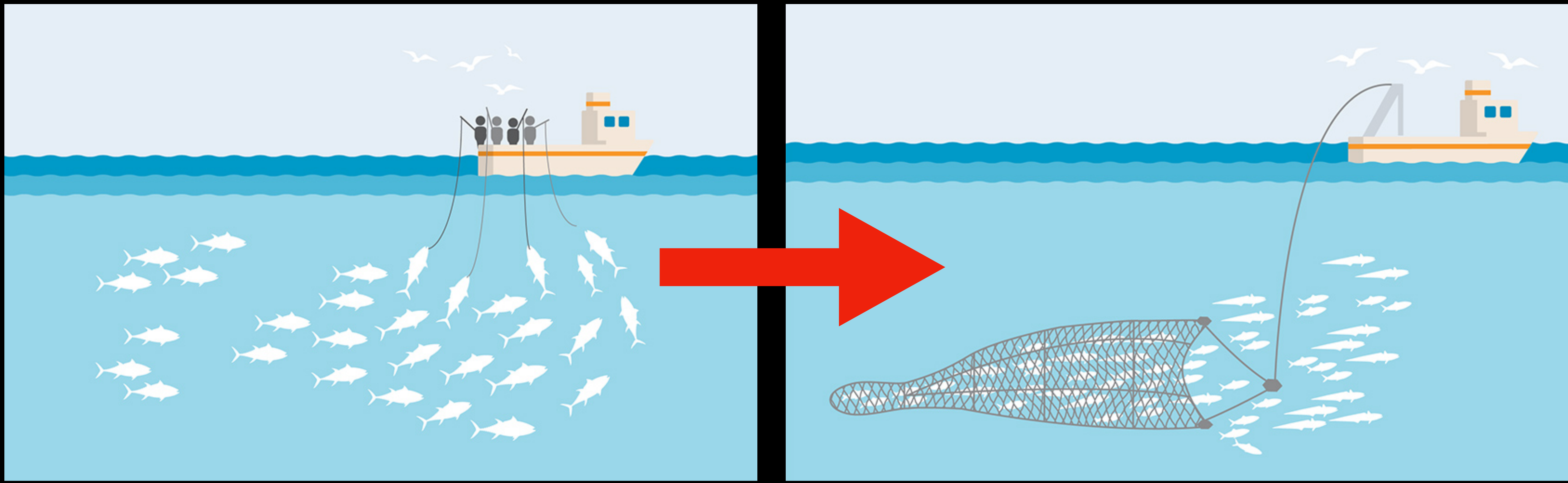
A global analysis approach

A single parameter quantifies
both Higgs Br precision and detector performance



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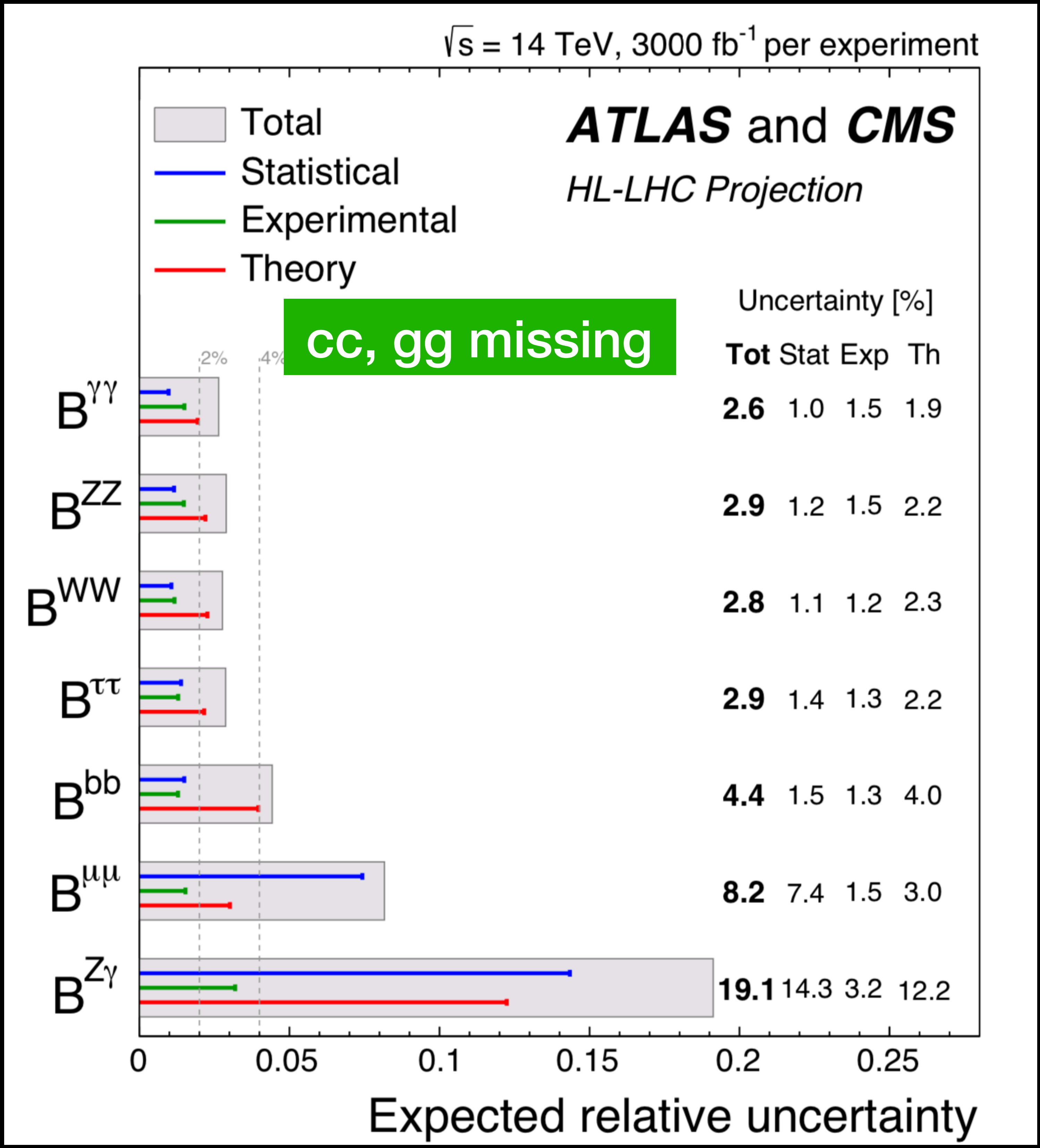
CEPC physics workshop， 2019/07/01-05，PKU

Outline

- Motivation
- Method
- Some numerical results with toy MC
- Discussion & summary

- Competition from both HL-LHC and FCC-ee

- FCC-ee
- ATLAS-CMS extrapolation range from 2 – 4%, with the exception of that on $B^{\mu\mu}$ at 8% and on $B^{Z\gamma}$ at 19%.

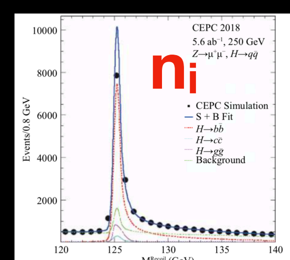


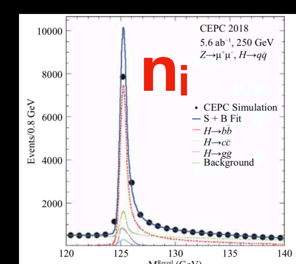
We possess what the LHC lacks（人无我有）

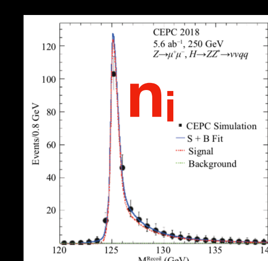
- Tagging method, absolute/model-independent
- All Higgs decays accessible except e and uds
- Multinomial distribution: statistical constraint
- Two types of backgrounds
  - Higgs background (crosstalk)
  - non-Higgs background (enlarge the stat. unc. of n_i)

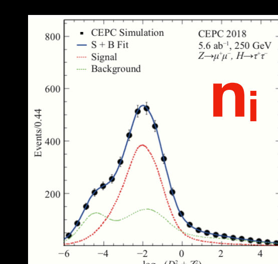
$$B_i = \frac{n_i/\epsilon_{ii}}{N}$$

B_i =

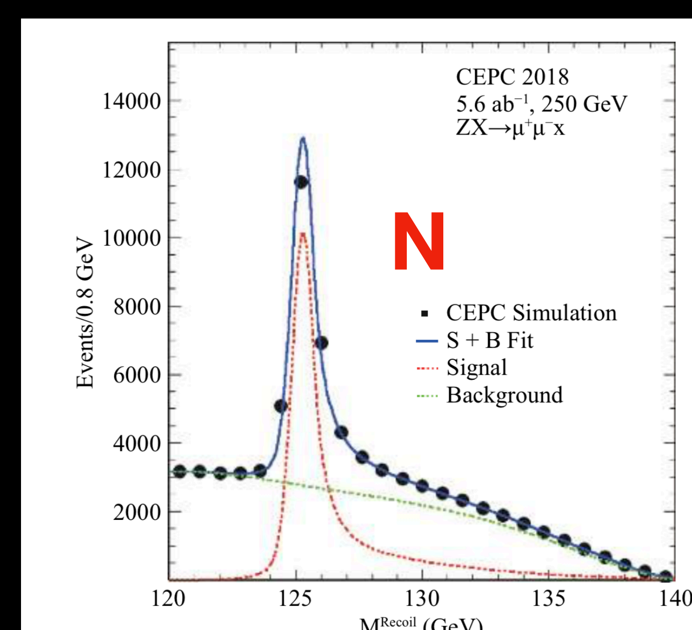








+ … …



non-Higgs background

— subtracted with fitting

— but enlarges σ_{n_i}

Take the simplest case as an example
—2 decay modes

Efficiency matrix

Based on MC, no dependence on Br's

$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix} = \begin{pmatrix} \epsilon_{11} & \epsilon_{12} \\ \epsilon_{21} & \epsilon_{22} \end{pmatrix} \times \begin{pmatrix} N_1 \\ N_2 \end{pmatrix}$$

Observation | MODULATION Matrix | Production

A produced final state j reconstructed as final state i

Measurement: DEMODULATION
All knowns on the right
Solve N and minimize its uncertainty

DEMODULATION

$$N = E^{-1} n$$

2 decays
p+q=1 — binomial distribution

Individual measurements

$$\sigma_{n_1} = \sqrt{N\epsilon_{11}}$$

$$\sigma_{n_2} = \sqrt{N\epsilon_{22}}$$

Global measurement

$$\sigma_{n_1} = \sqrt{Np(1-p)\epsilon_{11}}$$

$$\sigma_{n_2} = \sqrt{Nq(1-q)\epsilon_{22}}$$

$$\sum N_i = N$$

Based on text book, please read
https://en.wikipedia.org/wiki/Binomial_distribution
https://en.wikipedia.org/wiki/Multinomial_distribution

binomial / multinomial distributions

More on the full covariance

$$V = \begin{pmatrix} Npq & -Npq \\ -Npq & Npq \end{pmatrix}$$

100% anti-correlated between the two decays!
This can be used in data analysis to improve precisions.

Successful examples

- **Precision measurement of the D^{*0} decay branching fractions by BESIII, Phys. Rev. D91 (2015) no.3, 031101**
- **Branching ratios of tau decays by ALEPH, Physics Reports 421 (2005) 191–284**

Let’s see how it happens

$$\vec{\sigma}_n = \begin{pmatrix} \vec{\sigma}_{n_1} \\ \vec{\sigma}_{n_2} \end{pmatrix},$$

$$\sigma^2_{n_i} = \vec{\sigma}_{n_i} \cdot \vec{\sigma}_{n_i}, \;\; \sigma_{n_{ij}} = \sigma_{n_{ji}} = \sigma_{n_1}\sigma_{n_2}\rho_{ij} = \vec{\sigma}_{n_i} \cdot \vec{\sigma}_{n_j},$$

$$\rho_{ij} = \frac{\vec{\sigma}_{n_i} \cdot \vec{\sigma}_{n_j}}{\sigma_{n_1}\sigma_{n_2}},$$

Matrix: compact and easy to expand to higher dimension

$$\begin{aligned} \Sigma^n = \vec{\sigma}_n \vec{\sigma}_n^T &= \begin{pmatrix} \vec{\sigma}_{n_1} \\ \vec{\sigma}_{n_2} \end{pmatrix} \begin{pmatrix} \vec{\sigma}_{n_1} \\ \vec{\sigma}_{n_2} \end{pmatrix}^T \\ &= \begin{pmatrix} \sigma^2_{n_1} & \sigma_{n_{12}} \\ \sigma_{n_{21}} & \sigma^2_{n_2} \end{pmatrix} \end{aligned}$$

Space transformation

n(observable)→N(production)

$$N = E^{-1} n$$

$$\vec{\sigma}_N = E^{-1} \vec{\sigma}_n$$

$$J_{Nn} = E^{-1} = \frac{1}{|E|} \begin{pmatrix} \epsilon_{22} & -\epsilon_{12} \\ -\epsilon_{21} & \epsilon_{11} \end{pmatrix} \equiv \frac{J_N}{|E|}$$

Space transformation

N(production)→B(branching ratios)

$$J_{BN} = \frac{1}{N^2}\begin{pmatrix} N_2 & -N_1 \\ -N_2 & N_1 \end{pmatrix} = \frac{J_B}{N^2}$$

$$\vec{\sigma}_B = J_{BN} J_{Nn} \vec{\sigma}_n = \frac{\begin{pmatrix} n_2\vec{\sigma}_1 - n_1\vec{\sigma}_2 \\ -n_2\vec{\sigma}_1 + n_1\vec{\sigma}_2 \end{pmatrix}}{N^2|E|}$$

$$\begin{aligned}\Sigma_B &= \frac{\vec{\sigma}_B \vec{\sigma}_B^T}{N^4|E|^2} \\ &= \frac{[J_B J_N \vec{\sigma}_n][J_B J_N \vec{\sigma}_n]^T}{N^4|E|^2} \\ &= \frac{(n_2\sigma_{n_1} + n_1\sigma_{n_2})^2}{N^4|E|^2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\end{aligned}$$

- Features
  - ☑ Variance of B proportional to $1/(N^4|E|^2)$
  - ☑ N^4 : statistical power
  - ☑ $|E|^2$ proportional to the performance of Detector x Reconstruction x Analysis
  - ☑ Same uncertainties for both Br's

More than 2 decay modes

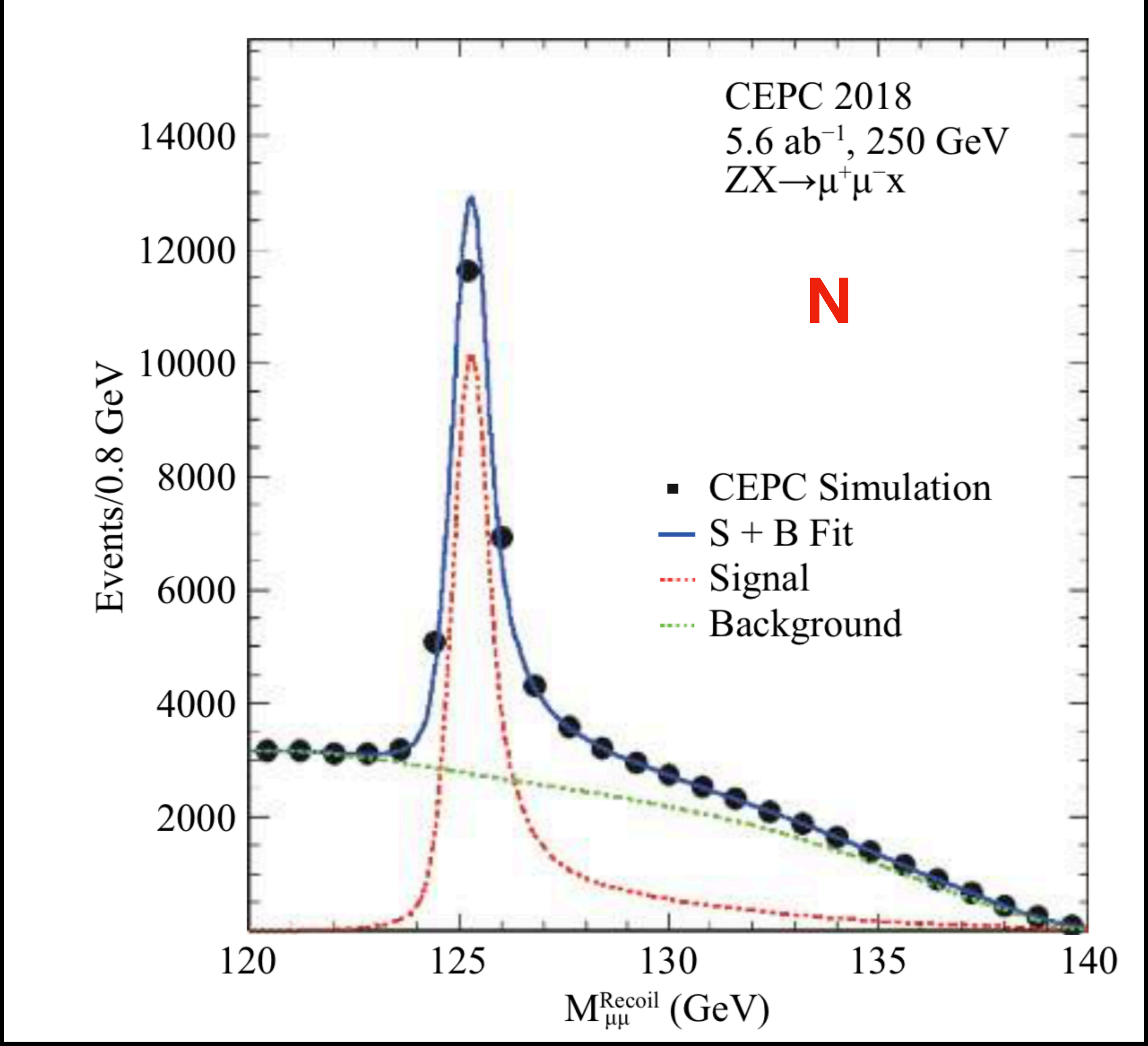
$$\Sigma_B = \frac{\vec{\sigma}_B \vec{\sigma}_B^T}{N^4 |E|^2} = \frac{1}{N^4\,|E|^2}\begin{pmatrix} \left\{\sum_j [N_1 \frac{\partial N}{\partial n_j} - N \frac{\partial N_1}{\partial n_j}]\overrightarrow{\sigma_{n_j}}\right\}^2 & .. & .. \\ .. & \left\{\sum_j [N_2 \frac{\partial N}{\partial n_j} - N \frac{\partial N_2}{\partial n_j}]\overrightarrow{\sigma_{n_j}}\right\}^2 & .. \\ .. & .. & \left\{\sum_j [N_3 \frac{\partial N}{\partial n_j} - N \frac{\partial N_3}{\partial n_j}]\overrightarrow{\sigma_{n_j}}\right\}^2 \end{pmatrix}$$

Similar features as N=2

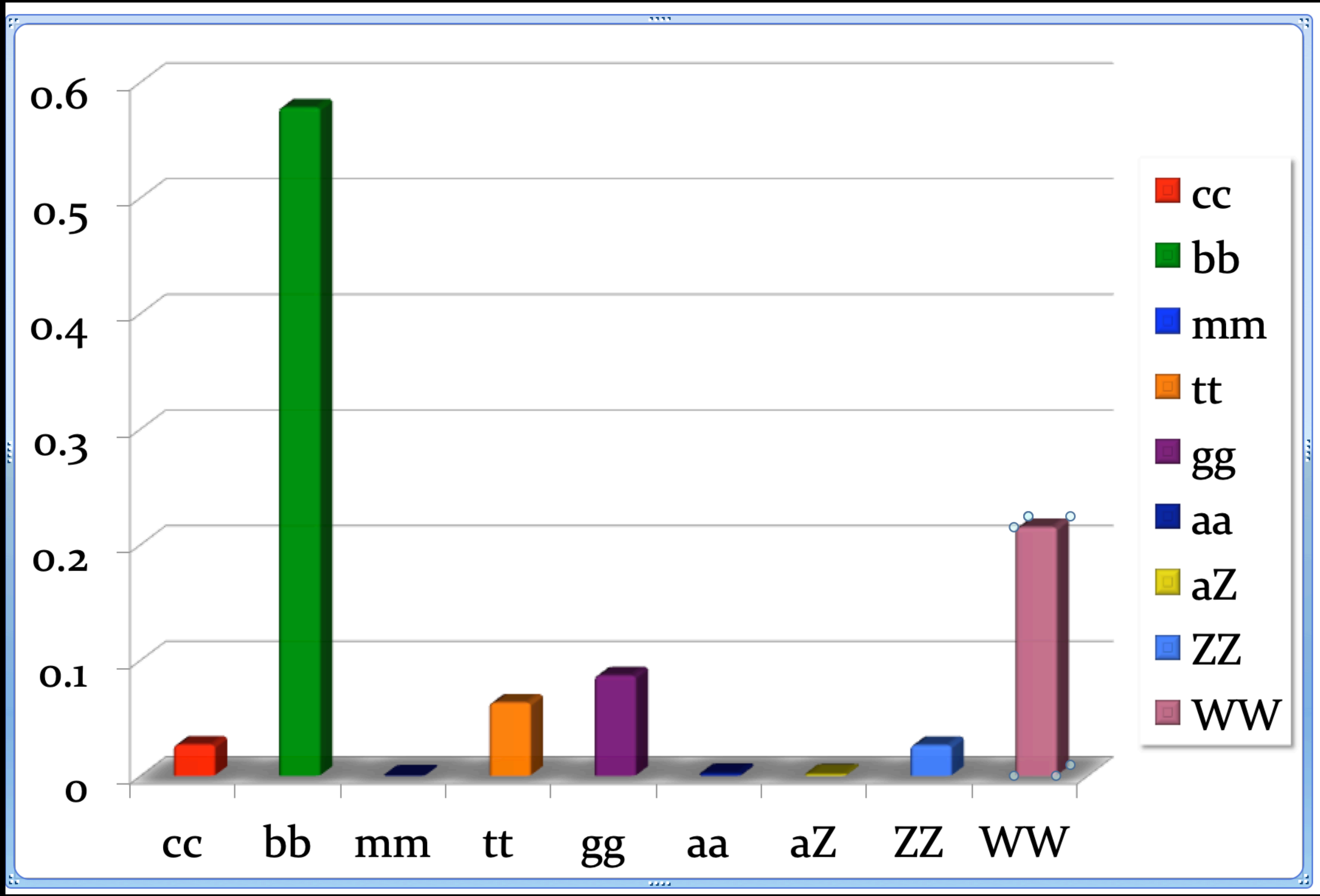
Numerical results with toy MC

On backgrounds

- Two type of backgrounds
  - Non-uuH backgrounds: subtracted by fitting, enlarging statistical uncertainty of n_i
  - uuH backgrounds(cross talk): the efficiency matrix dealing with them
  - N and n_i



- 9 Higgs decays accessible
- Di-muon, di-photon, and gamma Z are tiny: only 0.02%, 0.23%, and 0.15%, respectively
- cc contaminated by bb due to large bb Br
- ZZ important for Higgs Width



Solve N_i by minimizing the χ^2 with constraint

$$\chi^2 = \sum_i \frac{(\sum \epsilon_{ij} N_j - n_i)^2}{\sigma_{n_i}^2} + \frac{(\sum_l N_l - N)^2}{\sigma_N^2}$$

Higgs -> cc, bb, mm, tt, gg, aa, aZ, ZZ, WW

1 2 3 4 5 6 7 8 9

$$\begin{pmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5 \\ n_6 \\ n_7 \\ n_8 \\ n_9 \end{pmatrix} = \begin{pmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} & \epsilon_{14} & \epsilon_{15} & \epsilon_{16} & \epsilon_{17} & \epsilon_{18} & \epsilon_{19} \\ \epsilon_{21} & \epsilon_{22} & \epsilon_{23} & \epsilon_{24} & \epsilon_{25} & \epsilon_{26} & \epsilon_{27} & \epsilon_{28} & \epsilon_{29} \\ \epsilon_{31} & \epsilon_{32} & \epsilon_{23} & \epsilon_{34} & \epsilon_{35} & \epsilon_{36} & \epsilon_{37} & \epsilon_{38} & \epsilon_{39} \\ \epsilon_{41} & \epsilon_{42} & \epsilon_{33} & \epsilon_{44} & \epsilon_{45} & \epsilon_{46} & \epsilon_{47} & \epsilon_{48} & \epsilon_{49} \\ \epsilon_{51} & \epsilon_{52} & \epsilon_{43} & \epsilon_{54} & \epsilon_{55} & \epsilon_{56} & \epsilon_{57} & \epsilon_{58} & \epsilon_{59} \\ \epsilon_{61} & \epsilon_{62} & \epsilon_{53} & \epsilon_{64} & \epsilon_{65} & \epsilon_{66} & \epsilon_{67} & \epsilon_{68} & \epsilon_{69} \\ \epsilon_{71} & \epsilon_{72} & \epsilon_{63} & \epsilon_{74} & \epsilon_{75} & \epsilon_{76} & \epsilon_{77} & \epsilon_{78} & \epsilon_{79} \\ \epsilon_{81} & \epsilon_{82} & \epsilon_{73} & \epsilon_{84} & \epsilon_{85} & \epsilon_{86} & \epsilon_{87} & \epsilon_{88} & \epsilon_{89} \\ \epsilon_{91} & \epsilon_{92} & \epsilon_{83} & \epsilon_{94} & \epsilon_{95} & \epsilon_{96} & \epsilon_{97} & \epsilon_{98} & \epsilon_{99} \end{pmatrix} \begin{pmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \\ N_6 \\ N_7 \\ N_8 \\ N_9 \end{pmatrix}$$

Neglect e and uds decays — constraint feasible

$$\sum_i N_i = N^{tag} \text{ or } \sum_i B_i = 1$$

$$B_i = \frac{N_i}{N}$$

Statistical limit

- ➡99% efficiency,
- ➡no cross talk,
- ➡no other backgrounds
- ➡eeH and qqH as good as mumuH

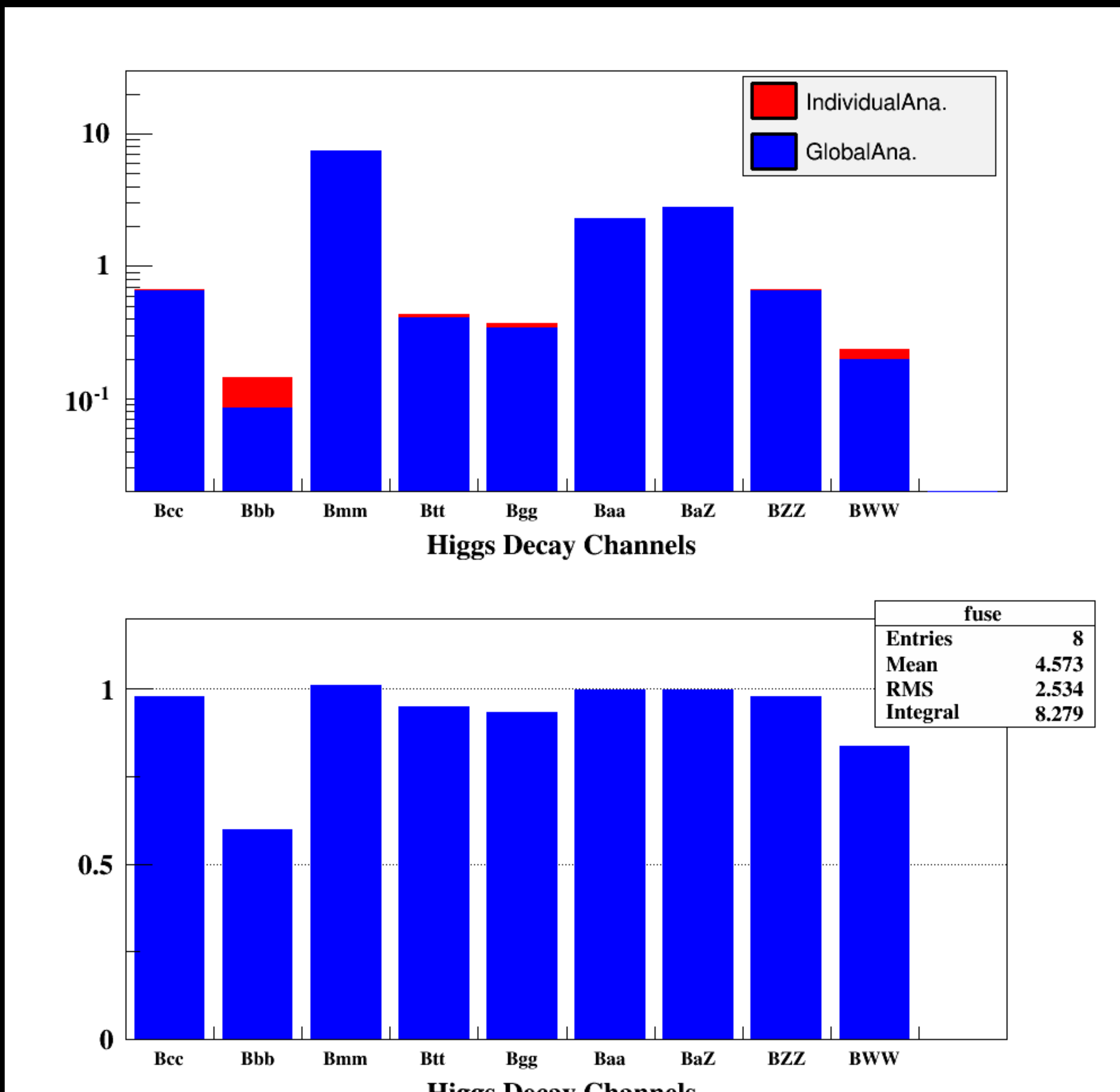
$$E = \begin{pmatrix} 0.99 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.99 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.99 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.99 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.99 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.99 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.99 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.99 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.99 \end{pmatrix}$$

$$N = L \times (\sigma_{\mu\mu H} + \sigma_{eeH} + \sigma_{qqH}) = 5600 \times (6.77 + 7.04 + 136.81) = 843,372$$

Ideal case:

eeH, qqh as good as uuH

No background, no cross talk, multinomial uncertainties, and constraint



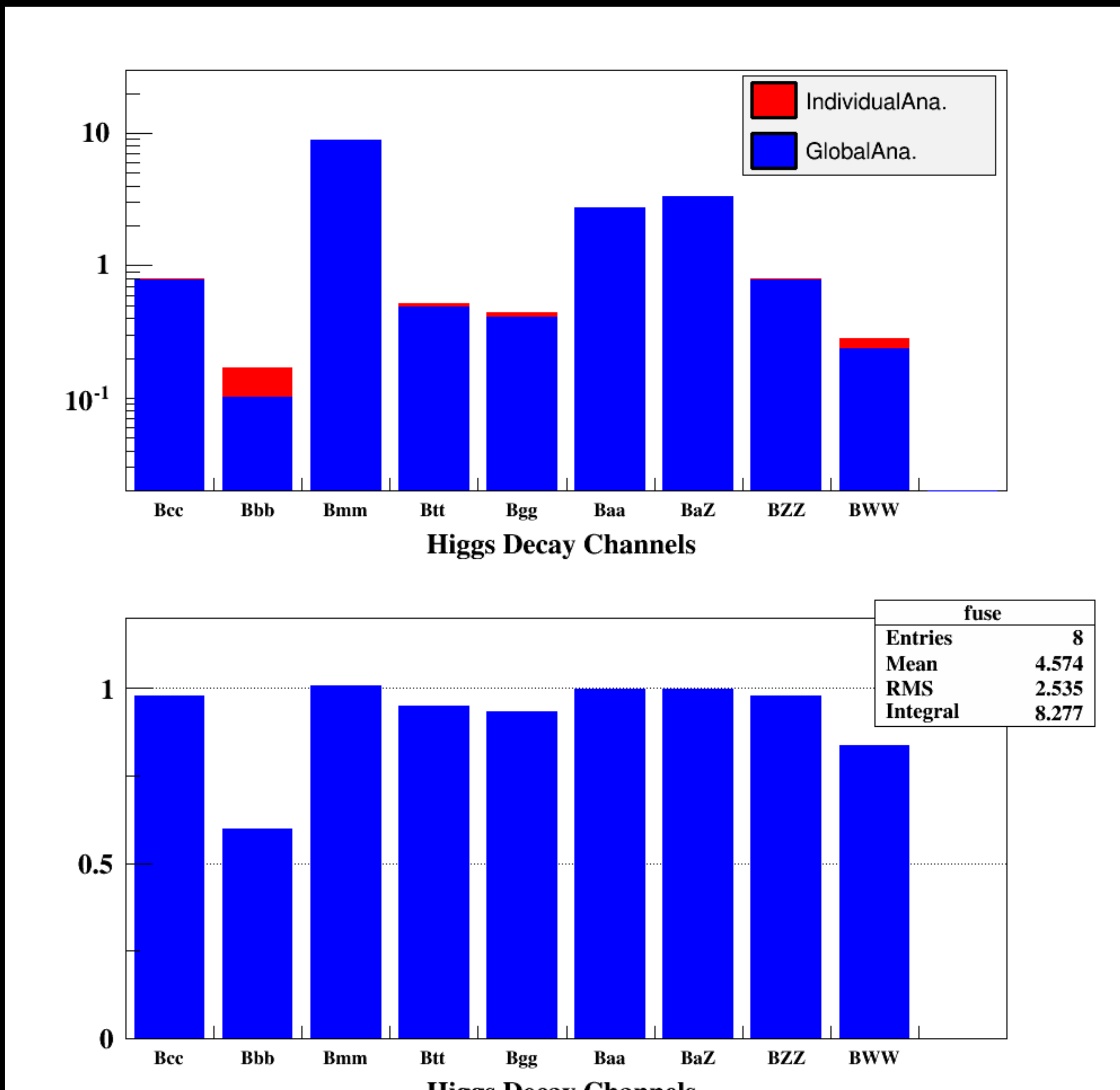
$$\sigma_{n_i} = \sqrt{Np(1-p)\epsilon_{ii}}$$

		MLT		POS
Bcc	2.713%	0.650%	(0.655%	0.664%)
Bbb	57.799%	0.086%	(0.094%	0.144%)
Bmm	0.023%	7.190%	(7.197%	7.198%)
Btt	6.319%	0.413%	(0.421%	0.435%)
Bgg	8.619%	0.347%	(0.356%	0.373%)
Baa	0.227%	2.294%	(2.296%	2.299%)
BaZ	0.150%	2.818%	(2.820%	2.822%)
BZZ	2.647%	0.658%	(0.664%	0.673%)
BWW	21.496%	0.198%	(0.209%	0.236%)

More realistic:

eeH, qqh as good as uuH

100% background, no cross talk, multinomial uncertainties, and constraint

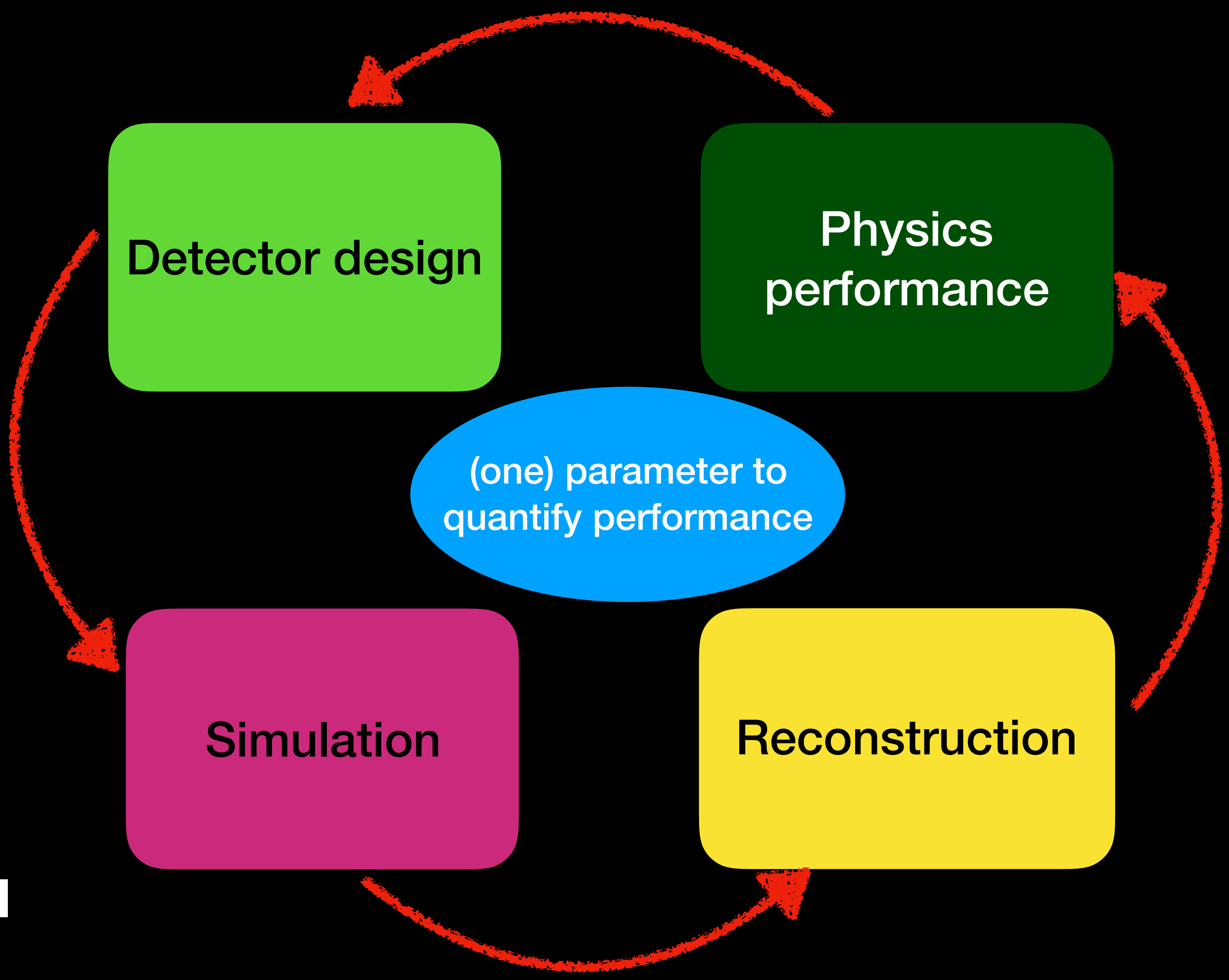


$$\sigma_{n_i} = \sqrt{Np(1-p)\epsilon_{ii}}$$

		MLT		POS
Bcc	2.713%	0.773%	(0.779%	0.790%)
Bbb	57.799%	0.102%	(0.111%	0.171%)
Bmm	0.023%	8.547%	(8.559%	8.560%)
Btt	6.319%	0.492%	(0.501%	0.518%)
Bgg	8.619%	0.413%	(0.424%	0.443%)
Baa	0.227%	2.728%	(2.731%	2.734%)
BaZ	0.150%	3.350%	(3.353%	3.356%)
BZZ	2.647%	0.783%	(0.789%	0.800%)
BWW	21.496%	0.235%	(0.249%	0.281%)

Short discussion

- This approach can improve Higgs branching ratio measurement and set a statistical limit
- qqH and eeH not good as uuH, but much more statistics
- Degrading in real analysis and lots of compromises
- No full cross talk information in current analyses



Good design:
- ★ **From top level**
- ★ **Break-down**

Detector design & Optimization

Multi-purpose optimization: a bunch of benchmarks —
A single parameter is favored, which means single-purpose optimization

Physics performance parameterized as a function of several parameters, or precision of a set of benchmark processes or determinant of efficiency matrix |E|

$$P = f(\sigma_p, \sigma_{E_\gamma}, PID, JID, JER, \ldots)$$
$$= |E|^2 \propto \frac{1}{|\Sigma_B|^2}$$

Difficult

Easy to minimize

Now problem successfully becomes how to Maximize $|E|^2$

Again on efficiency matrix

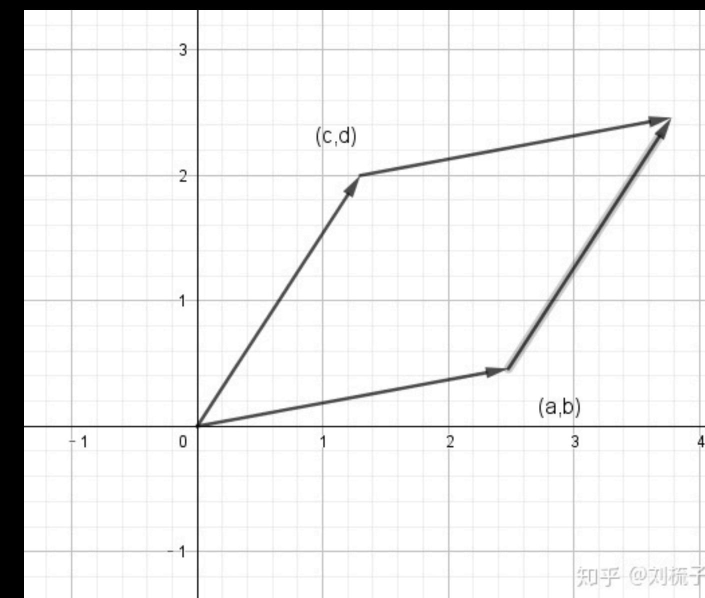
- *Not necessary to know the branching ratios of Higgs decays*
- *Quantifies the detector/software/analysis performance with* a single parameter det|E|
- *It could be useful for detector optimization*

A single purpose optimization instead of that of a bunch of benchmarks

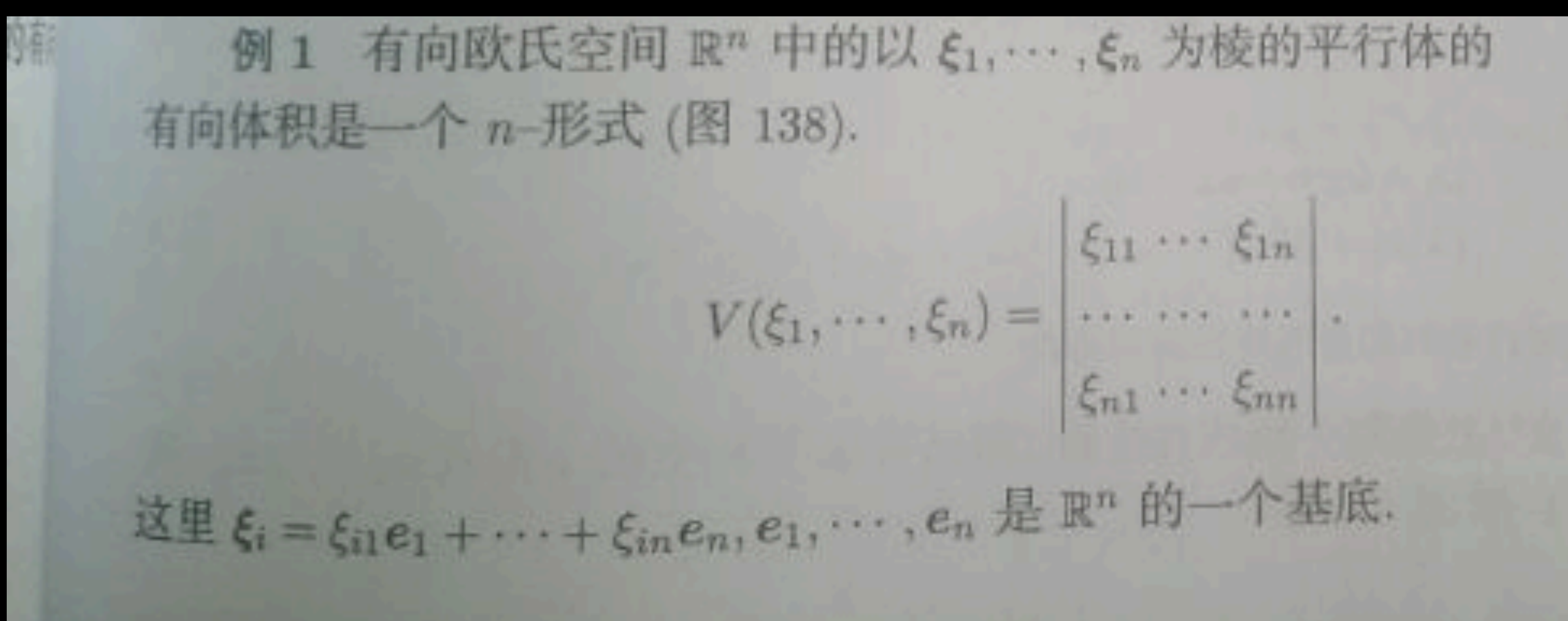
$$\Sigma_B \propto \frac{1}{|E|^2}$$

Geometrical interpretation of the efficiency matrix

- For a matrix $E = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, det(E) is the area of a quadrilateral
- $S = det(E) = ad - bc$



- For n=3, det(E) is volume of a parallelepiped
- For N>3, det(E) is hyper volume …



例 1 有向欧氏空间 $\mathbb{R}^n$ 中的以 $\xi_1,\cdots,\xi_n$ 为棱的平行体的有向体积是一个 n-形式 (图 138).

$$V(\xi_1,\cdots,\xi_n)=\begin{vmatrix}\xi_{11} & \cdots & \xi_{1n} \\ \cdots & \cdots & \cdots \\ \xi_{n1} & \cdots & \xi_{nn}\end{vmatrix}.$$

这里 $\xi_i=\xi_{i1}e_1+\cdots+\xi_{in}e_n$, $e_1,\cdots,e_n$ 是 $\mathbb{R}^n$ 的一个基底.

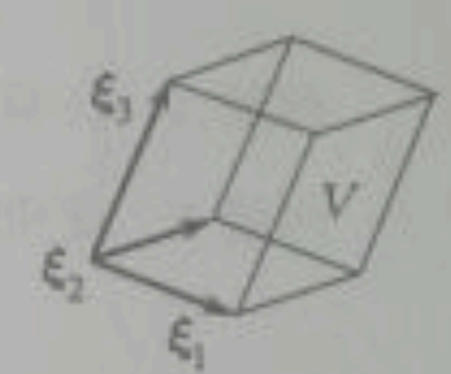
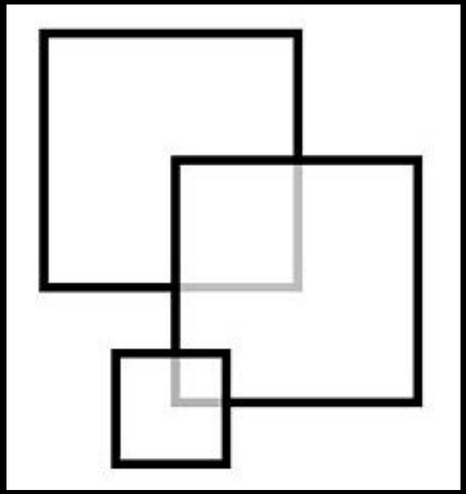


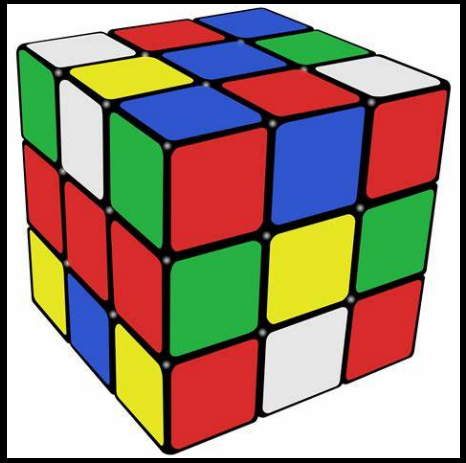
图 138 有向体积是一个 3-形式

Maximize $|E|^2$



- N=2, the maximum |E| → area of a square

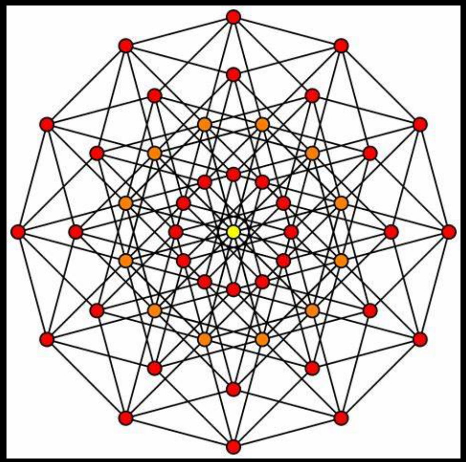
- N=3, the maximum |E| →volume of a cube



- N>3, … →volume of a HyperCube

- Hypercube efficiency matrix has the ideal/best performance, this is the dream of experimentalists

Detector: HC
HyperCube or HiggsCube



Summary

- CEPC dedicated for Higgs study, the Br's important
- Global analysis with constraint improves the precision
- Global analysis of $e^+e^- \to \mu^+\mu^- H$, $H \to$ all 9 decay modes serves as a "benchmark" to optimize detector, software, and analysis,
- Advantage : single parameter, easy to optimize, easily to realize in ML
- Using fast simulation + global analysis + machine learning to maximize |E| — fast iteration
- Including eeH and qqH much better but difficult, possible to do …, not very necessary at present