One-loop EW corrections for the processes $e^+e^- \to \gamma\gamma$, $\gamma\gamma \to l^+l^-$, $e\gamma \to e\gamma$

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CEPC Topical Workshop

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Future lepton collider projects

Linear collider (e+e-)

- ILC; CLIC
- ILC: technology at hand, realization in Japan??

E_{cm}

- 250GeV – 1TeV, 91GeV (ILC)
- 500GeV – 3TeV (CLIC)

$L \approx 2\times10^{34}cm^{-2}s^{-1}$ (~500fb^{-1}/year)

→ Stat. uncertainty ~ $10^{-3}\ldots10^{-2}$

Beam polarization

e- beam P = 80-90%

e+ beam

ILC: P = 30% baseline;
60% upgrade

CLIC: P ≥ 60% upgrade

Circular collider

- FCC-ee, TLEP
- CEPC

μ Collider

Projects under study

E_{cm}

91 GeV, 160GeV, 240GeV, 350GeV

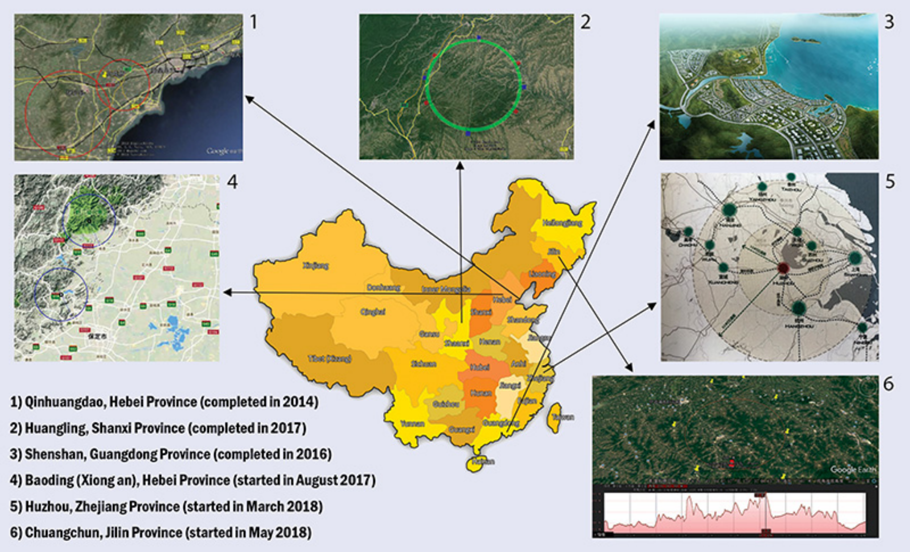
$L \approx 10^{36}cm^{-2}s^{-1}$ (4 experiments)

→ Stat. uncertainty $\leq10^{-3}$

Beam polarization

- Desired (?)

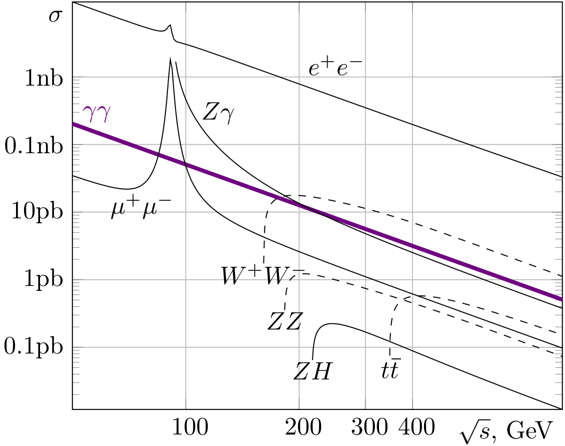
CEPC - The Circular Electron Positron Collider



MCSANC$_{ee}$: NLO EW RC for polarized scattering

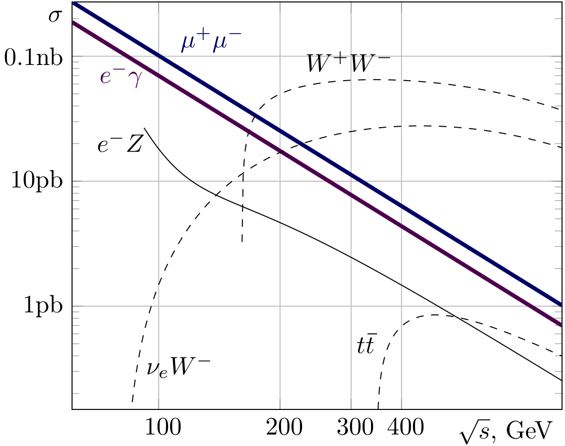
- NLO EW corrections for polarized e^+e^- scattering:
  - $e^+e^- \to e^+e^-$ (Bhabha) (Phys.Rev.D 98, 013001)
  - $e^+e^- \to ZH$ (arXiv:1812.10965)
  - $e^+e^- \to \mu^+\mu^-$(or $\tau^+\tau^-$) (preliminary)
  - $e^+e^- \to Z\gamma$ (preliminary)
  - $e^+e^- \to \gamma\gamma$ (preliminary)
  - $e^+e^- \to t\bar{t}$ (in progress)
  - $e^+e^- \to \nu\bar{\nu}H$ (in progress)
  - $e^+e^- \to ZZ$ (in progress)
  - $e^+e^- \to f\bar{f}\gamma$ (future plans)
  - $e^+e^- \to f\bar{f}H$ (future plans)
- NLO EW corrections for polarized $\gamma\gamma$ scattering:
  - $\gamma\gamma \to e^+e^-$ (in progress)
  - $\gamma\gamma \to \gamma\gamma$ (future plans)
  - $\gamma\gamma \to Z\gamma$ (future plans)
  - $\gamma\gamma \to ZZ$ (future plans)

Basic processes of SM for e^+e^- annihilation



The cross sections are given for polar angles between $10^o < \theta < 170^o$ in the final state.

Basic processes of SM for $e^{\pm}\gamma$ and $\gamma\gamma$ initial stat



The cross sections are given for polar angles between $10^o < \theta < 170^o$ in the final state.

Historical overview: Radiative corrections to Compton scattering

For the first time the process

$$e^+(p_1) + e^-(p_2) \longrightarrow \gamma(p_3) + \gamma(p_4), \qquad (1)$$

was considered in the classical papers:
1) L. M. Brown and R. P. Feynman, Radiative corrections to Compton scattering, Phys. Rev., 85, 1952
2) I. Harris and L. M. Brown, Radiative Corrections to Pair Annihilation, Phys. Rev., 105, 1957
3) Berends, Frits A. and Gastmans, R., Hard photon corrections for e+ e- —> gamma gamma, Nucl. Phys., B61, 1973.

MC generators at 1-loop level without polarization:

- Eur Phys J.(2011) 71:1597
  Monte-Carlo generator photon jets for the process $e^+e^- \to \gamma\gamma$
  S.I. Eidelman 1,2 , G.V. Fedotovich 1,2,a , E.A. Kuraev 3 , A.L. Sibidanov.
- Report "$e^+e^- \to \gamma\gamma$ for FCC$_{ee}$ lumi" C.M. Carloni Calame, M.Chiesa, G. Montagna, O. Nicrosini, F. Piccinini, in 11 FCC-ee workshop: Theory & Experiment

MC generators at tree level with polarization:

- WHIZARD
  W. Kilian, T. Ohl, J. Reuter, Eur.Phys.J.C71 (2011) 1742,
- CalcHEP
  A. Belyaev, N. Christensen, A. Pukhov,
  Comp. Phys. Comm. 184 (2013), pp. 1729-1769

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Motivation: an additional tool to measure luminosity

- Precise determination of luminosity is a key ingredient in all experiments.
  - **two-gamma-quantum annihilation**
    $e^+e^- \longrightarrow \gamma\gamma$,
  - **Bhabha scattering**
    $e^+e^- \longrightarrow e^+e^-$
  - **annihilation into a muon pair**
    $e^+e^- \longrightarrow \mu^+\mu^-$

Motivation: large cross section & no $\Pi_{\gamma\gamma}$ at NLO

• The cross section value estimated for large angles is **large enough**.
Events of this process have two collinear photons at large angles providing **a clean signature** for their selection among other detected particles.

• **No vacuum polarization effects.**
No source of uncertainty $\Pi_{\gamma\gamma}$.

Required accuracy

Cross section with radiative corrections (RC) at the level of per mill accuracy is needed (or even 10^{-4}):

- **1-loop level+polarization – done**
- **matching,** $e^+e^- \to \gamma\gamma + n\gamma$ – **soon** Implementation of works:
  - Frits A. Berends, R. Kleiss, et al., Nucl. Phys., B239, (1984)
  - E.A. Kuraev, V.S. Fadin, Sov.J.Nucl.Phys, **41**, 466 (1985)
- **NNLO corrections**

SANC: basics, procedures

- **Covariant amplitudes (CA) — $\mathcal{CA}$**
- **Scalar Form Factors (FF) — $\mathcal{F}_i$**
- **Helicity Amplitudes (HA) — $\mathcal{H}_{\{\lambda_i\}}(\mathcal{F}_i)$**

  standard approach: $\sigma \propto |\mathcal{CA}|^2$

  while in terms of HA: $\sigma \propto \sum_{\{\lambda_i\}} |\mathcal{H}_{\{\lambda_i\}}|^2$
- **Bremsstrahlung using HA: (BR)**
- **Analytical computing modules: FF, HA, BR**

Polarized scattering: two approaches

Longitudinal polarization, as example

- **First approach** — **covariant amplitude (CA) formalism**.
  The differential cross-section is proportional to square of the absolute value of covariant amplitude $\mathcal{A}$. For $2 \to 2$ kinematics

$$d\sigma = \frac{1}{32\pi s} |\mathcal{A}|^2 \, d\cos\theta,$$

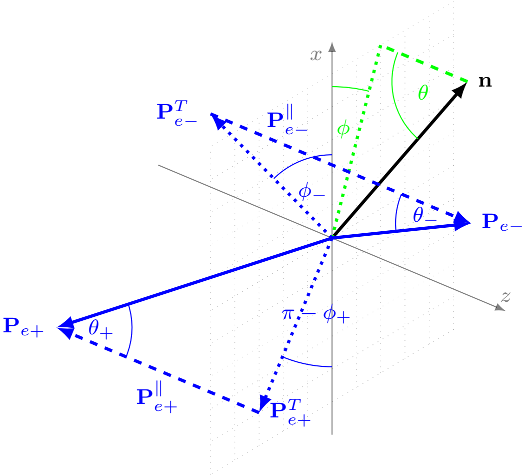
$$d\sigma = d\sigma_0 + (P_{e^+} - P_{e^-})d\sigma_1 + P_{e^+}P_{e^-}d\sigma_2$$

- **Second approach** — **helicity amplitude (HA) formalism**.
  To get the cross-section we sum squares of absolute values of non-interfering helicity amplitudes:

$$128\pi s \frac{d\sigma}{d\cos\theta}$$
$$= (1-P_{e^-})(1-P_{e^+})\sum_{ij}|\mathcal{H}_{++ij}|^2 + (1-P_{e^-})(1+P_{e^+})\sum_{ij}|\mathcal{H}_{+-ij}|^2$$
$$+(1+P_{e^-})(1-P_{e^+})\sum_{ij}|\mathcal{H}_{-+ij}|^2 + (1+P_{e^-})(1+P_{e^+})\sum_{ij}|\mathcal{H}_{--ij}|^2$$

(see eq. 1.15 in Phys. Rept. 460 (2008))

Decomposition of the $e^{\pm}$ polarization vectors



Matrix element squared

$$|\mathcal{M}|^2 = L^{\shortparallel}_{e^-} R^{\shortparallel}_{e^+} |\mathcal{H}_{-+}|^2 + R^{\shortparallel}_{e^-} L^{\shortparallel}_{e^+} |\mathcal{H}_{+-}|^2 + L^{\shortparallel}_{e^-} L^{\shortparallel}_{e^+} |\mathcal{H}_{--}|^2 + R^{\shortparallel}_{e^-} R^{\shortparallel}_{e^+} |\mathcal{H}_{++}|^2$$

$$- \frac{1}{2} P^{\perp}_{e^-} P^{\perp}_{e^+} \,\mathrm{Re}\Big[e^{i(\Phi_+ - \Phi_-)} \mathcal{H}_{++} \mathcal{H}^*_{--} + e^{i(\Phi_+ + \Phi_-)} \mathcal{H}_{+-} \mathcal{H}^*_{-+} \Big]$$
$$+ P^{\perp}_{e^-} \,\mathrm{Re}\Big[e^{i\Phi_-} \Big(L^{\shortparallel}_{e^+} \mathcal{H}_{+-} \mathcal{H}^*_{--} + R^{\shortparallel}_{e^+} \mathcal{H}_{++} \mathcal{H}^*_{-+} \Big) \Big]$$
$$- P^{\perp}_{e^+} \,\mathrm{Re}\Big[e^{i\Phi_+} \Big(L^{\shortparallel}_{e^-} \mathcal{H}_{-+} \mathcal{H}^*_{--} + R^{\shortparallel}_{e^-} \mathcal{H}_{++} \mathcal{H}^*_{+-} \Big) \Big],$$

where

$$L^{\shortparallel}_{e^\pm} = \frac{1}{2}(1 - P^{\shortparallel}_{e^\pm}), \quad R^{\shortparallel}_{e^\pm} = \frac{1}{2}(1 + P^{\shortparallel}_{e^\pm}), \quad \Phi_\pm = \phi_\pm - \phi,$$

$\mathcal{H}_{--}$, $\mathcal{H}_{++}$,$\mathcal{H}_{-+}$,$\mathcal{H}_{+-}$ — helicity amplitudes.

SANC: basics, scheme of FF calculation

- **One-loop accuracy level with using the renormalization scheme on the mass surface in R_ξ calibration with three calibration parameters ξ_A, ξ_Z and $\xi \equiv \xi_\mathsf{W}$.**
- **To parameterize the ultraviolet divergences used dimensional regularization.**
- **Loop integrals are expressed in terms of standard scalar Passarino-Veltman functions: A_0, B_0, C_0, D_0.**
- **The error criterion is the absence of ξ dependencies.**

SANC: basics, analytical calculations

- **Predictions: calculate in advance all the necessary single-loop diagrams and related quantities (for example, renormalization constants, etc.) and save the results as files.**
- **Covariant amplitudes (CA)**
- **Scalar Form Factors ($\mathcal{F}$)**
- **Helicity Amplitudes (HA)**
- **Related Photon Radiation (BR)**

Analytical calculations – FORM4.2

Foundation stone: $2f2b \to 0$ and channel reversal

$$\gamma(p_1) + \gamma(p_2) + l^+(p_3) + l^-(p_4) \longrightarrow 0.$$

$$\begin{aligned} \gamma + \gamma &\to l^+ + l^- \\ l^+ + l^- &\to \gamma + \gamma \\ l + \gamma &\to l + \gamma \end{aligned}$$

Numerical results $l = e, \mu$

Cross-section structure

The cross-section of this processes at one-loop can be devided into four parts:

$$\sigma^{\text{1-loop}} = \sigma^{\text{Born}} + \sigma^{\text{virt}}(\lambda) + \sigma^{\text{soft}}(\lambda, \omega) + \sigma^{\text{hard}}(\omega),$$

Contributions due to:
σ^{Born} — Born level cross-section,
σ^{virt} — virtual(loop) corrections,
σ^{soft} — soft photon emission,
σ^{hard} — hard photon emission (with energy $E_\gamma > \omega$).
Auxiliary parameters λ ("photon mass") and ω cancel out after summation.

SANC, from analytical results to numbers

- **Virtual corrections**

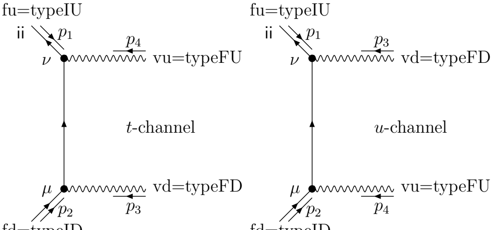
$$d\sigma = \sum_{\lambda_1\lambda_2\lambda_3\lambda_4} \left| \mathcal{H}\left(\mathcal{F}^{\mathbf{Born+1loop}}\right)_{\lambda_1\lambda_2\lambda_3\lambda_4} \right|^2$$

- **Real corrections**
  - **Soft bremsstrahlung**

$$d\sigma^{Soft} = K d\sigma^{Born}$$

  - **Hard bremsstrahlung**

Born level, $e^+e^- \to \gamma\gamma$, t and u channels



a) t-channel b) u-channel

fu=typeIU ... - “types” of external particles

$$s = -(p_1+p_2)^2, t = -(p_2+p_3)^2, u = -(p_2+p_4)^2,$$
$$s' = -(p_3+p_4)^2, t' = -(p_1+p_4)^2, u' = -(p_1+p_3)^2,$$
$$Z_1 = -2p_1p_5, Z_2 = -2p_2p_5, Z_3 = -2p_3p_5, Z_4 = -2p_4p_5.$$

Covariante amplitude

$$\mathcal{CA} = \sum_{i=1}^{8} Str(\mathcal{F}_{V_i})\mathcal{F}_{V_i}(s,t,u) + \sum_{i=1}^{4} Str(\mathcal{F}_{A_i})\mathcal{F}_{A_i}(s,t,u).$$

Vector structures general view

$$Str(\mathcal{F}_{V_0}) = (i\not{K}_2 - m_l)\gamma^\mu\gamma^\nu + 2i\frac{m_l^2 - t}{s}\left(\gamma^\mu p_4^\nu - \gamma^\nu p_3^\mu + \frac{1}{2}(\not{p}_3 - \not{p}_4)\delta^{\mu\nu}\right).$$

$$\not{K}_2 = \frac{1}{2}\left(\not{p}_3 - \not{p}_4 + \not{p}_2 - \not{p}_1\right).$$

Vector structures: minimized view

$$
\begin{aligned}
Str(\mathcal{F}_{V_1}) &= (i\not{K}_2 - m_l)\,\tau_7^{\mu\nu} - 2i\frac{m_l^2 - t}{s}\left(\tau_9^{\mu\nu} - \tau_{11}^{\mu\nu} + \frac{1}{2}(\not{p}_3 - \not{p}_4)\tau_{12}^{\mu\nu}\right), \\
Str(\mathcal{F}_{V_2}) &= i\not{K}_2\tau_3^{\mu\nu}, \\
Str(\mathcal{F}_{V_3}) &= -\frac{1}{2}\frac{m_l^2 - t}{s}\left[i\tau_1^{\mu\nu} + 2\tau_4^{\mu\nu} + 2m_l\left(\tau_5^{\mu\nu} - (m_l^2 - t)\tau_{12}^{\mu\nu}\right)\right] \\
&+ i\not{K}_2\tau_6^{\mu\nu}, \\
Str(\mathcal{F}_{V_4}) &= \frac{i}{2}\tau_1^{\mu\nu} + i\not{K}_2 2\tau_4^{\mu\nu} + (i\not{K}_2 + m_l)\,\tau_5^{\mu\nu} - m_l(m_l^2 - t)\tau_{12}^{\mu\nu},
\end{aligned}
$$

Vector structures: minimized view

$$Str(\mathcal{F}_{V_5}) = 4i\frac{m_l}{m_l^2 - t}\not{K}_2\tau_4^{\mu\nu} + \frac{1}{2}s\tau_7^{\mu\nu} + \not{p}_4\tau_9^{\mu\nu} + \not{p}_3\tau_{11}^{\mu\nu} - (m_l^2 + t)\tau_{12}^{\mu\nu},$$

$$Str(\mathcal{F}_{V_6}) = \frac{2}{s}\Big[(m_l^2 - t)\Big(\tau_5^{\mu\nu} + \frac{s}{4}\tau_7^{\mu\nu} - \frac{3m_l^2 - t}{2}\tau_{12}^{\mu\nu}\Big) + im_l\left(\tau_1^{\mu\nu} + 2\not{K}_2\tau_4^{\mu\nu}\right)\Big] + \not{p}_4\tau_8^{\mu\nu} + \not{p}_3\tau_{10}^{\mu\nu},$$

$$Str(\mathcal{F}_{V_7}) = \frac{1}{2}s\tau_3^{\mu\nu},$$

$$Str(\mathcal{F}_{V_8}) = \tau_6^{\mu\nu} + \frac{m_l^2 - t}{s}\left[\tau_5^{\mu\nu} - \frac{1}{2}(m_l^2 - t)\tau_{12}^{\mu\nu}\right].$$

Axial structures: minimized view

$$
\begin{aligned}
Str(\mathcal{F}_{A_1}) &= -i\not{K}_2\left(\tau_4^{\mu\nu} - \tau_7^{\mu\nu}\right) \\
&+ \frac{2i}{s}\left[\left(m_l^2 - t - im_l\right)\tau_9^{\mu\nu} + \left(m_l^2 - t + im_l\right)\tau_{11}^{\mu\nu}\right] \\
Str(\mathcal{F}_{A_2}) &= i\not{K}_2\frac{s}{m_l^2 - t}\tau_3^{\mu\nu}, \\
Str(\mathcal{F}_{A_3}) &= -\frac{1}{2}\tau_2^{\mu\nu} - i\not{K}_2\left(\tau_4^{\mu\nu} - \tau_6^{\mu\nu}\right), \\
Str(\mathcal{F}_{A_4}) &= \frac{1}{2}\frac{s}{m_l^2 - t}\tau_2^{\mu\nu} + i\left(2\tau_4^{\mu\nu} + \tau_5^{\mu\nu}\right).
\end{aligned}
$$

Strings

$$\tau_1^{\mu\nu} = \gamma^\mu \left[(m_l^2 - u)p_2^\nu - (m_l^2 - t)(p_1^\nu + p_3^\nu)\right]$$
$$-\gamma^\nu \left[(m_l^2 - u)p_1^\mu - (m_l^2 - t)(p_2^\mu + p_4^\mu)\right],$$

$$\tau_2^{\mu\nu} = (m_l \not{K}_2 - it)\,\gamma_\nu \left(p_1^\mu + \frac{m_l^2 - t}{s} p_3^\mu\right)$$
$$+ (m_l \not{K}_2 + it)\,\gamma_\mu \left(p_2^\nu + \frac{m_l^2 - t}{s} p_4^\nu\right),$$

$$\tau_3^{\mu\nu} = \delta^{\mu\nu} + 2\frac{p_3^\mu p_4^\nu}{s},$$

$$\tau_4^{\mu\nu} = \frac{m_l^2 - t}{s} p_3^\mu p_4^\nu,$$

$$\tau_5^{\mu\nu} = p_1^\mu p_4^\nu + p_2^\nu p_3^\mu,$$

$$\tau_6^{\mu\nu} = p_1^\mu p_2^\nu,$$

$$\tau_7^{\mu\nu} = \gamma^\mu \gamma^\nu,$$

$$\tau_8^{\mu\nu} = \gamma^\nu p_1^\mu,$$

$$\tau_9^{\mu\nu} = \gamma^\nu p_3^\mu,$$

$$\tau_{10}^{\mu\nu} = \gamma^\mu p_2^\nu,$$

$$\tau_{11}^{\mu\nu} = \gamma^\mu p_4^\nu,$$

$$\tau_{12}^{\mu\nu} = \delta^{\mu\nu}.$$

Massive basis

We introduce massive analogs of massless X,Y operators (14.58-14.59) and write structures using them.

- **CP-even**

$$X^1_{\pm\alpha\beta} = \frac{1}{\sqrt{s}}\,\bar{v}\gamma_\pm\gamma_\alpha\,(\not{p}_+ + \not{q}_+)\,\gamma_\beta u, \qquad X^2_{\pm\alpha\beta} = \frac{1}{\sqrt{s}}\,\bar{v}\gamma_\pm\not{q}_- u\delta_{\alpha\beta},$$
$$X^3_{\pm\alpha\beta} = \frac{1}{\sqrt{s}}\,\bar{v}\gamma_\pm\,(\gamma_\alpha q_{+\beta} - \gamma_\beta q_{-\alpha})\,u, \qquad X^4_{\pm\alpha\beta} = \frac{1}{\sqrt{s}}\bar{v}\gamma_\pm\,(\gamma_\alpha p_{-\beta} - \gamma_\beta p_{+\alpha})\,u,$$
$$X^5_{\pm\alpha\beta} = \frac{1}{s\sqrt{s}}\,\bar{v}\gamma_\pm\not{q}_- q_{-\alpha}q_{+\beta}u, \qquad X^6_{\pm\alpha\beta} = \frac{1}{s\sqrt{s}}\,\bar{v}\gamma_\pm\not{q}_- p_{+\alpha}p_{-\beta}u, \tag{14.58}$$
$$X^7_{\pm\alpha\beta} = \frac{1}{s\sqrt{s}}\,\bar{v}\gamma_\pm\not{q}_-\,(q_{-\alpha}p_{-\beta} + p_{+\alpha}q_{+\beta})\,u.$$

- **CP-odd**

$$Y^1_{\pm\alpha\beta} = \frac{1}{\sqrt{s}}\,\bar{v}\gamma_\pm\,(\gamma_\alpha q_{+\beta} + \gamma_\beta q_{-\alpha})\,u, \qquad Y^2_{\pm\alpha\beta} = \frac{1}{\sqrt{s}}\,\bar{v}\gamma_\pm\,(\gamma_\alpha p_{-\beta} + \gamma_\beta p_{+\alpha})\,u, \tag{14.59}$$
$$Y^3_{\pm\alpha\beta} = \frac{1}{s\sqrt{s}}\,\bar{v}\gamma_\pm\not{q}_-\,(q_{-\alpha}p_{-\beta} - p_{+\alpha}q_{+\beta})\,u.$$

D. Bardin and G. Passarino book 'The Standard Model in the Making'

Helicity amplitudes

Amplitudes combine to 4 sets:

Set 1

$$\mathcal{H}_{-++-}, \mathcal{H}_{+--+}, \mathcal{H}_{-+-+}, \mathcal{H}_{+-+-}.$$

Set 2

$$\mathcal{H}_{+-++}, \mathcal{H}_{-+++}, \mathcal{H}_{-+--}, \mathcal{H}_{+---}.$$

Set 3

$$\mathcal{H}_{---+}, \mathcal{H}_{+++-}, \mathcal{H}_{++-+}, \mathcal{H}_{--+-}.$$

Set 4

$$\mathcal{H}_{++++}, \mathcal{H}_{----}, \mathcal{H}_{--++}, \mathcal{H}_{++--}.$$

Helicity amplitudes $l^+l^- \to \gamma\gamma$. Example: set 1

$$
\begin{aligned}
\mathcal{A}_{-+-+} &= N_1 c_+ \left[-V_{11} + A_{11}\right], \\
\mathcal{A}_{+-+-} &= N_1 c_+ \left[V_{11} + A_{11}\right], \\
\mathcal{A}_{-++-} &= N_1 c_- \left[V_{12} + A_{12}\right], \\
\mathcal{A}_{+--+} &= N_1 c_- \left[-V_{12} + A_{12}\right]. \\
N_1 &= \frac{1}{8} s\beta \sin\theta_\gamma, \\
V_{11} &= 4\frac{s}{z_1 z_2}\mathcal{F}_{v_1} + \frac{s}{2}\left(2 - \beta c_+\right)\mathcal{F}_{v_3} - 2s\mathcal{F}_{v_4} + 4m_f\mathcal{F}_{v_6}, \\
A_{11} &= 4\beta\mathcal{F}_{a_1} - \frac{s}{2}\left[\left(\beta_-^2 - \beta^2 c_-\right)\mathcal{F}_{a_3} + \left(\beta_-^2 - 2\beta^2 c_-\right)\mathcal{F}_{a_4}\right], \\
V_{12} &= 4\frac{s}{z_1 z_2}\mathcal{F}_{v_1} + \frac{s}{2}\left(2 + \beta c_-\right)\mathcal{F}_{v_3} - 2s\mathcal{F}_{v_4} + 4m_f\mathcal{F}_{v_6}, \\
A_{12} &= -4\beta\mathcal{F}_{a_1} - \frac{s}{2}\left[\left(\beta_+^2 - \beta^2 c_+\right)\mathcal{F}_{a_3} + \left(\beta_+^2 - 2\beta^2 c_+\right)\mathcal{F}_{a_4}\right].
\end{aligned}
$$

Helicity amplitudes $l^+l^- \to \gamma\gamma$. Example: set 2

$$
\begin{aligned}
\mathcal{A}_{+-++} &= \mathcal{A}_{+---} = N_2(V_2 + A_2),\\
\mathcal{A}_{-+++} &= \mathcal{A}_{-+--} = N_2(-V_2 + A_2).\\
N_2 &= \frac{1}{8} s \sin\theta_\gamma,\\
V_2 &= -4\mathcal{F}_{v_2}\\
&+ s\beta\left[(\beta^c_- - \frac{1}{2}\beta c_+ c_-)\mathcal{F}_{v_3} + 2\cos\theta_\gamma\left(\mathcal{F}_{v_4} - 2\frac{m_f}{s}\mathcal{F}_{v_6}\right)\right],\\
A_2 &= s\beta\left[-\frac{1}{s}\left(k_1\mathcal{F}_{a_1} + \mathcal{F}_{a_2}\right) + \frac{1}{2}k_1^2\mathcal{F}_{a_3} + \frac{1}{2}(k_1^2 - \beta^2 c_- c_+)\mathcal{F}_{a_4}\right].
\end{aligned}
$$

Helicity amplitudes $\gamma\gamma \to l^+l^-$. Example: set 3

$$
\begin{aligned}
\mathcal{H}_{++++-} &= \mathcal{H}_{--+-} = N_1(V_1 - A_1), \\
\mathcal{H}_{---+} &= \mathcal{H}_{++-+} = N_1(V_1 + A_1). \\
N_1 &= -\frac{1}{8}s^{3/2}\beta c_+ c_-, \\
V_1 &= m_l\left[\frac{8}{s}\mathcal{F}_{v_1} + (2-\beta\beta_-^c)\mathcal{F}_{v_3} - 4\mathcal{F}_{v_4}\right] + 2(1-2\beta^2)\mathcal{F}_{v_6} + \beta^2\mathcal{F}_{v_8}, \\
A_1 &= m_l\left(\mathcal{F}_{a_3} + \mathcal{F}_{a_4}\right).
\end{aligned}
$$

Hard: $e^+e^- \to \gamma\gamma\gamma \& \gamma\gamma \to e^+e^-\gamma$

Descriptions of the pure QED reaction $\gamma\gamma \to e^+e^-\gamma$ using method of helicity amplitudes are:

- S. Dittmaier, Phys Rev D59, (1999), 016007, hep-ph/9805445
- T.V. Shishkina, V.V. Makarenko, (2003), hep-ph/0212409

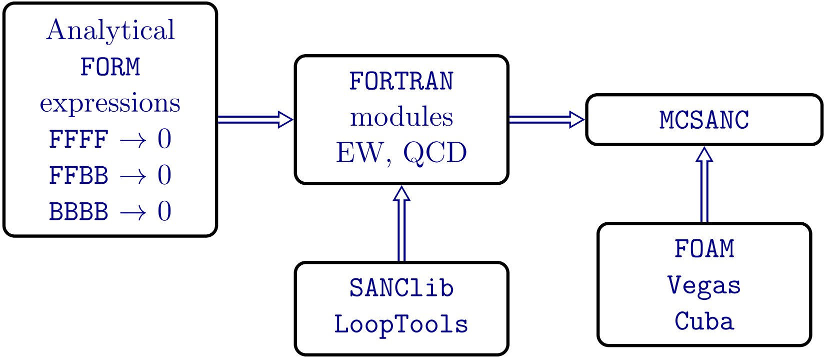
HA, example for interplay

$e^+(p_1) + e^-(p_2) \longrightarrow \gamma(p_3) + \gamma(p_4) + \gamma(p_5)$

C, P, Bose and crossing between final and initial symmetries:

$$\begin{aligned}
\mathcal{H}^{+---++} &= 2e^3 p_2 \cdot p_4 \sqrt{\frac{p_3 \cdot p_4}{p_1 \cdot p_3 \; p_1 \cdot p_4 \; p_3 \cdot p_5 \; p_4 \cdot p_5}} \\
\mathcal{H}^{+-+--} &= \mathcal{H}^{+---++}, (P + Bose) \\
\mathcal{H}^{+-+-+} &= \mathcal{H}^{+---++}, (C) \\
\mathcal{H}^{+--+-} &= \mathcal{H}^{+---++}, (CP + Bose) \\
\mathcal{H}^{+++--} &= \mathcal{H}^{+---++}, (C + crossing) \\
\mathcal{H}^{++-+-} &= \mathcal{H}^{+++--}, (C) \\
\mathcal{H}^{+++++} &= \mathcal{H}^{-----}, (P)
\end{aligned}$$

The SANC/ARIeL framework and MCSANC generator



We created Monte Carlo generator of unweighted events for the polarized events with NLO EW corrections stored in standard Les Houches format.

This generator uses adaptive algorithm mFOAM (CPC 177:441-458,2007) which is part of ROOT program.

Numerical results: Setup for tuned comparison

We performed a tuned comparison of our results for polarized Born and hard Bremsstrahlung with the results WHIZARD and CalcHEP programs.

Initial parameters

$$\begin{aligned}
\alpha^{-1}(0) &= 137.03599976, & M_W &= 80.451495 \text{ GeV}, & \Gamma_W &= 2.0836 \text{ GeV},\\
M_H &= 125.0 \text{ GeV}, & M_Z &= 91.1876 \text{ GeV}, & \Gamma_Z &= 2.49977 \text{ GeV},\\
m_e &= 0.5109990 \text{ MeV}, & m_\mu &= 0.105658 \text{ GeV}, & m_\tau &= 1.77705 \text{ GeV},\\
m_d &= 0.083 \text{ GeV}, & m_s &= 0.215 \text{ GeV}, & m_b &= 4.7 \text{ GeV},\\
m_u &= 0.062 \text{ GeV}, & m_c &= 1.5 \text{ GeV}, & m_t &= 173.8 \text{ GeV}.
\end{aligned}$$

$$\text{with cuts } |\cos\theta| < 0.9, \quad E_\gamma > 1 \text{ GeV}$$

WHIZARD and CalcHEP

- W. Kilian, T. Ohl, J. Reuter, Eur.Phys.J.C71 (2011) 1742,
- A.Belyaev, N.Christensen,A.Pukhov, Comp. Phys. Comm. 184 (2013), pp. 1729-1769

$e^+e^- \to \gamma\gamma$: CalcHep vs MCSANCee (Born), fb

$\sqrt{s}$=250 GeV

P_{e^-}, P_{e^+}	0, 0	-0.8, 0	-0.8, -0.6	-0.8, 0.6
WHIZARD	4262	4262	2216	6307
CalcHEP	4262	4262	2216	6307
MCSANCee	4261	4261	2216	6307

$\sqrt{s}$=500 GeV

P_{e^-}, P_{e^+}	0, 0	-0.8, 0	-0.8, -0.6	-0.8, 0.6
WHIZARD	1065	1065	554.0	1577
CalcHEP	1065	1065	554.0	1577
MCSANCee	1065	1065	554.0	1577

$\sqrt{s}$=1000 GeV

P_{e^-}, P_{e^+}	0, 0	-0.8, 0	-0.8, -0.6	-0.8, 0.6
WHIZARD	266.3	266.3	138.5	394.2
CalcHEP	266.4	266.4	138.5	394.2
MCSANCee	266.3	266.3	138.5	394.2

$e^+e^- \to \gamma\gamma$: CalcHep vs MCSANCee (Hard), fb

$\sqrt{s}$=250 GeV

P_{e^-}, P_{e^+}	0, 0	-0.8, 0	-0.8, -0.6	-0.8, 0.6
WHIZARD	594.1	594.1	308.9	879.2
CalcHEP	594.0	593.8	308.8	879.2
MCSANCee	594.0	594.2	308.9	879.1

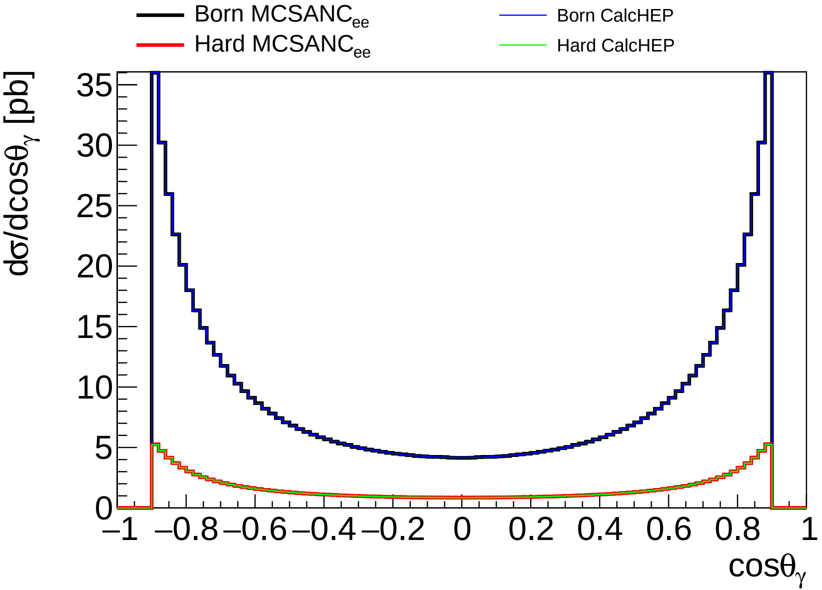
$\sqrt{s}$=500 GeV

P_{e^-}, P_{e^+}	0, 0	-0.8, 0	-0.8, -0.6	-0.8, 0.6
WHIZARD	158.6	158.5	82.50	234.5
CalcHEP	158.7	158.6	82.50	234.8
MCSANCee	158.7	158.7	82.50	234.8

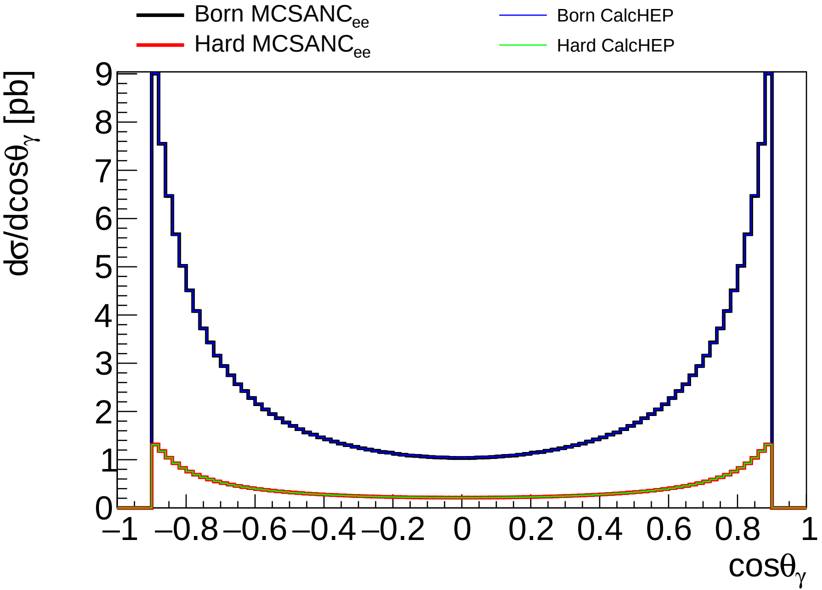
$\sqrt{s}$=1000 GeV

P_{e^-}, P_{e^+}	0, 0	-0.8, 0	-0.8, -0.6	-0.8, 0.6
WHIZARD	42.18	42.20	21.95	62.43
CalcHEP	42.19	42.18	21.94	62.45
MCSANCee	42.17	42.20	21.94	62.44

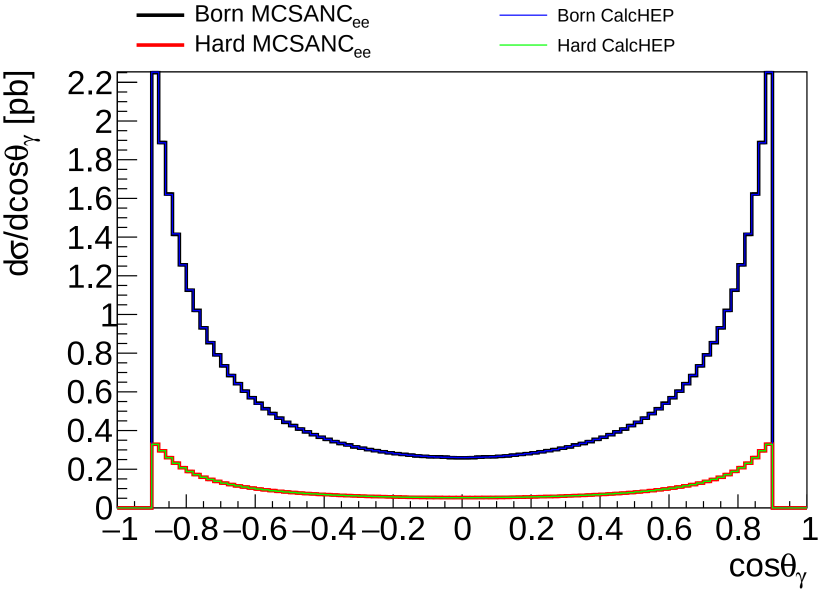
$e^+e^- \to \gamma\gamma$: σ distributions on $cos(\theta)$. $\sqrt{s}$=250 GeV



$e^+e^- \to \gamma\gamma$: σ distributions on $cos(\theta)$. $\sqrt{s}$=500 GeV



$e^+e^- \to \gamma\gamma$: σ distributions on $cos(\theta)$. $\sqrt{s}$=1000GeV



$e^+e^- \to \gamma\gamma$: Preliminary NLO EW results (QED)

P_{e^-}, P_{e^+}	0, 0	-0.8, 0	-0.8, 0.6	-0.8, -0.6
$\sqrt{s} = 500$ GeV				
σ^{Born}, pb	1065	1065	554.0	1577
$\sigma^{\text{1-loop}}$, pb	1074	1074	558.5	1589
δ, %	0.82	0.82	0.82	0.82

Angle cut $|cos(\theta)_{CM}| < 0.9$ on all photons.

RESUME: $\texttt{MCSANC}_{\texttt{ee}}$, April 2019

Implementation into Monte Carlo generator $\texttt{MCSANC}_{\texttt{ee}}$ of unweighted events the polarized $e^+e^- \to \gamma\gamma$ scattering with complete one-loop EW corrections and with possibility to produce events in Standard Les Houches Format.

Two schemes: $\alpha(M_Z), G_F$

Generator uses adaptive algorithm mFOAM (CPC 177:441-458,2007) which is part of ROOT program.