Three-point energy correlation in the coplanar limit

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Work in progress

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Hadron production at e^+e^- colliders

- Large cross section of $q\bar{q}$ production
- High luminosity at the CEPC, generate more than 10^{11} $q\bar{q}$ events at Z pole and more than 10^8 events at 240 Gev.
- These large number of events can reduce the statistical uncertainty dramatically.
- High energy can suppress the hadronization effects.

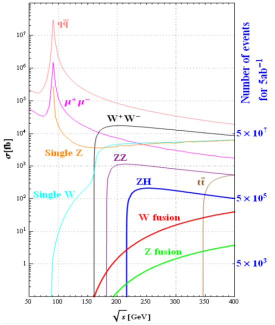


Figure: Total cross section at e^+e^- colliders

Operation mode	$\sqrt{s}$ (GeV)	L per IP (10^{34} cm^{-2}s^{-1})	Years	Total $\int L$ (ab^{-1}, 2 IPs)	Event yields
H	240	3	7	5.6	1×10^6
Z	91.2	32 (*)	2	16	7×10^{11}
W^+W^-	158–172	10	1	2.6	2×10^7 (†)

Figure: Luminosity and sample event yields at the CEPC[CEPC conceptual design report, 2018]

Event shape observables

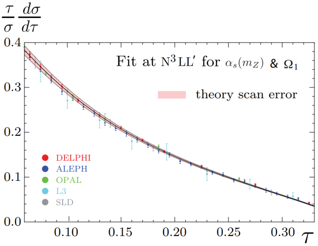
see also last talk of WanLi Ju

- Event shape observables were designed to obtain useful information from the large number of $q\bar{q}$ events.
- From experiment side, being only composed of kinematics of final state particles, they are easy to extract from data.
- From theory side, they are designed to be infrared and collinear safe therefore can be reliably calculated in pQCD.
- Can be used to extract α_s by comparing theory and data.
- Study of hadronization effects.
- For example, Thrust T [Brandt et all., 1964; Farhi, 1977], C-parameter [Parisi, 1978; Donoghue et al, 1979; Ellis, 1981], wide B_W and total B_T jet broadenings [Rakow and Webber, 1981; Ellis and Webber, 1986; Catani et al., 1992], normalized heavy jet mass M_H^2/s [clavelli, 1979], the energy-energy correlations (EEC) [Basham et al, 1978]

Event Shapes in the Dijet limit

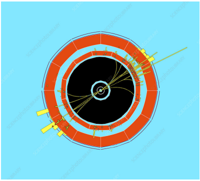
see also last talk of WanLi Ju

- At lepton colliders, event shape observables in the dijet limit have been well studied, to NNLO and N^{3}LL accuracy [Ridder et al, 2007; Weinzierl, 2008, 2009; Becher et al, 2008; Abbate et al, 2011; Chien et al, 2010; Hoang et al, 2014].
- Determination of α_s from thrust[Abbate, et al., 2010]

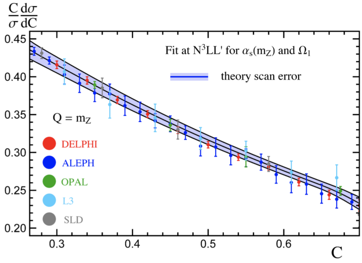


$\alpha_s(m_Z) = 0.1135 \pm (0.0002)_{\text{exp}} \pm (0.0005)_{\text{hadr}} \pm (0.0009)_{\text{pert}}$

- A typical two jets event from LEP



- Determination of α_s from C-parameter [Hoang et al., 2015]



$\alpha_s(m_Z) = 0.1123 \pm (0.0002)_{\text{exp}} \pm (0.0007)_{\text{hadr}} \pm (0.0014)_{\text{pert}}$

Event Shapes in the Trijet Limit

- However, only a few event shapes in the trijet limit were studied.
- Thrust minor T_m [Banfi et al, 2001]
- D-parameter: the three-jet coplanar region was studied[Banfi et al, 2001]; And recently all regions of $D \to 0$ were studied to perform the full resummation up to NLL[Larkoski and Procita, 2018]

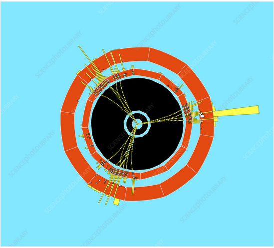
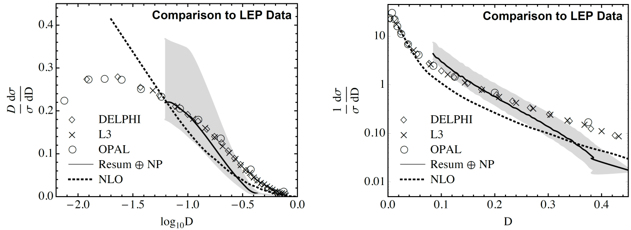


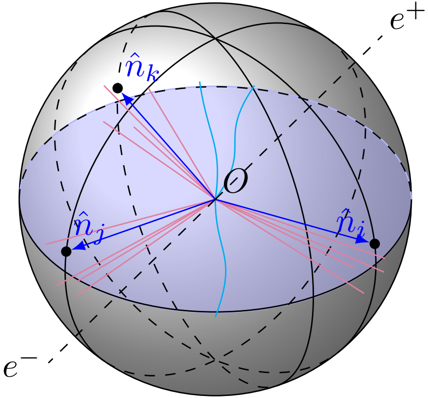
Figure: A typical three jets event from LEP



(a) (b) [Larkoski and Procita, 2018]

Why a *new* event shape: three-point energy correlation

- Allow an all order factorization formula in the coplanar limit
- Ingredients of factorization formula known to higher orders
- Allow us to get NLO+ NNLL accuracy
- Allow an operator definition, can be conveniently used to study non-perturbative effects



Review of the EEC

The Energy-Energy Correlation (EEC) is defined as [Basham et al, 1978]

$$\mathrm{EEC} = \frac{1}{\sigma_{\mathrm{tot}}} \sum_{ij} \int d\sigma \frac{E_i E_j}{Q^2} \delta\left(\cos\chi - \cos\theta_{ij}\right),$$

which measures the correlations of energy deposited in two detectors separated with angle χ.

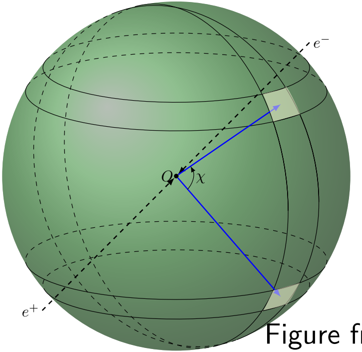


Figure from [Moult and Zhu, 2018]

From CMB to Colliders

- In the research of the Cosmic Microwave Background (CMB), two- and three-point correlation functions were studied
- Instead of only one universe at the CMB, we have numerous events at the lepton colliders
- There exists color evolutions of multiple Wilson lines and therefore more differential structures at the colliders.
- The general form of three-point correlations is a multivariable function

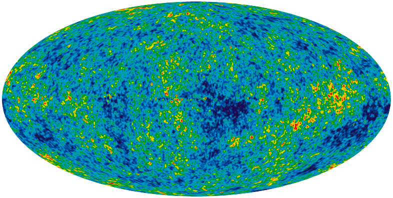


Figure: CMB [Wikipedia]

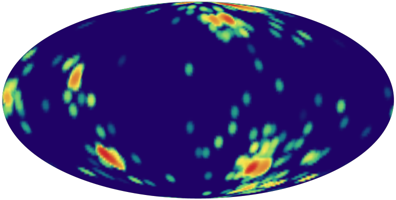


Figure: One Event at the Lepton Collider

Definition of the ETPC

- We define a *NEW* observable, the Energy Triple-Product Correlation (ETPC), it can be defined as a straightforward generalization of EEC

$$\sum_{ijk} \int d\sigma \frac{E_i E_j E_k}{Q^3 \sigma_{\text{tot}}} \delta(\cos\chi_1 - \cos\theta_{ij}) \delta(\cos\chi_2 - \cos\theta_{ik}) \delta(\cos\chi_3 - \cos\theta_{jk}).$$

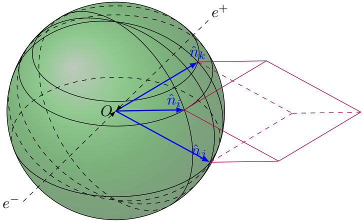
- i, j, k run over all the different final state particles
- It depends on three variables χ_1, χ_2 and χ_3
- It is not convenient to go on by using this definition
- We seek a one variable definition as a first step

Definition of the ETPC

- One variable definition of ETPC can be written as

$$ETPC = \frac{1}{\sigma_{\text{tot}}}\frac{d\Sigma}{d\tau_p} = \sum_{ijk}\int d\sigma \frac{E_i E_j E_k}{Q^3 \sigma_{\text{tot}}}\delta\left(\tau_p - \tau_{ijk}\right).$$

- $\tau_{ijk} = |(\hat{n}_i \times \hat{n}_j) \cdot \hat{n}_k|$ is the *volume* of the *parallelepiped* formed by $\hat{n}_i$, $\hat{n}_j$ and $\hat{n}_k$ which are unit vectors of three momentum p_i, p_j and p_k of final state particles
- $\tau_{ijk} \to 0$ corresponds that i, j, k are *in the coplanar limit* [This work]



The Relation between the ETPC and the D-Parameter

- The D-parameter is product of three eigenvalues $\lambda_1, \lambda_2, \lambda_3$ of the spherocity tensor $\Theta_{\alpha\beta} = \frac{1}{Q}\sum_i \frac{p_{i\alpha}p_{i\beta}}{E_i}$, α, β is the *spatial* component of four momentum [Parisi, 1978; Donoghue et al, 1979]
- In the case all the final particles are massless

$$
\begin{aligned}
D &= 27\lambda_1\lambda_2\lambda_3 \\
&= \frac{27}{6}\left\{(\lambda_1+\lambda_2+\lambda_3)\left[(\lambda_1+\lambda_2+\lambda_3)^2 - 3\left(\lambda_1^2+\lambda_2^2+\lambda_3^2\right)\right] + 2\left(\lambda_1^3+\lambda_2^3+\lambda_3^3\right)\right\} \\
&= \frac{27}{6}\left\{Tr\Theta\left[(Tr\Theta)^2 - 3Tr\Theta^2\right] + 2Tr\Theta^3\right\} \\
&= \frac{27}{Q^3}\sum_{i<j<k}\frac{\left|(\vec{p}_i\times\vec{p}_j)\cdot\vec{p}_k\right|^2}{E_iE_jE_k} \\
&= \frac{27}{Q^3}\sum_{i<j<k}E_iE_jE_k\tau_{ijk}^2
\end{aligned}
$$

- *The average of the D-parameter is the third moment of the ETPC*

$$
\langle D\rangle = \frac{9}{2}\int d\tau_p\ \tau_p^2\ ETPC\left(\tau_p\right)
$$

Trijet Coplanar limit

- The coplanar limit also contain collinear or back-to-back configurations, in this work, we only consider *trijet coplanar limit*, this means the coplanar three jets are well separated.
- In the work of [Banfi et al., 2001], the trijet resolution variable y_3 is required to be larger than a parameter

$$y_3 > y_{\text{cut}}$$

- y_3 is defined to be the minimum value of $y_{hh'}$ according to the k_T (Durham) algorithm [Catani, 1991]

$$y_{hh'} = 2\left(1 - \cos\theta_{hh'}\right)\min\left(E_h^2, E_{h'}^2\right)/Q^2$$

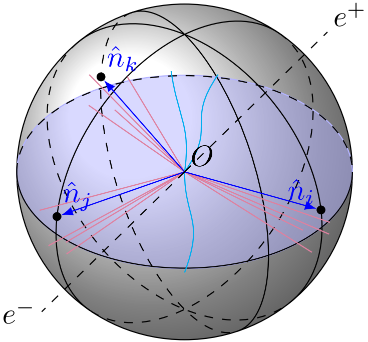


Figure: Trijet Coplanar Limit

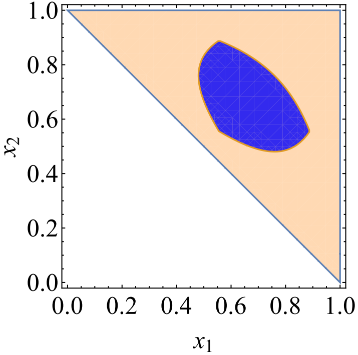
Trijet Coplanar Limit for the ETPC

For the ETPC, we apply two methods to approach the trijet coplanar limit

1. In each event, choose the three particles set $\{i, j, k\}$ such that

$$\sin\theta_{ij} > a_{\text{cut}}\,,\ \ \sin\theta_{jk} > a_{\text{cut}}\,,\ \ \sin\theta_{ki} > a_{\text{cut}}$$

   where $0 < a_{\text{cut}} < \frac{\sqrt{3}}{2}$ is a parameter that control the size of the allowed phase space.



The phase space of three particles, with $a_{\text{cut}} = 0.6$
Here $x_i = 2E_i/Q$

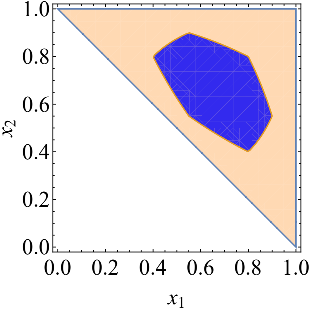
Trijet Coplanar Limit for the ETPC

2 Use the k_T algorithm to find three jets, we keep the event only if $y_3 > y_{\text{cut}}$, and modify the definition of the ETPC to

$$\sum_{\substack{i\in J_1\\ j\in J_2\\ k\in J_3}} \int d\sigma \frac{E_i E_j E_k}{E_{J_1} E_{J_2} E_{J_3} \sigma_{\text{tot}}} \delta\left(\tau_p - \tau_{ijk}\right)$$

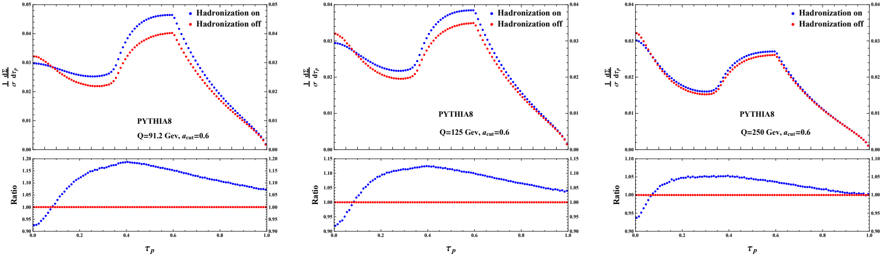
where J_1, J_2, J_3 denote three jets.

The phase space of three particles, with $y_{\text{cut}} = 0.1$
Here $x_i = 2E_i/Q$



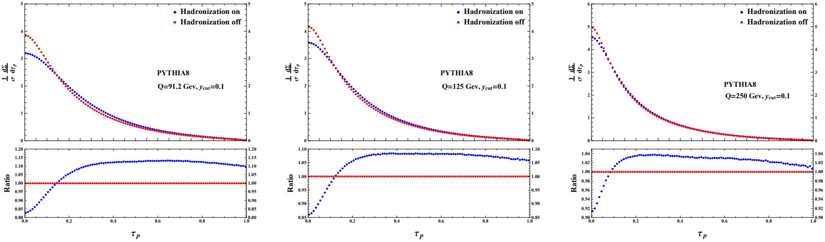
Using PYTHIA8.2 *Preliminary!*

- Use PYTHIA8.2 [Sjöstrand et al., 2015] to generate events
- Turn on/off hadronization
- The hadronization effect is roughly $\frac{1}{Q}$-dependent
- $a_{\text{cut}} = 0.6$



Using PYTHIA8.2 *Preliminary!*

- $y_{\text{cut}} = 0.1$



- This Pythia results can only be used as a reference.
- Pythia only contains lowest order hard matrix element, $e^+e^- \to q\bar{q}$ here, and LL resummation.
- Pythia can not estimate the scale uncertainty.
- We seek an analytical method to study ETPC in the coplanar limit.

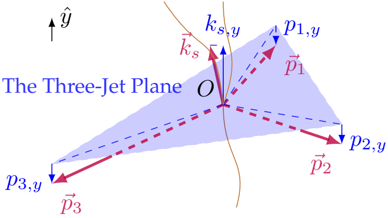
An all order factorization formula in the coplanar limit

$$\frac{1}{\hat{\sigma}_0}\frac{d\sigma}{d\tau_p} = \int_D d\upsilon d\omega H \int_{-\infty}^{\infty} \frac{db}{2\pi\xi} 2\cos\left(b\tau_p/\xi\right) S J_q J_{\bar{q}} J_g$$

Factorization: Kinematics

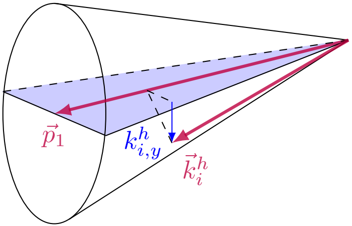
There are two sources dominate the ETPC in the trijet coplanar limit

- The recoil effect of soft radiations.



- The soft radiations *don't change the energy* of three jets, only make the three jets deviate the trijet plane slightly in the *opposite* direction.

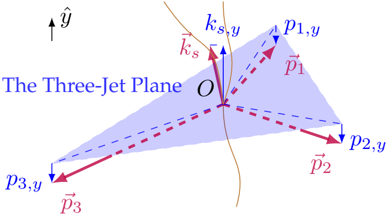
- Collinear fragmentation



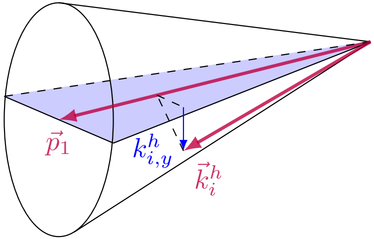
- The final state hardron carrys longitudinal fraction momentum z^h_i from it's parent jet, i.e. $\vec{k}^h_i = z^h_i \vec{p}_i$.

There are two sources dominate the ETPC in the trijet coplanar limit

- The recoil effect of soft radiations.



- Collinear fragmentation



- $\sum_{i=1}^{3} p_{i,y} = -k_{s,y}$, $\sin\theta_1 = \frac{p_{1,y}}{|\vec{p}_i|} = \frac{p_{1,y}}{E_i}$

- $\sin\theta_i = \frac{k^h_{i,y}}{|\vec{k}^h_i|} = \frac{k^h_{i,y}}{z^h_i E_i}$

$$\Rightarrow \quad \tau_{ijk} = \underbrace{\boxed{\frac{|\vec{p}_1 \times \vec{p}_2|}{E_1 E_2 E_3}}}_{\xi} \left| \frac{k^h_{i,x}}{z^h_i} + \frac{k^h_{j,y}}{z^h_j} + \frac{k^h_{k,y}}{z^h_k} - k_{s,y} \right| + \mathcal{O}\left(\tau_p^2\right)$$

The Factorization Formula

$$\frac{1}{\sigma_b}\frac{d\sigma}{d\tau_p} = \int_D dv d\omega\ H(v,\omega,\mu) \sum_{ijk} \int dk^h_{i,y} \int dk^h_{j,y} \int dk^h_{k,y} \int dk_{s,y}$$

$$\times \int dz^h_i dz^h_j dz^h_k\ z^h_i z^h_j z^h_k S(k_{s,y},\mu,\nu)\ \delta\left(\tau_p - \xi\left|\frac{k^h_{i,y}}{z^h_i} + \frac{k^h_{j,y}}{z^h_j} + \frac{k^h_{k,y}}{z^h_k} - k_{s,y}\right|\right)$$

$$\times F_{1\to i}(k^h_{i,y}, z^h_i, \mu, \nu) F_{2\to j}(k^h_{j,y}, z^h_j, \mu, \nu) F_{3\to k}(k^h_{k,y}, z^h_k, \mu, \nu) + \text{power corr.},$$

- $u = (p_1+p_2)^2, v = (p_1+p_3)^2$, $\omega = (p_2+p_3)^2$; i, j, k belong to the three different jets; σ_b is the born cross section for $e^+e^- \to q\bar{q}$

D The domain of the integrals, constrained by the phase space cuts

H The hard function

S The soft function

F TMD fragmentation functions

$$F_{q\to h}(b, z_h) = \frac{1}{4 z_h N_c} \sum_X \int \frac{d\xi^+}{2\pi} e^{-ip_h^-\xi^+/z_h} \langle 0 | \overline{\chi}_n(\xi) | X, h\rangle \bar{\not{n}} \langle X, h | \chi(0) | 0\rangle$$

$$\xi = (\xi^+, ib_0/\nu, b, 0)$$

Factorization Formula

Use Fourier representation of delta function, Fourier transformed TMD FFs and TMD SF

$$(1) \quad \delta\left(\tau - \xi\left|\frac{k_{i,y}^h}{z_i^h} + \frac{k_{j,y}^h}{z_j^h} + \frac{k_{k,y}^h}{z_k^h} - k_{s,y}\right|\right)$$

$$= \int \frac{db}{2\pi\xi} 2\cos(b\tau/\xi) \exp\left[ib\left(\frac{k_{i,y}^h}{z_i^h} + \frac{k_{j,y}^h}{z_j^h} + \frac{k_{k,y}^h}{z_k^h} - k_{s,y}\right)\right],$$

$$(2) \quad F_{ij}(\frac{b}{z_i}, z_i, \mu, \nu) = \int dk_{y,i}^h \exp\left(i\frac{b}{z_i}k_{i,y}^h\right)F_{ij}(k_{i,y}^h, z_i, \mu, \nu),$$

$$(3) \quad S(b, \mu, \nu) = \int dk_{s,y} \exp\left(ibk_{s,y}\right)S(k_{s,y}, \mu, \nu).$$

We rewrite the factorization formula as

$$\frac{1}{\sigma_b}\frac{d\sigma}{d\tau_p} = \int_D dv d\omega H(v, \omega, \mu) \sum_{ijl} \int \frac{db}{2\pi\xi} 2\cos(b\tau/\xi) \int dz_i dz_j dz_k$$

$$(z_i z_j z_k) S(b, \mu, \nu) F_{i1}(\frac{b}{z_i}, z_i, \mu, \nu) F_{j2}(\frac{b}{z_j}, z_j, \mu, \nu) F_{l3}(\frac{b}{z_k}, z_k, \mu, \nu).$$

Simplicity of the Factorization Formula

By operator product expansion, the TMDFF can be written as the convolution of the standard FF and matching coefficient

$$F_{h1}(\frac{b}{z_h}, z_h, \mu, \nu) = \sum_m f_{h/m}(z_h) \otimes \mathcal{I}_{m1}(\frac{b}{z_h}, z_h) \cdot \left(1 + \mathcal{O}(\Lambda^2_{\mathrm{QCD}} b^2)\right).$$

Using the following equation

$$\begin{aligned}
\sum_i \int dz_i z_i F_{i1}(\frac{b}{z_i}, z_i, \mu, \nu) &= \sum_{i,m} \int dz_i z_i \int d\tau_m dx_i f_{i/m}(x_i) \mathcal{I}_{m1}(\frac{b}{\tau_m}, \tau_m) \delta(z_i - \tau_m x_i) \\
&= \sum_m \int d\tau_m \tau_m \mathcal{I}_{m1}(\frac{b}{\tau_m}, \tau_m) \left\{ \sum_i \int dx_i x_i f_{i/m}(x_i) = 1 \right\} \\
&= \sum_m \int d\tau_m \tau_m \mathcal{I}_{m1}(\frac{b}{\tau_m}, \tau_m) \equiv J_1(b, \mu, \nu),
\end{aligned}$$

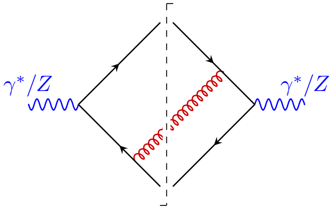
we simplify our formula to

$$\frac{1}{\hat{\sigma}_0} \frac{d\sigma}{d\tau_p} = \int_D d\upsilon d\omega H(\upsilon, \omega, \mu) \int \frac{db}{2\pi\xi} 2\cos\left(b\tau_p/\xi\right) S(b, \mu, \nu) J_q\left(b, \mu, \nu\right) J_{\bar{q}}\left(b, \mu, \nu\right) J_g\left(b, \mu, \nu\right)$$

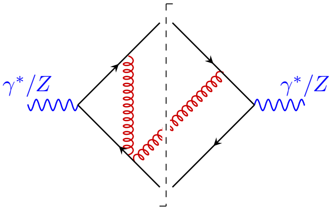
Hard Function: process dependent part

The hard function incorporates virtual correlations for $e^+e^- \to 3$ Jets

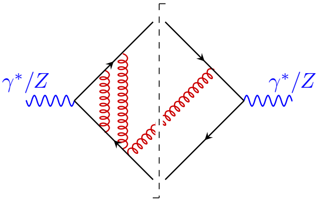
- LO



- NLO [Ellis et al., 1981; Fabricius et al., 1981]



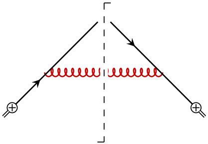
- NNLO(for future work) [Garland et all., 2001]



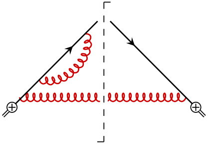
Jet Functions: process independent

The jet functions encode the collinear fragmentations, they are universal and same as for EEC [Moult and Zhu, 2018].

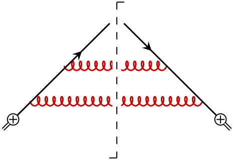
- NLO



- NNLO(RV) [Ming-xing Luo et al., appear soon]



- NNLO(RR) [Ming-xing Luo et al., appear soon]



The soft function: process independent

$$S\left(n_q, n_{\overline{q}}, n_g, b_x\right) = \mathrm{tr}\langle 0|\mathrm{T}\left[Y_{n_q} Y_{n_{\overline{q}}} Y_g\left(0, b_x, 0, 0\right)\right]\overline{\mathrm{T}}\left[Y^\dagger_{n_q} Y^\dagger_{n_{\overline{q}}} Y^\dagger_g(0,0,0,0)\right]|0\rangle$$

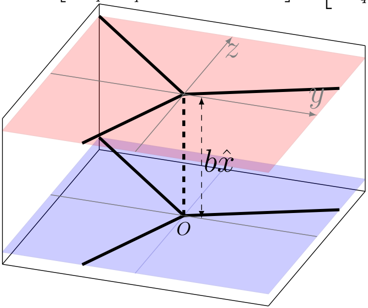


Figure: The spatial structure of the ETPC soft function. Each set of Wilson lines lies in the trijet plane, and their relative displacement is perpendicular to the plane

- Use the exponential regulator to deal with the rapidity divergences [Li et al., 2016]

$$\int d^d k \theta\left(k^0\right)\delta\left(k^2\right) \to \int d^d k \theta\left(k^0\right)\delta\left(k^2\right) e^{-2k^0 \tau e^{-\gamma_E}}\,, \quad \nu = \frac{1}{\tau}$$

Soft function: Factorization

In general, soft functions involve correlations of multiple Wilson lines and are very complicated. In our case, the soft functions factorize into multiplication of three dipole soft functions(at least to two loops)

$$S\left(n_q, n_{\bar{q}}, n_g, b_x, \mu, \nu\right) = \hat{S}_{q\bar{q}}\left(b_x, \mu, \nu, n_q, n_{\bar{q}}\right) \hat{S}_{qg}\left(b_x, \mu, \nu, n_q, n_g\right) \hat{S}_{\bar{q}g}\left(b_x, \mu, \nu, n_{\bar{q}}, n_g\right),$$

this is the crucial reason that we can go to analytic high order calculations.

- Impossible to construct scaling invariant variables from three light-like vectors.

- NLO

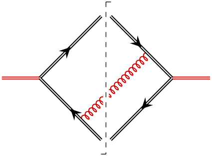


Figure: Only involve two Wilson lines

- NNLO(RR)

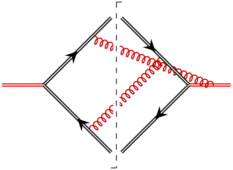


Figure: Can involve three Wilson lines, contribution cancles when summing all diagrams together [Catani and Grazzini, 1999]

Soft function: Factorization

- NNLO(RV)[Catani and Grazzini, 2000]

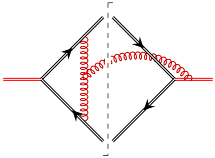


Figure: The triple color correlations vanish due to the color algebra

- Tree-level current

$$J_a^{\mu(0)}(q) = \sum_k T_k^a \frac{p_k^\mu}{p_k \cdot q}$$

- One-loop soft current

$$i f_{abc} \sum_{i \neq j} T_i^b T_j^c \left(\frac{p_i^\mu}{p_i \cdot q} - \frac{p_j^\mu}{p_j \cdot q} \right) \left(\frac{p_i \cdot p_j}{(p_i \cdot q)\,(p_j \cdot q)} \right)^\epsilon$$

- Color conservation $T_3^a = -T_1^a - T_2^a$ and

$$i f_{abc} T_i^a T_j^b (T_i^c + T_j^c) = -\delta_{ij} C_A T_i^2$$

$$\Longrightarrow i f_{abc} T_1^a T_2^b T_3^c = -i f_{abc} T_1^a T_2^b (T_1^c + T_2^c) = 0$$

Furthermore, $\hat{S}_{ij}\left(b_x, \mu, \nu, n_i, n_j\right) = S_{ij}\left(b_x, \mu, \nu\sqrt{n_i \cdot n_j/2}\right)$

where S_{ij} the back-to-back dipole soft functions calculated to three loops[Li and Zhu, 2017].

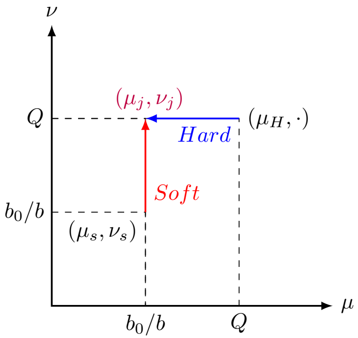
Renormalization Group Equations

$$
\begin{aligned}
\frac{dH}{d\ln\mu^2} &= \left[\frac{C_A+2C_F}{2}\gamma_{\rm cusp}\left(\alpha_s\right)\ln\frac{Q^2}{\mu^2}+\gamma_H\left(y,z,\alpha_s\right)\right]H\,,\\
\frac{d\ln S}{d\ln\mu^2} &= \left[\frac{2C_F+C_A}{2}\left(\gamma_{\rm cusp}\left[\alpha_s\right]\ln\frac{\mu^2}{\nu^2}-\gamma_s\left[\alpha_s\right]\right)+\frac{C_A}{2}\gamma_{\rm cusp}\left[\alpha_s\right]\ln\frac{(1-u)^2u}{\upsilon\omega}\right.\\
&\quad\left.+C_F\gamma_{\rm cusp}\left[\alpha_s\right]\ln\frac{(1-\upsilon)(1-\omega)}{u}\right]\,,\\
\frac{d\ln S}{d\ln\nu^2} &= \frac{2C_F+C_A}{2}\left(\int_{\mu^2}^{b_0^2/b^2}\frac{d\overline{\mu}^2}{\overline{\mu}^2}\gamma_{\rm cusp}\left(\alpha_s[\overline{\mu}]\right)+\gamma_r\left(\alpha_s\left[b_0/b\right]\right)\right)\,,\\
\frac{dJ_i}{d\ln\mu^2} &= \left(-\frac{1}{2}C_i\gamma_{\rm cusp}\ln\frac{\left(2p_i^0\right)^2}{\nu^2}+\gamma_{J,i}\right)J_i\,,\\
\frac{dJ_i}{d\ln\nu^2} &= \frac{C_i}{2}\left(\int_{b_0^2/b^2}^{\mu^2}\frac{d\overline{\mu}^2}{\overline{\mu}^2}\gamma_{\rm cusp}\left[\alpha_s(\overline{\mu})\right]-\gamma_r\left[\alpha_s\left(b_0/b\right)\right]\right)J_i\,.
\end{aligned}
$$

- All the anomalous dimensions are known to at least three loops
- RG invariant condition

$$
\gamma_H-\frac{C_A+2C_F}{2}\gamma_s-2\gamma_{J,q}-\gamma_{J,g}=0
$$

Setting $\mu = \mu_j = b_0/b$, $\nu = \nu_j = Q$,



There is rapidity evolution for the soft function

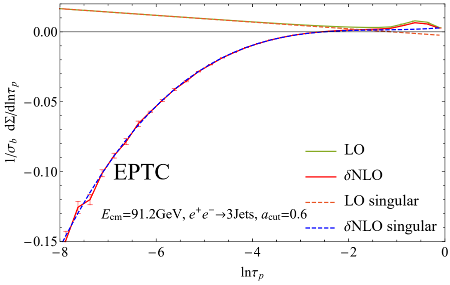
Numerical Implementation

- Apply the numerical code NLOJET++ [Nagy, 2001, 2003] to calculate the fixed-order ETPC: 4-jet LO + (5-jet real + 4-jet virtual) NLO
- Use two different settings:
  1. $a_{\text{cut}} = 0.6$ $(\sin\theta_{ij} > a_{\text{cut}}\,,\ \ \sin\theta_{jk} > a_{\text{cut}}\,,\ \ \sin\theta_{ki} > a_{\text{cut}})$
  2. $y_{\text{cut}} = 0.1$ $(y_3 > y_{\text{cut}})$
- Verify our factorization formula by comparing the predicted singular fixed-order results with NLOJET++
- Resummation + power corrections

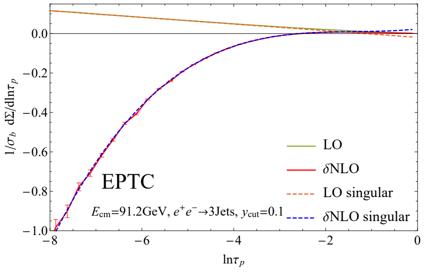
- Expanding the factorization formula with $\frac{\alpha_s}{4\pi}$, then integrate over b analytically

$$\frac{d\Sigma}{d\ln\tau_p} = \int_D dv d\omega \Big\{ \left(\frac{\alpha_s}{4\pi}\right)^2 (c_1 \ln\tau_p + c_2) + \left(\frac{\alpha_s}{4\pi}\right)^3 (c_3 \ln^3\tau_p + c_4 \ln^2\tau_p + c_5 \ln\tau_p + c_6) + \mathcal{O}(\tau_p^2) \Big\}.$$

- The predicted singular results of $\frac{d\Sigma}{d\ln\tau_p}$ is consistent with the full fixed-order results given by NLOJet++, in the $\tau_p \to 0$ limit. *So our factorization formula is correct*.

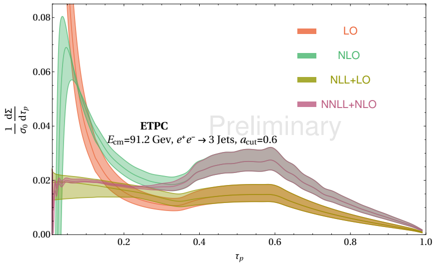


$a_{\text{cut}} = 0.6$

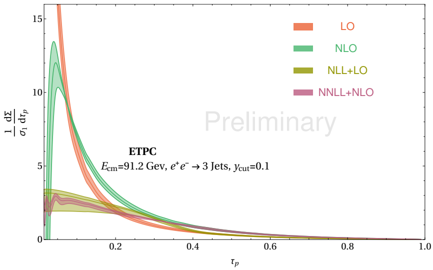


$y_{\text{cut}} = 0.1$

Resummation Preliminary!



$a_{\text{cut}} = 0.6$



$y_{\text{cut}} = 0.1$

- The fixed-order results are unreasonable in the $\tau_p \to 0$ limit, resummation is necessary in this region
- The reduction of scale uncertainties from NLO to NNLL+NLO
- The perturbative corrections from NLL+LO to NNLL+NLO are large
- NLO LO do not overlap; NLL NNLL overlap, converge

Conclusions

- We initiated the study of a new event shape observable called the Energy Triple-Product Correlation
- Derived an all order factorization formula for the ETPC in the coplanar limit
- Presented the results of NNLL matching with NLO

Thank You!