Thrust distribution in Higgs decay

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Overview

Introduction

- Higgs discovery in 2012, the Higgs characteristic detections
- Higgs Factory
  1. Future leptonic colliders, such as ILC [1], CEPC [2], CLIC [3] and FCC-*ee* [4].
  2. CEPC: $H \to b\bar{b}$ Estimated precision 0.56% [2]
     $H \to gg$ Estimated precision 1.4% [2]
     $\cdots$
  3. The opportunities of precise measurements.

- Requirements for the accurate theoretical calculations

Introduction

- Theoretical progress in the recent years
  1. Partial width for $H \to b\bar{b}$ — N^{4}LO (no m_b)[5]
  2. Partial width for $H \to gg$ — N^{3}LO (heavy m_t limit) [6]
  3. Fully differential results $H \to b\bar{b}$ — NNLO (no m_b)[9, 1]
     NNLO (with m_b) [2]

     ...

     ...
- Event shapes of the Higgs decay
  1. IRC finite   perturbative calculation
  2. Improve the measurement of $H \to gg$ and $H \to q\bar{q}$ couplings [3, 4].

Fixed-order Calculation: NLO and beyond

- The definition of the thrust τ

$$\tau \equiv \min_{\vec{n}} \left[1 - \frac{\sum_i |\vec{n} \cdot \vec{p}_i|}{\sum_i |\vec{p}_i|} \right], \tag{1}$$

where $\vec{p}_i$ runs over the 3-momenta of the final state particles, and $\vec{n}$ is a 3-vector with unit norm.

The limit $\tau \to 0$ manifests the final-state configuration of two back-to-back jets, and the limit $\tau \to 1/2$ corresponds to a nearly isotropic event.

Fixed-order Calculation: NLO and beyond

- Effective Lagrangian after integrating out the top quark

$$\begin{aligned}\mathcal{L}_{\text{eff}} &= \frac{\alpha_s(\mu)C_t(m_t,\mu)}{12\pi v}O_g + \sum_q \frac{y_q(\mu)}{\sqrt{2}}O_q \\ &\equiv \frac{\alpha_s(\mu)C_t(m_t,\mu)}{12\pi v}HG^{\mu\nu,a}G^a_{\mu\nu} + \sum_q \frac{y_q(\mu)}{\sqrt{2}}H\bar{\psi}_q\psi_q,\end{aligned} \tag{2}$$

where μ is the renormalization scale, v is the vacuum expectation value of the Higgs field.

$C_t(m_t,\mu)$ stands for the Wilson coefficient from integrating out the top quark, while $y_q(\mu)$ denotes the Yukawa coupling.

Fixed-order Calculation: NLO and beyond

- The approximation of vanishing light quark masses, i.e., $\tau \gg m_q^2/m_H^2$
  1. The chirality conservation of QCD interactions forbids the interference between O_g and O_q up to orders in the strong coupling α_s, like

$$\langle 0|G^{\mu\nu,a}G^a_{\mu\nu}|X\rangle \langle X|\bar{\psi}_q\psi_q|0\rangle \to 0. \tag{3}$$

  2. The mixture of O_g and O_q vanishes in renormalization process. Therefore, their Wilson coefficients would evolve independently under RGE, namely,

$$\begin{aligned} \frac{d}{d\ln\mu} C_t(m_t,\mu) &= \gamma^t(\alpha_s(\mu))\, C_t(m_t,\mu), \\ \frac{d}{d\ln\mu} y_q(\mu) &= \gamma^y(\alpha_s(\mu))\, y_q(\mu). \end{aligned} \tag{4}$$

Fixed-order Calculation: NLO and beyond

- The straightforward LO Calculation

$$\frac{1}{\Gamma_0^q}\frac{d\Gamma_{\rm LO}^q}{d\tau} = \frac{y_q^2(\mu)}{y_q^2(m_H)}\frac{\alpha_s(\mu)}{2\pi}C_F\frac{1}{\tau(\tau-1)}\left[3(1-3\tau)(1-\tau)^2-2\left(2-3\tau+3\tau^2\right)\ln\frac{1-2\tau}{\tau}\right],$$

$$\frac{1}{\Gamma_0^g}\frac{d\Gamma_{\rm LO}^g}{d\tau} = \frac{\alpha_s^2(\mu)}{\alpha_s^2(m_H)}\frac{\alpha_s(\mu)}{2\pi}\left\{C_A\frac{1}{3\tau(\tau-1)}\left[(1-3\tau)(1-\tau)(11-24\tau+15\tau^2)\right.\right.$$
$$\left.-12\left(1-\tau+\tau^2\right)^2\ln\frac{1-2\tau}{\tau}\right]$$
$$\left.+T_F n_f\frac{2}{3\tau}\left[(1-3\tau)(2-15\tau+15\tau^2)+6\tau\left(1-2\tau+2\tau^2\right)\ln\frac{1-2\tau}{\tau}\right]\right\}, \quad (5)$$

where $\Gamma_0^i(i=q,g)$ stands for the LO decay widths.

Fixed-order Calculation: NLO and beyond

- The NLO calculation and numerical method
  1. The dipole-subtraction method

$$\begin{aligned}\Gamma^i_{\mathrm{V+R}} &= \int_{n+1} d\Gamma^i_{\mathrm{real}} + \int_n d\Gamma^i_{\mathrm{virt}} \\ &= \int_{n+1} \left(d\Gamma^i_{\mathrm{real}} - d\Gamma^i_A\right) + \int_n \left(d\Gamma^i_{\mathrm{virt}} + \int_1 d\Gamma^i_A\right),\end{aligned} \tag{6}$$

  2. The Monte-Carlo integration

Fixed-order Calculation: NLO and beyond

Numerical NLO results

The significant NLO corrections, considerable scale uncertainties.

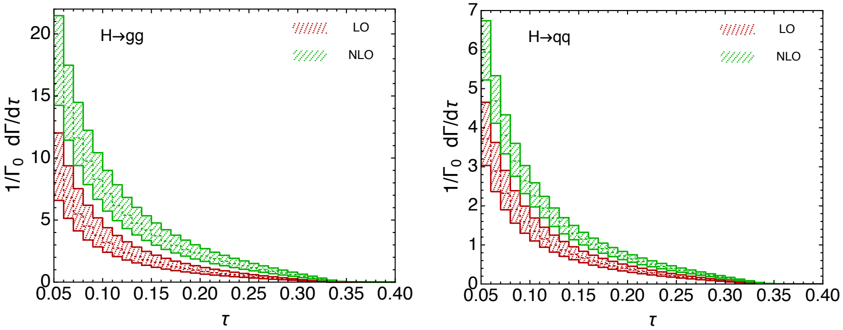

Figure: Thrust distributions at LO and NLO in the Hgg (left plot) and $Hq\bar{q}$ (right plot) channels.

Fixed-order Calculation: NLO and beyond

- Factorization formula at small τ within SCET [6, 7, 8, 9, 11, 12]

$$\frac{d\Gamma^i}{d\tau} = \Gamma_0^i(\mu)\,|C_t^i(m_t,\mu)|^2\,|C_S^i(m_H,\mu)|^2 \int dp_n^2\, dp_{\bar{n}}^2\, dk \times \delta\left(\tau - \frac{p_n^2 + p_{\bar{n}}^2}{m_H^2} - \frac{k}{m_H}\right) J_n^i(p_n^2,\mu)\, J_{\bar{n}}^i(p_{\bar{n}}^2,\mu)\, S^i(k,\mu)\,, \tag{7}$$

where $i = q, g$ denote the $Hq\bar{q}$ and Hgg channels, respectively.

1. Comprising the most singular terms, like $1/\tau$ and $\log^n \tau/\tau (n \geq 1)$.
2. Hard functions $C_S^{q,g}$ extracted from the $Hq\bar{q}(gg)$ form factors.
3. Jet functions $J_n^i(p_n^2,\mu)$ at N^{3}LO [13, 14, 15, 16].
4. Soft functions $S^i(k,\mu)$ at N^{3}LO (numeric uncertainties) [15, 17, 18, 19, 20].

Numerical LO results

Agreements in the small τ area

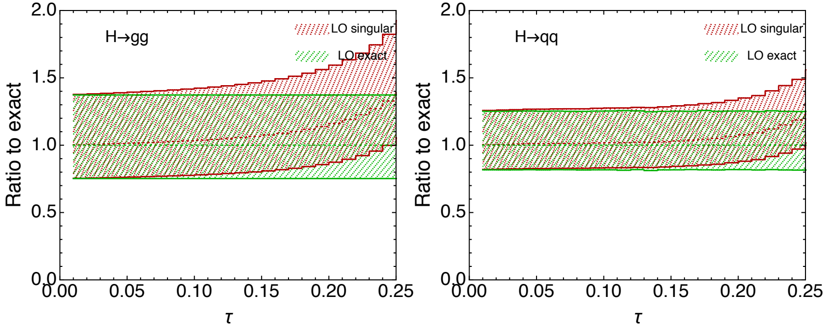

Figure: Comparison between the exact results and the singular terms at LO.

Fixed-order Calculation: NLO and beyond

Numerical NLO results

Agreements in the small τ area

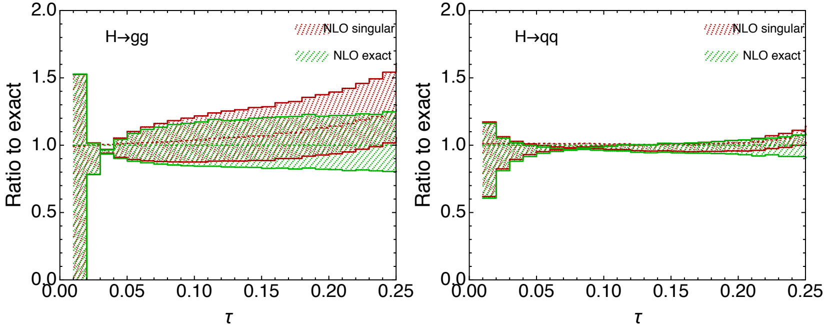

Figure: Comparison between the exact results and the singular terms at NLO.

Approximate NNLO results

1) Still large corrections, 2) Scale variations are relatively small.

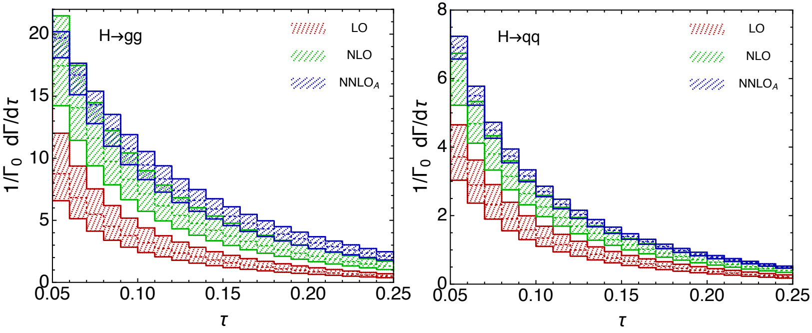

Figure: Thrust distributions at LO, NLO and approximate NNLO (NNLO$_A$).

Approximate NNLO results in the bin $\tau \in [0.1, 0.2]$

Scale variations are relatively small

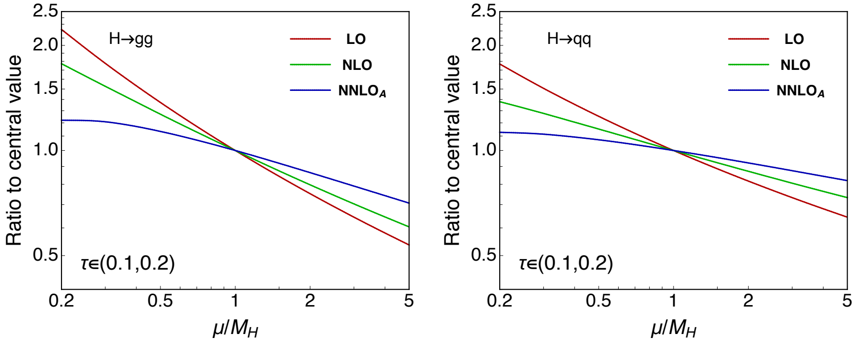

Figure: The ratios of the integrated cross sections in the bin $\tau \in [0.1, 0.2]$ to their central values at $\mu = m_H$, as a function of μ/m_H.

Resummation

- Factorization formula in $\tau \to 0$

$$\frac{d\Gamma^i}{d\tau} = \Gamma_0^i(\mu)\,|C_t^i(m_t,\mu)|^2\,|C_S^i(m_H,\mu)|^2 \int dp_n^2\,dp_{\bar{n}}^2\,dk \times \delta\left(\tau - \frac{p_n^2+p_{\bar{n}}^2}{m_H^2} - \frac{k}{m_H}\right) J_n^i(p_n^2,\mu)\,J_{\bar{n}}^i(p_{\bar{n}}^2,\mu)\,S^i(k,\mu)\,, \tag{8}$$

1. Straightforward expansion results in the singular terms like $1/\tau$, $\log\tau/\tau$, ..., decreasing the perturbativity.
2. Evolving the typical scales to one common scale exploiting RGEs.

Resummation

- Evolution in RGEs

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}\ln\mu} C_t(m_t^2,\mu^2) &= \gamma_t(\alpha_s) C_t(\mu^2), \\
\frac{\mathrm{d}}{\mathrm{d}\ln\mu} C_S^{q,g}(-M_h^2 - i\epsilon,\mu^2) &= \left[C_{F,A}\Gamma_{\mathrm{cusp}}(\alpha_s)\ln\frac{-M_h^2 - i\epsilon}{\mu^2} + \gamma_H^{q,g}(\alpha_s)\right] C_S^{q,g}(-M_h^2 - i\epsilon,\mu^2), \\
\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\tilde{j}_{q,g}(\ln\frac{Q^2}{\mu^2},\mu^2) &= \left[-2C_{F,A}\Gamma_{\mathrm{cusp}}(\alpha_s)\ln\frac{Q^2}{\mu^2} - 2\gamma_j^{q,g}(\alpha_s)\right]\tilde{j}_{q,g}(\ln\frac{Q^2}{\mu^2},\mu^2), \\
\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\tilde{s}_{q,g}(\ln\frac{Q}{\mu},\mu^2) &= \left[4C_{F,A}\Gamma_{\mathrm{cusp}}(\alpha_s)\ln\frac{Q}{\mu} - 2\gamma_s^{q,g}(\alpha_s)\right]\tilde{s}_{q,g}(\ln\frac{Q}{\mu},\mu^2),
\end{aligned} \tag{9}$$

where $\tilde{j}_{q,g}$ and $\tilde{s}_{q,g}$ are the Laplace-transformed jet and soft functions, Γ_{cusp} and $\gamma_i^{q.g}$ are the abnormal dimensions, respectively.

Resummation

- Plugging the solutions of these RGEs into the factorization formula in Eq, (8), we achieve the resummation,

$$
\begin{aligned}
\frac{\mathrm{d}\Gamma_q^{\mathrm{resum}}}{\mathrm{d}\tau} =& \Gamma_B^q U_q\left(\mu_m,\mu_h,\mu_j,\mu_s\right)|C_S^q(M_h,\mu_h)|^2 \int \mathrm{d}M_n^2 \mathrm{d}M_{\bar{n}}^2 \mathrm{d}k \\
& \times \left[\tilde{j}_q\left(\ln\frac{\mu_s M_h}{\mu_j^2}+\partial_{\eta_q},\mu_j\right)\right]^2 \tilde{s}_q\left(\partial_{\eta_q},\mu_s\right)\left[\frac{1}{\tau\Gamma(\eta_q)}\left(\frac{\tau M_h}{\mu_s e^{\gamma_E}}\right)^{\eta_q}\right], \\
\frac{\mathrm{d}\Gamma_g^{\mathrm{resum}}}{\mathrm{d}\tau} =& \Gamma_B^g U_g\left(\mu_t,\mu_h,\mu_j,\mu_s\right)|C_t(m_t,\mu_t)|^2|C_S^g(M_h,\mu_h)|^2 \int \mathrm{d}M_n^2 \mathrm{d}M_{\bar{n}}^2 \mathrm{d}k \\
& \times \left[\tilde{j}_g\left(\ln\frac{\mu_s M_h}{\mu_j^2}+\partial_{\eta_g},\mu_j\right)\right]^2 \tilde{s}_g\left(\partial_{\eta_g},\mu_s\right)\left[\frac{1}{\tau\Gamma(\eta_g)}\left(\frac{\tau M_h}{\mu_s e^{\gamma_E}}\right)^{\eta_g}\right],
\end{aligned}
\tag{10}
$$

where $U_{q,g}\left(\mu_m,\mu_h,\mu_j,\mu_s\right)$ and $\eta_{q,g}$ represent the evolution functions.

- Matching condition

$$\frac{\mathrm{d}\Gamma^{\mathrm{matched}}_{q,g}}{\mathrm{d}\tau} = \frac{\mathrm{d}\Gamma^{\mathrm{resum}}_{q,g}}{\mathrm{d}\tau} - \left(\frac{\mathrm{d}\Gamma^{\mathrm{app}}_{q,g}}{\mathrm{d}\tau} - \frac{\mathrm{d}\Gamma^{\mathrm{FO}}_{q,g}}{\mathrm{d}\tau}\right), \tag{11}$$

- Definition of orders in resummation

logarithmic accuracy	$\Gamma_{\mathrm{cusp}}, \beta$	$\gamma_{t,m,H,j,s}$	$C_t, C_S, \tilde{j}, \tilde{s}$	fixed-order matching
NLL	2-loop	1-loop	tree	
N^{2}LL	3-loop	2-loop	1-loop	LO
N^{3}LL	4-loop	3-loop	2-loop	NLO

Resummation

- Discussions on the resummed thrust distributions in Eq. (10)
  1. Resummed thrust distributions involve scales like $\mu_{m,t},\ \mu_h,\ \mu_j,\ \mu_s$
  2. These scales induce the typical logarithms in C_t, $C_S^{q,g}$, $\tilde{j}_{q,g}$ and $\tilde{s}_{q,g}$, for instance, $\log\left[(-M_h^2 - i\epsilon)/\mu^2\right]$ in $C_S^{q,g}$ ...
  3. Proper choice of these scales enhances perturbativity for the results.
  4. In this work, we would determine these scales in the Laplace space.

The choice of Scales in the Laplace space

- The Laplace-transformed thrust distributions,

$$\begin{aligned}\frac{\mathrm{d}\tilde{\Gamma}_q}{\mathrm{d}\tau}(\nu) =&\Gamma_B^q U_q\left(\mu_m,\mu_h,\mu_j,\mu_s\right)|C_S^q(M_h,\mu_h)|^2\int \mathrm{d}M_n^2\mathrm{d}M_{\bar{n}}^2\mathrm{d}k\left[\tilde{j}_q\left(\ln\frac{\mu_s M_h}{\mu_j^2}+\partial_{\eta_q},\mu_j\right)\right]^2\\ &\times\tilde{s}_q\left(\partial_{\eta_q},\mu_s\right)\left(\frac{M_h}{\nu\mu_s e^{\gamma_E}}\right)^{\eta_q},\\ \frac{\mathrm{d}\tilde{\Gamma}_g}{\mathrm{d}\tau}(\nu) =&\Gamma_B^g U_g\left(\mu_t,\mu_h,\mu_j,\mu_s\right)|C_t(m_t,\mu_t)|^2|C_S^g(M_h,\mu_h)|^2\int \mathrm{d}M_n^2\mathrm{d}M_{\bar{n}}^2\mathrm{d}k\\ &\times\left[\tilde{j}_g\left(\ln\frac{\mu_s M_h}{\mu_j^2}+\partial_{\eta_g},\mu_j\right)\right]^2\tilde{s}_g\left(\partial_{\eta_g},\mu_s\right)\left(\frac{M_h}{\nu\mu_s e^{\gamma_E}}\right)^{\eta_q}.\end{aligned} \tag{12}$$

1. To eliminate the logarithms in the Wilson coefficients, hard, Laplace-transformed soft and jet functions, we take

$$\mu_m=\mu_t=e_t m_t,\quad \mu_h=e_h M_h,\quad \mu_j=\sqrt{m_h\mu_s},\quad \mu_s=m_h e^{-\gamma_E}/\nu. \tag{13}$$

The choice of Scales in the Laplace space

- Then, we perform the inverse Laplace transformation

$$\frac{\mathrm{d}\Gamma^{\mathrm{Lresum}}_{q,g}}{\mathrm{d}\tau} = \frac{1}{2\pi i}\int_c \mathrm{d}\nu \; e^{\nu\tau}\frac{\mathrm{d}\tilde{\Gamma}_{q,g}}{\mathrm{d}\tau}. \tag{14}$$

- Ultimately, we match $\frac{\mathrm{d}\Gamma^{\mathrm{Lresum}}_{q,g}}{\mathrm{d}\tau}$ onto the NLO results, like Eq. (11).

$$\frac{\mathrm{d}\Gamma^{\mathrm{Lmatched}}_{q,g}}{\mathrm{d}\tau} = \frac{\mathrm{d}\Gamma^{\mathrm{Lresum}}_{q,g}}{\mathrm{d}\tau} - \left(\frac{\mathrm{d}\Gamma^{\mathrm{app}}_{q,g}}{\mathrm{d}\tau} - \frac{\mathrm{d}\Gamma^{\mathrm{FO}}_{q,g}}{\mathrm{d}\tau}\right), \tag{15}$$

Numerical results(Preliminary)

Manifest decline of the scale variations in both cases

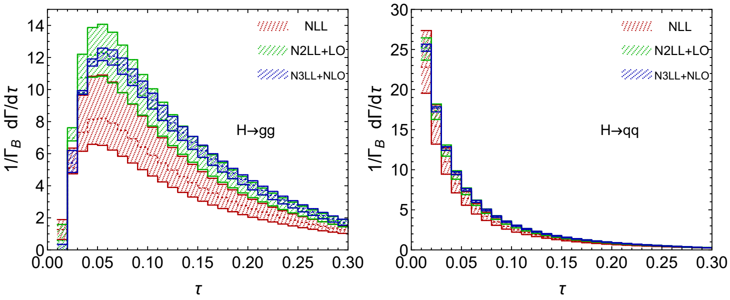

Figure: The resummed thrust distributions induced by *Hgg* or *Hqq* vertex, respectively.

Summary

The significant corrections from NLO

1. Our calculation is based on a low energy effective theory with Hgg effective coupling and $Hq\bar{q}$ Yukawa couplings.
2. We have calculated the NLO QCD corrections to both channels and observe significant corrections to the thrust distributions. Especially for the $H \to gg$ case, the NLO corrections can be as large as the LO results (corresponding to a K-factor $\sim 200\%$).

The approximate NNLO

1. Approximate NNLO is obtained by means of expanding the factorization formula in the soft-collinear effective theory.
2. We find that the NNLO corrections are still sizable, although they reduce the scale uncertainties significantly.

Summary

The resummation

1. We perform the resummation based on the SCET factorization formula, and determine the typical scales within the Laplace-transformed thrust distributions.
2. In comparison with fixed-order results, we observe the manifest decline of the scale variations in the resummed results, and find the N^3LL corrections become mild.

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