Spin effects from quantum transport theory

Jian-Hua Gao (高建华)

Shandong University at Weihai

ArXiv:1902.06510 JHG and Z.T. Liang

The Workshop on QCD Physics & Study of the QCD Phase Diagram and New-type Topologic Effect

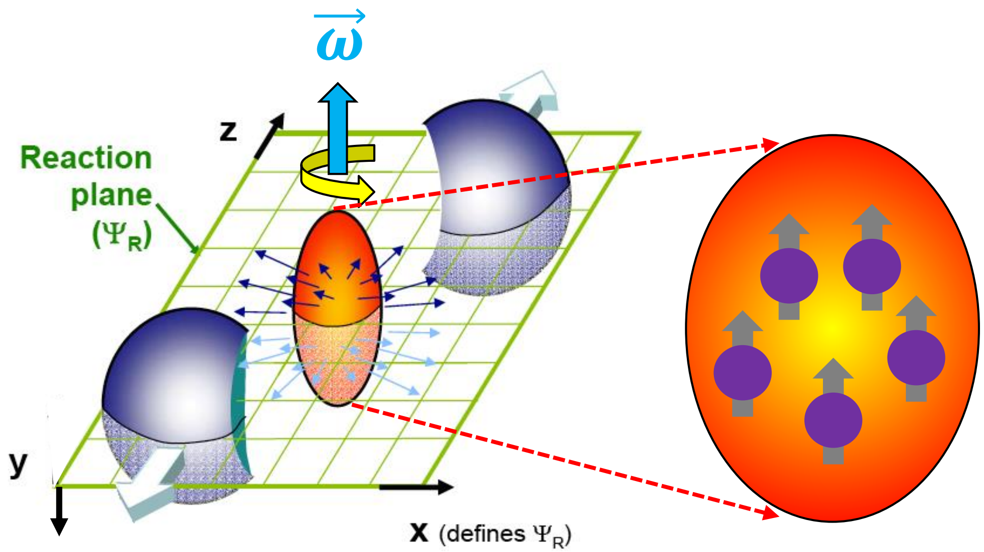
Weihai，China， July 17 – July 25，2019

Outline

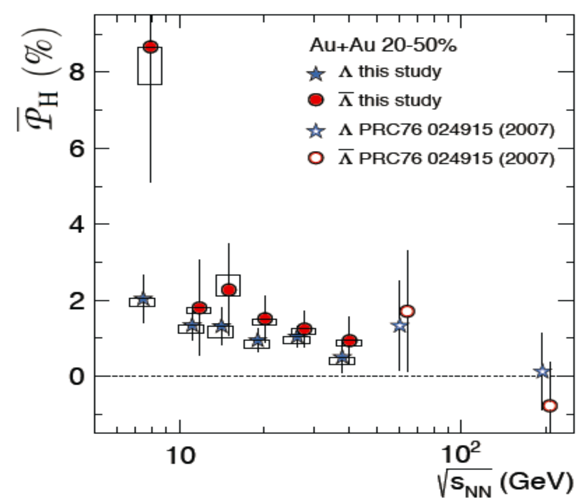
- **Introduction**
- **Quantum Transport Theory**
- **Spin Effects from Quantum Transport Theory**
- **Summary and Outlook**

Global Polarization in HIC

Global polarization induced by the global strong vorticity:



Z.T. Liang, X.N. Wang, PRL 94 (2005) 102301

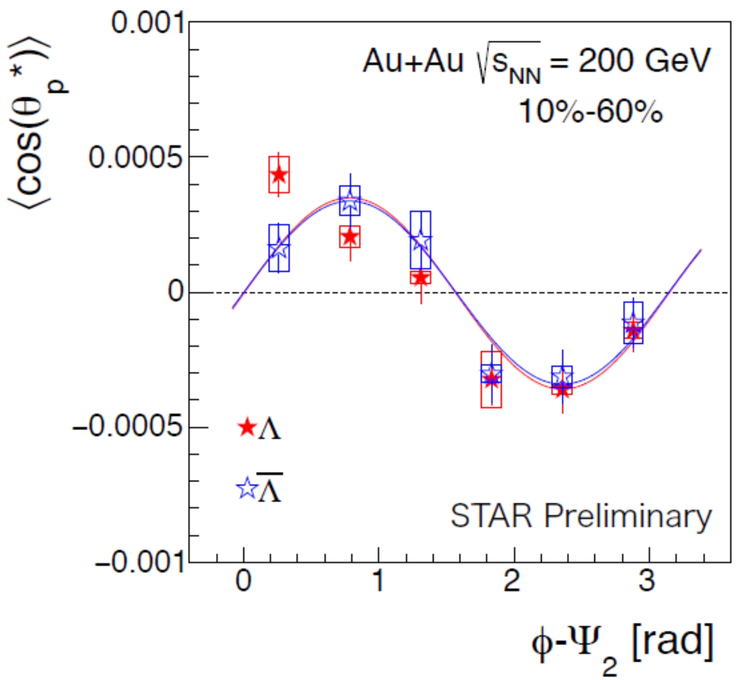


STAR collaboration, Nature 548 (2017) 62-65

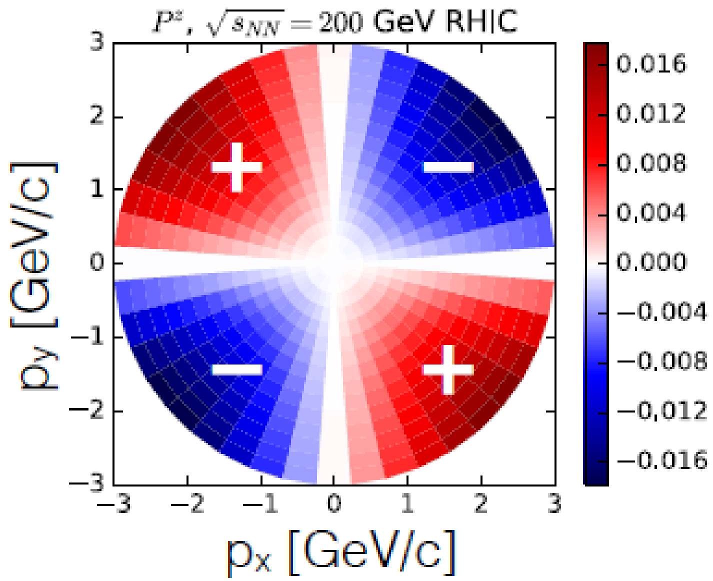
See Wang', Niida's and Huang' talks

Local Polarization in HIC

Local polarization induced by the local strong vorticity:

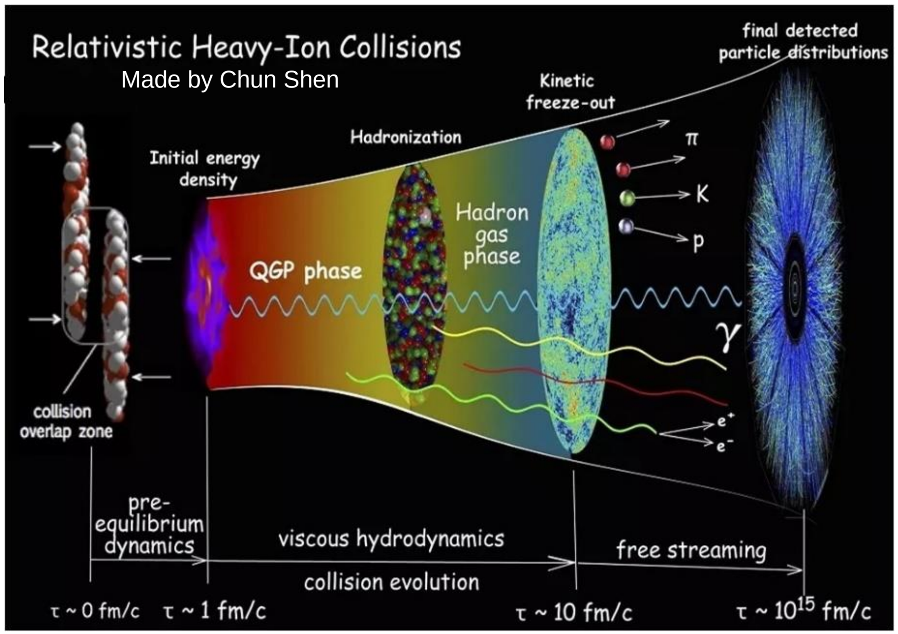


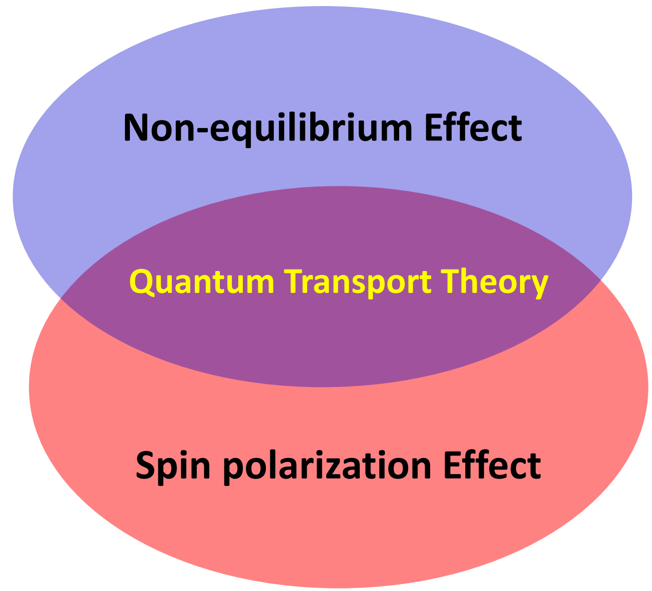
S.Voloshin SQM2017
F.Becattini and I. Karpenko PRL 120.012302(2018)



See Wang', Niida's and Huang' talks

Quantum Transport Theory





- Hydrodynamics with Spin:

Florkowski, Friman, Jaiswal & Speranza PRC2018; Becattini & Karpendo PRL2018; Florkowski, Speranza & Becattini APPB2018; Becattini, Florkowski & Speranza PLB2019; Hattori, Hongo, Huang, Matsuo & Taya 1901.06615 … … … …

See Huang' talk

Quantum Transport Theory

- ……
- Heinz PRL1983
- Vasak, Gyulassy, Elze NPB 1986; AP1987
- Elze, Heinz, Phys. Rept.1989
- Zhuang, Heinz, AP1996; PRD1998
- Ochs,Heinz, AP1998
- ……
- Gao, Liang, Pu, Wang, Wang PRL2012
- Chen, Pu, Wang, Wang PRL 2013
- Hidaka, Pu, Yang PRD2017
- Huang, Shi, Jiang, Liao, Zhuang PRD2018
- Gao, Liang, Wang, Wang PRD 2018
- ……
- Gao, Liang 1902.06510;
- Weickgenannt, Sheng, Speranza, Wang 1902.06510;
- Wang, Guo, Shi, Zhuang 1903.03461;
- Hattori, Hidaka, Yang 1903.01653
- ……

…….

Stephanov, Yin PRL2012;

Son & Yamamoto PRD2013;

Manuel & Torres-Rincon PRD2014;

Chen, Son & Stephanov PRL2015

Mueller & Venugopalan PRD2018;

Lin & Shukla JHEP 2019

…….

Classical VS Quantum Transport

Classical transport theory:

1 distribution function : $f(t, \vec{x}, \vec{p})$

1 Boltzmann equation: $p^\mu \left(\partial_\mu - F_{\mu\nu}\partial_p^\nu\right) f(t, \vec{x}, \vec{p}) \;=\; \mathcal{C}[f]$

Quantum transport theory:

scalar, pseudo, vector, axial, tensor

16 Wigner functions:

Spin 1/2

$$W = \frac{1}{4}\left[\mathscr{F} + i\gamma^5\mathscr{P} + \gamma^\mu\mathscr{V}_\mu + \gamma^5\gamma^\mu\mathscr{A}_\mu + \frac{1}{2}\sigma^{\mu\nu}\mathscr{S}_{\mu\nu}\right]$$

32 Wigner equations:

Background approximation

$$\begin{aligned}
\nabla^\mu\mathscr{V}_\mu = 0, \quad p^\mu\mathscr{A}_\mu &= 0 & m\mathscr{F} &= p^\mu\mathscr{V}_\mu \quad m\mathscr{P} = -\frac{1}{2}\nabla^\mu\mathscr{A}_\mu \\
\frac{1}{2}\nabla_\mu\mathscr{F} - p^\nu\mathscr{S}_{\mu\nu} &= 0 & m\mathscr{V}_\mu &= p_\mu\mathscr{F} + \frac{1}{2}\nabla^\nu\mathscr{S}_{\mu\nu} \\
p_\mu\mathscr{P} + \frac{1}{4}\epsilon_{\mu\nu\rho\sigma}\nabla^\nu\mathscr{S}^{\rho\sigma} &= 0 & m\mathscr{A}_\mu &= \frac{1}{2}\nabla_\mu\mathscr{P} - \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}p^\nu\mathscr{S}^{\rho\sigma} \\
p_{[\mu}\mathscr{V}_{\nu]} + \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\nabla^\rho\mathscr{A}^\sigma &= 0 & m\mathscr{S}_{\mu\nu} &= \frac{1}{2}\nabla_{[\mu}\mathscr{V}_{\nu]} - \epsilon_{\mu\nu\rho\sigma}p^\rho\mathscr{A}^\sigma
\end{aligned}$$

Vasak, Gyulassy, Elze, Annals Phys. 1987

Wigner functions

Physical meaning of Wigner functions:

- $\mathscr{F}$ Mass density, particle number distribution function
- $\mathscr{A}_\mu$ space components: spin density, spin polarization vector
- $\mathscr{V}_\mu$ Charge density and current density, current vector
- $\mathscr{S}_{\mu\nu}$ space components: magnetic moment density
- $\mathscr{P}$ Pseudo scalar density

Vasak, Gyulassy, Elze, Annals Phys. 1987;
Bialynicki-Birula, Gornicki, Rafelski PRD 1991

Choose $\mathscr{F}$ and $\mathscr{A}^\mu$ as the independent fundamental components

Eleven of 32 provide the expressions of other components:

$$\mathscr{P} = -\frac{\hbar}{2m}\nabla^\mu \mathscr{A}_\mu, \quad \mathscr{V}_\mu = \frac{1}{m}p_\mu \mathscr{F} - \frac{\hbar}{2m^2}\epsilon_{\mu\nu\rho\sigma}\nabla^\nu p^\rho \mathscr{A}^\sigma$$

$$\mathscr{S}_{\mu\nu} = -\frac{1}{m}\epsilon_{\mu\nu\rho\sigma}p^\rho \mathscr{A}^\sigma + \frac{\hbar}{2m^2}\left(\nabla_\mu p_\nu - \nabla_\nu p_\mu\right)\mathscr{F}$$

Other works: Weickgenannt, Sheng, Speranza, Wang & Rischke 1902.06513;
Hattori, Hidaka & Yang 1903.01653; Wang, Guo, Shi & Zhuang 1903.03461

New Wigner functions

Five of 32 give the modified on-shell conditions:

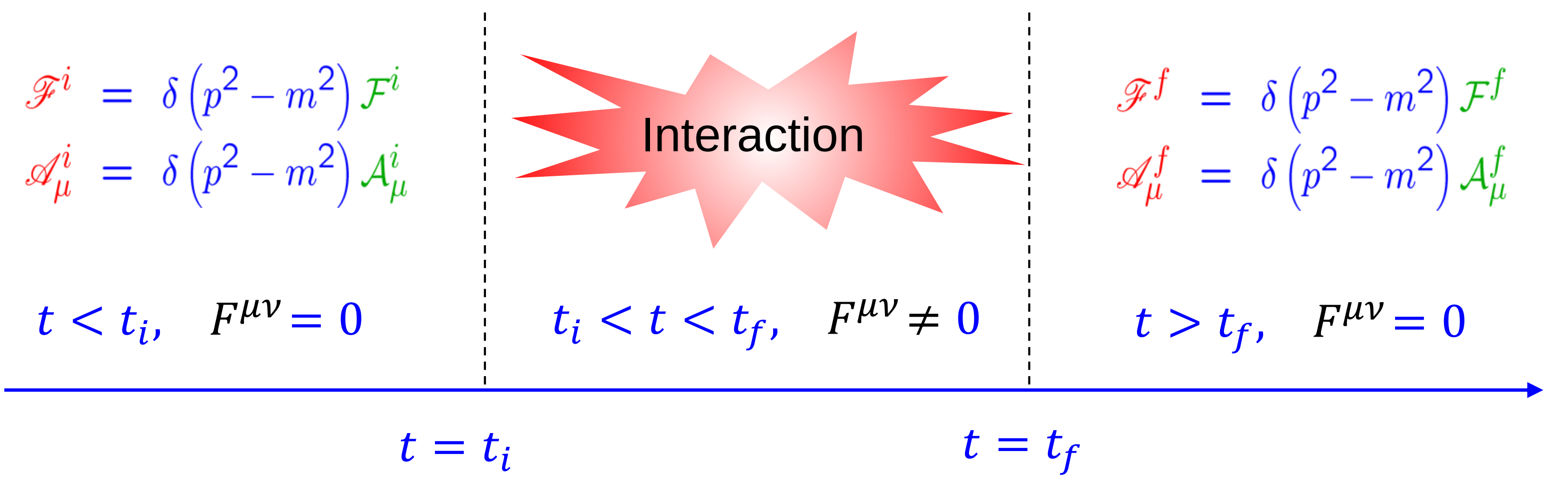
$$\mathscr{F} = \delta\left(p^2 - m^2\right)\mathcal{F} + \frac{\hbar}{m}\tilde{F}_{\mu\nu}p^{\mu}\mathcal{A}^{\nu}\delta'\left(p^2 - m^2\right)$$
$$\mathscr{A}_{\mu} = \delta\left(p^2 - m^2\right)\mathcal{A}_{\mu} + \frac{\hbar}{m}p^{\nu}\tilde{F}_{\mu\nu}\mathcal{F}\delta'\left(p^2 - m^2\right)$$

Regard $\mathcal{F}$ and $\mathcal{A}^{\mu}$ as new independent Wigner functions.

Convenient to deal with transient EM field or scattering process:

$$\mathscr{F}^{i} = \delta\left(p^2 - m^2\right)\mathcal{F}^{i}$$
$$\mathscr{A}^{i}_{\mu} = \delta\left(p^2 - m^2\right)\mathcal{A}^{i}_{\mu}$$

$$\mathscr{F}^{f} = \delta\left(p^2 - m^2\right)\mathcal{F}^{f}$$
$$\mathscr{A}^{f}_{\mu} = \delta\left(p^2 - m^2\right)\mathcal{A}^{f}_{\mu}$$



Independent Functions and Equations

Five of 32 lead to coupled transport equation for $\mathscr{F}$ and $\mathscr{A}^\mu$:

$$
\begin{aligned}
p\cdot\nabla\mathscr{F} &= \frac{\hbar}{2m}p^\mu(\partial_\lambda^x\tilde{F}_{\mu\nu})\partial_p^\lambda\mathscr{A}^\nu \\
p\cdot\nabla\mathscr{A}_\mu &= F_{\mu\nu}\mathscr{A}^\nu + \frac{\hbar}{2m}p^\nu(\partial_\lambda^x\tilde{F}_{\mu\nu})\partial_p^\lambda\mathscr{F}
\end{aligned}
$$

$$
\begin{aligned}
p\cdot\nabla\left[\mathcal{F}\delta\left(p^2-m^2\right)+\frac{\hbar}{m}\tilde{F}_{\mu\nu}p^\mu\mathcal{A}^\nu\delta'\left(p^2-m^2\right)\right] &= \frac{\hbar}{2m}(\partial_\lambda^x\tilde{F}_{\mu\nu})\partial_p^\lambda\left[p^\mu\mathcal{A}^\nu\delta\left(p^2-m^2\right)\right] \\
p\cdot\nabla\left[\mathcal{A}_\mu\delta\left(p^2-m^2\right)+\frac{\hbar}{m}p^\nu\tilde{F}_{\mu\nu}\mathcal{F}\delta'\left(p^2-m^2\right)\right] &= F_{\mu\nu}\left[\mathcal{A}^\nu\delta\left(p^2-m^2\right)+\frac{\hbar}{m}p_\lambda\tilde{F}^{\nu\lambda}\mathcal{F}\delta'\left(p^2-m^2\right)\right]+\frac{\hbar}{2m}(\partial_\lambda^x\tilde{F}_{\mu\nu})\partial_p^\lambda\left[p^\nu\mathcal{F}\delta\left(p^2-m^2\right)\right]
\end{aligned}
$$

One of 32 provide a subsidiary condition:

$$p_\mu\mathscr{A}^\mu = 0 \quad \Rightarrow \quad p_\mu\mathcal{A}^\mu\,\delta\left(p^2-m^2\right) = 0$$

All the rest **10** of the 32 Wigner equations are satisfied automatically !

4 independent Wigner functions + 4 transport equations

Vasak, Gyulassy, Elze Annals Phys. 1987； ArXiv:1902.06510 JHG and Liang

Integrated Kinetic Equations

Unintegrated kinetic equations in **4**-vector form:

$$p\cdot\nabla\left[\mathcal{F}\delta\left(p^2-m^2\right)+\frac{\hbar}{m}\tilde{F}_{\mu\nu}p^{\mu}\mathcal{A}^{\nu}\delta'\left(p^2-m^2\right)\right] = \frac{\hbar}{2m}(\partial_\lambda^x\tilde{F}_{\mu\nu})\partial_p^\lambda\left[p^\mu\mathcal{A}^\nu\delta\left(p^2-m^2\right)\right]$$

$$p\cdot\nabla\left[\mathcal{A}_\mu\delta\left(p^2-m^2\right)+\frac{\hbar}{m}p^\nu\tilde{F}_{\mu\nu}\mathcal{F}\delta'\left(p^2-m^2\right)\right] = F_{\mu\nu}\left[\mathcal{A}^\nu\delta\left(p^2-m^2\right)+\frac{\hbar}{m}p_\lambda\tilde{F}^{\nu\lambda}\mathcal{F}\delta'\left(p^2-m^2\right)\right]+\frac{\hbar}{2m}(\partial_\lambda^x\tilde{F}_{\mu\nu})\partial_p^\lambda\left[p^\nu\mathcal{F}\delta\left(p^2-m^2\right)\right]$$

$$p_\mu\mathcal{A}^\mu\,\delta\left(p^2-m^2\right) = 0$$

Manifest Lorentz Covariance ! Singular Dirac delta function !

Integrated kinetic equations in **3**-vector form:

$$\mathcal{A}^0 = \vec{v}\cdot\vec{\mathcal{A}}$$

$$\left(\nabla_t+\vec{v}\cdot\vec{\nabla}\right)\mathcal{F} = -\frac{\hbar}{2mE_p}\left[(\vec{B}+\vec{E}\times\vec{v})(\vec{v}\cdot\vec{\nabla}+E_p\overleftarrow{\nabla}_x\cdot\vec{\nabla}_p)-(\vec{B}\cdot\vec{v})(\vec{v}\cdot\vec{\nabla}+E_p\overleftarrow{\nabla}_x\cdot\vec{\nabla}_p)\vec{v}\right]\cdot\vec{\mathcal{A}}$$

$$\left(\nabla_t+\vec{v}\cdot\vec{\nabla}\right)\vec{\mathcal{A}} = \vec{B}\times\vec{\mathcal{A}}-\vec{E}(\vec{v}\cdot\vec{\mathcal{A}})-\frac{\hbar}{2mE_p}(\vec{B}+\vec{E}\times\vec{v})(\vec{v}\cdot\vec{\nabla}+E_p\overleftarrow{\nabla}_x\cdot\vec{\nabla}_p)\mathcal{F}$$

$$\vec{v}=\vec{p}/E_p,\quad \nabla_t=\partial_t+\vec{E}\cdot\vec{\nabla}_p,\quad \vec{\nabla}=\vec{\nabla}_x+\vec{B}\times\vec{\nabla}_p,$$

Manifest Lorentz Covariance Broken! More suitable for simulation!

Simplified Version

Vector current and energy-momentum tensor at $O(\hbar)$:

$$j^\mu = \int d^4p\, \mathscr{V}^\mu, \qquad T^{\mu\nu} = \int d^4p\, p^\nu \mathscr{V}^\mu$$

$$\mathscr{V}_\mu = \frac{1}{m} p_\mu \mathscr{F} - \frac{\hbar}{2m^2} \epsilon_{\mu\nu\rho\sigma} \nabla^\nu p^\rho \mathscr{A}^\sigma,$$

Covariant unintegrated kinetic equations for $\mathcal{A}^\mu$ at $O(1)$:

$$p\cdot\nabla\left[\mathcal{A}_\mu \delta\left(p^2-m^2\right) + \frac{\hbar}{m}\cancel{p^\nu \tilde{F}_{\mu\nu}\mathcal{F}\delta'\left(p^2-m^2\right)}\right] = F_{\mu\nu}\left[\mathcal{A}^\nu\delta\left(p^2-m^2\right) + \frac{\hbar}{m}\cancel{p_\lambda \tilde{F}^{\nu\lambda}\mathcal{F}\delta'\left(p^2-m^2\right)}\right] + \frac{\hbar}{2m}\cancel{(\partial^x_\lambda \tilde{F}_{\mu\nu})\partial^\lambda_p\left[p^\nu\mathcal{F}\delta\left(p^2-m^2\right)\right]}.$$

⇓

$$p\cdot\nabla\left[\mathcal{A}_\mu\delta\left(p^2-m^2\right)\right] = F_{\mu\nu}\mathcal{A}^\nu\delta\left(p^2-m^2\right)$$

+

$$p_\mu \mathcal{A}^\mu\, \delta\left(p^2-m^2\right) = 0$$

Inserting the solved $\mathcal{A}^\mu$ into the transport equation for $\mathcal{F}$

$$p\cdot\nabla\left[\mathcal{F}\delta\left(p^2-m^2\right) + \frac{\hbar}{m}\tilde{F}_{\mu\nu}p^\mu\mathcal{A}^\nu\delta'\left(p^2-m^2\right)\right] = \frac{\hbar}{2m}(\partial^x_\lambda\tilde{F}_{\mu\nu})\partial^\lambda_p\left[p^\mu\mathcal{A}^\nu\delta\left(p^2-m^2\right)\right]$$

Simplified Version

Define:

$$\frac{\mathcal{A}_\mu}{\mathcal{F}} = P s_\mu \qquad s^2 = -1, \quad p \cdot s = 0$$

P : Spin polarization magnitude

s^μ : Spin polarization direction

Decoupled equations for P and s_μ :

Vlasov equation: $$p \cdot \nabla \left[P \delta \left(p^2 - m^2 \right) \right] = 0,$$

BMT equation: $$p \cdot \nabla \left[s_\mu \delta \left(p^2 - m^2 \right) \right] = F_{\mu\nu} s^\nu \delta \left(p^2 - m^2 \right).$$

Rewrite the transport equations for $\mathcal{F}$ as:

$$p \cdot \nabla \left[\mathcal{F} \delta \left(p^2 - m^2 - 2E_p \Delta E \right) \right] = \frac{\hbar}{2m} (\partial_\lambda^x \tilde{F}^{\rho\sigma}) \partial_p^\lambda \left[p_\rho s_\sigma P \mathcal{F} \delta \left(p^2 - m^2 - 2E_p \Delta E \right) \right]$$

The effective interaction energy:

$$\Delta E = -\frac{\hbar P}{2m E_p} \tilde{F}^{\rho\sigma} p_\rho s_\sigma$$

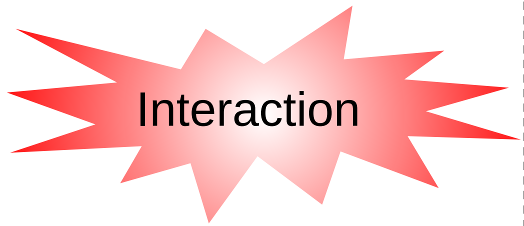
Spin Polarization Generation

Transient EM field process:

$$\mathscr{A}_\mu = \delta\left(p^2 - m^2\right)\mathcal{A}_\mu + \frac{\hbar}{m} p^\nu \tilde{F}_{\mu\nu}\mathcal{F}\delta'\left(p^2 - m^2\right)$$

$$\mathscr{F}^i = \delta\left(p^2 - m^2\right)\mathcal{F}^i \neq 0$$

$$\mathscr{A}^i_\mu = \delta\left(p^2 - m^2\right)\mathcal{A}^i_\mu = 0$$



$$\mathscr{A}^f_\mu = \delta\left(p^2 - m^2\right)\mathcal{A}^f_\mu = \textbf{?}$$

$t < t_i, \quad F^{\mu\nu} = 0$

$t_i < t < t_f, \quad F^{\mu\nu} \neq 0$

$t > t_f, \quad F^{\mu\nu} = 0$

$t = t_i$

$t = t_f$

Evolution equation for spin polarization vector up to $O(1)$:

$$\left(\nabla_t + \vec{v}\cdot\vec{\nabla}\right)\vec{\mathcal{A}} = \vec{B}\times\vec{\mathcal{A}} - \vec{E}(\vec{v}\cdot\vec{\mathcal{A}})$$

near t_i

$\vec{\mathcal{A}}(t_i) = 0$

$$\frac{\partial\vec{\mathcal{A}}}{\partial t} = 0$$

No way to generate the polarization from a zero initial value !

Spin Polarization Generation

Evolution equation for spin vector up to $O(\hbar)$:

$$\left(\nabla_t + \vec{v}\cdot\vec{\nabla}\right)\vec{A} = \vec{B}\times\vec{A} - \vec{E}(\vec{v}\cdot\vec{A}) - \frac{\hbar}{2mE_p}(\vec{B}+\vec{E}\times\vec{v})(\vec{v}\cdot\vec{\nabla} + E_p\overleftarrow{\nabla}_x\cdot\vec{\nabla}_p)\mathcal{F}$$

near t_i ⇩ $\vec{A}(t_i) = 0$

$$\frac{\partial\vec{A}}{\partial t} = -\frac{\hbar}{2mE_p}(\vec{B}+\vec{E}\times\vec{v})(\vec{v}\cdot\vec{\nabla} + E_p\overleftarrow{\nabla}_x\cdot\vec{\nabla}_p)\mathcal{F}(t_i)$$

Polarization seed: EM field + inhomogeneous $\mathcal{F}(t_i)$

Self-consistent background EM field: Vasak, Gyulassy, Elze, Annals Phys. 1987

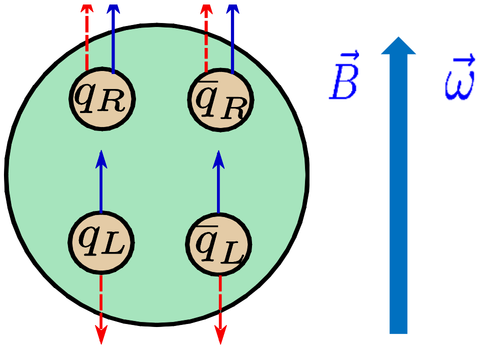
$$\partial_\mu F^{\mu\nu} = j^\nu \quad \Rightarrow \quad \partial_\lambda\partial^\lambda F_{\mu\nu} = (\partial_\mu j_\nu - \partial_\nu j_\mu)$$

Global polarization: **Vorticity → EM field → polarization**

Particle scattering generation: arxiv:1904.09152, Wang' talk

CSE with mass correction

Chiral separation effect:



$$j_5^\mu = \frac{\mu}{2\pi^2}B^\mu + \frac{1}{2\pi^2}\left(\frac{\pi^2 T^2}{3} + \mu^2 + \mu_5^2\right)\omega^\mu$$

Global equilibrium solution with constant $\Omega_{\mu\nu}$ & $F_{\mu\nu}$

$$\mathcal{A}_\mu = 0, \quad \mathcal{F} = \frac{m}{2\pi^3}\left[\frac{\theta(u\cdot p)}{e^{(u\cdot p-\mu)/T}+1} + \frac{\theta(-u\cdot p)}{e^{-(u\cdot p-\mu)/T}+1}\right]$$

$$\beta_\mu = u_\mu/T$$

$$\Omega_{\mu\nu} = \partial_\mu\beta_\nu - \partial_\nu\beta_\mu$$

$$\mathscr{A}_\mu = \frac{\hbar}{m}p^\nu \tilde{F}_{\mu\nu}\mathcal{F}\delta'\left(p^2 - m^2\right) \Rightarrow j_5^\mu = \int d^4p\mathscr{A}^\mu = \sigma B^\mu$$

$$\Rightarrow \sigma = \frac{\hbar}{2\pi^2}\int_0^\infty dp\left(n_+ - n_-\right), \quad n_\pm = \frac{1}{e^{(E_p\mp\mu)/T}+1}$$

Lin & Yang PRD2018

Chiral limit: $\sigma|_{m=0} = \frac{\hbar\mu}{2\pi^2}$ Zero temperature limit: $\sigma|_{T\to 0} = \frac{\hbar\sqrt{\mu^2 - m^2}}{2\pi^2}$

Quantum magnetization effect

Wigner function associated to spin magnetic moment density:

$$\mathscr{S}_{\mu\nu} = -\frac{1}{m}\epsilon_{\mu\nu\rho\sigma}p^{\rho}\mathscr{A}^{\sigma} + \frac{\hbar}{2m^2}\left(\nabla_{\mu}p_{\nu} - \nabla_{\nu}p_{\mu}\right)\mathscr{F}$$

Spin magnetic moment vector:

$$M_{\mu} = \frac{1}{2}\epsilon_{\nu\mu\alpha\beta}u^{\nu}\int d^4p\,\mathscr{S}^{\alpha\beta}$$

Global equilibrium solution with constant $\Omega_{\mu\nu}$ & $F_{\mu\nu}$:

$$\mathcal{A}_{\mu} = 0 \qquad \mathcal{F} = \frac{m}{2\pi^3}\left[\frac{\theta(u\cdot p)}{e^{(u\cdot p-\mu)/T}+1} + \frac{\theta(-u\cdot p)}{e^{-(u\cdot p-\mu)/T}+1}\right]$$

Quantum magnetization effect

$$M_{\mu} = \hbar\kappa B_{\mu} - \frac{\hbar\rho}{m}\omega_{\mu}$$

Susceptibility:

$$\kappa = \frac{m}{2\pi^2}\int\frac{dp}{E_p}\left(n_+ + n_-\right)$$

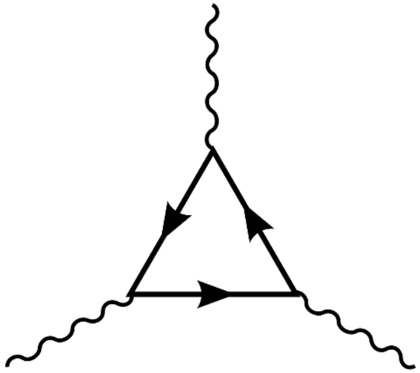
Charge density:

$$\rho = \frac{1}{\pi^2}\int dp\,p^2\left(n_+ - n_-\right)$$

Chiral Anomaly

Chiral anomaly:

$$\partial_\mu j_5^\mu = -2mj_5 - \frac{e^2}{2\pi^2} E \cdot B$$



Pseudo scalar Wigner function:

$$\mathscr{P} = -\frac{\hbar}{2m} \nabla^\mu \mathscr{A}_\mu$$

$$\mathscr{A}_\mu = \delta\left(p^2 - m^2\right) \mathcal{A}_\mu + \frac{\hbar}{m} p^\nu \tilde{F}_{\mu\nu} \mathcal{F} \delta'\left(p^2 - m^2\right)$$

Integrate over momentum:

$$j_5^\mu = \int d^4p \mathscr{A}^\mu \qquad j_5 = \int d^4p \mathscr{P}$$

$$\partial_\mu j_5^\mu = -\frac{2m}{\hbar} j_5 + \hbar C E \cdot B$$

$$C = \frac{1}{2m} \int d^4p \, \partial^\lambda \left[\mathcal{F} \partial_\lambda \delta(p^2 - m^2)\right]$$

Summary and outlook

- Relativistic quantum kinetic theory for particle with spin-1/2 up to first order in $\hbar$ is derived from the quantum transport theory.

- Spin polarization generation, chiral separate effect, quantum magnetization effect, and chiral anomaly can be described in quantum transport theory.

- Introduce quantum gauge fields systematically ?

Thanks for your attention !