中法核工程与技术学院
Institut Franco–Chinois de l'Energie Nucléaire

# 从轻核到重核的壳模型研究及不确定度初步探索

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中山大学中法核工程与技术学院
2019.10.10@武汉

- **2009年12月21日，两国总理的见证下，中山大学与法国民用核能工程师教学联盟签署协议**
- **结合法国工程师教育和中国高等教育资源，培养核能领域的一流工程师**
- **2010年9月正式招生，通过法国工程师职衔委员会(CTI)认证和欧洲工程教育认证(EUR-ACE)，合格毕业生不出国门就能拿到欧洲认可的证书**

  - Total hours for Bachelor at IFCEN = 5331 hours representing 314 credits
  - Total hours for Bachelor at SPE = 2815 hours representing 147 credits

| Nb of teaching hours | SYSU IFCEN | SYSU Physics | SYSU Chemistry | SYSU Biology | SYSU Maths | SYSU Environment |
|---|---|---|---|---|---|---|
| Total for Bachelor | 5331 | 2815 | 3565 | 2896 | 2790 | 2790 |
| % | 189% | 100% | 127% | 103% | 99% | 99% |
| Representing n credits | 314 | 147 | 155 | 155 | 155 | 155 |
| % | 214% | 100% | 105% | 105% | 105% | 105% |

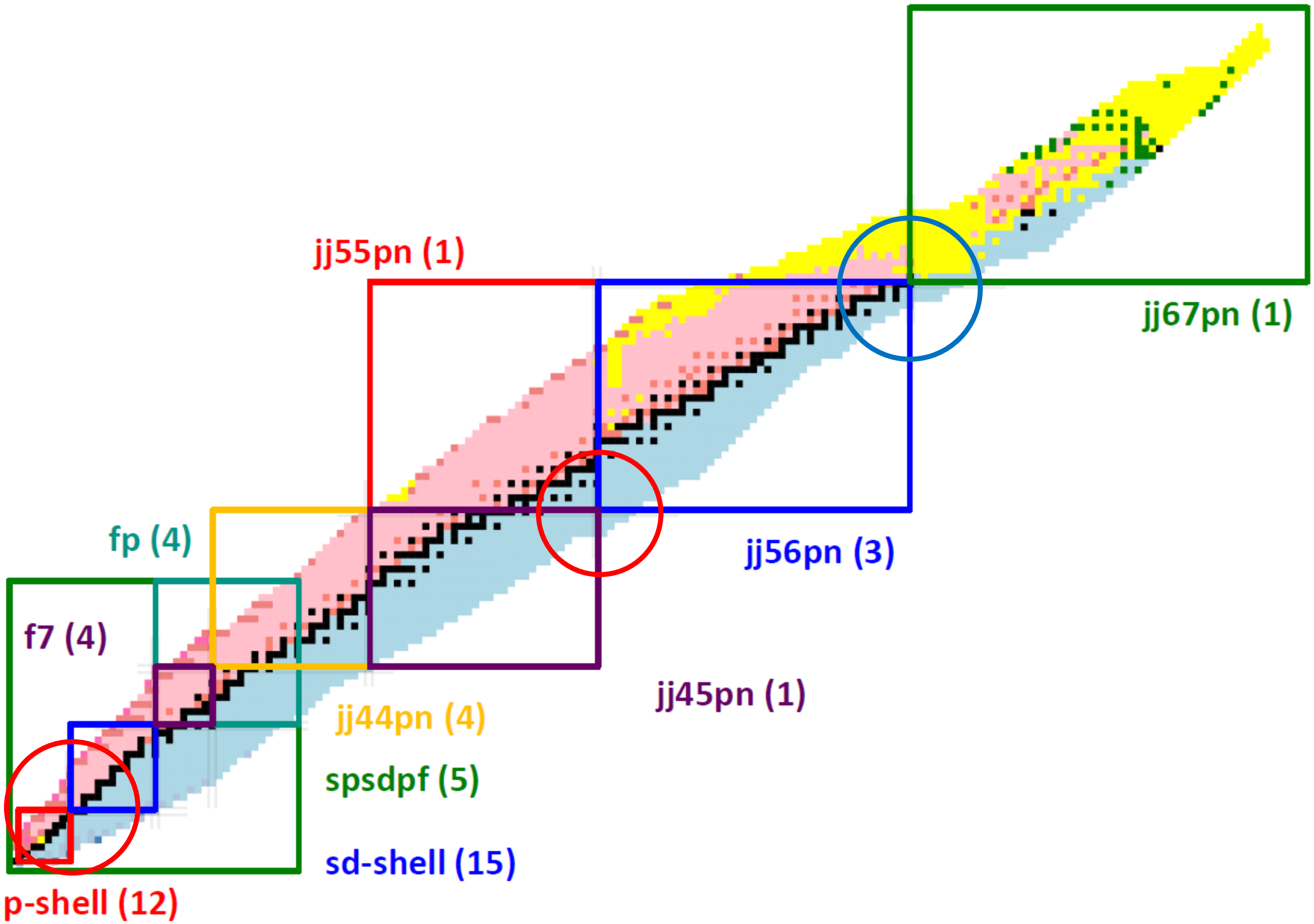
# Outline

- **Motivation and shell model**
- **Light nuclei**
- **Medium mass nuclei**
- **Heavy nuclei**
- **Uncertainty**
- **Summary and perspective**

**Force: realistic, phenomenological; Model: *Ab initio*, Mean Field**

**Huge amount of nuclear structure data: BE, level, EM, GT**

**Phenomenological shell model: deduce force from data**



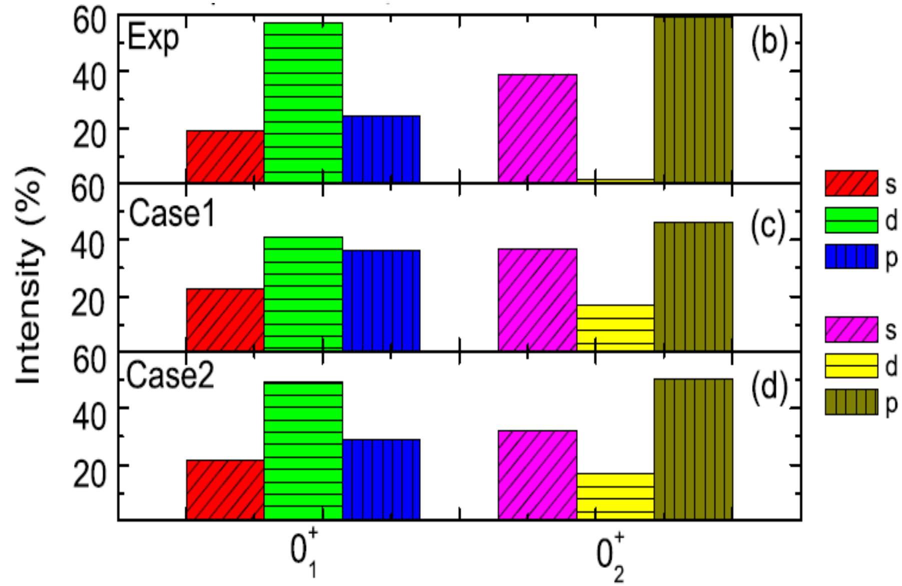

**Traditional shell VS dynamic shell**

## YSOX, enlarged model space, 0-3hw compared with 0-1hw in WBT…

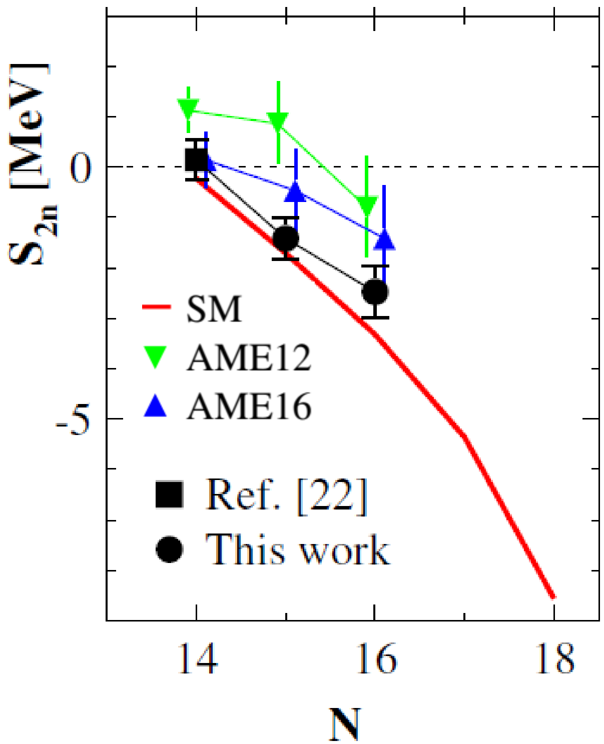
## Cross-shell interaction is reconsidered

CXY, et al., PRC 85 (2012) 064324

### *sd* component in $^{12}$Be beyond N=8




J. Chen, et al., PLB 781(2018)412

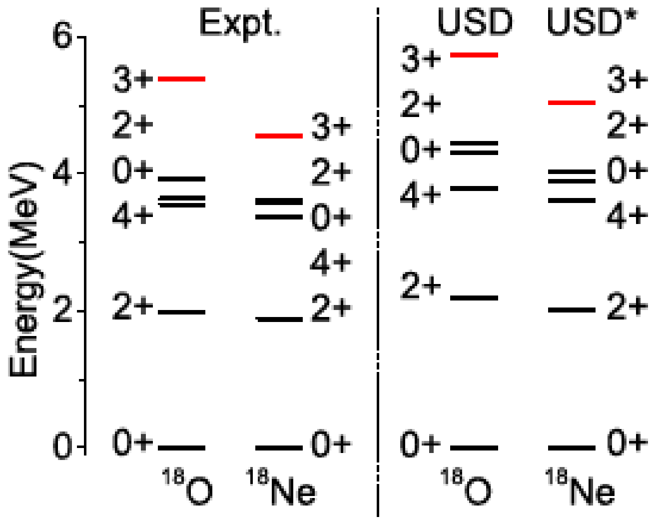



PHYSICAL REVIEW LETTERS **121,** 262502 (2018)

**First Observation of $^{20}$B and $^{21}$B**

We wish to extend our thanks to the accelerator staff of the RIKEN Nishina Center for their efforts in delivering the intense $^{48}$Ca beam, and to C. Yuan for the matrix elements of the YSOX interaction. N. L. A., F. D., J. G., F. M. M., and N. A. O. acknowledge partial support from the Franco-

[32] C. Yuan, T. Suzuki, T. Otsuka, F. Xu, and N. Tsunoda, Phys. Rev. C **85**, 064324 (2012).




# IAS ($0^+$) of $^{22}$Al,

# Expt. 8829 (406) keV, USD* 9020 keV

# Observed properties of decay of $^{22}$Si

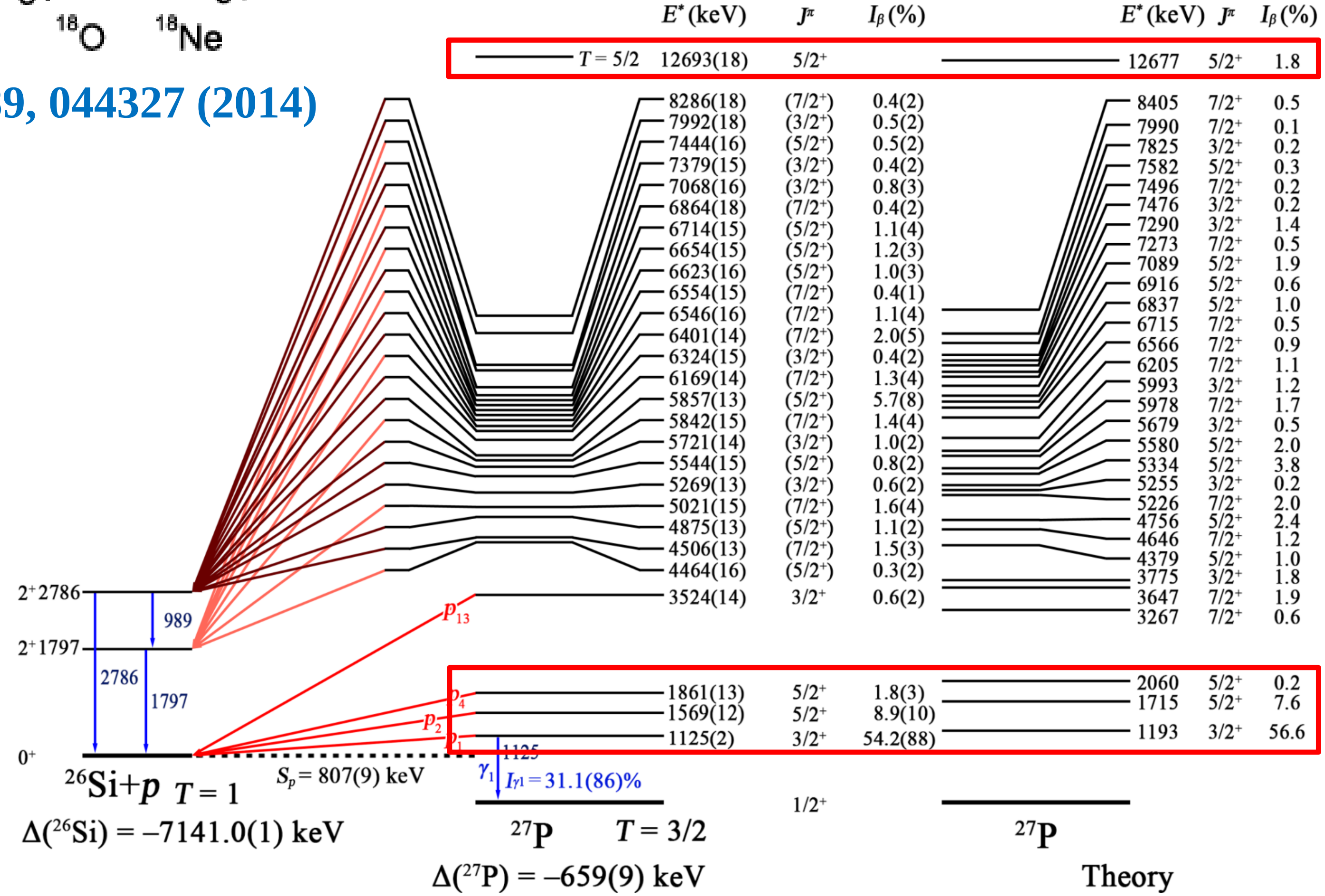
## CIAE group (PLB766(2017)312, arXiv:1610.08291)

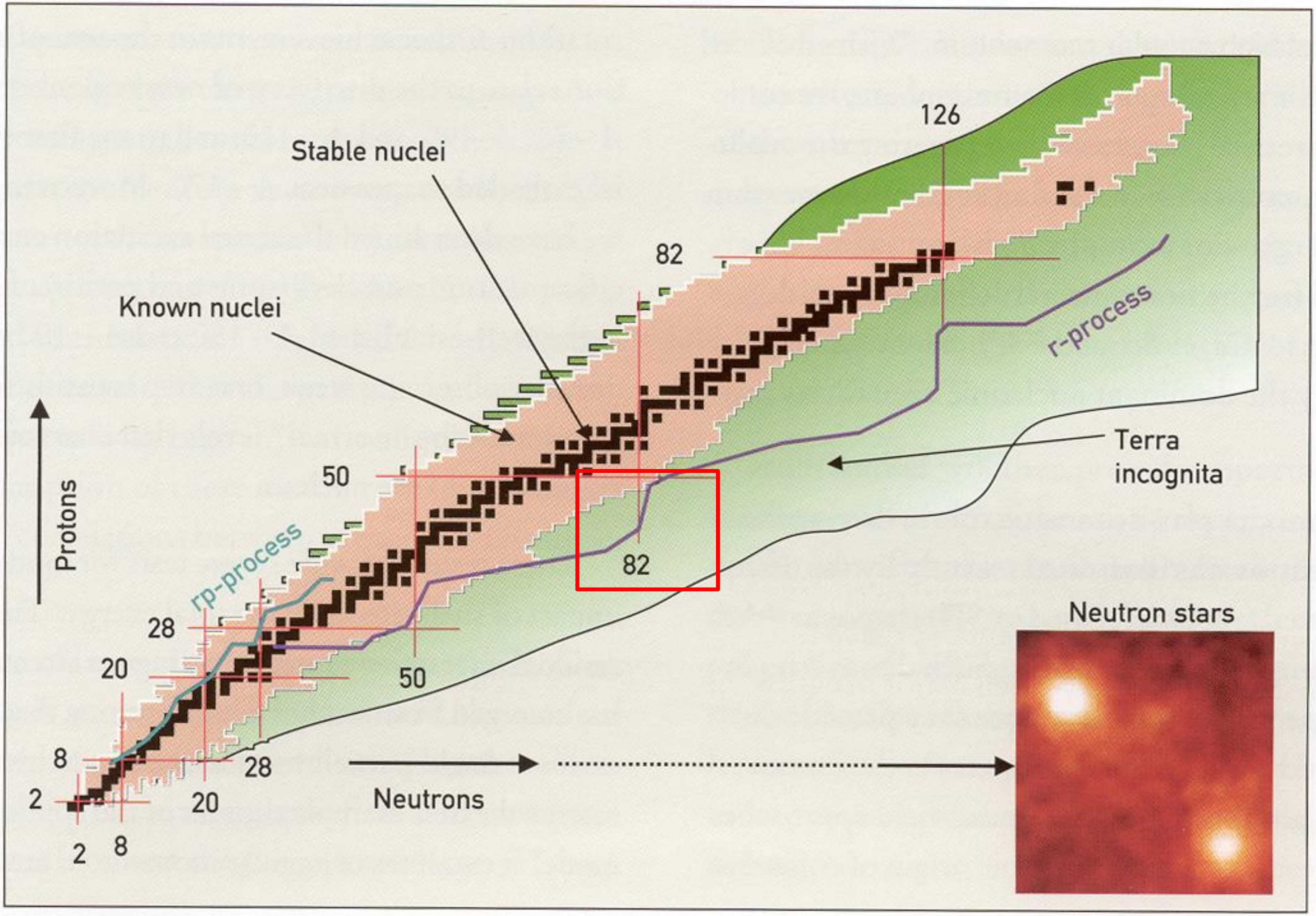
CXY, et al., PRC 89, 044327 (2014)

PHYSICAL REVIEW C **99**, 064312 (2019)

**β-decay spectroscopy of $^{27}$S**

(RIBLL Collaboration)





Stable nuclei
Known nuclei
126
82
50
28
20
8
2
rp-process
r-process
Terra incognita
82
50
28
20
8
2
Protons
Neutrons
Neutron stars

# Possible isomer in "South east" region of $^{132}$Sn

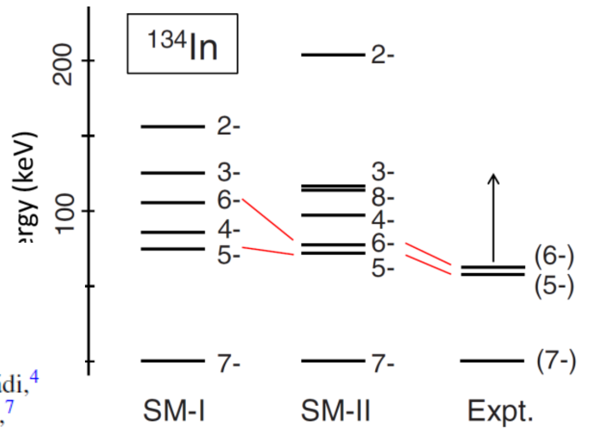
| Nuclei | $J_i^\pi \rightarrow J_f^\pi$ | $E_i$ | $\Delta E$ | $B(E2)_{th}$ | $\tau_{E2}$ | $B(E2)_{Expt}$ | Configuration |
|---|---|---|---|---|---|---|---|
| $^{134}$Sn | $6^+ \rightarrow 4^+$ | 1.267 | 0.149 | 41.72 | 0.266 | 35(6) | $\nu(1f_{7/2})^2(96.3\%)$ |
| $^{135}$Sn | $21/2^- \rightarrow 17/2^-$ | 2.288 | 0.147 | 93.96 | 0.127 | | $\nu(1f_{7/2})^2(0h_{9/2})(96.7\%)$ |
| $^{136}$Sn | $6^+ \rightarrow 4^+$ | 1.388 | 0.222 | 15.43 | 0.098 | 24(4) | $\nu(1f_{7/2})^4(75.5\%)$ |
| $^{138}$Sn | $6^+ \rightarrow 4^+$ | 1.544 | 0.183 | 12.80 | 0.311 | 19(4) | $\nu(1f_{7/2})^6(53.13\%)$ |
| $^{132}$In | $5^- \rightarrow 7^-$ | 0.067 | 0.067 | 1.75 | 345.693 | | $\mu(0g_{9/2})^{-1}\nu(1f_{7/2})(99.0\%)$ |
| $^{133}$In | $17/2^- \rightarrow 13/2^-$ | 0.972 | 0.257 | 48.36 | 0.015 | | $\mu(0g_{9/2})^{-1}\nu(1f_{7/2})^2(93.9\%)$ |
| $^{134}$In | $5^- \rightarrow 7^-$ | 0.074 | 0.074 | 27.69 | 13.282 | | $\mu(0g_{9/2})^{-1}\nu(1f_{7/2})^3(72.5\%)$ |
| $^{130}$Cd | $8^+ \rightarrow 6^+$ | 2.123 | 0.106 | 59.07 | 1.032 | 66(13)/50(10) | $\mu(0g_{9/2})^{-2}(100.0\%)$ |
| $^{131}$Cd | $19/2^- \rightarrow 15/2^-$ | 1.789 | 0.139 | 100.03 | 0.157 | | $\mu(0g_{9/2})^{-2}\nu(1f_{7/2})(99.7\%)$ |
| $^{130}$Ag | $5^- \rightarrow 7^-$ | 0.072 | 0.072 | 18.06 | 23.350 | | $\mu(0g_{9/2})^{-3}\nu(1f_{7/2})(74.1\%)$ |
| $^{128}$Pd | $8^+ \rightarrow 6^+$ | 2.198 | 0.100 | 13.43 | 6.076 | 8.43(0.25) | $\mu(0g_{9/2})^{-4}(69.3\%)$ |
| $^{129}$Pd | $19/2^- \rightarrow 15/2^-$ | 1.913 | 0.146 | 13.70 | 0.898 | | $\mu(0g_{9/2})^{-4}\nu(1f_{7/2})(72.7\%)$ |
| $^{130}$Pd | $6^+ \rightarrow 4^+$ | 1.328 | 0.211 | 184.24 | 0.011 | | $\mu(0g_{9/2})^{-4}\nu(1f_{7/2})^2(53.5\%)$ |

Isomerism in the "south-east" of $^{132}$Sn and a predicted neutron-decaying isomer in $^{129}$Pd

Cenxi Yuan[a,*], Zhong Liu[b,*], Furong Xu[c,d], P.M. Walker[e], Zs. Podolyák[e], C. Xu[f], Z.Z. Ren[f], B. Ding[b], M.L. Liu[b], X.Y. Liu[b], H.S. Xu[b], Y.H. Zhang[b], X.H. Zhou[b], W. Zuo[b]

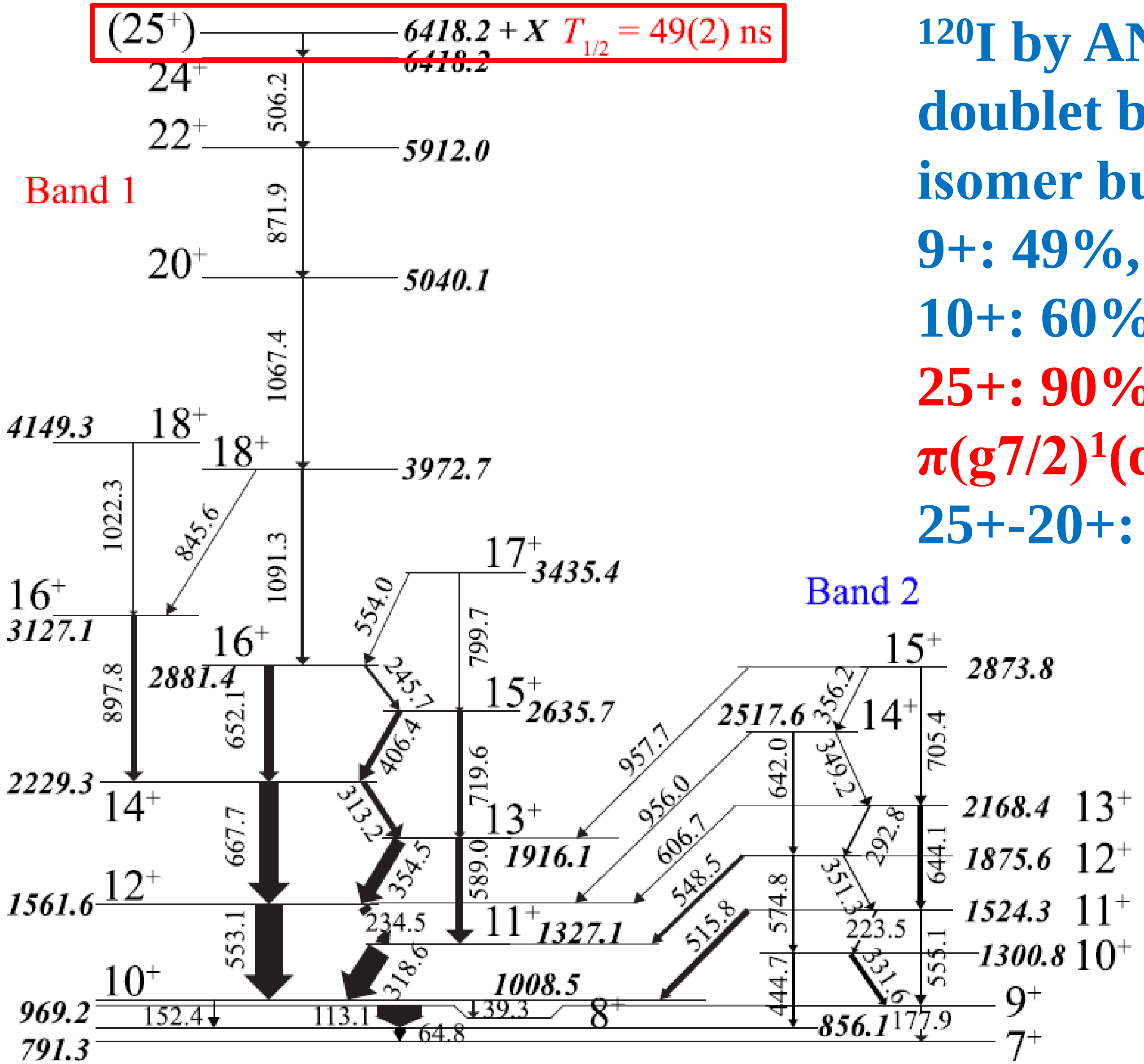
**First spectroscopic information from even-even nuclei in the region "southeast" of $^{132}$Sn: Neutron-excitation dominance of the $2_1^+$ state in $^{132}$Cd**

H. Wang,[1,*] N. Aoi,[2] S. Takeuchi,[1] M. Matsushita,[3] T. Motobayashi,[1] D. Steppenbeck,[1] K. Yoneda,[1] H. Baba,[1] Zs. Dombrádi,[4] K. Kobayashi,[5] Y. Kondo,[6] J. Lee,[1,†] H. Liu,[1,7] R. Minakata,[6] D. Nishimura,[8] H. Otsu,[1] H. Sakurai,[1,9] D. Sohler,[4] Y. Sun,[7] Z. Tian,[7] R. Tanaka,[6] Zs. Vajta,[4] Z. Yang,[7,‡] T. Yamamoto,[2] Y. Ye,[7] and R. Yokoyama[3]




**Phong et al., PRC 100, 011302(R) (2019)**

**$2^+$ state of $^{132}$Cd: Our 708 keV, observed 618(8) keV**




# $^{120}$I by ANU Group

**doublet bands and a high-spin isomer built on $\pi h_{11/2} \upsilon h_{11/2}$**

**9+: 49%, 1004 keV**

**10+: 60%, 1003 keV**

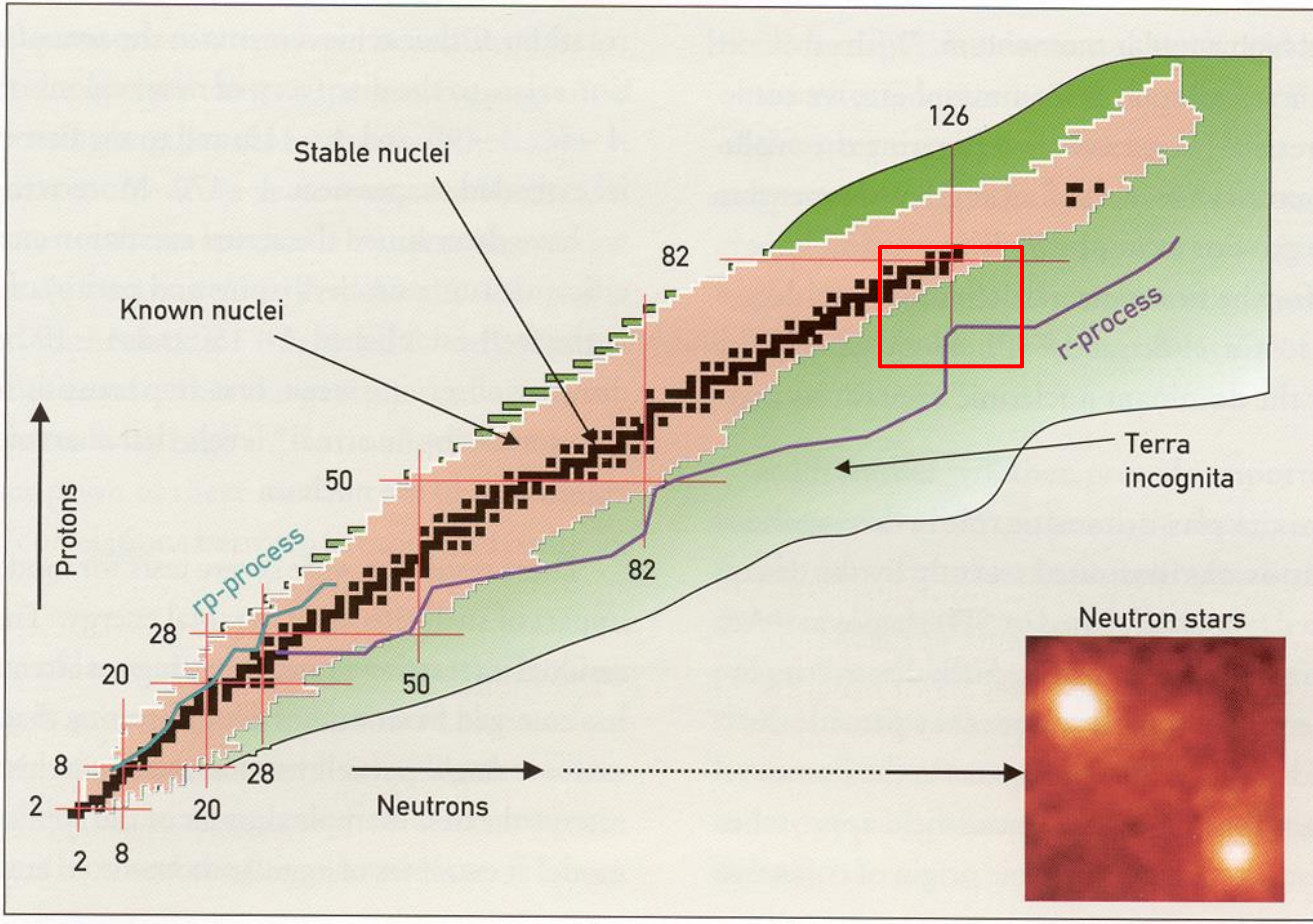
**25+: 90%**

**$\pi(g7/2)^1(d5/2)^1(h11/2)^1\upsilon(h11/2)^3$**

**25+-20+: 1.4 MeV**

transitions from the $25^+$ state are highly forbidden in terms of the reduced transition rates, which are $0.02\mu_N^2$ and $5.746e^2\text{fm}^4$ for the $M1$ and $E2$ transitions, respectively. This calculated $25^+$ state is

**B. Moon et al., PLB 782 (2018) 602**


Stable nuclei
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82
rp-process
r-process
Terra incognita
Protons
Neutrons
Neutron stars

**Pb South**

**khhe**

**khpe**

**$V_{MU}$**

**45 BE RMS**

**0.09 MeV**

| nuclide | $BE_{SM}$ | $BE_{Expt.}$ | $\Delta BE$ |
|---|---|---|---|
| $^{215}$Pb | 1666.803 | 1666.839 | 0.036 |
| $^{214}$Pb | 1663.220 | 1663.292 | 0.072 |
| $^{213}$Pb | 1658.194 | 1658.241 | 0.047 |
| $^{212}$Pb | 1654.466 | 1654.516 | 0.050 |
| $^{211}$Pb | 1649.367 | 1649.388 | 0.021 |
| $^{210}$Pb | 1645.522 | 1645.553 | 0.031 |
| $^{209}$Pb | 1640.368 | 1640.367 | -0.001 |
| $^{208}$Pb | 1636.430 | 1636.430 | 0.000 |
| $^{207}$Pb | 1629.062 | 1629.062 | 0.000 |
| $^{206}$Pb | 1622.323 | 1622.325 | 0.002 |
| $^{205}$Pb | 1614.291 | 1614.238 | -0.053 |
| $^{204}$Pb | 1607.564 | 1607.506 | -0.058 |
| $^{203}$Pb | 1599.222 | 1599.112 | -0.110 |
| $^{202}$Pb | 1592.319 | 1592.194 | -0.125 |
| $^{201}$Pb | 1583.648 | 1583.454 | -0.194 |
| $^{200}$Pb | 1576.602 | 1576.362 | -0.240 |
| $^{213}$Tl | 1654.249 | 1654.037 | -0.212 |
| $^{211}$Tl | 1645.832 | 1645.756 | -0.076 |
| $^{210}$Tl | 1640.891 | 1640.854 | -0.037 |
| $^{209}$Tl | 1637.209 | 1637.180 | -0.029 |
| $^{208}$Tl | 1632.220 | 1632.214 | -0.006 |
| $^{207}$Tl | 1628.428 | 1628.427 | -0.001 |
| $^{206}$Tl | 1621.583 | 1621.575 | -0.008 |
| $^{205}$Tl | 1615.072 | 1615.071 | -0.001 |
| $^{204}$Tl | 1607.519 | 1607.525 | 0.006 |
| $^{203}$Tl | 1600.931 | 1600.869 | -0.062 |
| $^{202}$Tl | 1593.022 | 1593.016 | -0.006 |
| $^{201}$Tl | 1586.284 | 1586.145 | -0.139 |
| $^{200}$Tl | 1578.051 | 1577.941 | -0.110 |
| RMS | | | 0.089 |

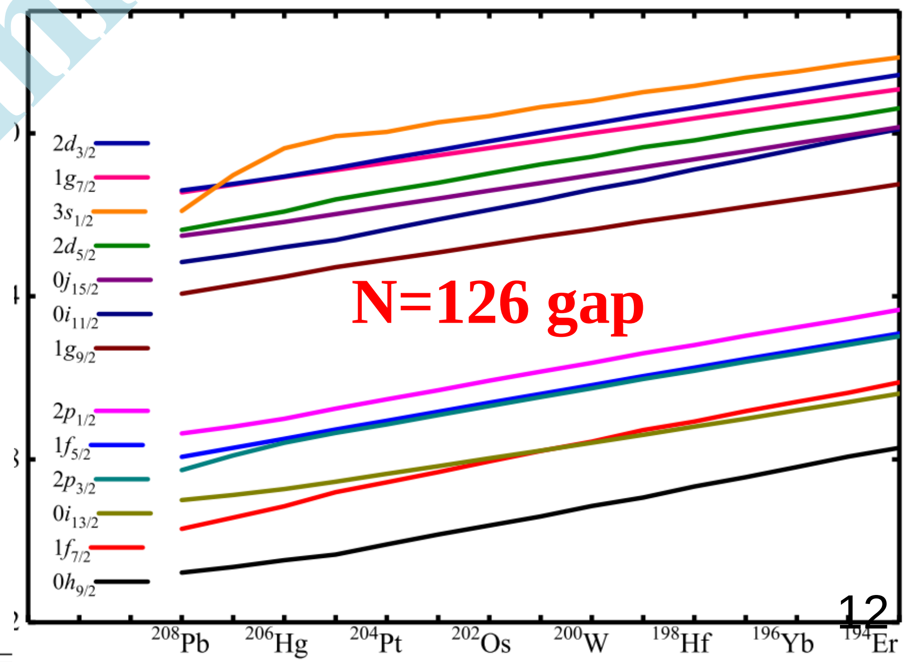
| nuclide | $BE_{Expt.}$ | $BE_{KHHE}$ | $BE_{d3d5g7h11}$ | $\Delta BE_{KHHE}$ | $\Delta BE_{d3d5g7h11}$ |
|---|---|---|---|---|---|
| $^{208}$Hg | 1629.512 | 1629.556 | 1629.457 | -0.044 | 0.055 |
| $^{207}$Hg | 1624.662 | 1624.690 | 1624.594 | -0.028 | 0.068 |
| $^{206}$Hg | 1621.049 | 1621.071 | 1620.977 | -0.022 | 0.072 |
| $^{205}$Hg | 1614.320 | 1614.336 | 1614.196 | -0.016 | 0.124 |
| $^{204}$Hg | 1608.651 | 1608.644 | 1608.502 | 0.007 | 0.149 |
| $^{203}$Hg | 1601.159 | 1601.164 | 1601.025 | -0.005 | 0.134 |
| $^{202}$Hg | 1595.164 | 1595.241 | 1595.100 | -0.077 | 0.064 |
| $^{201}$Hg | 1587.410 | 1587.449 | 1587.313 | -0.039 | 0.097 |
| $^{200}$Hg | 1581.179 | 1581.297 | 1581.156 | -0.118 | 0.023 |
| $^{203}$Au | 1599.815 | 1600.051 | 1599.901 | -0.236 | -0.086 |
| $^{202}$Au | 1592.954 | 1593.147 | 1592.897 | -0.193 | 0.057 |
| $^{201}$Au | 1586.930 | 1587.296 | 1587.127 | -0.366 | -0.197 |
| $^{200}$Au | 1579.698 | 1580.001 | 1579.774 | -0.303 | -0.076 |
| $^{202}$Pt | 1592.075 | 1592.517 | 1591.883 | -0.442 | 0.192 |
| $^{201}$Pt | 1585.052 | 1585.684 | 1585.044 | -0.632 | 0.008 |
| $^{200}$Pt | 1579.840 | 1580.513 | 1579.877 | -0.673 | -0.037 |
| RMS | | | | 0.295 | 0.105 |

| nuclide | $BE_{Expt.}$ | $BE_{KHHE}$ | $BE_{d3d5g7h11}$ | $\Delta BE_{KHHE}$ | $\Delta BE_{d3d5g7h11}$ |
|---|---|---|---|---|---|
| $^{217}$Pb | 1675.023 | 1675.141 | 1675.141 | -0.118 | -0.118 |
| $^{216}$Pb | 1671.840 | 1671.749 | 1671.749 | 0.091 | 0.091 |
| $^{212}$Tl | 1649.360 | 1649.379 | 1649.379 | -0.019 | -0.019 |
| $^{211}$Hg | 1641.158 | 1641.215 | 1641.117 | -0.057 | 0.041 |
| $^{210}$Hg | 1637.790 | 1637.875 | 1637.776 | -0.085 | 0.014 |
| $^{209}$Hg | 1632.917 | 1633.051 | 1632.953 | -0.134 | -0.036 |
| $^{209}$Au | 1627.692 | 1628.037 | 1627.942 | -0.345 | -0.250 |
| $^{208}$Au | 1623.232 | 1623.455 | 1623.367 | -0.223 | -0.135 |
| $^{207}$Au | 1619.775 | 1620.096 | 1620.008 | -0.321 | -0.233 |
| $^{206}$Au | 1615.040 | 1615.499 | 1615.420 | -0.459 | -0.380 |
| $^{205}$Au | 1611.505 | 1611.996 | 1611.918 | -0.491 | -0.413 |
| $^{204}$Au | 1605.480 | 1605.657 | 1605.487 | -0.177 | -0.007 |
| $^{207}$Pt | 1614.186 | 1614.735 | 1614.140 | -0.549 | 0.046 |
| $^{206}$Pt | 1611.332 | 1611.613 | 1611.007 | -0.281 | 0.325 |
| $^{205}$Pt | 1606.585 | 1607.072 | 1606.467 | -0.487 | 0.118 |
| $^{204}$Pt | 1603.440 | 1603.812 | 1603.191 | -0.372 | 0.249 |
| $^{203}$Pt | 1597.001 | 1597.645 | 1596.997 | -0.644 | 0.004 |
| $^{204}$Ir | 1596.096 | 1597.002 | 1596.162 | -0.906 | -0.066 |
| $^{203}$Ir | 1592.941 | 1593.890 | 1592.968 | -0.949 | -0.027 |
| $^{202}$Ir | 1586.912 | 1588.082 | 1587.114 | -1.170 | -0.202 |
| $^{201}$Ir | 1582.071 | 1583.037 | 1582.181 | -0.966 | -0.110 |
| $^{203}$Os | 1586.648 | 1588.002 | 1586.666 | -1.354 | -0.018 |
| $^{202}$Os | 1584.084 | 1585.095 | 1583.720 | -1.011 | 0.364 |
| $^{201}$Os | 1578.051 | 1579.437 | 1578.046 | -1.386 | 0.005 |
| $^{200}$Os | 1573.600 | 1574.854 | 1573.536 | -1.254 | 0.064 |
| $^{202}$Re | | 1577.062 | 1575.306 | | |
| $^{201}$Re | | 1574.248 | 1572.491 | | |
| $^{200}$Re | | 1569.016 | 1567.174 | | |
| $^{199}$Re | 1562.349 | 1564.531 | 1562.797 | -2.182 | -0.448 |
| $^{201}$W | | 1567.529 | 1565.188 | | |
| $^{200}$W | | 1564.967 | 1562.566 | | |
| $^{199}$W | | 1559.803 | 1557.387 | | |
| $^{198}$W | | 1555.738 | 1553.435 | | |
| RMS | | | | 0.811 | 0.201 |

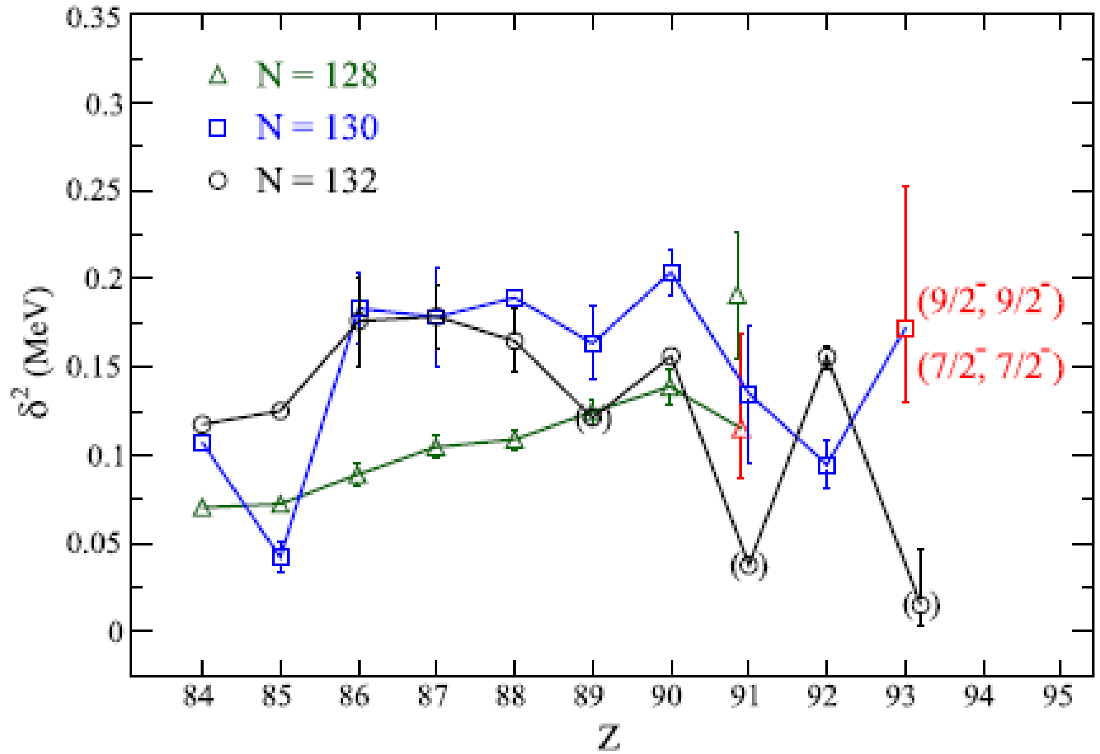
# Prediction for more properties

TABLE IX. The prediction of possible isomers including transition energy ($\delta$E), B(E2), and half-lives.

| nuclide | $J_i^\pi$ | $J_f^\pi$ | $\delta$E (MeV) | B(E2)($e^2fm^4$) | half-life ($\mu$s) |
|---|---|---|---|---|---|
| $^{215}$Pb | $17/2^+$ | $13/2^+$ | 0.33 | 31.69 | 0.0041 |
| | $21/2^+$ | $17/2^+$ | 0.12 | 35.47 | 0.16 |
| $^{213}$Pb | $17/2^+$ | $13/2^+$ | 0.29 | 37.78 | 0.0061 |
| | $21/2^+$ | $17/2^+$ | 0.12 | 0.51 | 10.74 |
| $^{213}$Tl | $13/2^+$ | $9/2^+$ | 0.09 | 0.01 | 1812.75 |
| | $17/2^+$ | $13/2^+$ | 0.05 | 0.03 | 352.57 |
| $^{212}$Tl | $11^+$ | $9^+$ | 0.11 | 2.24 | 2.92 |
| $^{211}$Tl | $13/2^+$ | $9/2^+$ | 0.09 | 67.56 | 0.14 |
| | $17/2^+$ | $13/2^+$ | 0.05 | 18.89 | 0.66 |
| $^{210}$Tl | $11^+$ | $9^+$ | 0.11 | 145.95 | 0.052 |
| $^{210}$Hg | $6^+$ | $4^+$ | 0.10 | 73.99 | 0.12 |
| | $8^+$ | $6^+$ | 0.06 | 27.40 | 0.44 |
| $^{209}$Hg | $17/2^+$ | $13/2^+$ | 0.38 | 416.80 | 0.0002 |
| | $21/2^+$ | $17/2^+$ | 0.12 | 212.33 | 0.030 |



# Ground state spin parity of $^{223}$Np compared with alpha decay results, by Zhong Liu@IMP

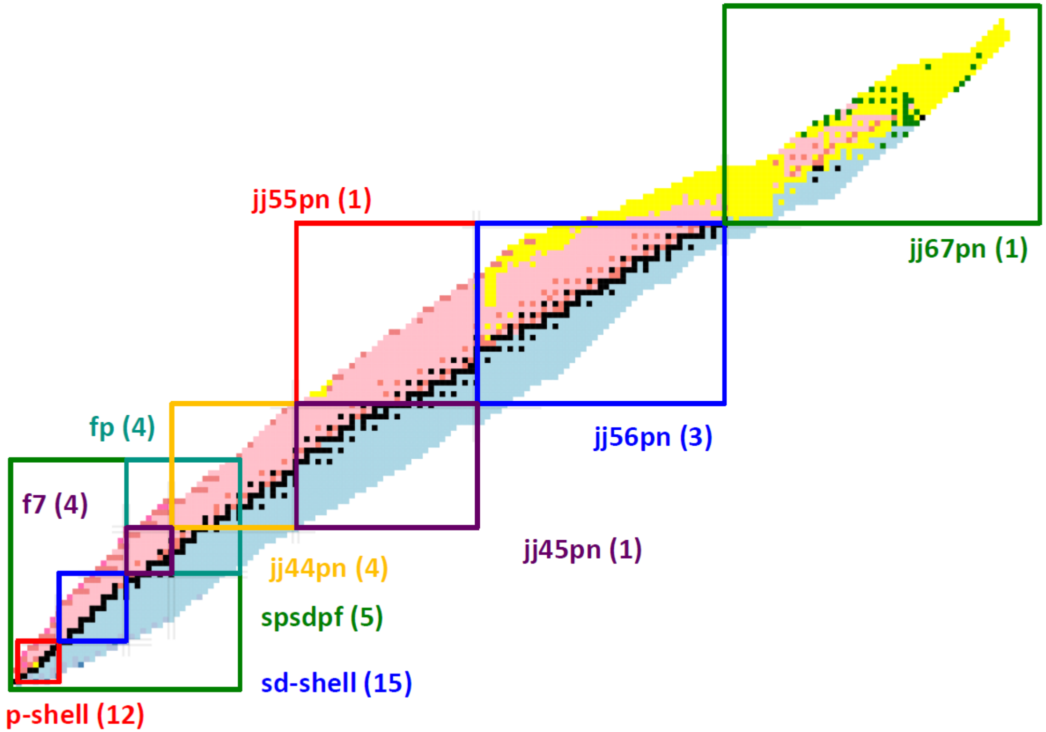



change for Np, i.e. in case of a $7/2^-$ ground state? Thus, I would like the authors to clarify this point, and eventually include *both* options (compatible with the experimental decay observations) in Figs. 3 and 4; that the shell-model predictions later-on prefer one of them is just fine.

$Z = 92$ subshell closure. The spin and parity of $^{223}$Np are proposed to be $9/2^-$ by combining the reduced $\alpha$-decay width and large-scale shell-model calculations in truncated model space, negating

**M.D. Sun *et al.* PLB771(2017)303**

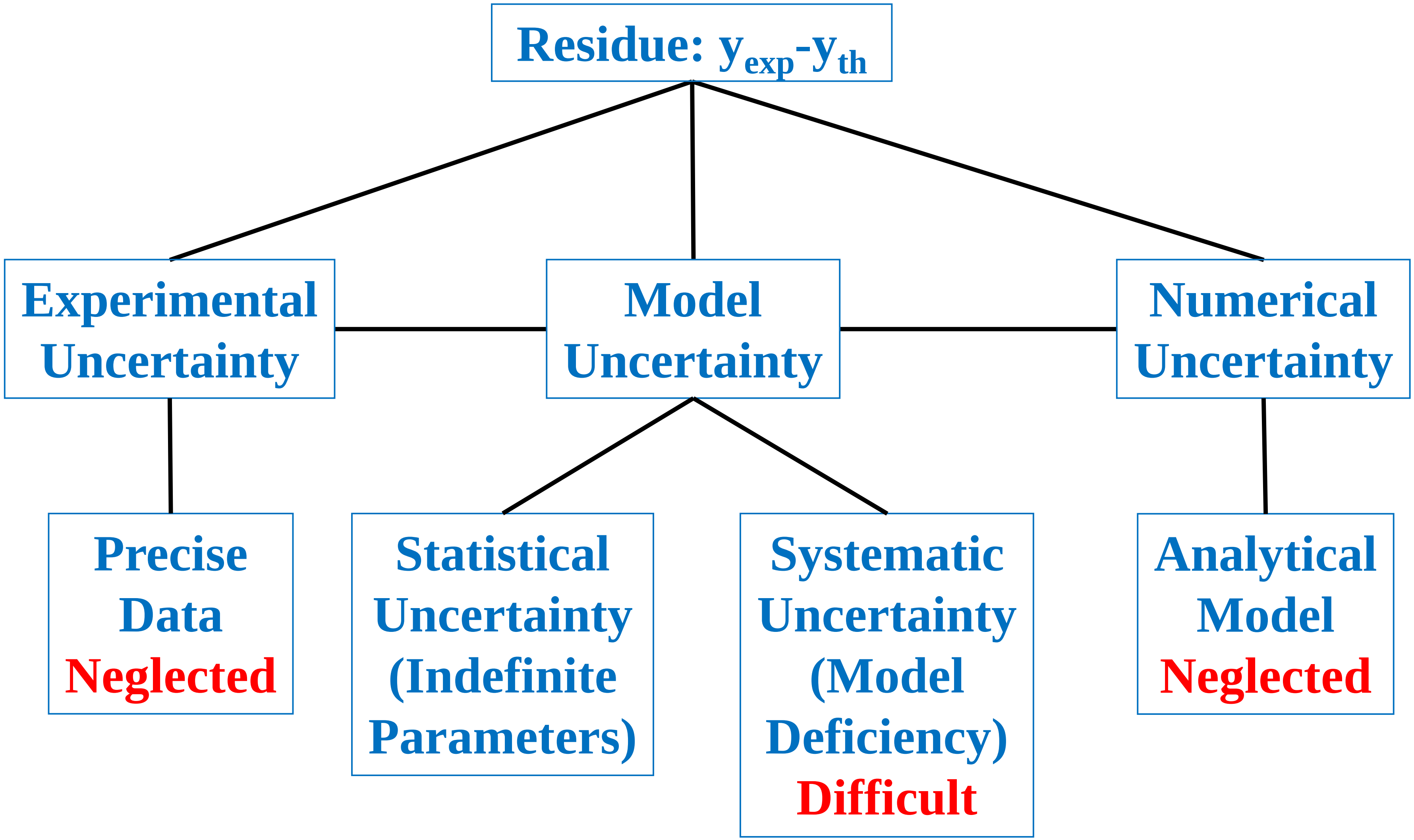
- **Confidence:** nice results on BE, level, EM, GT
- **Confusion:** different H, quality of predicted power




- **Uncertainty MUST be included in shell model to evaluate predictions**
- **One effective force for all possible nuclei (RMS for BE, level, EM. GT)**

# Theoretical model: $y_{th}=f(x_1, x_2\ldots\ p_1,p_2\ldots)$

# $y_{th}$: unknown, f(): model, $x_1,x_2$ : known, $p_1,p_2$: parameters



- **Uncertainty Decomposition Method (UDM) for simple model under simple statistical assumption.**
- **Distribution of residue is assumed to be the sum of two normal distribution:**

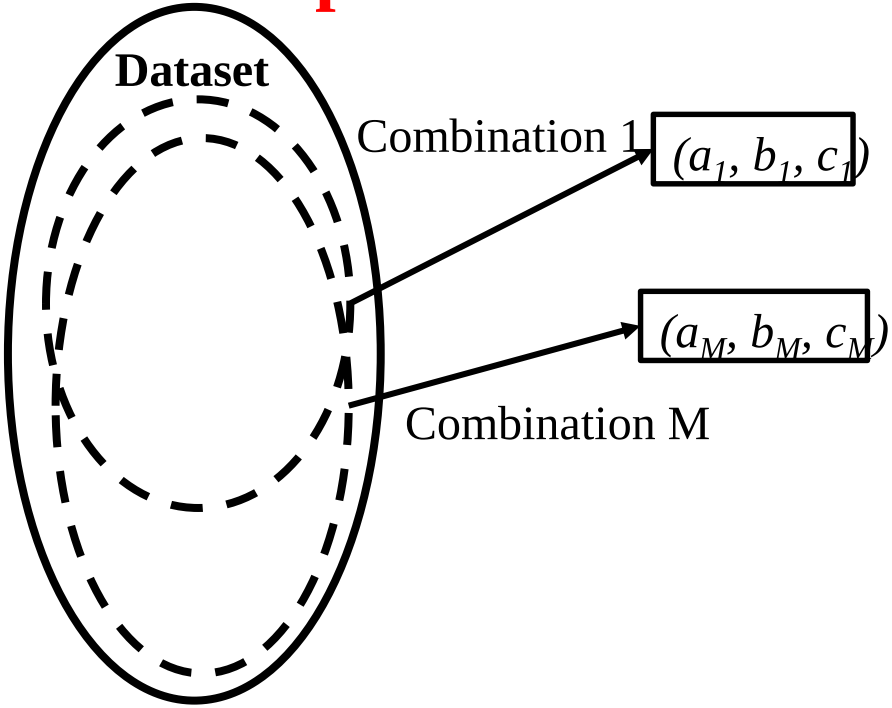
$$f(e) = f(stat) + f(syst) = \frac{1}{2}N(m_{stat}, \sigma_{stat}) + \frac{1}{2}N(m_{syst}, \sigma_{syst}).$$

- **After inclusion of shell term both statistical and systematic uncertainty reduce:**

| Model | data | $m_{stat}$ | $\sigma_{stat}$ | $m_{syst}$ | $\sigma_{syst}$ | $\frac{1}{2}(m^2_{stat}+\sigma^2_{stat})$ | $\frac{1}{2}(m^2_{syst}+\sigma^2_{syst})$ |
|---|---|---|---|---|---|---|---|
| LD6 | AME1995 | 1.02 | 1.63 | -1.00 | 2.98 | 1.84 | 4.95 |
| LD6 | AME2003 | 1.01 | 1.66 | -0.86 | 2.77 | 1.89 | 4.22 |
| LD6 | AME2012 | 1.05 | 1.66 | -0.88 | 2.85 | 1.93 | 4.45 |
| LD8 | AME1995 | 0.94 | 0.91 | -1.03 | 1.17 | 0.86 | 1.22 |
| LD8 | AME2003 | 0.81 | 0.92 | -0.74 | 1.33 | 0.75 | 1.16 |
| LD8 | AME2012 | 0.91 | 0.86 | -0.96 | 1.22 | 0.78 | 1.20 |

**CXY, PRC, 93, 034310 (2016)**

# • Non-parametric Bootstrap Method




**Universal Decay Law (by Qi *et al.* in 2009):**

$$\log T_{1/2} = aZ_cZ_d\sqrt{\frac{\mu}{Q}} + b\sqrt{\mu Z_cZ_d\left(A_c^{1/3} + A_d^{1/3}\right)} + c$$

**New Geiger-Nuttall Law (by Ren *et al.* in 2012):**

$$\log T_{1/2} = aZ_cZ_d\sqrt{\frac{\mu}{Q}} + b\sqrt{\mu Z_cZ_d} + c + S + Pl(l+1)$$

| | | a | | b | | c | | d | | Uncertainty | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Mean | S.D. | Mean | S.D. | Mean | S.D. | Mean | S.D. | total | stat | sys |
| UDL | (Total) | 0.4094 | 0.0022 | -0.4256 | 0.0043 | -21.6122 | 0.2870 | | | 0.3442 | 0.0504 | 0.3405 |
| | $N \leqslant 126$ | 0.4188 | 0.0031 | -0.3943 | 0.0054 | -24.7439 | 0.5106 | | | 0.2922 | 0.0626 | 0.2854 |
| | $N \geqslant 127$ | 0.4013 | 0.0013 | -0.3707 | 0.0066 | -24.7370 | 0.5221 | | | 0.1737 | 0.0356 | 0.1700 |
| | $N \geqslant 127^*$ | | | | | | | | | 0.7330 | 0.0939 | 0.7270 |
| NGNL | (Total) | 0.4074 | 0.0019 | -1.3232 | 0.0123 | -17.7678 | 0.2162 | | | 0.2658 | 0.0406 | 0.2626 |
| | $N \leqslant 126$ | 0.4126 | 0.0030 | -1.3574 | 0.0143 | -17.6370 | 0.4536 | | | 0.2736 | 0.0577 | 0.2674 |
| | $N \geqslant 127$ | 0.3982 | 0.0013 | -1.2622 | 0.0213 | -18.0730 | 0.5863 | | | 0.1720 | 0.0344 | 0.1686 |
| | $N \geqslant 127^*$ | | | | | | | | | 0.3079 | 0.0814 | 0.2970 |
| uncorrected (Eq. 11) | (Total) | 0.4031 | 0.0024 | -1.5054 | 0.0190 | -11.9837 | 0.4932 | | | 0.3939 | 0.0594 | 0.3894 |
| corrected (Eq. 17) | (Total) | 0.4069 | 0.0020 | -1.3682 | 0.0142 | -16.4383 | 0.3919 | 1.7018 | 0.1278 | 0.2657 | 0.0466 | 0.2616 |

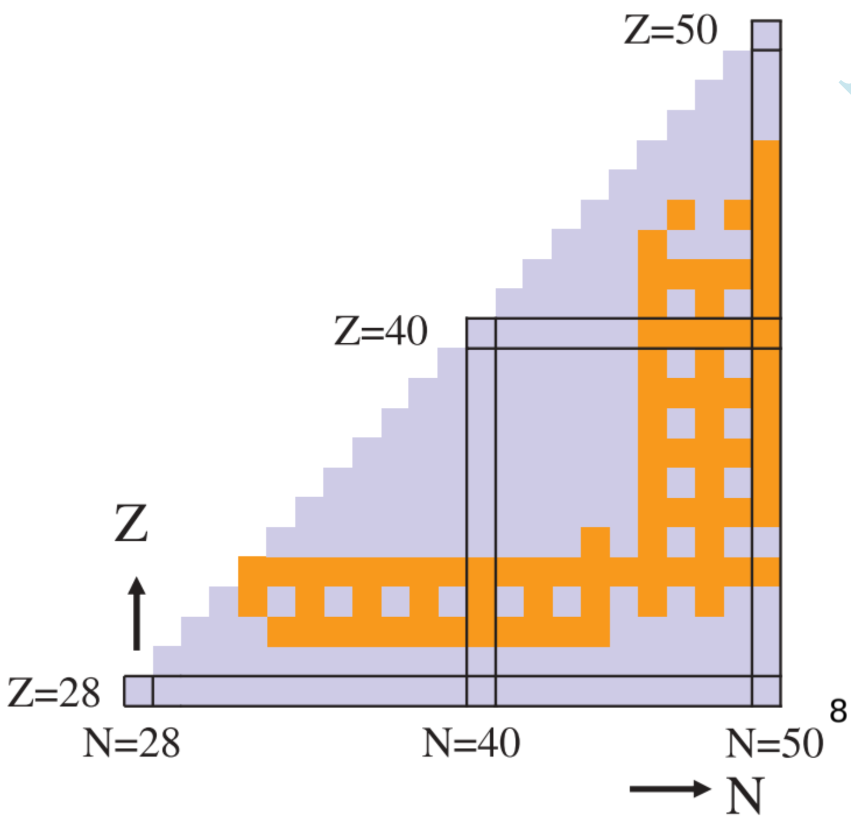


**BS Cai, GS Chen, CXY, *et al.*, submitted to PRC**

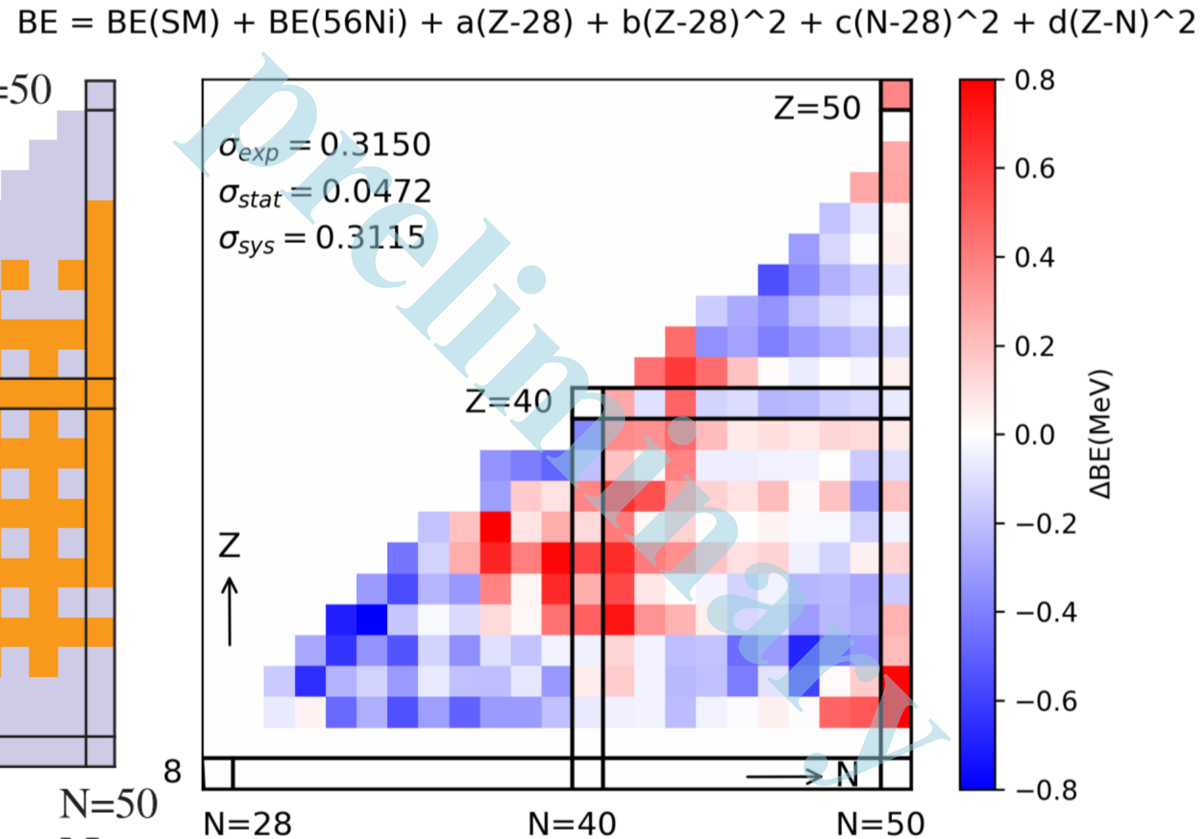
# • Full model space calculation:

– 2≤Z,N≤8; 8≤Z,N≤20; 20≤Z,N≤28

– 28≤Z,N≤50? JUN45 interaction dimension~$10^{10}$




M Honma *et al.*, PRC, 80, 064323 (2009)




CXY, *et al.*, in preparation

# The effective interaction between nucleons deduced from nuclear spectra*

John P. Schiffer

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William W. True

*University of California, Davis, California 95616*

**Configuration mixing**

**Semi magic T=1**

**C10+C11+LS+T**

**Uncertainty of C10 and C11**

Two-body matrix elements of the residual nucleon–nucleon interaction are extracted from experimental data throughout the periodic table and are used to determine the ranges and well depths of various components of a local interaction. The $T=1$ even and odd components of the central interaction both definitely require two wells with different ranges; a shorter-range attractive well with a longer-range repulsive one. The need for a tensor interaction and a two-body spin–orbit interaction is also explored and their inclusion improves the fit slightly.

$$V_{\rm CEN} = U_{\rm SO}(r)P^{\rm SO} + U_{\rm SE}(r)P^{\rm SE} + U_{\rm TO}(r)P^{\rm TO} + U_{\rm TE}(r)P^{\rm TE} ,$$

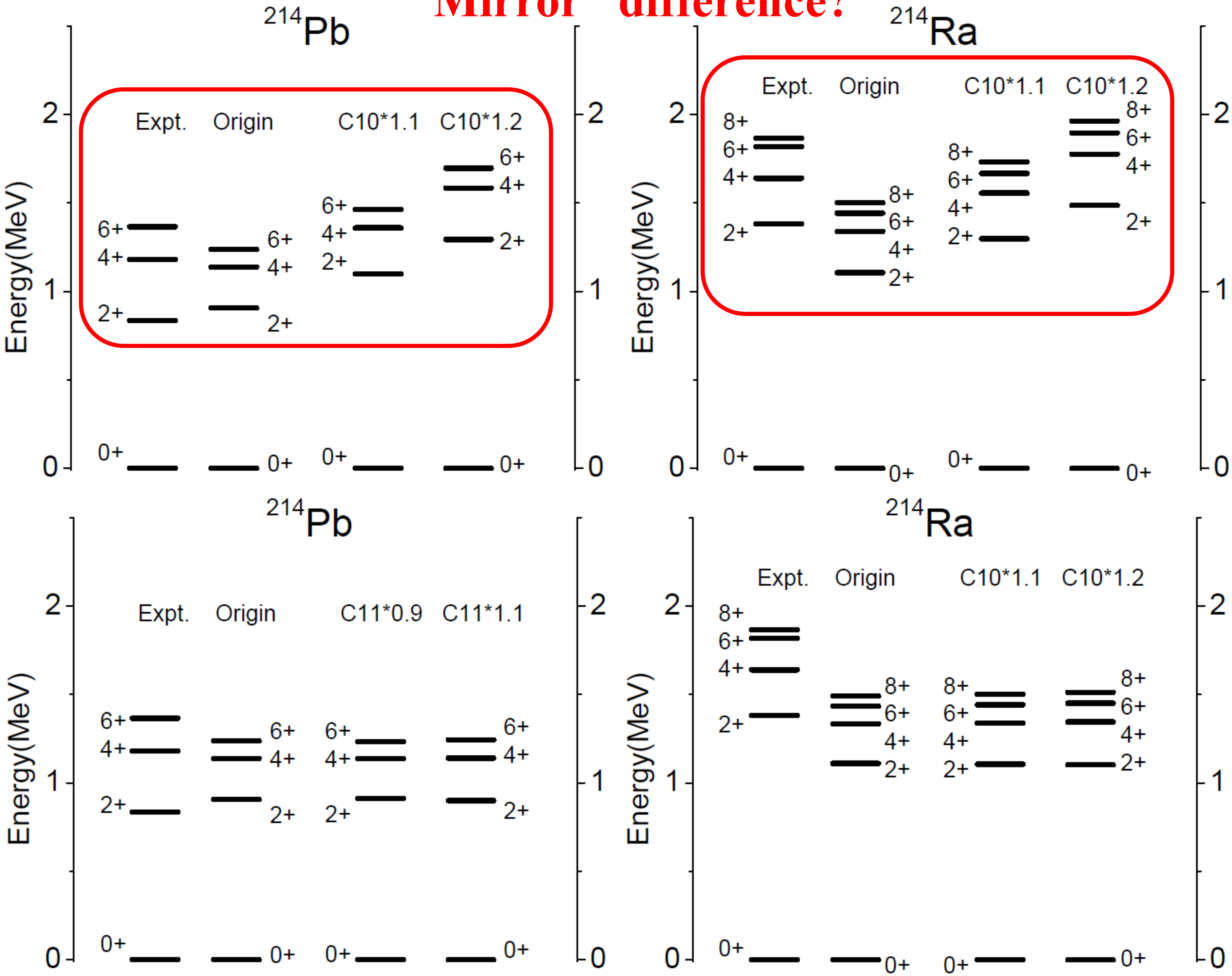
$$V_{\rm LS} = U_{\rm LS}(r)\vec{L}\cdot\vec{S}$$

$$V_{\rm Tensor} = U_{\rm Tensor}(r)\left[\frac{3(\vec{\sigma}_1\cdot\vec{r})(\vec{\sigma}_2\cdot\vec{r}) - r^2(\vec{\sigma}_1\cdot\vec{\sigma}_2)}{r^2}\right]$$

TABLE XV. Typical errors in the interaction strengths in MeV with $r_1 = 1.415$ fm and $r_2 = 2.0$ fm for a 1% change in $\chi^2$.

| $T=0$ S. O.$_A$ | S. O.$_B$ | Tr. E.$_A$ | Tr. E.$_B$ | Tens. E. | LS. E. |
|---|---|---|---|---|---|
| 3.7 ±9 | 122 ±73 | −56 ±2 | −63 ±20 | −43 ±10 | −0.4 ±2 |
| $T=1$ S. E.$_A$ | S. E.$_B$ | Tr. O.$_A$ | Tr. O.$_B$ | Tens. O. | LS. O. |
| −13.5 ±1.2 | −36 ±13 | 15.2 ±1.1 | −171 ±12 | −6.1 ±1.4 | 3.4 ±0.8 |

# “Mirror” difference?

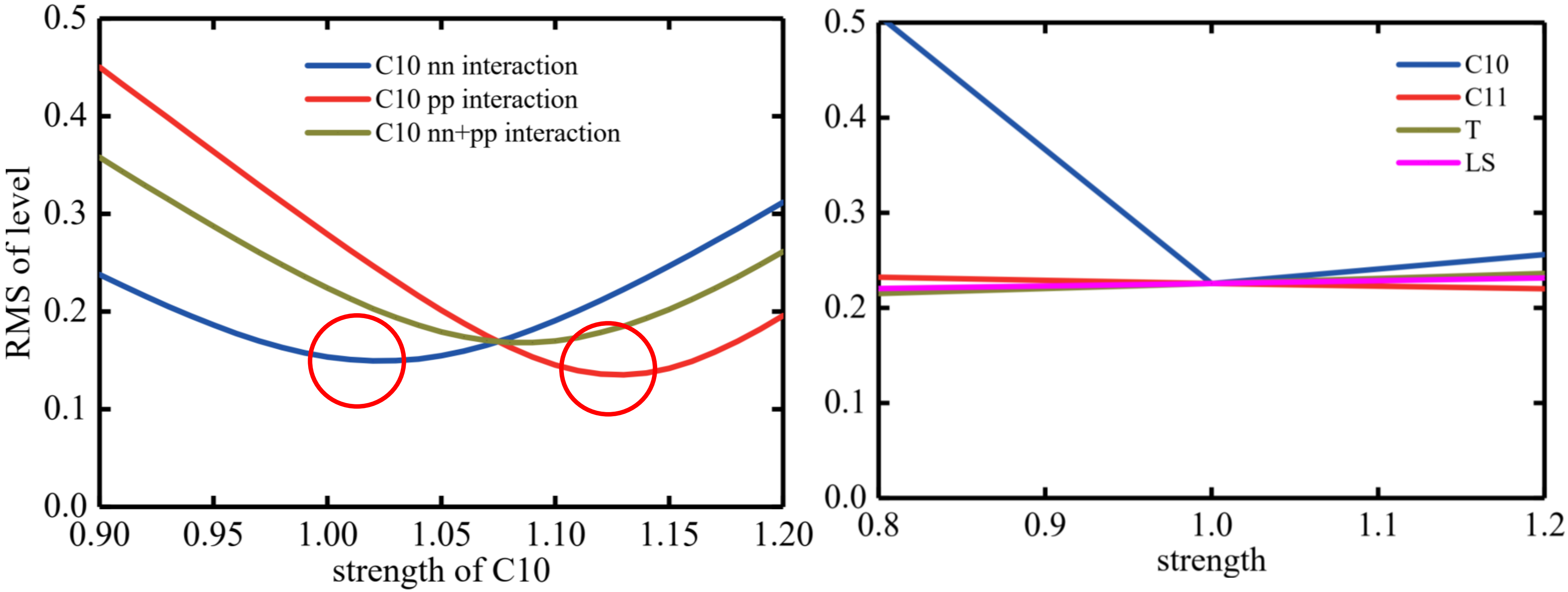



CXY, TX Du, NPR35.4(2018)537

# RMS from nuclei around $^{208}$Pb and $^{132}$Sn

**nn: 101 levels in Pb and Sn isotopes, $^{201-214}$Pb, $^{125-138}$Sn**

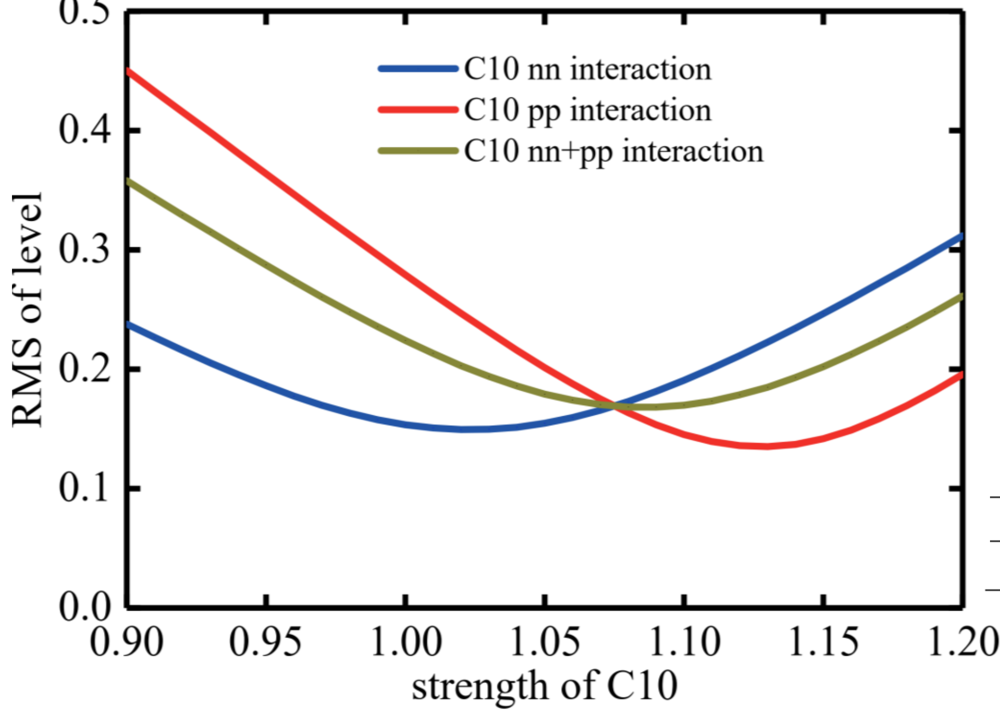
**pp: 97 levels in N=126 and N=82 isotones, $^{204}$Pt-$^{214}$Ra, $^{128}$Pd-$^{139}$La**




**Different pp and nn interaction for C10 channel**

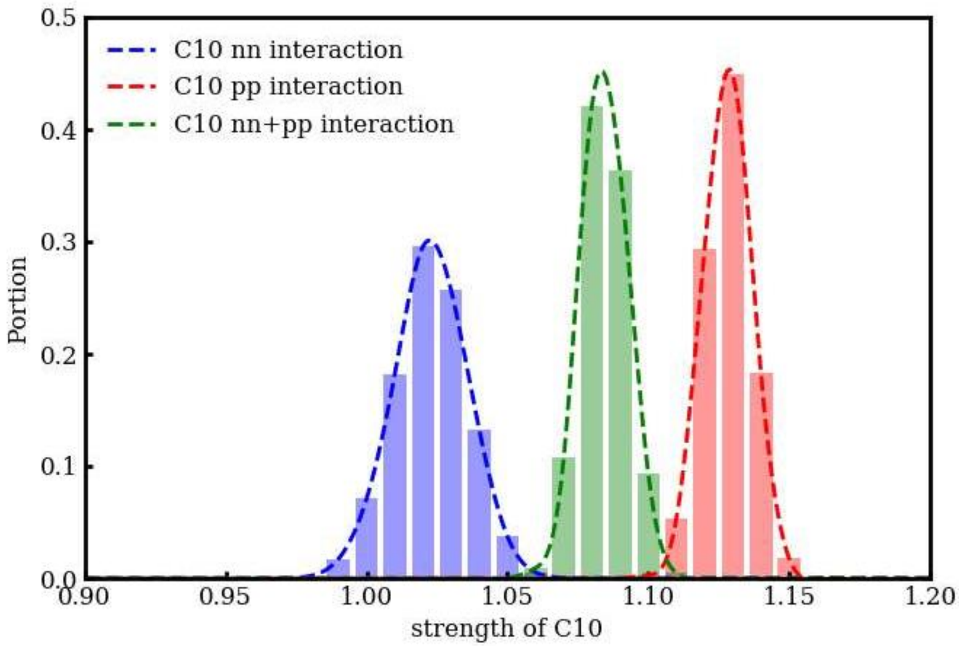
**Few effect from C11, T, and LS channel**

**CXY, TX Du, NPR35.4(2018)537 and CXY, et al., ND2019 proceedings**



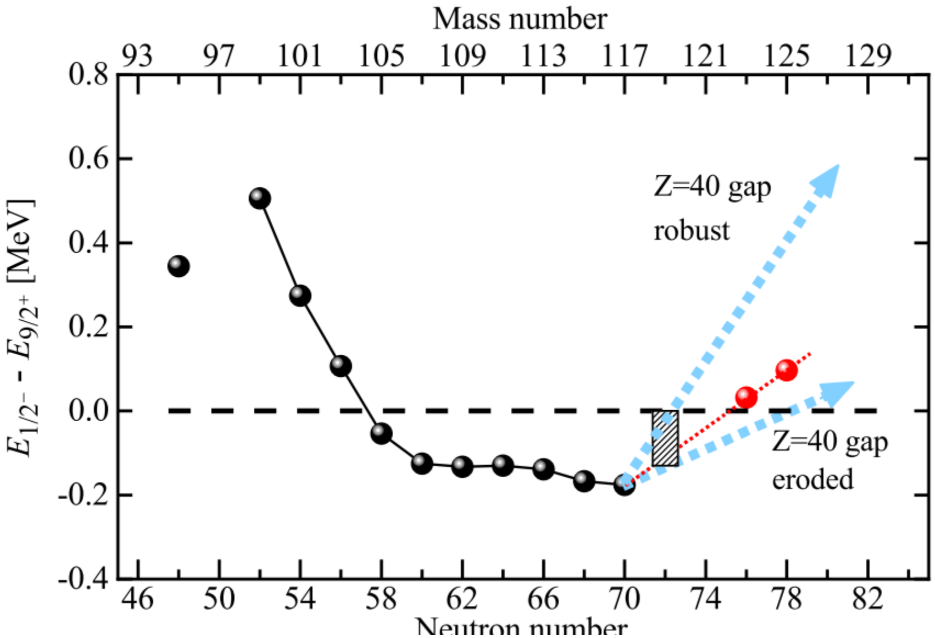

Comparing with 88 levels in 15 nuclei, the RMS of levels are 0.17, 0.27, and 0.47 for $V_{MU}$+LS, jj45pnm, and jj45pna, respectively. Simple nuclear force $V_{MU}$+LS gives

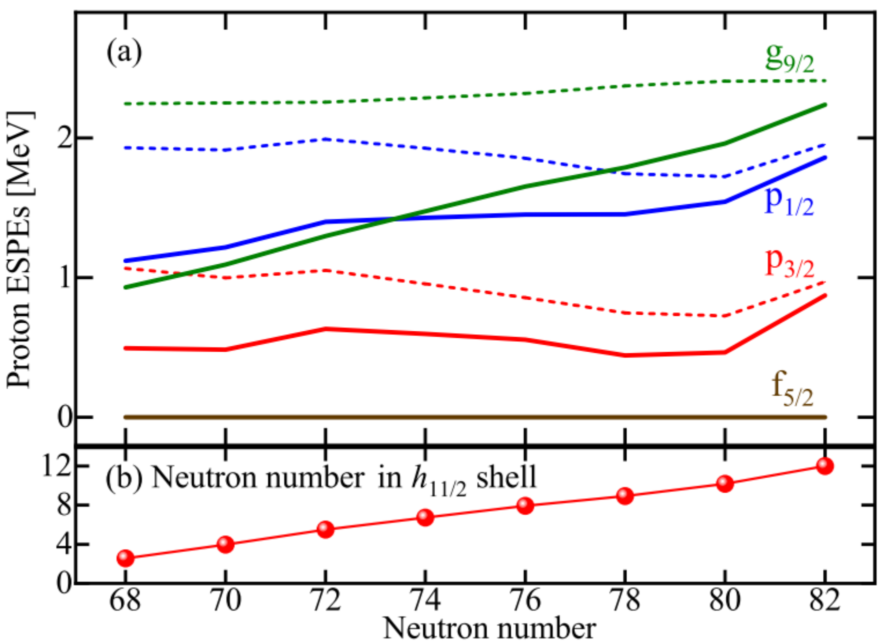
| nucleus | Jπ | Expt. | $V_{MU}$+LS | jj45pnm | jj45pna |
|---|---|---|---|---|---|
| RMS | | | 0.17 | 0.27 | 0.47 |

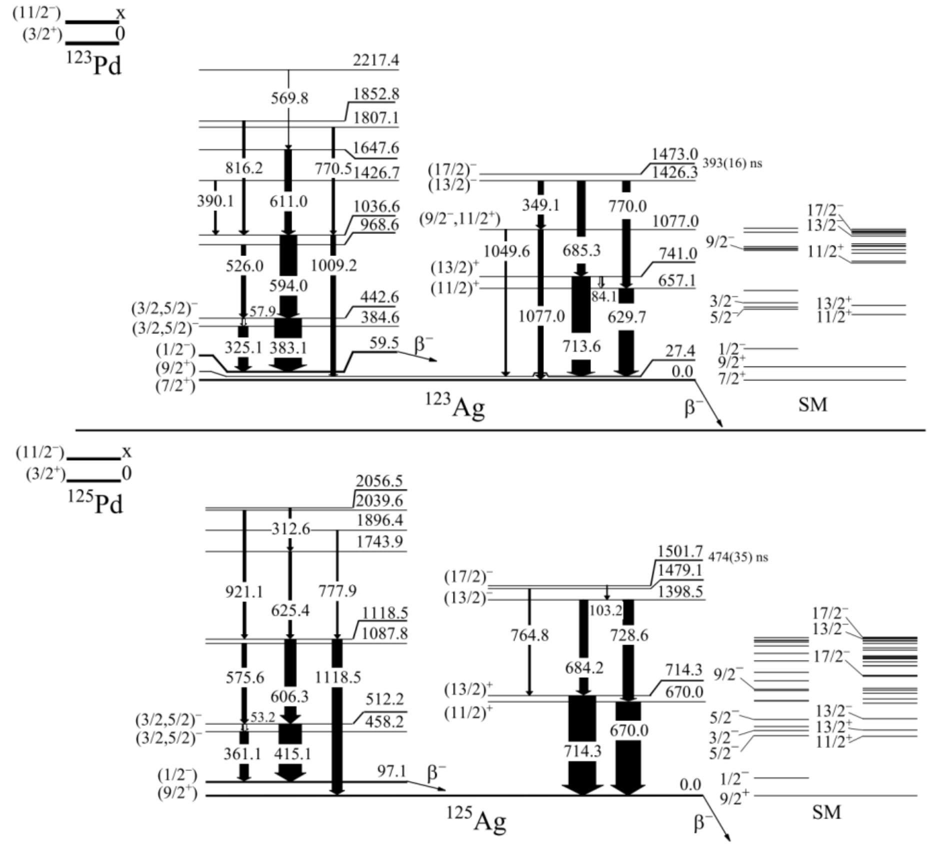



## Uncertainty of strength of C10 from bootstrap method

**CXY, et al., ND2019 proceedings**




**Such simple nuclear force give both nice description on levels, ESPE, and the mechanism of shell evolution**

**ZQ Chen, et al., PRL, 122, 212502 (2019)**

# Summary and perspective

- **Summary and perspective**
  - **Systematic study for proton- and neutron-rich nuclei from light to heavy nuclei**
  - **More data and more computational power! Uncertainty of shell model from more than 1000 nuclei and more than 10000 spectroscopic properties, BE, levels, EM, GT and FF**

# Collaboration

**Furong Xu (PKU)**

**Chong Qi (KTH)**

**Takaharu Otsuka (U of Tokyo)**

**Toshio Suzuki (Nihon U)**

**Noritaka Shimizu (CNS)**

**Zhong Liu (IMP)**

**Phil Walker, Zsolt Podolyak (Surrey U)**

**Zhongzhou Ren, Chang Xu (Nanjing U)**

**Many others …**

# Thank You for Attending