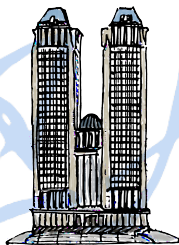


Baryon form factors

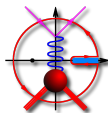
Rinaldo Baldini Ferroli, Alessio Mangoni and Simone Pacetti



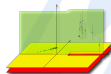
**Workshop on form factor, polarization and CP violation
in quantum-correlated hyperon-antihyperon production**

Fudan University, Shanghai, People's Republic of China

July 7th and 8th, 2019



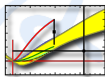
Baryon form factors and dispersion relations



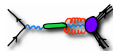
Space-like and time-like G_E^p/G_M^p via DR's



Asymptotic G_M^p from a DR sum rule



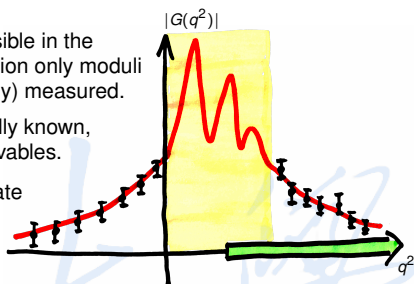
Hints for the ratio of Λ form factors



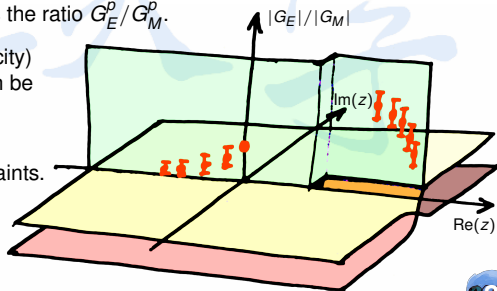
Clues from the J/ψ decays

About proton form factors

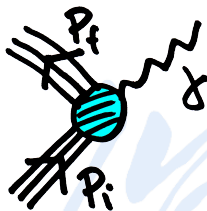
- * Proton form factors are completely accessible in the space-like region while in the time-like region only moduli above the physical threshold can be (easily) measured.
- ▲ In the space-like region they are individually known, especially by means of polarization observables.
- ◆ Only recently, attempts to measure separate value of moduli in the time-like region have become stronger.



- ◎ So far, the better known quantity is the ratio G_E^p/G_M^p .
- * Using dispersion relations (analyticity) space-like and time-like values can be exploited to extract information on phase and asymptotic behavior.
- * Analyticity imposes serious constraints. A space-like zero for the ratio G_E^p/G_M^p does require an asymptotic phase of 180° .



Proton-photon vertex



Nucleon electromagnetic four-current ($q = p_f - p_i$)

$$\langle P_f | J_{EM}^\mu(0) | P_i \rangle = e \bar{u}(p_f) \left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2M_p} F_2(q^2) \right] u(p_i)$$

$F_1(q^2)$ and $F_2(q^2)$ are the Dirac and Pauli form factors

$$F_1(0) = Q_p$$

$$F_2(0) = \kappa_p$$

$Q_p =$ electric charge

$\kappa_p =$ anomalous magnetic moment

Breit frame

$$p_f = (E, \vec{q}/2)$$

$$q = (0, \vec{q})$$

$$p_i = (E, -\vec{q}/2)$$

$$\langle P_f | J_{EM}^\mu(0) | P_i \rangle \equiv J_{EM}^\mu = (J_{EM}^0, \vec{J}_{EM})$$

$$\odot J_{EM}^0 = e \left(F_1(q^2) + \frac{q^2}{4M_p^2} F_2(q^2) \right)$$

$$\diamond \vec{J}_{EM} = e \bar{u}(p_f) \vec{\gamma} u(p_i) (F_1(q^2) + F_2(q^2))$$

Sachs form factors

$$\odot G_E(q^2) = F_1(q^2) + \frac{q^2}{4M_p^2} F_2(q^2)$$

$$\diamond G_M(q^2) = F_1(q^2) + F_2(q^2)$$

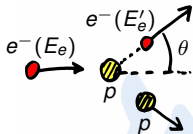
Normalizations

$$\odot G_E(0) = Q_p$$

$$\diamond G_M(0) = \mu_p = \kappa_p + Q_p$$

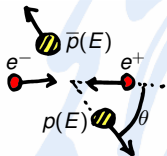
$\mu_p =$ total magnetic moment

Cross sections and Coulomb correction



Elastic scattering cross section (Rosenbluth)

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 E'_\theta \cos^2\left(\frac{\theta}{2}\right)}{4E_\theta^3 \sin^4\left(\frac{\theta}{2}\right)} \left[G_E^2 - \tau \left(1 + 2(1-\tau) \tan^2\left(\frac{\theta}{2}\right) \right) G_M^2 \right] \frac{1}{1-\tau}$$



Annihilation cross section


$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta C}{16E^2} \left[(1 + \cos^2(\theta)) |G_M|^2 + \frac{1}{\tau} \sin^2(\theta) |G_E|^2 \right]$$


$$\tau = E^2 / M_p^2$$

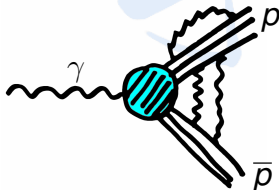
$$\beta = \sqrt{1 - 1/\tau}$$

Coulomb correction

$$C = \frac{\pi\alpha/\beta}{1 - e^{-\pi\alpha/\beta}}$$

 $p\bar{p}$ Coulomb interaction as FSI

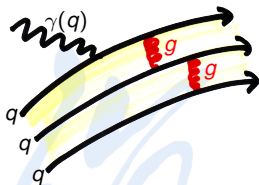
 Only S-wave



pQCD asymptotic behavior

Space-like region

V.A. Matveev, R.M. Muradian, A.N. Tavkhelidze,
 LNC7 (1973) 719
 S. J. Brodsky, G. R. Farrar, PRL31 (1973) 1153
 M. V. Galinsky, E. A. Kuraev JETPL96 (2012) 6



- ⚠ **pQCD:** as $q^2 \rightarrow -\infty$, F_1 , F_2 , G_E , G_M follow power laws driven by counting rules
- 🌀 Valence quarks exchange gluons to distribute the photon momentum transfer q

Non-helicity-flip current $J^{\lambda, \lambda}(q^2)$

- ⚠ $J^{\lambda, \lambda}(q^2) \propto G_M(q^2)$
- 🔍 Two gluon propagators
- 🌀 $G_M(q^2) \underset{q^2 \rightarrow -\infty}{\sim} (q^2)^{-2}$

Dirac and Pauli form factors

- 🔍 $F_1(q^2) \underset{q^2 \rightarrow -\infty}{\sim} (q^2)^{-2}$
- 🌀 $F_2(q^2) \underset{q^2 \rightarrow -\infty}{\sim} (q^2)^{-3}$

Helicity-flip current $J^{\lambda, -\lambda}(q^2)$

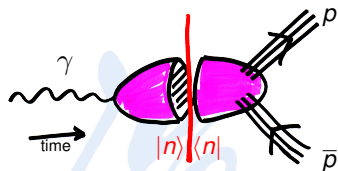
- ⚠ $J^{\lambda, -\lambda}(q^2) \propto G_E(q^2)/\sqrt{-q^2}$
- 🔍 [Two gluon propagators]/ $\sqrt{-q^2}$
- 🌀 $G_E(q^2) \underset{q^2 \rightarrow -\infty}{\sim} (q^2)^{-2}$

Ratio of Sachs form factors

- ⚠ $\frac{G_E(q^2)}{G_M(q^2)} \underset{q^2 \rightarrow -\infty}{\sim} \text{constant}$

Nucleon form factors

Time-like region ($q^2 > 0$)



◇ Crossing symmetry:

$$\langle P(p') | j^\mu | P(p) \rangle \rightarrow \langle \bar{P}(p') P(p) | j^\mu | 0 \rangle$$

◎ Form factors are complex functions of q^2

Optical theorem

$$\text{Im} \langle \bar{P}(p') P(p) | j^\mu | 0 \rangle \sim \sum_n \langle \bar{P}(p') P(p) | j^\mu | n \rangle \langle n | j^\mu | 0 \rangle \implies \begin{cases} \text{Im} F_{1,2} \neq 0 \\ \text{for } q^2 > 4M_\pi^2 \end{cases}$$

$|n\rangle$ are on-shell intermediate states: $2\pi, 3\pi, 4\pi, \dots$

Time-like asymptotic behavior

Phragmén Lindelöf theorem

If $f(z) \rightarrow a$ as $z \rightarrow \infty$ along a straight line, and $f(z) \rightarrow b$ as $z \rightarrow \infty$ along another straight line, and $f(z)$ is regular and bounded in the angle between, then $a = b$ and $f(z) \rightarrow a$ uniformly in this angle.

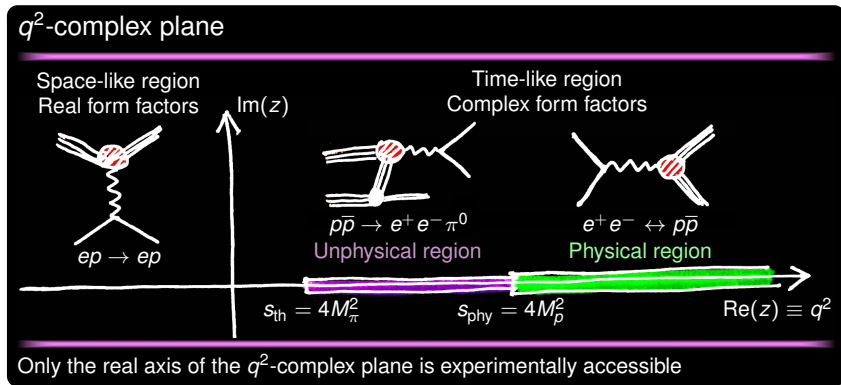
$$\underbrace{\lim_{q^2 \rightarrow -\infty} G_{E,M}(q^2)}_{\text{space-like}} = \lim_{q^2 \rightarrow +\infty} \underbrace{G_{E,M}(q^2)}_{\text{time-like}}$$

$$\triangle G_{E,M} \sim (q^2)^{-2}$$

Must be real



Analyticity of form factors



Space-like region $q^2 < 0$	Time-like region* $s_{th} < q^2 \leq s_{phy}$	Time-like region $q^2 > s_{phy}$	
$ep \rightarrow ep$	$p\bar{p} \rightarrow e^+ e^- \pi^0$	$e^+ e^- \leftrightarrow p\bar{p}$	$e^+ e^- \leftrightarrow p\bar{p}$ (pol.)
G_E, G_M	$ G_E , G_M $	$ G_E , G_M $	$ G_E , G_M , \arg(G_E/G_M)$

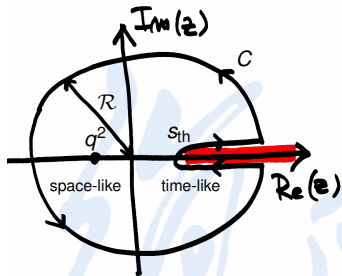
* C. Adamuscin, E.A. Kuraev, E. Tomasi-Gustafsson, F. Maas PRC75, 045205

E. A. Kuraev et al., JETP115, 93

G. I. Gakh, E. Tomasi-Gustafsson, A. Dbeyssi, A.G. Gakh PRC86, 025204



Dispersion relations



- * The form factors are **analytic** on the q^2 -plane with a **multiple cut** ($s_{\text{th}} = 4M_{\pi}^2, \infty$)
- * **Dispersion relation for the imaginary part** ($q^2 < 0$)

$$G(q^2) = \lim_{R \rightarrow \infty} \frac{1}{2\pi i} \oint_C \frac{G(z) dz}{z - q^2} = \frac{1}{\pi} \int_{s_{\text{th}}}^{\infty} \frac{\text{Im} G(s) ds}{s - q^2}$$

- * **Dispersion relation for the logarithm** ($q^2 < 0$)

B.V. Geshkenbein, Yad. Fiz. 9 (1969) 1232.

$$\ln G(q^2) = \frac{\sqrt{s_{\text{th}} - q^2}}{\pi} \int_{s_{\text{th}}}^{\infty} \frac{\ln |G(s)| ds}{(s - q^2) \sqrt{s - s_{\text{th}}}}$$

Experimental inputs

- ⊙ Space-like data on the **real values** of form factors from: $ep \rightarrow ep$ and $e^{\uparrow} p \rightarrow e^{-} p^{\uparrow}$, with polarization
- ⊠ Time-like data on form factor **moduli** from: $e^+ e^- \leftrightarrow p \bar{p}$
- ⚠ Time-like data on G_E/G_M **phase** from: $e^+ e^- \leftrightarrow p^{\uparrow} \bar{p}$ (pol.)

Theoretical ingredients

- ⊙ Analyticity \Rightarrow convergence relations
- ⊠ Normalization and threshold values
- ⚠ Asymptotic behavior \Rightarrow super-convergence relations

Advantages and drawbacks of dispersive approaches

Advantages



DR's are based on unitarity and analyticity \Rightarrow **model-independent approach**



DR's relate data from different processes in different energy regions

$$\left[\begin{array}{c} \text{space-like} \\ \text{form factor} \\ ep \rightarrow ep \end{array} \right] = \int_{s_{\text{th}}}^{\infty} \left[\begin{array}{c} \text{Im(form factor) or } \ln|\text{form factor}| \\ \text{over the time-like cut } (s_{\text{th}}, \infty) \\ e^+e^- \rightarrow p\bar{p} + \text{theory} \end{array} \right]$$



Normalizations and theoretical constraints can be directly implemented



Form factors can be computed in the whole q^2 -complex plane

Drawbacks



Very long-range integration

Remedy #1

pQCD power laws

Remedy #2

Subtracted DR's



No data in the unphysical region, crucial in dispersive analyses

Advantages and drawbacks of dispersive approaches

Advantages



DR's are based on unitarity and analyticity \Rightarrow **model-independent approach**



DR's relate data from different processes in different energy regions

$$\left[\begin{array}{c} \text{space-like} \\ \text{form factor} \\ ep \rightarrow ep \end{array} \right] = \int_{s_{\text{th}}}^{\infty} \left[\begin{array}{c} \text{Im(form factor) or } \ln|\text{form factor}| \\ \text{over the time-like cut } (s_{\text{th}}, \infty) \\ e^+ e^- \rightarrow p\bar{p} + \text{theory} \end{array} \right]$$



Normalizations and theoretical constraints can be directly implemented



Form factors can be computed in the whole q^2 -complex plane



Poles cancel out in the ratio!

Drawbacks



Very long-range integration

Remedy #1

pQCD power laws

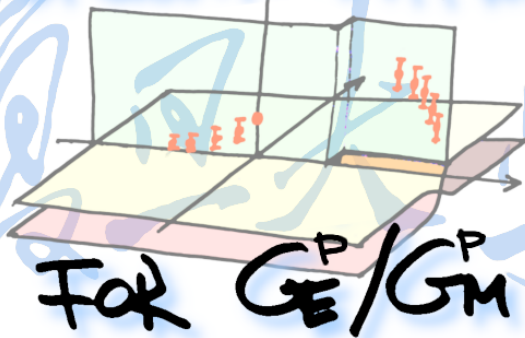
Remedy #2

Subtracted DR's



No data in the unphysical region, crucial in dispersive analyses

A DISPERSIVE APPROACH

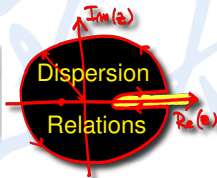


Dispersive approach for the ratio $R = \mu_p G_E^p / G_M^p$

We start from the imaginary part of the ratio $R(q^2)$, written in the most general and model-independent way as

$$I(q^2) \equiv \text{Im}[R(q^2)] = \text{series of orthogonal polynomials}$$

Theoretical constraints can be applied directly on this function $I(q^2)$



The function $R(q^2)$ is reconstructed in time and space-like regions

Additional theoretical conditions and the experimental constraints can be imposed on the obtained analytic expression of $R(q^2)$



Parametrization for $R = \mu_p G_E^p / G_M^p$

The imaginary part of $R(q^2)$ is parametrized by two series of orthogonal polynomials

$$\text{Im} [R(q^2)] \equiv I(q^2) = \begin{cases} \sum_i C_i T_i(x) & x = \frac{2q^2 - s_{\text{phy}} - s_{\text{th}}}{s_{\text{phy}} - s_{\text{th}}} \quad s_{\text{th}} \leq q^2 \leq s_{\text{phy}} \\ \sum_j D_j T_j(x') & x' = \frac{2s_{\text{phy}}}{q^2} - 1 \quad q^2 > s_{\text{phy}} \end{cases}$$

Theoretical conditions on $\text{Im} [R(q^2)]$

- ① $R(4M_\pi^2)$ is real $\implies I(4M_\pi^2) = 0$
- ② $R(4M_\rho^2)$ is real $\implies I(4M_\rho^2) = 0$
- ③ $R(\infty)$ is real $\implies I(\infty) = 0$

Theoretical conditions on $R(q^2)$

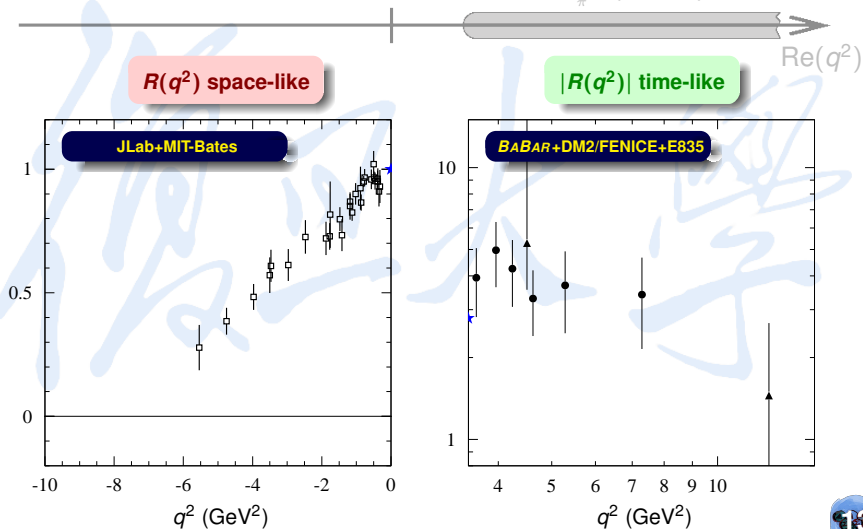
- ④ Continuity at $q^2 = 4M_\pi^2$
- ④ $R(4M_\rho^2)$ is real and $\text{Re} [R(4M_\rho^2)] = \mu_p$

Experimental conditions on $R(q^2)$ and $|R(q^2)|$

- ⚠ Space-like region ($q^2 < 0$) data for R from JLab and MIT-Bates
- ⚠ Time-like region ($q^2 \geq 4M_\rho^2$) data for $|R|$ from FENICE+DM2, BABAR, and E835



$$R(q^2) = R(0) + \frac{q^2}{\pi} \int_{4M_\pi^2}^{\infty} \frac{\text{Im}[R(s)]}{s(s - q^2)} ds$$

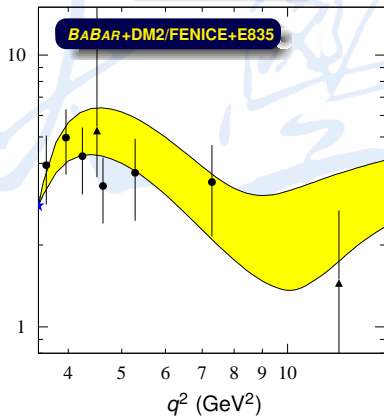
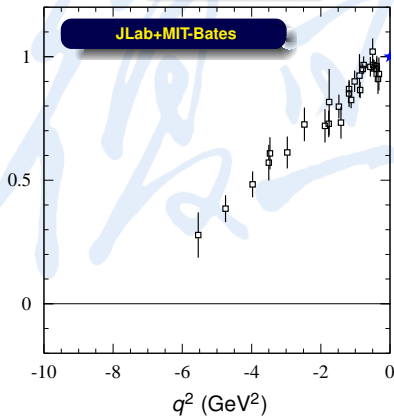


$$R(q^2) = R(0) + \frac{q^2}{\pi} \int_{4M_\pi^2}^{\infty} \frac{\text{Im}[R(s)]}{s(s - q^2)} ds$$

 $\text{Re}q^2$

$R(q^2)$ space-like

$|R(q^2)|$ time-like

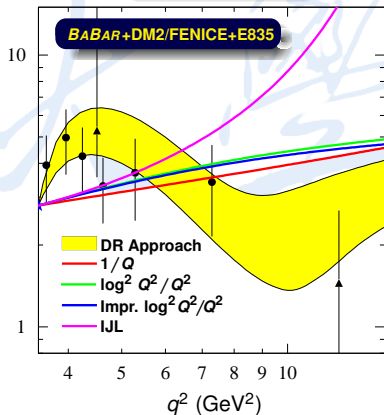
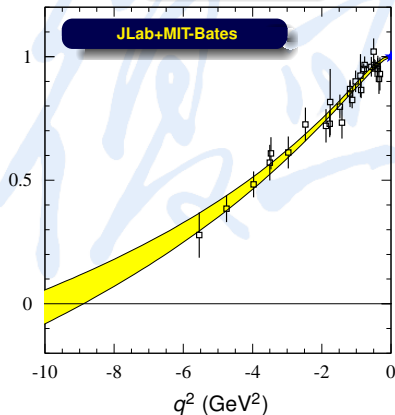


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$R(q^2)$ space-like

$|R(q^2)|$ time-like

$\text{Re} q^2$

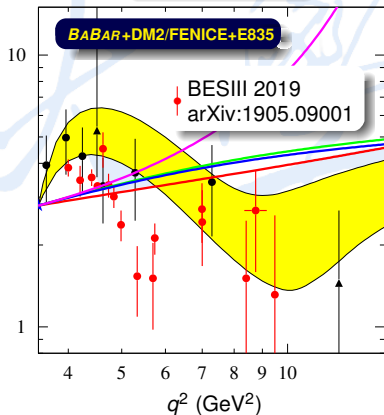
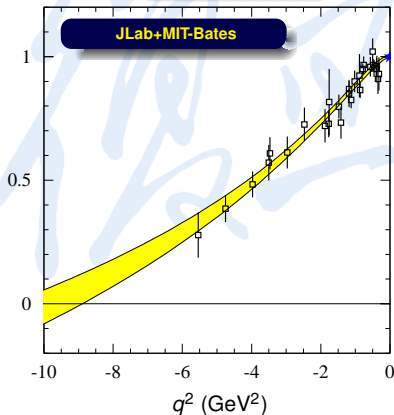


$$R(q^2) = R(0) + \frac{q^2}{\pi} \int_{4M_\pi^2}^{\infty} \frac{\text{Im}[R(s)]}{s(s - q^2)} ds$$

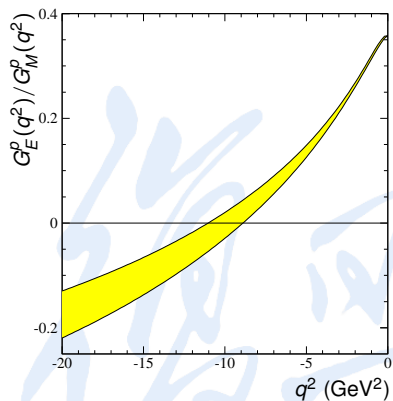
$R(q^2)$ space-like

$|R(q^2)|$ time-like

$\text{Re}q^2$

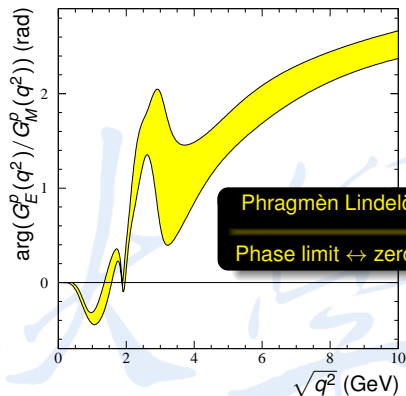


Space-like zero and phase



Space-like zero

$$t_0^{BABAR} = (-10 \pm 1) \text{ GeV}^2$$



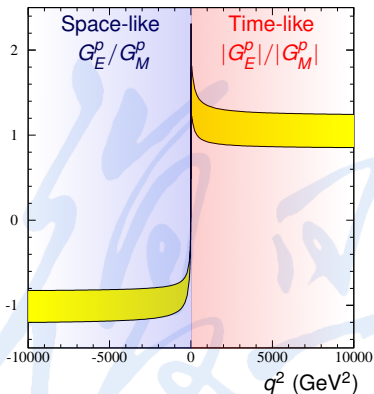
Phragmén Lindelöf

Phase limit \leftrightarrow zeros

Phase from dispersion relations

$$\phi(q^2) = -\frac{\sqrt{q^2 - s_{th}}}{\pi} \text{Pr} \int_{s_{th}}^{\infty} \frac{\ln |R(s)| ds}{\sqrt{s - s_{th}}(s - q^2)}$$

Asymptotic G_E^p/G_M^p



Real asymptotic values for G_E^p/G_M^p

$$\diamond \frac{G_E^p}{G_M^p} \Big|_{|q^2| \rightarrow \infty} \rightarrow -1.0 \pm 0.2$$

Asymptotic behaviour of F_2^p/F_1^p

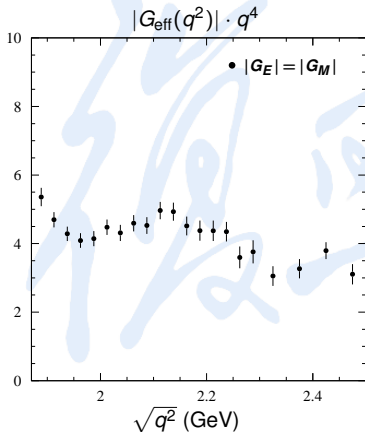
$$\circlearrowleft \left| \frac{q^2}{4M_N^2} \frac{F_2}{F_1} \right| \Big|_{|q^2| \rightarrow \infty} \rightarrow \left| \frac{G_E^p}{G_M^p} - 1 \right| = 2.0 \pm 0.2$$

pQCD prediction

$$\left| \frac{G_E^p(q^2)}{G_M^p(q^2)} \right| \Big|_{|q^2| \rightarrow \infty} \rightarrow 1$$

$|G_E^p|$ and $|G_M^p|$ from $p\bar{p}$ cross section and dispersion relations

EPJA32, 421 (2007)

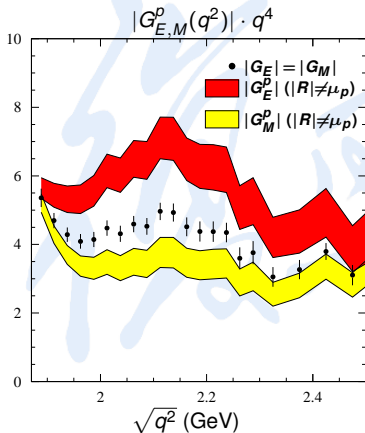


$$|G_{\text{eff}}(q^2)|^2 = \frac{\sigma_{p\bar{p}}(q^2)}{\frac{4\pi\alpha^2\beta C}{3s}} \left(1 + \frac{1}{2\tau}\right)^{-1}$$

- Usually what is extracted from the cross section $\sigma(e^+e^- \rightarrow p\bar{p})$ is the effective time-like form factor $|G_{\text{eff}}^p|$ obtained assuming $|G_E^p| = |G_M^p|$ i.e. $|R| = \mu_p$

$|G_E^p|$ and $|G_M^p|$ from $p\bar{p}$ cross section and dispersion relations

EPJA32, 421 (2007)

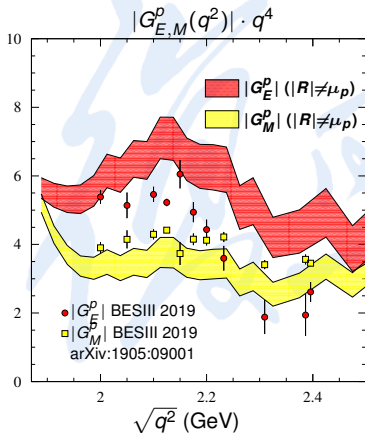


$$|G_M^p(q^2)|^2 = \frac{\sigma_{p\bar{p}}(q^2)}{4\pi\alpha^2\beta C} \left(1 + \frac{|R(q^2)|}{2\mu_p\tau}\right)^{-1}$$

- ◆ Usually what is extracted from the cross section $\sigma(e^+e^- \rightarrow p\bar{p})$ is the effective time-like form factor $|G_{\text{eff}}^p|$ obtained assuming $|G_E^p| = |G_M^p|$ i.e. $|R| = \mu_p$
- * Using our parametrization for R and the **BABAR** data on $\sigma(e^+e^- \rightarrow p\bar{p})$, $|G_E^p|$ and $|G_M^p|$ may be disentangled

$|G_E^p|$ and $|G_M^p|$ from $p\bar{p}$ cross section and dispersion relations

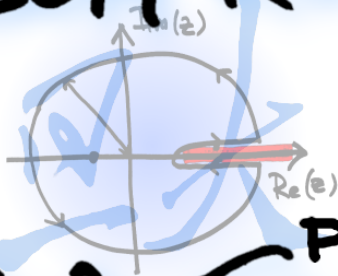
EPJA32, 421 (2007)



$$|G_M^p(q^2)|^2 = \frac{\sigma_{p\bar{p}}(q^2)}{4\pi\alpha^2\beta C} \left(1 + \frac{|R(q^2)|}{2\mu_p\tau}\right)^{-1}$$

- ◆ Usually what is extracted from the cross section $\sigma(e^+e^- \rightarrow p\bar{p})$ is the effective time-like form factor $|G_{\text{eff}}^p|$ obtained assuming $|G_E^p| = |G_M^p|$ i.e. $|R| = \mu_p$
- * Using our parametrization for R and the **BABAR** data on $\sigma(e^+e^- \rightarrow p\bar{p})$, $|G_E^p|$ and $|G_M^p|$ may be disentangled

A SUM RULE



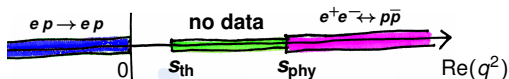
FOR G_N^P

Dispersion relations and sum rules

Geshkenbein, Ioffe, Shifman Yad. Fiz. 20, 128 (1974)

- * DR's connect space and time values of a form factor $G(q^2)$

$$G(q^2) = \frac{1}{\pi} \int_{s_{\text{th}}}^{\infty} \frac{\text{Im}G(s) ds}{s - q^2}$$



Drawbacks

- * The imaginary part is not experimentally accessible
- * There are no data in the unphysical region $[s_{\text{th}}, s_{\text{phy}}]$
- * We need to know the asymptotic behavior

- * They applied the DR for the imaginary part to the function

$$\phi(z) = f(z) \frac{\ln G(z)}{z\sqrt{s_{\text{th}} - z}} \quad \text{with} \quad \int_0^{s_{\text{phy}}} f^2(z) dz \ll 1$$

Advantages

- 🌀 The DR integral contains the modulus $|G(s)|$
- 🌀 The unphysical region contribution is suppressed

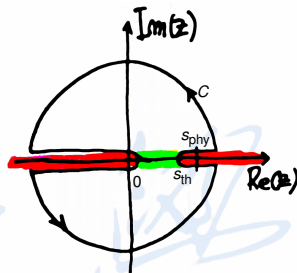
Drawback

- 🌀 Zeros of $G(z)$ are poles for $\phi(z)$

Assuming $G(q^2) \neq 0$ and using the Cauchy theorem, we have the new DR

$$\oint_C \phi(z) dz = 0$$

$$\underbrace{-\int_{-\infty}^0 \frac{\text{Im}[f(t)] \ln G(t)}{t\sqrt{s_{\text{th}} - t}} dt}_{\text{Space-like}} \Downarrow \underbrace{\int_{s_{\text{th}}}^{\infty} \frac{f(s) \ln |G(s)|}{s\sqrt{s - s_{\text{th}}}} ds}_{\text{Time-like}}$$

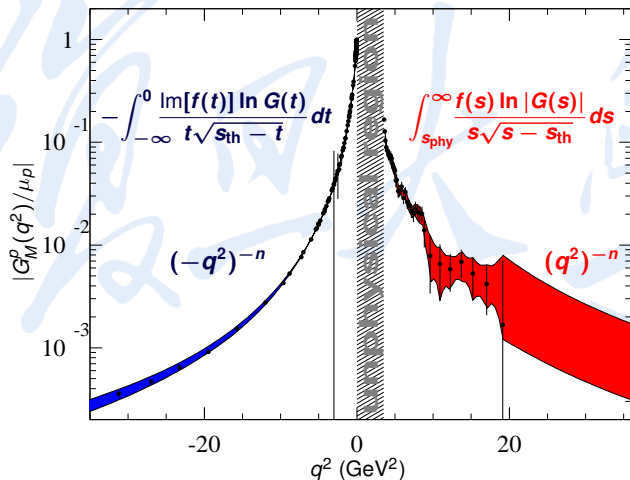


Convergence relation to find the asymptotic power-law behavior of G_M

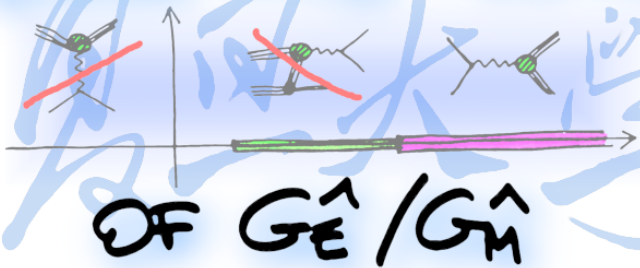
$$\underbrace{-\int_{-\infty}^0 \frac{\text{Im}[f(t)] \ln G(t)}{t\sqrt{s_{\text{th}} - t}} dt}_{\text{Space-like data} + (-t)^{-n}} = \int_{s_{\text{th}}}^{\infty} \frac{f(s) \ln |G(s)|}{s\sqrt{s - s_{\text{th}}}} ds \approx \underbrace{\int_{s_{\text{phy}}}^{\infty} \frac{f(s) \ln |G(s)|}{s\sqrt{s - s_{\text{th}}}} ds}_{\text{Time-like data} + s^{-n}}$$

n is the only free parameter

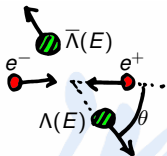
$$G_M^p(q^2) \underset{|q^2| \rightarrow \infty}{\propto} |q^2|^{-(2.27 \pm 0.36)}$$



PHASE AND MODULUS



Λ form factors



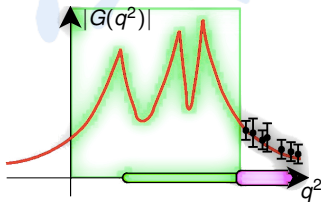
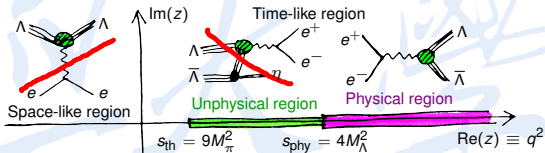
Same definitions, but for labels and Coulomb factor...
Annihilation cross section

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta \zeta}{16E^2} \left[(1 + \cos^2(\theta)) |G_M^\Lambda|^2 + \frac{1}{\tau} \sin^2(\theta) |G_E^\Lambda|^2 \right]$$

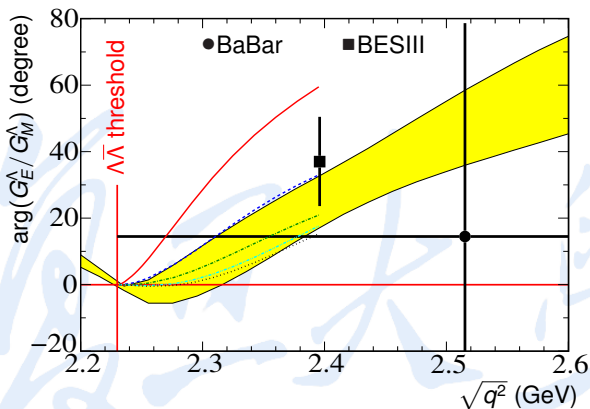
$$\tau = E^2 / M_\Lambda^2$$

$$\beta = \sqrt{1 - 1/\tau}$$

- * Same analyticity as for nucleons.
- * Difficult to measure in space-like and unphysical regions.
- * Relative phase from weak decay.



- ◆ Same unitarity and intermediate states contributions, but for the isospin.
- ◆ Form factors have not vanishing imaginary part above the theoretical threshold.

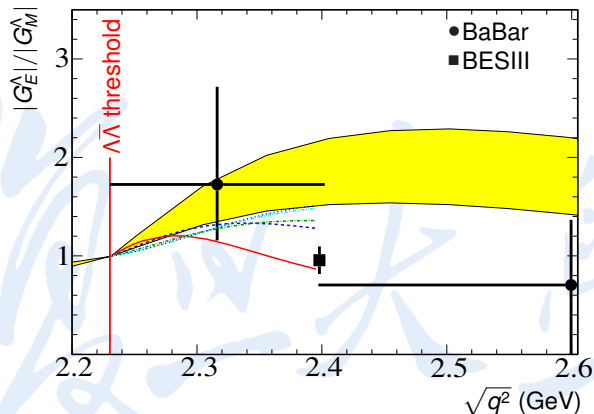


▲ Theoretical prediction based considering only $\Lambda\bar{\Lambda}$ FSI
 [J. Haidenbauer, U.-G. Meissner, PLB 761 (2016) 456]

◎ Data from BaBar and BESIII (preliminary)
 [PRD 76 (2007) 092006, arXiv:1903.09421 [hep-ex]]

* "Lambdization" of proton, i.e., proton results with $\sqrt{q^2} \rightarrow \sqrt{q^2} + 2(M_\Lambda - M_p)$
 [EPJA32 421 (2007)]

Modulus of $G_E^\Lambda / G_M^\Lambda$



Theoretical prediction based considering only $\Lambda\bar{\Lambda}$ FSI

[J. Haidenbauer, U.-G. Meissner, PLB 761 (2016) 456]



Data from BaBar and BESIII (preliminary)

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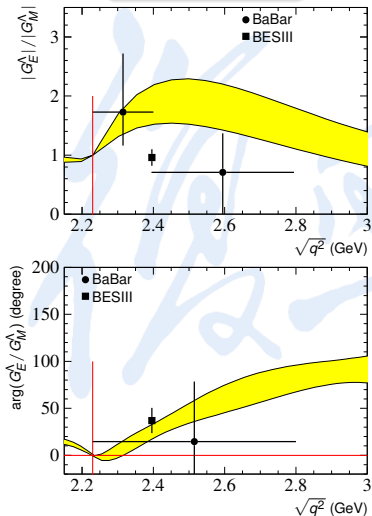


"Lambdization" of proton, i.e., proton results with $\sqrt{q^2} \rightarrow \sqrt{q^2} + 2(M_\Lambda - M_p)$

[EPJA32 421 (2007)]

Modulus and phase of G_E^Λ/G_M^Λ

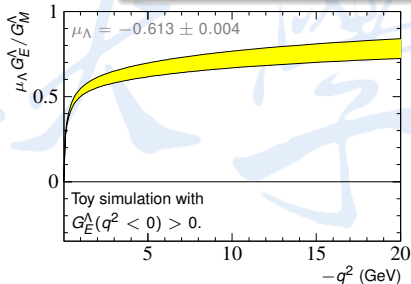
Time-like region



Space-like region

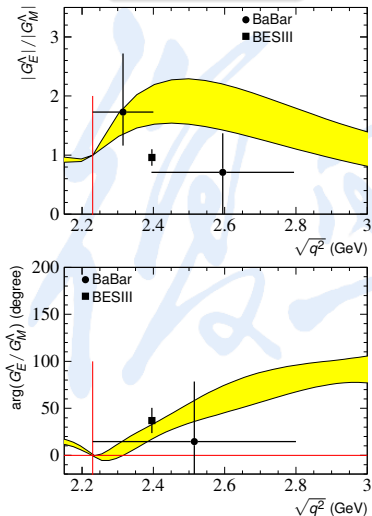
- The space-like FF ratio can be obtained by means of the **two** dispersion relations for the logarithm and phase.
- The cancellation of poles in the ratio reduces the dependence on the unknown unphysical region.
- Since $G_E^\Lambda(0) = 0$ the ratio has a zero at $q^2 = 0$.

$$\Rightarrow \arg \left[G_E^\Lambda(q^2)/G_M^\Lambda(q^2) \right] \xrightarrow{q^2 \rightarrow \infty} \pi$$



Modulus and phase of $G_E^\Lambda / G_M^\Lambda$

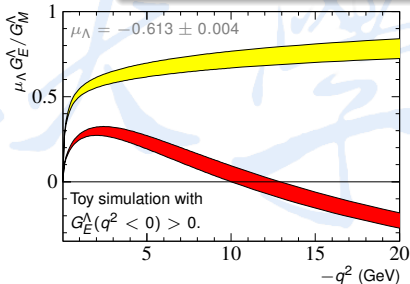
Time-like region



Space-like region

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- The cancellation of poles in the ratio reduces the dependence on the unknown unphysical region.
- Since $G_E^\Lambda(0) = 0$ the ratio has a zero at $q^2 = 0$.


$$\Rightarrow \arg \left[G_E^\Lambda(q^2) / G_M^\Lambda(q^2) \right] \xrightarrow{q^2 \rightarrow \infty} \pi$$



In case of an additional zero at $q^2 < 0$

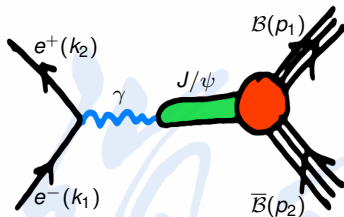
$$\arg \left[G_E^\Lambda(q^2) / G_M^\Lambda(q^2) \right] \xrightarrow{q^2 \rightarrow \infty} 2\pi$$

CLUES



FROM THE J/ψ DECAYS

J/ψ decay amplitude



Baryon four-current ($q = p_1 + p_2$)

$$J_B^\mu = \bar{u}(p_1) \left[\gamma^\mu f_1^B + \frac{i\sigma^{\mu\nu} q_\nu}{2M_B} f_2^B \right] v(p_2)$$

f_1^B and f_2^B are the Dirac and Pauli couplings

Amplitude of $e^+ e^- \rightarrow J/\psi \rightarrow B\bar{B}$

$$\mathcal{M} = -ie^2 J_B^\mu D_\psi(q^2) \bar{v}(k_2) \gamma_\mu u(k_1)$$

$D_\psi(q^2)$ is the J/ψ propagator

Differential cross section of $e^+ e^- \rightarrow J/\psi \rightarrow B\bar{B}$

$$\frac{d\sigma_{e^+e^- \rightarrow J/\psi \rightarrow B\bar{B}}}{d\cos(\theta)} = \frac{\pi\alpha^2\beta}{2M_\psi^2} \left(|g_M^B|^2 + \frac{4M_B^2}{M_\psi^2} |g_E^B|^2 \right) (1 + \alpha_\psi^B \cos^2(\theta))$$

Polarization parameter

$$\alpha_\psi^B = \frac{M_\psi^2 |g_M^B|^2 - 4M_B^2 |g_E^B|^2}{M_\psi^2 |g_M^B|^2 + 4M_B^2 |g_E^B|^2}$$

Sachs couplings

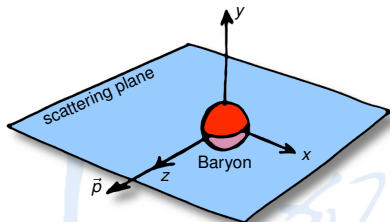
$$g_E^B = f_1^B + \frac{M_\psi^2}{4M_B^2} f_2^B \quad \Bigg| \quad g_M^B = f_1^B + f_2^B$$

Polarization formulae in the time-like region

The ratio $R(q^2)$ is complex for $q^2 \geq 4M_\pi^2$

$$\frac{G_E(q^2)}{G_M(q^2)} = \frac{|G_E(q^2)|}{|G_M(q^2)|} e^{i\rho(q^2)}$$

The polarization depends on the phase ρ



[A.Z. Dubnickova, S. Dubnicka, M.P. Rekalo, NCA109,241(96)]

$$\diamond P_y = - \frac{\sin(2\theta) \sin(\rho)}{D\sqrt{\tau}} \frac{|G_E|}{|G_M|} = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} \equiv \mathcal{A}_y \left. \vphantom{\frac{\sin(2\theta) \sin(\rho)}{D\sqrt{\tau}}} \right\} \text{Does not depend on } P_e$$

$$\diamond P_x = - P_e \frac{2 \sin(2\theta) \cos(\rho)}{D\sqrt{\tau}} \frac{|G_E|}{|G_M|}$$

$$\diamond P_z = P_e \frac{2 \cos(\theta)}{D} \left. \vphantom{\frac{2 \cos(\theta)}{D}} \right\} \text{Does not depend on } \rho$$

$$D = 1 + \cos^2(\theta) + \frac{|G_E|^2 \sin^2(\theta)}{|G_M|^2 \tau}$$

$$\tau = \frac{q^2}{4M_B^2}$$

* P_e is the electron polarization

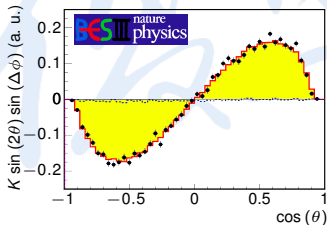
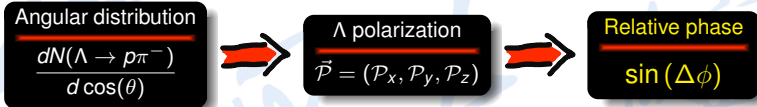
* θ is the scattering angle

$J/\psi\Lambda\bar{\Lambda}$ Sachs coupling constants

- 🌀 If the relative phase between g_E^Λ and g_M^Λ is not an integer multiple of π it follows

$$\Delta\phi \equiv \arg\left(g_E^\Lambda/g_M^\Lambda\right) \neq k\pi \quad \forall k \in \mathbb{Z} \quad \Rightarrow \quad \sin(\Delta\phi) \neq 0 \quad \Rightarrow \quad \mathcal{P}_y \neq 0$$

- 🌀 The polarization of Λ baryons can be measured through their weak decay



🌀 $\Delta\phi = 42.4 \pm 0.6 \pm 0.5^\circ$

🌀 $\alpha_\psi^\Lambda = 0.461 \pm 0.006 \pm 0.007$

🌀 $\frac{|g_E^\Lambda|}{|g_M^\Lambda|} = \frac{M_\psi}{2M_\Lambda} \sqrt{\frac{1 - \alpha_\psi^\Lambda}{1 + \alpha_\psi^\Lambda}} = 0.84 \pm 0.01$

Sachs coupling constants g_E^Λ and g_M^Λ of the J/ψ are complex

$\text{Im}(g_E^\Lambda/g_M^\Lambda) \neq 0$



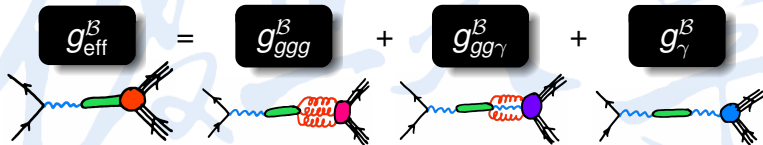
❖ Total cross section of $e^+e^- \rightarrow J/\psi \rightarrow B\bar{B}$

$$\sigma_{e^+e^- \rightarrow J/\psi \rightarrow B\bar{B}} = \frac{4\pi\alpha^2\beta}{3M_\psi^2} \left(|g_M^B|^2 + \frac{2M_B^2}{M_\psi^2} |g_E^B|^2 \right) = \frac{4\pi\alpha^2\beta}{3M_\psi^2} \left(1 + \frac{2M_B^2}{M_\psi^2} \right) |g_{\text{eff}}^B|^2$$

❖ Decay rate of $J/\psi \rightarrow B\bar{B}$

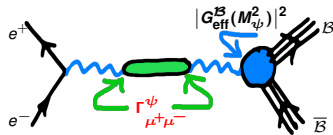
$$\Gamma_{J/\psi \rightarrow B\bar{B}} = \frac{M_\psi\beta}{12\pi} \left(|g_M^B|^2 + \frac{2M_B^2}{M_\psi^2} |g_E^B|^2 \right) = \frac{M_\psi\beta}{12\pi} \left(1 + \frac{2M_B^2}{M_\psi^2} \right) |g_{\text{eff}}^B|^2$$

❖ The effective coupling constant g_{eff}^B has three contributions



❖ $\Gamma_{J/\psi \rightarrow B\bar{B}}(g_\gamma^B) = \Gamma_{\mu^+\mu^-}^\psi \frac{\sigma_{B\bar{B}}(M_\psi^2)}{\sigma_{\mu^+\mu^-}(M_\psi^2)}$

$$|G_{\text{eff}}^B(M_\psi^2)|^2 = \frac{\Gamma_{J/\psi \rightarrow B\bar{B}}(g_\gamma^B)}{\beta \Gamma_{\mu^+\mu^-}^\psi}$$



Effective Lagrangian density

$$\mathcal{L} = \mathcal{L}(B; G_0, D_e, D_m, F_e, F_m, R)$$

▶ Spin 1/2 SU(3) baryon octet

$$B = \begin{pmatrix} \frac{\Lambda}{\sqrt{6}} + \frac{\Sigma^0}{\sqrt{2}} & \Sigma^+ & p \\ \Sigma^- & \frac{\Lambda}{\sqrt{6}} - \frac{\Sigma^0}{\sqrt{2}} & n \\ \Xi^- & \Xi^0 & \frac{-2\Lambda}{\sqrt{6}} \end{pmatrix}$$

▶ Coupling constants: G_0, D_e, D_m, F_e, F_m

$J/\psi \rightarrow p\bar{p}$

$$g_{ggg}^p = (G_0 - D_m + F_m)e^{i\varphi}$$

$$g_{gg\gamma}^p = R(G_0 - D_m + F_m)e^{i\varphi}$$

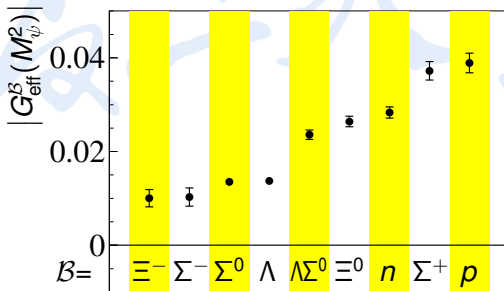
$$g_{\gamma}^p = D_e + F_e$$

$J/\psi \rightarrow n\bar{n}$

$$g_{ggg}^n = (G_0 - D_m + F_m)e^{i\varphi}$$

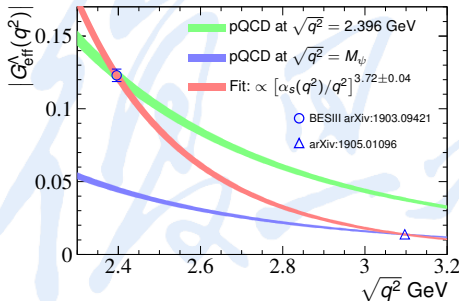
$$g_{gg\gamma}^n = 0$$

$$g_{\gamma}^n = -2D_e$$



The model

- Same $g_{gg\gamma}^B/g_{ggg}^B$ for all SU(3) octet baryons.
- $g_{gg\gamma}^B = 0$ for all neutral baryons.
- $g_{gg\gamma}^B$ and g_{ggg}^B have the same phase.



$J/\psi \rightarrow \Lambda \bar{\Lambda}$

- $g_{ggg}^\Lambda = (G_0 - 2D_m)e^{i\varphi}$
- $g_{gg\gamma}^\Lambda = 0$
- $g_\gamma^\Lambda = -D_e$

Perturbative QCD

$$|G_{\text{eff}}^\Lambda(q^2)| \propto \left(\frac{\alpha_s(q^2)}{q^2}\right)^2$$

Observed behavior

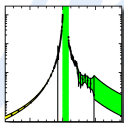
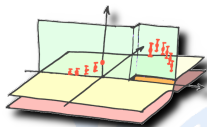
$$|G_{\text{eff}}^\Lambda(q^2)| \propto \left(\frac{\alpha_s(q^2)}{q^2}\right)^{3.72 \pm 0.04}$$

The asymptotic regime is not attained

There is still dynamical activity

This is proven by the rising trend of the relative phase

Final considerations



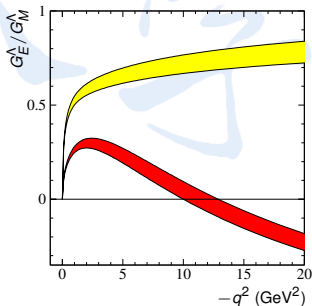
- Space-like zero for G_E^D
- Time-like phase of G_E^D/G_M^D goes to 180°
- Time-like form factors separation

Space-like and time-like "fixed" data on $|G_M^D|$ and analyticity



Confirmation of the pQCD asymptotic behavior

- Relative phase and modulus for the ratio G_E^Λ/G_M^Λ agree with (only) FSI interaction
- However, due to the zero of the ratio G_E^Λ/G_M^Λ at the $q^2 = 0$, **the phase has to tend to 180° as $q^2 \rightarrow \infty$**
- An asymptotic relative phase of 360° would imply the presence of an **additional space-like zero** for G_E^Λ



“To do and to do better” list



Time-like $|G_E| - |G_M|$ separation

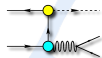
DR and data



Understanding threshold effect(s):



Dispersive analyses: integral equation, sum rule, ...



Experimental observation in $p\bar{p} \rightarrow \pi^0 l^+ l^-$

[PRC75,045205(07)]



Asymptotic behavior: DR and data for the phase



Zeros \leftrightarrow phases

DR and data



“To do and to do better” list



Time-like $|G_E| - |G_M|$ separation

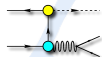
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Thank you

