attentions!

Backup

Baryon number probability distribution



K.-x. Sun, R. Wen, WF, PRD 98, 074028, (2018)

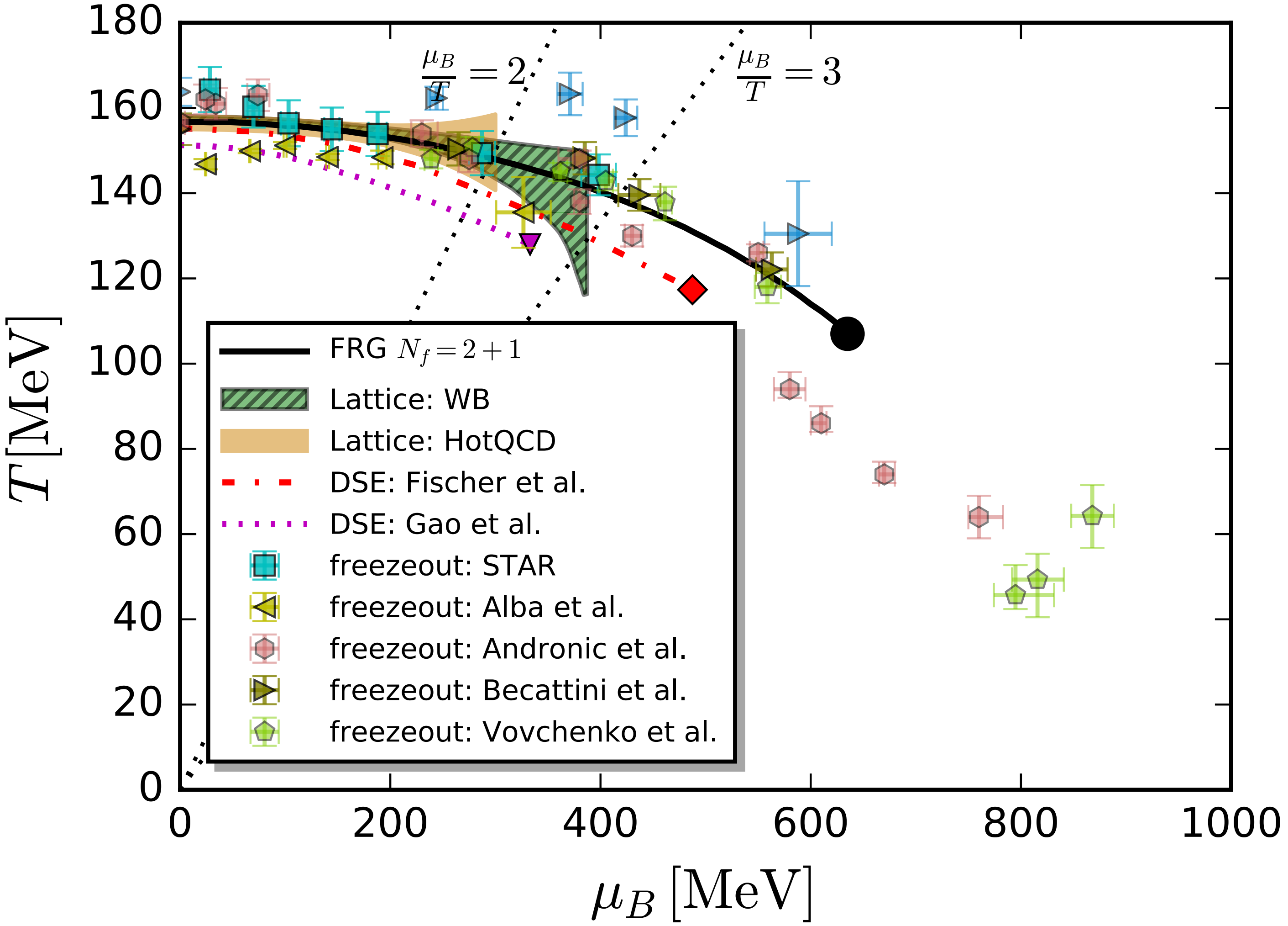
Probability distribution:

canonical partition function

$$P(N_B; T, V, \mu_B) = \frac{Z(T, V, N_B)}{\mathcal{Z}(T, V, \mu_B)} \exp\left(\frac{\mu_B N_B}{T}\right),$$

grand canonical partition function

Comparison with other approaches and experiments



Systematic error:

- Overestimation of mesonic interactions delays the occurrence of the CEP.
- Fierz complete basis for the four-fermion couplings are needed.
- DSE overestimates the chemical potential dependence and predict a larger curvature.
- Reducing the curvature for the DSE sets a lower bound for the occurrence of the CEP.