2019年第十七届全国核物理大会，CCNU，2019,10

Evaluation of pion-nucleon sigma term in Dyson-Schwinger equation approach of QCD

Jing-Hui Huang[1] (黄鲸珲)，Ting-Ting Sun[1]（孙婷婷)，Huan Chen[1]（陈欢）

1. School of Mathematics and Physics, China University of Geosiciences, Lumo Road 388, 430074 Wuhan, China.

Abstract

We calculate the variation of the chiral condensate in medium with respect to the quark chemical potential and evaluate the pion-nucleon sigma term via the Hellmann-Feynman theorem. The variation of chiral condensate in medium are obtained by solving the truncated Dyson-Schwinger equation for quark propagator at finite chemical potential, with different models for the quark-gluon vertex and gluon propagator. We obtain the value of the sigma term $\sigma_{\pi N}$ = 62(1)(2) MeV, where the first represents the systematic error due to our different model for the quark-gluon vertex and gluon propagator and the second represents a statistical error in our linear fitting procedure.

Intorduction

The pion-nucleon sigma term , which is usually written via the Hellmann-Feynman theorem as

$$\sigma_{\pi N} = m_q \frac{\partial M_N}{\partial m_q}$$

is of fundamental importance for understanding the chiral symmetry breaking effects in nucleon, and the origin of mass of the observable matter. Several recent analysis in the pion-nucleon scattering or pionic atom experiments [1,2] give the value around 60 MeV with quite small error bars. However, theoretical results varies largely with different methods. Notably, the values from lattice QCD are around 30 to 40 MeV, which are much smaller.

It has been known that the DSEs of QCD provide a natural approach to investigate the DCSB and the chiral symmetry restoration in both vacuum and medium , we restudy the pion--nucleon sigma term in the DSE approach with both the widely used rainbow approximation and the Ball-Chiu vertex [2,3] for the effective quark-gluon vertex, and two different infrared dominant models for the effective interaction.

DSEs for quark propagater

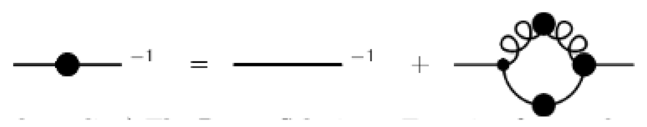

FIG.1 The Dyson-Schwigner Equation for quark propagater

The quark propagator at finite chemical potentials S (p , μ) satisfies the Dyson-Schwinger equation in Fig.(1). The gluon propagator and the quark-gluon vertex in vacuum are usually taken as

$$Z_1 g^2 D_{\rho\sigma}(k)\Gamma_\sigma^a(q,p) = G(k^2)\frac{1}{k^2}[\delta_{\rho\sigma} - \frac{k_\rho k_\sigma}{k^2}]\frac{\lambda^a}{2}\Gamma_\sigma(q,p)$$

where $G(k^2)$ is a model effective interaction, and $\Gamma_\sigma(q,p)$ is the effective quark-gluon vertex. In the following, we take two widely used model for the effective quark-gluon vertex, i.e. the rainbow approximation: $\Gamma_\sigma(q,p) = \gamma_\sigma$ and the extended form [5] of Ball-Chiu (BC) vertex [3],

$$i\Gamma_\sigma^{BC}(q,p,\mu) = i\Sigma_A(q,p,\mu)\gamma_\sigma^\perp + i\Sigma_C(q,p,\mu)\gamma_\sigma^\parallel + (\tilde{q}+\tilde{p})_\sigma[\frac{i}{2}\gamma^\perp\cdot(\tilde{q}+\tilde{p})\Delta_A(q,p,\mu)]$$

$$+(\tilde{q}+\tilde{p})_\sigma[\frac{i}{2}\gamma^\parallel\cdot(\tilde{q}+\tilde{p})\Delta_C(q,p,\mu) + (\tilde{q}+\tilde{p})\Delta_B(q,p,\mu)]$$

$$\gamma_\sigma^\parallel = (0,\gamma_4), \gamma_\sigma^\perp = \gamma - \gamma_\sigma^\parallel, F = A,B,C$$

$$\Sigma_F(q,p,\mu) = \frac{1}{2}[F(q^2,q_4,\mu) + F(p^2,p_4,\mu)], \Delta_F(q,p,\mu) = \frac{F(q^2,q_4,\mu) - F(p^2,p_4,\mu)}{q^2 - p^2}$$

For the model effective-interaction, we employ two infrared dominant models, noted as the ``GS" and the ``QC" model, which express the long-range behavior of the renormalization-group-improved Maris-Tanday model [6], and the Qin-Chang model [7]. The two models are expressed as:

$$G^{GS}(s) = \frac{4\pi^2}{\omega^6} Dse^{-s/\omega^2} \qquad G^{QC}(s) = \frac{8\pi^2}{\omega^4} Dse^{-s/\omega^2}$$

neglecting the dependence of the effective interaction G(s) and the gluon propagator on the chemical potential.

The pion-nucleon sigma term

Making use of the Hellmann-Feynman theorem in nuclear matter [8], one obtains

$$2m_q[<\bar{q}q>_n - <\bar{q}q>_0] = m_q\frac{d\varepsilon}{dm_q} = m_q\frac{dM_N}{dm_q}n_B = n_B\sigma_{\pi N}$$

where $\bar{q}q = (\bar{u}u+\bar{d}d)/2, m_q = (m_u+m_d)/2$, and approximate the energy density of the baryon matter at low densities as $\varepsilon = M_N n_B$.

Replacing with $n_q = 3n_B$,we obtain the linear dependence of the variation of chiral condensate on the quark number density, with “k” as the slope.

$$[<\bar{q}q>_n - <\bar{q}q>_0] = \frac{\sigma_{\pi N}}{6m_q} = kn_q$$

Conversely, we can evaluate the pion-nucleon sigma term as

$$\sigma_{\pi N} = 6km_q = 3k(-m_\pi^2 f_\pi^2 / <\bar{q}q>_0^0)$$

where the Gell-Mann--Oakes--Renner relation for light quarks is used and $<\bar{q}q>_0^0$ is the quark condensate in chiral limit (represented by the superscript `0') in vacuum.

Parameters setting

DSE	vertex	interaction	(ω,D)	$-\langle\bar{q}q\rangle_0^{1/3}$	m_q	k	$\sigma_{\pi N}$
DSE1	RB	GS	500,1.00	252	5.2	1.95±0.03	61 ± 1
DSE2	RB	QC	678,1.10	253	5.2	2.04±0.03	63 ± 1
DSE3	BC	GS	678,0.50	258	4.7	2.22±0.01	63 ± 1
DSE4	RB	GS	500,1.00	–	5.2	1.94±0.05	61 ± 2

Tabel 1. Parameters ω,d and some characterized numericalresults (dimensional quantities in unit of MeV). DES1, DSE2 and DSE3 are in chiral limit, while DSE4 investigates variation of chiral condensate beyond chiral limit.

We choose the set of parameters that can fit meson properties in vacuum well or fit the chiral quark condensate and the pion decay constant [9]in vacuum approximately.

"DSE1" : rainbow approximation and the 'GS' model .
"DSE2" : rainbow approximation and the 'QC' model.
"DSE3" : BC vertex and the 'GS' model .
"DSE4" : DSE1 + the variation of the chiral condensate beyond chiral limit.

Result

The quark number density n_q and chiral condensate in chiral limit can be calculated as:

$$n_q = N_c N_f Z_2 Tr[-\gamma_4 S(p,\mu)]$$
$$<\bar{q}q> = -N_c Z_2 Z_m Tr[S(p,\mu)]$$

where Tr represents the trace in color and Dirac space and integration in momentum space.

We also investigate the variation of the chiral condensate beyond chiral limit, defined as:

$$\Delta<\bar{q}q>_n^{m_q} = -N_c Z_2 Z_m Tr[S(p,\mu) - S(p,\mu=0)]$$

where the quark propagator are calculated with finite current quark mass.

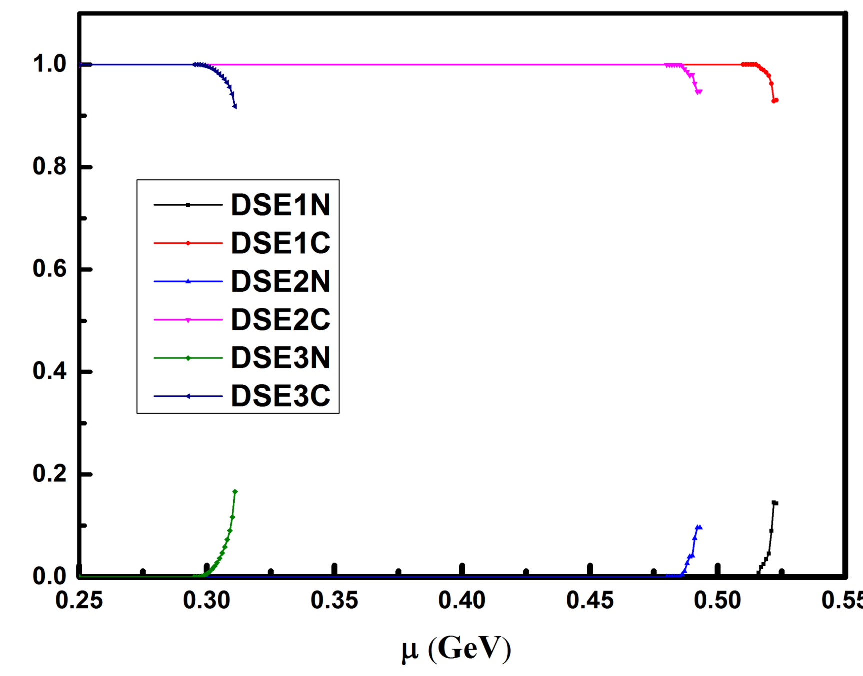

Fig.2 The quark chemical potential dependence of quark number density (scaled with 0.0038GeV) and chiral condensate (scaled with the value in vacuum).

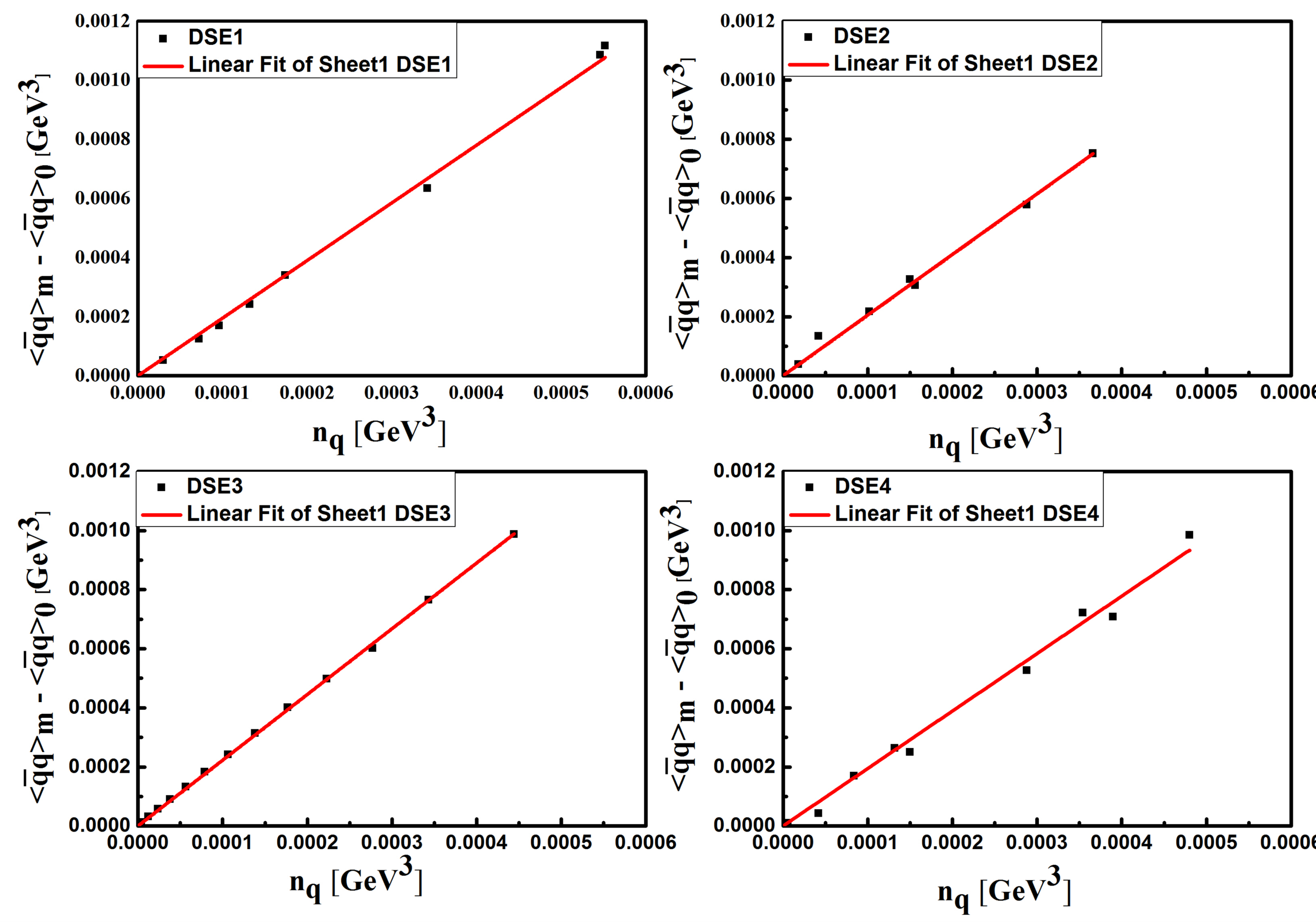

Fig .3 The variation behavior of the chiral condensate with respect to the quark number density. The linear relation is manifested well.

Summary

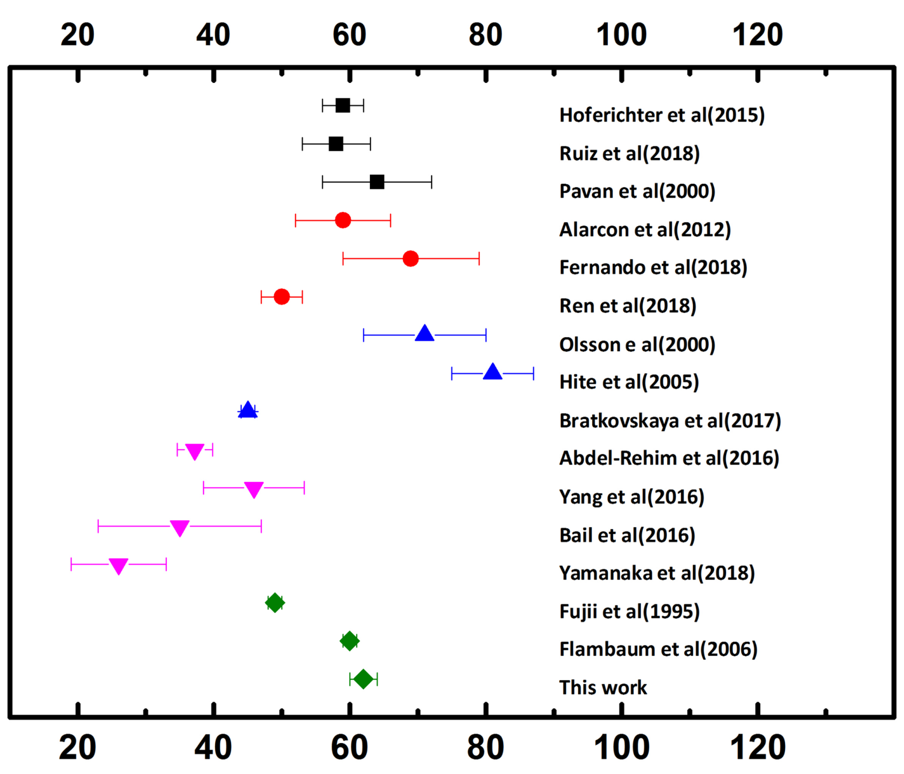

Fig.4 Comparison of our result for the pion-nucleon sigma term with others

Fig .4 display our result for the pion-nucleon sigma term, comparing with others from experimental analysis (black), chiral perturbation theory (red), Lattice QCD (magenta), some other models (blue) and the DSE approach (green).

In summary, with the DSEs approach of QCD, we calculate the chiral quark condensate in matter at low density, and then evaluate the pion-nucleon sigma term via the Hellmann-Feynman theorem. Our result 62(1)(2) MeV favors a relatively large value and is consistent very well with the recent data obtained by analyzing the pion-nucleon scattering and pionic atom experiments[1,2].

Reference

1. M. Hoferichter, J. Ruiz de Elvira, B. Kubis, and U.-G. MeißnePhys. Rev. Lett. **115**,092301 (2015). 2. J. Ruiz de Elvira, M. Hoferichter, B. Kubis, and U.-G. Meißner, J. Phys. **G45**, 024001 (2018). 3. J. S. Ball and T.-W. Chiu, Phys. Rev. **D22**, 2542 (1980). 4. J. S. Ball and T.-W. Chiu, Phys. Rev. **D22**, 2550 (1980). 5. H. Chen, W. Yuan, L. Chang, Y.-X. Liu, T. Klahn, and C. D. Roberts, Phys. Rev. **D78**, 116015 (12008). 6. P. Maris and P. C. Tandy, Phys. Rev. **C60**, 055214 (1999). 7. S.-x. Qin, L. Chang, Y.-x. Liu, C. D. Roberts, and D. J. Wilson,Phys. Rev. **C84**, 042202 (2011) . 8. T. D. Cohen, R. J. Furnstahl, and D. K. Griegel, Phys.Rev.**C45**, 1881 (1992). 9. D. W. McKay, H. J. Munczek, and B.-L. Young, Phys. Rev. **D37**, 195 (1988)