Bhabha scattering

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Outline

Motivation

Motivation:

- Development of the physical program for future high-energy e^+e^- colliders
- Having high-precision theoretical description of Bhabha scattering is of crucial importance
- Many of the future e^+e^- colliders foresee running with polarized beam(s)

QUESTIONS:

- What we have?
- What we need?
- What to do?
- How to do?

FUTURE e^+e^- COLLIDER PROJECTS

Linear Colliders

- ILC, CLIC
- ILC: technology is ready, to be built in Japan (?)

E_{tot}

- ILC: 91; 250 GeV — 1 TeV
- CLIC: 500 GeV — 3 TeV

$\mathcal{L} \approx 2 \cdot 10^{34}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$

Stat. uncertainty $\sim 10^{-3}$

Beam polarization:
e^-beam: $P = 80 - 90\%$
e^+beam: $P = 30 - 60\%$

Circular Colliders

- FCC-ee, TLEP
- CEPC
- muon collider (?)

E_{tot}

- 91; 160; 240; 350 GeV

$\mathcal{L} \approx 2 \cdot 10^{36}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$ (4 exp.)

Stat. uncertainty $< 10^{-3}$

Beam polarization: desirable

Super Charm-Tau Factory Projects

Budker Institute of Nuclear Physics in Novosibirsk and/or China

Colliding electron-positron beams with c.m.s. energies from 2 to 5 GeV with unprecedented high luminosity $10^{35} cm^{-2} c^{-1}$

The electron beam will be longitudinally polarized

The main goal of experiments at the Super Charm-Tau factory is to study the processes charmed mesons and tau leptons, using a data set that is 2 orders of magnitude more than the one collected by BESIII

ESTIMATED EXPERIMENTAL PRECISION

Now:

Quantity	Theory error	Exp. error
M_W [MeV]	4	15
$\sin^2\theta^l_{eff}[10^{-5}]$	4.5	16
Γ_Z [MeV]	0.5	2.3
$R_b[10^{-5}]$	15	66

Quantity	ILC	FCC-ee	CEPC	Projected theory error
M_W [MeV]	3–4	1	3	1
$\sin^2\theta^l_{eff}[10^{-5}]$	1	0.6	2.3	1.5
Γ_Z [MeV]	0.8	0.1	0.5	0.2
$R_b[10^{-5}]$	14	6	17	5–10

The estimated error for the theoretical predictions of these quantities is given, under the assumption that $O(\alpha\alpha_s^2)$, fermionic $O(\alpha^2\alpha_s)$, fermionic $O(\alpha^3)$, and leading four-loop corrections entering through the ρ-parameter will become available.

FCC-EE: THE TERA-Z

Report on the 1st Mini workshop: Precision EW and QCD calculations for the FCC studies: methods and tools: A. Blondel *et al.*, "Standard Model Theory for the FCC-ee: The Tera-Z," arXiv:1809.01830 [hep-ph].

Having high-precision luminosity measurements is crucial for extraction of electroweak quantities. The most sensitive are: the cross section of $\sigma(e^+e^- \to hadrons)$ and the number of (light) neutrinos N_ν

In general, QED (mostly) and QCD radiative corrections to cross-sections and angular distributions that are needed to convert experimentally measured cross-sections back to pseudo-observables: couplings, masses, partial widths, asymmetries, etc.

PERTURBATIVE QED (I)

Fortunately, in our case the general perturbation theory can be applied:

$$\frac{\alpha}{2\pi} \approx 1.2 \cdot 10^{-3}, \quad \left(\frac{\alpha}{2\pi}\right)^2 \approx 1.4 \cdot 10^{-6}$$

Moreover, other effects: hadronic vacuum polarization, (electro)weak contributions, hadronic pair emission, etc. are small in Bhabha scattering and can be treated one-by-one separately

Nevertheless, there are some enhancement factors:

1) First of all, the large logarithm $L \equiv \ln \frac{\Lambda^2}{m_e^2}$ where $\Lambda^2 \sim Q^2$ is the momentum transferred squared, e.g., $L(\Lambda = 1\,\mathrm{GeV}) \approx 16$ and $L(\Lambda = M_Z) \approx 24$.

2) The energy region at the Z boson peak ($s \sim M_Z^2$) requires a special treatment since factor M_Z/Γ_Z appears in the annihilation channel

PERTURBATIVE QED (II)

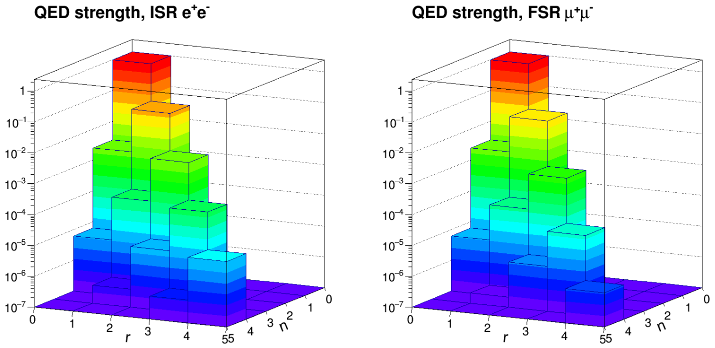


Fig.: The parameter γ_{nr} characterizing the size of the QED corrections,

$$\gamma_{nr} = \left(\frac{\alpha}{\pi}\right)^n \left(2\ln\frac{M_Z^2}{m_f^2}\right)^r, \qquad 1 \le r \le n$$

Figure from [S.Jadach and M.Skrzypek, arXiv:1903:09895]

Perturbative QED (III)

Methods of resummation of QED corrections

- Resummation of vacuum polarization corrections (geometric series)
- Yennie–Frautschi–Suura (YFS) soft photon exponentiation its extensions. see e.g. PHOTOS
- Resummation of leading logarithms via QED structure functions (Kuraev and Fadin 1985)

N.B. Resummation of real photon radiation is good for inclusive observables...

RADIATIVE CORRECTIONS

- Add contributions to observed quantities and complicate extraction of physical results
- Provide information of physics at the quantum (loop) level
- Affect kinematics of signal and background events
- Require involved Monte Carlo codes
- Add to resulting uncertainty
- The goal is to have theoretical uncertainty $\leq 1/3$ of the experimental one

Introduction to SANC

- The SANC system implements calculations of complete (real and virtual) NLO QCD and EW corrections for various processes at the partonic level

- All calculations are performed within the OMS (on-mass-shell) renormalization scheme in the R_ξ gauge which allows an explicit control of the gauge invariance by examining cancellation of the gauge parameters in the analytic expression of the matrix element

- Cross-sections of the processes at hadron level obtained by convolution the partonic level cross-sections with PDFs

- The list of processes implemented in the MCSANC integrator includes Drell-Yan processes (inclusive), associated Higgs and gauge boson production and single-top quark production in s- and t-channel (v1.01 – CPC 184 (2013) 2343), photon-induced contribution, EW corrections beyond NLO approximation to DY (v1.20 – JETP Lett. 103 (2016) 131)

ROOTS OF SANC: ZFITTER

ZFITTER is a Fortran program for the calculation of fermion pair production and radiative corrections at high energy e^+e^- colliders. It is also suitable for other applications where electroweak radiative corrections appear.

Authors: D. Bardin et al.

http://zfitter.com, http://sanc.jinr.ru/users/zfitter

T. Riemann (spokesperson since 2005)

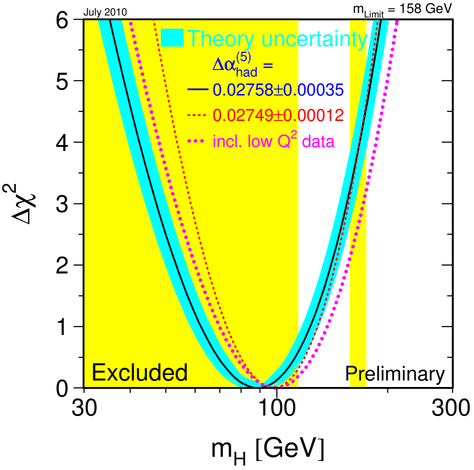
ZFITTER code v.6.42 is described in *Comp.Phys.Comm.*'2006.

Review and status of the project: *Phys.Part.Nucl.*'2014.

ZFITTER is a semi-analytic code

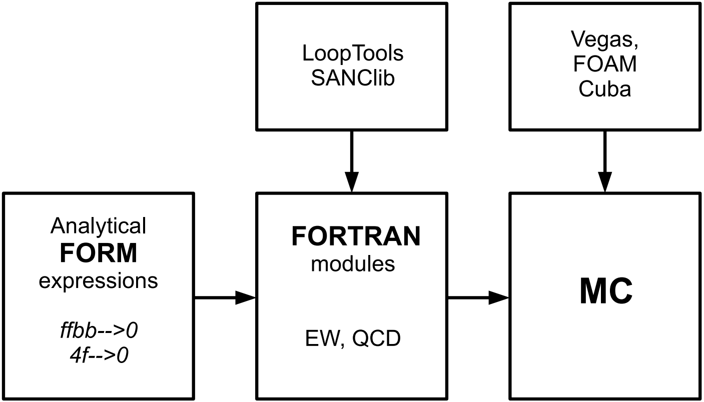
The code is still supported and used

ROOTS OF SANC: ZFITTER



LEP EW working group report '2010.

SANC FRAMEWORK SCHEME



SANC FOR PROCESSES WITH POLARIZED BEAMS

- NLO EW corrections for polarized e^+e^- scattering:
  - Bhabha scattering (PRD 2018)
  - $e^+e^- \to ZH$ (arXiv:1812.10965)
  - $e^+e^- \to \mu^+\mu^-$(or $\tau^+\tau^-$) (preliminary)
  - $e^+e^- \to Z\gamma$ (preliminary)
  - $e^+e^- \to \gamma\gamma$ (preliminary)
  - $e^+e^- \to t\bar{t}$ (in progress)
  - $e^+e^- \to ZZ$ (in progress)
  - $e^+e^- \to f\bar{f}\gamma$ (future plans)
  - $e^+e^- \to f\bar{f}H$ (future plans)
- NLO EW corrections for polarized $\gamma\gamma$ scattering:
  - $\gamma\gamma \to \gamma\gamma$ (future plans)
  - $\gamma\gamma \to Z\gamma$ (future plans)
  - $\gamma\gamma \to ZZ$ (future plans)

BHABHA SCATTERING: GENERAL REMARKS

- Bhabha scattering is the basic QED process which is widely used for luminosity measurements at e^+e^- colliders (but no only)
- We got a record accuracy in small-angle Bhabha measurement and description at LEP. That was one of keystones for the high-precision verification of the Standard Model at LEP
- Since that time a considerable progress has been achieved:
  1) calculations of higher order radiative corrections
  2) computer tools, including adaptive Monte Carlo
  3) detector techniques, data analysis, etc.
- Still a lot has to be done to meet requirements of experiments at future e^+e^- colliders
- The aim of the talk is to overview the past achievements and to indicate and discuss the present problems (=tasks)

BHABHA: THE BORN

Pay attention to the choice of the Born level definition

— Either pure QED or QED+EW

— Either the "standard" or "improved" Born approximation

— EW scheme choices: $\alpha(0)$, $\alpha(M_Z)$, $G_{\rm Fermi}$, ...

The main point is to avoid double counting when Born is matched to higher order corrections

Another point is to be prepared for comparisons with other codes which might have a different Born definition

$\mathcal{O}(\alpha)$ CORRECTIONS

$\mathcal{O}(\alpha)$ corrections in QED and EW are known for years:
F.A. Redhead, Proc.Roy.Soc.'1953 R.V. Polovin, ZhETF'1956
F.A. Berends, K.J.F. Gaemers, R. Gastmans, Nucl.Phys.B '1974
M. Consoli, Nucl.Phys B '1979
M. Böhm, A. Denner, W. Hollik, Nucl.Phys B '1988
D. Bardin et al. [SANC], Phys. Rev. D '2018 (EW with polarization)

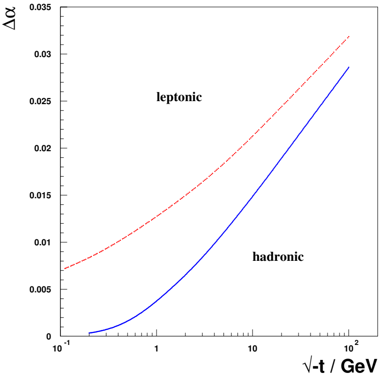
Remarks:
1) The t channel dominates everywhere even at very large scattering angles, except large angle scattering at narrow peak regions: Φ, J/Ψ, Z etc.
2) The pure QED Born contribution dominates everywhere below the Z peak, and also well above the peak for small scattering angles
3)Among radiative corrections, QED photonic contributions and vacuum polarization are the most important ones
4) Large logs $L = \ln(-t/m_e^2)$ provide the bulk of RC

VACUUM POLARIZATION

A.A. et al. EPJC' 2004

$\sqrt{s}$ (GeV)	91.187	91.2	189	206	500	1000	3000
	45 mrad $<\theta<$ 110 mrad						
$\langle\sqrt{-t}\rangle$ (GeV)	3.4	3.4	7.1	7.7	18.8	37.5	112.6
QED	51.428	51.413	11.971	10.077	1.7105	0.42763	0.047514
QED_t	51.484	51.469	11.984	10.088	1.7124	0.42809	0.047566
EW	51.436	51.413	11.965	10.072	1.7105	0.42871	0.049507
EW+VP_t	54.041	54.018	12.743	10.745	1.8590	0.47303	0.055748
EW+VP	54.036	54.013	12.742	10.744	1.8588	0.47296	0.055742
	5 mrad $<\theta<$ 50 mrad						
$\langle\sqrt{-t}\rangle$ (GeV)	1.1	1.1	2.2	2.4	5.8	11.6	34.8
QED	4963.4	4962.0	1155.4	972.54	165.08	41.271	4.5857
QED_t	4963.5	4962.1	1155.4	972.57	165.09	41.272	4.5858
EW	4963.4	4962.0	1155.4	972.53	165.08	41.272	4.5885
EW+VP_t	5075.0	5073.5	1190.6	1003.3	172.51	43.647	4.9603
EW+VP	5075.0	5073.5	1190.6	1003.3	172.51	43.646	4.9605

RUNNING OF $\alpha_{\rm QED}$



$$\alpha(q^2) = \frac{\alpha(0)}{1 - \Delta\alpha(q^2)}$$

See, e.g., review S.Actis, A.A. et al., EPJC 66 (2010) 585

SECOND ORDER CORRECTIONS

The complete $\mathcal{O}\left(\alpha^2 L\right)$ analytic result was first received in A.A., V. Fadin, E. Kuraev, L. Lipatov, N. Merenkov, L. Trentadue [Nucl.Phys.B '1997]

Two-loop virtual pure QED RC were computed by A. Penin [PRL'2005, NPB'2006] and [T. Becher, K. Melnikov, JHEP'2007]

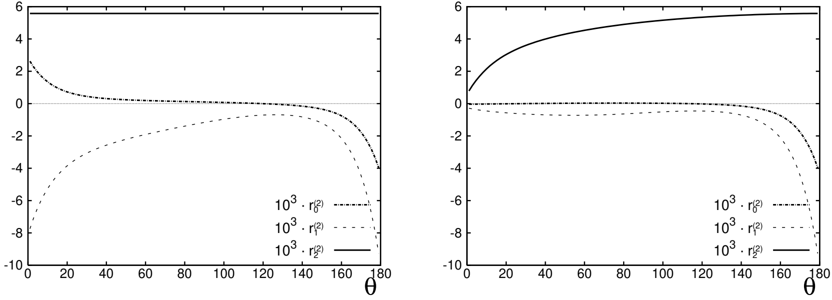
Emission of one or two real photons was also added, see e.g. C. Carloni Calame, H. Czyz, J. Gluza, M. Gunia, G. Montagna, O. Nicrosini, F. Piccinini, T. Riemann, M. Worek *NNLO leptonic and hadronic corrections to Bhabha scattering and luminosity monitoring at meson factories*, JHEP 1107 (2011) 126

A. Penin and G. Ryan, *Two-loop electroweak corrections to high energy large-angle Bhabha scattering*, JHEP'2011

SIZE OF SECOND ORDER RC

Let's look at soft + virtual $\mathcal{O}\left(\alpha^2\right)$ RC [A. Penin, PRL'2005, NPB'2006]:

$\Delta = \sum_{n=0,1,2} C_n \ln^n \frac{M^2}{m_e^2} = \sum_{n=0,1,2} \eta_n$



Soft and virtual second order photonic relative radiative corrections in permil *versus* the scattering angle in degrees for $\Delta = 1$, $\sqrt{s}$=1 GeV; $M = \sqrt{s}$ on the left side and $M = \sqrt{-t}$ on the right side.

SOFT-HARD SEPARATOR

Beware of the proper separation of soft and hard radiation in higher orders: the sum of photon energies or for each photon separately. Note that exponentiation of soft radiation assumes independent emission of each photon

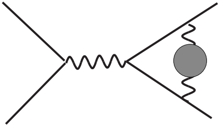
See, e.g., A.A., T.V. Kopylova, *On higher order radiative corrections to elastic electron-proton scattering*, EPJC 2015

$\mathcal{O}(\alpha^2)$ Virtual (loop) pair corrections

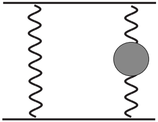
e^+e^- pair corrections
R. Bonciani, A. Ferroglia, P. Mastrolia, E. Remiddi, J. van der Bij, NPB'2004

$\mu^+\mu^-$ (heavy fermion) pair corrections
S. Actis, M. Czakon, J. Gluza, T. Riemann, Acta Phys. Polon. B'2007; PRL'2008
R. Bonciani, A. Ferroglia, A. Penin, JHEP'2008

Virtual Hadronic NNLO corrections
S. Actis, M. Czakon, J. Gluza, T. Riemann, PRL'2008; PRD'2008
J. H. Kühn, S. Uccirati, NPB'2009



(c)



(d)

$\mathcal{O}(\alpha^2)$ HEAVY-FERMION AND HADRONIC CONTRIBUTIONS (II)

Real pair emission corrections

Relevant semi-analytic results are known (should be updated), see LEPEWWG studies: M. Kobel et al. CERN Yellow Rep. '1999; A.A. JHEP '2001

Monte Carlo simulation for emission of real leptonic pairs is straightforward

Question: what are requirements for description of processes like $e^+e^- \to e^+e^- + \pi^+\pi^-$?

Leading two-loop (electro)weak corrections

In `MCSANC v.1.20` we follow the recipe introduced by o J. Fleischer, O. Tarasov, and F. Jegerlehner in 1993 and implemented further in `ZFITTER`

The $\Delta\rho$ parameter as the ratio of the neutral current to charged current amplitudes at zero momentum transfer

$$\rho = \frac{G_{NC}(0)}{G_{CC}(0)} = \frac{1}{1-\Delta\rho}$$

The leading in $G_\mu m_t^2$ NLO EW contribution is

$$\Delta\rho^{(1)} = 3x_t = \frac{3\sqrt{2}G_\mu m_t^2}{16\pi^2}$$

At the two-loop level

$$\Delta\rho = N_c \frac{\sqrt{2}G_\mu m_t^2}{16\pi^2}\left[1+\rho^{(2)}\left(M_H^2/m_t^2\right)\, x_t\right]\left[1-\frac{2\alpha_s(M_Z^2)}{9\pi}(\pi^2+3)\right]$$

BHLUMI (I)

Currently (since LEP times) the best Monte Carlo code to describe small-angle Bhabha scattering is BHLUMI by S. Jadach et al.

It is based on YFS exponentiation providing $\sim \mathcal{O}(\alpha^{1.5})$ precision

Status for 1999

	LEP1		LEP2	
Type of correction/error	1996	1999	1996	1999
(a) Missing photonic $\mathcal{O}(\alpha^2)$	0.10%	0.027%	0.20%	0.04%
(b) Missing photonic $\mathcal{O}(\alpha^3 L_e^3)$	0.015%	0.015%	0.03%	0.03%
(c) Vacuum polarization	0.04%	0.04%	0.10%	0.10%
(d) Light pairs	0.03%	0.03%	0.05%	0.05%
(e) Z and s-channel γ	0.015%	0.015%	0.0%	0.0%
Total	0.11%	0.061%	0.25%	0.12%

Table from arXiv:1809.01830 (FCC-ee workshop)

BHLUMI (II)

Status for 2018

Type of correction / Error	1999	Update 2018
(a) Photonic $\mathcal{O}(L_e\alpha^2)$	0.027%	0.027%
(b) Photonic $\mathcal{O}(L_e^3\alpha^3)$	0.015%	0.015%
(c) Vacuum polariz.	0.040%	0.013%
(d) Light pairs	0.030%	0.010%
(e) Z and s-channel γ exchange	0.015%	0.015%
(f) Up-down interference	0.0014%	0.0014%
(f) Technical Precision	–	(0.027)%
Total	0.061%	0.038%

Table from arXiv:1809.018305 (FCC-ee workshop)

Upgrades to reach the 0.01% precision level are planned

LARGE-ANGLE BHABHA AT LEP

Agreement between different calculations was much worse than in the small-angle case

E_{CM}, GeV	BHWIDE	TOPAZ0	BHAGENE3	UNIBAB	SABSPV	BHAGEN95	LABSMC
	$\vartheta_{\text{acol}} = 10^\circ$						
175	35.257	35.455	34.690	34.498	35.740	35.847	35.337
190	29.899	30.024	28.780	29.189	30.270	30.352	29.941
205	25.593	25.738	24.690	25.976	25.960	26.007	25.687
	$\vartheta_{\text{acol}} = 25^\circ$						
175	39.741	40.487	39.170	39.521	40.240	40.505	40.029
190	33.698	34.336	32.400	33.512	34.100	34.331	33.954
205	28.929	29.460	27.840	28.710	29.280	29.437	29.178

Table from [A.A. arXiv:hep-ph/9910280]
see also CERN Yellow Report 96-01, vol. 2, 1996, p.229

N.B. A large correction comes from radiative return to the Z boson peak

BHABHA IN SANC: HA FOR BORN AND VIRTUAL CONTRIBUTIONS

At one-loop level we have 6 non-zero HAs (4 independent):

$$\mathcal{H}_{++++} = \mathcal{H}_{----} = -2e^2\,\frac{s}{t}\left[\mathcal{F}^{(\gamma,Z)}_{QQ}(t,s,u) - \chi^t_Z\delta_e\mathcal{F}^Z_{QL}(t,s,u)\right],$$

$$\mathcal{H}_{+-+-} = \mathcal{H}_{-+-+} = -e^2\,c_-\left[\mathcal{F}^{(\gamma,Z)}_{QQ}(s,t,u) - \chi^s_Z\delta_e\mathcal{F}^Z_{QL}(s,t,u)\right],$$

$$\mathcal{H}_{+--+} = -e^2\,c_+\left(\left[\mathcal{F}^{(\gamma,Z)}_{QQ}(s,t,u) + \chi^s_Z\left(\mathcal{F}^Z_{LL}(s,t,u) - 2\delta_e\mathcal{F}^Z_{QL}(s,t,u)\right)\right]\right.$$
$$\left. + \frac{s}{t}\left[\mathcal{F}^{(\gamma,Z)}_{QQ}(t,s,u) + \chi^t_Z\left(\mathcal{F}^Z_{LL}(t,s,u) - 2\delta_e\mathcal{F}^Z_{QL}(t,s,u)\right)\right]\right),$$

$$\mathcal{H}_{-++-} = -e^2\,c_+\left(\left[\mathcal{F}^{(\gamma,Z)}_{QQ}(s,t,u)\right] + \frac{s}{t}\left[\mathcal{F}^{(\gamma,Z)}_{QQ}(t,s,u)\right]\right),$$

where $\quad c_+ = 1+\cos\theta, \quad c_- = 1-\cos\theta$

$$\chi^s_Z = \frac{1}{4s^2_W c^2_W}\,\frac{s}{s - M^2_Z + iM_Z\Gamma_Z}, \quad \chi^t_Z = \frac{1}{4s^2_W c^2_W}\,\frac{t}{t - M^2_Z}, \quad \delta_e = v_e - a_e = 2s^2_W,$$

$$\mathcal{F}^{(\gamma,Z)}_{QQ}(a,b,c) = \mathcal{F}^{\gamma}_{QQ}(a,b,c) + \chi^a_Z\delta^2_e\mathcal{F}^Z_{QQ}(a,b,c)$$

We get the Born level HAs by replacing $\mathcal{F}^Z_{LL}\to 1$, $\mathcal{F}^Z_{QL}\to 1$, $\mathcal{F}^Z_{QQ}\to 1$ and $\mathcal{F}^{\gamma}_{QQ}\to 1$

SANC MONTE CARLO GENERATOR FOR BHABHA

See the next talks

We created a Monte Carlo generator of unweighted events for polarized Bhabha scattering $e^+e^- \to e^+e^-$ with complete one-loop EW corrections and with possibility to produce events in the standard Les Houches format.

This generator uses adaptive algorithm `mFOAM` (CPC 177:441-458,2007) which is part of the `ROOT` program

Setup for tuned comparison

We performed a tuned comparison of polarized Born and hard Bremsstrahlung by WHIZARD. The unpolarized soft and virtual parts were compared with the results of AİTALC.

Initial parameters

$$\alpha^{-1}(0) = 137.03599976,$$

$$M_W = 80.451495\ \text{GeV}, \quad M_Z = 91.1876\ \text{GeV}, \quad \Gamma_Z = 2.49977\ \text{GeV},$$

$$m_e = 0.5109990\ \text{MeV}, \quad m_\mu = 0.105658\ \text{GeV}, \quad m_\tau = 1.77705\ \text{GeV},$$

$$m_d = 0.083\ \text{GeV}, \quad m_s = 0.215\ \text{GeV}, \quad m_b = 4.7\ \text{GeV},$$

$$m_u = 0.062\ \text{GeV}, \quad m_c = 1.5\ \text{GeV}, \quad m_t = 173.8\ \text{GeV}.$$

Cuts

$$|\cos\theta| < 0.9,$$

$$E_\gamma > 1\ \text{GeV} \quad \text{(for comparison of hard Bremsstrahlung)}.$$

$e^+e^- \to e^+e^-$: WHIZARD VS SANC (HARD)

P_{e^-}, P_{e^+}	0, 0	-0.8, 0	-0.8, -0.6	-0.8, 0.6
$\sqrt{s} = 250$ GeV				
$\sigma^{\text{hard}}_{e^+e^-}$, pb	48.62(1)	49.58(1)	48.74(1)	50.40(1)
$\sigma^{\text{hard}}_{e^+e^-}$, pb	48.65(1)	49.56(1)	48.78(1)	50.44(1)
$\sqrt{s} = 500$ GeV				
$\sigma^{\text{hard}}_{e^+e^-}$, pb	15.14(1)	15.81(1)	13.54(1)	18.07(1)
$\sigma^{\text{hard}}_{e^+e^-}$, pb	15.12(1)	15.79(1)	13.55(1)	18.11(2)
$\sqrt{s} = 1000$ GeV				
$\sigma^{\text{hard}}_{e^+e^-}$, pb	4.693(1)	4.976(1)	3.912(1)	6.041(1)
$\sigma^{\text{hard}}_{e^+e^-}$, pb	4.694(1)	4.975(1)	3.913(1)	6.043(1)

$e^+e^- \to e^+e^-$: ÅITALC VS SANC $\sqrt{s} = 500 GeV$

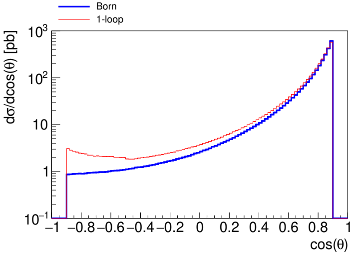
$\cos\theta$	$\sigma^{\text{Born}}_{e^+e^-}$, pb	$\sigma^{\text{Born+virt+soft}}_{e^+e^-}$, pb
-0.9	$2.16999 \cdot 10^{-1}$ $2.16999 \cdot 10^{-1}$	$1.93445 \cdot 10^{-1}$ $1.93445 \cdot 10^{-1}$
-0.5	$2.61360 \cdot 10^{-1}$ $2.61360 \cdot 10^{-1}$	$2.38707 \cdot 10^{-1}$ $2.38707 \cdot 10^{-1}$
0	$5.98142 \cdot 10^{-1}$ $5.98142 \cdot 10^{-1}$	$5.46677 \cdot 10^{-1}$ $5.46677 \cdot 10^{-1}$
$+0.5$	$4.21273 \cdot 10^{0}$ $4.21273 \cdot 10^{0}$	$3.81301 \cdot 10^{0}$ $3.81301 \cdot 10^{0}$
$+0.9$	$1.89160 \cdot 10^{2}$ $1.89160 \cdot 10^{2}$	$1.72928 \cdot 10^{2}$ $1.72928 \cdot 10^{2}$
$+0.99$	$2.06556 \cdot 10^{4}$ $2.06555 \cdot 10^{4}$	$1.90607 \cdot 10^{4}$ $1.90607 \cdot 10^{4}$
$+0.999$	$2.08236 \cdot 10^{6}$ $2.08236 \cdot 10^{6}$	$1.91624 \cdot 10^{6}$ $1.91624 \cdot 10^{6}$
$+0.9999$	$2.08429 \cdot 10^{8}$ $2.08429 \cdot 10^{8}$	$1.91402 \cdot 10^{8}$ $1.91402 \cdot 10^{8}$

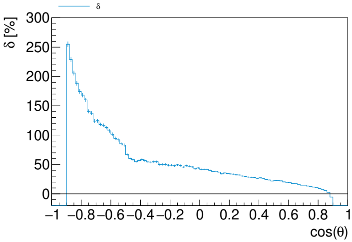
Born vs 1-loop (SANC)

P_{e^-}, P_{e^+}	0, 0	-0.8, 0	-0.8, -0.6	-0.8, 0.6
$\sqrt{s} = 250$ GeV				
$\sigma^{\text{Born}}_{e^+e^-}$, pb	56.6763(1)	57.7738(1)	56.2725(4)	59.2753(5)
$\sigma^{\text{1-loop}}_{e^+e^-}$, pb	61.731(6)	62.587(6)	61.878(6)	63.287(7)
δ, %	8.92(1)	8.33(1)	9.96(1)	6.77(1)
$\sqrt{s} = 500$ GeV				
$\sigma^{\text{Born}}_{e^+e^-}$, pb	14.3789(1)	15.0305(1)	12.7061(1)	17.3550(2)
$\sigma^{\text{1-loop}}_{e^+e^-}$, pb	15.465(2)	15.870(2)	13.861(1)	17.884(2)
δ, %	7.56(1)	5.59(1)	9.09(1)	3.05(1)
$\sqrt{s} = 1000$ GeV				
$\sigma^{\text{Born}}_{e^+e^-}$, pb	3.67921(1)	3.90568(1)	3.03577(3)	4.77562(5)
$\sigma^{\text{1-loop}}_{e^+e^-}$, pb	3.8637(4)	3.9445(4)	3.2332(3)	4.6542(7)
δ, %	5.02(1)	0.99(1)	6.50(1)	-2.54(1)

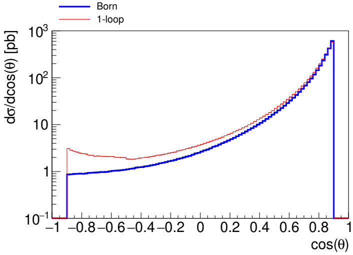
DISTRIBUTIONS IN $\cos\theta$

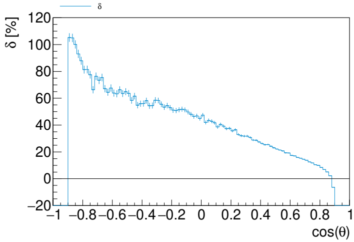
$\sqrt{s} = 250$ GeV





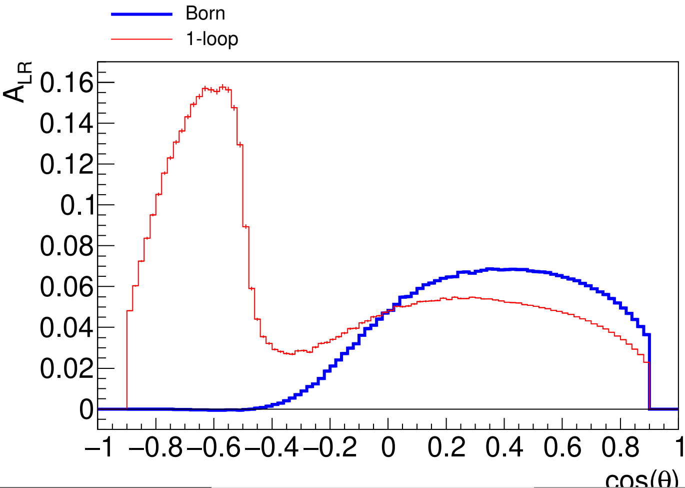
$\sqrt{s} = 500$ GeV





LEFT-RIGHT ASYMMETRY ($E_{CM} = 250$ GEV)

$$A_{LR} \equiv \frac{\sigma_{LR} - \sigma_{RL}}{\sigma_{LR} + \sigma_{RL}}$$



LEADING AND NEXT-TO-LEADING LOGS IN QED

The QED leading (LO) logarithmic corrections

$$\sim \left(\frac{\alpha}{2\pi}\right)^n \ln^n \frac{s}{m_e^2}$$

were relevant for LEP measurements of Bhabha, $e^+e^- \to \mu^+\mu^-$ etc.
for $n \leq 3$ since $\ln(M_Z^2/m_e^2) \approx 24$

NLO contributions

$$\sim \left(\frac{\alpha}{2\pi}\right)^n \ln^{n-1} \frac{s}{m_e^2}$$

with $n = 3$ are required for future e^+e^- colliders

In the collinear approximation we can get them within
the NLO QED structure function formalism

- F.A.Berends, W.L. van Neerven, G.J.Burgers, NPB'1988
- A.A., K.Melnikov, PRD'2002; A.A. JHEP'2003

QED NLO master formula

The NLO Bhabha cross section reads

$$
\begin{aligned}
d\sigma &= \sum_{a,b,c,d=e,\bar{e},\gamma} \int_{\bar{z}_1}^{1} dz_1 \int_{\bar{z}_2}^{1} dz_2 \mathcal{D}^{\rm str}_{ae}(z_1) \mathcal{D}^{\rm str}_{b\bar{e}}(z_2) \\
&\times \left[d\sigma^{(0)}_{ab\to cd}(z_1,z_2) + d\bar{\sigma}^{(1)}_{ab\to cd}(z_1,z_2) \right] \\
&\times \int_{\bar{y}_1}^{1} \frac{dy_1}{Y_1} \int_{\bar{y}_2}^{1} \frac{dy_2}{Y_2} \mathcal{D}^{\rm frg}_{ec}\left(\frac{y_1}{Y_1}\right) \mathcal{D}^{\rm frg}_{\bar{e}d}\left(\frac{y_2}{Y_2}\right) + \mathcal{O}(\alpha^n L^{n-2})
\end{aligned}
$$

$\alpha^2 L^1$ terms are completely reproduced [A.A., E.Scherbakova, JETP Lett. 2006; PLB 2008]

$\alpha^3 L^2$ terms are required at the Z peak and for e^+e^- annihilation channels at higher energies

QED NLO EVOLUTION

$$\frac{d\mathcal{D}_{ee}(x,\mu_f,m_e)}{d\ln\mu_f^2} = \sum_{a=e,\gamma,\bar{e}} \int_z^1 \frac{dz}{z} P_{ea}(z,\bar{\alpha}(\mu_f)) \mathcal{D}_{ae}\left(\frac{x}{z},\mu_f,m_e\right)$$

with perturbative splitting functions

$$P_{ea}(z,\bar{\alpha}(\mu_f)) = \frac{\bar{\alpha}(\mu_f)}{2\pi} P_{ea}^{(0)}(z) + \left(\frac{\bar{\alpha}(\mu_f)}{2\pi}\right)^2 P_{ea}^{(1)}(z) + \mathcal{O}(\alpha^3)$$

and initial conditions

$$\begin{aligned}
\mathcal{D}_{ee}^{\text{ini}}(x,\mu_0,m_e) &= \delta(1-x) + \frac{\bar{\alpha}(\mu_0)}{2\pi} d_1(x,\mu_0,m_e) + \mathrm{O}(\alpha^2) \\
d_1(x,\mu_0,m_e) &= \left[\frac{1+x^2}{1-x}\left(\ln\frac{\mu_0^2}{m_e^2} - 2\ln(1-x) - 1\right)\right]_+ \\
\mathcal{D}_{\gamma e}^{\text{ini}}(x,\mu_0,m_e) &= \frac{\bar{\alpha}(\mu_0)}{2\pi} P_{\gamma e}^{(0)}(x) + \mathrm{O}(\alpha^2)
\end{aligned}$$

ITERATIVE SOLUTION

The pure photonic non-singlet part of the electron structure function

$$\mathcal{D}_{ee}^{(\gamma)}(x,\mu_f,m_e) = \delta(1-x) + \frac{\alpha}{2\pi}d_1(x,\mu_0,m_e) + \frac{\alpha}{2\pi}L_f P_{ee}^{(0)}(x)$$
$$+ \left(\frac{\alpha}{2\pi}\right)^2 \left(\frac{1}{2}L_f^2 P_{ee}^{(0)} \otimes P_{ee}^{(0)}(x) + L_f P_{ee}^{(0)} \otimes d_1(x,\mu_0,m_e) + L_f P_{ee}^{(1,\gamma)}(x)\right)$$
$$+ \left(\frac{\alpha}{2\pi}\right)^3 \left(\frac{1}{6}L_f^3 P_{ee}^{(0)} \otimes P_{ee}^{(0)} \otimes P_{ee}^{(0)}(x) + L_f^2 P_{ee}^{(0)} \otimes P_{ee}^{(1,\gamma)}(x)\right.$$
$$\left.+ L_f^2 P_{ee}^{(0)} \otimes P_{ee}^{(0)} \otimes d_1(x,\mu_0,m_e)\right) + \mathcal{O}(\alpha^3 L^1)$$

The large logarithm $L_f \equiv \ln \frac{\mu_f^2}{m_e^2}$ with factorization scale $\mu_f^2 \sim s$ or $\sim -t$

NLO contributions of e^+e^- pairs are recovered in the same way

Required convolution integrals are listed in [A.A. hep-ph/0304063]

Example of convolution (I)

$$P_{ee}^{(0)}(x) = \left[\frac{1+x^2}{1-x}\right]_+$$

$$P_{ee}^{(1,\gamma)\mathrm{str}}(x) = \delta(1-x)\left(\frac{3}{8} - 3\zeta(2) + 6\zeta(3)\right) + \frac{1+x^2}{1-x}\left(-2\ln x\ln(1-x) + \ln^2 x + 2\mathrm{Li}_2(1-x)\right) - \frac{1}{2}(1+x)\ln^2 x + 2\ln x + 3 - 2x$$

Plus prescription

$$\int_{x_{\min}}^1 dx\,[V(x)]_+\,W(x) = \int_0^1 dx V(x)\Big[W(x)\Theta(x-x_{\min}) - W(1)\Big]$$

Convolution

$$A\otimes B(x) = \int_0^1 dz\int_0^1 dz'\delta(x-zz')A(z)B(z')$$

EXAMPLE OF CONVOLUTION (II) PRELIMINARY

$$
\begin{aligned}
P_{ee}^{(0)} \otimes P_{ee}^{(1,\gamma)\mathrm{str}}(x) = \Bigg[& \frac{1+x^2}{1-x} \Bigg(-4\mathrm{S}_{12}(x) + 4\mathrm{Li}_2(1-x)(\ln(1-x) - \ln(x)) \\
& -4\ln(x)\ln^2(1-x) + 4\ln^2(x)\ln(1-x) - \frac{2}{3}\ln^3(x) + 3\mathrm{Li}_2(1-x) \\
& -3\ln(x)\ln(1-x) + \frac{3}{2}\ln^2(x) + 4\zeta(2)\ln(x) + 6\zeta(3) - 3\zeta(2) + \frac{3}{8} \Bigg) \\
& +4(1+x)\mathrm{S}_{12}(x) + \frac{1+x}{2}\ln^3(x) - 2(1+x)\ln^2(x)\ln(1-x) \\
& +(6x-2)\mathrm{Li}_2(1-x) + (6-2x)\ln(x)\ln(1-x) + \left(\frac{11}{4}x - \frac{9}{4}\right)\ln^2(x) \\
& +(6-4x)\ln(1-x) + (5x-3)\ln(x) + 2x - \frac{1}{2} \Bigg]_+
\end{aligned}
$$

This is a part of universal collinear radiation factors

OUTLOOK

- Having high theoretical precision for the normalization processes $e^+e^- \to e^+e^-$, $e^+e^- \to \mu^+\mu^-$, and $e^+e^- \to 2\gamma$ with polarized beams is crucial for future e^+e^- colliders
- There are several two-loop QED results, but most of them are w/o polarization yet
- Two-loop EW RC are in progress, polarization to be foreseen from the beginning
- The SANC computer system is being upgraded
- New Monte Carlo codes are required
- $O(\alpha^3 L^2)$ collinear radiator factors are derived. First, they will be implemented into the ZFITTER code

Statement of the problem

General task: we need a high-precision description of polarized small-angle and large-angle Bhabha scattering at high energies

General results: Monte Carlo generator(s), Monte Carlo integrator(s), and semi-analytic codes for cross-checks

Input: the beam energy distribution (for linear colliders), hadronic vacuum polarization, analytic results from the literature, and new calculations

Output: various distributions and inclusive observables with estimates of uncertainties

Set-up: collaboration of several groups is definitely required to solve the problems