Phase structure of QCD matter with magnetic field and Rotation

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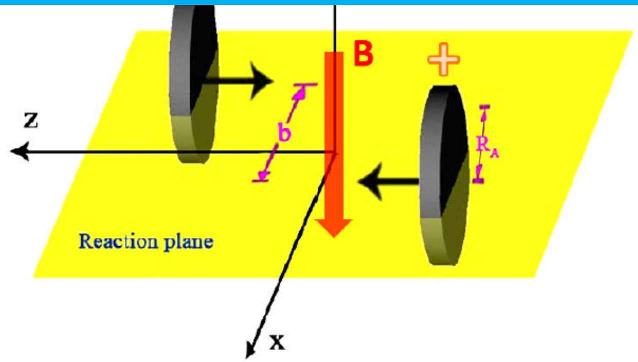
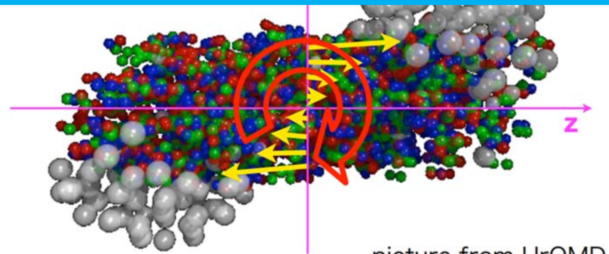
workshop on QCD Physics & Study of the QCD Phase Diagram and New-type Topologic Effect , Weihai, July 17-25, 2019

Outlines

- Motivations
- Phase structure with magnetic field (B)
- FRG study on xPT of dense QCD matter
- Rotation effects on phase structure
- Summary

Phase structure under new extrem condition





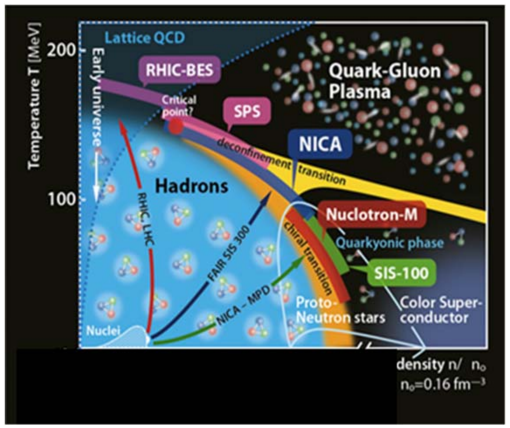
picture from UrQMD

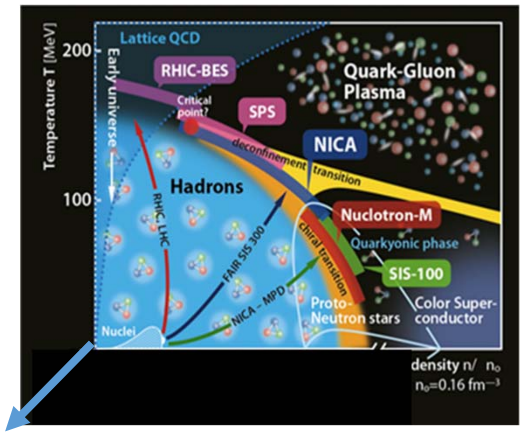
Strongest EM fields

Largest local rotation

What are their effects on the QCD phase structure

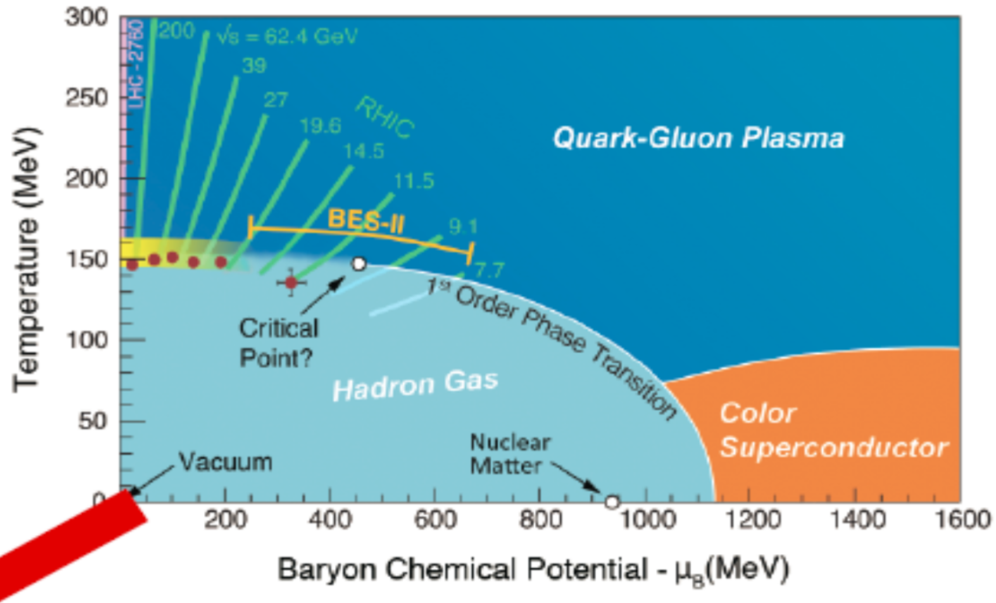
Explore the new dimensions of the QCD phase diagram





B, ω

New dimensions of phase structure



Putting the strongly interaction matter under strong magnetic field or fluid rotation!

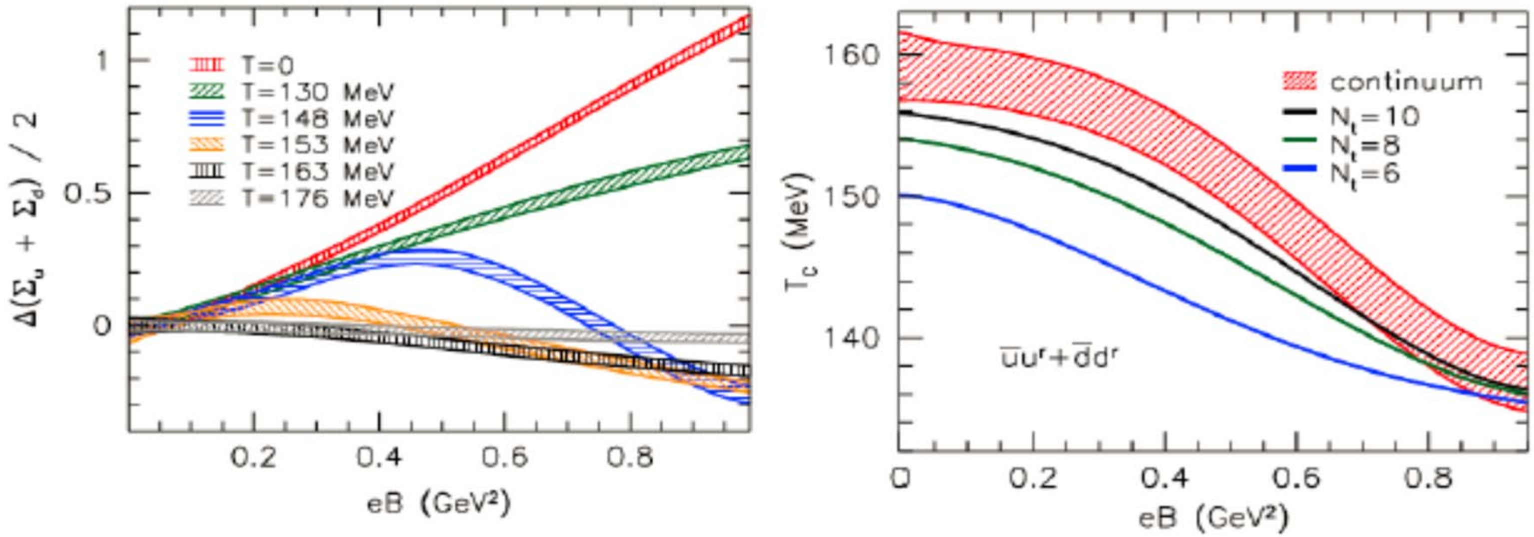
$\vec{B}$

$\vec{\omega}$

Opening up new dimensions: Toward Hyper-Phase-Diagram!

Magnetic field and rotation effects

We know that magnetic fields can change the thermodynamic properties and phase structure of QCD matter.



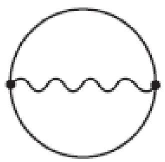
[Lattice results by Bali, et al]

And we know the similarity between B field and rotation.
It is thus tempting to ask:
influence of rotation on phase structure?
[BTW: it could be studied on lattice,
c.f. Yamamoto & Hirono, 2013]

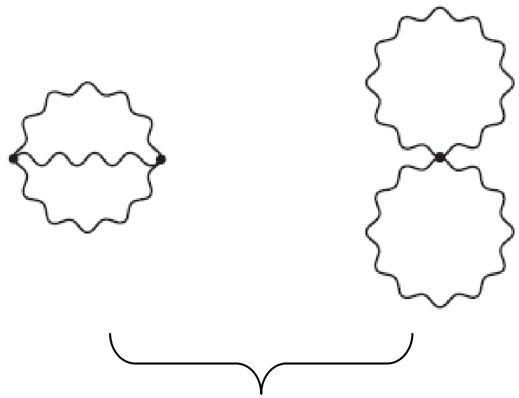
CJT effective action of thermal dense QCD

$$\Gamma[\bar{D},\bar{S}] = \frac{1}{2}\{Tr\ln \bar{D}^{-1} + Tr(D^{-1}\bar{D}-1) - Tr\ln \bar{S}^{-1} - Tr(S^{-1}\bar{S}) - 2\Gamma_2[\bar{D},\bar{S}]$$

The two-loop approximation
to \gamma_2



Order of g^2mu^4



Powers of T

Stationary points

$$\left.\frac{\delta\Gamma}{\delta\bar{\mathcal{D}}}\right|_{\bar{\mathcal{D}}=\mathcal{D},\bar{\mathcal{S}}=\mathcal{S}} = 0\,, \qquad \left.\frac{\delta\Gamma}{\delta\bar{\mathcal{S}}}\right|_{\bar{\mathcal{D}}=\mathcal{D},\bar{\mathcal{S}}=\mathcal{S}} = 0$$

$$\Downarrow \qquad \Downarrow$$

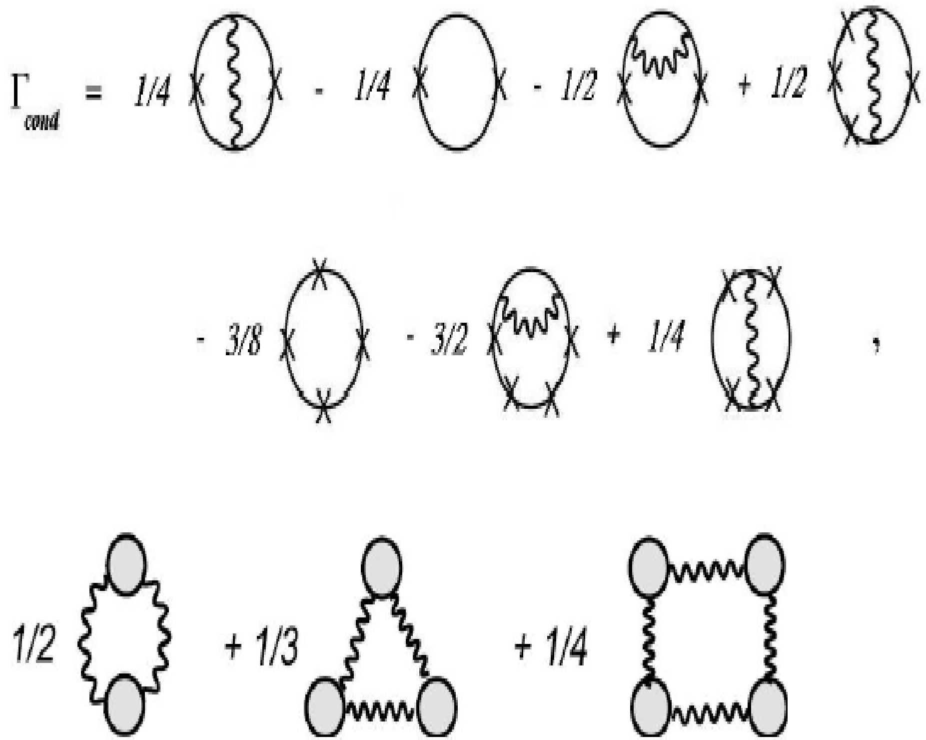
$$\mathcal{D}^{-1} = D^{-1} + \Pi[\mathcal{S}] \qquad \mathcal{S}^{-1} = \mathcal{S}_0^{-1} + \Sigma$$

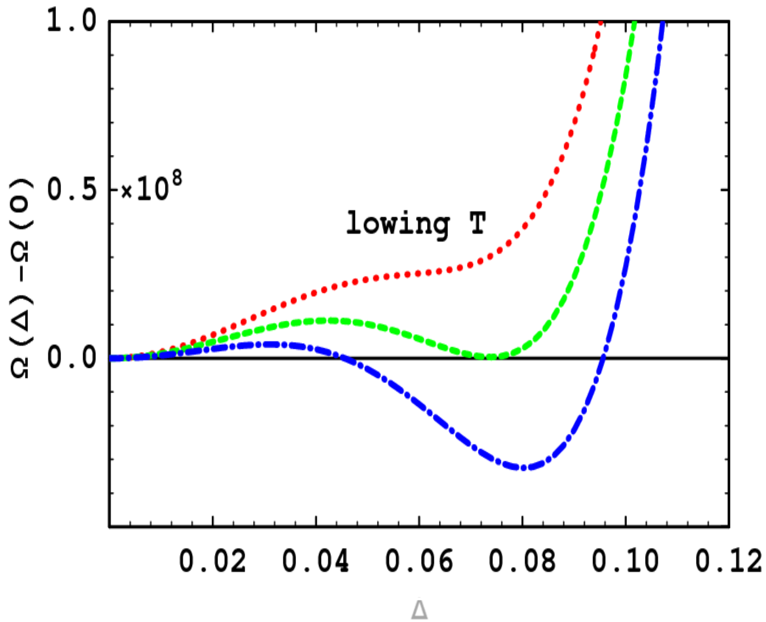
$$\Gamma_2[\bar{\mathcal{D}},\bar{\mathcal{S}}] = -\frac{1}{2}\,\mathrm{Tr}\{\bar{\mathcal{D}}\,\Pi[\bar{\mathcal{S}}]\}$$

D. Rischke Prog. Part. Nucl. Phys. 52 197 (2004)

Gauge field fluc. induce 1st order PT of CSC in dense QCD

Ginnakis, Hou, Ren, Rischke, PRL 93 (04) ; PRD73 (06)





Introduction of Δ^3 term in free energy by flucts. Inducing 1st order PT in stead of 2nd order PT in MFA

Color Superconductor with B

Oscillation, decrease the gap at low B, and increase gap at high B

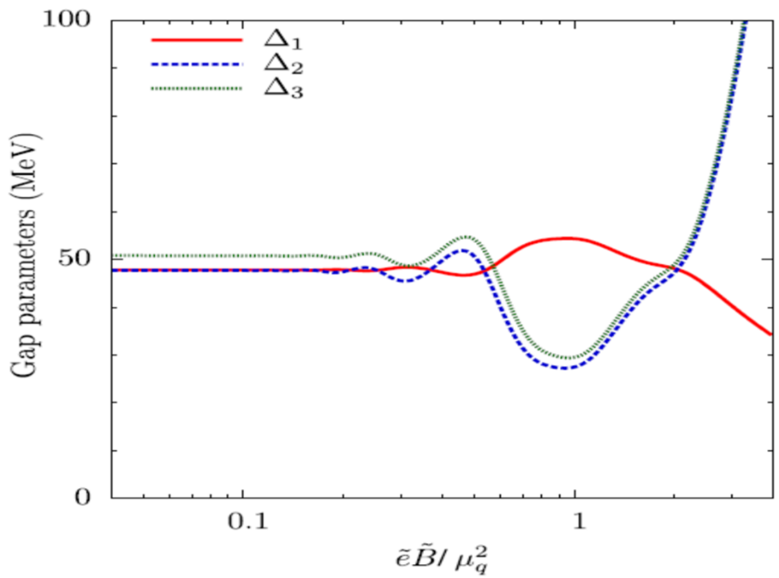


FIG. 1 (color online). Gap parameters as a function of $\tilde{e}\tilde{B}/\mu_q^2$ for $\mu_q = 500$ MeV and $M_s = 100$ MeV without neutrality.

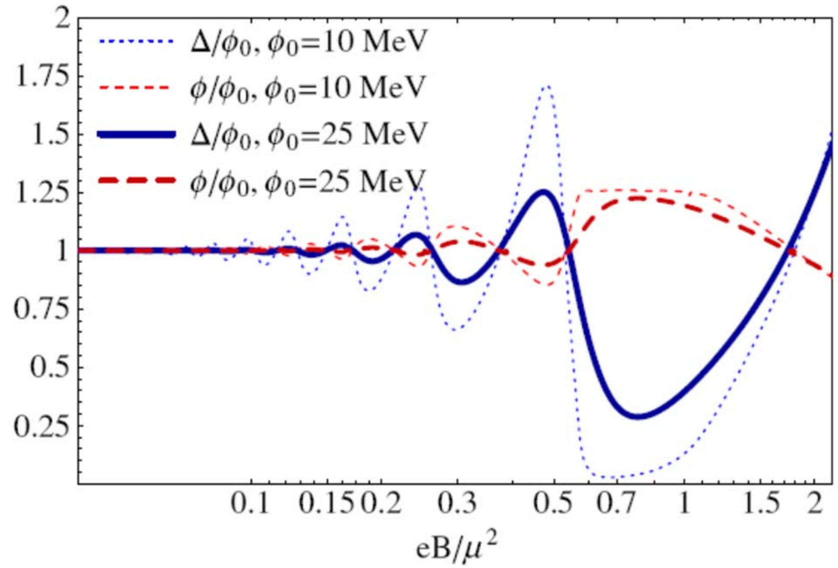


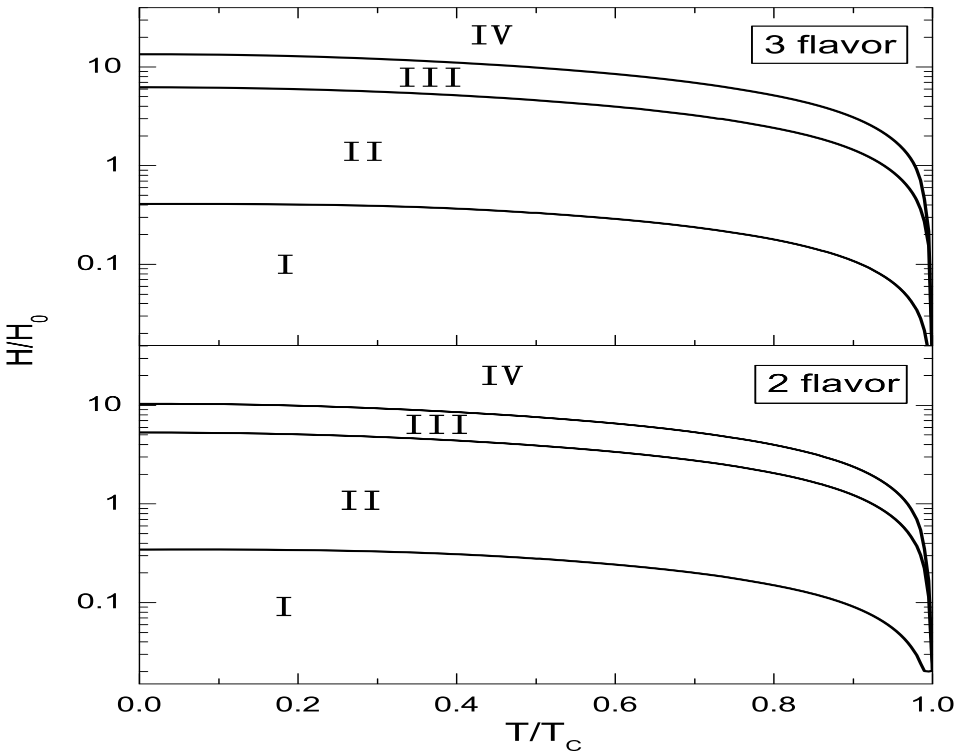
FIG. 1 (color online). Ratios Δ/ϕ_0 and ϕ/ϕ_0 versus eB/μ^2 for two sets of parameters that yield $\phi_0 = 10$ MeV and $\phi_0 = 25$ MeV.

K.Fukushima,etc.
PRL 100(2008)032007, CFL

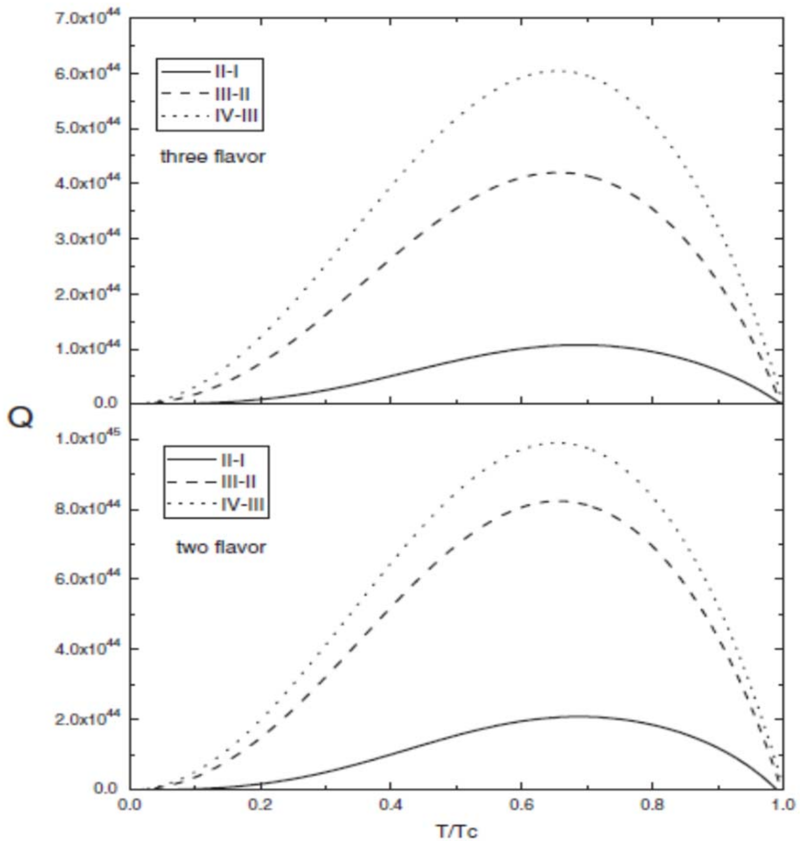
J.Noroha,etc. PRD 76(2007)105030, CFL

Nonspherical states in dense QCD with B

	I	II	III	IV	$T_C(10^{-1}$ MeV)
Two-flavor	CSL_u, CSL_d	$(\mathrm{polar})_u$, $(\mathrm{planar})_d$	$(\mathrm{normal})_u$, $(\mathrm{polar})_d$	$(\mathrm{normal})_u$, $(\mathrm{normal})_d$	1.35
Three-flavor	CSL_u, $\mathrm{CSL}_{d,s}$	$(\mathrm{polar})_u$, $(\mathrm{planar})_{d,s}$	$(\mathrm{normal})_u$, $(\mathrm{polar})_{d,s}$	$(\mathrm{normal})_u$, $(\mathrm{normal})_{d,s}$	0.49



Feng, Hou, Ren,Wu, PRL 105(2010)



Wu, He, Hou, Ren, PRD84 (2011)

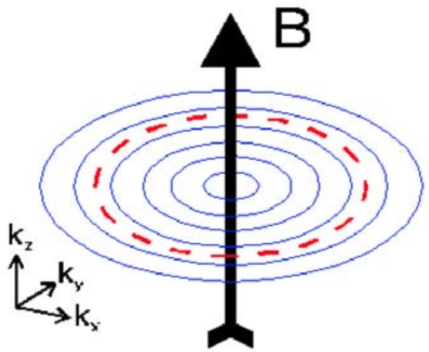
Chiral Magnetic Catalysis

- Chiral magnetic catalysis: Gusyin, Miransky & Shovkovy (1994)

$$< 0 | \bar{\psi}\psi | 0 > \overset{\lim\ m=0}{=} -\frac{|eB|}{2\pi}$$

Dynamical breakdown of chiral symmetry takes place at $m = 0$ and $B \neq 0$ even without any additional interactions between fermions.

The essence of this effect is the dimensional reduction 3+1->1+1 in the dynamics of fermion pairing in a magnetic field.



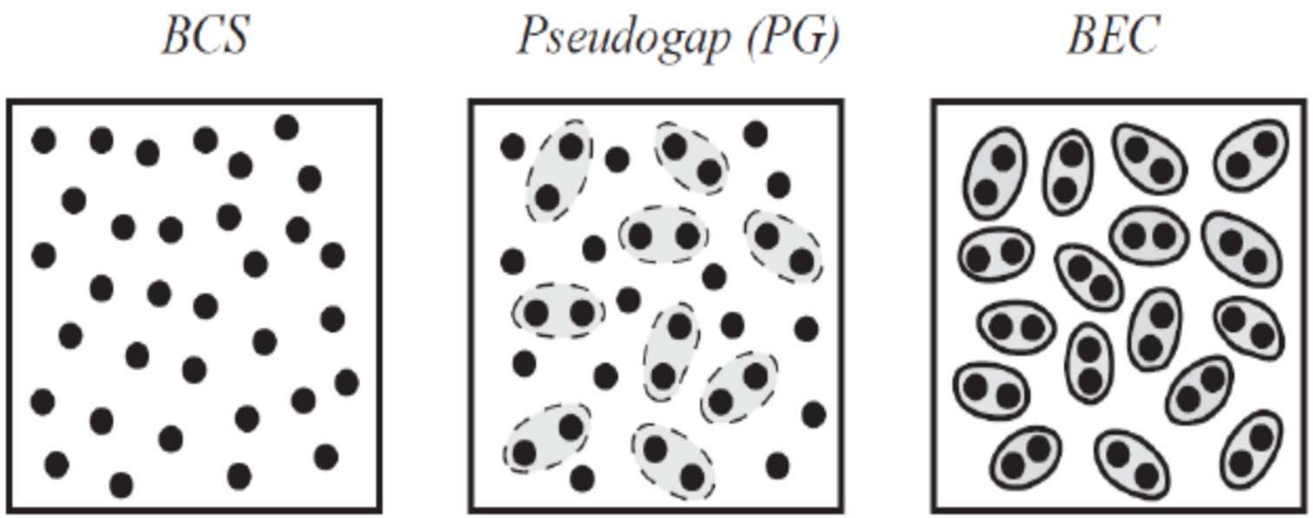
$$E_n(p_3) = \pm\sqrt{m_{dyn}^2 + 2|eB|n + p_3^2}, \quad n = 0,1,2......$$

In a strong magnetic field, all charged fermions will be restricted in lowest Landau level only, thus effectively reduce the dimension of the system.

$$m_{dyn} \approx C\sqrt{eB}\exp\left[-\left(\frac{\pi}{\alpha}\right)^{1/2}\right]$$

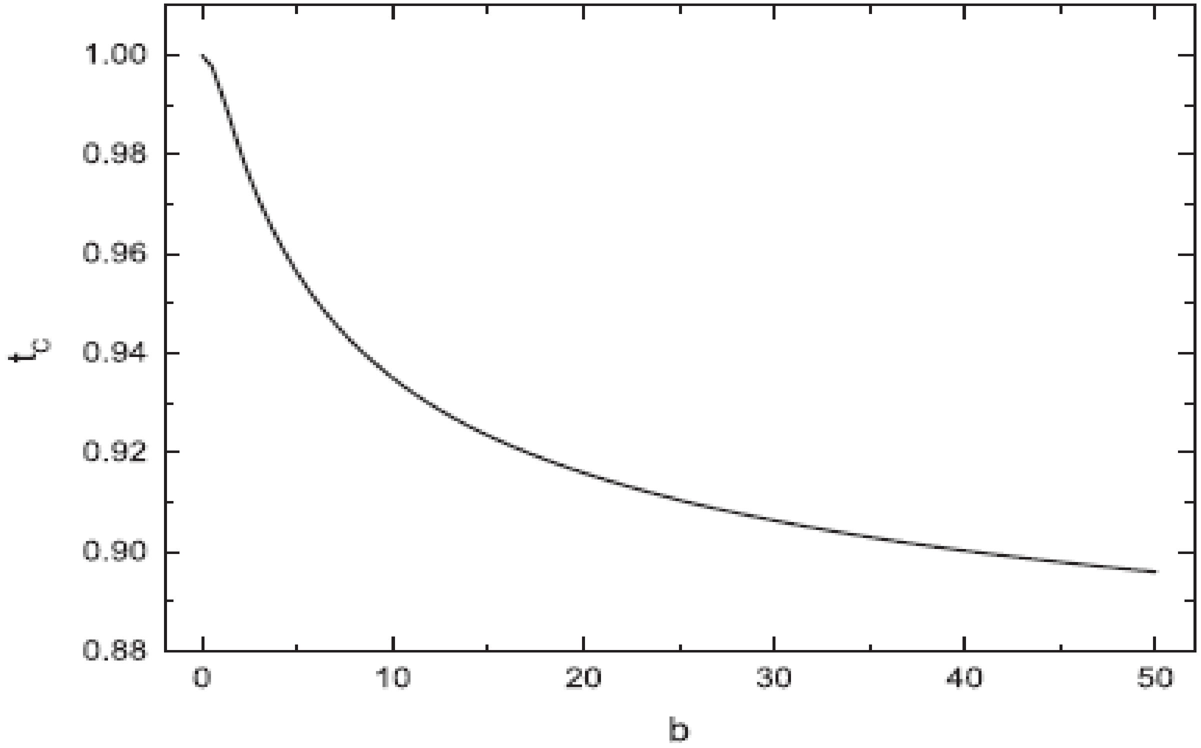
BEC with Magnetic field B

1. BEC shares the same physics as chiral condensate.
2. Chiral condensates correspond to the BEC limit in BCS/BEC crossover.



BEC with Magnetic field B

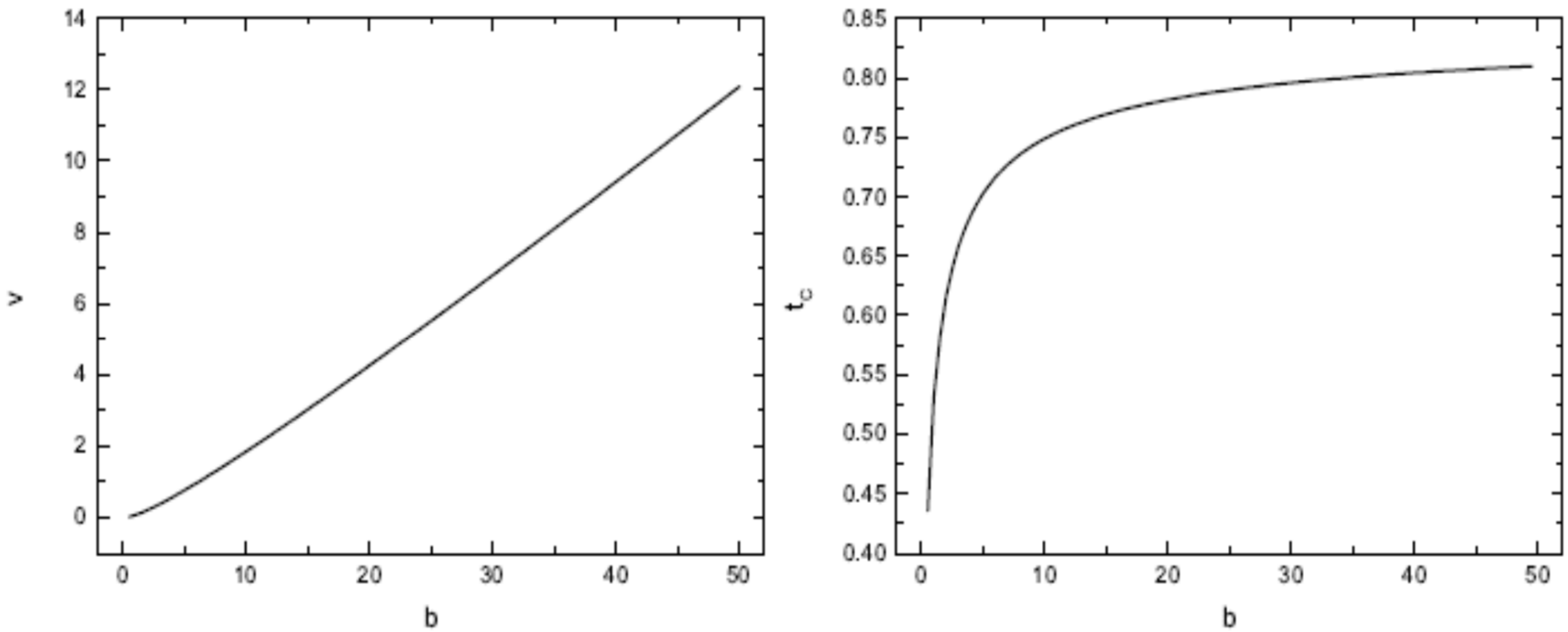
Inverse chiral catalysis in strong coupling



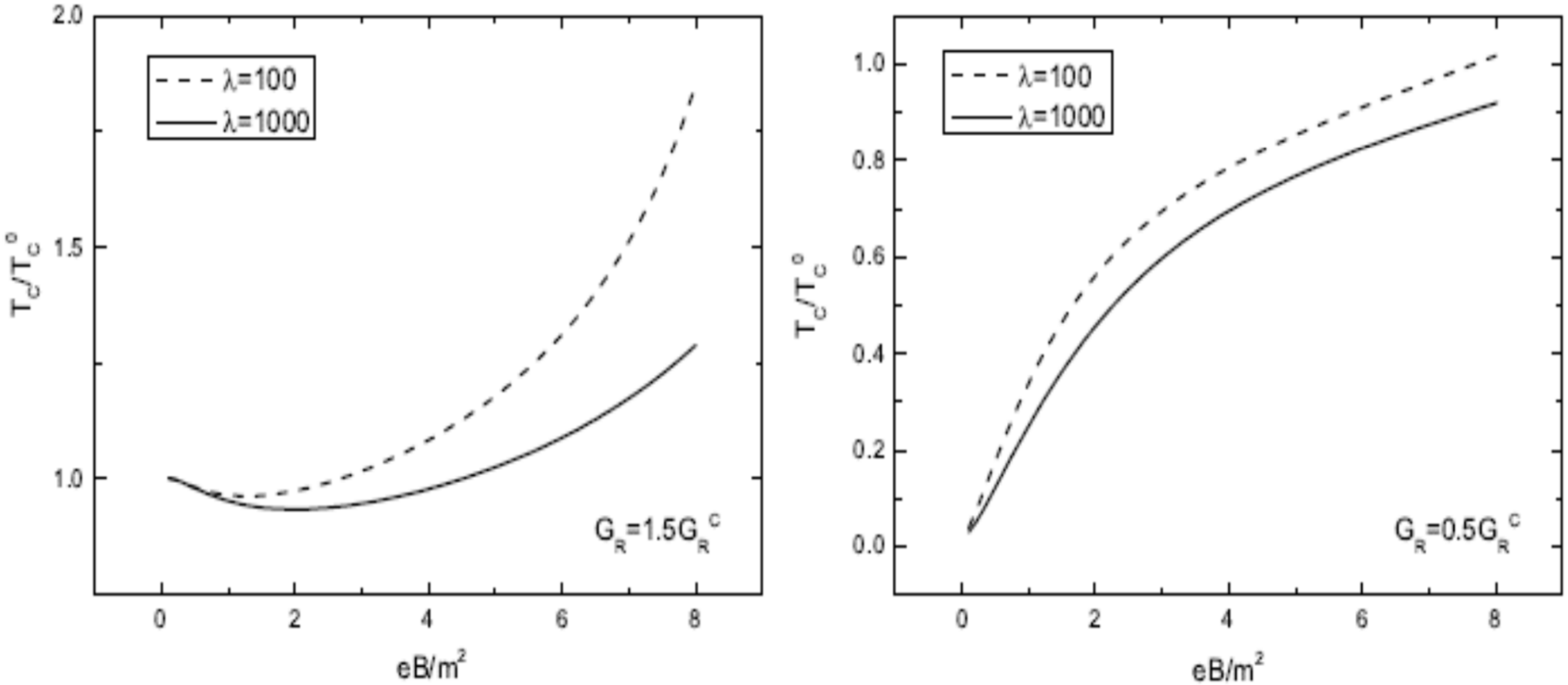
The ratio of BEC temperature tc versus b

Feng, Hou, Ren PRD 92(2015)

Chiral catalysis in weak coupling



Feng, Hou, Ren PRD 92(2015)



Condensation temperature versus the dimensionless magnetic field

Feng, Hou, Ren , Wu, _**PRD 93 (2016)085019**

FRG and phase structute

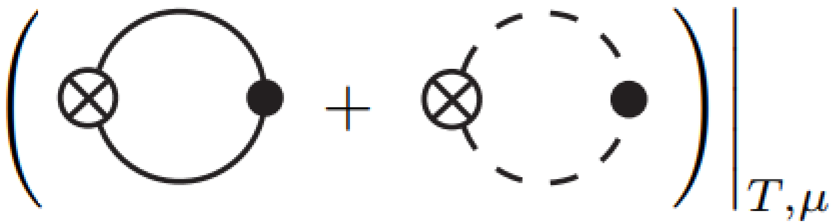
See Wei-Jie Fu’s talk

- For continuum field theory
- Non-perturbative
- (known) microscopic laws → complex macroscopic phenomena
- Flow from classical action S[φ] to effective action Γ[φ]
- Scale dependent effective action $\Gamma_k[\varphi]$

Wetterich, PLB301, 90 (1993).

$$\partial_k \Gamma_k = \frac{1}{2}\,\mathrm{Tr}\left[\frac{\partial_k R_k}{\Gamma_k^{(2)} + R_k}\right]$$

FRG Flow Equation



FRG study of phase structure at finite density

$$\mathcal{L} = \bar{\psi}\Big[i\gamma_\mu\partial^\mu - g_s(\sigma + i\gamma_5\,\boldsymbol{\tau}\cdot\boldsymbol{\pi}) - g_v\gamma_\mu\omega^\mu - \gamma_0\mu\Big]\psi$$
$$+ \tfrac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \tfrac{1}{2}\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi} - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}$$
$$- U(\sigma, \boldsymbol{\pi}, \omega)$$

$$F_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu \qquad \psi = (u, d)^T$$

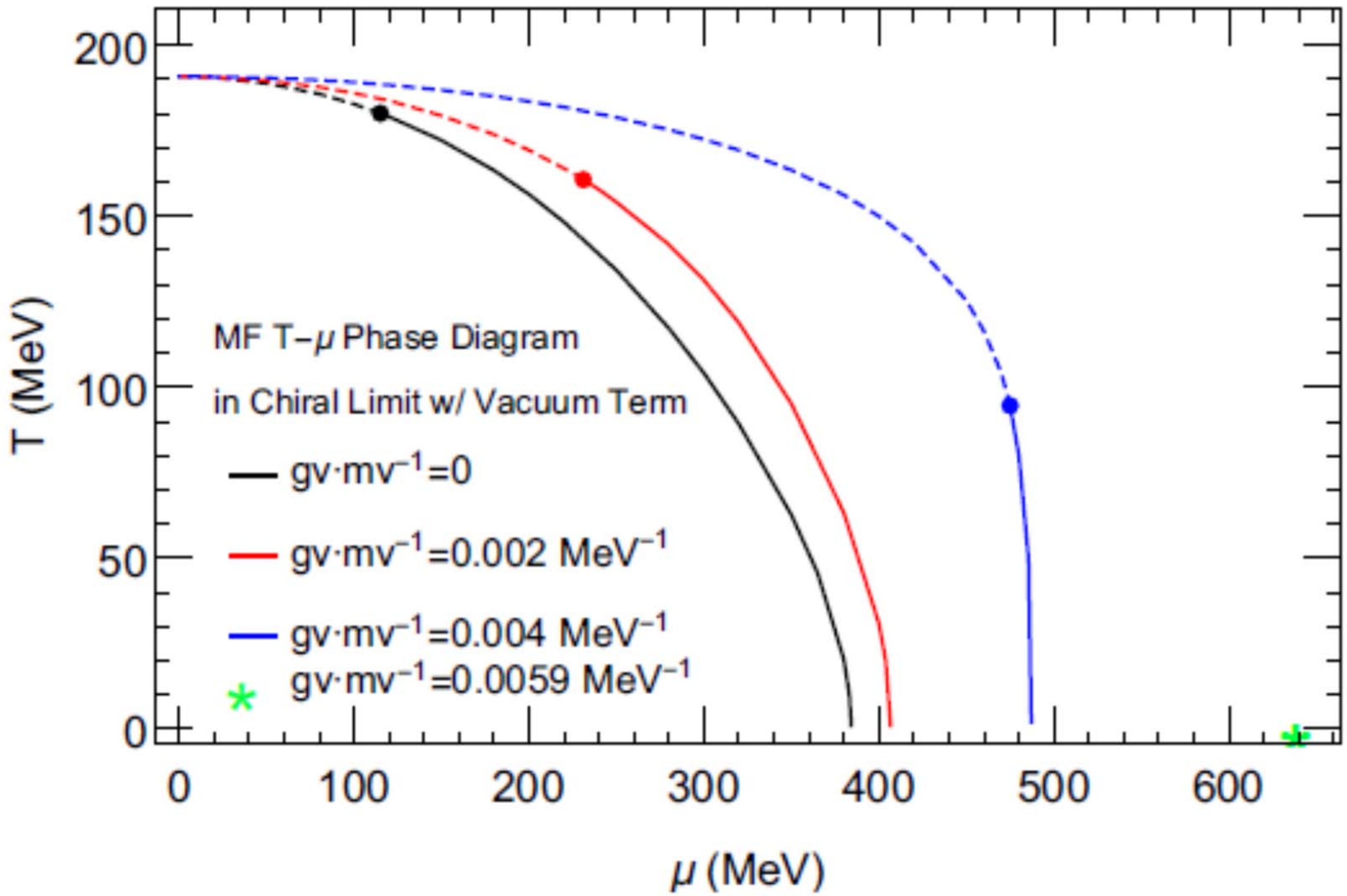
The potential for σ, π, and ω is

$$U(\sigma, \boldsymbol{\pi}, \omega) = \frac{\lambda}{4}(\sigma^2 + \boldsymbol{\pi}^2 - f_\pi^2)^2 - \frac{m_v^2}{2}\omega_\mu\omega^\mu,$$ For chiral limit

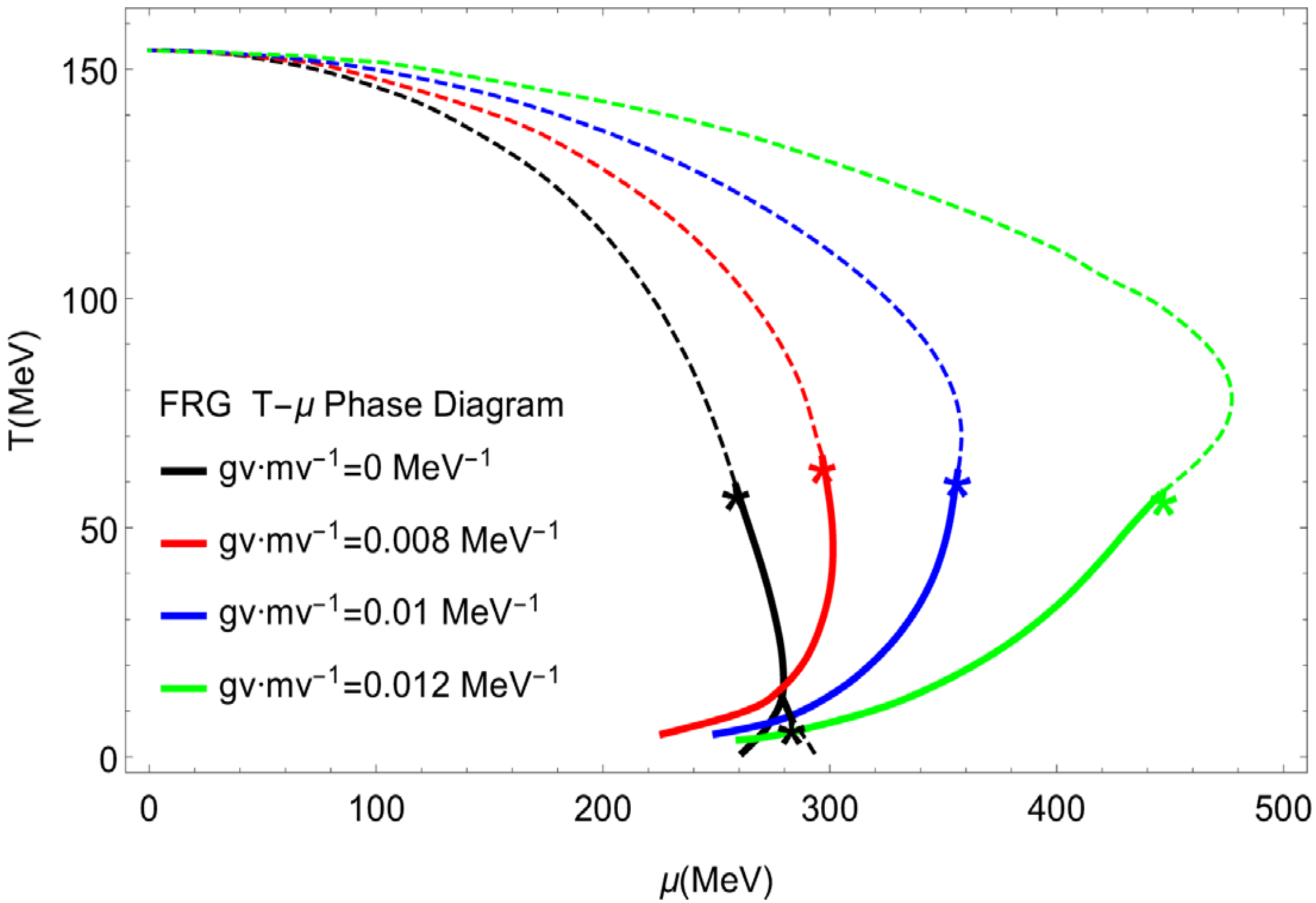
$$U(\sigma, \boldsymbol{\pi}, \omega) = \frac{\lambda}{4}(\sigma^2 + \boldsymbol{\pi}^2 - \nu^2)^2 + H\sigma - \frac{m_v^2}{2}\omega_\mu\omega^\mu,$$ For explicit SB

Phase structure

Zhang, Hou, Kojo , Qin Phys.Rev. D96,114029 (2017)



Zhang, Hou, Kojo , Qin Phys.Rev. D96,114029 (2017)



(i) At high T the fluctuations turn the 1st order line in the MF into 2nd order, yielding the TCP

(ii) While the critical μ of the TCP is sensitive to the vector coupling, its critical T is similar

Order parameter and baryon density

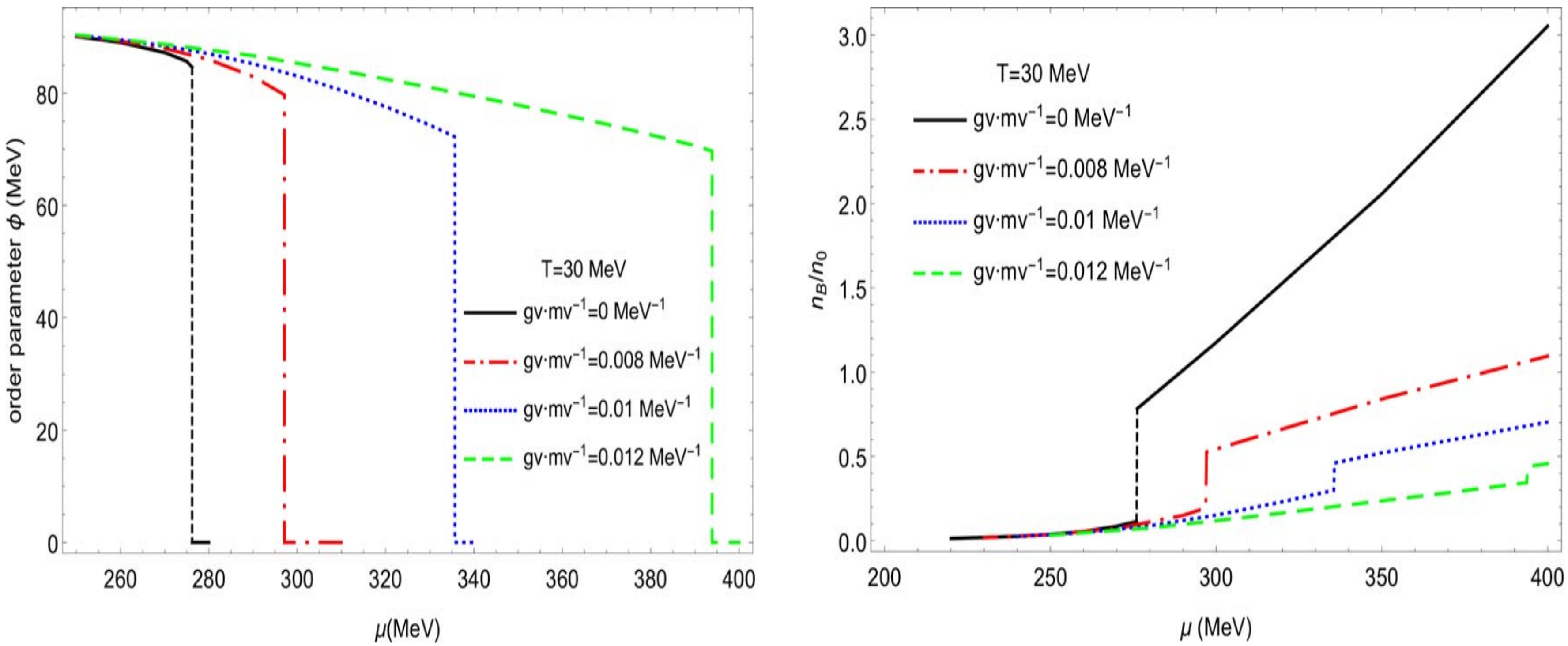


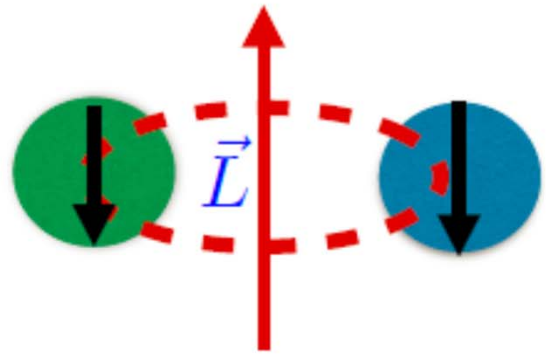
Figure: The μ-dependence of the baryon density considerably deviates from $\sim \mu^3$ behavior expected from the single particle contributions.
$n_B = n_B^{\mathrm{single}} + n_B^{\mathrm{fluct}}$

Rotation suppression of scalar pairing

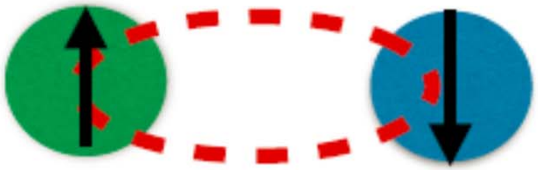
Let us consider pairing phenomenon in fermion systems.
There are many examples:
superconductivity, superfluidity, chiral condensate, diquark, ...

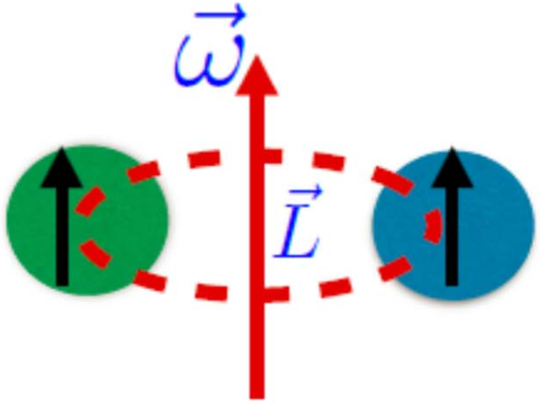
We consider scalar pairing state, with J=0.

$\vec{S} = \vec{s}_1 + \vec{s}_2 \qquad \vec{J} = \vec{L} + \vec{S}$



Rotation tends to polarize ALL angular momentum, both L and S, thus suppressing scalar pairing.





[Yin Jiang, JL, PRL2016;
See also: Chen, Fukushima, Huang, Mameda, arXiv:1512.08974]

Description of rotating system

Dirac Lagrangian in rotating frame:

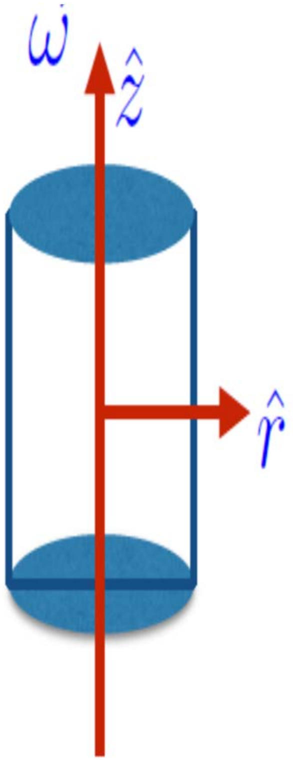
$$g_{\mu\nu} = \begin{pmatrix} 1-\vec{v}^2 & -v_1 & -v_2 & -v_3 \\ -v_1 & -1 & 0 & 0 \\ -v_2 & 0 & -1 & 0 \\ -v_3 & 0 & 0 & -1 \end{pmatrix}$$

$$\vec{v} = \vec{\omega} \times \vec{x}.$$

$$\bar{\gamma}^\mu = e_a{}^\mu \gamma^a$$

$$\Gamma_\mu = \frac{1}{4} \times \frac{1}{2}[\gamma^a, \gamma^b]\Gamma_{ab\mu}$$

$$\mathcal{L} = \bar{\psi}\left[i\bar{\gamma}^\mu(\partial_\mu + \Gamma_\mu) - m\right]\psi$$



Under slow rotation:

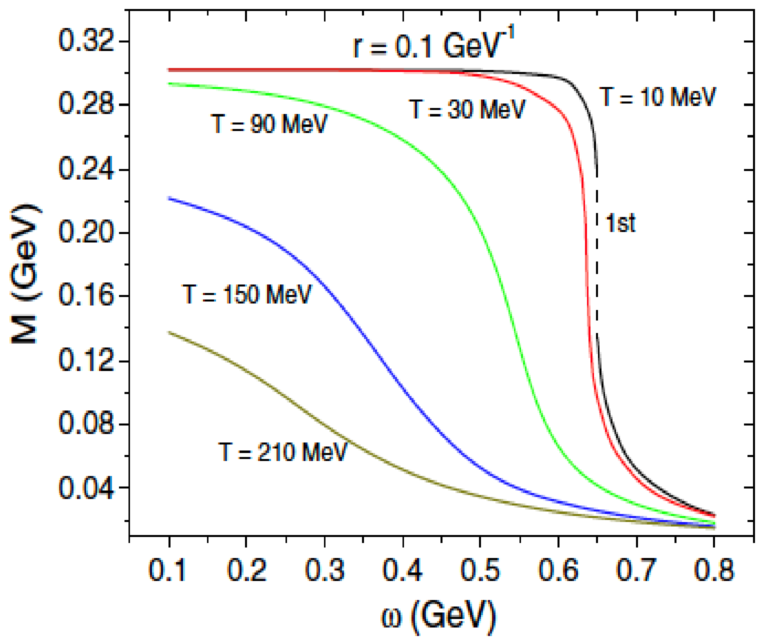
$$\mathcal{L} = \psi^\dagger \left[i\partial_0 + i\gamma^0\vec{\gamma}\cdot\vec{\partial} + (\vec{\omega}\times\vec{x})\cdot(-i\vec{\partial}) + \vec{\omega}\cdot\vec{S}_{4\times 4} \right]\psi$$

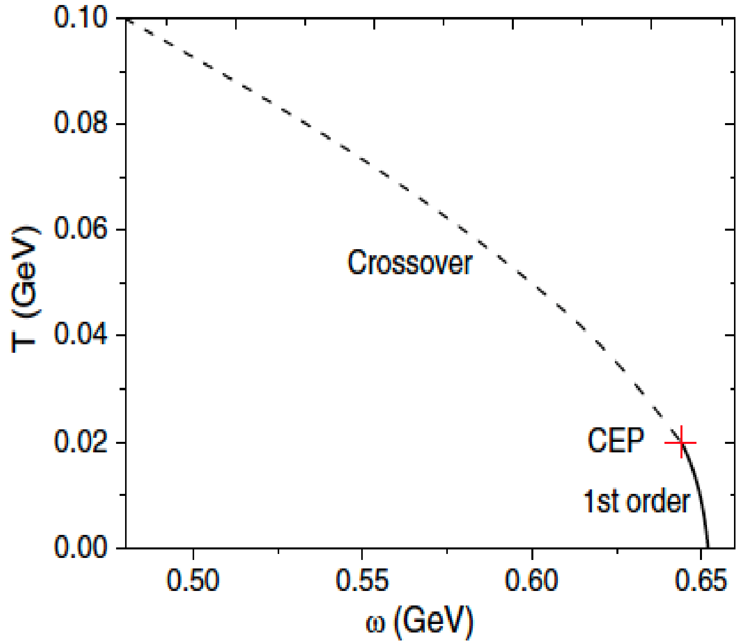
$$\hat{H} = \gamma^0(\vec{\gamma}\cdot\vec{p} + m) - \vec{\omega}\cdot(\vec{x}\times\vec{p} + \vec{S}_{4\times 4}) = \hat{H}_0 - \vec{\omega}\cdot\hat{\vec{J}}$$

Rotational polarization effect!

Phase structure under rotation

Jiang,Liao: PRL117(2016)192303





Mesonic superfluidity under rotation

$$\mathcal{L} = \bar{\psi}(i\gamma_\mu\partial^\mu - m_0 + \frac{\mu_I}{2}\gamma_0\tau_3)\psi \quad + G_s\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi\right)^2\right]$$
$$-G_v\left[\left(\bar{\psi}\gamma_\mu\boldsymbol{\tau}\psi\right)^2 + \left(\bar{\psi}\gamma_\mu\gamma_5\boldsymbol{\tau}\psi\right)^2\right]$$

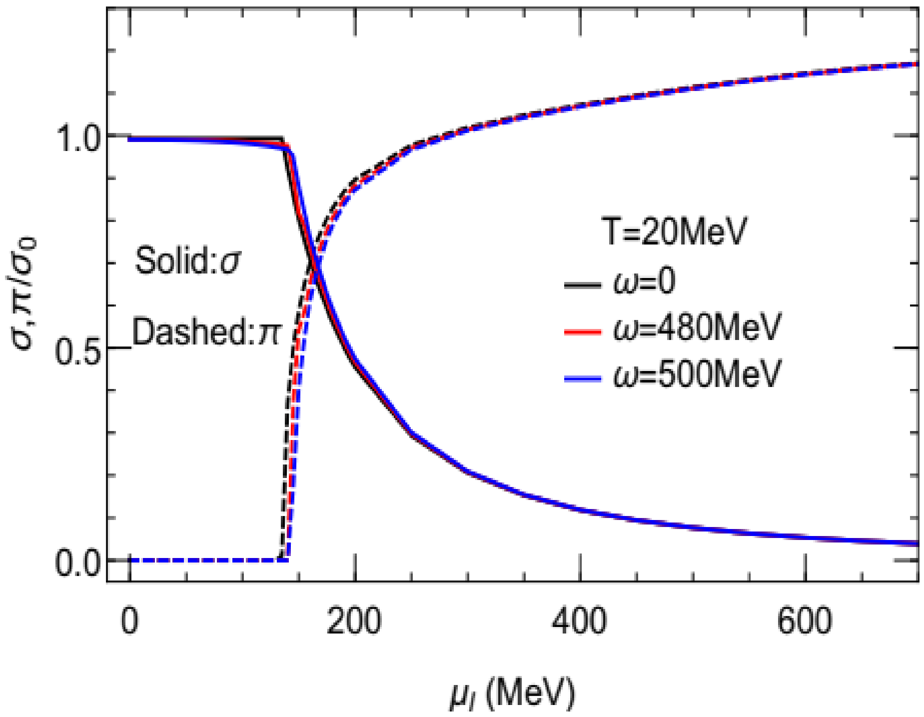
MF approximation: $\sigma = \langle\bar{\psi}\psi\rangle,\ \pi = \langle\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi\rangle,\ \rho = \langle\bar{\psi}i\gamma_0\gamma_5\tau_3\psi\rangle$

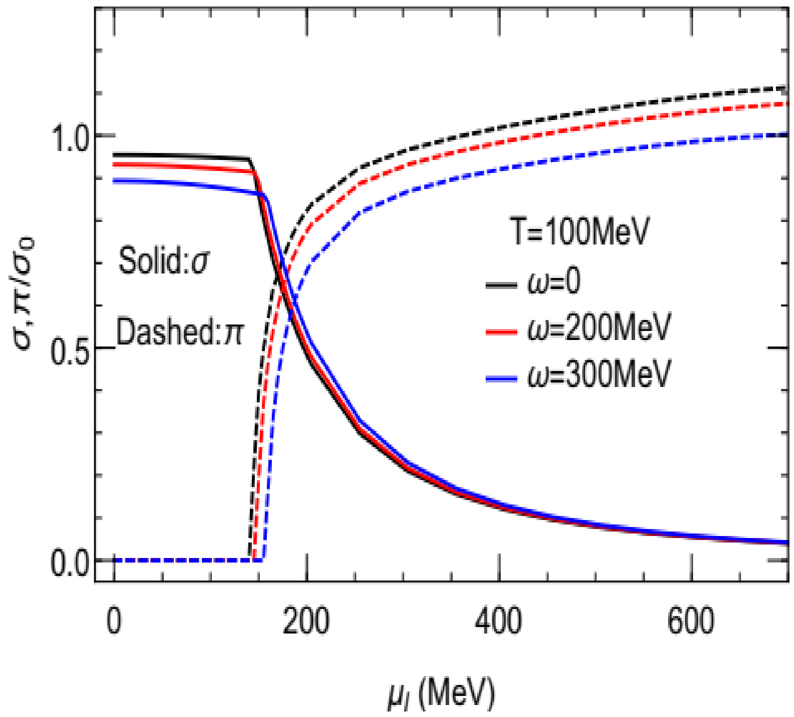
$$\begin{aligned}\Omega &= G(\sigma^2+\pi^2) - G\rho^2 - \frac{\mathrm{N_c N_f}}{16\pi^2}\sum_n \int \mathrm{d}k_t^2 \int \mathrm{d}k_z [J_{n+1}(k_t r)^2 + J_n(k_t r)^2] \\ &\times T\Big[\ln\Big(1+\exp(-\frac{\omega^+ - (n+\frac{1}{2})\omega}{T})\Big) + \ln\Big(1+\exp(\frac{\omega^+ - (n+\frac{1}{2})\omega}{T})\Big) \\ &\quad + \ln\Big(1+\exp(-\frac{\omega^- - (n+\frac{1}{2})\omega}{T})\Big) + \ln\Big(1+\exp(\frac{\omega^- - (n+\frac{1}{2})\omega}{T})\Big)\Big]\end{aligned}$$

Rotational suppression of Pion superfluid

Rotation weaken spin 0 condensate, inverse catalysis effect

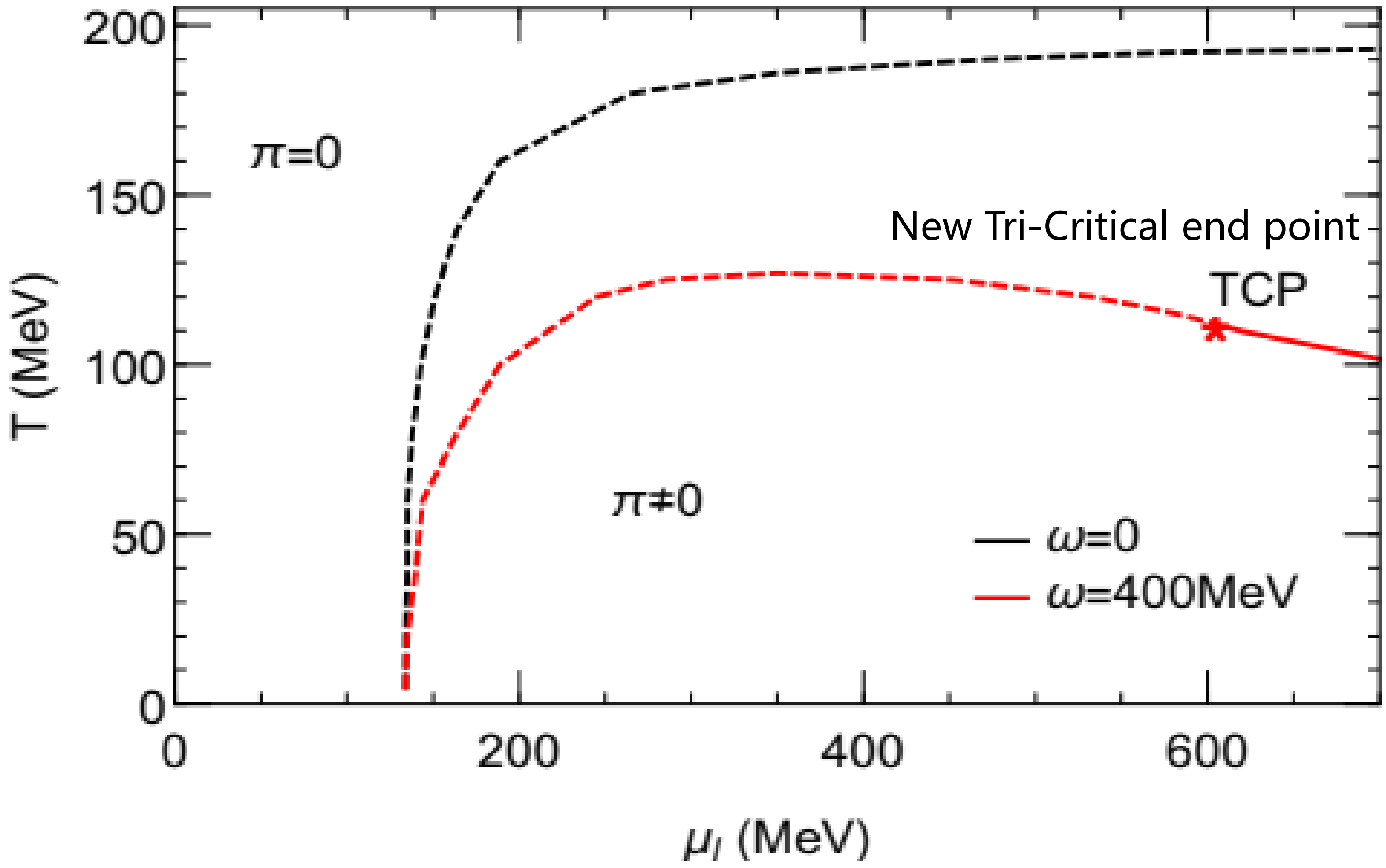
H. Zhang, DF Hou, JF Liao, arxiv 1812.11787





He, M. Jin and P. Zhuang, Phys. Rev. D 71,116001 (2005);
L. He and P. Zhuang, Phys. Lett. B 615, 93 (2005)

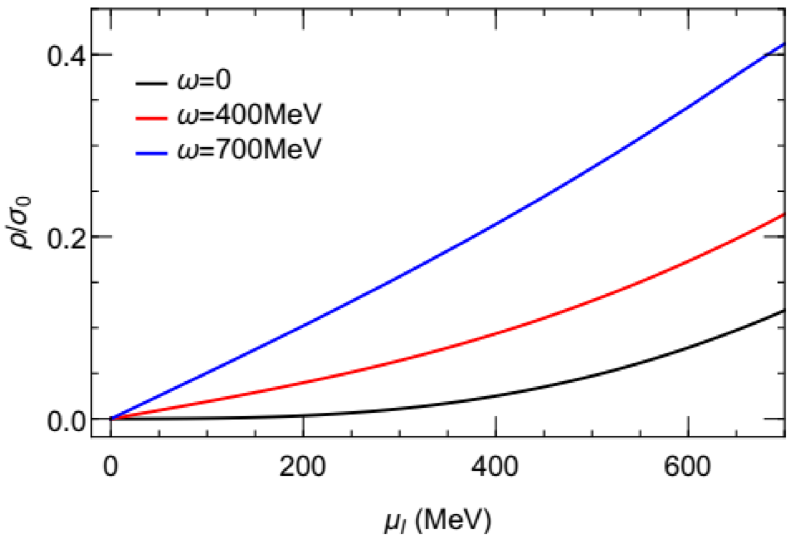
Pion superfluidity phase diagram in T-μI

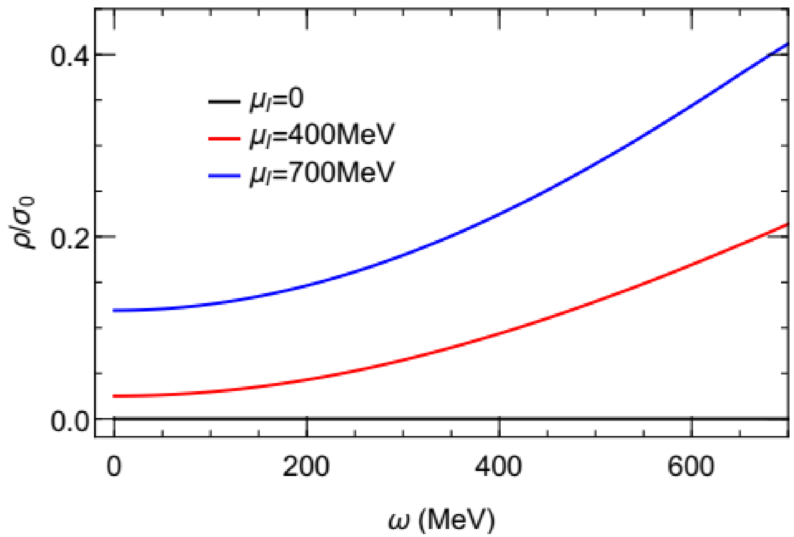


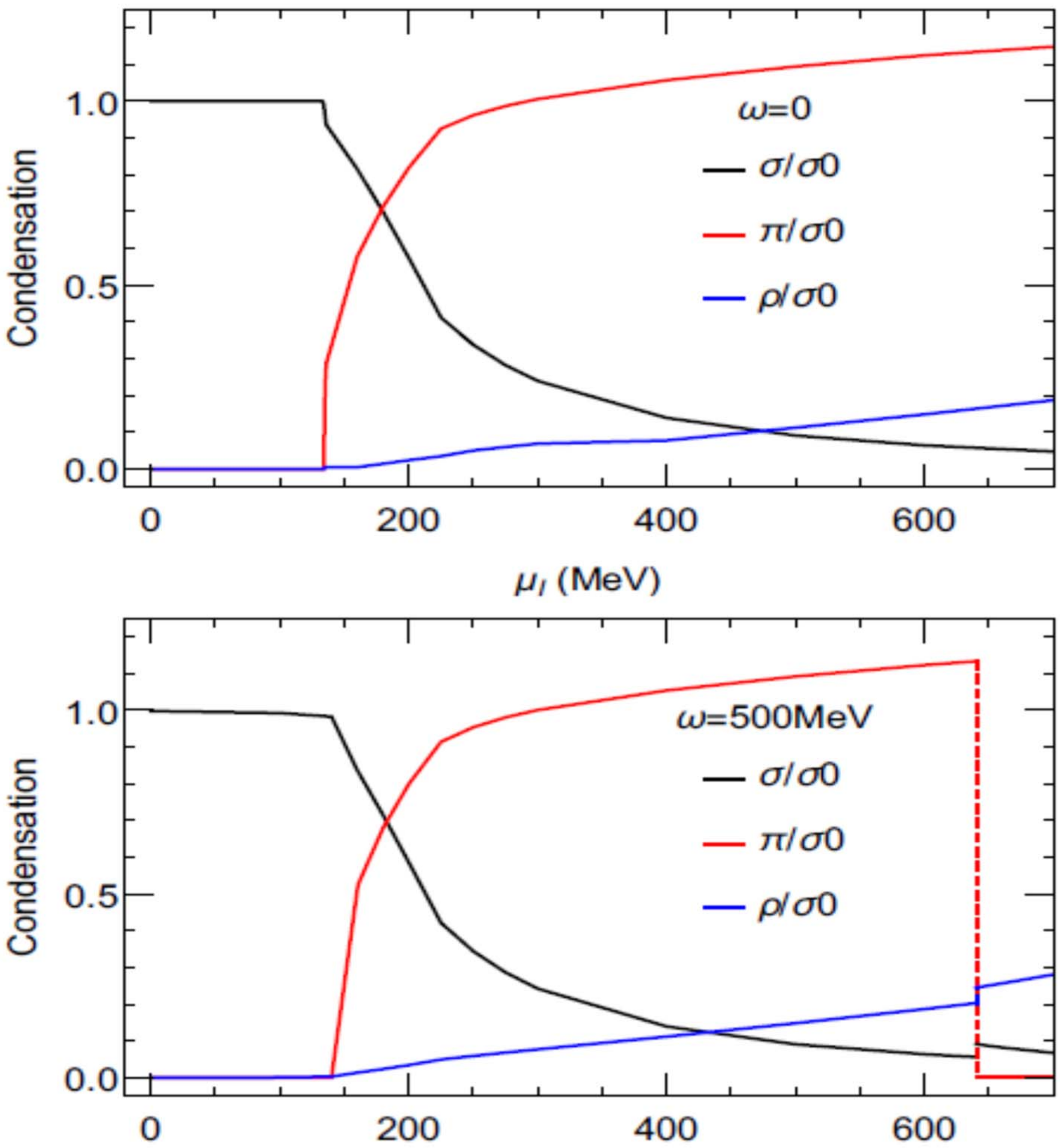
Enhenced Rho Superfluid under rotation

Rotation enhances spin 1 condensate ρ channel

Rho condensate at T=mu=0 with none zero isospin chemical potential under rotation

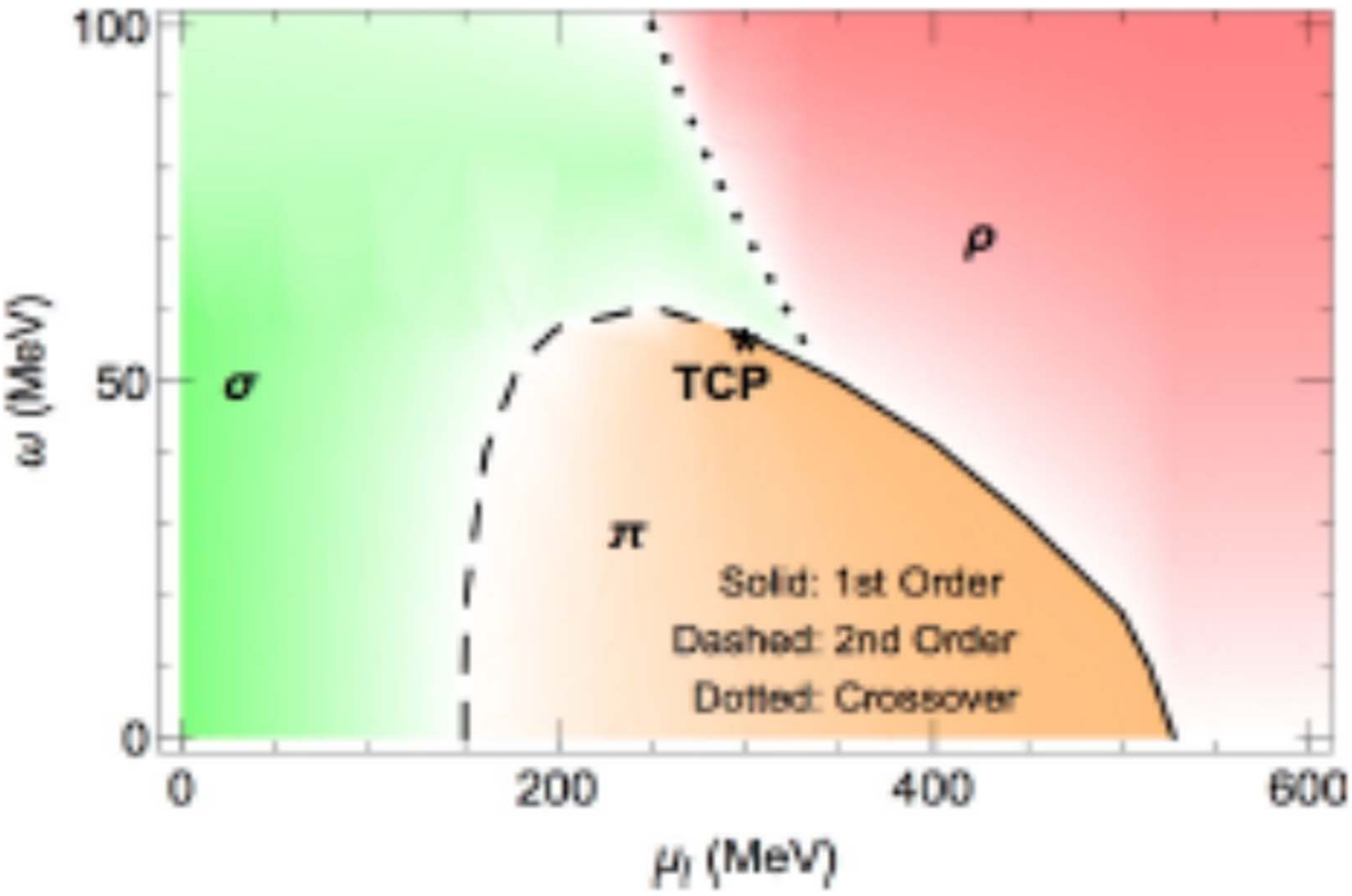






Condensation
1.0
0.5
0.0
ω=0
σ/σ0
π/σ0
ρ/σ0
0
200
400
600
μ_I (MeV)
Condensation
ω=500MeV
σ/σ0
π/σ0
ρ/σ0

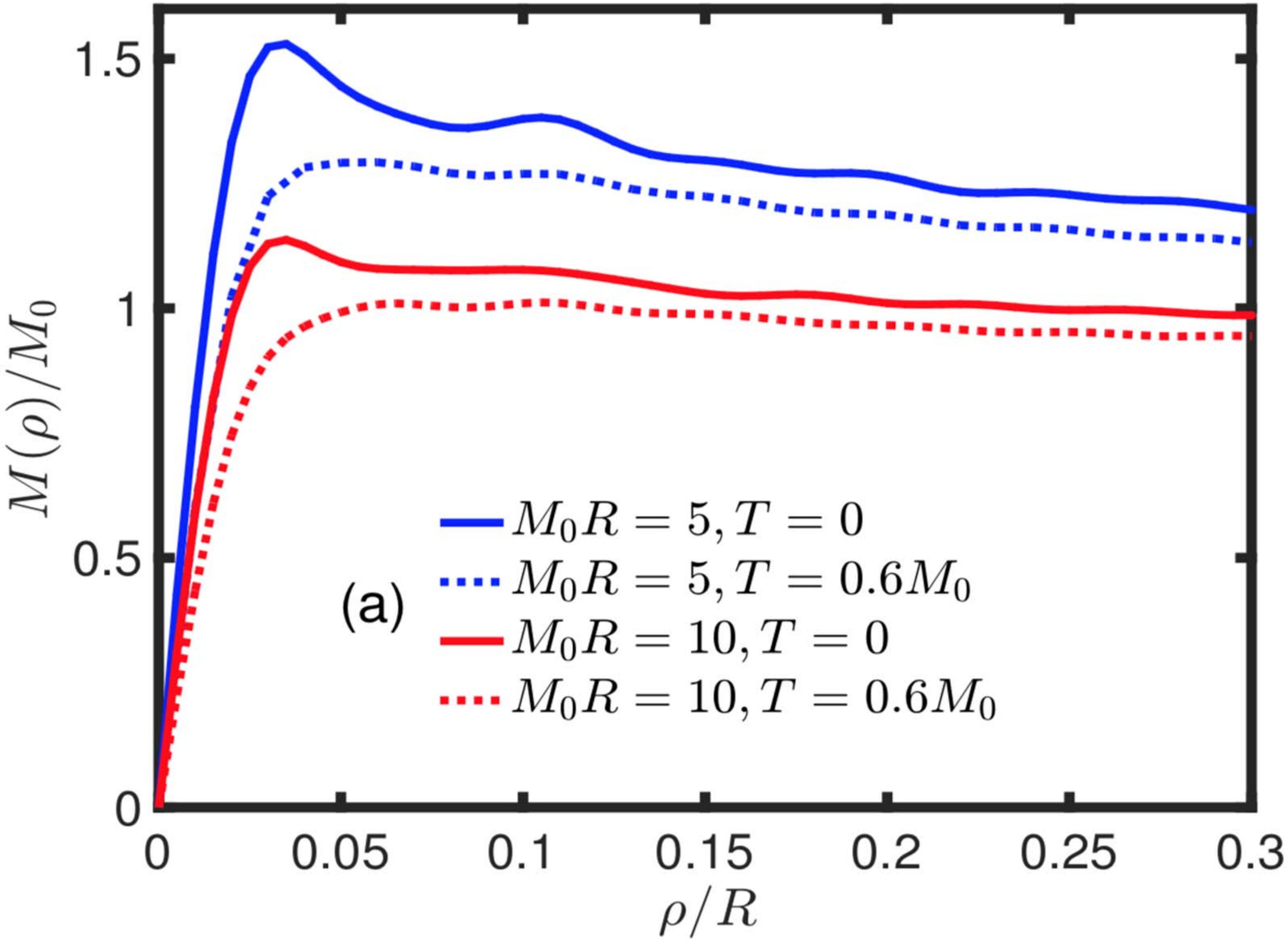
New mesonic superfluid phase diagram



$\Gamma = 10\text{MeV}$ and baryon chemical potential is $\mu = 250\text{MeV}$.

H. Zhang, DF Hou, JF Liao, arxiv 1812.11787

Chiral vortex state



L.Y. He et al arXiv : 1901.04697

Summary and outlook

- QCD matter has very rich phase structures with B and rotation

- Fluctuations are important for TCP of QCD, FRG provides an useful tool to study

- Magnetic field has nontrivial effects on phase structure (Magnetic Catalyse & inverse Magetic catalyse)

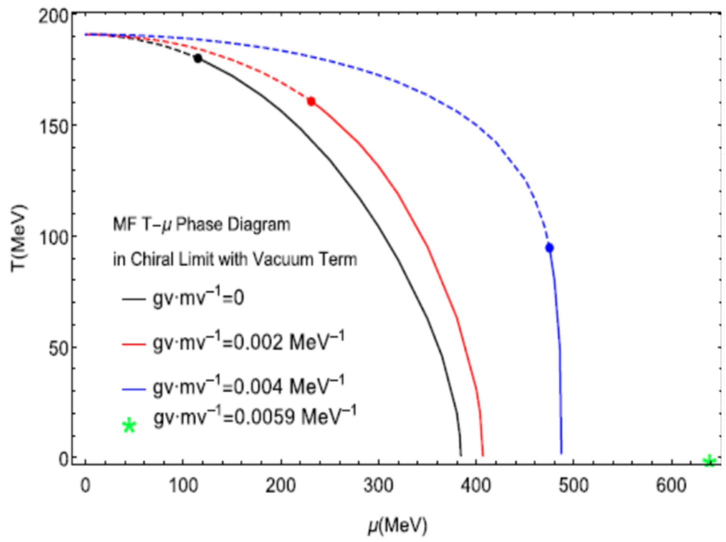
- Rotation suppresses spin 0 condensate , enhances nonzero spin ones due to the rotation polarization

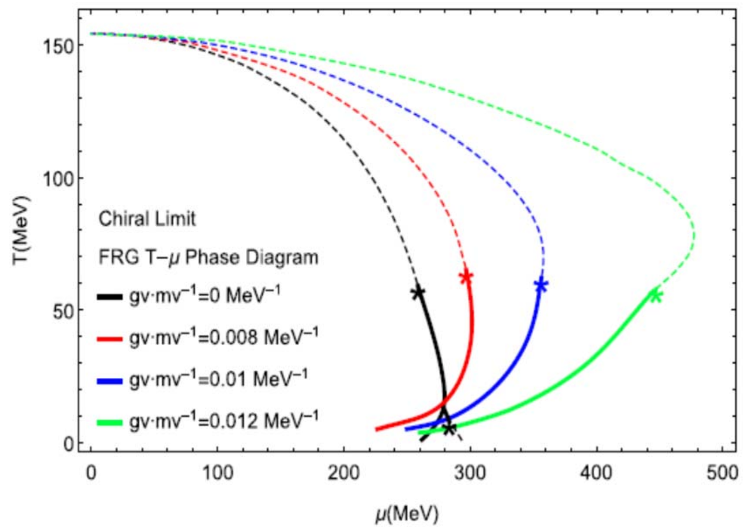
- A new phase diagram for isospin matter under rotation with a new TCP due to the interplay between rotation and isospin chemical potential

Thank you very much for your attention!

FRGE study of phase diagram: Flucts on CEP

Zhang, Hou , Kojo, Qin, PRD96 (2017)



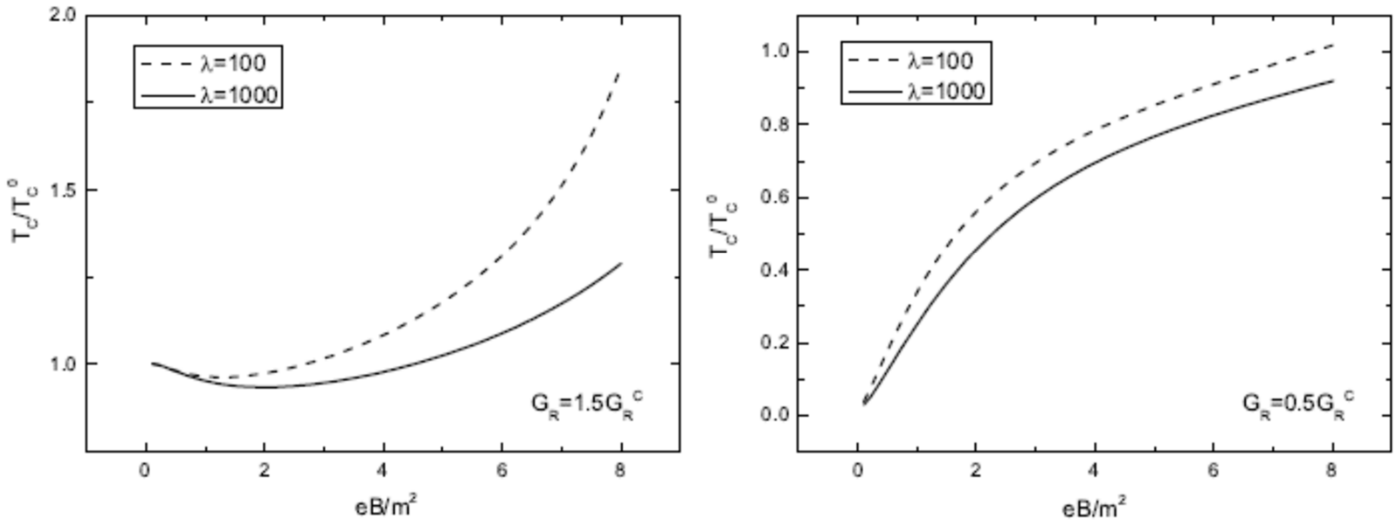


Nematic Isotropic (NI) Puzzle with FRGE

Qin, Hou, Huang, Zhang, PRB98, (2018)

Magnetic (Inverse)chiral catalysis at weak (strong) coupling

Feng, Hou, Ren , Wu, _**PRD 93 (2016)085019**

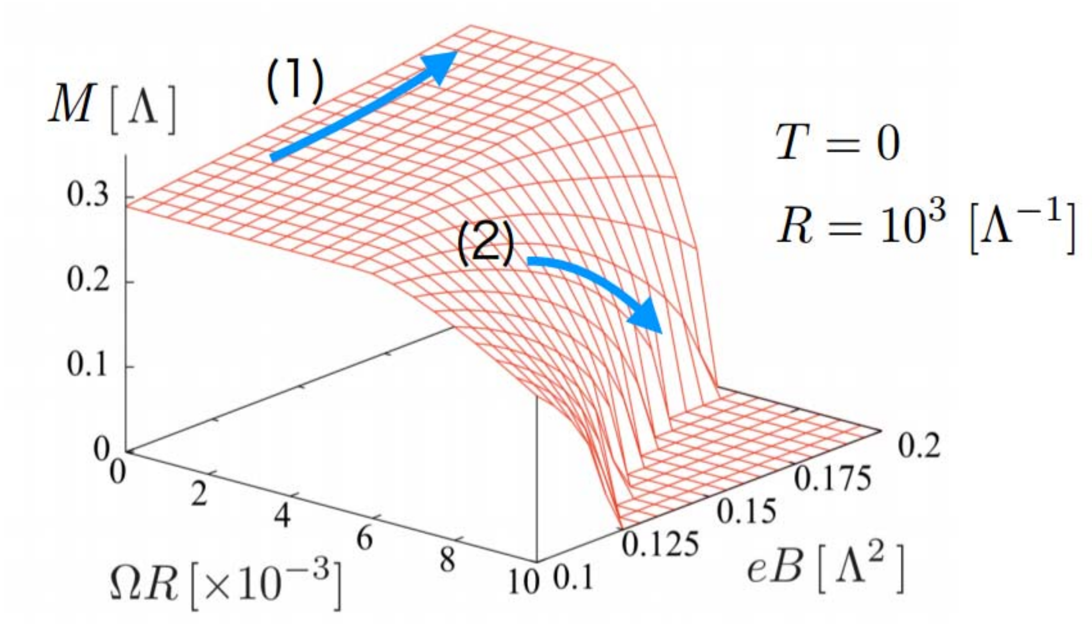


Condensation temperature versus the dimensionless magnetic field

Summary for BEC under B

- We point out that magnetic field has two effects: dimension reduction and enhancement of fluctuations.
- We elaborate this mechnism via a simpe example: BEC of neutral composite bosons.
- We find that in NR case the fluctuations play a signifcant role and inverse magnetic catalysis arises in strong coupling domain.
- In relativistic case, the fluctuations are *NOT* as signifcant as that in NR. The inverse magnetic catalysis found in Lattice QCD may due to the complex in the dynamics in QCD.

Dirac fermion in rotation and B field



(1) eB increases → M increases Magnetic Catalysis

(2) eB increases → M decreases Inverse of MC

'Rotational magnetic inhibition'

Chen-Fukushima-XGH-Mameda PRD2016
See also: Jiang-Liao 2016; Chernodub-Gongyo 2016; Liu-Zahed 2017; Chen-Fukushima-XGH-Mameda 2017; Wang-Wei-Li-Huang 2019