Novel method for direct measurement of the τ dipole moments

Jinlin Fu
INFN Sezione di Milano

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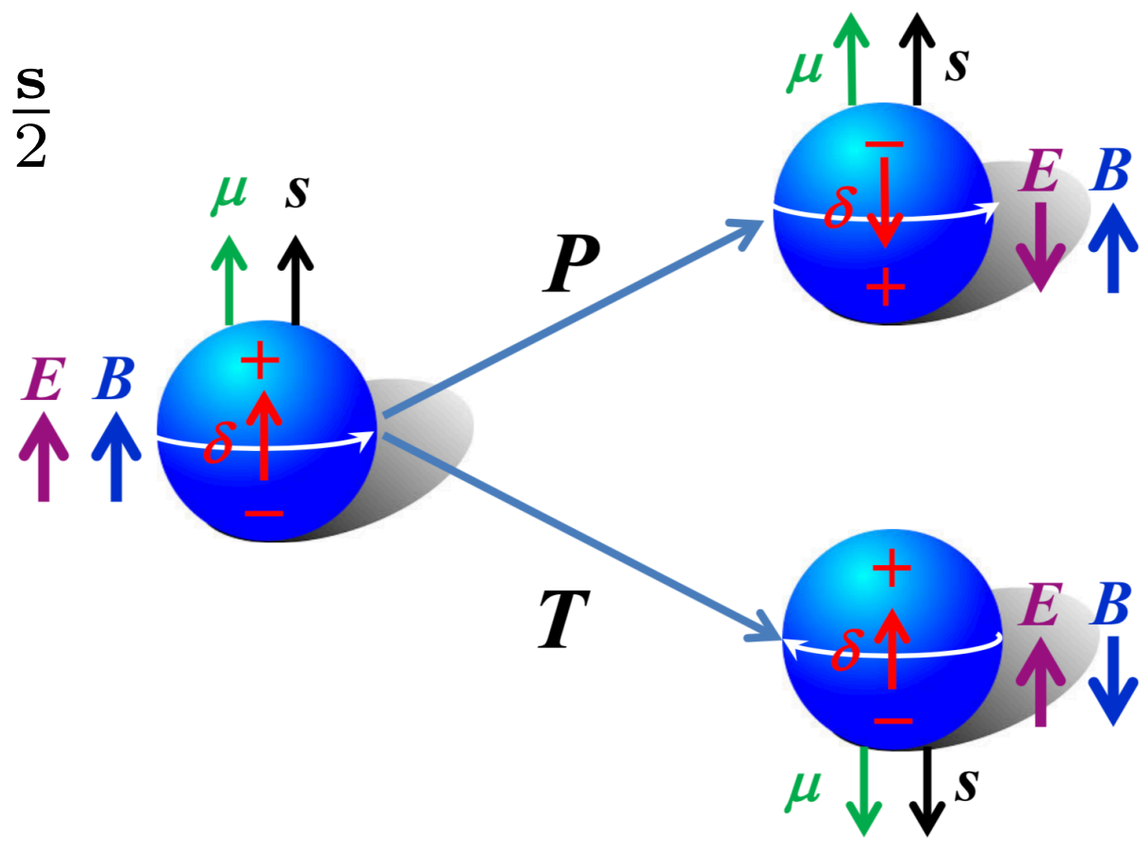
Istituto Nazionale di Fisica Nucleare

Electromagnetic dipole moments

- Electric and magnetic dipole moments, Gaussian units

EDM: $\boldsymbol{\delta} = d\frac{e\hbar}{2m_\tau c}\frac{\mathbf{s}}{2} = d\mu_N\frac{\mathbf{s}}{2}$

MDM: $\boldsymbol{\mu} = g\frac{e\hbar}{2m_\tau c}\frac{\mathbf{s}}{2} = g\mu_N\frac{\mathbf{s}}{2}$



- CP violation

$$\mathcal{H} = -\boldsymbol{\mu}\cdot\boldsymbol{B} - \boldsymbol{\delta}\cdot\boldsymbol{E}$$

$$\mathcal{H} \xrightarrow{\mathrm{P,T}} \mathcal{H} = -\boldsymbol{\mu}\cdot\boldsymbol{B} + \boldsymbol{\delta}\cdot\boldsymbol{E}$$

CPT theorem

$\boldsymbol{\delta} \neq 0 \implies$ P and T violation $\implies$ CP violation

- Anomalous MDM: $a = (g-2)/2$

Motivation: g-2

- a=(g-2)/2 of e and μ, among the most stringent QED test

$$a_e = (1159652180.73 \pm 0.28) \times 10^{-12}$$ most accurate determination of α

$$a_\mu = (11659208.9 \pm 6.3) \times 10^{-10}$$

more sensitive to EW corrections from virtual heavier states, scale as $(m_\mu/m_e)^2$

- Long standing 3σ discrepancy between theory and experiment for a_μ, QCD or NP?

$$a_\mu^{\mathrm{SM}} = (11659184.4 \pm 5.3) \times 10^{-10}$$

- g-2 of τ has an enhanced sensitivity to NP due to large mass.

In SM the EW correction scale roughly as $(m_\tau/m_\mu)^2$=283

$$a_\tau^{\mathrm{SM}} = (117721 \pm 5) \times 10^{-8} \equiv 0.00117721 \pm 0.00000005$$

S. Eidelman et al., Mod. Phys. Lett. A22, 159 (2017)

Motivation: g-2 (cont'd)

- Measurement of $\sigma(e^+e^- \to e^+e^-\tau^+\tau^-)$ at 183 and 208 GeV from DELPHI at LEP2: $-0.052 < a_\tau < 0.013$ @95% CL **PDG2018**
- One order of magnitude worse than QED contribution => huge room to improve
- Several methods suggest in the literature, none implemented:
  A. Pich, Prog.Part.Nucl.Phys. 75 (2014) 41-85
  - ✓ At B factories, using the decay $\tau^- \to \upsilon_\tau \ell^- \overline{\upsilon}_\ell \gamma$
  - ✓ Using radiative $W^- \to \tau^- \overline{\upsilon}_\tau \gamma$ decays at LHC
  - ✓ Through $\gamma\gamma \to \tau^+\tau^-$, i.e. $e^+e^- \to e^+\gamma^*\gamma^* e^- \to e^+\tau^+\tau^- e^-$
  - ✓ From angular distribution of the τ decay products at Super B factory $e^+e^- \to \gamma^* \to \tau^+\tau^-$
    $\sigma(a_\tau) \approx 10^{-5}$ Belle2, 50ab^{-1} **Chen, Wu, arXiv:1803.00501**
- Combination of experimental information from LEP1, LEP2

$$-0.007 < a_\tau^{\mathrm{NP}} < 0.005$$

Gonzalez, Santamaria, Vidal, NPB 582 (2003) 3

Motivation: EDM

- For e and μ, there are strong experimental limits, which provides stringent constraints on BSM:
  $|\delta_e| < 8.7\times10^{-29}\, e\ \mathrm{cm}\ (90\%\ \mathrm{CL})$ $\qquad \delta_\mu = (-0.1\pm0.9)\times10^{-19}\, e\ \mathrm{cm}$
- For τ, several techniques have been proposed:
  A. Pich, Prog.Part.Nucl.Phys. 75 (2014) 41-85
  - ✓ Using cross sections for
    $e^+e^-\to\tau^+\tau^-,\ e^+e^-\to\tau^+\tau^-\gamma,\ e^+e^-\to e^+e^-\tau^+\tau^-,\ Z\to\tau^+\tau^-,\ Z\to\tau^+\tau^-\gamma$
    rather poor sensitivity since $F_3(s)$ dependence is quadratic
  - ✓ From angular distribution of the τ decay products at Super B factory $e^+e^-\to\gamma^*\to\tau^+\tau^-$
    $\sigma(\delta_\tau)\approx10^{-19}\ e\ \mathrm{cm}$ Belle2, 50ab^{-1} Chen, Wu, arXiv:1803.00501
  - ✓ Sensitivity could be improved with a longitudinally polarised e beam
- Current limit from Belle using $e^+e^-\to\gamma^*\to\tau^+\tau^-$

$$-0.22 < \mathrm{Re}(\delta_\tau) < 0.45 \quad (\times10^{-16} e\ \mathrm{cm},\ 95\%\ \mathrm{CL})$$
$$-0.25 < \mathrm{Im}(\delta_\tau) < 0.08 \quad (\times10^{-16} e\ \mathrm{cm})$$

Belle collaboration, Pays. Lett. B551, 16 (2003)

Motivation summary

- Potentially very sensitive to NP
- Strong theoretical interest
- Still essentially unknown experimentally

How to access EDM/MDM

- **Spin precession** induced by interaction of its EDM and MDM with external EM field

- **Time revolution of spin-polarisation vector**

$$\frac{d\mathbf{s}}{dt} = \mathbf{s} \times \boldsymbol{\Omega}$$

$$\boldsymbol{\Omega} = \boldsymbol{\Omega}_{\mathrm{MDM}} + \boldsymbol{\Omega}_{\mathrm{EDM}} + \boldsymbol{\Omega}_{\mathrm{TH}}$$

$$\boldsymbol{\Omega}_{\mathrm{MDM}} = \boxed{\frac{g\mu_B}{\hbar}}\left(\mathbf{B} - \frac{\gamma}{\gamma+1}(\boldsymbol{\beta}\cdot\mathbf{B})\boldsymbol{\beta} - \boldsymbol{\beta}\times\mathbf{E}\right)$$

$$\Omega_{\mathrm{EDM}} = \boxed{\frac{d\mu_B}{\hbar}}\left(\mathbf{E} - \frac{\gamma}{\gamma+1}(\boldsymbol{\beta}\cdot\mathbf{E})\boldsymbol{\beta} - \boldsymbol{\beta}\times\mathbf{B}\right)$$

$$\boldsymbol{\Omega}_{\mathrm{TH}} = \frac{\gamma^2}{\gamma+1}\boldsymbol{\beta}\times\frac{d\boldsymbol{\beta}}{dt} = \frac{q}{mc}\left[\left(\frac{1}{\gamma}-1\right)\mathbf{B} + \frac{\gamma}{\gamma+1}(\boldsymbol{\beta}\cdot\mathbf{B})\boldsymbol{\beta} - \left(\frac{1}{\gamma+1}-1\right)\boldsymbol{\beta}\times\mathbf{E}\right]$$

Experimental difficulties

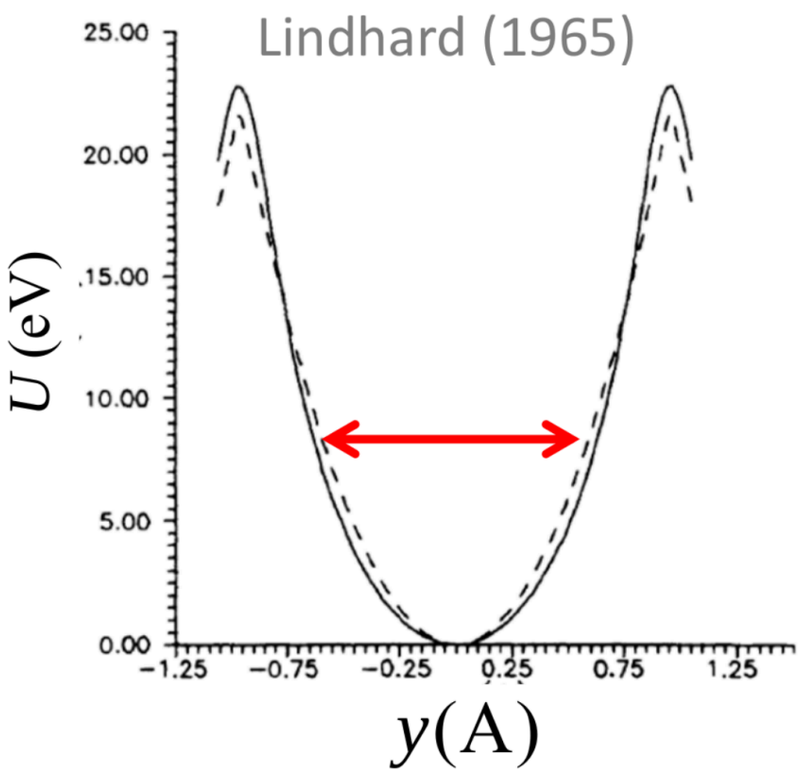
- short lifetime 10^{-13}s of tau
- large source of highly-boosted and polarised tau
- neutrino in tau decays

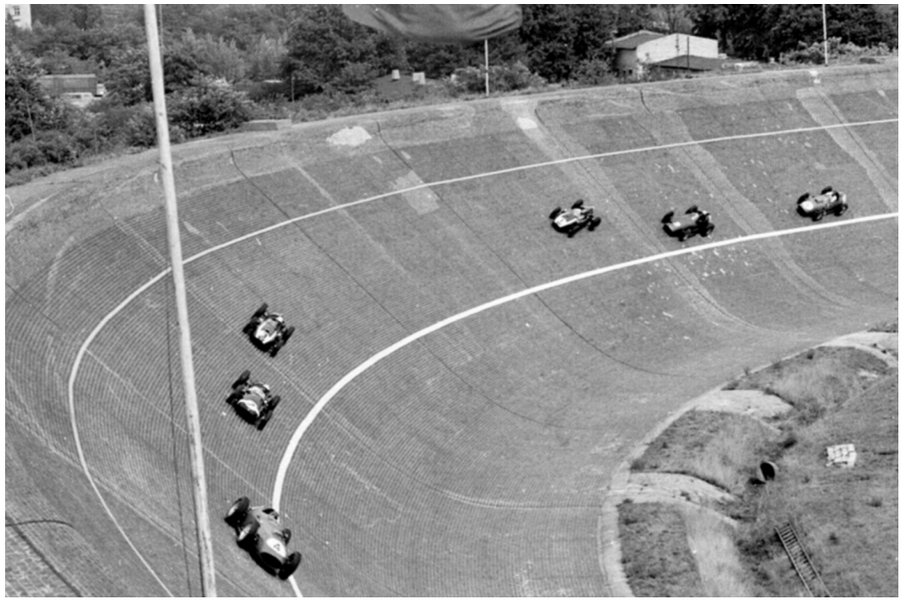
Experimental difficulties

- short lifetime 10^{-13}s of tau
- large source of highly-boosted and polarised tau
- neutrino in tau decays
- **Intense EM field:** bent crystal.
- **New tau production:** fixed target collision
- **New analysis technique**

Channeling in Bent Crystals

Potential well between crystallographic planes





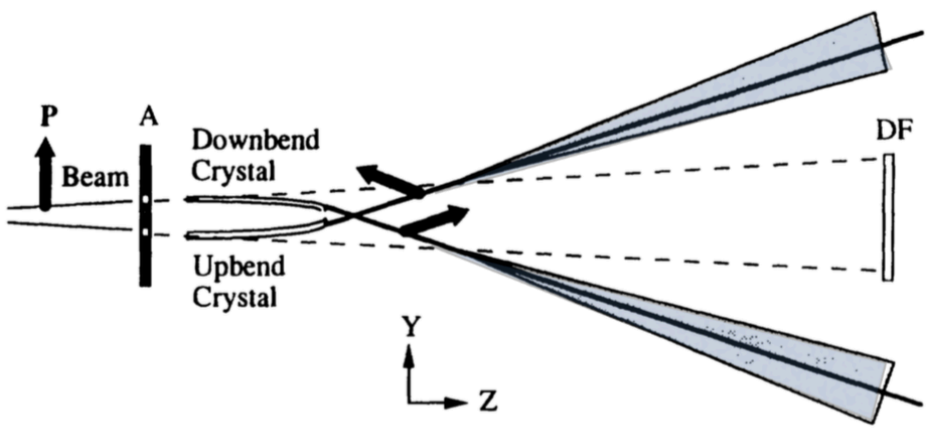
- Incident positively-charged particles with small transverse energy can be trapped ⇒ small incident angle w.r.t. crystal planes (few μrad)
- Intense E in crystal transforming to stronger EM field in τ^+ rest frame
- τ^+ pathways adiabatically bent following crystal curvature, resulting net deflection of incoming direction

Experiments with bent crystal

- Spin process first observed in bend crystals at E761 Fermilab experiment and measure MDM of Σ^+ **Phys. Rev. Lett. 69 (1992) 3286**

VOLUME 69, NUMBER 23 PHYSICAL REVIEW LETTERS 7 DECEMBER 1992

First Observation of Magnetic Moment Precession of Channeled Particles in Bent Crystals



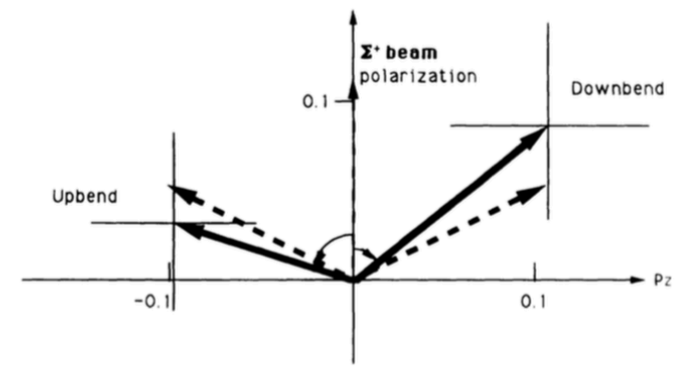


FIG. 3. Measured polarizations and uncertainties (1σ statistical errors) after spins have been precessed by the two crystals. The dashed arrows show the expected precessions.

- Experiments proposed to measure MDM/EDM of heavier baryon using bent crystal

E. Bagli et al., Electromagnetic dipole moments of charged baryons with bent crystals at LHC. EPJC (2017) 77: 828

F.J.Botella et al., On the search for the electric dipole moment of strange and charm baryons at LHC. EPJC (2017) 77: 181

Tau production

- Vast majority of tau produced in fixed-target collisions comes from $D_s^+ \to \tau^+ \nu_\tau$ BF=5.55%, Fragmentation fraction c→Ds ~9.25%

  M. Lisovyi et al., EPJC76, 397 (2016)

  L. Gladilin et al., EPJC75, 19 (2015)

- Convention factor for 7 TeV proton on T=1 cm thickness tungsten(W) target to produce $\tau^+ \to 3\pi\overline{\nu}_\tau$

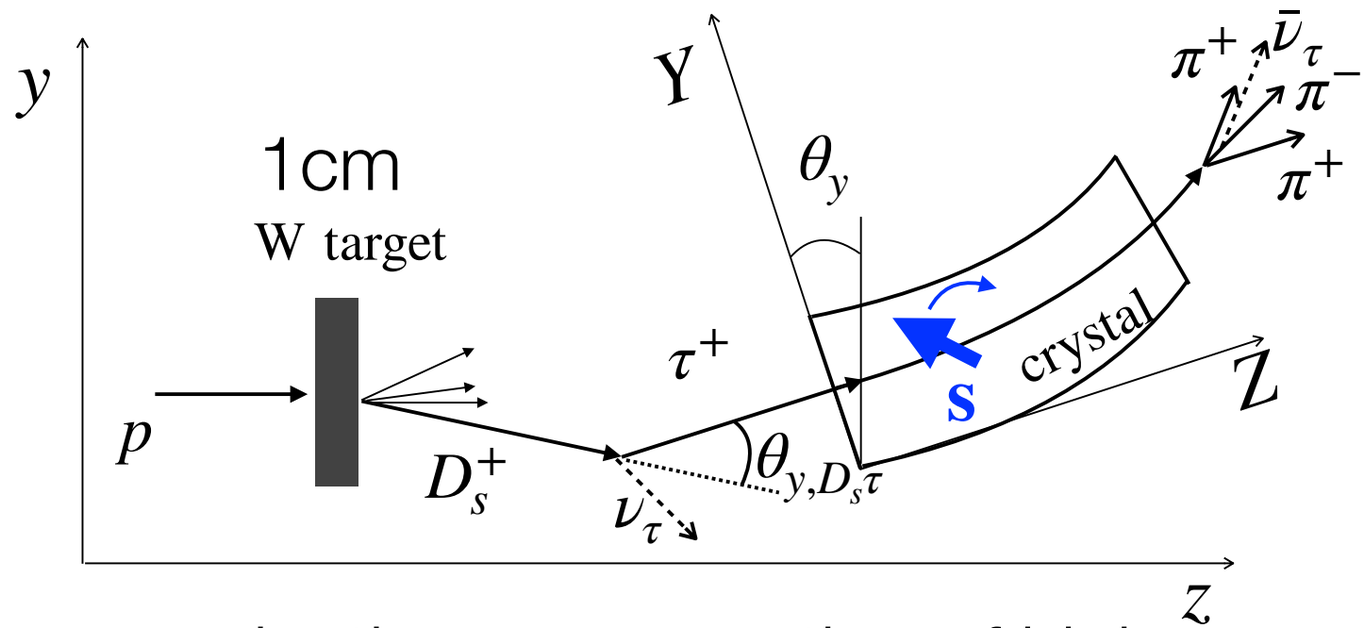
$$\sigma[pp \to D_s^+(\to \tau^+\nu_\tau)X]N_A \frac{\rho T A_N}{A_T} \mathcal{B}(\tau^+ \to 3\pi\overline{\nu}_\tau) \approx 2.1 \times 10^{-6},$$

✓ σ estimated using rescaled charm production cross section measured in proton-helium collision $\sqrt{s}$ =86.6 GeV at LHCb

R. Aaij et al., LHCb collaboration, arXiv1810.07907

Experimental layout

J. Fu et al., arXiv:1901.04003, submitted to PRL



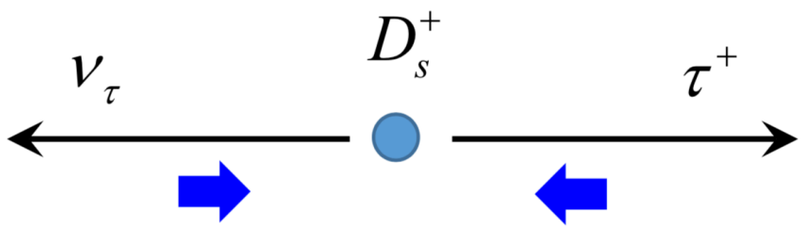
crystal frame: (*X*, *Y*, *Z*)

lab: (*x*, *y*, *z*)

- Large production cross section of high-energy polarised τ^+, originating in proton fixed-target collisions at the LHC
- $D_s^+ \to \tau^+\nu_\tau, \tau^+ \to \pi^+\pi^-\pi^+\bar{\nu}_\tau$ decay is considered
- A bent crystal employed to exploit channeling phenomenon of τ^+ aligned with crystal atomic planes
- YZ bending => Y polarisation perpendicular to crystal plane; Lorentz boost => Z polarisation along crystal Z axis
- MDM (EDM) signature given by spin rotation in YZ bending plane (appearance of x component)

Initial polarisation

- In D_s^+ rest frame, τ^+ produced with negative helicity



- For spin precession is the polarisation in τ^+ rest frame

$$\mathbf{s}_0 = \frac{1}{\omega}\left(m_\tau \mathbf{q} - q_0 \mathbf{p} + \frac{\mathbf{q}\cdot\mathbf{p}}{p_0 + m_\tau}\mathbf{p}\right)$$

unit vector of D_s^+ momentum in τ^+ rest frame

p(**q**) and $p_0(q_0)$: $\tau^+(D_s^+)$ momentum and energy in Lab

$$\omega = (m_{D_s}^2 - m_\tau^2)/2$$

Initial polarisation

- Coordinate frame: channeled moving τ^+ direction for Z, crystal edges for Y

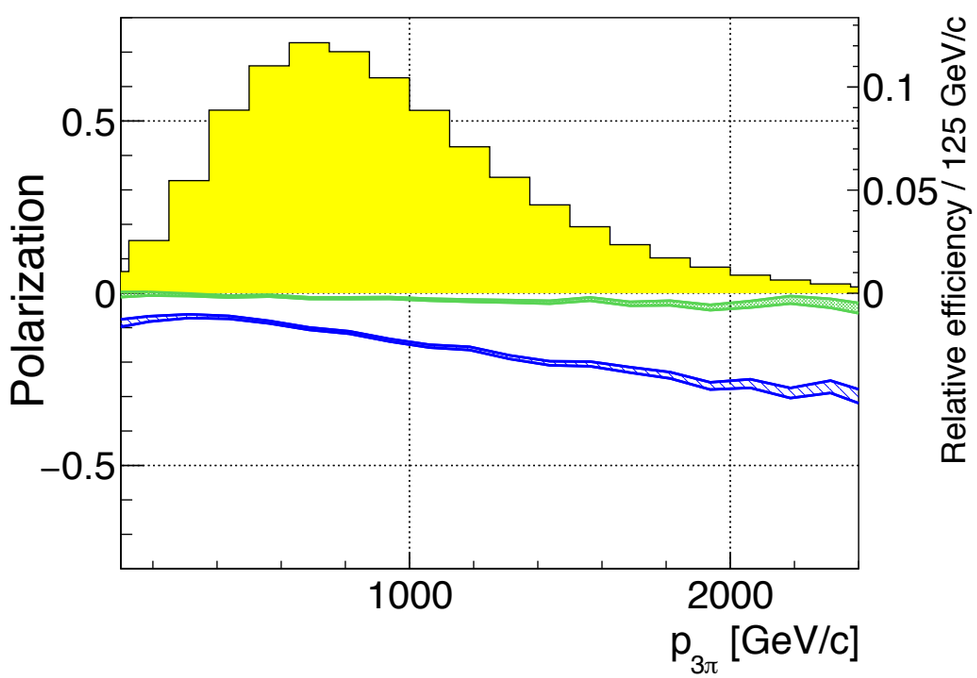
$$s_{0,X} \approx \frac{m_\tau |\mathbf{q}|}{\omega}\theta_{x,D_s\tau}$$

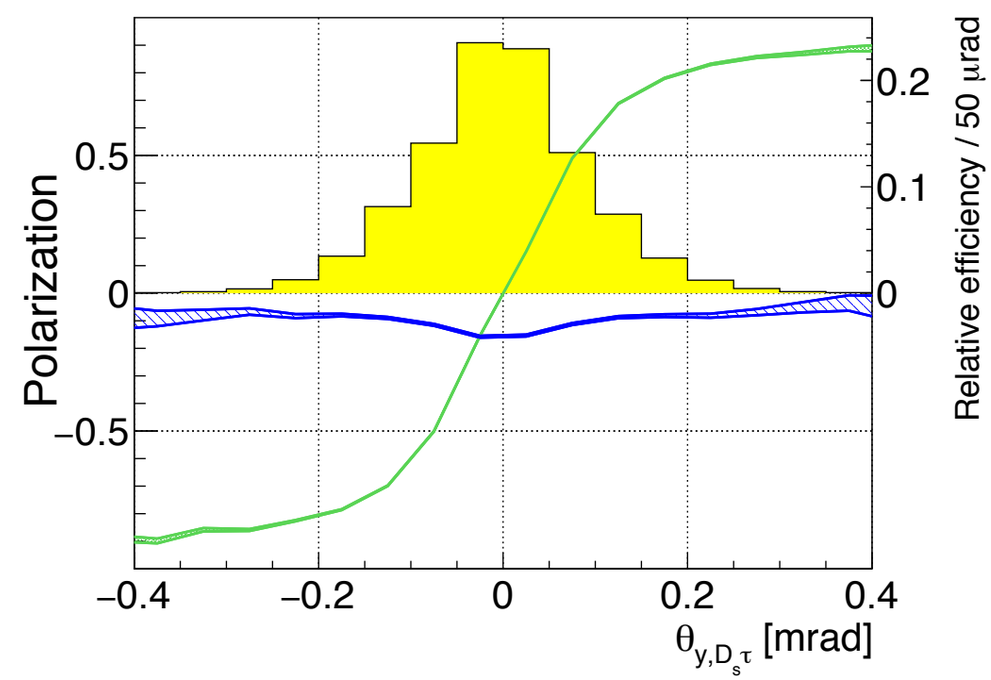
$$\boxed{s_{0,Y}} \approx \frac{m_\tau |\mathbf{q}|}{\omega}\theta_{y,D_s\tau}$$

$$\boxed{s_{0,Z}} \approx \frac{1}{\omega}\left(|\mathbf{q}|\, p_0 - q_0\, |\mathbf{p}|\right)$$

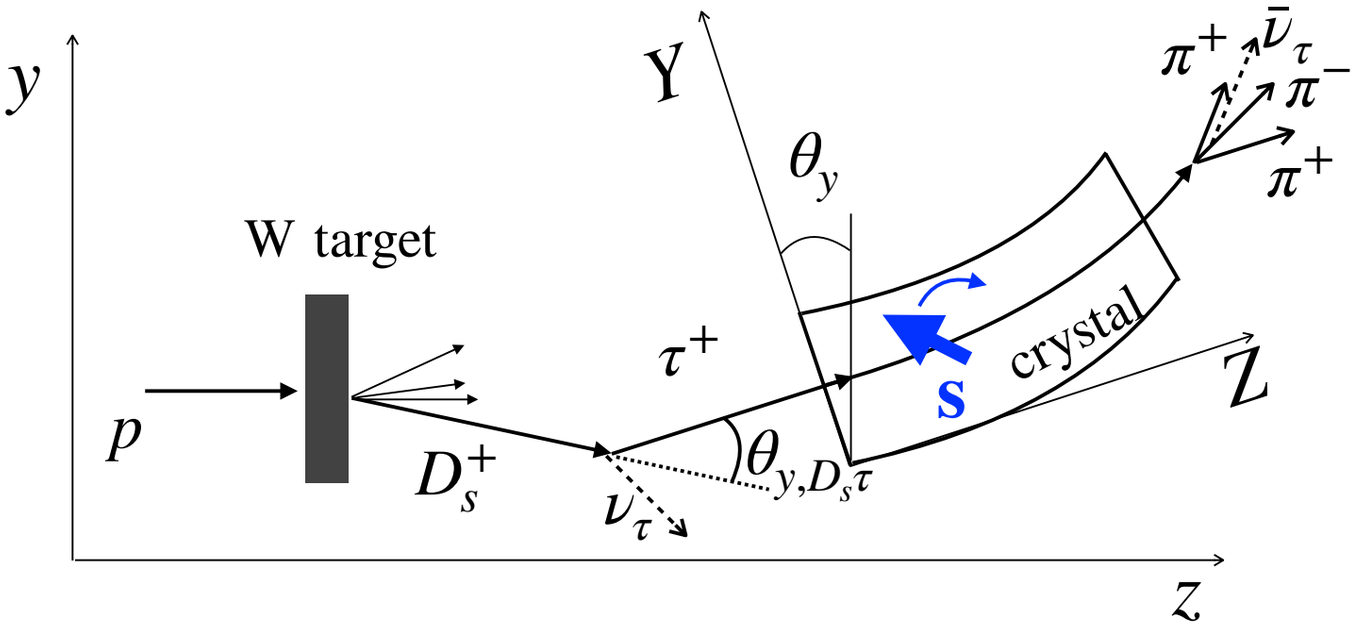
θ ~ mrad

$\langle s_{0,X}\rangle = 0$





Polarisation after precession in crystal



precession angle:

$$\Phi = \gamma \theta_C a'_d$$

$$a' = a + \frac{1}{1+\gamma}$$
$$d' = d/2$$
$$a'_d = \sqrt{a'^2 + d'^2}$$

θ_C: crystal bending angle

For expression at 10^{-2} precision:

$$\boxed{s_X} \approx -s_{0,Z}\frac{d'}{a'_d}\sin\Phi + s_{0,Y}\frac{d'a'}{a'^2_d}(1-\cos\Phi)$$

$$\boxed{s_Y} \approx s_{0,Z}\frac{a'}{a'_d}\sin\Phi + s_{0,Y}\left(\frac{d'^2}{a'^2_d} + \frac{a'^2}{a'^2_d}\cos\Phi\right)$$

$$\boxed{s_Z} \approx s_{0,Z}\cos\Phi - s_{0,Y}\frac{a'}{a'_d}\sin\Phi$$

Measure s_Y and s_Z (s_X)
to extract MDM (EDM)

Polarisation determination

M. Davier et al., PLB306, 411 (1993)
A. Rouge et al., Z. Phys. C48, (1990) 75
K. Hagiwara et al., PLB235, (1990) 178
Kuhn, Z. Phys. C56, (1992) 661, PLB286, (1992) 381
ALEPH Collab., Z.Phys.C59(1993)369, EPJC20(2001)40

- For any τ decay, generally angular distribution:

$W(\boldsymbol{\xi}) = f(\boldsymbol{\xi}) + pg(\boldsymbol{\xi})$ $\boldsymbol{\xi}$:set of kinematic variables

- (Longitudinal) polarisation found by maximising likelihood:

$$L(p,\{\boldsymbol{\xi}\}_i) = \prod_{i=1}^{n} W(\boldsymbol{\xi}_i)$$

$$\frac{\partial}{\partial p}\log L(p,\{\boldsymbol{\xi}\}_i) = \frac{\partial}{\partial p}\sum_{i=1}^{n}\log\left[f(\boldsymbol{\xi}_i) + pg(\boldsymbol{\xi}_i)\right] = \sum_{i=1}^{n}\frac{g(\boldsymbol{\xi}_i)}{f(\boldsymbol{\xi}_i) + pg(\boldsymbol{\xi}_i)} = \sum_{i=1}^{n}\frac{w_i}{1+pw_i} = 0 \qquad w = \frac{g(\boldsymbol{\xi})}{f(\boldsymbol{\xi})}$$

- So ML fit to $W(\boldsymbol{\xi})$ equivalent to ML fit to w

$$W(\boldsymbol{\xi}) = f(\boldsymbol{\xi}) + pg(\boldsymbol{\xi}) = \frac{1}{2}\left[(1+p)W^{+}(\boldsymbol{\xi}) + (1-p)W^{-}(\boldsymbol{\xi})\right] = \frac{1}{2}\left[\left(W^{+}(\boldsymbol{\xi}) + W^{-}(\boldsymbol{\xi})\right) + p\left(W^{+}(\boldsymbol{\xi}) - W^{-}(\boldsymbol{\xi})\right)\right]$$

$$w = \frac{g(\boldsymbol{\xi})}{f(\boldsymbol{\xi})} = \frac{W^{+}(\boldsymbol{\xi}) - W^{-}(\boldsymbol{\xi})}{W^{+}(\boldsymbol{\xi}) + W^{-}(\boldsymbol{\xi})}$$

the fractional difference between angular distributions of +1 and -1 polarisation

- Multi-d to1-d angular distribution:

$$\widehat{W}(w) = \frac{1}{2}\left[(1+p)\widehat{W}^{+}(w) + (1-p)\widehat{W}^{-}(w)\right]$$

evaluate $\widehat{W}^{\pm}(w)$ from MC, fit $\widehat{W}(w)$ from data

Polarisation determination: MVA

- A technique based on MVA explored to extract polarisation without prior knowledge of detailed decay dynamics and of τ energy
- Using fully polarised τ simulation sample for training
- Variables ζ, sensitivity to τ^+ spin polarisation
  - ✓ angles between p(3π) in τ^+ rest frame and crystal frame axes
  - ✓ angles describing 3π decay plane in 3π rest frame w.r.t crystal frame axes
  - ✓ m(2π) and m(3π)
- A approximate τ^+ rest frame to calculate ζ
  - ✓ p(τ^+) estimated by applying correction, determined from simulation, to measured p(3π) as a function of magnitude and direction
  - ✓ Assuming τ^+ flight direction to be connecting D_s^+ production vertex and τ^+ decay vertex lying in crystal plane

Polarisation determination: MVA (cont'd)

- Polarisation component s_i along i-th (i=X,Y,Z) crystal frame axis is extracted by fitting classifier distribution on data (simulation)

$$\mathcal{W}_i(\eta) = \frac{1}{2}\left[(1+s_i)\mathcal{W}_i^+(\eta) + (1-s_i)\mathcal{W}_i^-(\eta)\right]$$

$\eta \equiv \eta(\boldsymbol{\zeta})$ classifier response

$\mathcal{W}_i^{\pm}(\eta)$ templates representing response for ±1 polarisations

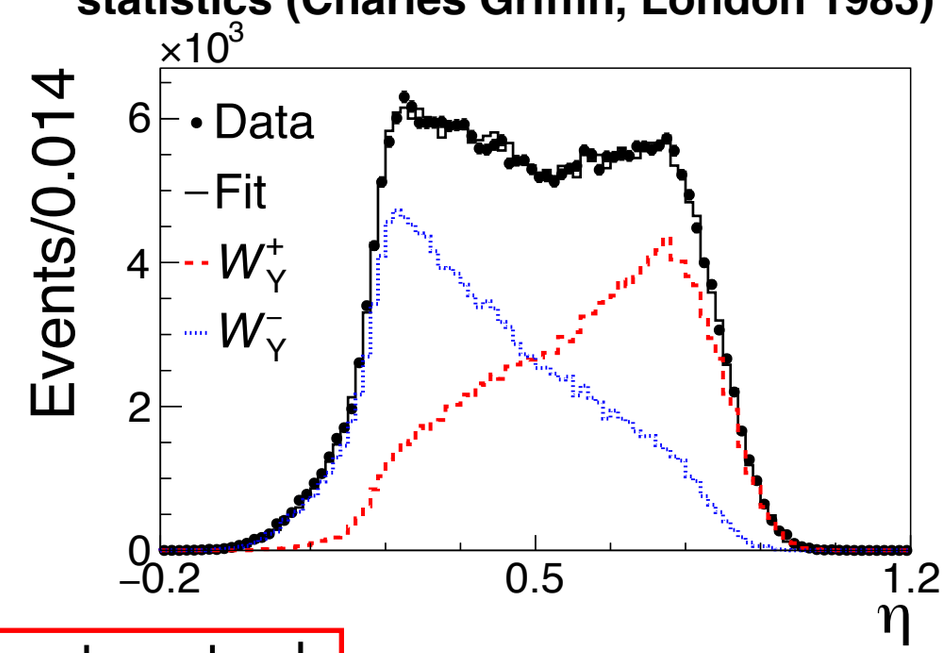
- Squared average event information of the polarisation

$$\boxed{S_i^2} = \frac{1}{N_{\tau^+}^{\rm rec}\sigma_i^2} = \left\langle \left(\frac{\mathcal{W}_i^+(\eta) - \mathcal{W}_i^-(\eta)}{\mathcal{W}_i^+(\eta) + \mathcal{W}_i^-(\eta)} \right)^2 \right\rangle$$

σ_i uncertainty on s_i

$N_{\tau^+}^{\rm rec}$ channeled and reconstructed τ^+

M. Kendall et al, The advanced theory of statistics (Charles Griffin, London 1983)



Results (S_Z) affected by undetected $\overline{\nu}_\tau$

$S_X \approx S_Y \approx 0.42$ $\qquad$ $S_Z \approx 0.23$

with complete kinematics of τ^+ decay reconstructed

$S_X \approx S_Y \approx S_Z \approx 0.58$

M. Davier et al, Phys. Lett. B306, 411 (1993)

Sensitivity

- Statistical uncertainties on a and d estimated from spin-polarisation projection after crystal

- $\gamma\theta_C \sim 10$ and $a'_d \sim 10^{-3}$ => small $\mathbf{\Phi}$

- For $s_{0,Z}$ initial polarisation:

$$\sigma_a \approx \frac{1}{S_Y s_{0,Z}\gamma\theta_C}\frac{1}{\sqrt{N^{\rm rec}_{\tau^+}}}, \quad \sigma_d \approx \frac{2}{S_X s_{0,Z}\gamma\theta_C}\frac{1}{\sqrt{N^{\rm rec}_{\tau^+}}}$$

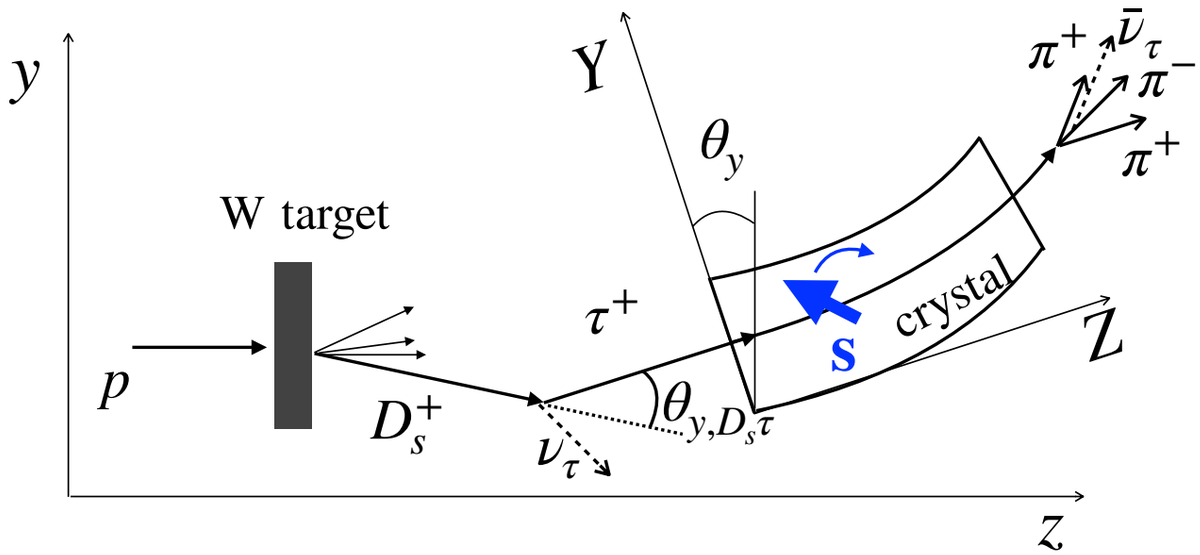
- For $s_{0,Y}$ initial polarisation:

$$\sigma_a \approx \frac{1}{S_Z s_{0,Y}\gamma\theta_C}\frac{1}{\sqrt{N^{\rm rec}_{\tau^+}}}, \quad \sigma_d \approx \frac{2}{S_X s_{0,Y}(\gamma\theta_C)^2 a'}\frac{1}{\sqrt{N^{\rm rec}_{\tau^+}}}$$

disfavoured by $1/(\gamma\theta_C a') \sim 100$

Experimental layout optimisation

- Find region of minimal uncertainty on a and d by scan in ($\theta_C, L, \theta_y, L_{tar}$) space in case of $s_{0,Z}$ initial polarisation using simulation



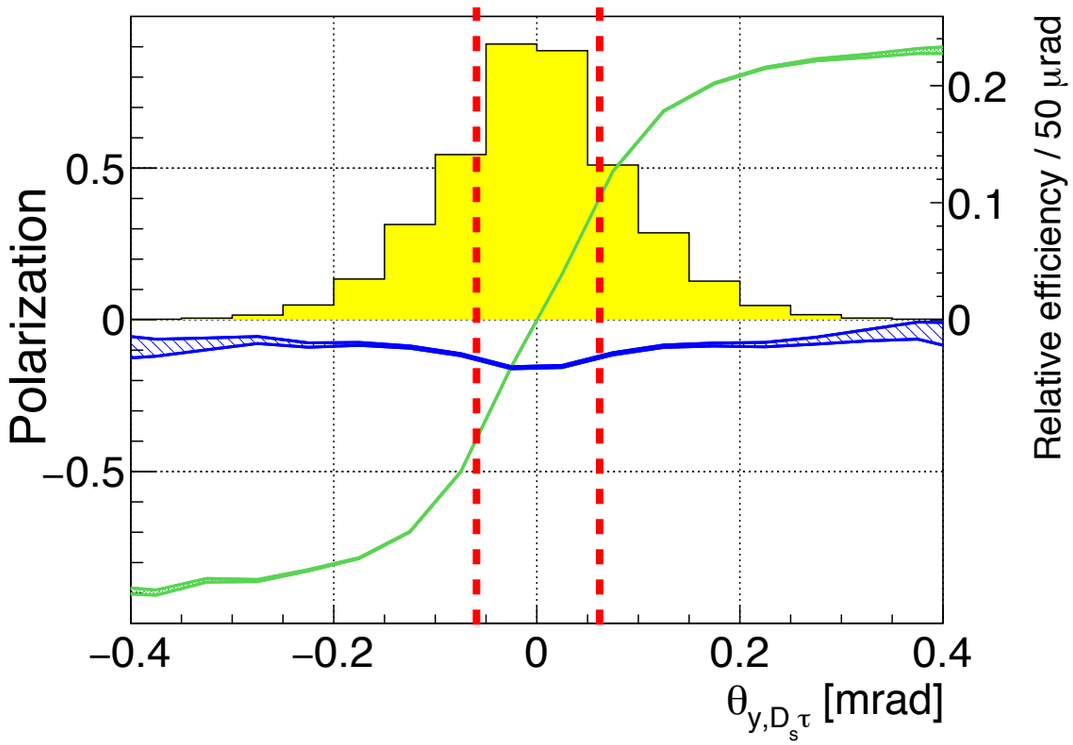
- ✓ θ_C (L) crystal bending angle (length)
- ✓ L_{tar} distance between target and crystal
- ✓ crystal tilted by θ_y=0.1mrad

- Channeled τ^+ with p(3π)>800 GeV to enhance $s_{0,Z}$
- Maximum $\mathbf{\Phi}$: channeled τ^+ originated before crystal, decay after crystal
- Optimal layout:

	θ_C	L	L_{tar}	channeling eff.
Ge(Si)	16mrad	8(11)cm	12cm	6.3×10^{-6}(lose factor of 3)

Crystal prototype with similar θ_C, L tested on beam at CERN SPS last Oct.

Alternative measurement with θ_y-tagging

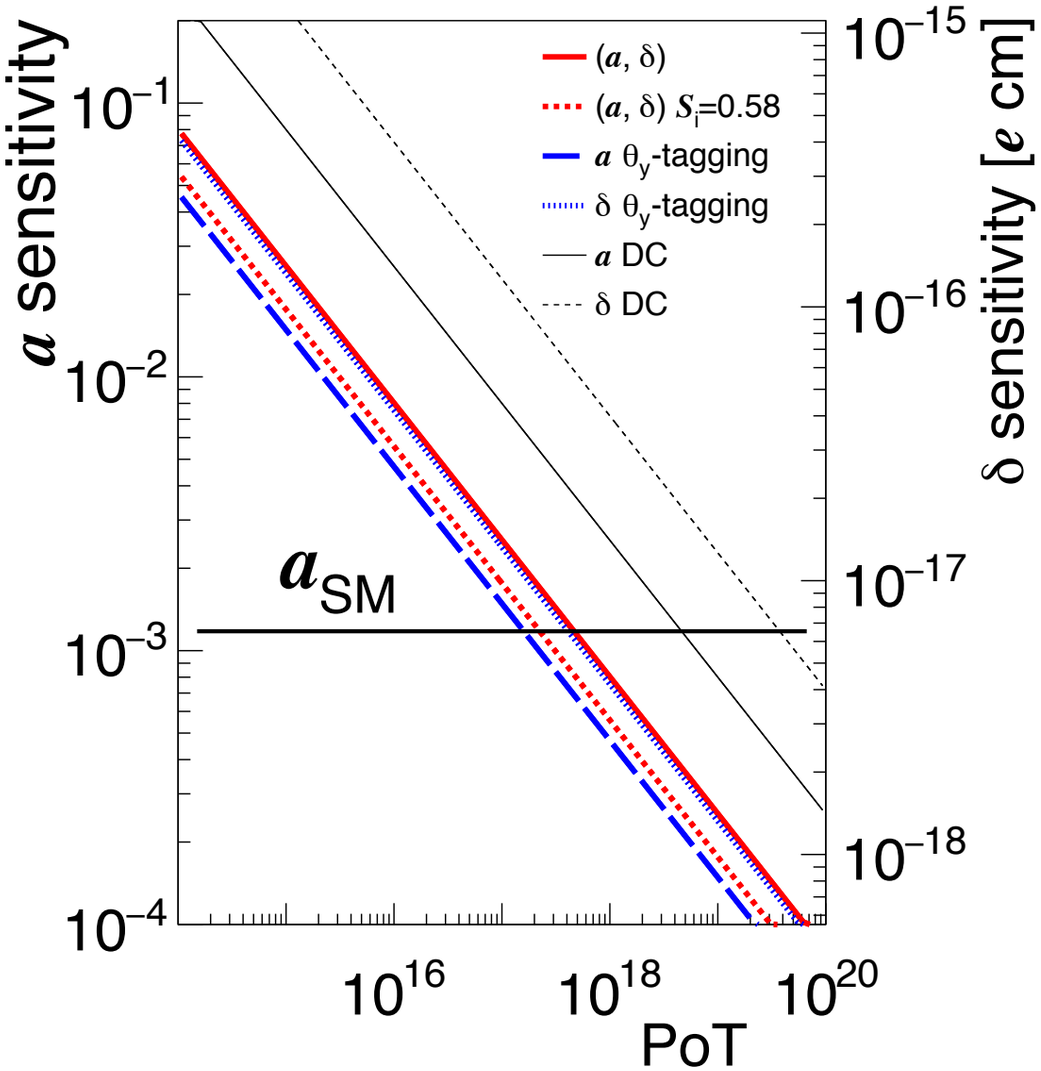


- θ_y-tagging: select a region of positive or negative $\theta_{y,D_s\tau}$ angles
- Large $s_{0,Y}$ polarisation achieved

- Information statistically correlated with $\theta_{y,D_s\tau}$ is required for θ_y-tagging
  - ✓ exploit global event topology, i.e. kinematic distributions of particles associated with Ds+ produced interaction point
  - ✓ pixel radiation-hard diamond sensors to reconstruct D_s^+ trajectory
  - ✓ put second bend crystal to channel Ds, inducing large $s_{0,Y}$, but eff. lost much **A. Fomin et al., arXiv:1810.06699**

Dipole moment sensitivities

- Sensitivities assessed by toys:
  - ✓ using $\mathcal{W}_{i(\eta)}$ distributions and final spin-polarisation to generate samples with number of events per sample corresponding to 10^{16}PoT
  - ✓ fit each sample, and estimate sensitivity from rms of fitted values
  - ✓ scale result for 10^{16}PoT according to 1/sqrt(N(PoT))



- Ge, optimal layout

  verify SM prediction of MDM with 10^{18}PoT

  search for EDM below 10^{-17} *e* cm

- Assuming 40% detector reconstruction eff.
- Si, a factor of 2 worse

Signal, backgrounds and systematics

- p(τ^+) unchanged while τ^+ deflected at the bending angle, this can be identified by reconstruction of p(3π) and vertex of 3π
- Highly-boosted τ^+ direction with an 0.5 mrad uncertainty due to missing $\bar{\nu}_\tau$
- Negligible contribution from non-channeled τ^+ due to kinematic selection of 3π and $L+L_{tar}$>20cm from interaction point
- τ^+ channeled with a fraction of crystal length:

  D_s^+ decay inside crystal or τ^+ not reach the end of crystal introduce a small bias on $\mathbf{\Phi}$, ~1.4%
- Backgrounds from channeled hadrons, D^+, D_s^+, Λ_c^+ with 3π final state, vetoed by reconstructed mass and dedicated detectors
- Systematics effects: limit knowledge of crystal position and orientation, initial polarisation, τ^+ momentum
- Possible effects due to τ^+ week interactions with crystal are negligible

V.G.Baryshevsky arXiv1803.05770

Summary

- A novel method for direct measurement of τ MDM and EDM presented with interesting perspective for stringent test of SM and search for NP
- Fixed-target setup and analysis technique discussed for possible future scenarios
- Could verify SM prediction for τ MDM with a sample of 10^{18}PoT and search for τ EDM below 10^{-17} e cm
- The possibility of a test or an experiment at CERN SPS will be explored in future

Thank you!

Backup

General description

A. Pich, Prog.Part.Nucl.Phys. 75 (2014) 41-85

- General description of EM coupling of on-shell spin-1/2 charged lepton to a virtual γ

$$M_{l\bar{l}\gamma^*} = eQ_l\varepsilon_\mu(q)\bar{u}_l(\vec{p}\,')\left[F_1(q^2)\gamma^\mu + i\frac{F_2(q^2)}{2m_l}\sigma^{\mu\nu}q_\nu + \frac{F_3(q^2)}{2m_l}\sigma^{\mu\nu}\gamma_5 q_\nu\right]u_l(\vec{p})$$

$q^\mu = (p'-p)^\mu$ incoming photon momentum $\qquad Q_l = -1$

- ✓ Follows from Lorentz invariance and EM current conservation (gauge invariance)
- ✓ From electric charge conservation, $F_1(0) = 1$
- ✓ At $q^2 = 0$, the other two factors are the lepton magnetic and electric DMs:

$$\mu_l \equiv \frac{e}{2m_l}\frac{g_l}{2} = \frac{e}{2m_l}\left[1 + F_2(0)\right] \qquad \delta_l \equiv \frac{e}{2m_l}F_3(0)$$

- ✓ Similar expressions can be defined for Z coupling (weak dipole moments)

Equation of motion

$$\frac{d\mathbf{s}}{dt} = \mathbf{s} \times \boldsymbol{\Omega}\,, \quad \boldsymbol{\Omega} = \boldsymbol{\Omega}_{\mathrm{MDM}} + \boldsymbol{\Omega}_{\mathrm{EDM}} + \boldsymbol{\Omega}_{\mathrm{TH}},$$

$$\boldsymbol{\Omega}_{\mathrm{MDM}} = \frac{g\mu_B}{\hbar}\left(\mathbf{B} - \frac{\gamma}{\gamma+1}(\boldsymbol{\beta}\cdot\mathbf{B})\boldsymbol{\beta} - \boldsymbol{\beta}\times\mathbf{E}\right),$$

$$\boldsymbol{\Omega}_{\mathrm{EDM}} = \frac{d\mu_B}{\hbar}\left(\mathbf{E} - \frac{\gamma}{\gamma+1}(\boldsymbol{\beta}\cdot\mathbf{E})\boldsymbol{\beta} + \boldsymbol{\beta}\times\mathbf{B}\right),$$

$$\begin{aligned}\boldsymbol{\Omega}_{\mathrm{TH}} &= \frac{\gamma^2}{\gamma+1}\boldsymbol{\beta}\times\frac{d\boldsymbol{\beta}}{dt} \\ &= \frac{q}{mc}\left[\left(\frac{1}{\gamma}-1\right)\mathbf{B} + \frac{\gamma}{\gamma+1}(\boldsymbol{\beta}\cdot\mathbf{B})\boldsymbol{\beta} \right. \\ &\quad \left. -\left(\frac{1}{\gamma+1}-1\right)\boldsymbol{\beta}\times\mathbf{E}\right], \end{aligned} \tag{18}$$

For $\mathbf{B} = 0$ and $q = +1$, Eq. (18) simplifies to

$$\begin{aligned}\boldsymbol{\Omega} &= \frac{2\mu'}{\hbar}(\mathbf{E}\times\boldsymbol{\beta}) + \frac{d\mu_B}{\hbar}\mathbf{E} + \frac{1}{\gamma+1}\frac{2\mu_B}{\hbar}(\mathbf{E}\times\boldsymbol{\beta}) \\ &\quad -\frac{d\mu_B}{\hbar}\frac{\gamma}{\gamma+1}(\boldsymbol{\beta}\cdot\mathbf{E})\,\boldsymbol{\beta},\end{aligned}$$

where

$$\mu' = \frac{g-2}{2}\frac{e\hbar}{2mc},$$

$$\Omega_x \approx \frac{2\mu'}{\hbar}(E_y\beta_z - E_z\beta_y) = \frac{2\mu'}{\hbar}\left(-\frac{dV}{d\rho}\frac{\rho\Omega}{c}\right),$$

$$\begin{aligned}\Omega_y &\approx \frac{d\mu_B}{\hbar}[E_y - (\boldsymbol{\beta}\cdot\mathbf{E})\beta_y] \\ &= -\frac{d\mu_B}{\hbar}\frac{dV}{d\rho}\cos(\Omega t) \\ &\quad +\frac{d\mu_B}{\hbar}\frac{dV}{d\rho}\frac{\dot\rho}{c^2}[-\rho\Omega\sin(\Omega t) + \dot\rho\cos(\Omega t)]\,,\end{aligned}$$

$$\begin{aligned}\Omega_z &\approx \frac{d\mu_B}{\hbar}\left[E_z - (\boldsymbol{\beta}\cdot\mathbf{E})\,\beta_z\right] \\ &= -\frac{d\mu_B}{\hbar}\frac{dV}{d\rho}\sin(\Omega t) \\ &\quad +\frac{d\mu_B}{\hbar}\frac{dV}{d\rho}\frac{\dot\rho}{c^2}[\rho\Omega\cos(\Omega t) + \dot\rho\sin(\Omega t)]\,.\end{aligned} \tag{32}$$