Nuclear Science Computing Center at CCNU

QCD phase structure from Lattice QCD

Heng-Tong Ding (丁亨通)

Central China Normal University

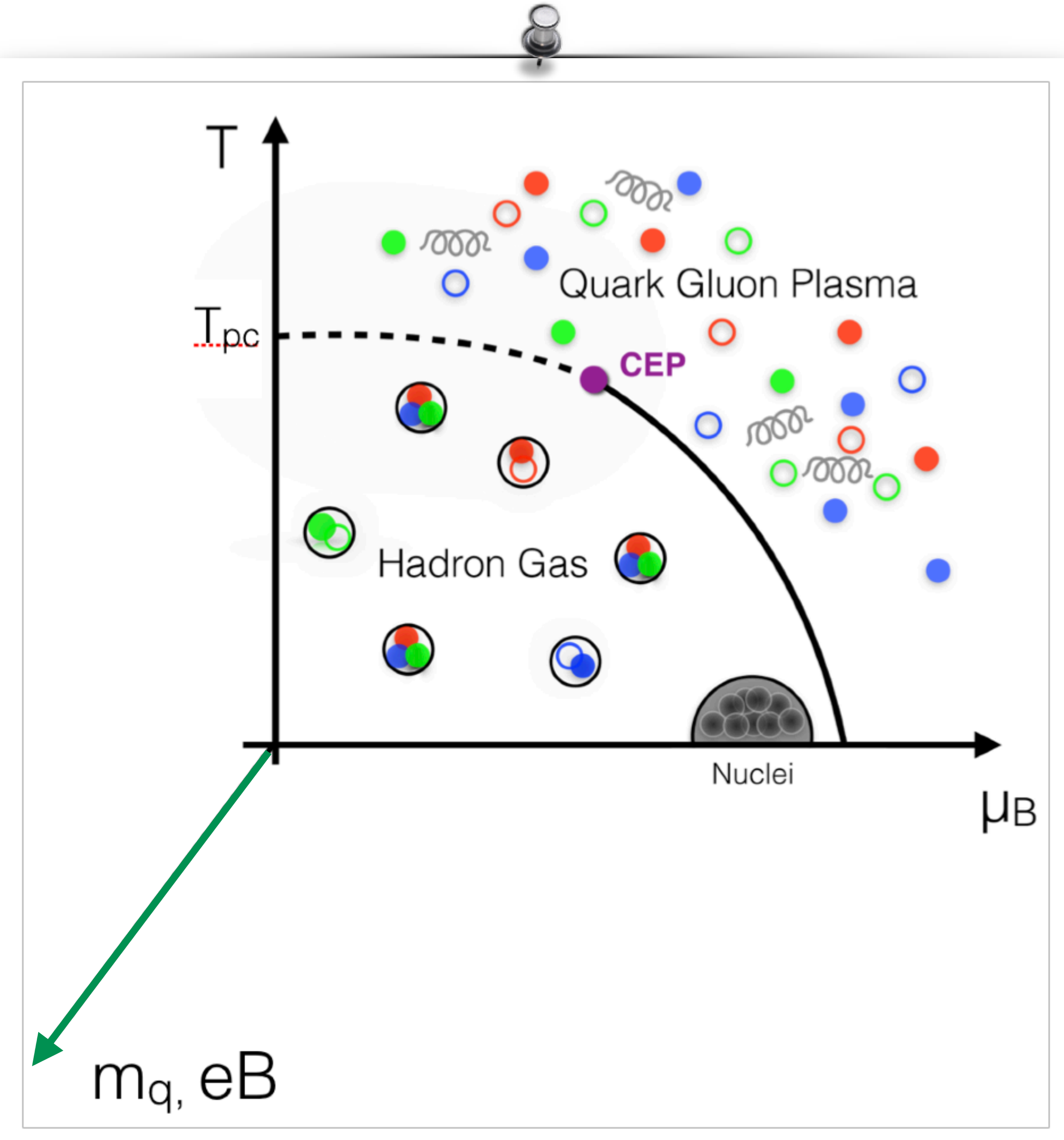
workshop on

QCD Physics & Study of the QCD Phase Diagram and New-type Topologic Effect

QCD物理暨国家自然科学基金重大项目交流会

17-25 July, 2019@Shangdong University

Outline: QCD phase structure



HTD, F. Karsch, S. Mukherjee, arXiv: 1504.05274
Int.J.Mod.Phys. E24 (2015) no.10, 1530007

- Chiral crossover at zero and small μ_B

  A. Bazavov, HTD, P. Hegde et al. [HotQCD], Phys. Lett. B795 (2019) 15 (arXiv:1812.08235)

- Chiral phase transition temperature

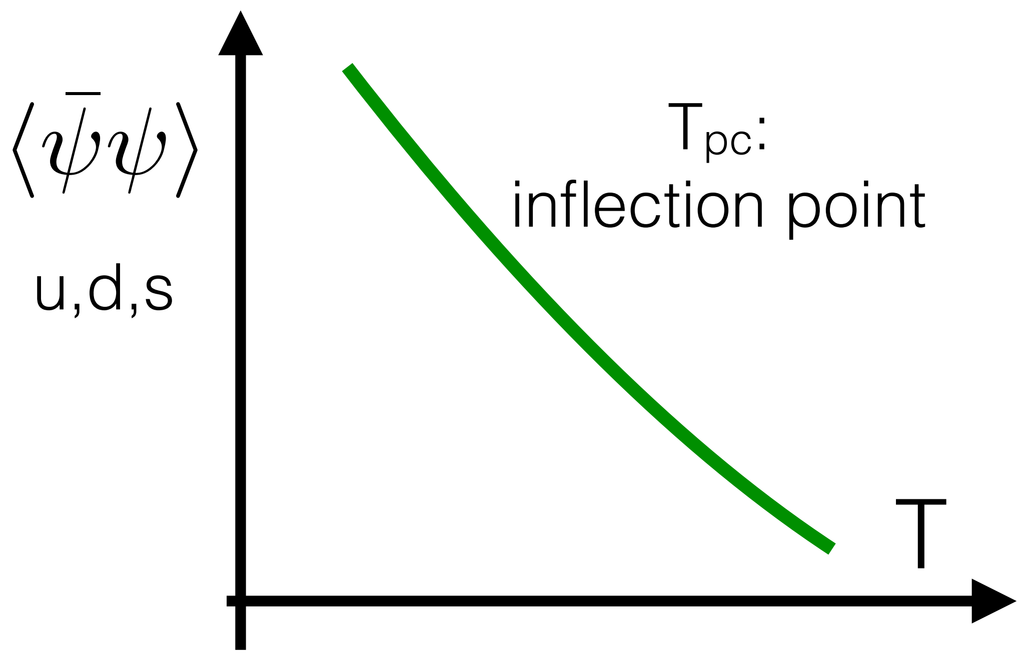
  HTD, P. Hegde, O. Kaczmarek et al. [HotQCD], arXiv:1903.04801, PRL in press

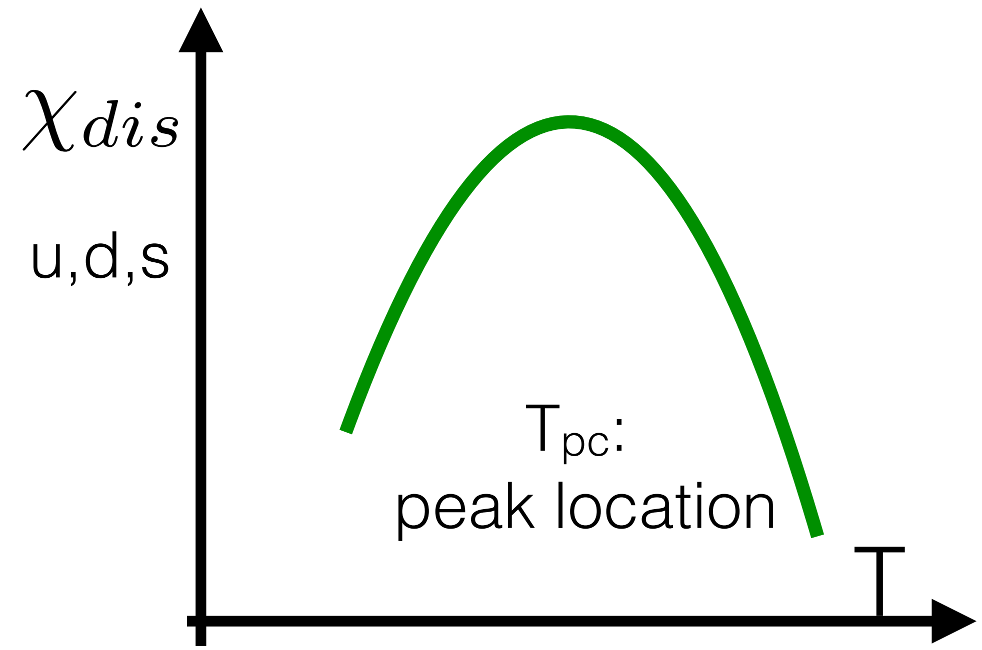
- QCD transition in external B

  Xiao-Dan Wang (汪晓丹) et al., work in progress & arXiv: 1904.01276

Crossover transition temperature T_{pc} in the real world

- Crossover nature of the transition





- Chiral phase transition: most likely 2nd order, 3d O(4)

Ejiri et al., PRD 80(2009)094505,
HTD et al. [HotQCD], arXiv:1903.04801…

- A well-defined **chiral crossover transition temperature**: based on scaling properties of QCD

HTD, P. Hegde, O. Kaczmarek et al.
[HotQCD], arXiv:1903.04801, PRL in press

Scaling behavior of chiral observables

chiral condensate: $\Sigma(T, \mu_B) \sim m^{1/\delta} f_G$

chiral susceptibility: $\chi^{\Sigma}(T, \mu_B) \sim m^{1/\delta - 1} f_\chi$

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Taylor expansions:

$$\Sigma(T,\mu_X) = \sum_{n=0}^{\infty} \frac{C_{2n}^{\Sigma}(T)}{(2n)!}\left(\frac{\mu_X}{T}\right)^{2n} \qquad \chi(T,\mu_X) = \sum_{n=0}^{\infty} \frac{C_{2n}^{\chi}(T)}{(2n)!}\left(\frac{\mu_X}{T}\right)^{2n}$$

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$\partial_T \chi^{\Sigma}(T)$, $\partial_T C_0^{\chi}(T)$, $C_2^{\chi}(T)$ $\sim \; m^{1/\delta-1-1/\beta\delta} f'_\chi(z)$

$\partial_T^2 C_0^{\Sigma}(T)$, $\partial_T C_2^{\Sigma}(T)$ $\sim \; m^{1/\delta-2/\beta\delta} f''_G(z)$

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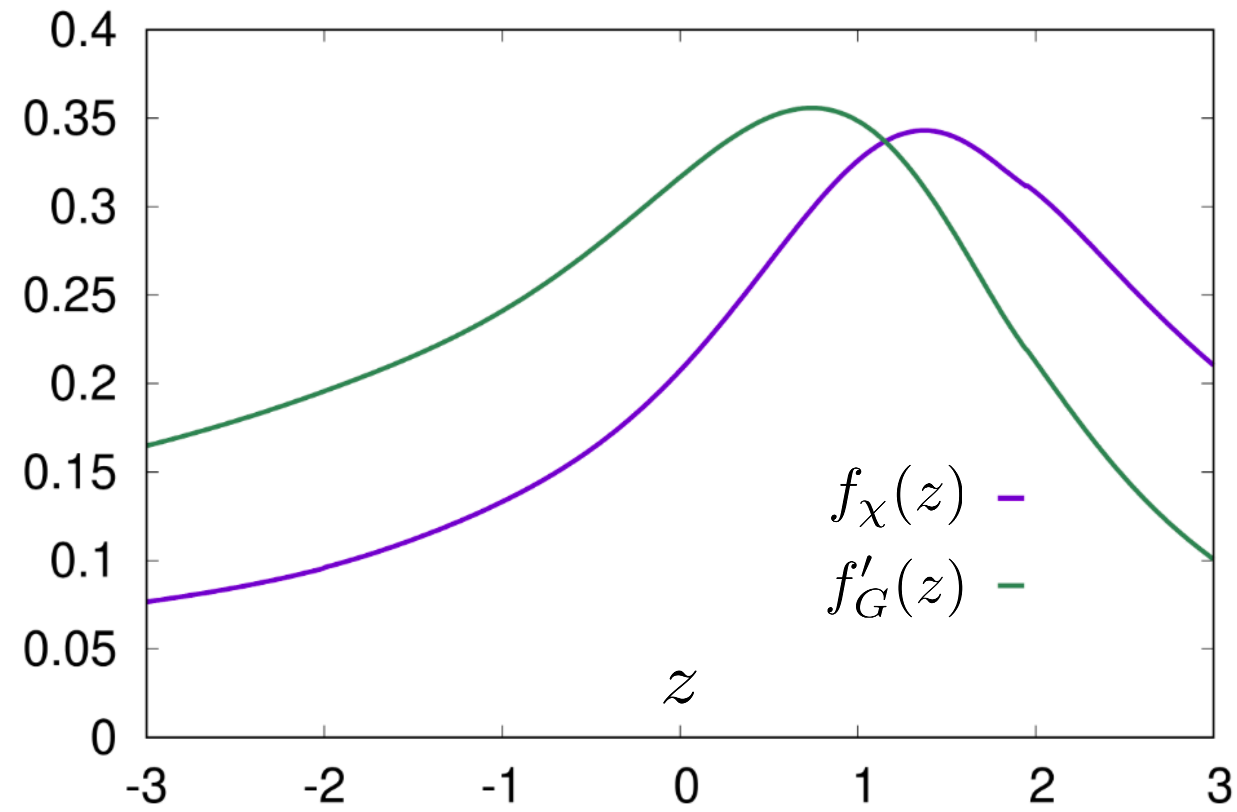
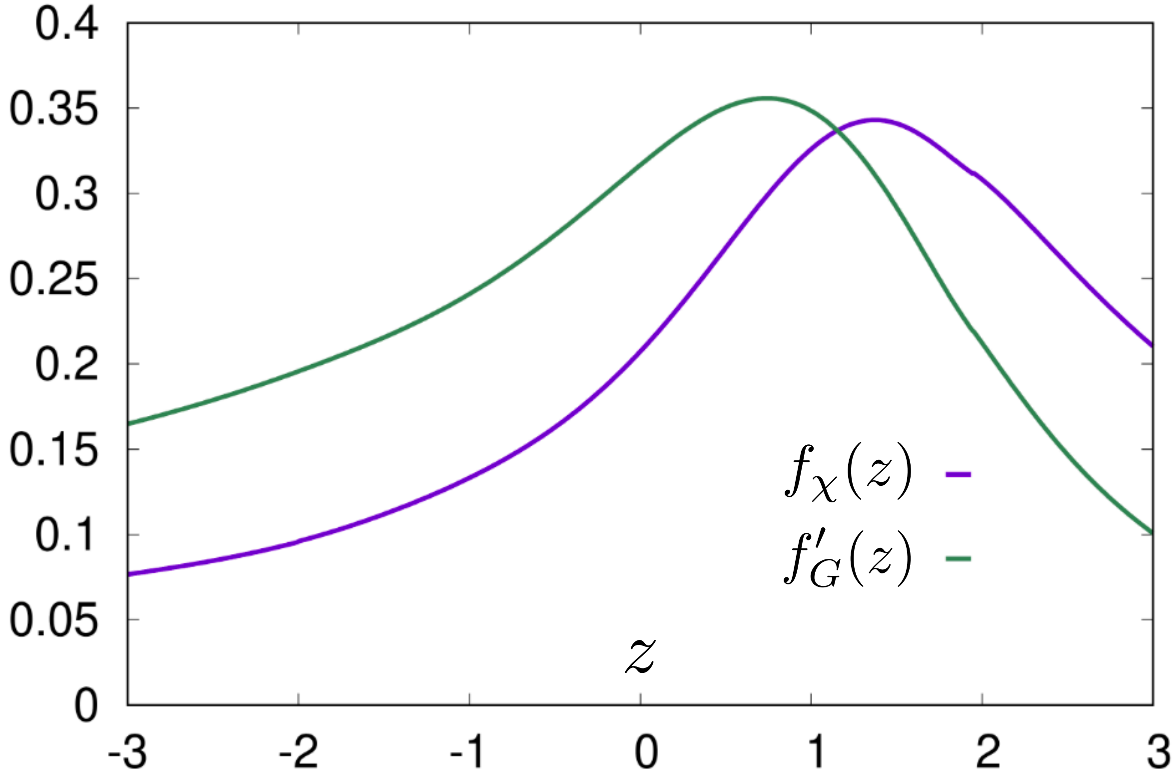
$$\left.\begin{matrix} \partial_T \chi^{\Sigma}(T) \\ \partial_T C_0^{\chi}(T) \\ C_2^{\chi}(T) \end{matrix}\right\} \sim m^{1/\delta-1-1/\beta\delta} f'_\chi(z)$$

$$\left.\begin{matrix} \partial_T^2 C_0^{\Sigma}(T) \\ \partial_T C_2^{\Sigma}(T) \end{matrix}\right\} \sim m^{1/\delta-2/\beta\delta} f''_G(z)$$



$z \sim \left((T-T_c^0)/T_c^0 + K(\mu_B/T)^2\right)/m^{\frac{1}{\beta\delta}}$

Well-defined notation of chiral crossover transition temperature



$z \sim \left((T - T_c^0)/T_c^0 + K(\mu_B/T)^2\right)/m^{\frac{1}{\beta\delta}}$

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$$\left.\begin{matrix}\partial_T^2 C_0^\Sigma(T) \\ \partial_T C_2^\Sigma(T)\end{matrix}\right\} \sim \quad m^{1/\delta - 2/\beta\delta} f''_G(z)$$

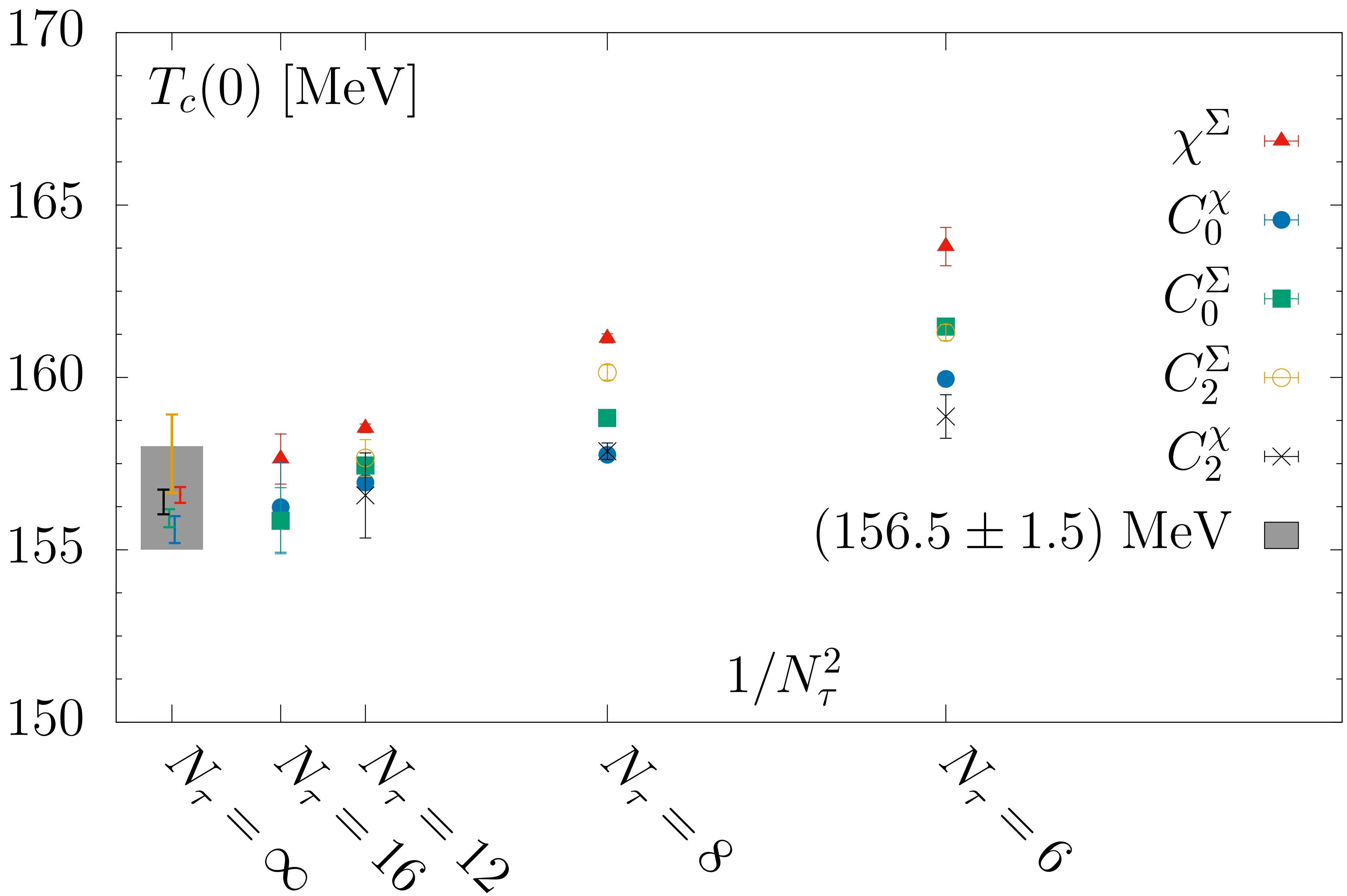
- 5 conditions to extract Tc: maxima of f_χ and f'_G

$\partial_T \chi^\Sigma(T) = 0$ $\quad \partial_T C_0^\chi(T) = 0$ $\quad C_2^\chi(T) = 0$ $\quad \partial_T^2 C_0^\Sigma(T) = 0$ $\quad \partial_T C_2^\Sigma(T) = 0$

- m=0: all these susceptibilities diverge at a unique T

- m=/=0: non-unique temperatures, crossover

QCD transition with m_π =140 MeV at μ_B=0



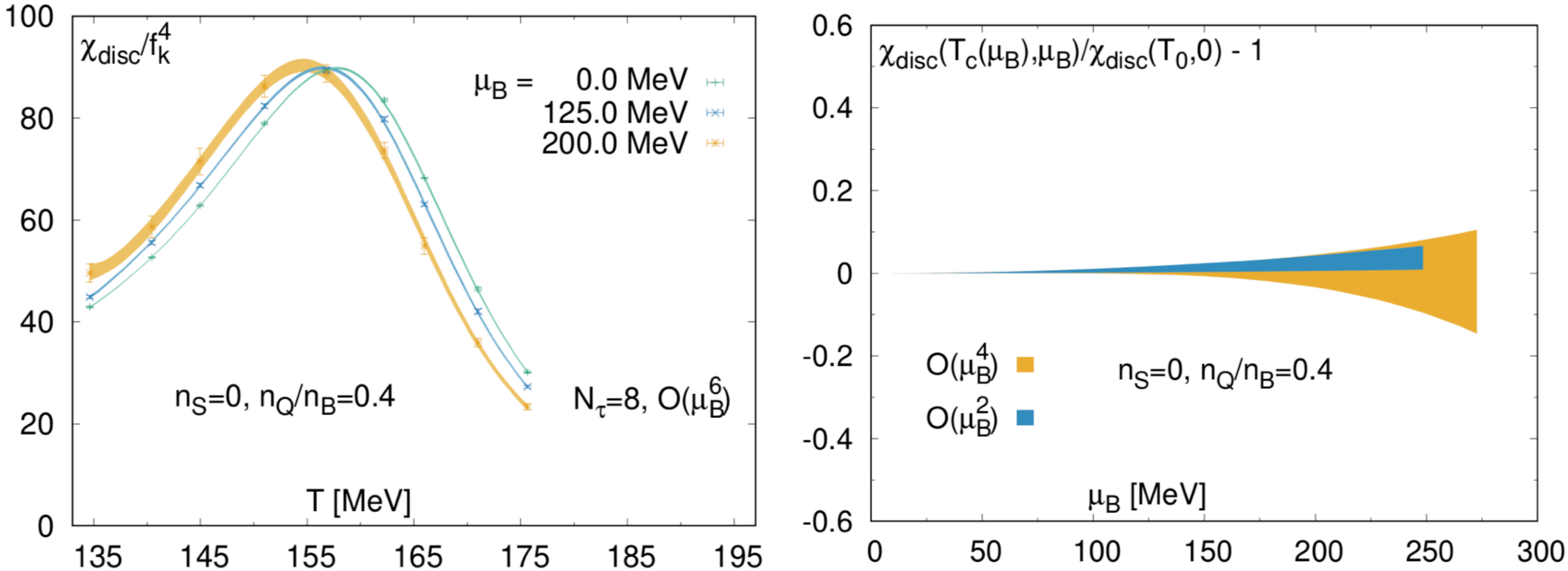
A. Bazavov, HTD, P. Hegde et al. [HotQCD], Phys. Lett. B795 (2019) 15

Higher precision in the continuum limit:

T_{pc} =156.5(1.5)MeV

Previous results: T_{pc} =155(9) MeV, [HotQCD] PRL 113(2014)082001

Order Parameter Susceptibility at μ_B=/=0

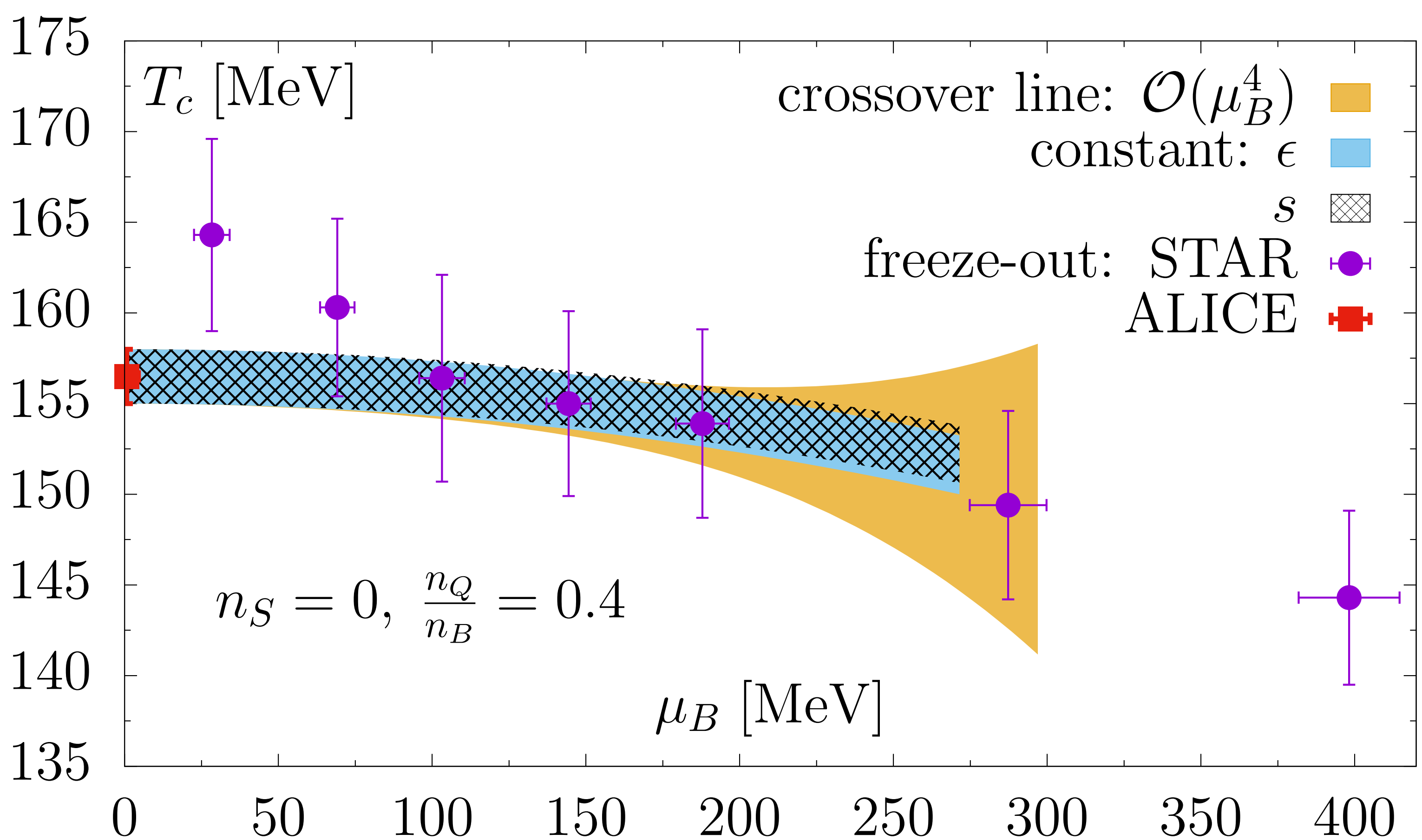


A. Bazavov, HTD, P. Hegde et al. [HotQCD], Phys. Lett. B795 (2019) 15

No indication of a stronger transition at larger μ_B

Crossover, line of constant physics & freeze-out

$$T(\mu_B) = T(0)\left(1 - \kappa_2\left(\frac{\mu_B}{T}\right)^2 - \kappa_4\left(\frac{\mu_B}{T}\right)^4 + \mathcal{O}\left(\frac{\mu_B}{T}\right)^6\right)$$



A. Bazavov, HTD, P. Hegde et al. [HotQCD], Phys. Lett. B795 (2019) 15

curvature of crossover line

$$\kappa_2 = 0.0123 \pm 0.003$$

$$\kappa_4 = 0.000131 \pm 0.0041$$

curvature at constant b:

$$0.006 \leq \kappa_2^b \leq 0.012, \quad b = P, \epsilon, s$$

Bielefeld-BNL-CCNU, PRD95 (2017) no.5, 054504

Radius of convergence

Taylor expansion of the pressure: $$\frac{P}{T^4} = \sum_0^\infty \frac{1}{n!}\chi_n^B(T)\left(\frac{\mu_B}{T}\right)^n$$

radius of convergence = $$\lim_{n\to\infty} r_{2n}^{\chi,a} = \lim_{n\to\infty} r_{2n-2}^{\chi,b}$$

$$r_{2n}^{\chi,a} = \left|\frac{2n(2n-1)\chi_{2n}^B}{\chi_{2n+2}^B}\right|^{1/2} \quad , \qquad r_{2n-2}^{\chi,b} = \left|\frac{(2n)!\,\chi_2^B}{\chi_{2n}^B}\right|^{1/2n}$$

- The Radius of Convergence corresponds to a critical point only if all $\chi_n > 0$ for all $n > n_0$

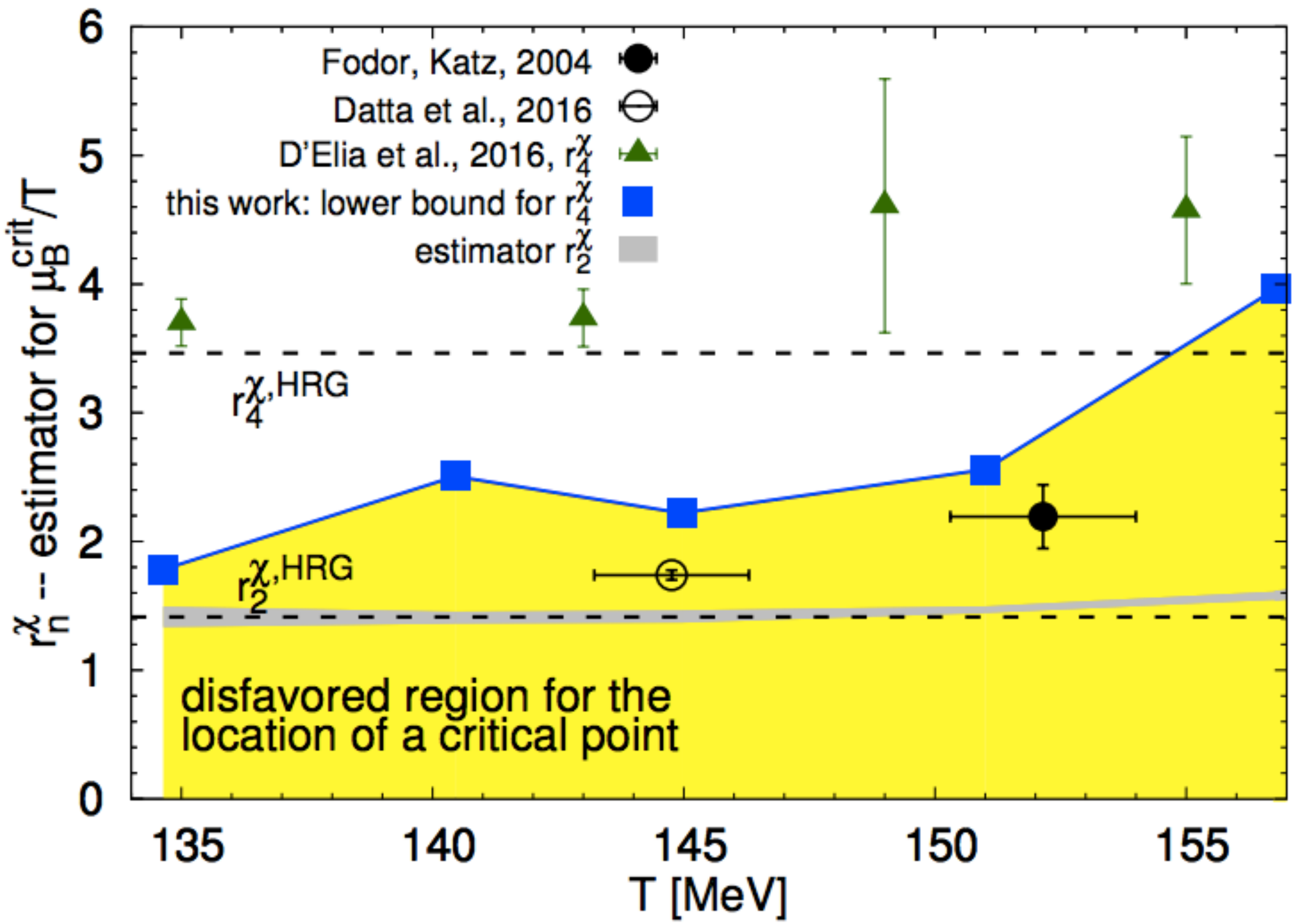
This forces P/T^4 and $\chi_{n,\mu}^B$ grows monotonically with μ_B/T

$$(\kappa\sigma^2)_B = \chi_{4,\mu}^B/\chi_{2,\mu}^B > 1$$

- Otherwise: 1) the ROC does not determine a critical point 2) Taylor expansion is not applicable near the critical point

Estimates of the radius of convergence

radius of convergence = $\lim_{n\to\infty} r_{2n}^{\chi} = \lim_{n\to\infty} \left| \frac{2n(2n-1)\chi_{2n}^{B}}{\chi_{2n+2}^{B}} \right|^{1/2}$



HISQ + Taylor Exp. (this work):
Nf=2+1, Nt=8
Bielefeld-BNL-CCNU,
PRD 95 (2017) no.5, 054504

stout + Img. mu:
Nf=2+1,Nt=8
D'Elia et al., PRD 95 (2017) 094503

unimproved staggered + Taylor Exp.:
Nf=2, Nt=4,6,8
Datta et al., PRD 95 (2017) 054512

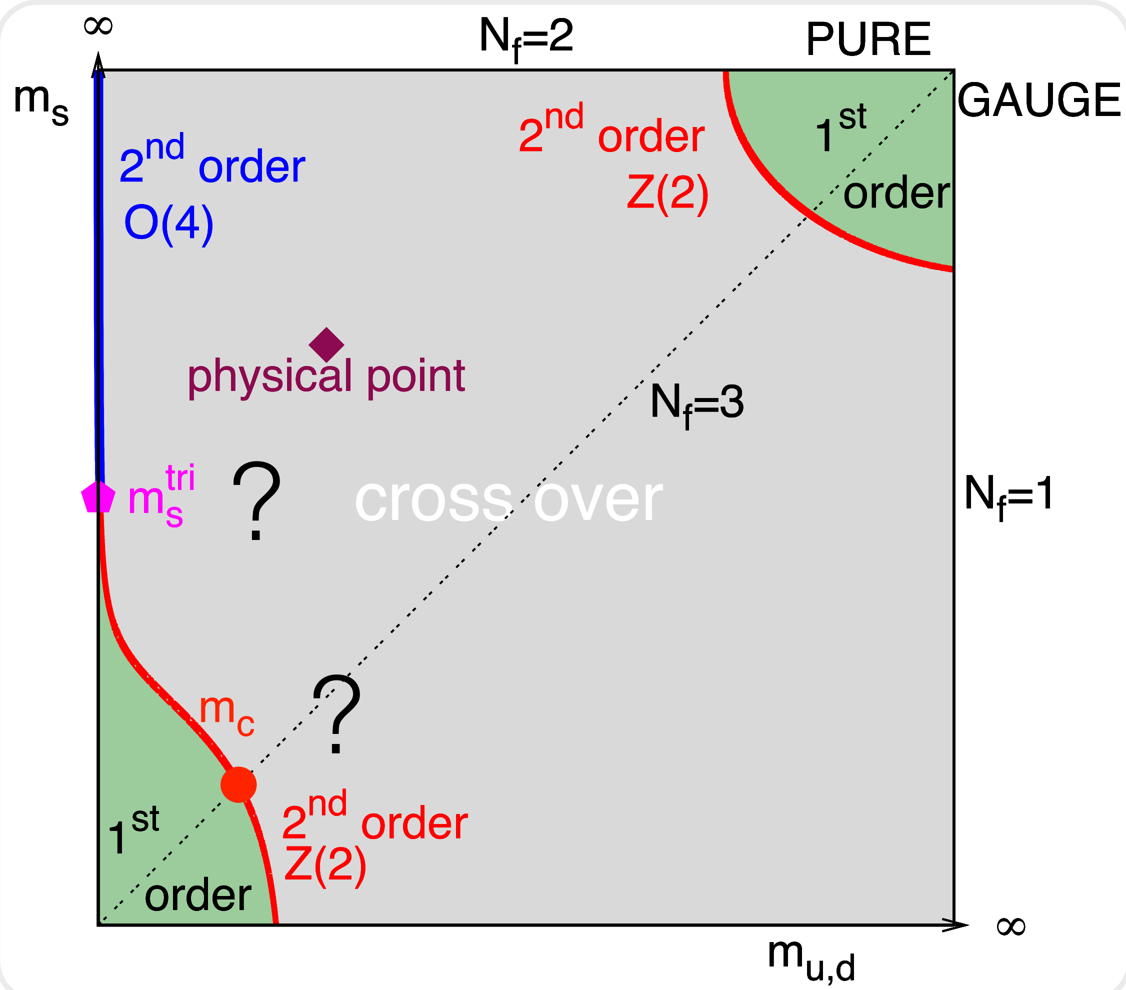
unimproved staggered + Reweighting:
Nf=2+1, Nt=4
Fodor and Katz, JHEP 0404 (2004) 050

A QCD critical point is disfavored at $\mu_B/T \lesssim 2$ at $T \gtrsim 135$ MeV

A. Bazavov, HTD et al., [Bielefeld-BNL-CCNU], Phys.Rev. D95 (2017) no.5, 054504

QCD phase diagram in the quark mass plane

Columbia plot:

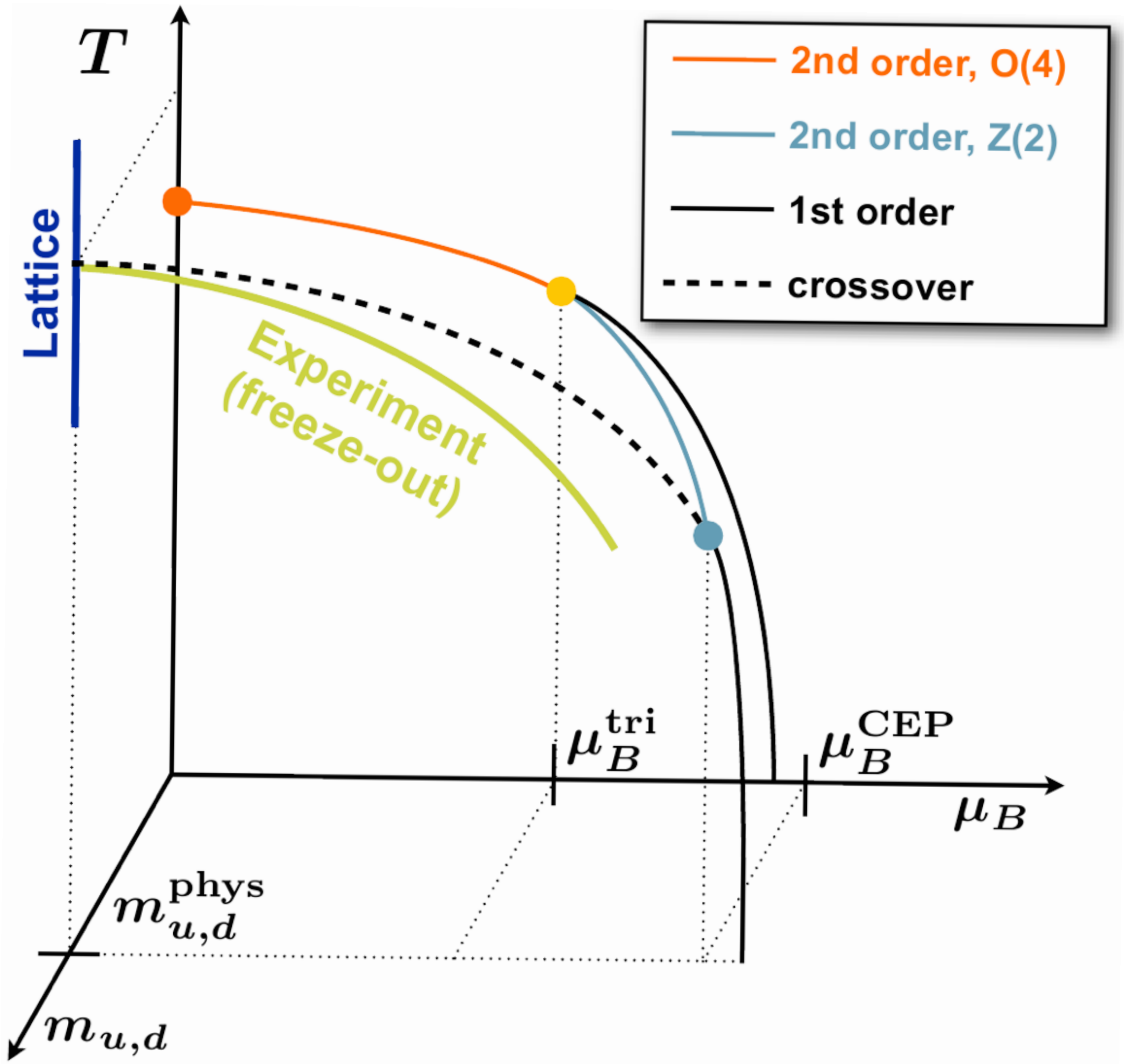


- At physical point: cross over, T_{pc} = 156.5(1.5) MeV HotQCD, arXiv:1812.08235
- N_f=2(+1): $U_A(1)$ remains broken at $T_{\chi SB}$
  JLQCD '13,'14,'15, HotQCD '13,'14
- Critical lines of second order transition
  Pisarski & Wilczek PRD '84
  N_f=2: O(4) universality class Kogut & Sinclair, PRD '06
  N_f=3: Ising universality class Karsch, Laermann, Schmidt PLB '04,...

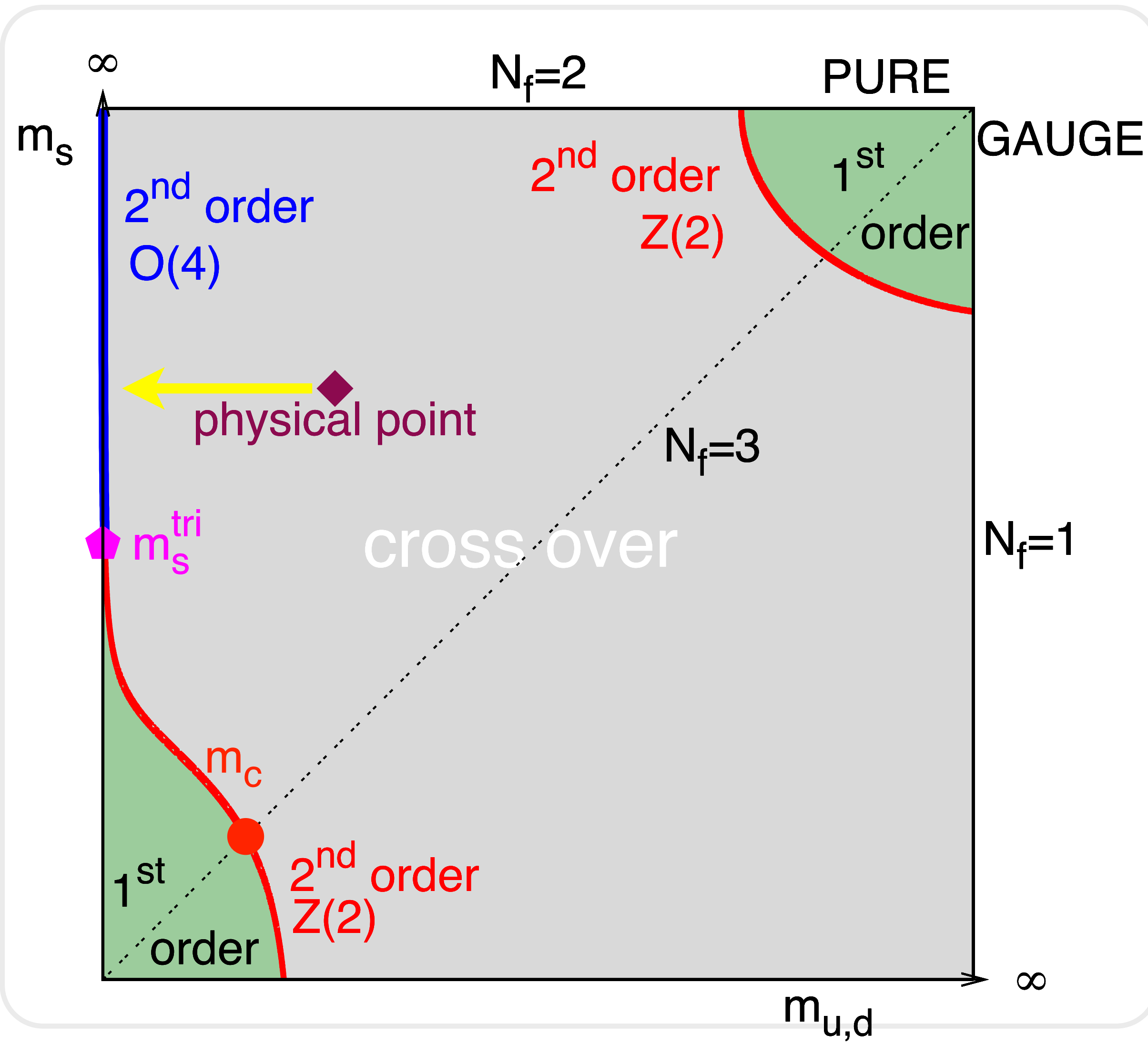
Towards the chiral limit:

- N_f=2+1 QCD: m_s^{tri} ? m_s^{phy}
- Fundamental scale of QCD: chiral T_c^0 ?
- Relation between chiral T_c^0 and T_{CEP}

QCD Phase Diagram



Towards chiral limit of (2+1)-flavor QCD



- HISQ/tree action
- **N_f=2+1:**

$m_u = m_d \rightarrow 0$

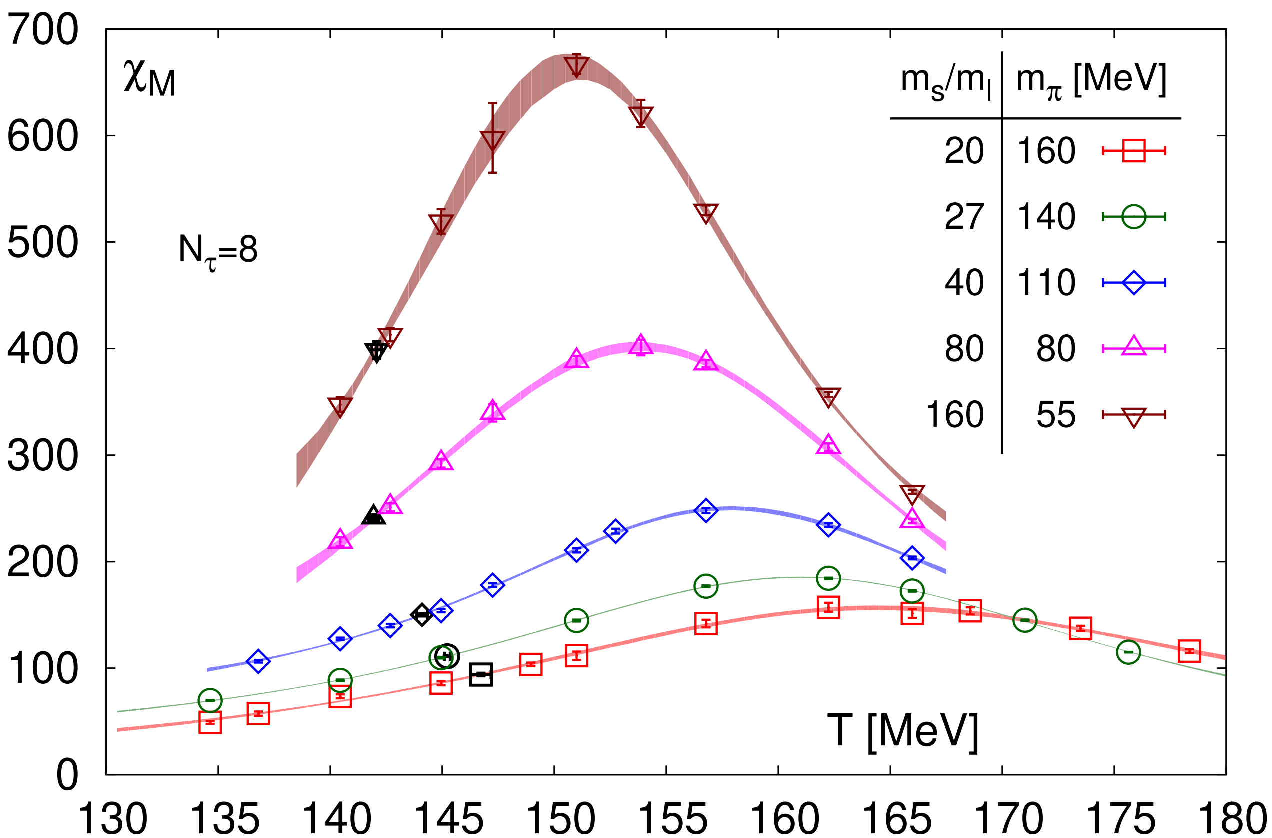
$m_s = m_s^{phy}$

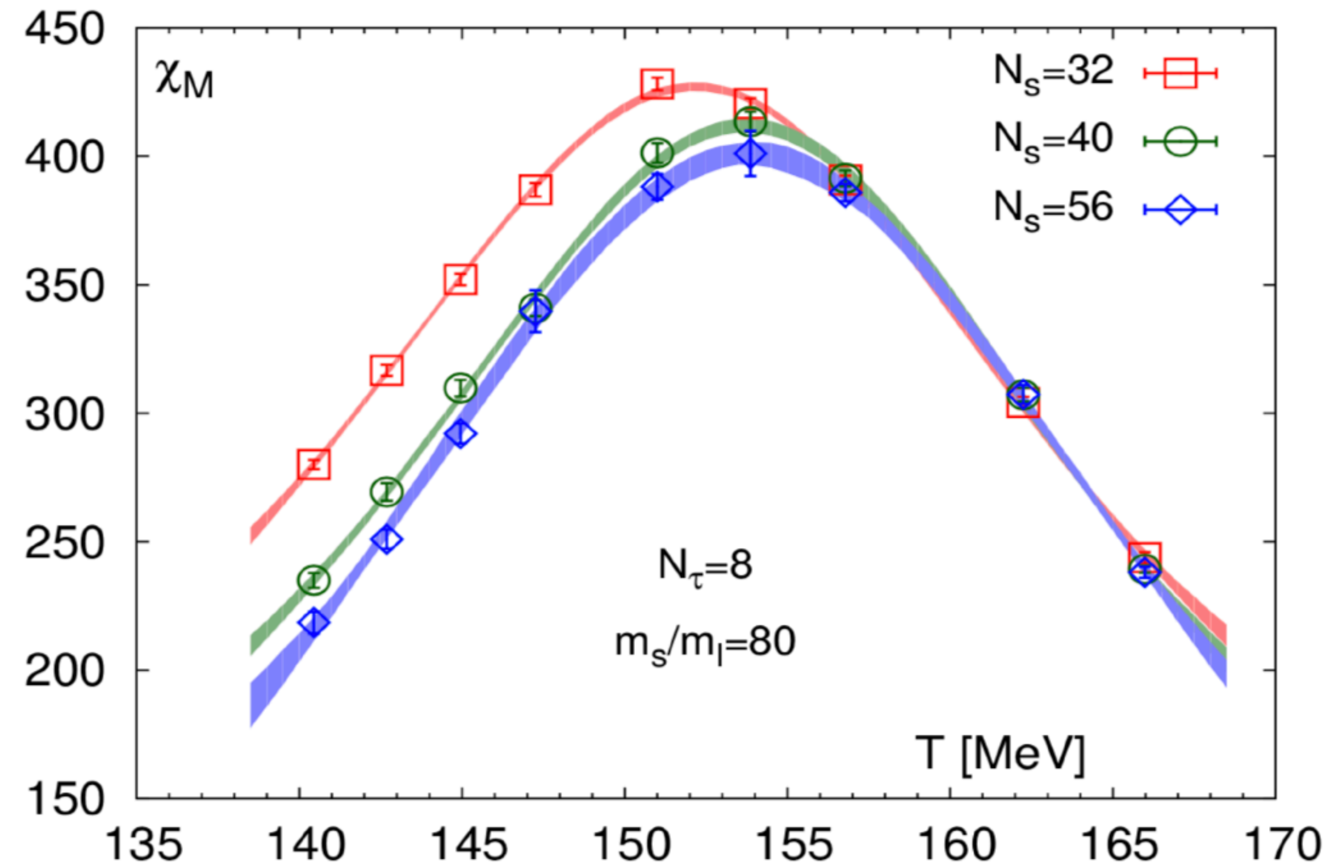
- ☑ Nt=6,8,12
- ☑ $m_s^{phy}/\mathbf{m_l}$ = 20, 27, 40, 60, 80,160

  $m_\pi \approx$160,140,110,90,80,55 MeV
- ☑ $7 \geq N_s/N_t \geq 4 \Leftrightarrow 5 \gtrsim m_\pi L \gtrsim 3$

This allows us to perform
infinite volume, continuum and then chiral extrapolation!

Quark mass and volume dependences of chiral susceptibility





- Susceptibility increases as $m_l^{1/\delta-1}$+const, here $\delta \simeq 4.8$
- Peak height of susceptibility slightly changes with Volume
- Consistent with a continuous phase transition with O(N) universality class in the chiral limit of m_l

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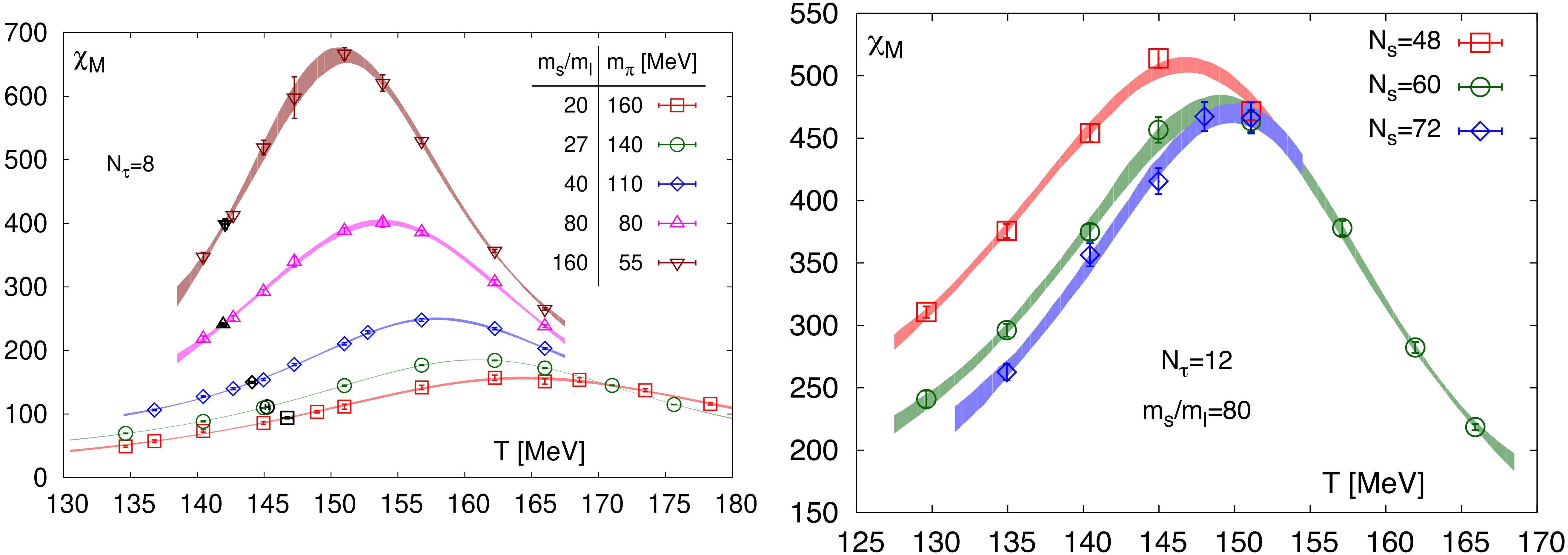

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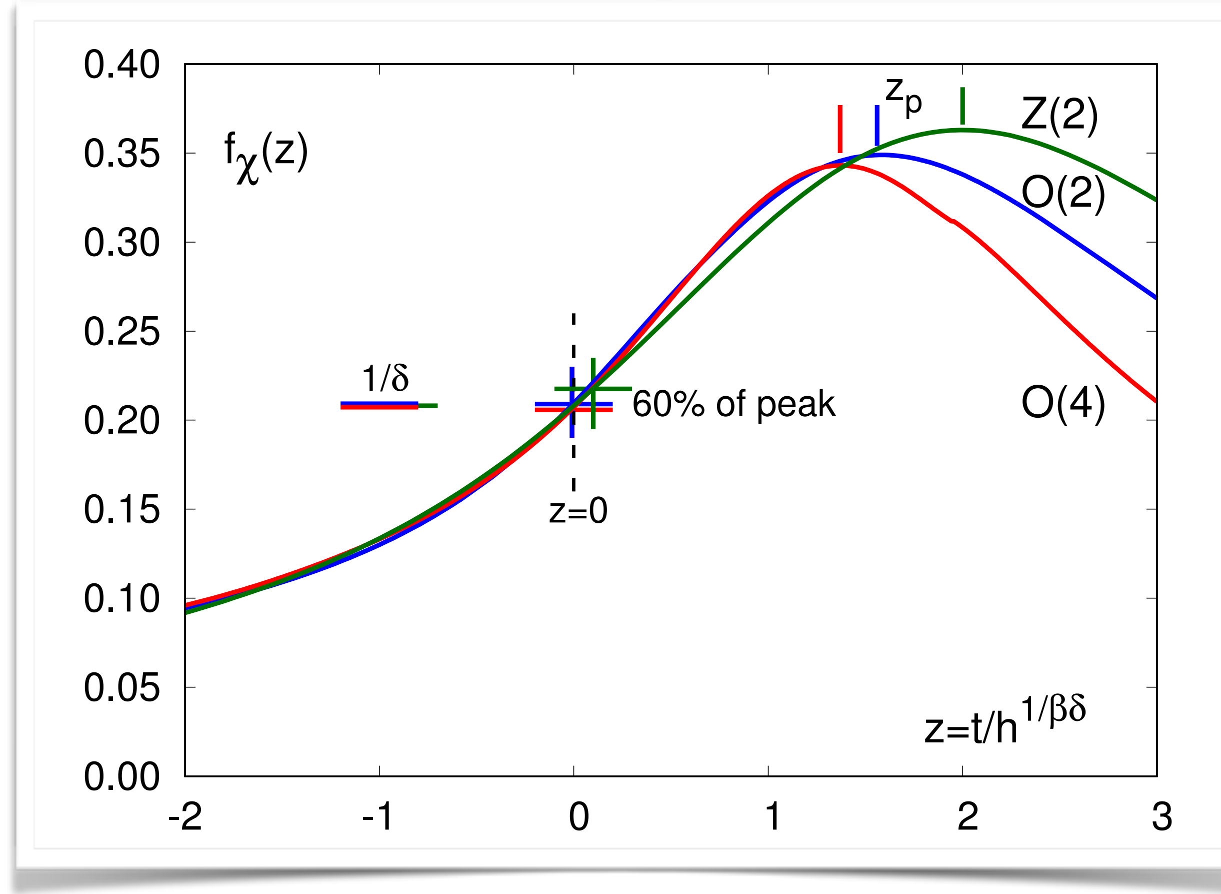
A novel approach to estimate T_c^0

- Pseudo-critical temperature at H

$$T_{pc}(H) = T_c^0\left(1 + \frac{z_p}{z_0} H^{\frac{1}{\beta\delta}}\right)$$

$$z = \frac{1}{t_0}\frac{T-T_c^0}{T_c^0}\left(\frac{H}{h_0}\right)^{-1/\beta\delta} = z_0\frac{T-T_c^0}{T_c^0}H^{-1/\beta\delta}$$



- Estimate of the chiral transition T_c^0

$$\frac{H\chi_M(T_\delta, H, L)}{M(T_\delta, H, L)} = \frac{1}{\delta} \quad \Longleftrightarrow \quad z(T_\delta)=0$$

$$\chi_M(T_{60}, H) = 0.6\chi_M^{max} \quad \Longleftrightarrow \quad z(T_{60})\simeq 0$$

z_p: peak location of the susceptibility

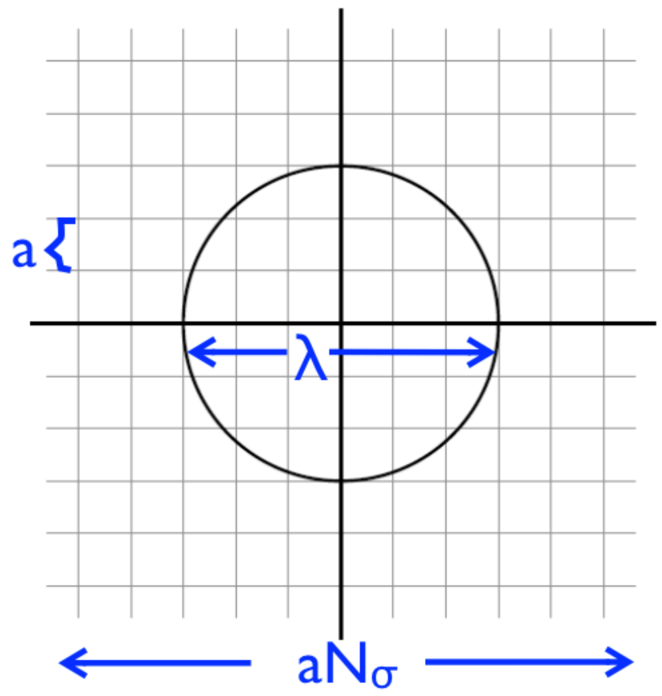
z_{60}: location of 60% of peak height from left

- ☑ small quark mass dependence
- ☑ small variations among universality classes

	δ	z_p	z_{60}^-
Z(2)	4.805	2.00(5)	0.10(1)
O(2)	4.780	1.58(4)	-0.005(9)
O(4)	4.824	1.37(3)	-0.013(7)

Things need to be taken care of

- Thermodynamic limit
- Continuum limit
- Chiral limit



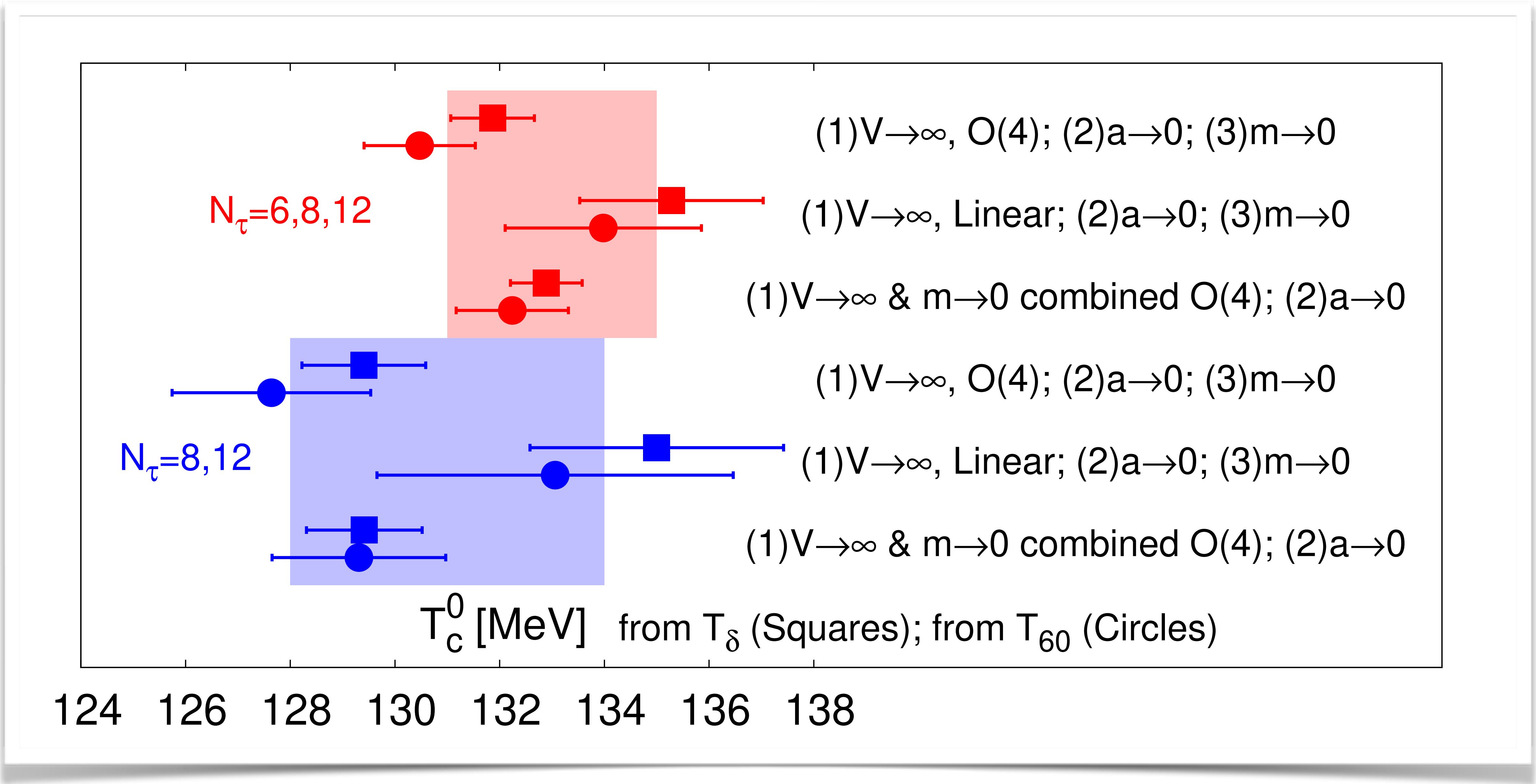
$$T_X(H,L) = T_c^0 \left(1 + \left(\frac{z_X(z_L)}{z_0}\right) H^{1/\beta\delta}\right) + c_X H^{1-1/\delta+1/\beta\delta}$$

Singular

Regular

X=60,δ

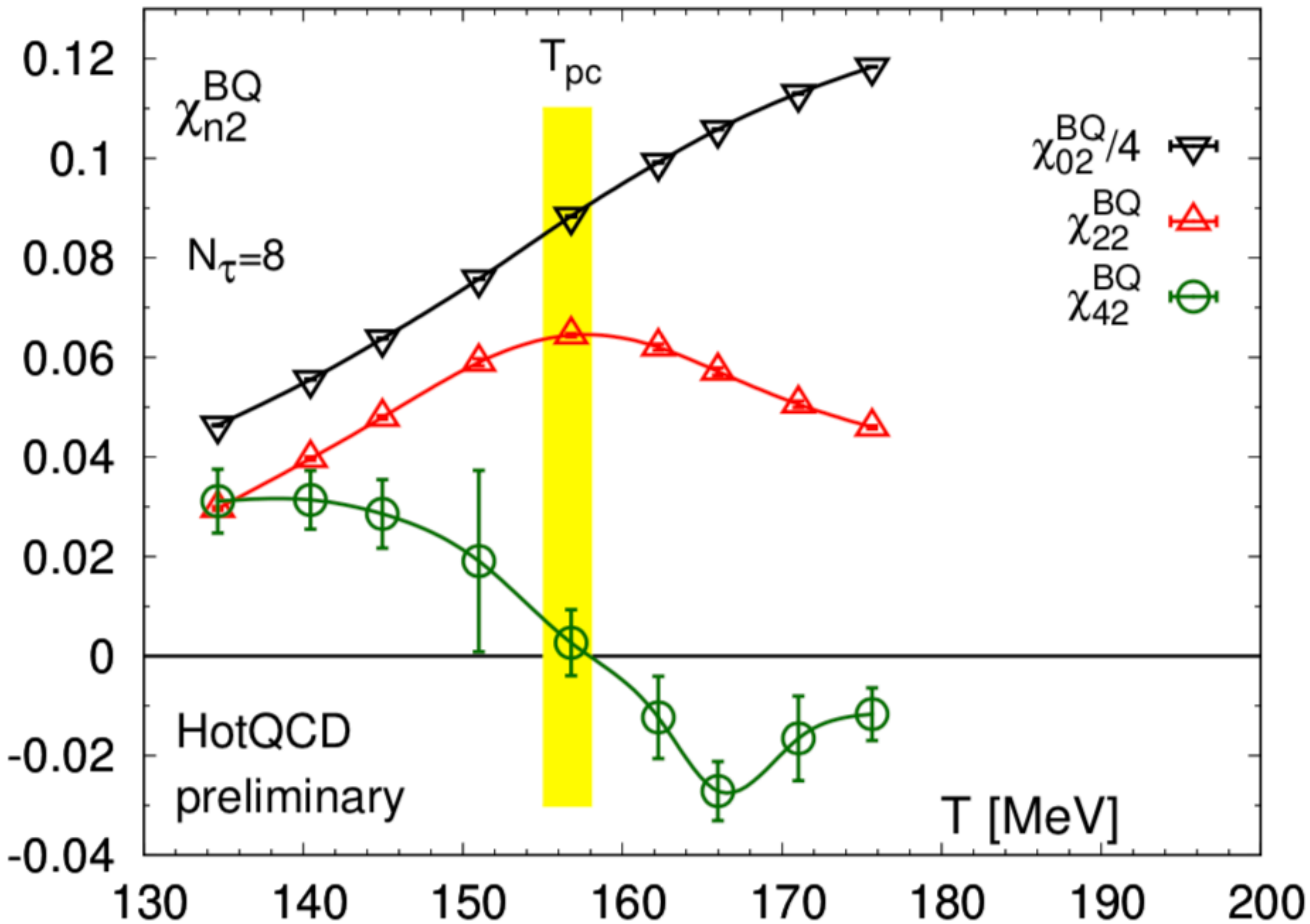
Chiral phase transition temperature: $T_c^0 = 132^{+3}_{-6}$ MeV



HTD, P. Hegde, O. Kaczmarek et al.[HotQCD], arXiv:1903.04801, PRL in press

- T_{60} and T_δ give consistent results
- About 25 MeV lower than T_{pc} at the physical point!
- Indication of $T_{CEP} \lesssim 132$ MeV

Expansion coefficients of net electric charge fluctuations



$$\chi_2^Q(T,\mu_B) = \chi_{02}^{BQ}(T) + \frac{1}{2}\chi_{22}^{BQ}(T)\hat{\mu}_B^2 + \frac{1}{24}\chi_{42}^{BQ}(T)\hat{\mu}_B^4 + \mathscr{O}(\mu_B^6)$$

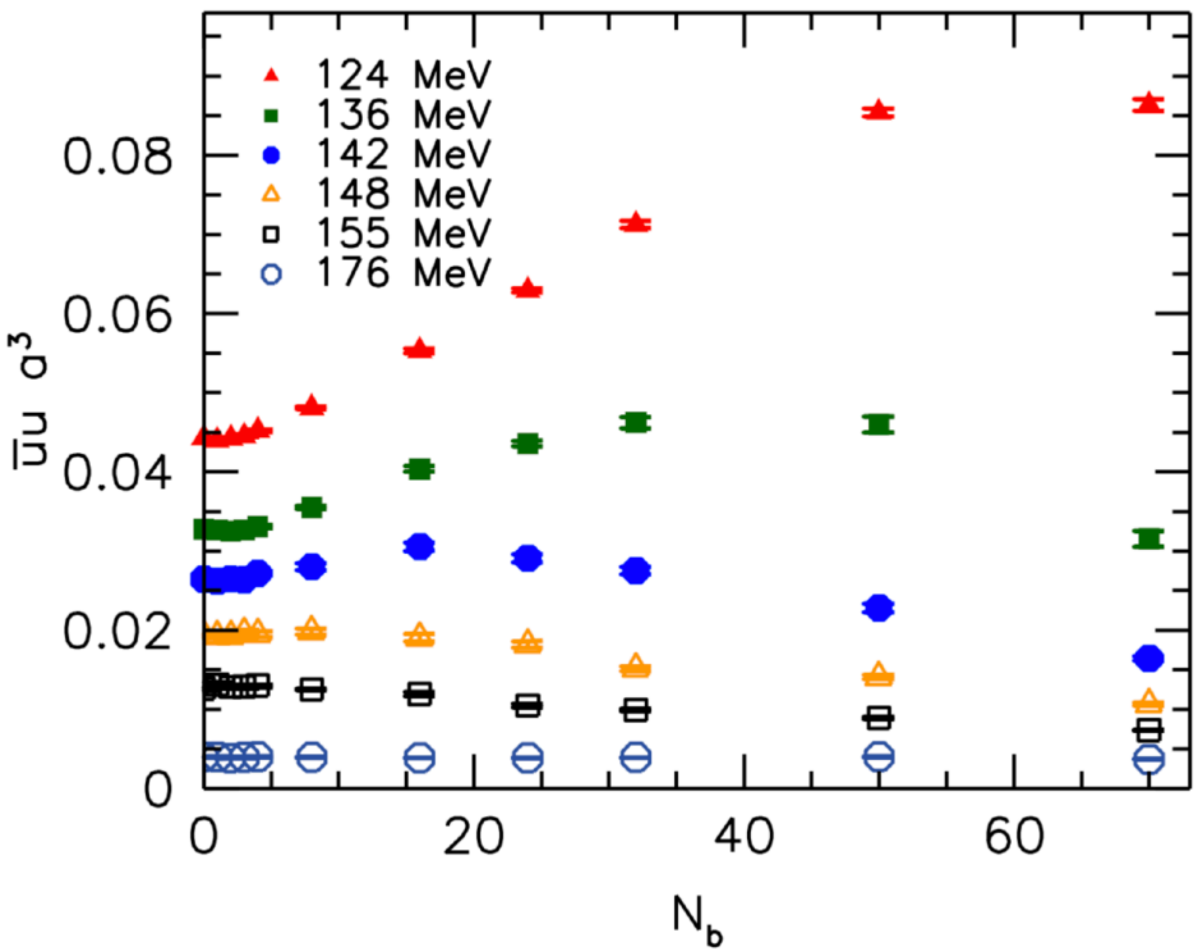
$$\chi_{ijk}^{BQS} \equiv \chi_{ijk}^{BQS}(T) = \left.\frac{\partial P(T,\hat{\mu})/T^4}{\partial\hat{\mu}_B^i \partial\hat{\mu}_Q^j \partial\hat{\mu}_S^k}\right|_{\hat{\mu}=0}$$

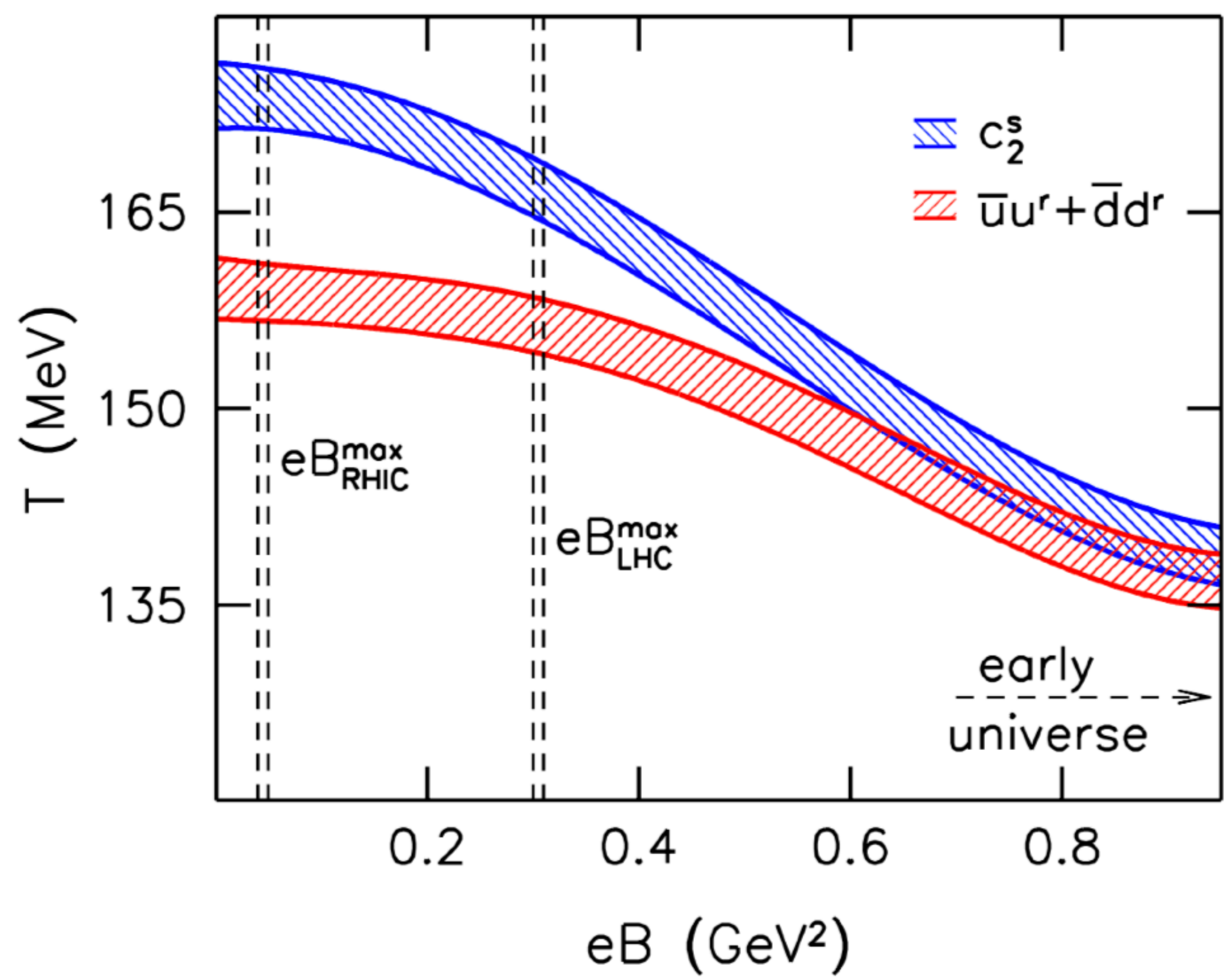
$$t \sim \frac{T-T_c^0}{T_c^0} + \kappa_2^{B,0}\left(\frac{\mu_B}{T}\right)^2$$

- In the scaling regime, two derivatives wrts μ_B ∝ one derivative wrt T
- Irregular sign change seen at $T>T_{pc}$ in χ_{42}^{BQ}
- Irregular sign change expected at T≳ 135 MeV in χ_{62}^{BQ}

More support for $T_{CEP} < T_c^0$

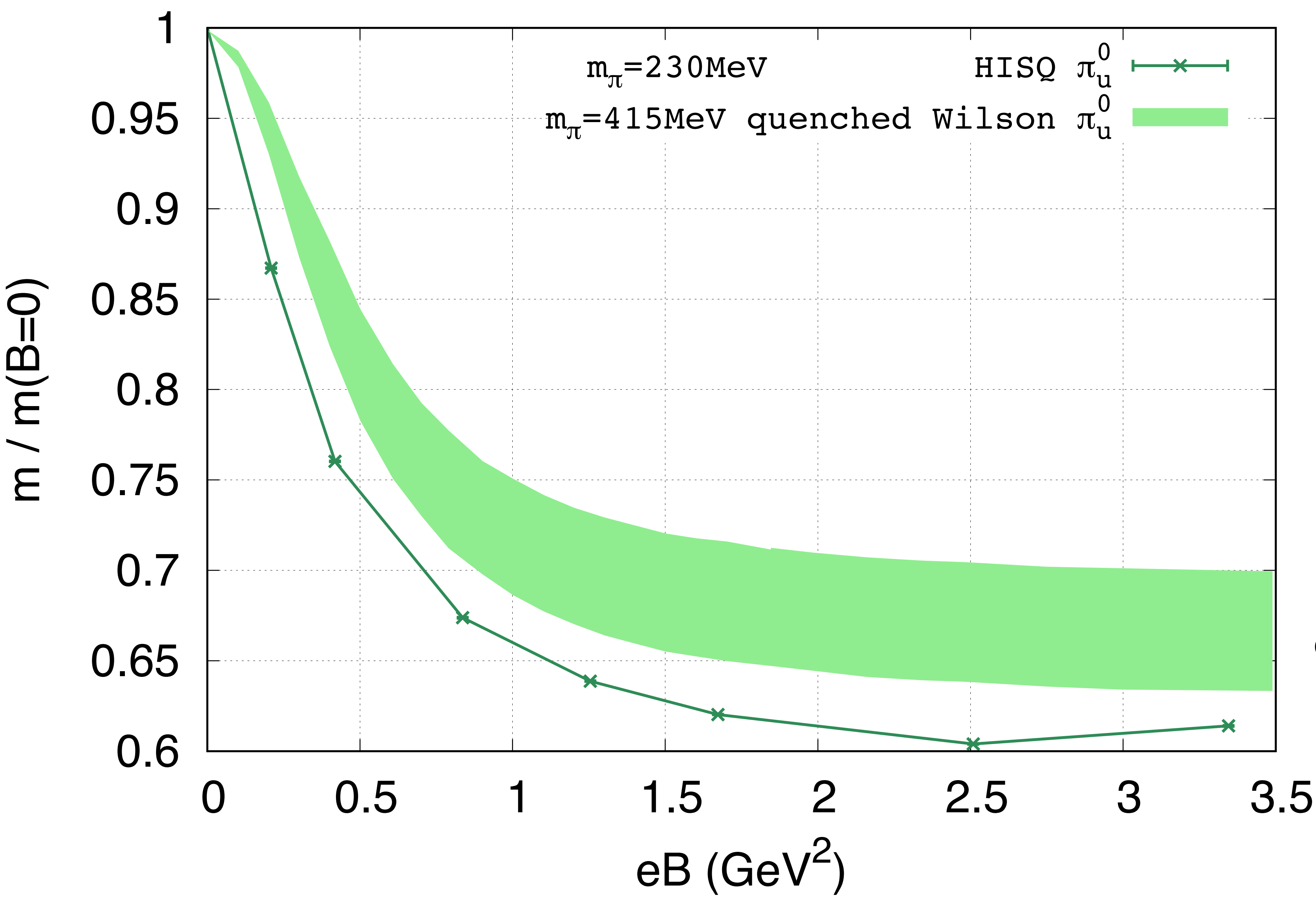
Inverse magnetic catalyses v.s. $T_C(B)$





Bali et al., JHEP02(2012)044

QCD in the external Magnetic field



T=0

Nf=2+1 QCD

HISQ fermion

32^3x96 lattices

M_π = 230 MeV

quenched Wilson/dyn stout results from Bali et al PRD97, 034505 (2018)

Xiao-Dan Wang(汪晓丹), HTD et al, Lattice 2019, Work in progress

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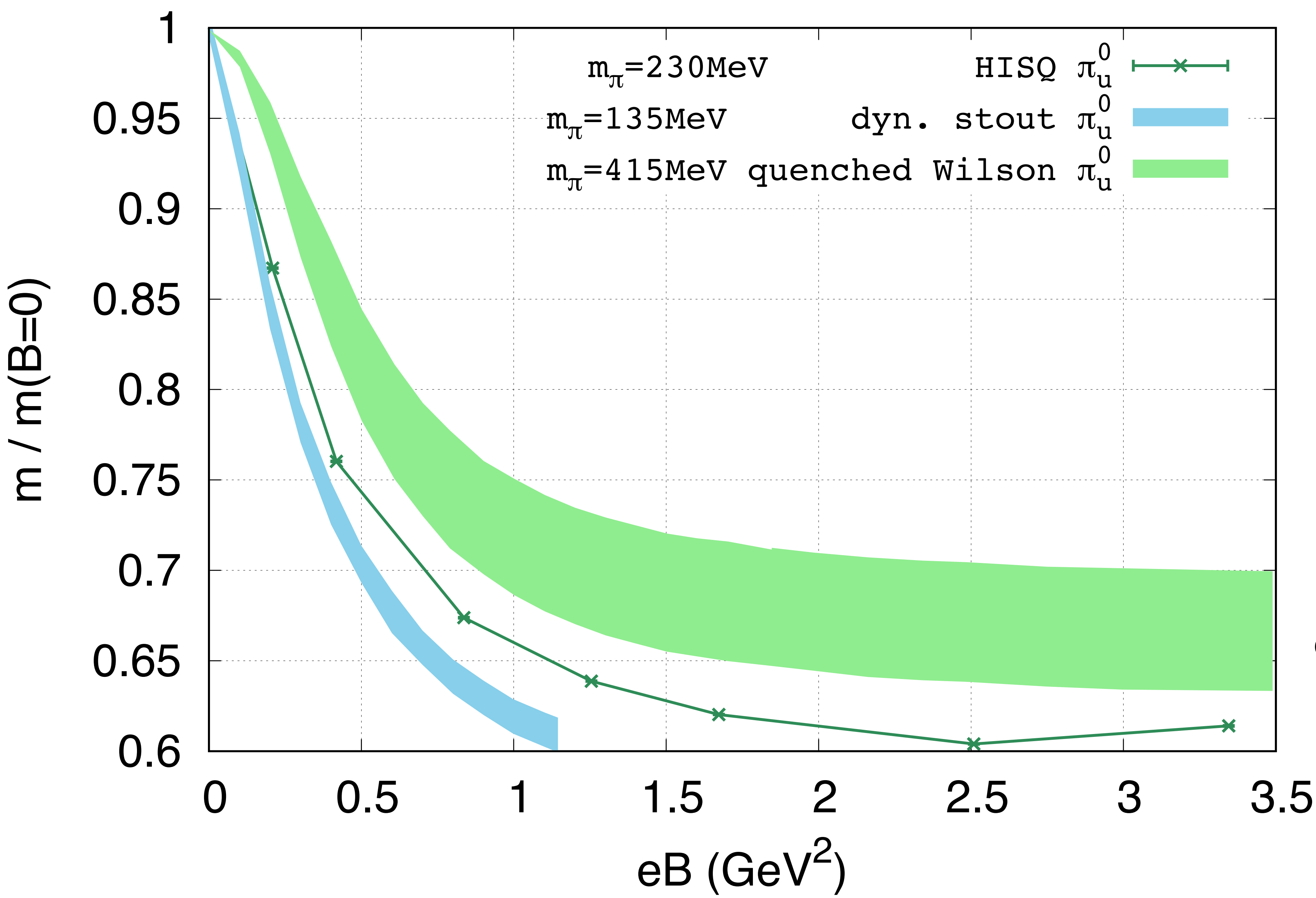

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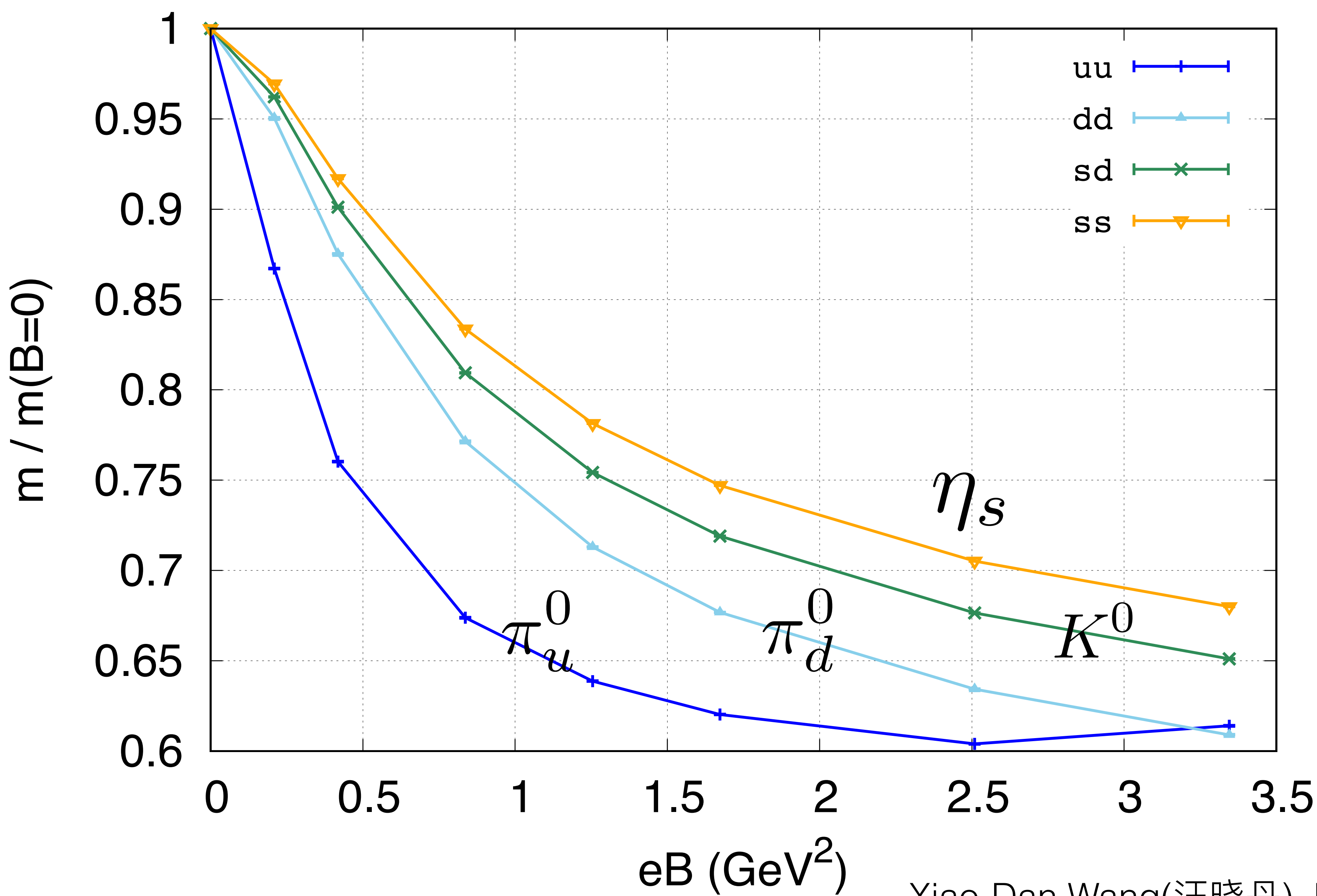
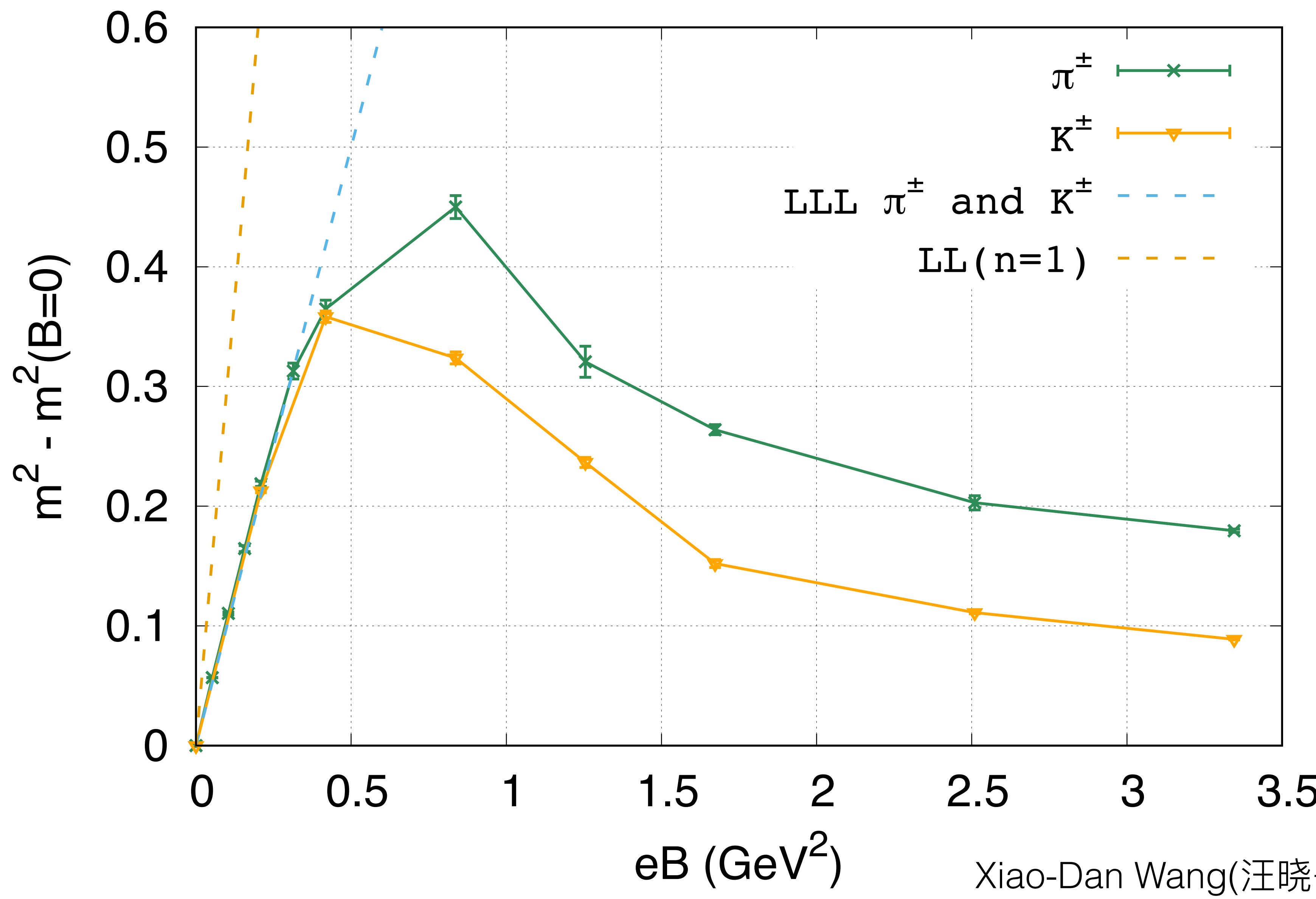
eB dependences of neutral pseudo mesons



Xiao-Dan Wang(汪晓丹), HTD et al, Lattice 2019

- Mass decreases as eB grows
- Lighter mesons are more influenced by magnetic field

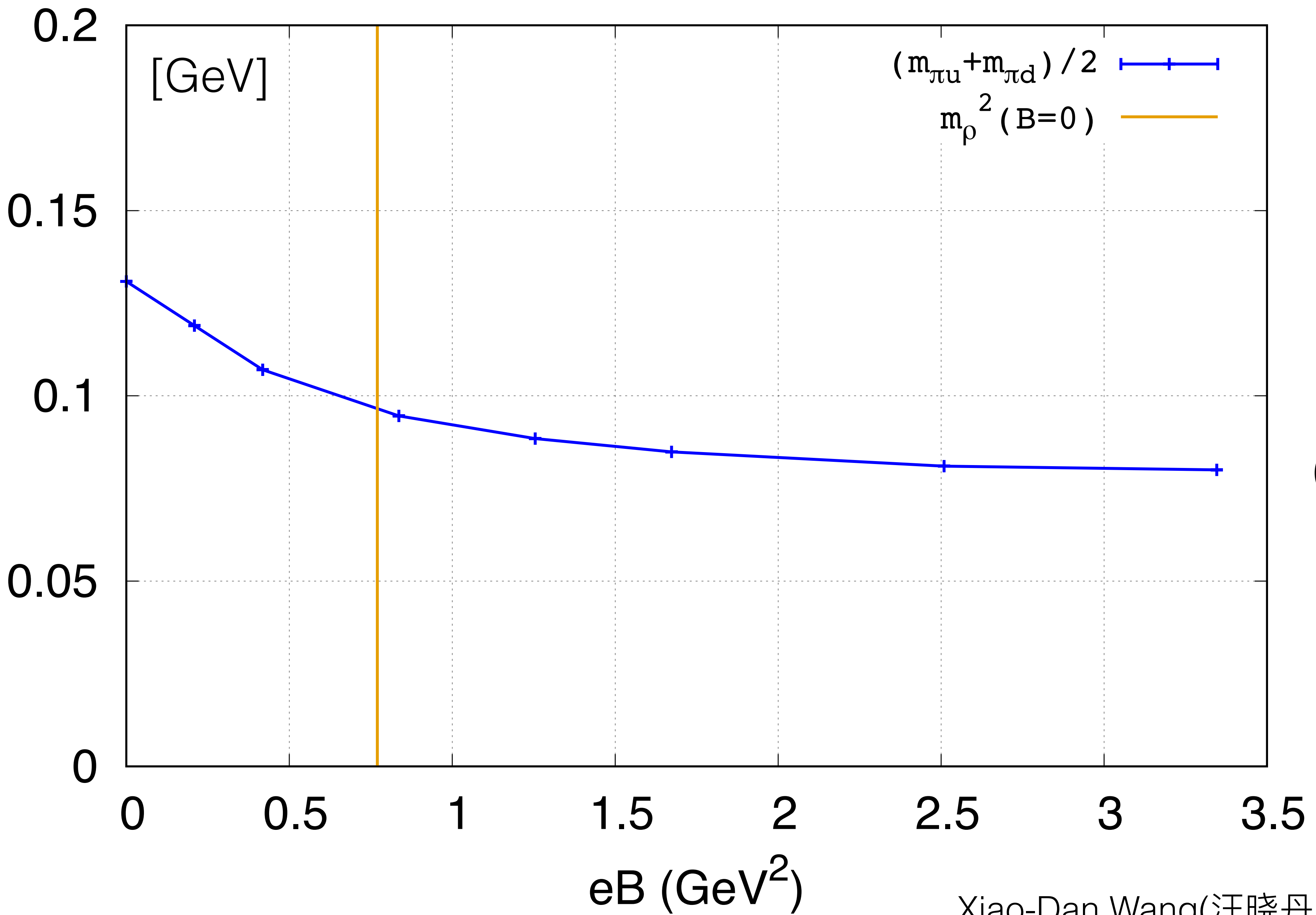
eB dependences of charged pseudo mesons



Xiao-Dan Wang(汪晓丹), HTD et al, Lattice 2019

- Non-monotonic dep. on eB, not observed in quenched QCD
- At eB ≲ 0.5 GeV² mass of charged particle well described by LLL approximation: $m^2(B) = m^2(B=0) + (2n+1)|qB|$

QCD superconductivity induced by magnetic field ?



NJL model:

$$eB_c \sim m_\rho^2(B=0)$$

Chernodub, PRL106 (2011) 142003

QCD inequality:

$$m_{\rho^\pm} \geq (m_{\pi_u^0} + m_{\pi_d^0})/2$$

Y. Hidaka and A. Yamamoto ,
Phys. Rev. D 87 (2013) 094502

Xiao-Dan Wang(汪晓丹), HTD et al, Lattice 2019

No rho condensation is observed

Conclusions

- chiral crossover temperature is determined with better precision, i.e. **T_{pc}** = 156.5(1.5) MeV, while chiral phase transition temperature is determined to be about 25 MeV smaller, i.e. **T_c^0** $= 132^{+3}_{-6}$ MeV

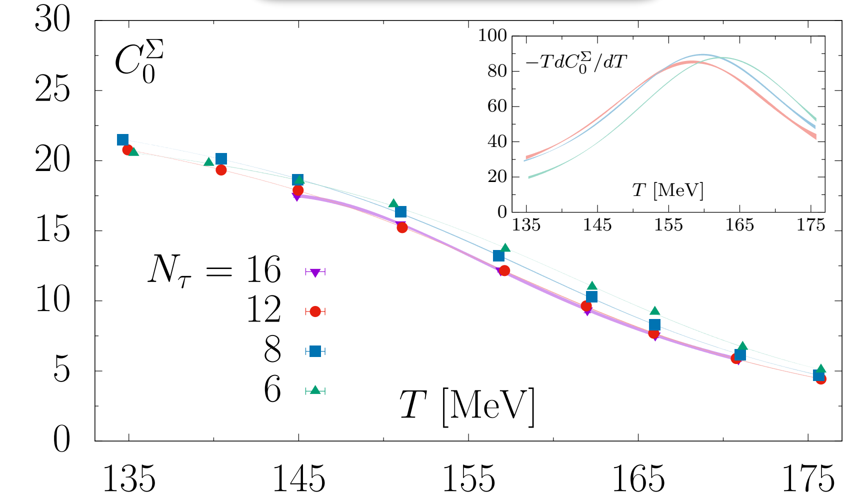
- Negative 6th order cumulants, radius of convergence and the low chiral phase transition T suggests that a possible existing critical end point can only be found at **$T_{CEP} \lesssim 135$ MeV**

- Decreasing of T_{pc} with B may be relevant with the reduction of neutral pion mass. No condensation of rho is found.

谢谢!

Thanks for your attention!

$\partial_T^2 C_0^\Sigma(T) = 0$



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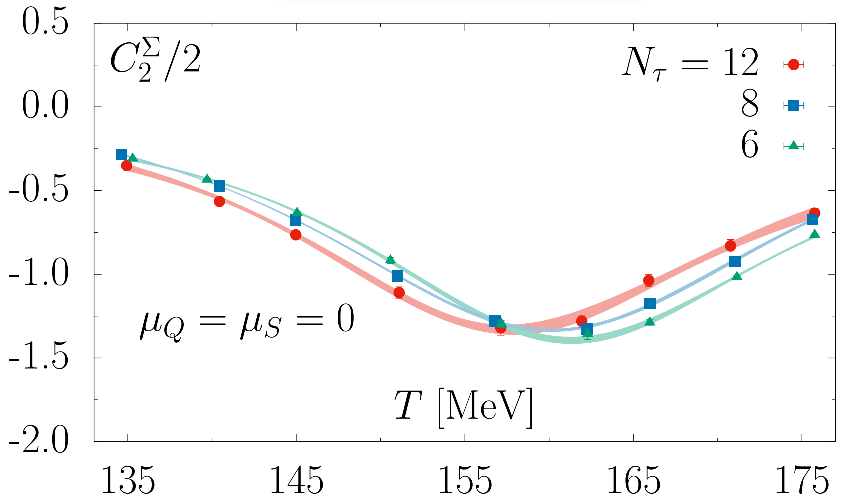
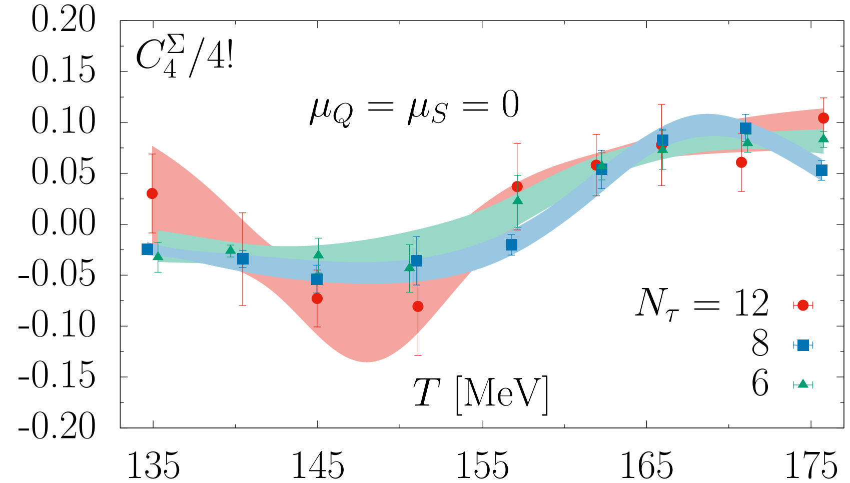
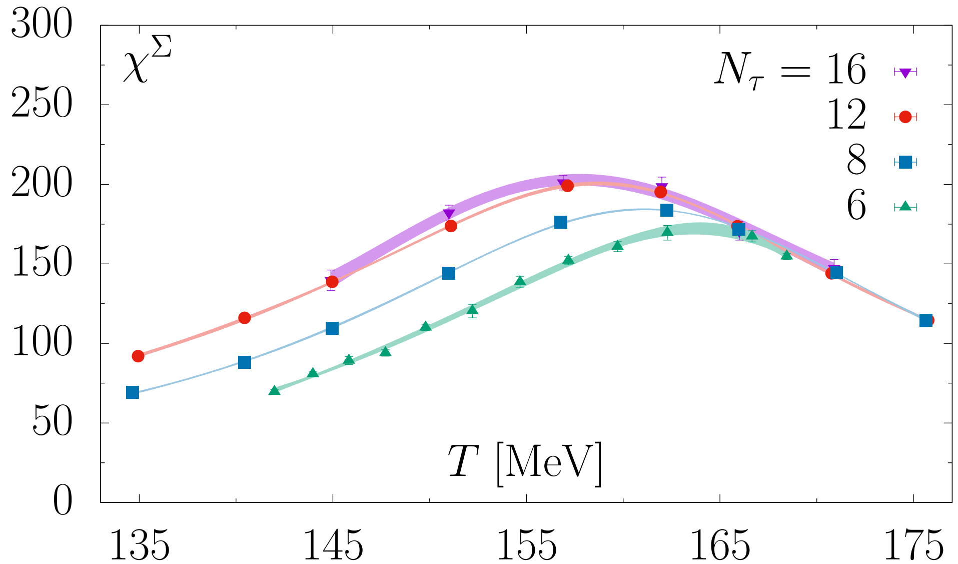


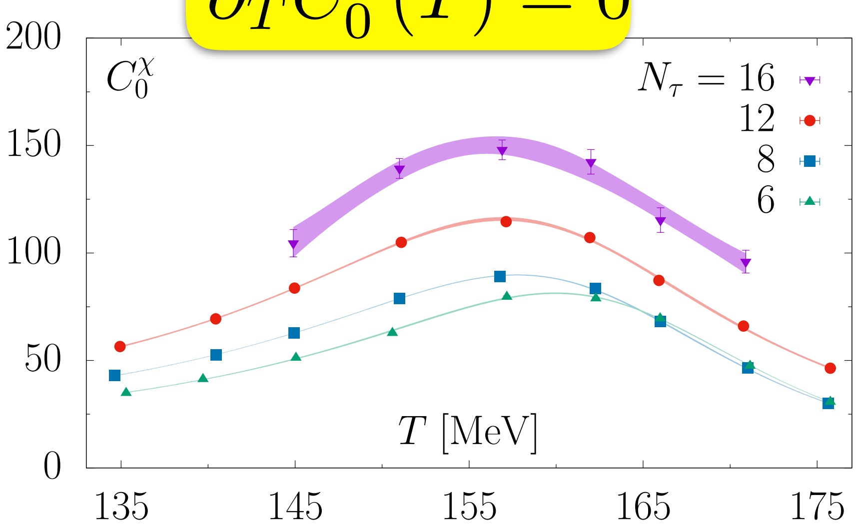
$C_4^\Sigma(T = T_{pc}) = 0$



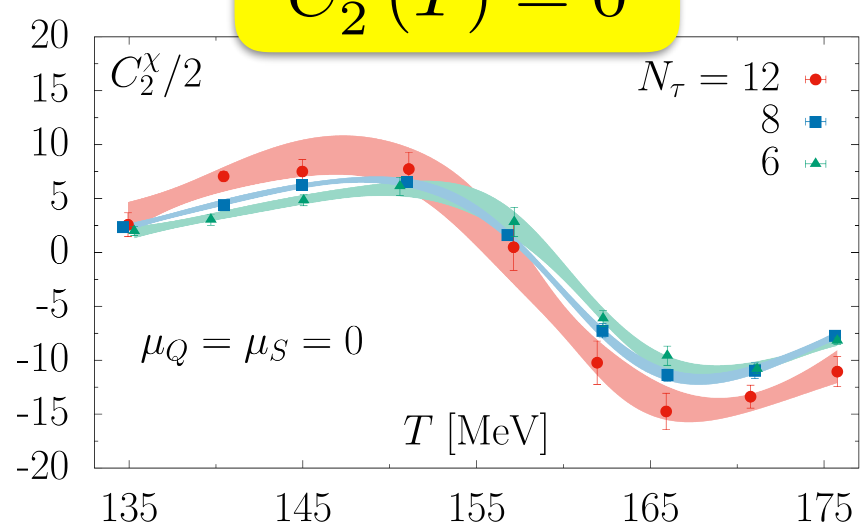


$\partial_T \chi^\Sigma(T) = 0$

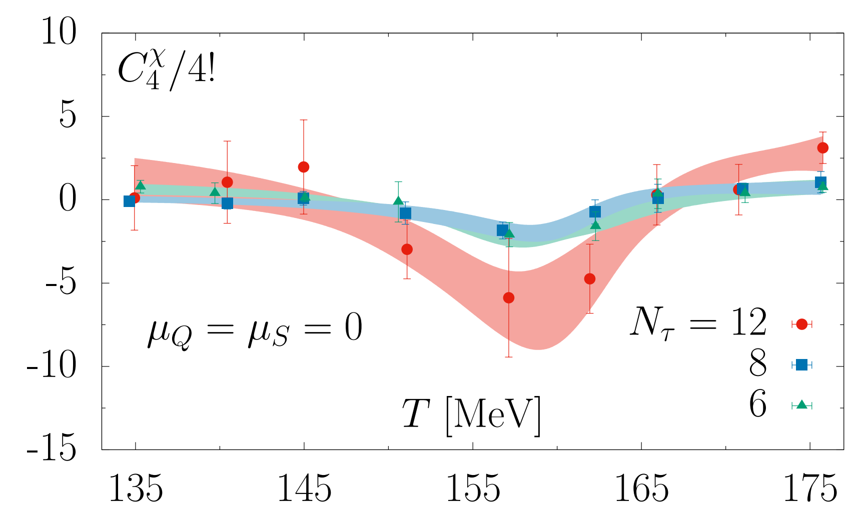
$\partial_T C_0^\chi(T) = 0$



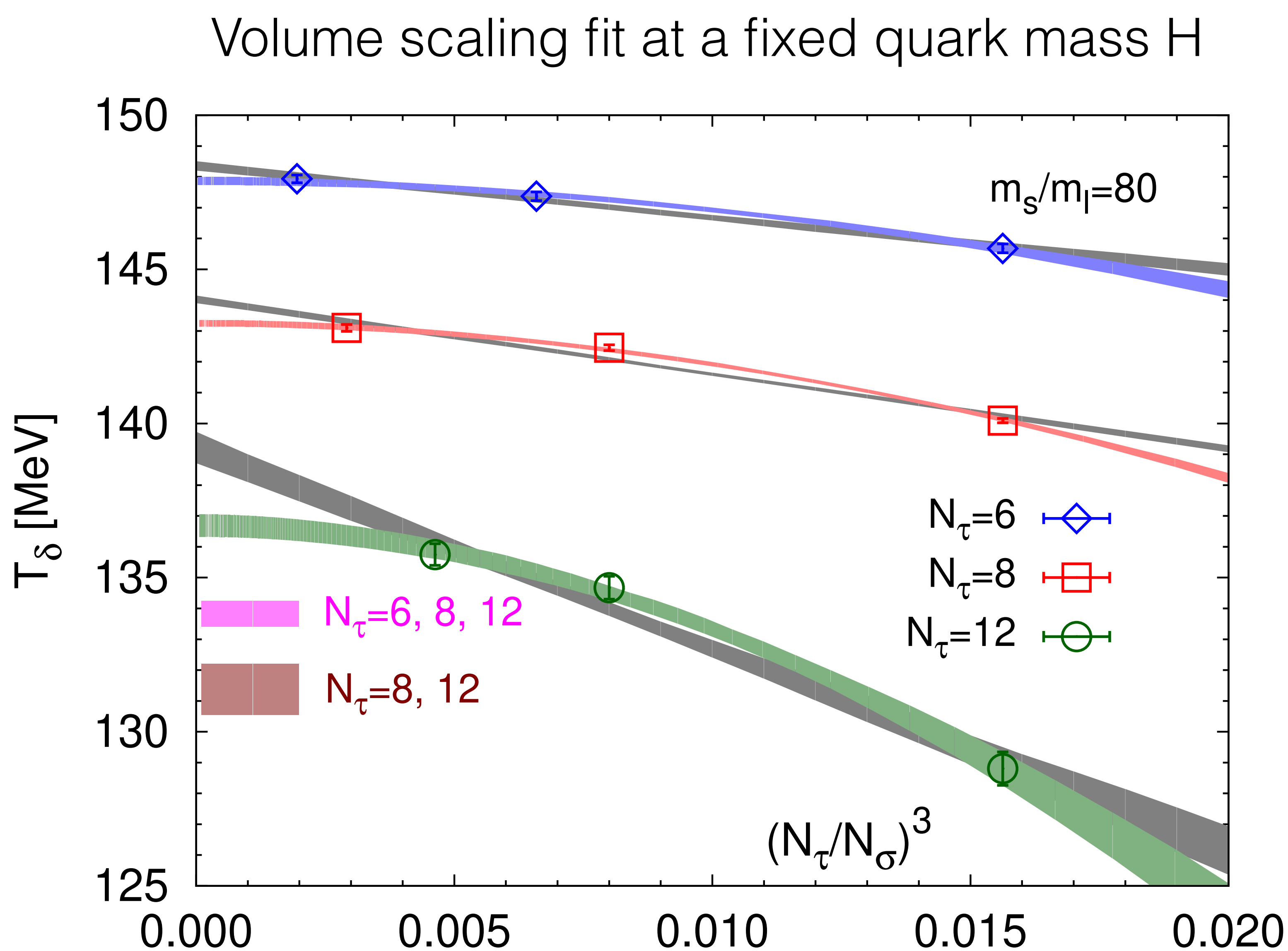
$C_2^\chi(T) = 0$



$\partial_T C_4^\chi(T = T_{pc}) = 0$



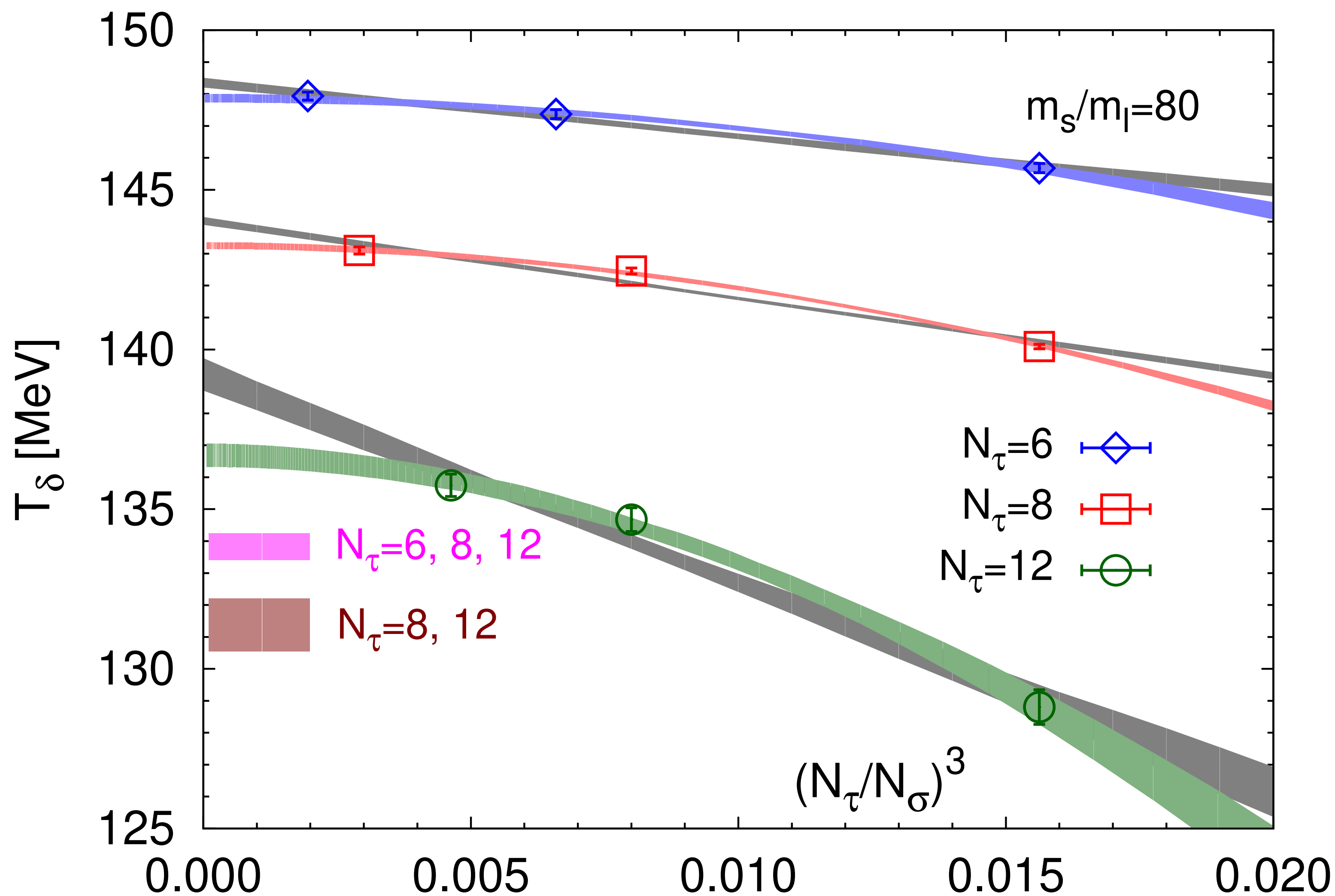
T_δ: Infinite V limit → continuum limit → chiral limit



	T_δ(H,V,a) [MeV] with Nt=6,8&12	T_δ(H,V,a) [MeV] with Nt= 8&12
V→∞, a→0, H=1/80	133.8(4)	131.4(8)

T_δ: Infinite V limit → continuum limit → chiral limit

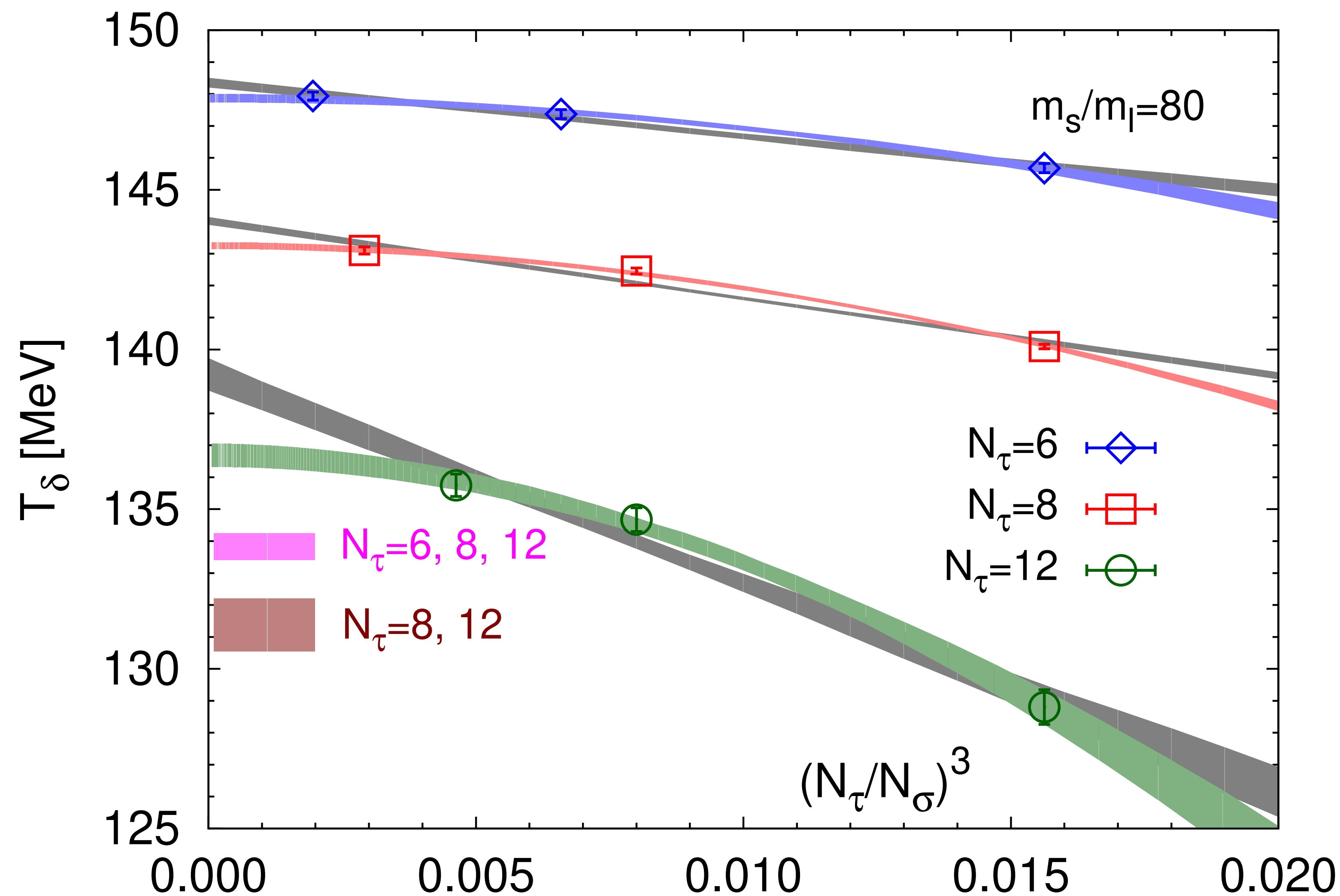
Volume scaling fit at a fixed quark mass H



	T_δ(H,V,a) [MeV] with Nt=6,8&12	T_δ(H,V,a) [MeV] with Nt= 8&12
V→∞, a→0, H=1/80	133.8(4)	131.4(8)
V→∞, a→0, H=1/40	136.9(5)	135.5(8)

T_δ: Infinite V limit → continuum limit → chiral limit

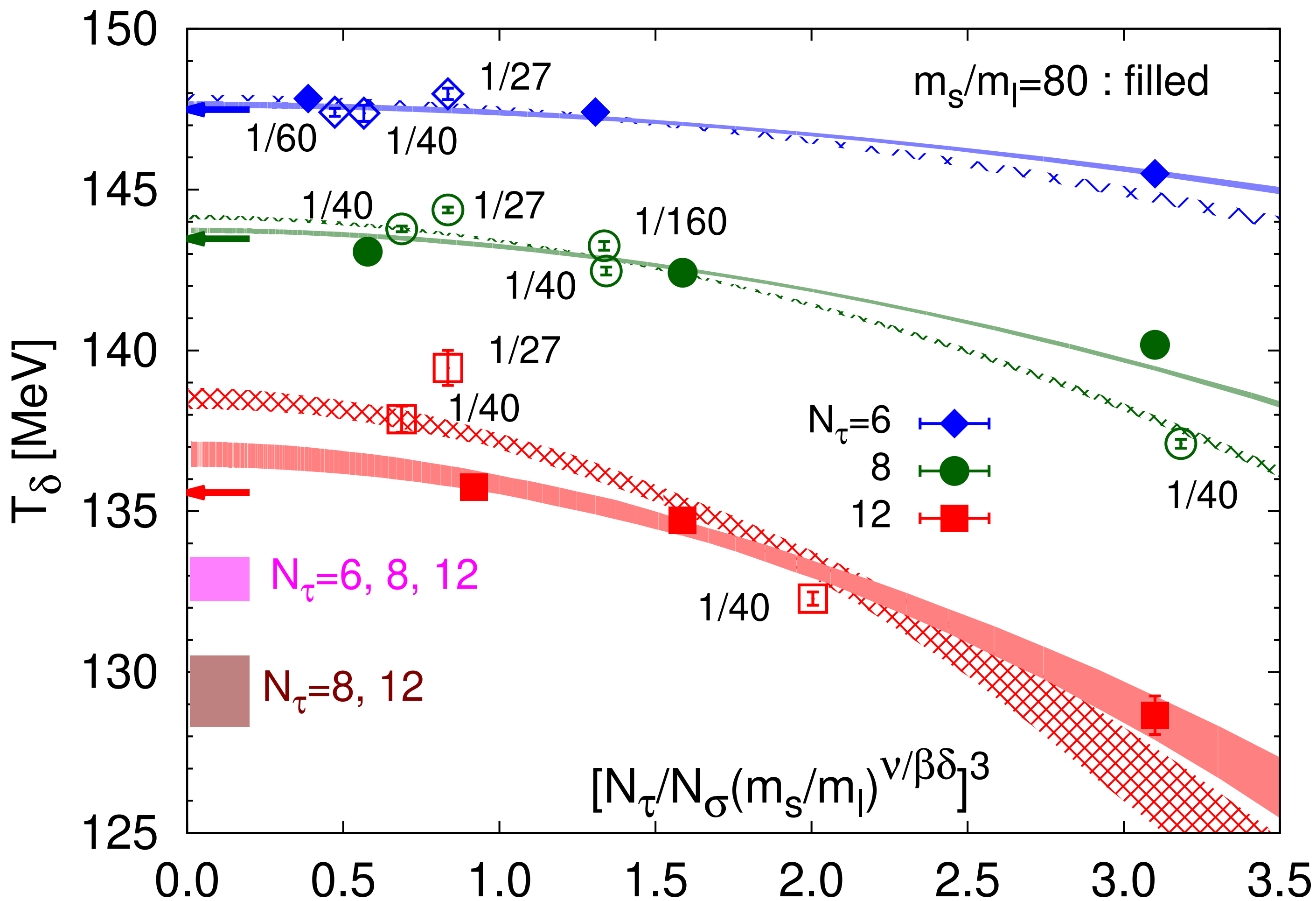
Volume scaling fit at a fixed quark mass H



	$T_\delta(H,V,a)$ [MeV] with Nt=6,8&12	$T_\delta(H,V,a)$ [MeV] with Nt= 8&12
V→∞, a→0, H=1/80	133.8(4)	131.4(8)
V→∞, a→0, H=1/40	136.9(5)	135.5(8)
V→∞, a→0, H→0	132.8(1.4)	130.6(2.4)

T_δ: Infinite V limit → chiral limit → continuum limit

Joint volume scaling fit with all quark masses



$T_\delta(H,V,a)$ [MeV]

	Nt=6,8&12	Nt=8&12
V→∞, H→0, a→0	132.9(6)	128.6(1.1)
V→∞, a→0, H→0	132.8(1.4)	130.6(2.4)

Chiral and continuum limits are Interchangeable

chiral phase transition and universal scaling

Behavior of the free energy close to critical lines

$f(m,T)=h^{1+1/\delta} f_s(z)$, $z=t/h^{1/\beta\delta}$

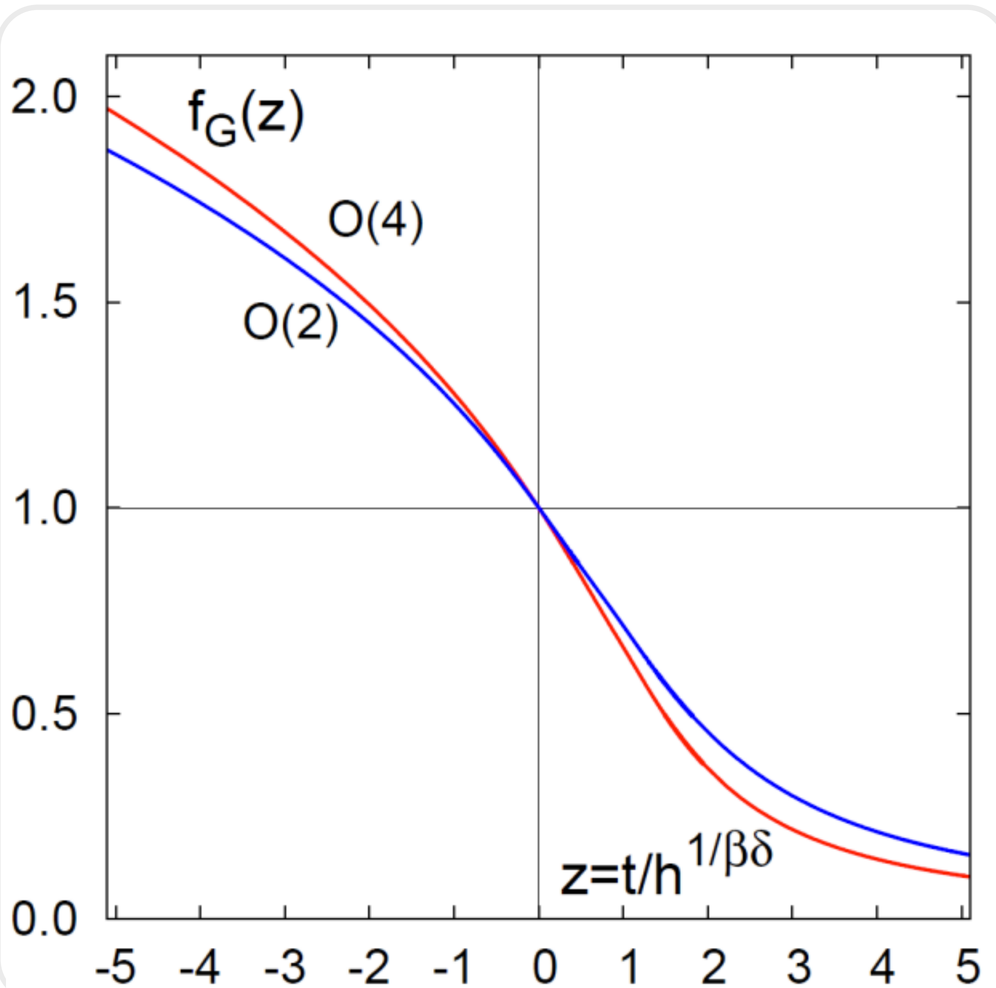
h: external field, t: reduced temperature, β,δ: universal critical exponents

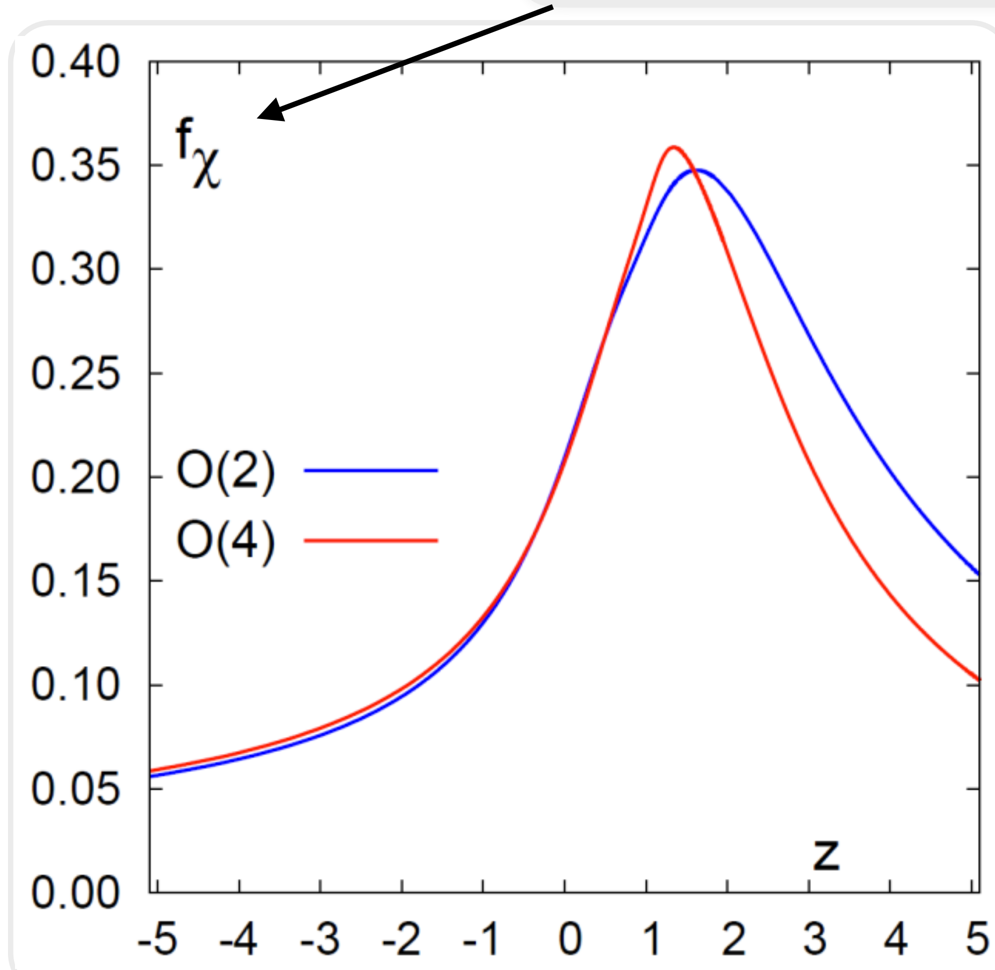
$f_s(z)$: universal scaling function, O(N) etc.

$h = \frac{1}{h_0}\frac{m_l}{m_s}$ $t = \frac{1}{t_0}\frac{T-T_c}{T_c}$

Magnetic Equation of State (MEoS):

$$M = -\partial f_s(t,h)/\partial H = h^{1/\delta} f_G(z) \qquad \chi_M = \partial M/\partial H = \frac{h^{1/\delta}}{H}\frac{1}{\delta}\left(f_G(z) - \frac{z}{\beta}\frac{df_G(z)}{dz}\right)$$





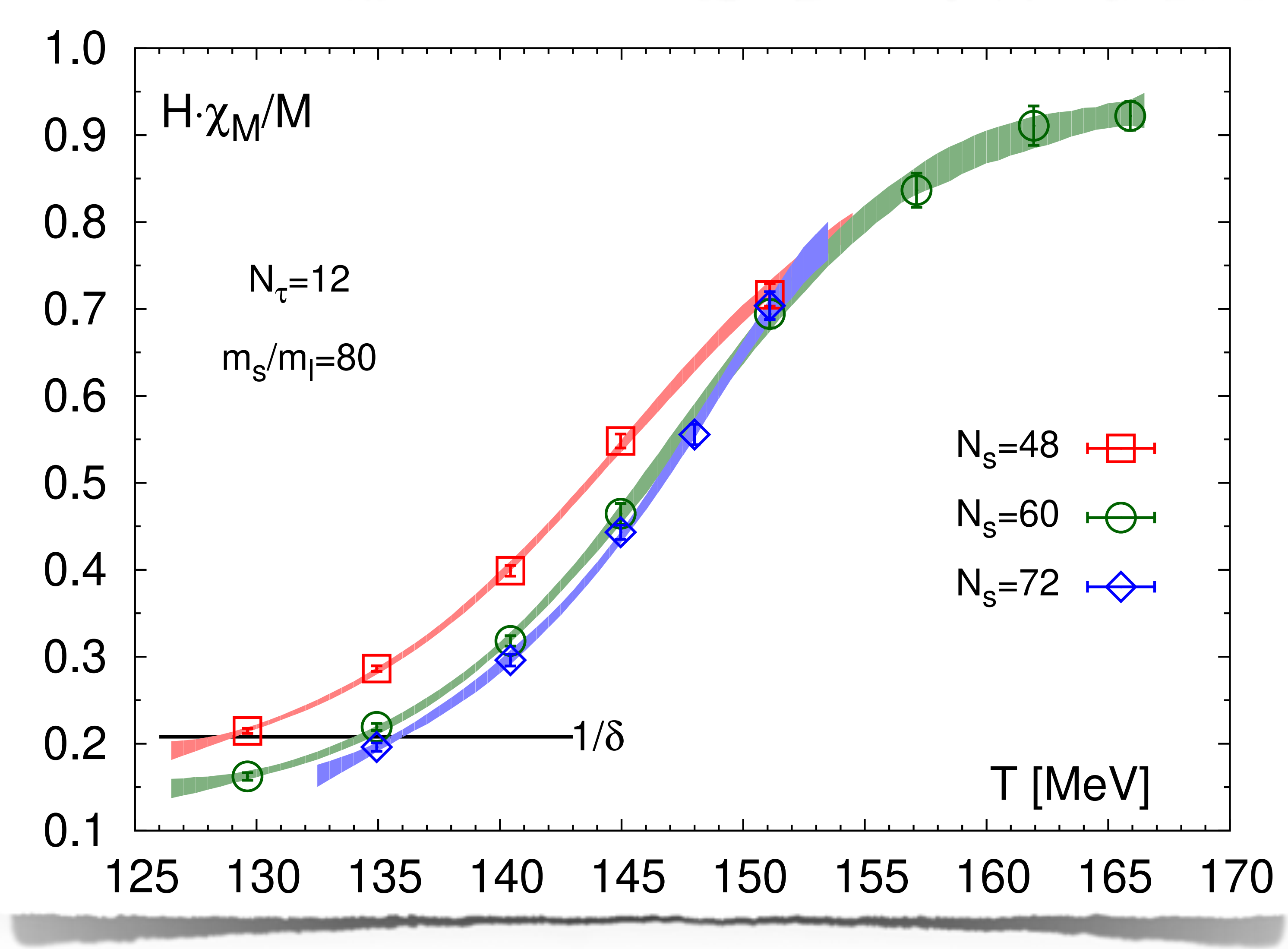
Chiral phase transition temperature T_c^0

$$M = -\partial f_s(t,h)/\partial H = h^{1/\delta} f_G(z)$$

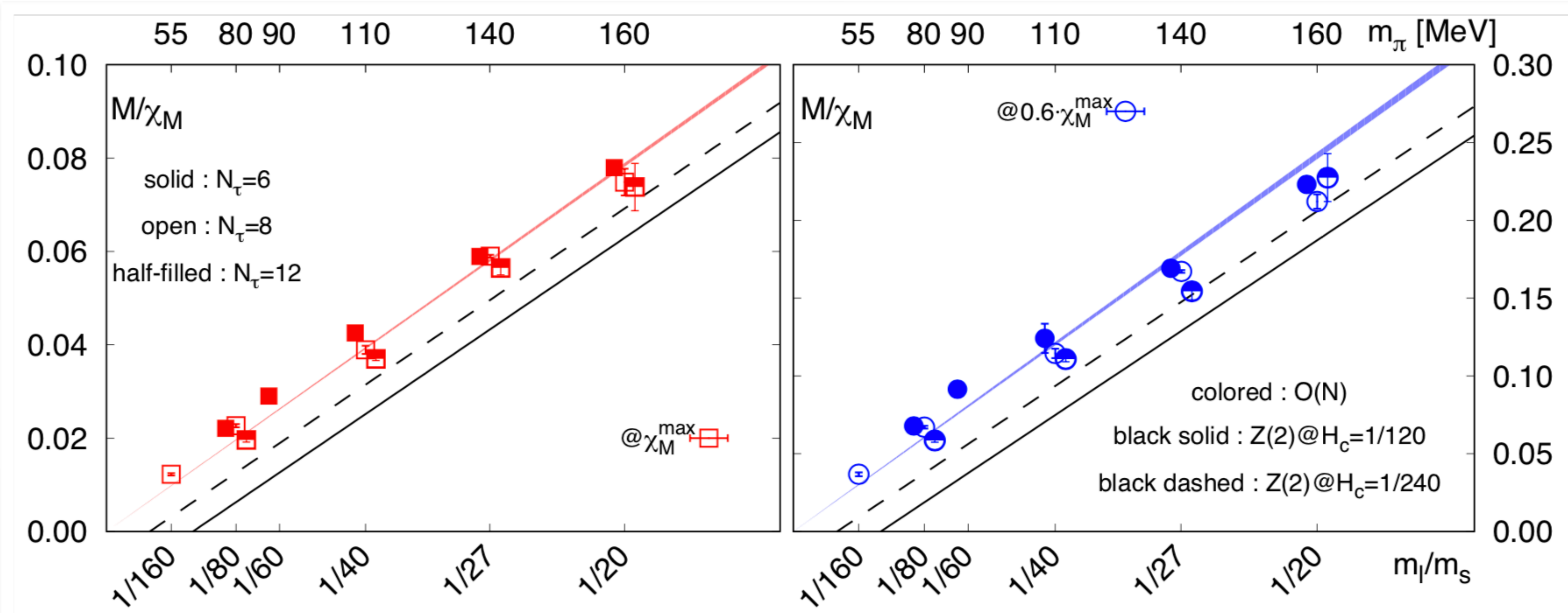
$$\chi_M = \partial M/\partial H = \frac{h^{1/\delta}}{H}\,\frac{1}{\delta}\,\left(f_G(z) - \frac{z}{\beta}\frac{df_G(z)}{dz}\right)$$

$H\chi_M/M \rightarrow 1/\delta$ @ T_c^0

H: m_l/m_s
M: chiral condensate
χ_M: chiral susceptibility



Consistency of QCD chiral phase transition with O(N) universality class



S.-T. Li(李胜泰), Lattice 2018, A. Lahiri, QM 2018

$$M/\chi_M = \frac{m_l - m_l^{critical}}{m_s^{phys}} \frac{f_M}{f_{\chi_M}}$$