Thermodynamics and susceptibilities of isospin imbalanced QCD matter

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arXiv:1907.11497

第十七届全国核物理大会 (华中师范大学, 武汉)

2019.10.08-12

Outline

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3. Susceptibilities
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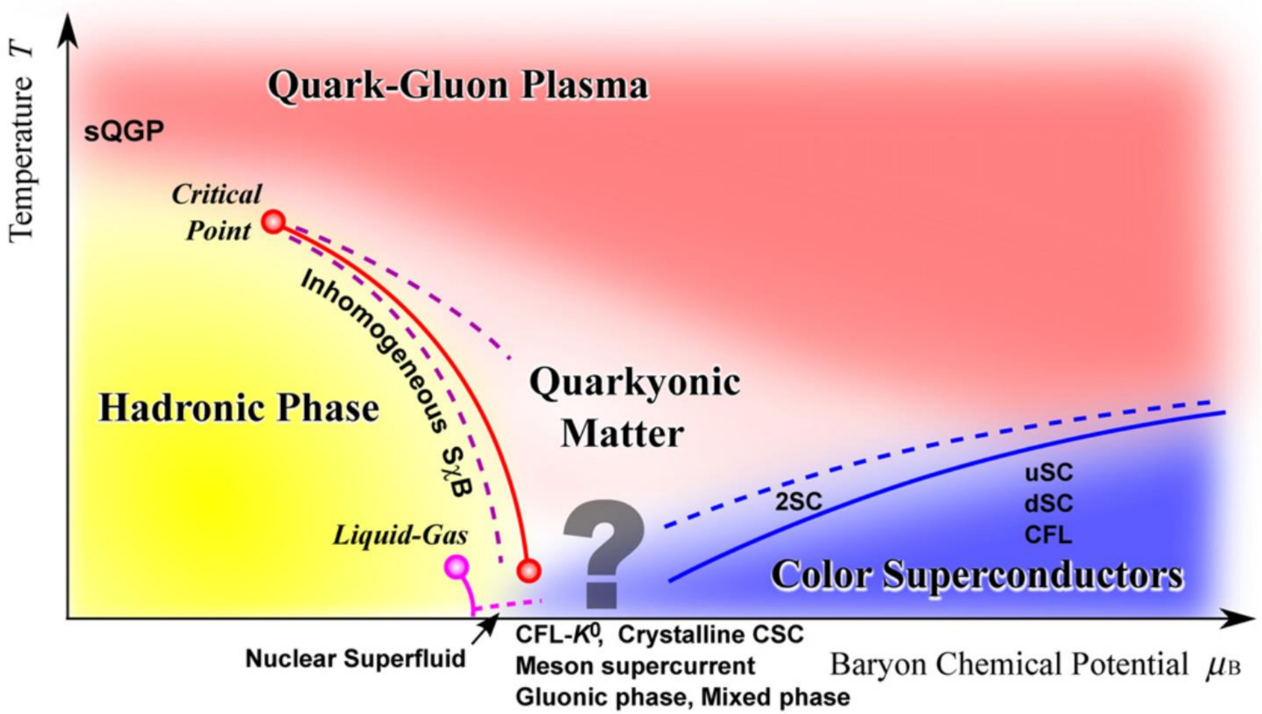
QCD phase diagram at finite μ_B and T



Conjectured QCD phase diagram

- Sign problem
- ……

Fukushima, K. & Hatsuda, T. Rept. Prog. Phys., 2011, 74, 014001

Why QCD at finite μ_I?

$$\begin{cases} \mu_{\mathrm{u}} = \dfrac{\mu_B}{3} + \dfrac{\mu_I}{2} \\ \mu_{\mathrm{d}} = \dfrac{\mu_B}{3} - \dfrac{\mu_I}{2} \end{cases} \Rightarrow \begin{cases} \mu_I = \mu_u - \mu_d \\ \mu_B = \dfrac{3}{2}(\mu_u + \mu_d) \end{cases}$$

- **Why we consider finite μ_I?**

- **Isospin asymmetry environments**
  → **compact stars, heavy ion collisions, etc**
- **No sign problem → first principle calculations**
  → **model constraining, tests for effective models, etc**
- **……**

QCD at finite isospin chemical potential

- A Bose-Einstein condensate of charged pions happens at $\mu_I^c(T=0) = m_\pi$
- The normal to pion superfluid phase transition is <u>**second order**</u>

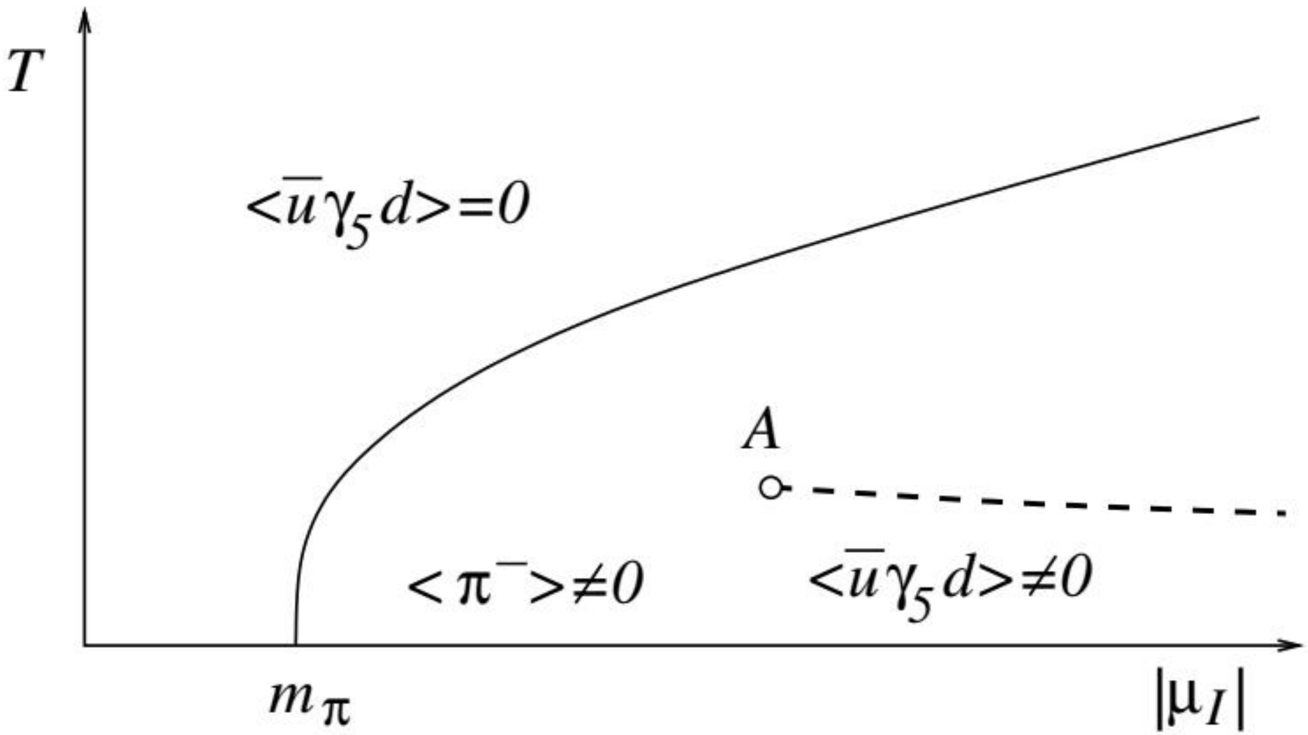
CHPT:
Son, D. T. & Stephanov, M. A. Phys. Rev. Lett., 2001, 86, 592-595; (LO)
Splittorff, K.; Son, D. T. & Stephanov, M. A. Phys. Rev. D, 2001, 64, 016003; (LO)
Adhikari, P.; Andersen, J. O. & Kneschke, P. arXiv:1904.03887. (NLO)

NJL:
He, L.-y.; Jin, M. & Zhuang, P.-f. Phys. Rev. D, 2005, 71, 116001;
He, L. & Zhuang, P. Phys. Lett. B, 2005, 615, 93-101;
Warringa, H. J.; Boer, D. & Andersen, J. O. Phys. Rev. D, 2005, 72, 014015.

Lattice simulations:
Kogut, J. B. & Sinclair, D. K. Phys. Rev. D, 2002, 66, 034505;
Detmold, W.; Orginos, K. & Shi, Z. Phys. Rev. D, 2012, 86, 054507.

Phase diagram in $\mu_I - T$ plane

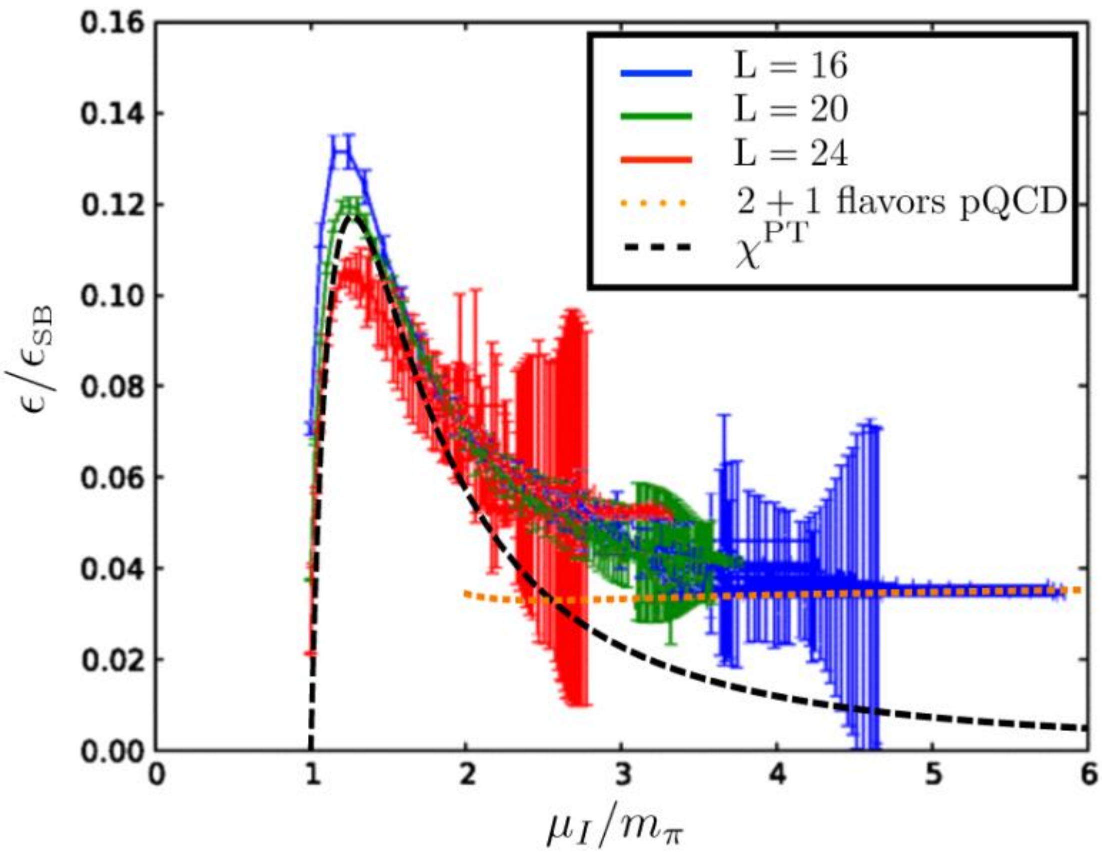
$\mu_B = 0$



- <u>No sign problem</u>
- Lattice QCD works well

Son, D. T. & Stephanov, M. A. Phys. Rev. Lett., 2001, 86, 592-595

Energy density



Carignano, S.; Mammarella, A. & Mannarelli, M.
Phys. Rev. D, 2016, 93, 051503

- Chiral Perturbation theory → Low densities ✓
- Perturbative QCD → High densities ✓
- Lattice simulations ✓

NJL model at finite isospin chemical potential

The SU(2) NJL model Lagrangian density

$$\mathcal{L} = \bar{q}(i\gamma^\mu\partial_\mu - m + \hat{\mu}\gamma_0)q + G[(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau q)^2]$$

Mean field approximation

The mean field Lagrangian density

$$\mathcal{L} = \bar{q}\big[i\gamma^\mu\partial_\mu - M_q + \hat{\mu}\gamma^0 + 2G\Pi i\gamma^5\tau_1\big]q + G(\sigma^2 + \Pi^2)$$

Thermodynamic potential (in mean field approximation)

$$\Omega = G(\sigma^2 + \Pi^2) - \frac{2N_c}{\beta}\int\frac{d^3p}{(2\pi)^3} \times\Big\{\ln\Big[(1+e^{-\beta E_p^+})(1+e^{\beta E_p^+})\Big] + \ln\Big[(1+e^{-\beta E_p^-})(1+e^{\beta E_p^-})\Big]\Big\}$$

$$E_p = \sqrt{p^2 + M^2} \qquad E_p^{\pm} = \sqrt{(E_p \pm \mu_I/2)^2 + 4G^2\Pi^2}$$

Gap equations $\frac{\partial\Omega}{\partial\sigma} = \frac{\partial\Omega}{\partial\Pi} = 0$

Equation of state

$$\begin{cases} P = -(\Omega - \Omega_0) \\ E = -P - T\frac{\partial\Omega}{\partial T} - \mu_I\frac{\partial\Omega}{\partial\mu_I} \end{cases}$$

CHPT at finite isospin chemical potential

Chiral Lagrangian at the lowest order

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}(D_\nu \Sigma D^\nu \Sigma^\dagger) + \frac{f_\pi^2}{4}\mathrm{Tr}(X\Sigma^\dagger + \Sigma X^\dagger)$$

covariant derivative $D_\mu \Sigma = \partial_\mu \Sigma - \frac{i}{2}[v_\mu, \Sigma], \qquad v_\mu = -2\mu\delta_{\mu 0}$

SU(2) CHPT

SU(3) CHPT

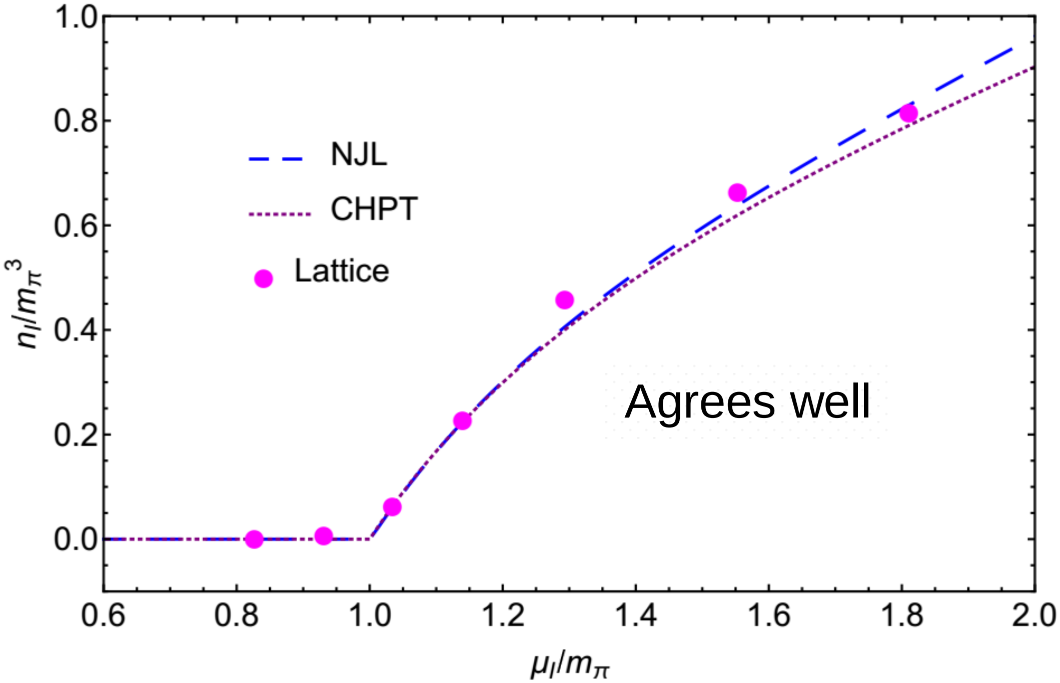
In the pion superfluid phase

$$E^{\mathrm{LO}} = \frac{f_\pi^2}{2\mu_I^2}(\mu_I^4 + 2\mu_I^2 m_\pi^2 - 3m_\pi^4)$$

$$P^{\mathrm{LO}} = \frac{f_\pi^2}{2\mu_I^2}(\mu_I^2 - m_\pi^2)^2$$

$$n_I = \frac{\partial P^{\mathrm{LO}}}{\partial \mu_I} = \frac{f_\pi^2}{\mu_I^3}(\mu_I^4 - m_\pi^4)$$

Isospin number density

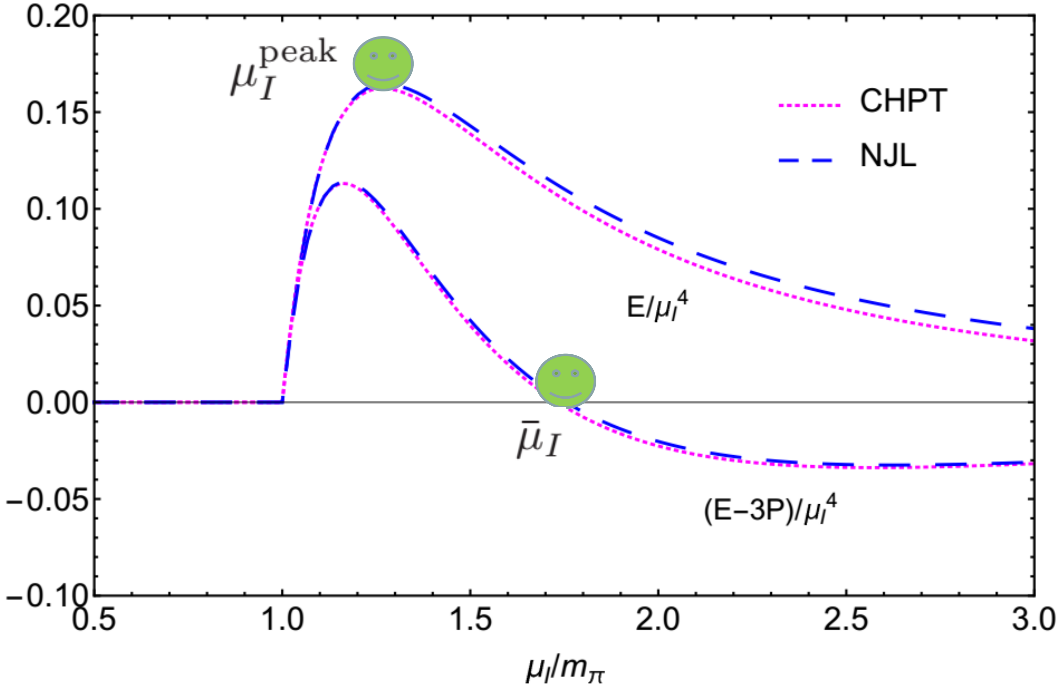


Isospin number density:

$$n_I = -\frac{\partial \Omega}{\partial \mu_I}$$

● The normalized isospin number density as a function of μ_I/m_π at fixed T = 0. The blue dashed line corresponds to the result obtained from the NJL model.

Normalized energy density and trace anomaly

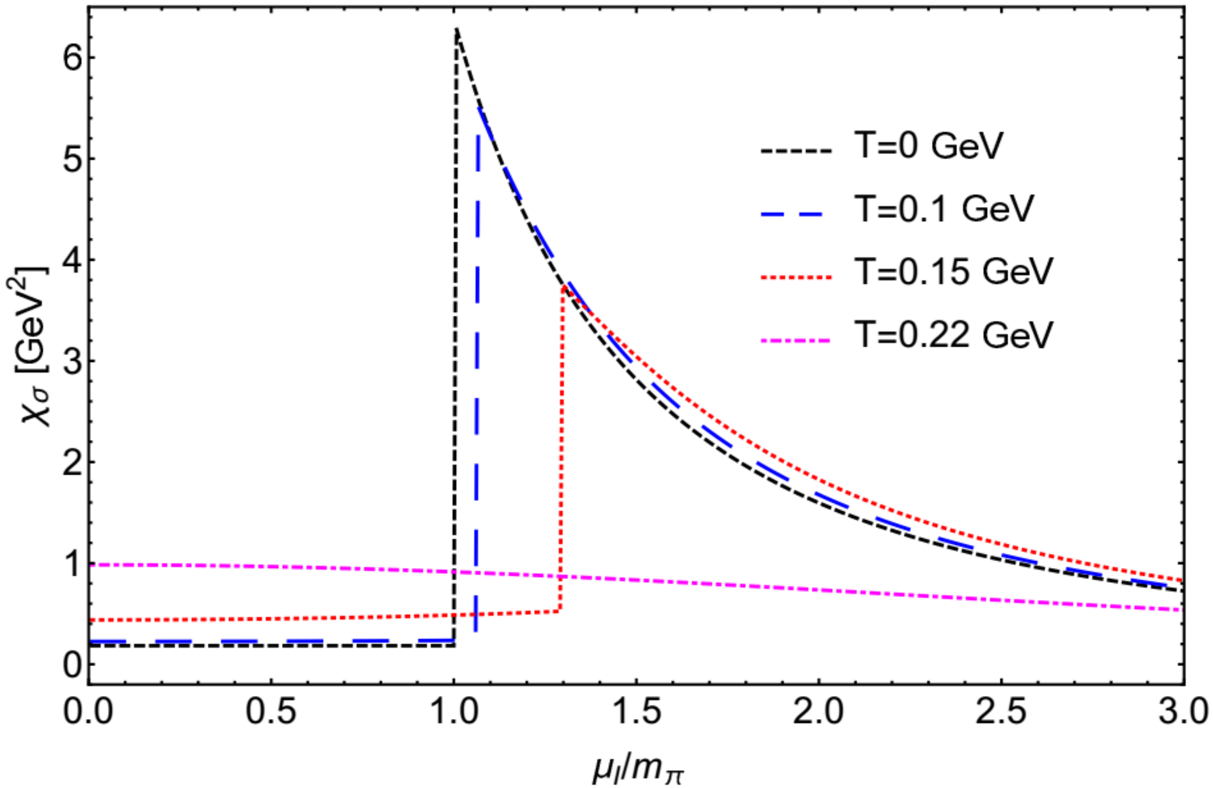


$$\left\{\begin{array}{ll} \bar{\mu}_I \simeq 1.754 m_\pi & \text{NJL} \\ \bar{\mu}_I = \sqrt{3} m_\pi & \text{CHPT} \end{array}\right.$$

(the NJL results)
<u>In good agreement</u> with the results obtained in <u>CHPT</u> and <u>lattice simulation</u>

$$\mu_I^{\rm peak} \simeq \begin{cases} 1.274 m_\pi, & \text{NJL} \\ 1.276 m_\pi, & \text{CHPT} \\ \{1.20, 1.25, 1.275\} m_\pi, & \text{Lattice data} \end{cases}$$

μ_I − Susceptibility plane



Chiral susceptibility

$$\chi_\sigma = -\frac{\partial \sigma}{\partial m}$$

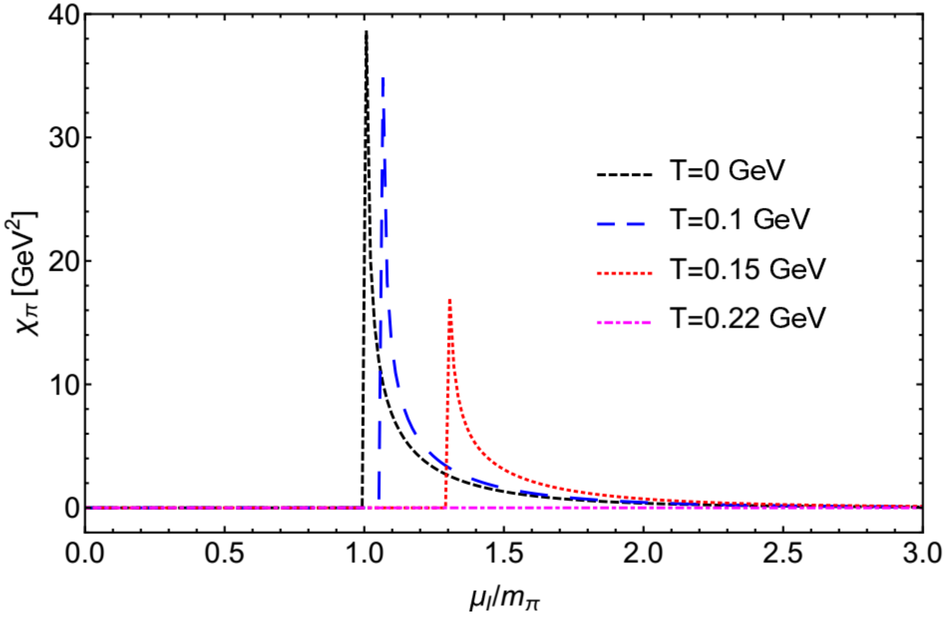
● The variation behaviors of chiral susceptibility with respect to μ_I/m_π for several values of the temperature.

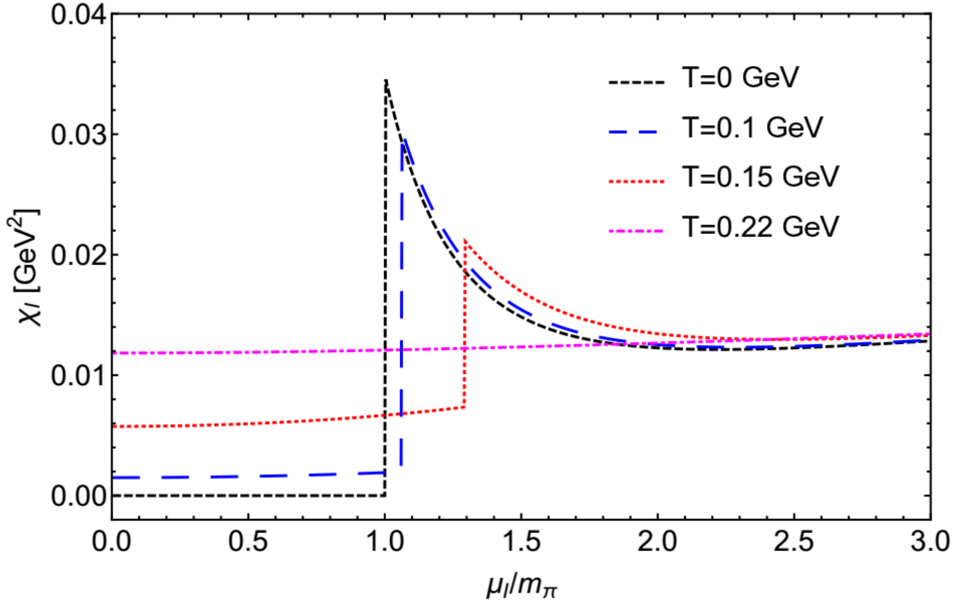
Next

Pion susceptibility $\chi_\pi = \frac{\partial \Pi}{\partial m}$

Isospin susceptibility $\chi_I = \frac{d n_I}{d \mu_I}$

μ_I − Susceptibility plane

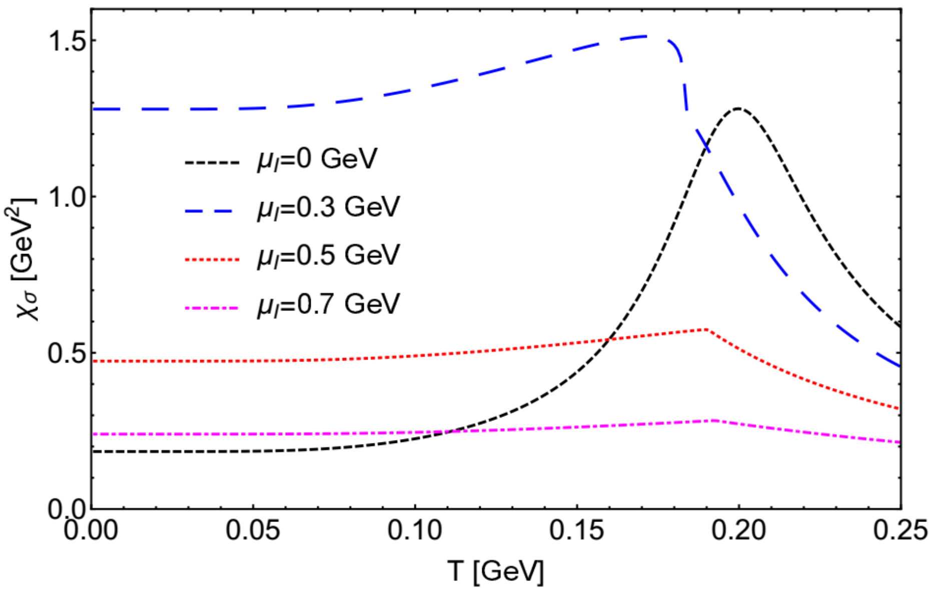


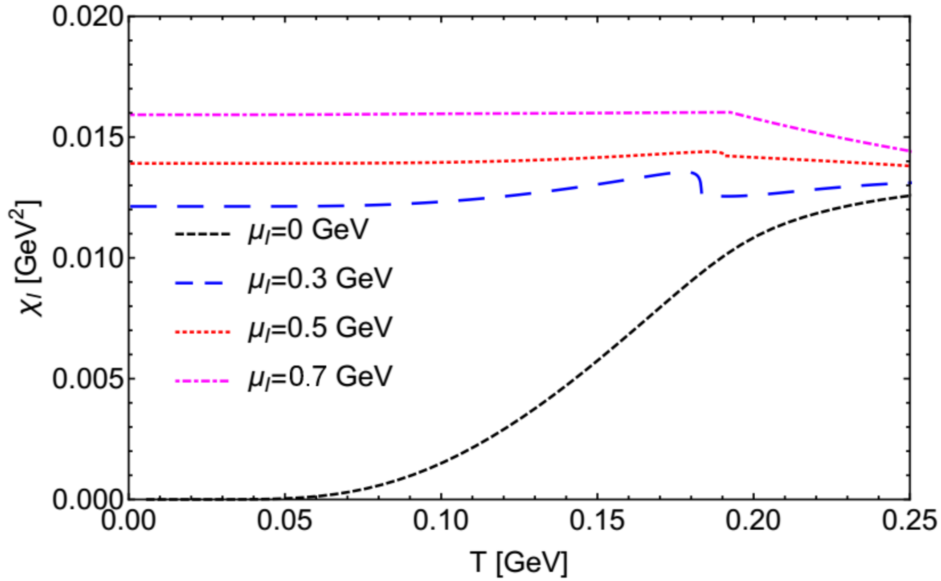


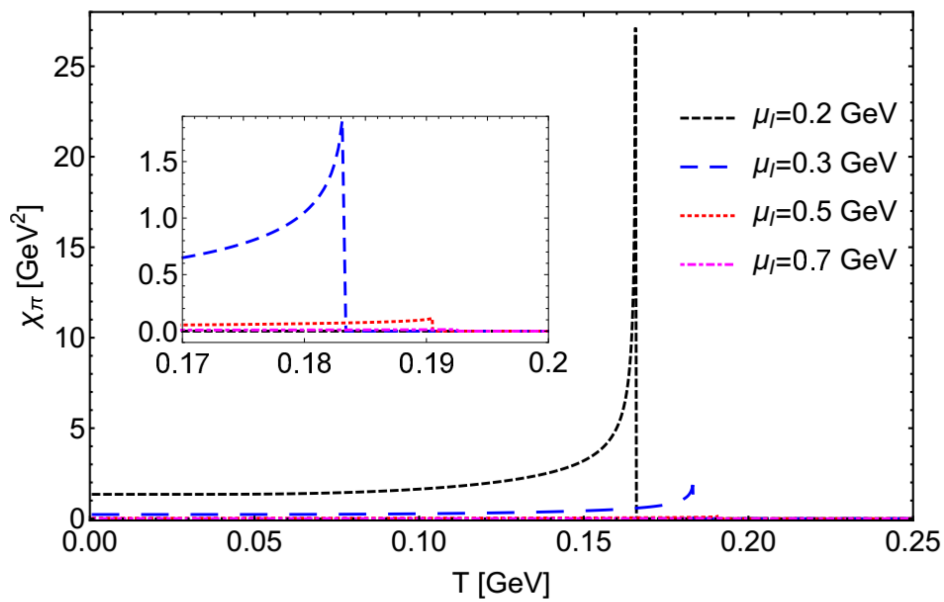
● The variation behaviors of pion susceptibility and isospin number susceptibility with respect to μ_I/m_π for several values of the temperature.

Discontinuities at the critical point

T – Susceptibility plane







- The variation behaviors of the susceptibilities against temperature for several values of μ_I.

Conclusions

— we have studied the isospin chemical potential and temperature dependence of several thermodynamic quantities (in particular of several susceptibilities, of isospin-imbalanced QCD matter)

● Isospin number density, normalized energy density and trace anomaly are shown to be in good agreement with the *available lattice data* as well as with the results from *chiral perturbation theory* at zero temperature.

● We *find a peak* for the chiral, pion, and isospin susceptibilities at the critical isospin chemical potential, $\mu_{\mathrm{I}}^{c}(T)$, at the boundary of the phase transition between the normal and pion superfluid phase.

● In this study we have ignored the role of baryon and strangeness chemical potentials.

Thank You !