Example 3. Experimental data on p_T-spectra are sometimes fitted to the exponential Boltzmann-type function given by

$$f(p_T) = \frac{1}{p_T}\frac{dN}{dp_T} \simeq C\, e^{-m_T/T_{\rm eff}}. \qquad (61)$$

The $\langle m_T \rangle$ could be obtained by

$$\langle m_T \rangle = \frac{\int_0^\infty p_T\, dp_T\, m_T\, \exp(-m_T/T_{\rm eff})}{\int_0^\infty p_T\, dp_T\, \exp(-m_T/T_{\rm eff})} = \frac{2T_{\rm eff}^2 + 2m_0 T_{\rm eff} + m_0^2}{m_0 + T_{\rm eff}}, \qquad (62)$$

where m_0 is the rest mass of the particle. It can be seen from the above expression that for a massless particle

$$\langle m_T \rangle = \langle p_T \rangle = 2T_{\rm eff}. \qquad (63)$$

This also satisfies the principle of equipartition of energy which is expected for a massless Boltzmann gas in equilibrium.

However, in experiments the higher limit of p_T is a finite quantity. In that case the integration will involve an incomplete gamma function.

2.7 Energy in CMS and LS

2.7.1 For Symmetric Collisions ($A + A$)

Consider the collision of two particles. In LS, the projectile with momentum $\mathbf{p}_1$, energy E_1, and mass m_1 collides with a particle of mass m_2 at rest. The 4-momenta of the particles are

$p_1 = (E_1, \mathbf{p}_1), \qquad p_2 = (m_2, \mathbf{0})$.

In CMS, the momenta of both the particles are equal and opposite, the 4-momenta are

$p_1^* = (E_1^*, \mathbf{p}_1^*), \qquad p_2^* = (E_2^*, -\mathbf{p}_1^*)$.

The total 4-momentum of the system is a conserved quantity in the collision.

In CMS,

$(p_1 + p_2)^2 = (E_1 + E_2)^2 - (\mathbf{p}_1 + \mathbf{p}_2)^2$

$= (E_1 + E_2)^2 = E_{\rm cm}^2 \equiv s$.

$\sqrt{s}$ is the total energy in the CMS which is the invariant mass of the CMS.

In LS,

$(p_1 + p_2)^2 = m_1^2 + m_2^2 + 2E_1 m_2$.

Hence

$$E_{\rm cm} = \sqrt{s} = \sqrt{m_1^2 + m_2^2 + 2E_{\rm proj} m_2}, \qquad (64)$$

where $E_1 = E_{proj}$, the projectile energy in LS. Hence it is evident here that the CM frame with an invariant mass $\sqrt{s}$ moves in the laboratory in the direction of $\mathbf{p}_1$ with a velocity corresponding to
Lorentz factor,

$$\gamma_{cm} = \frac{E_1 + m_2}{\sqrt{s}} \tag{65}$$

$$\Rightarrow \sqrt{s} = \frac{E_{lab}}{\gamma_{cm}}, \tag{66}$$

this is because $E = \gamma m$ and

$$y_{cm} = \cosh^{-1} \gamma_{cm}. \tag{67}$$

Note 1. We know

$$s = E_{cm}^2 = m_1^2 + m_2^2 + 2(E_1 + E_2 + \mathbf{p}_1.\mathbf{p}_2). \tag{68}$$

For a head-on collision with $m_1, \ m_2 \ll E_1, E_2$,

$$E_{cm}^2 \simeq 4E_1E_2. \tag{69}$$

For two beams crossing at an angle θ,

$$E_{cm}^2 = 2E_1E_2(1 + \cos\theta). \tag{70}$$

The CM energy available in a collider with equal energies (E) for new particle production rises linearly with E, i.e.,

$$E_{cm} \simeq 2E. \tag{71}$$

For a fixed-target experiment the CM energy rises as the square root of the incident energy

$$E_{cm} \simeq \sqrt{2m_2E_1}. \tag{72}$$

Hence the highest energy available for new particle production is achieved at collider experiments. For example, at SPS fixed-target experiment to achieve a CM energy of 17.3 AGeV the required incident beam energy is 158 AGeV.

Note 2. Most of the times the energy of the collision is expressed in terms of nucleon–nucleon center of mass energy. In the nucleon–nucleon CM frame, two nuclei approach each other with the same boost factor γ. The nucleon–nucleon CM is denoted by $\sqrt{s_{NN}}$ and is related to the total CM energy by

$$\sqrt{s} \ = \ A \ \sqrt{s_{NN}}. \tag{73}$$

This is for a symmetric collision with number of nucleons in each nuclei as A. The colliding nucleons approach each other with energy $\sqrt{s_{NN}}/2$ and with equal and opposite momenta. The rapidity of the nucleon–nucleon center of mass is $y_{NN} = 0$ and taking $m_1 = m_2 = m_N$, the projectile and target nucleons are at equal and opposite rapidities.

$$y_{\text{proj}} \ = \ - \, y_{\text{target}} \ = \ \cosh^{-1} \frac{\sqrt{s_{NN}}}{2m_N} \ = \ y_{\text{beam}}. \tag{74}$$

Note 3. Lorentz factor

$$\begin{aligned} \gamma &= \frac{E}{M} = \frac{\sqrt{s}}{2A \ m_N} \\ &= \frac{A \ \sqrt{s_{NN}}}{2A \ m_N} = \frac{\sqrt{s_{NN}}}{2 \ m_N} \\ &= \frac{E_{\text{beam}}^{\text{CMS}}}{m_N}, \end{aligned} \tag{75}$$

where E and M are energy and mass in CMS, respectively. Assuming mass of the nucleon $m_N \sim 1$ GeV, the Lorentz factor is of the order of beam energy in CMS for a symmetric collision.

2.7.2 For Asymmetric Collisions ($A + B$)

During the early phase of relativistic nuclear collision research, the projectile mass was limited by accelerator-technical conditions (^{38}Ar at the Bevalac, ^{28}Si at the AGS, ^{32}S at the SPS). Nevertheless, collisions with mass ≈ 200 nuclear targets were investigated. Analysis of such collisions is faced with the problem of determining an "effective" center of mass frame, to be evaluated from the numbers of projectile and target participant nucleons, respectively. Their ratio – and thus the effective CM rapidity – depends on impact parameter. Moreover, this effective CM frame refers to soft hadron production only whereas hard processes are still referred to the frame of nucleon–nucleon collisions. The light projectile on heavy target kinematics is described in [43].

2.8 Luminosity

The luminosity is an important parameter in collision experiments. The reaction rate in a collider is given by

$$R \ = \ \sigma L, \tag{76}$$

where