



National Natural Science
Foundation of China



Tale of coherent photon products: from **UPC** to **HHIC**

Wangmei Zha

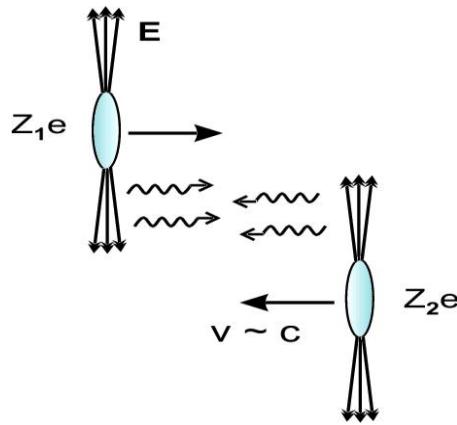
University of Science and Technology of China

Collaborators: Lijuan Ruan, Zebo Tang, Zhangbu Xu, Shuai Yang, Chi Yang, Spencer Klein, Rongrong Ma, Qian Yang, Zhen Liu, Zehua Cao

第十七届全国核物理大会暨第十三次会员代表大会
华中师范大学，武汉，2019年10月8-12日

Coherent photons as “partons” in heavy-ion collisions

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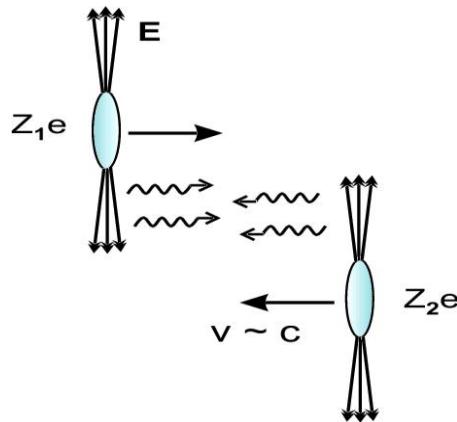
Coherent limitation: $Q^2 \leq 1/R^2 \Rightarrow$ quasi-real !

Photon four momentum: $q^u = (\omega, \vec{q}_T, \omega/v)$

$$Q^2 = \frac{\omega^2}{\gamma^2} + q_T^2$$

$$\omega \leq \omega_{max} \sim \frac{\gamma}{R}$$

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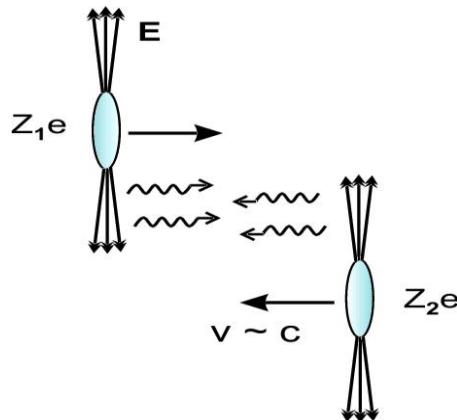
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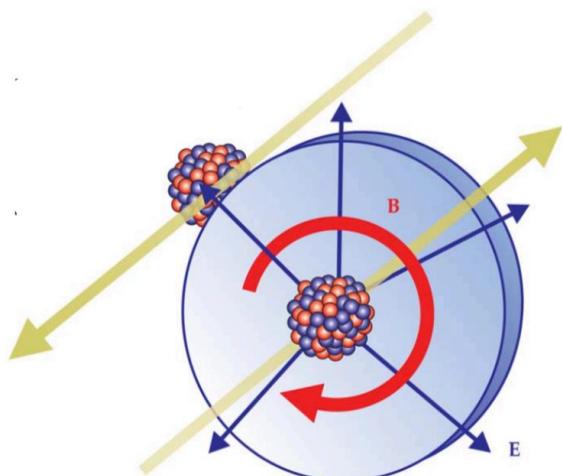
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- View photons as “partons” being present with fast moving ions!

The extent of photons swarming about the ions:

The radius of nuclear matter $R_{Nuc} \sim 6.3$ fm (Au)
 $R_{photons} \gg R_{Nuc}$

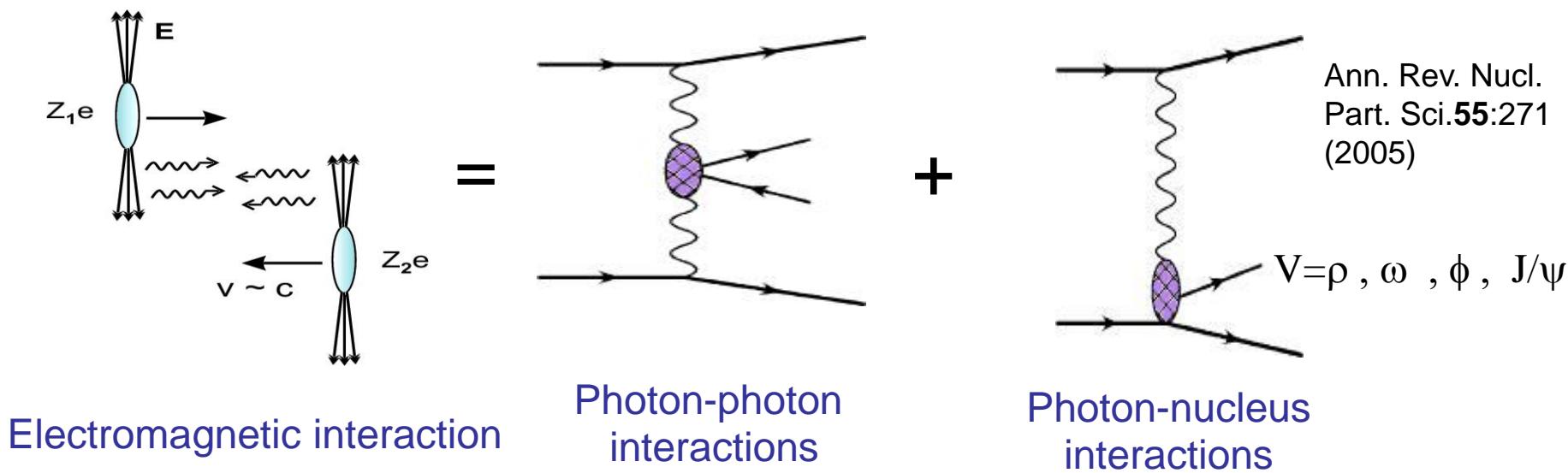


Take the photoproduction of ρ (Au+Au 200 GeV) in ultra-peripheral collisions (UPCs) as example: $\langle R_{production} \rangle \sim 40$ fm

Physics Today 70, 10, 40 (2017)

Photon interactions in A+A

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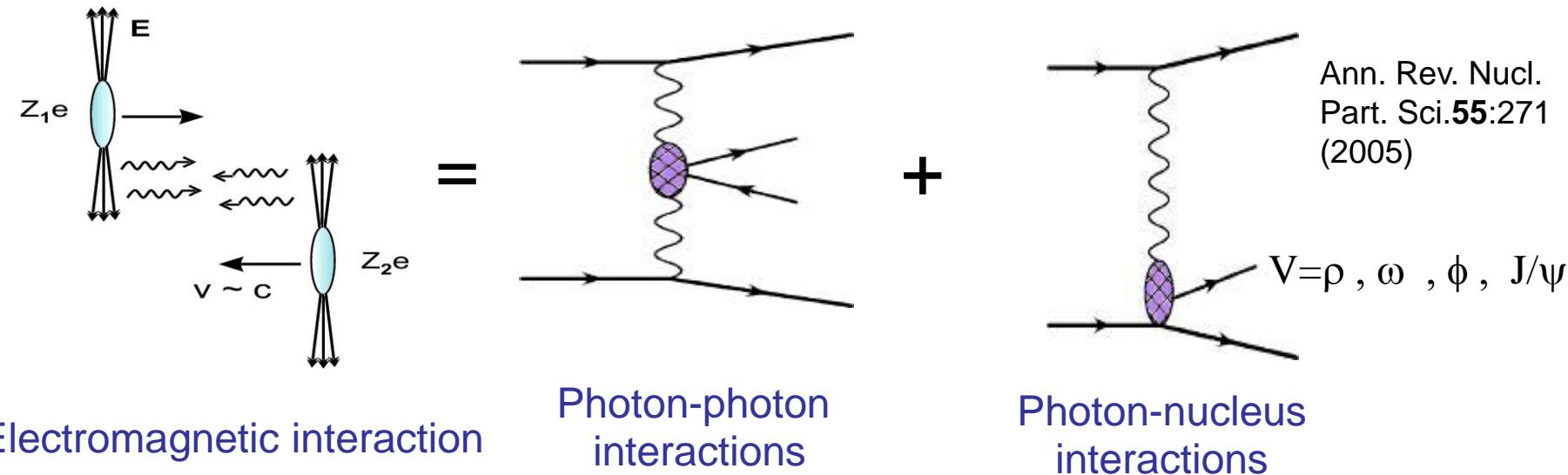


Electromagnetic interaction

Photon-photon
interactions

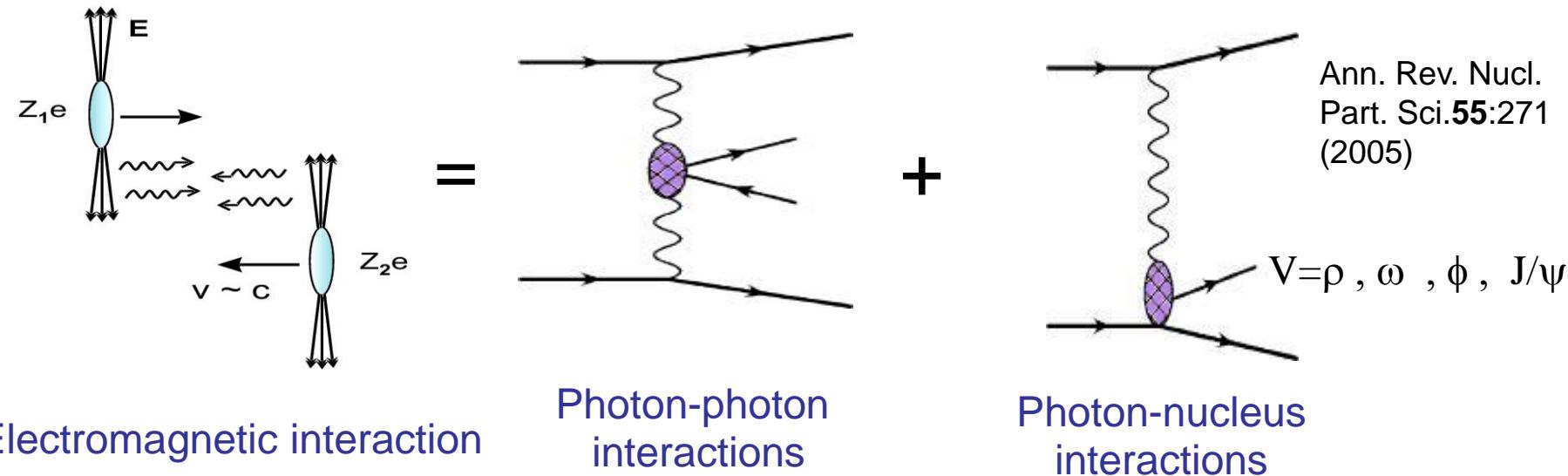
Photon-nucleus
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Photon interactions in A+A



- This large flux of quasi-real photons makes a hadron collider also a photon collider!
 - ✓ Photon-nucleus interactions: Vector meson
 - ✓ Photon-photon interactions: dileptons ...

Photon interactions in A+A



Electromagnetic interaction

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Photon-nucleus interactions

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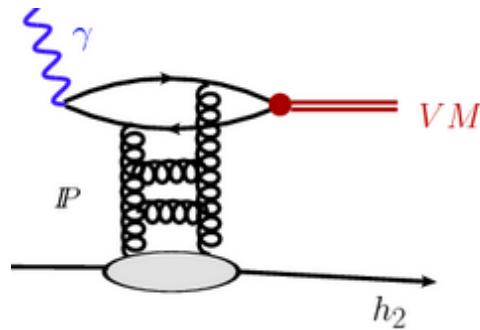
- Conventionally believed to be only exist in ultra-peripheral collisions (UPC) to keep “coherent”!

Vector meson photon-production

Vector meson photon-production

- Vector meson production:

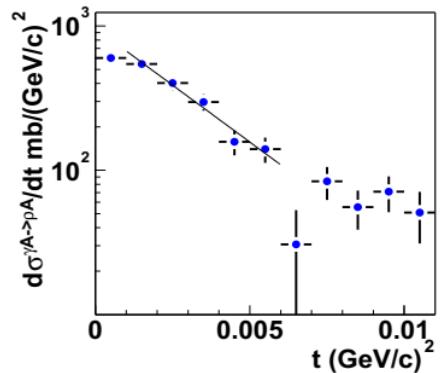
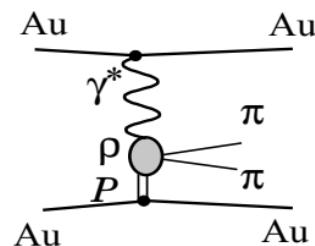
- ✓ chargeless ‘Pomeron exchange’
- ✓ Light meson production is usually treated via vector meson dominance model:
 ρ , direct $\pi^+\pi^-$, ω
- ✓ Heavy quarkonia production could be treated with pQCD :
 J/ψ , ψ' , $Y(1S)$, $Y(2S)$, $Y(3S)$...



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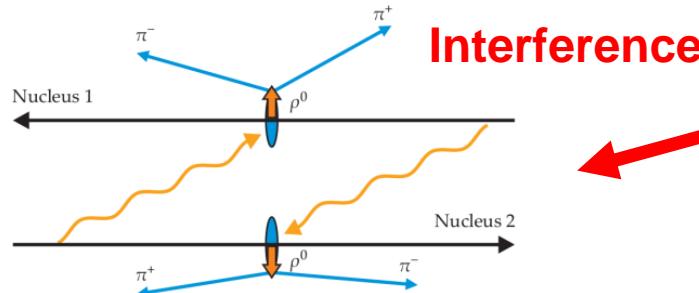


STAR, PRL 89 (2002) 272302

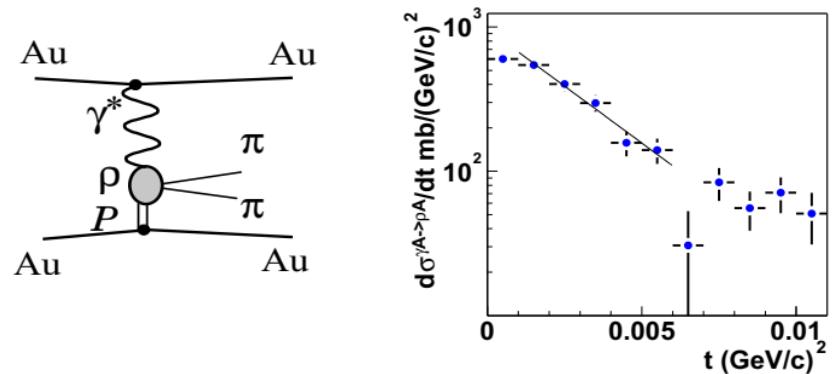
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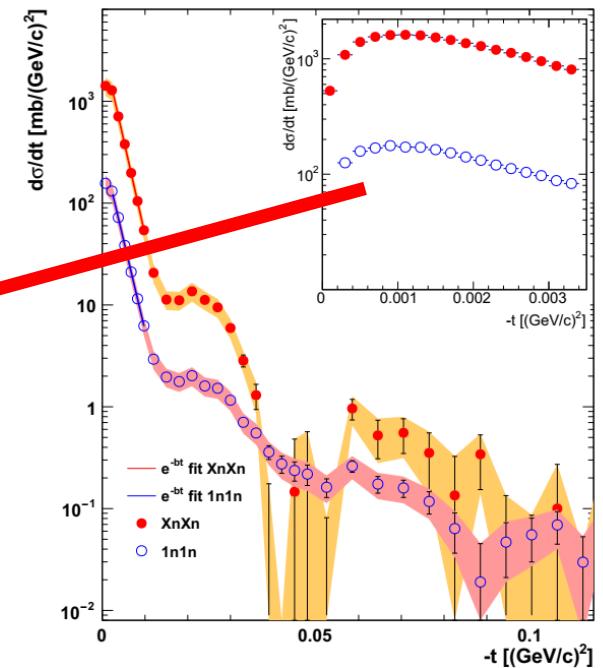
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STAR, PRC 96, (2017) 054904



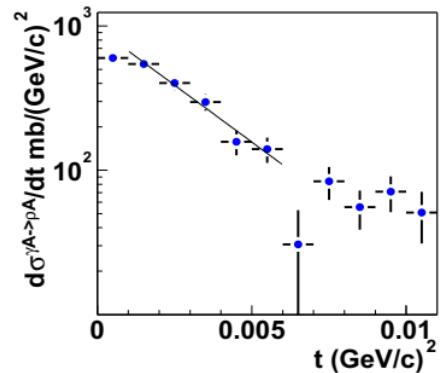
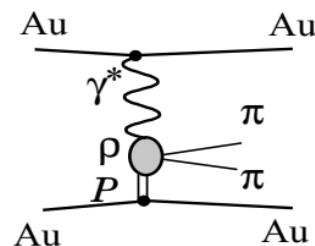
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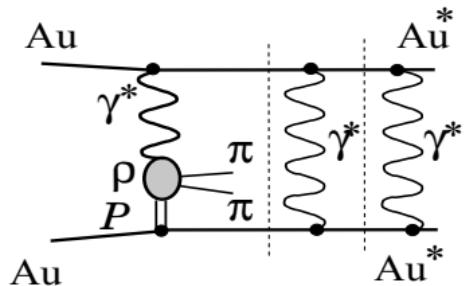
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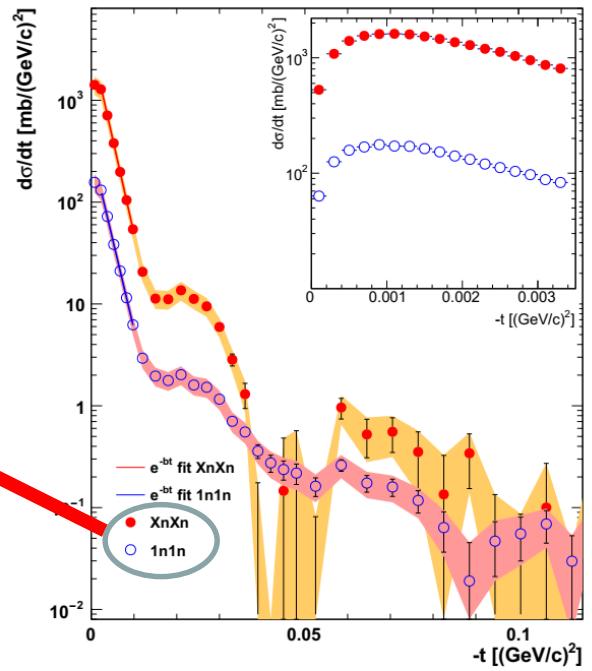


STAR, PRL 89 (2002) 272302



STAR, PRC 96, (2017) 054904

When the nucleus break, coherent photoproduction can still occur!



Vector meson photon-production

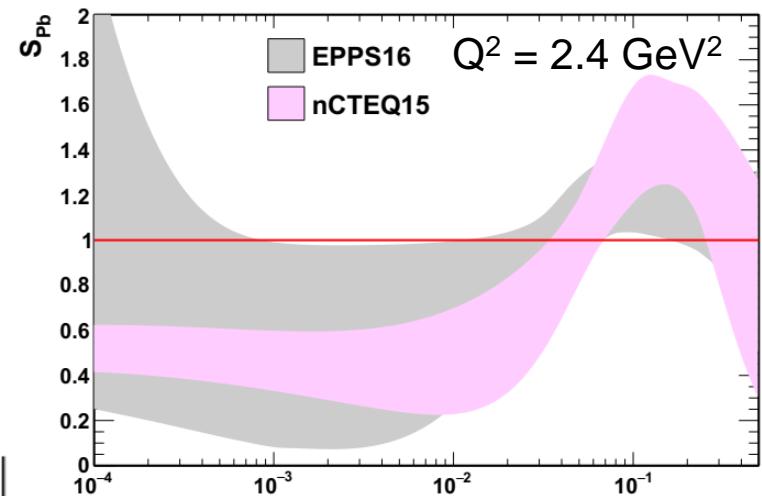
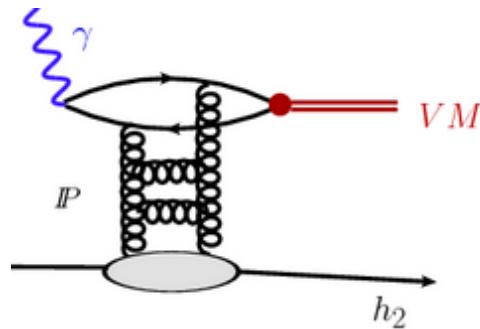
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- Sensitive to the gluon distribution:

$$\frac{d\sigma(\gamma A \rightarrow VA)}{dt} \Big|_{t=0} = \frac{\alpha_s^2 \Gamma_{ee}}{3\alpha M_V^5} 16\pi^3 [xG_A(x, Q^2)]$$

$$x = \frac{M_V e^{\pm y}}{\sqrt{s}} \quad Q^2 = M_V^2/4$$

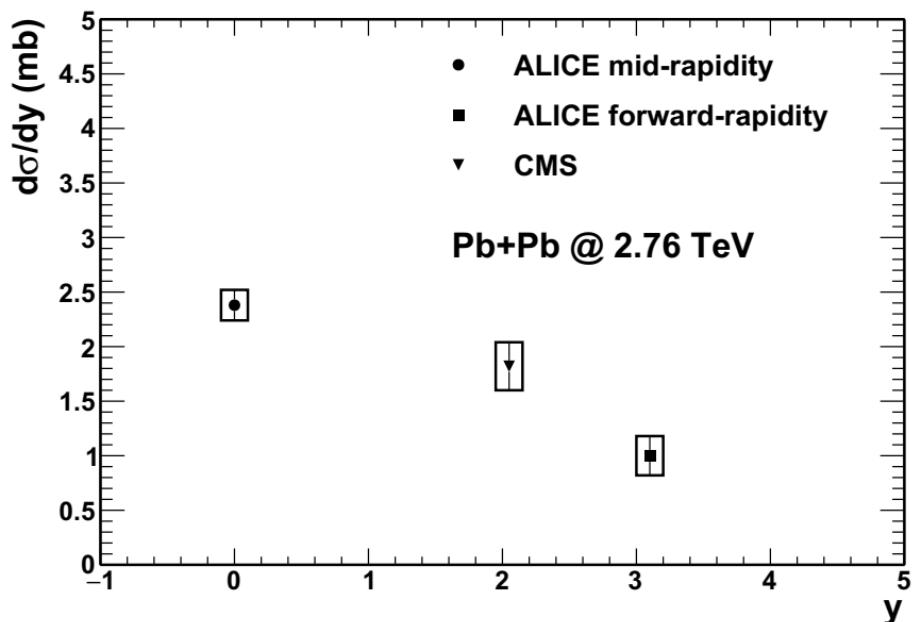


EPPS16: EPJC **77** (2017) 163

nCTEQ15: PRD **93** (2016) 085037

Nuclear shadowing from J/ ψ measurements in UPCs

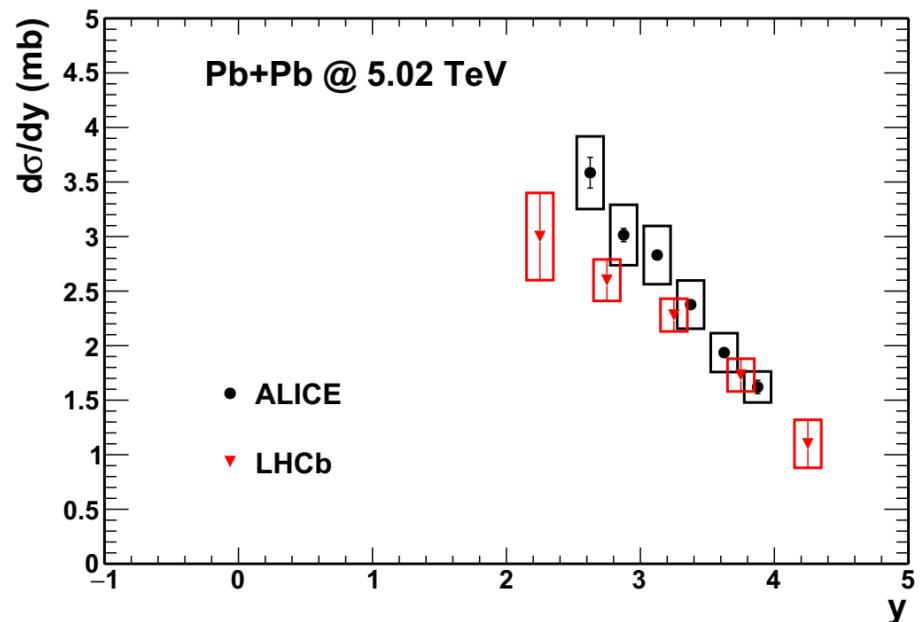
Nuclear shadowing from J/ψ measurements in UPCs



ALICE: EPJC **73** (2013) 2617

ALICE: PLB **718** (2013) 1273

CMS: PLB **772** (2017) 489

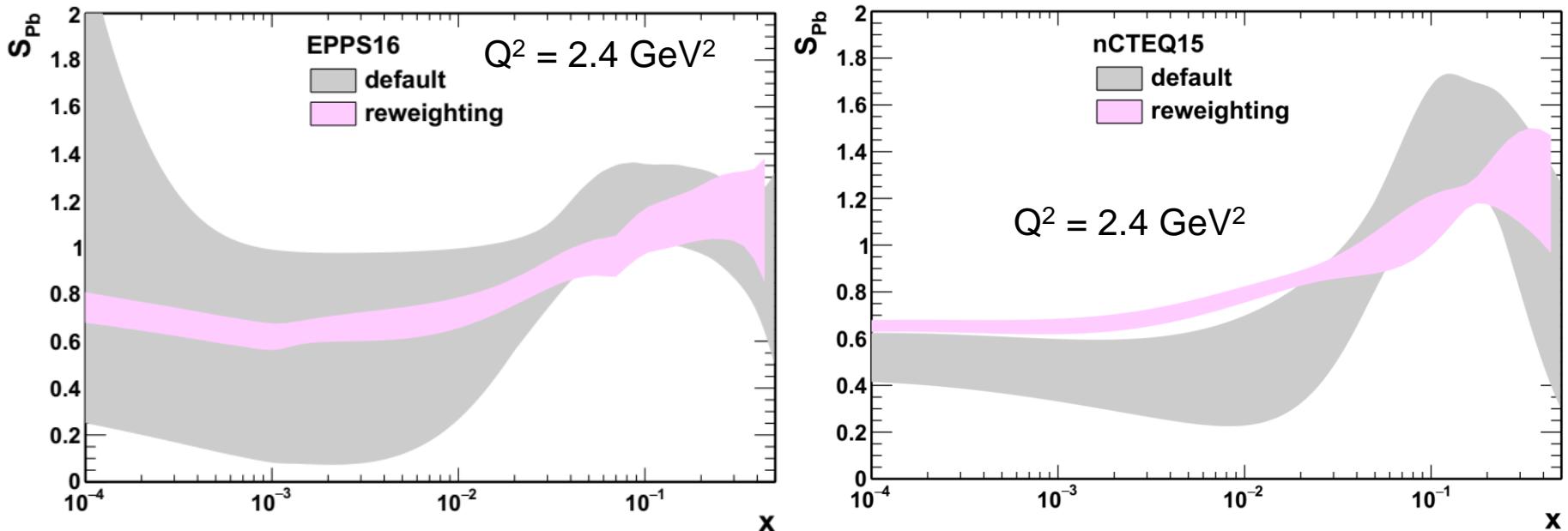


LHCb: LHCb-CONF-2018-003

ALICE: arXiv: 1904.06272

Various precise measurements!
Powerful to constrain nPDF

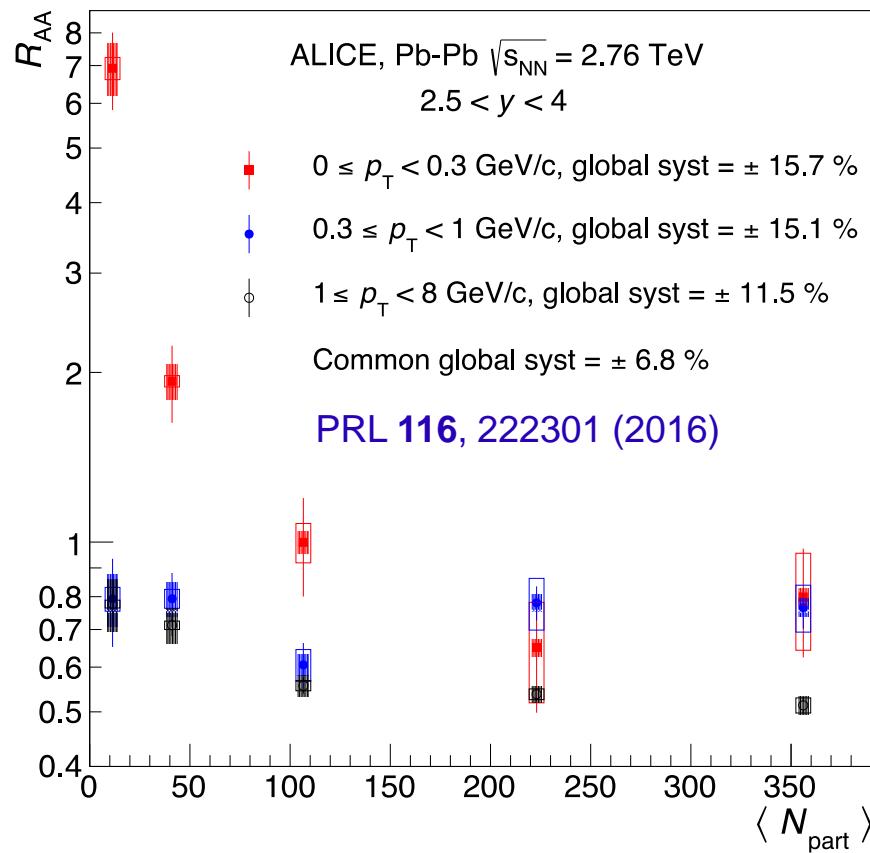
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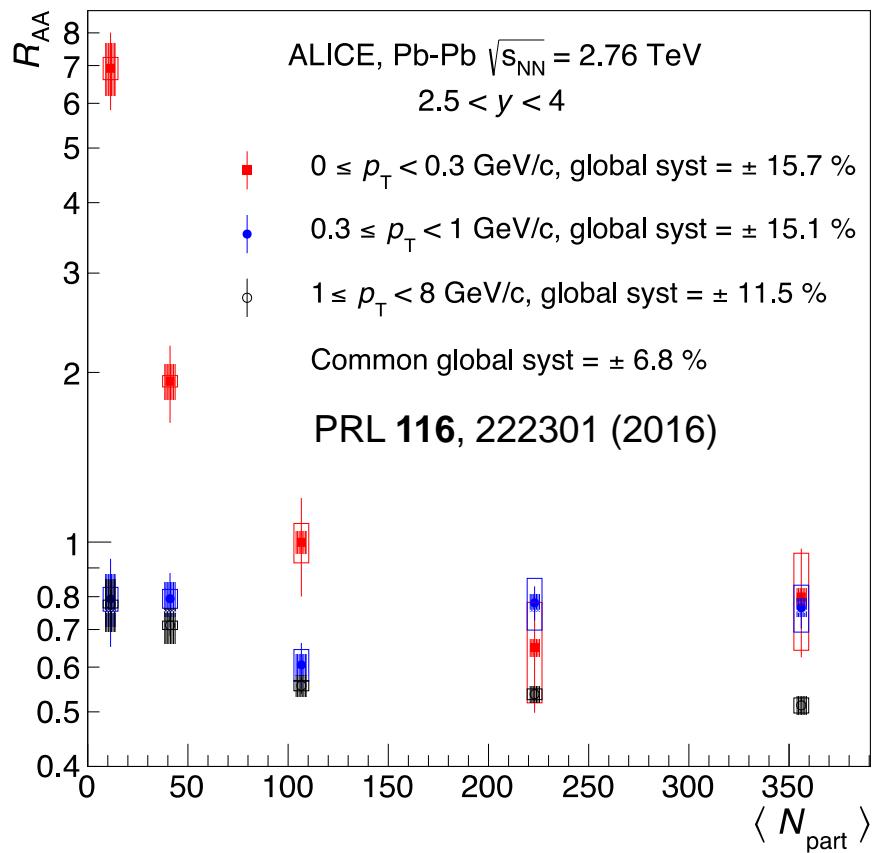
- The UPC measurements **dramatically reduce** the uncertainty band of EPPS16 and nCTEQ15 PDF sets.
- **Significant shadowing effect** has been observed in both PDF sets at small x .

Please see Zehua Cao's talk for detail
Aug. 19th Parallel I.5 15:00

Anomalous excess of J/ ψ production observed at ALICE

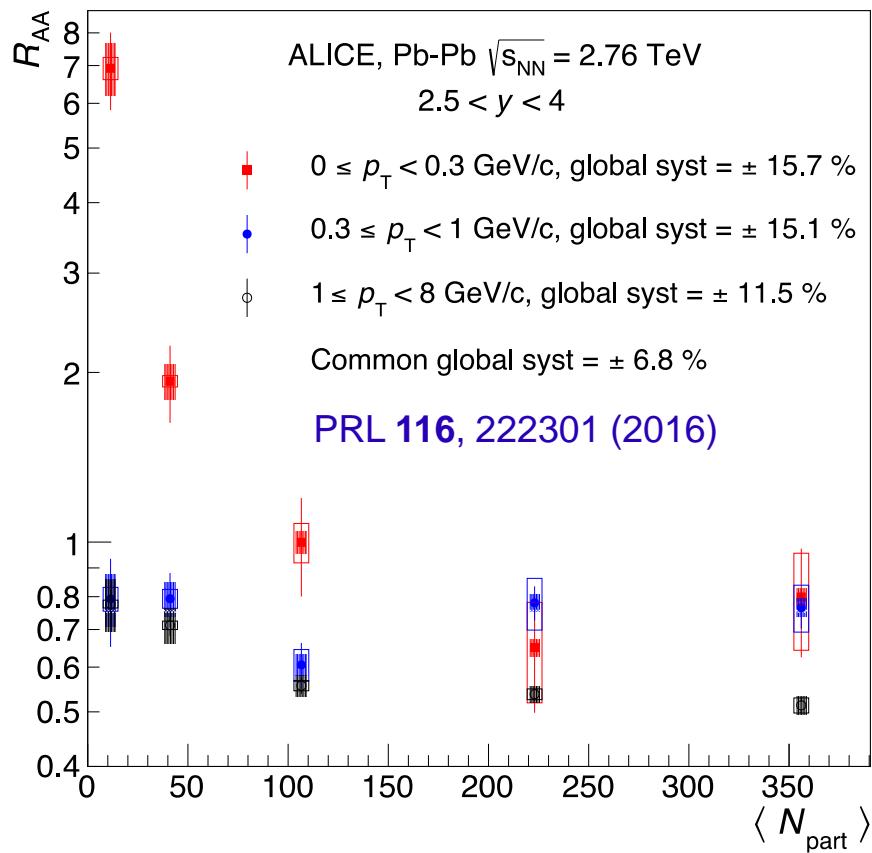


Anomalous excess of J/ ψ production observed at ALICE



- Significant enhancement of J/ψ yield observed in p_T interval 0 – 0.3 GeV/c for peripheral collisions (50 – 90%).
- Can not be described by hadronic production modified by the hot medium or cold nuclear matter effects!

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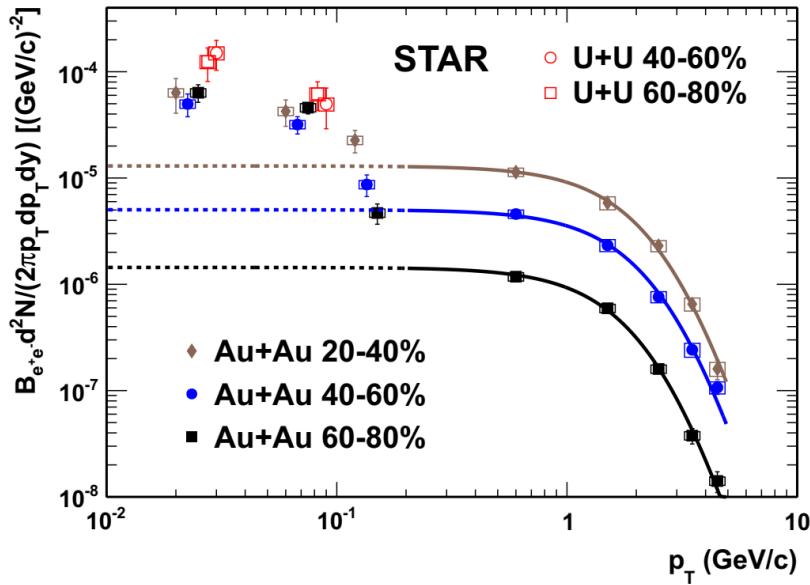
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- Origin from **coherent photon-nucleus interactions?**

What does STAR say for the excess?

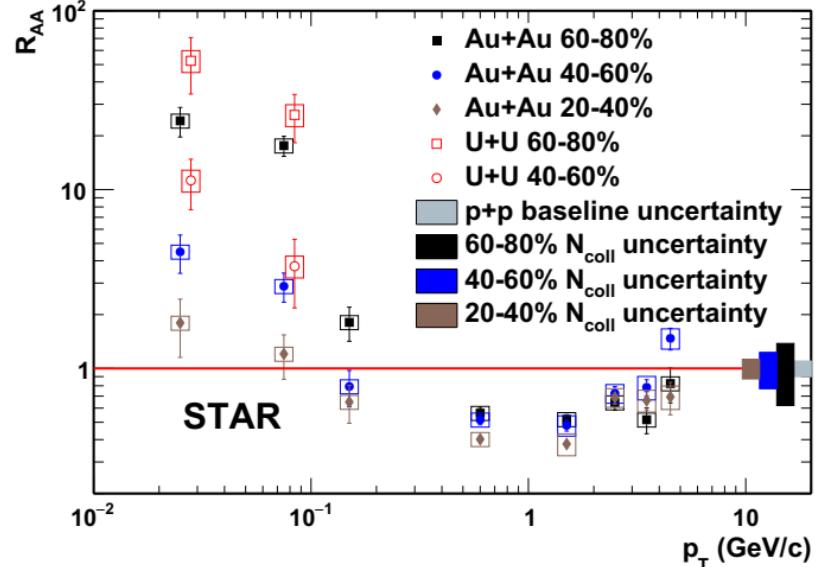
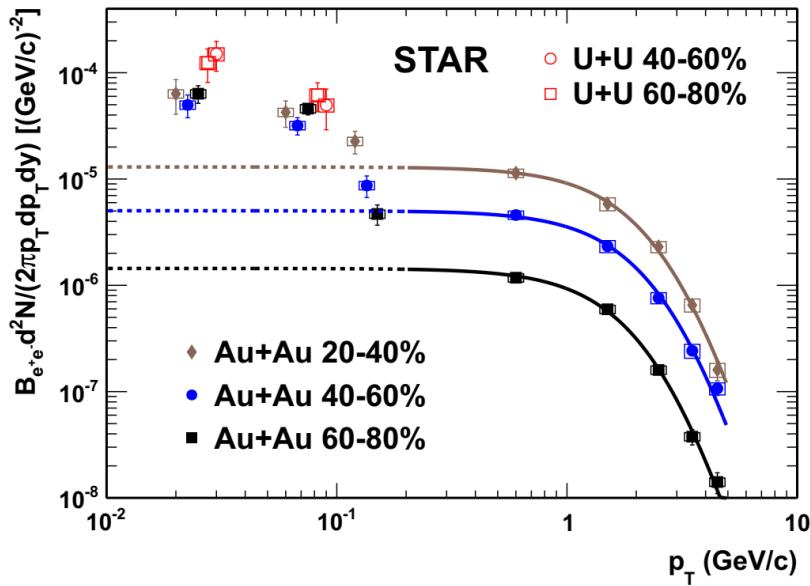
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PRL 123 (2019) 132302



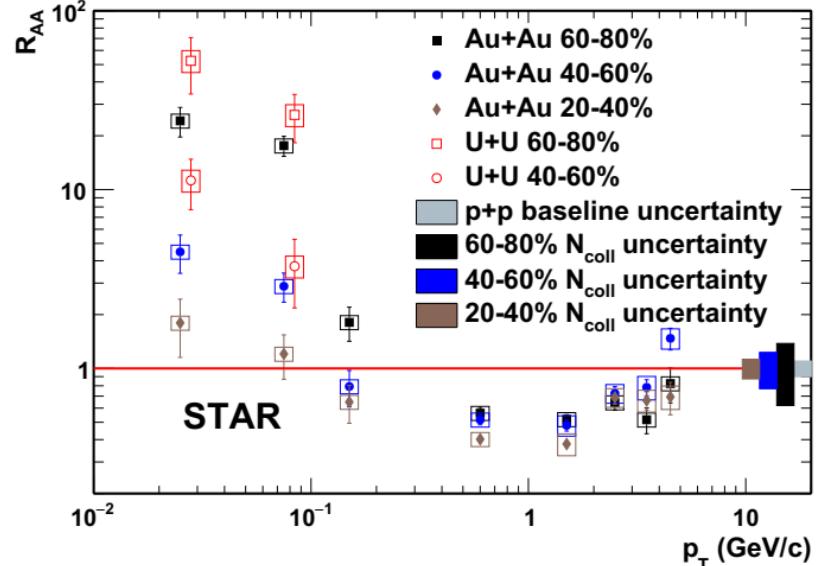
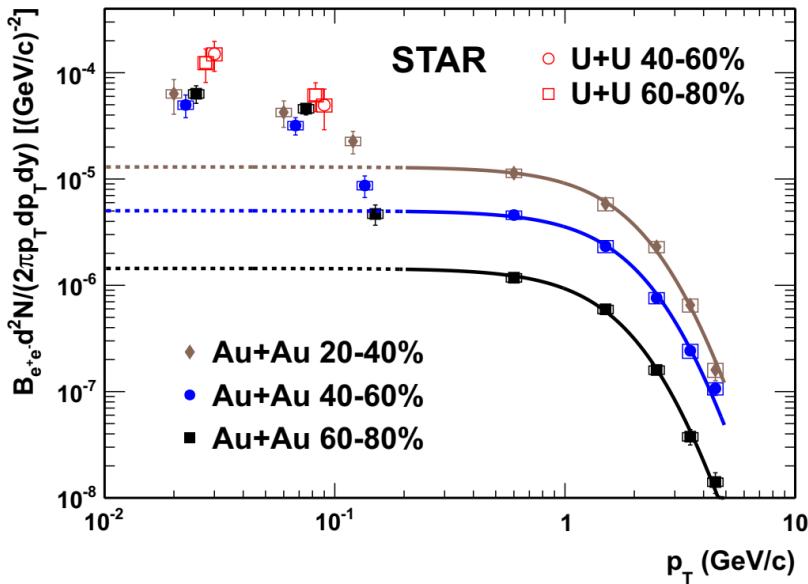
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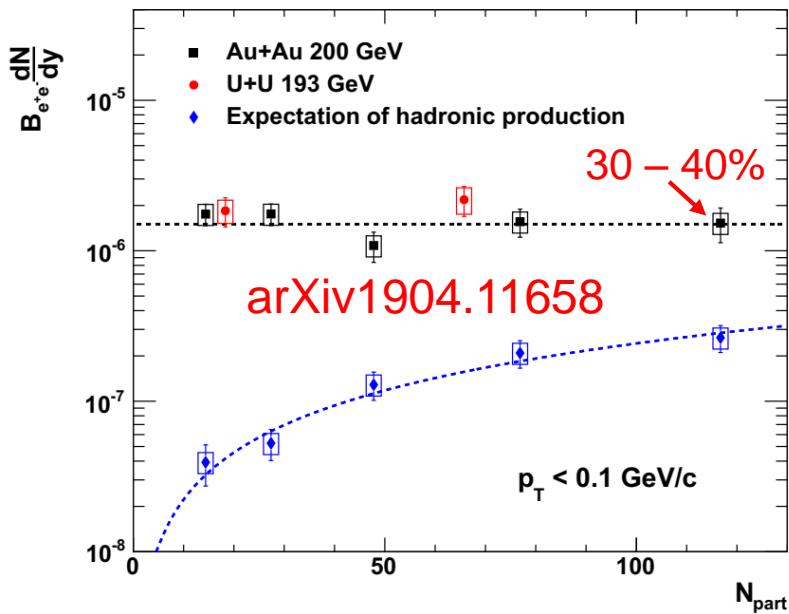
PRL 123 (2019) 132302



- Significant enhancement of J/ψ yield observed at p_T interval 0 – 0.2 GeV/c for peripheral collisions (40 – 80 %)!
- No significant difference between Au+Au and U+U collisions.

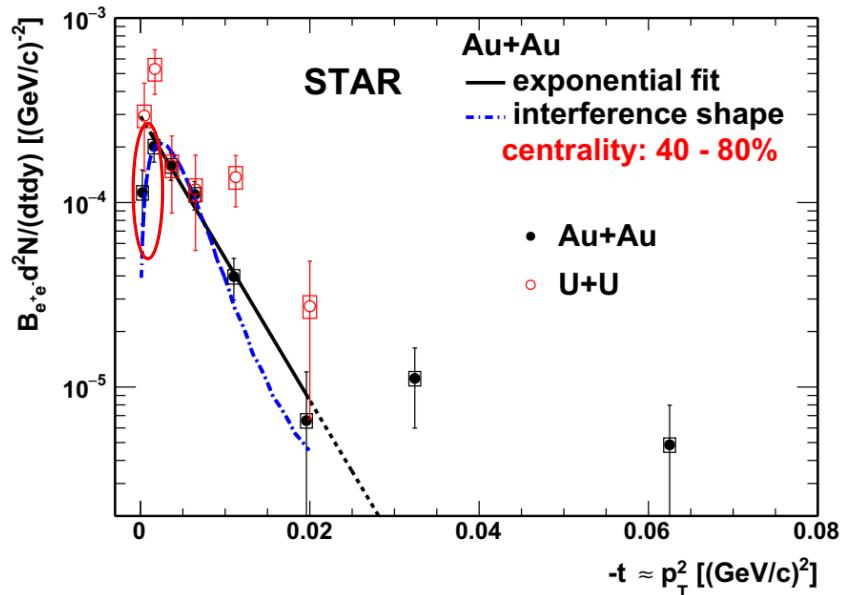
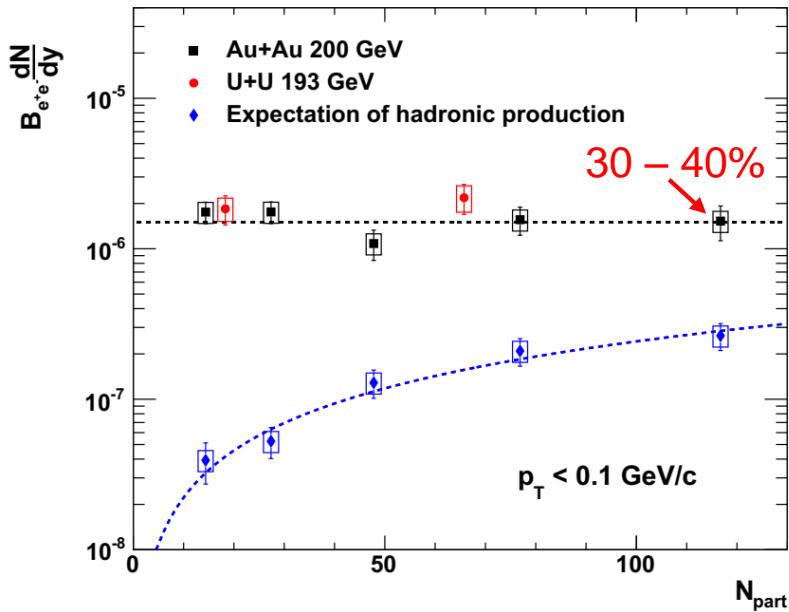
The excess yield and dN/dt distribution

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- No significant centrality dependence of the excess yield!

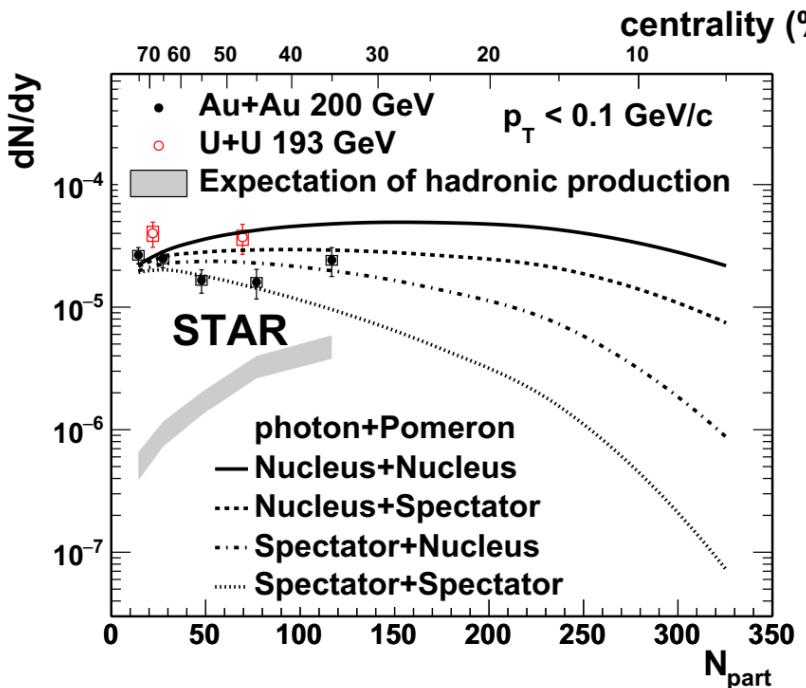
The excess yield and dN/dt distribution



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- Similar structure to that in UPC case!
- Indication of interference!
 - ✓ Interference shape from calculation PRC 97 (2018) 044910
- Similar slope parameter!
 - ✓ Slope from STARLIGHT prediction in UPC case $- 196 (\text{GeV}/c)^{-2}$
 - ✓ Slope w/o the first point: $177 \pm 23 (\text{GeV}/c)^{-2}$
 $\chi^2/NDF = 1.7/2$

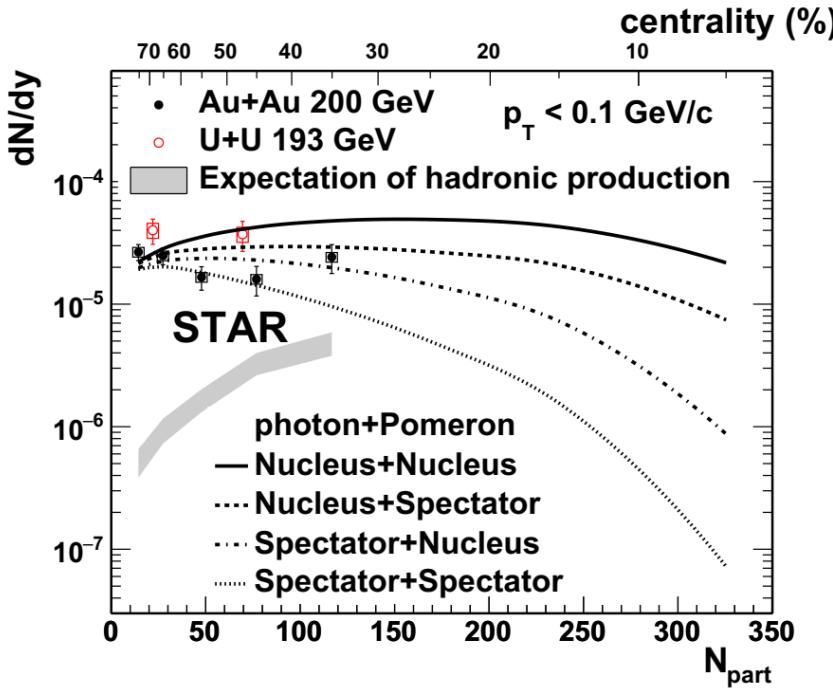
Comparison with theoretical calculations



W. Zha et al., PRC **97** (2018) 044910

W. Zha et al., PRC **99**, (2019) 061901(R)

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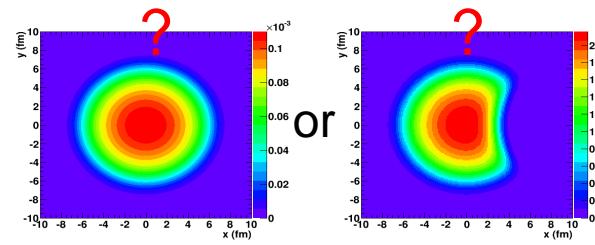
W. Zha et al., PRC **97** (2018) 044910
W. Zha et al., PRC **99**, (2019) 061901(R)

How coherence keep? --- Time scale matters

Collision (production) time > GeV

Fragment of spectator and nucleus excitation ~ MeV

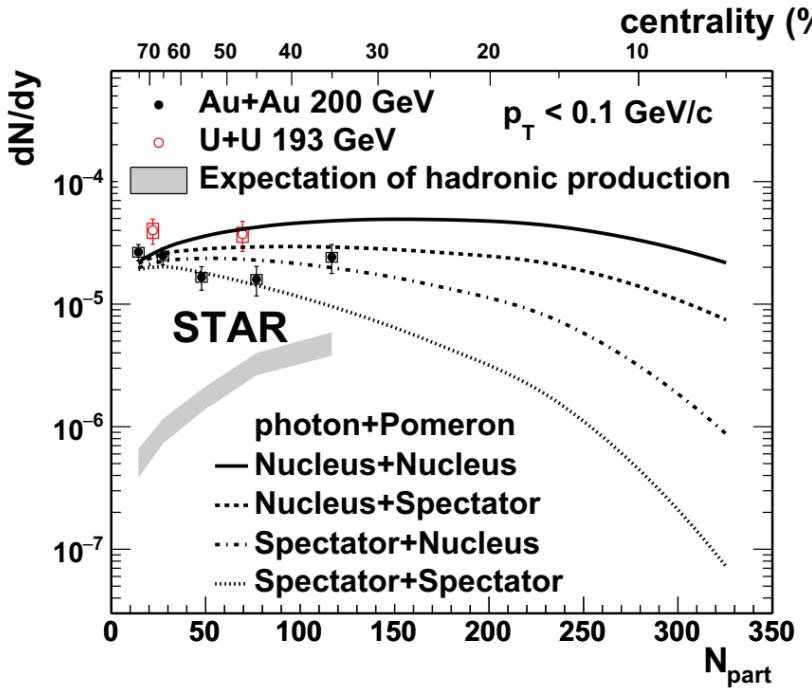
Photon emitter:



Pomeron emitter

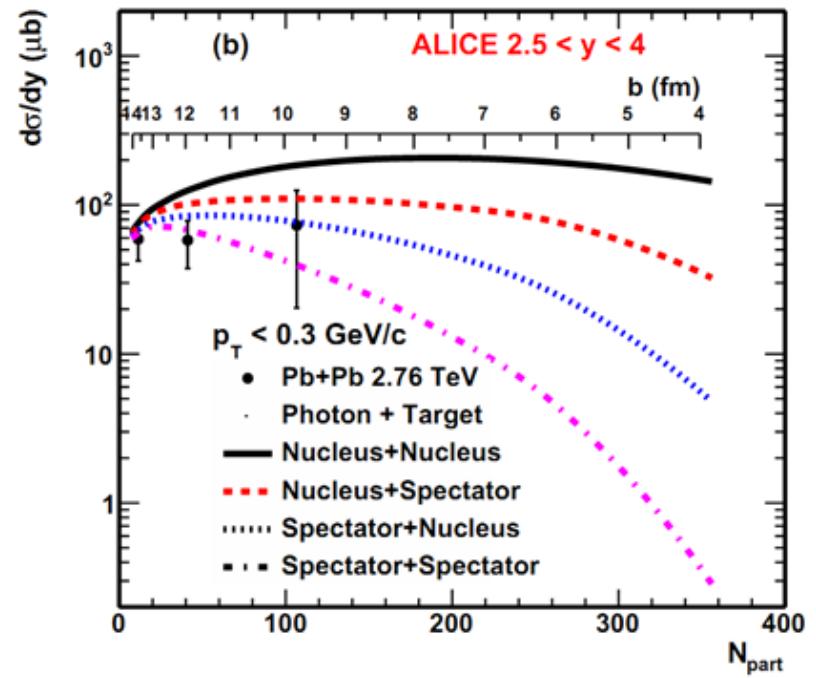
or

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W. Zha et al., PRC 97 (2018) 044910

W. Zha et al., PRC 99, (2019) 061901(R)



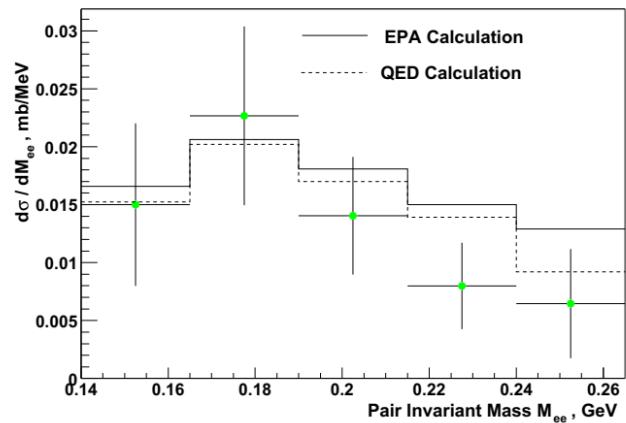
- All scenarios **describe** the data very well in peripheral collisions!
- Nuclues+Nucleus: **overestimate** the data in semi-central collisions.
- Spectator+Spectator: **under predict** the data in semi-central collisions.
- To distinguish the different scenarios, measurements at central collisions are needed!

Photon-photon interactions in UPC

Photon-photon interactions in UPC

● Test QED --- $\gamma\gamma \rightarrow$ Dileptons

- ✓ $Z\alpha \sim 0.6$, so perturbation theory might fail
- ✓ Data is in excellent agreement with lowest order QED



STAR, PRC 70 (2004) 031902

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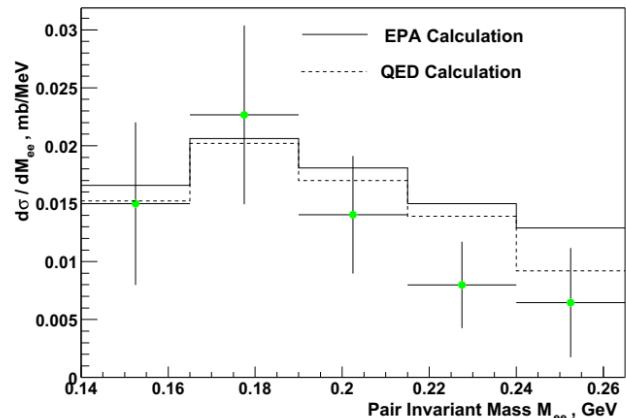
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meson	mass [MeV]	σ^{RHIC} [mb]	σ^{LHC} [mb]
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η	547	1.0	16
η'	958	0.75	21
$f_2(1270)$	1275	0.54	22
$a_2(1320)$	1318	0.19	8.2
η_c	2981	3.3×10^{-3}	0.61
χ_{0c}	3415	0.63×10^{-3}	0.16
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No measurements
in UPCs yet!

Ann. Rev. Nucl.
Part. Sci. **55**:271
(2005)



STAR, PRC **70** (2004) 031902

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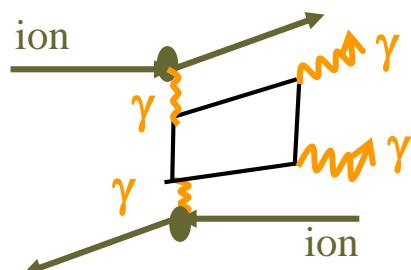
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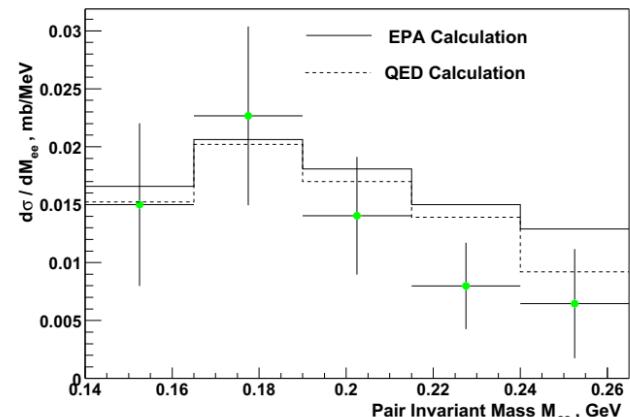
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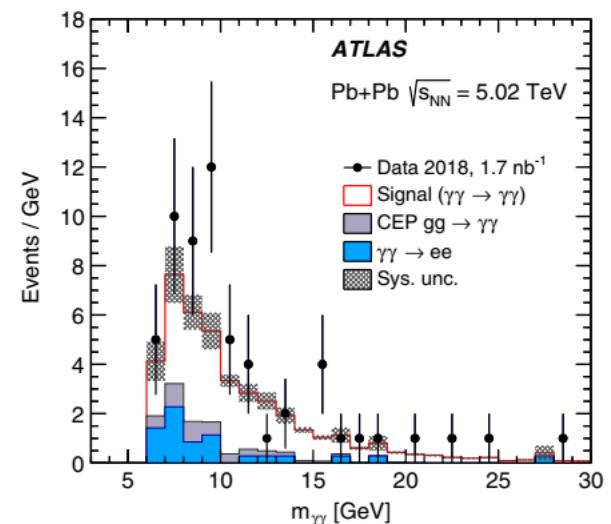


- From “virtual” to “real”

- ✓ Light-by-light scattering seen by ATLAS

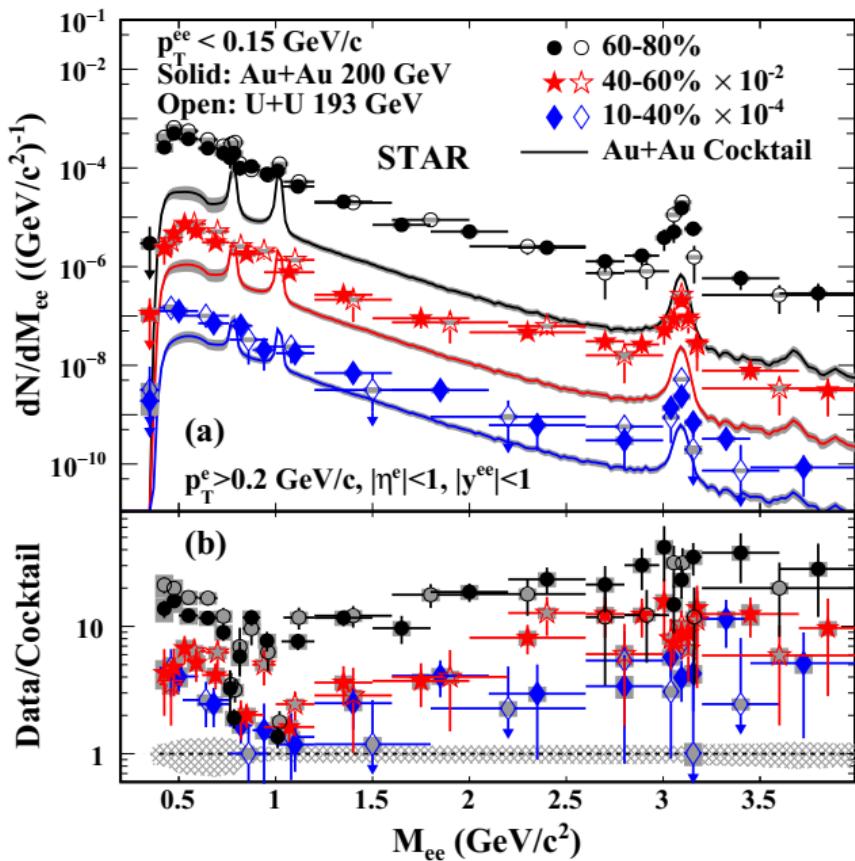


STAR, PRC **70** (2004) 031902



Nature Phys. 13 (2017) 852
PRL 123 (2019) 052001

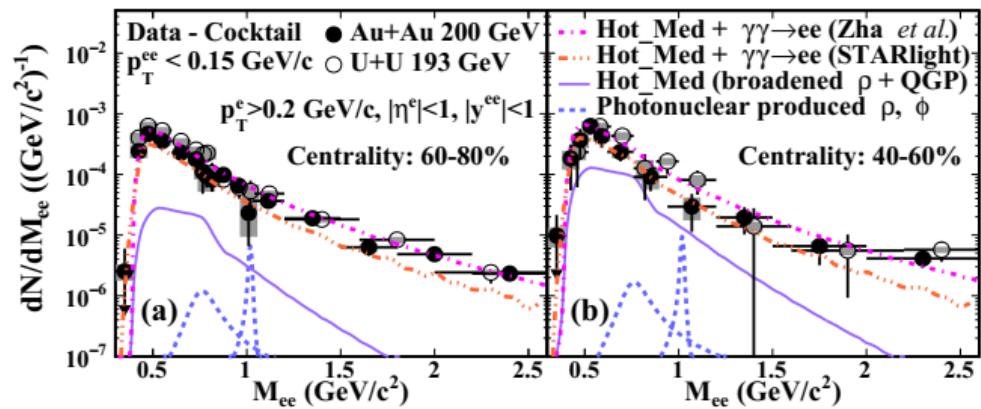
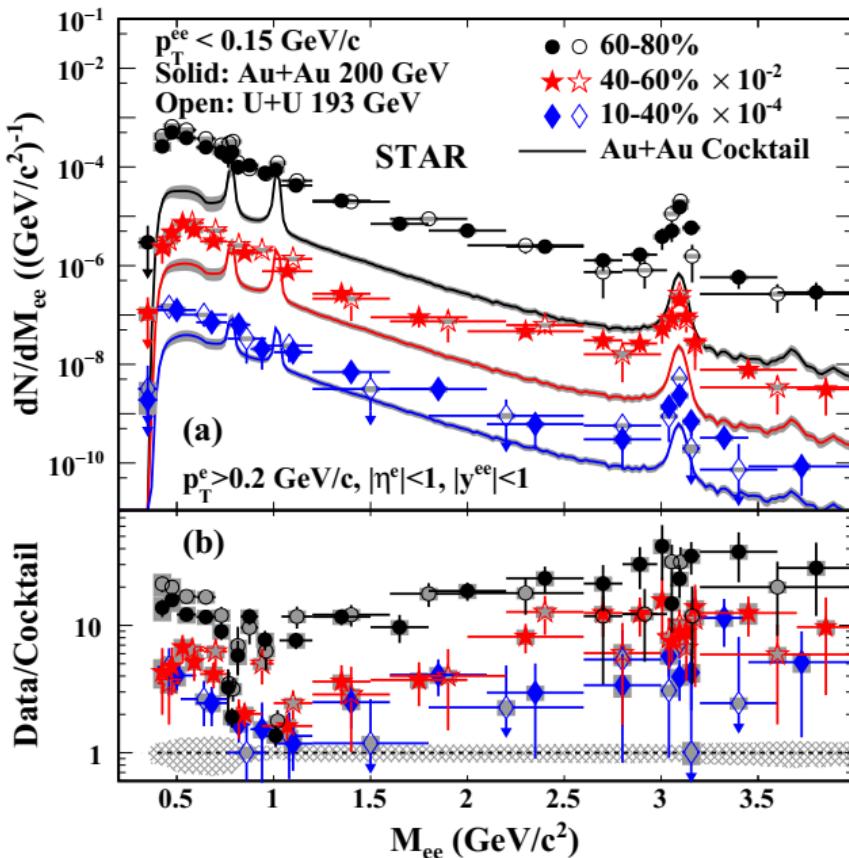
How about the contribution in HHIC?



STAR, PRL 121 (2018) 132301

- Significant excess in 60-80% central Au + Au and U + U collisions for the whole invariant mass range!

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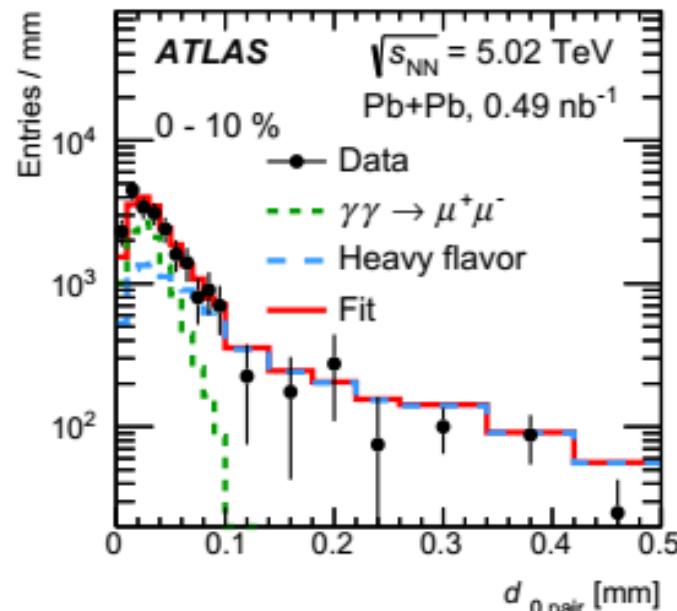
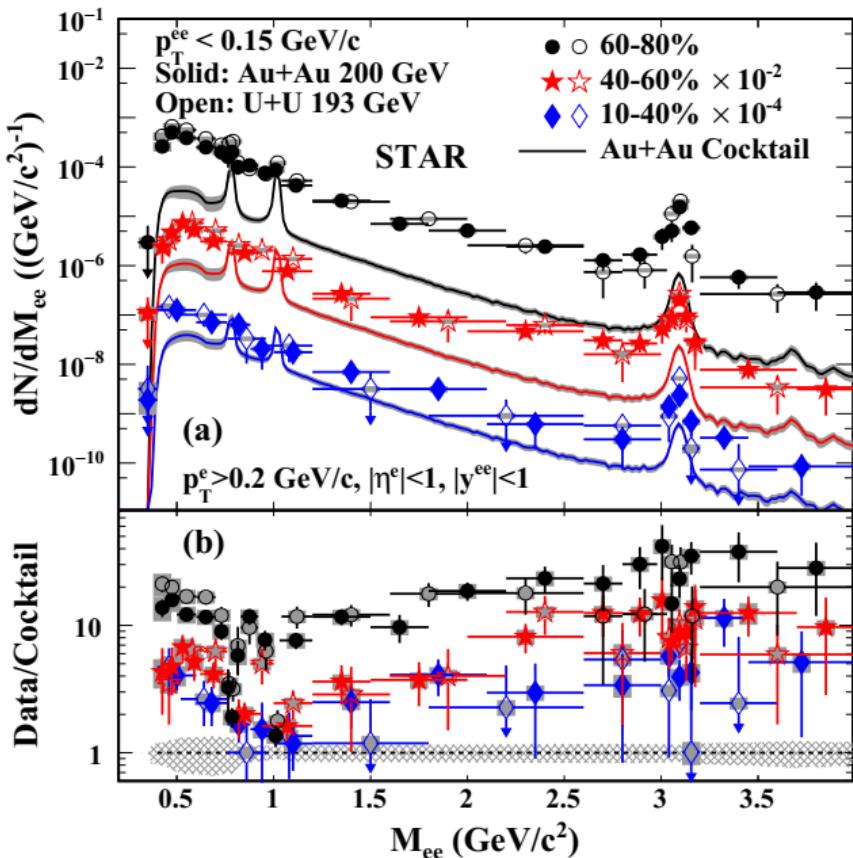
STAR, PRL 121 (2018) 132301

W. Zha et al., PLB 781 (2018) 182

W. Zha et al., PLB 789 (2019) 238

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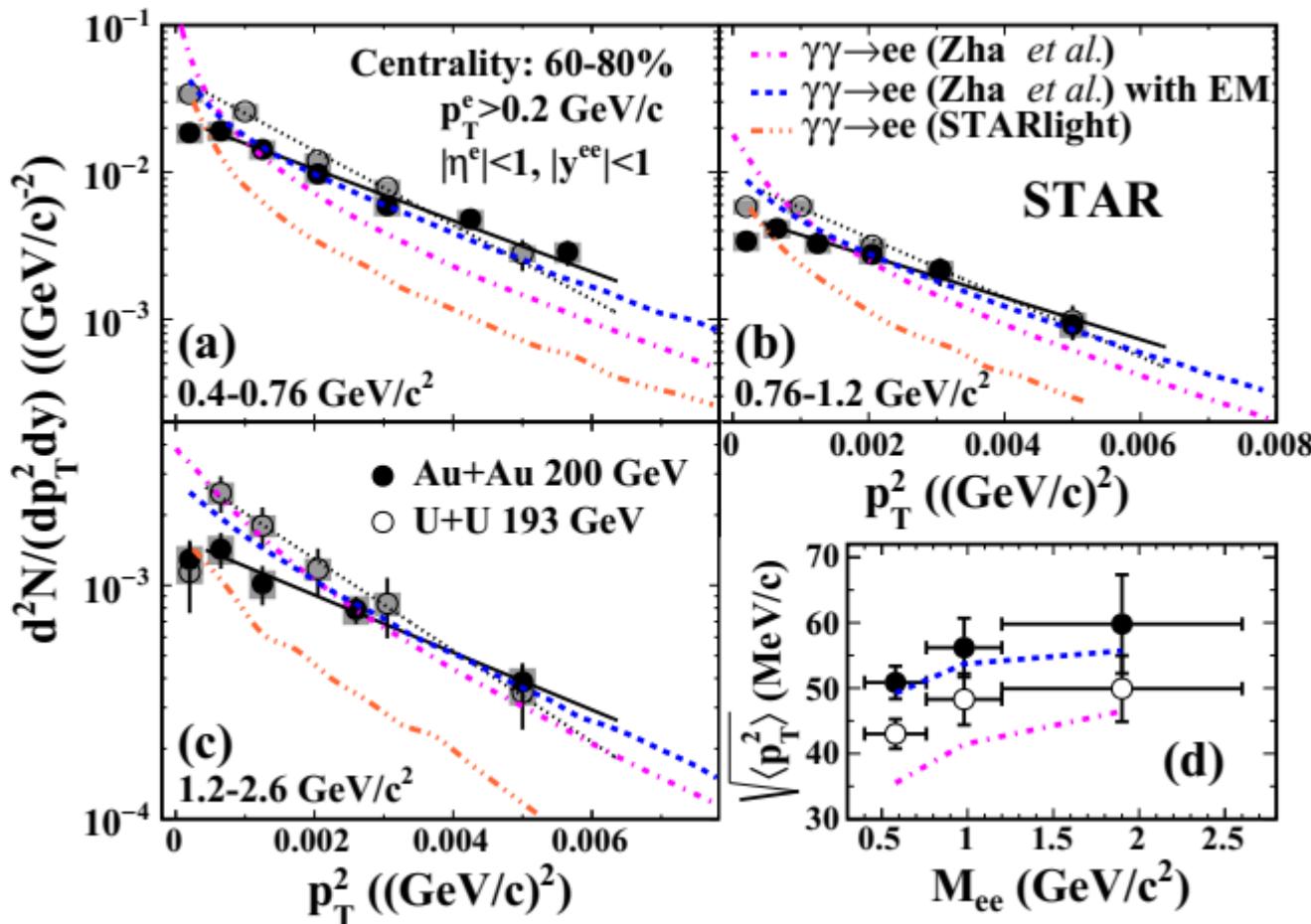
How about the contribution in HHIC?



ATLAS, PRL 121 (2018) 212 301

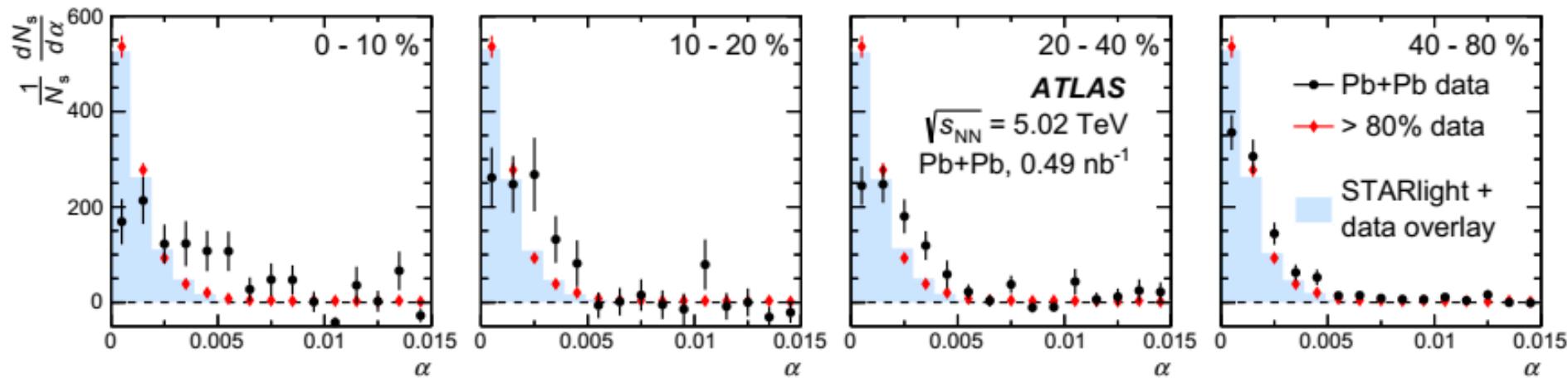
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The puzzle: pair p_T broadening



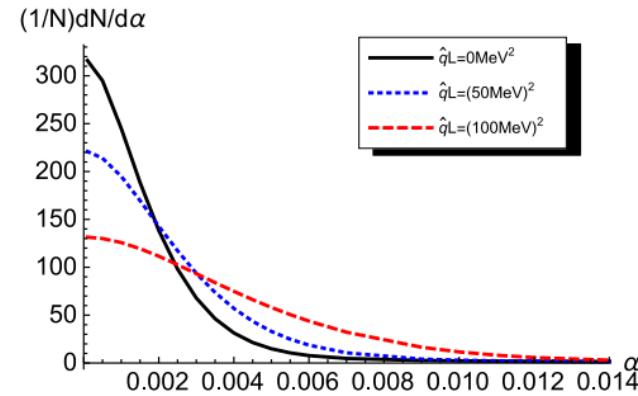
- The equivalent photon approximation could not describe the pair p_T distribution
- Possible medium effects --- magnetic field trapped in the QGP?

The puzzle: pair p_T broadening



ATLAS, PRL 121 (2018) 212 301

- The broadening **increases towards central collisions**
- Possible medium effects --- QED multiple scattering?



S.R. Klein et al, PRL 122 (2019)
132301

The initial impact parameter dependence

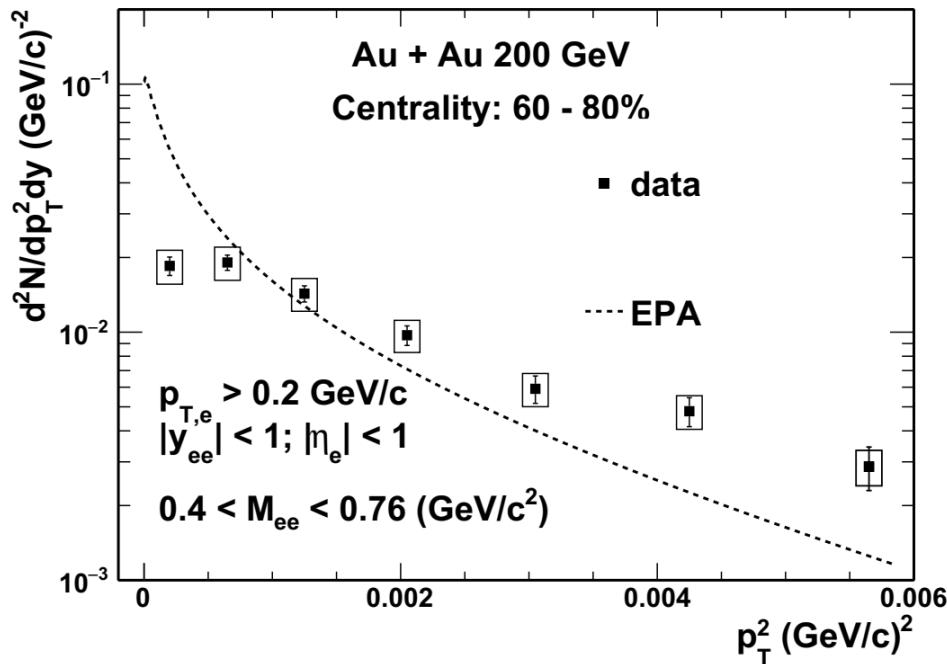
EPA approach

The photon k_T spectrum for fixed k :

The final-state p_T is the vector sum of the two photons.

$$\frac{dN}{dk_\perp} = \frac{2Z^2\alpha F^2(k_\perp^2 + k^2/\gamma^2)k_\perp^3}{\pi[k_\perp^2 + k^2/\gamma^2]^2}$$

No impact parameter dependence!



Fail to reproduce the pair p_T !

The initial impact parameter dependence

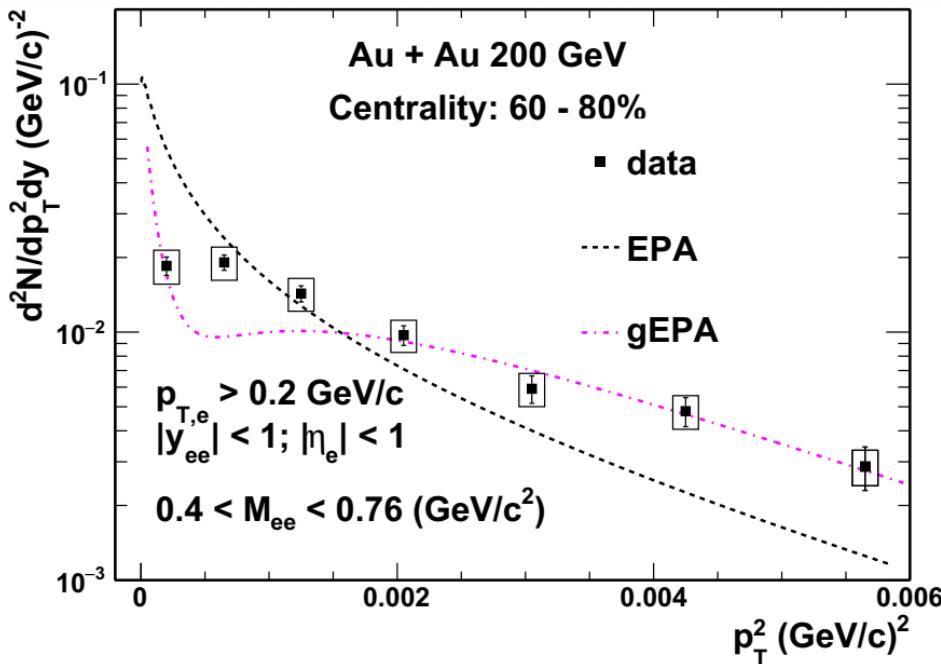
gEPA approach

$$\sigma = 16 \frac{Z^4 e^4}{(4\pi)^2} \int d^2 b \int \frac{d\omega_1}{\omega_1} \int \frac{d\omega_2}{\omega_2} \int \frac{d^2 k_{1\perp}}{(2\pi)^2} \int \frac{d^2 k_{2\perp}}{(2\pi)^2} \int \frac{d^2 q_\perp}{(2\pi)^2} e^{-i\mathbf{b}\cdot\mathbf{q}_\perp}$$

Impact parameter dependence!

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$$\begin{aligned} & \times \mathcal{F}_1(\mathbf{k}_{1\perp}, \omega_1) \mathcal{F}_2(\mathbf{k}_{2\perp}, \omega_2) \mathcal{F}_1^*(\mathbf{k}_{1\perp} - \mathbf{q}_\perp, \omega_1) \mathcal{F}_2^*(\mathbf{k}_{2\perp} + \mathbf{q}_\perp, \omega_2) \\ & \times \{(\mathbf{k}_{1\perp} \cdot \mathbf{k}_{2\perp}) ((\mathbf{k}_{1\perp} - \mathbf{q}_\perp) \cdot (\mathbf{k}_{2\perp} + \mathbf{q}_\perp)) \sigma_s(\omega_1, \omega_2) \\ & + (\mathbf{k}_{1\perp} \times \mathbf{k}_{2\perp}) \cdot ((\mathbf{k}_{1\perp} - \mathbf{q}_\perp) \times (\mathbf{k}_{2\perp} + \mathbf{q}_\perp)) \sigma_{ps}(\omega_1, \omega_2)\} \end{aligned}$$



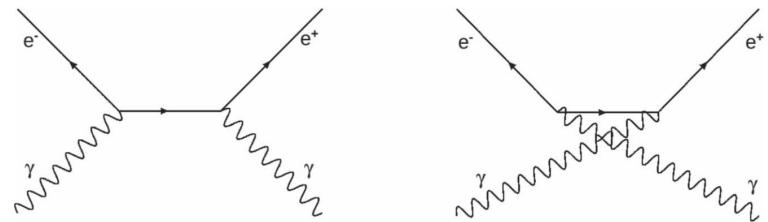
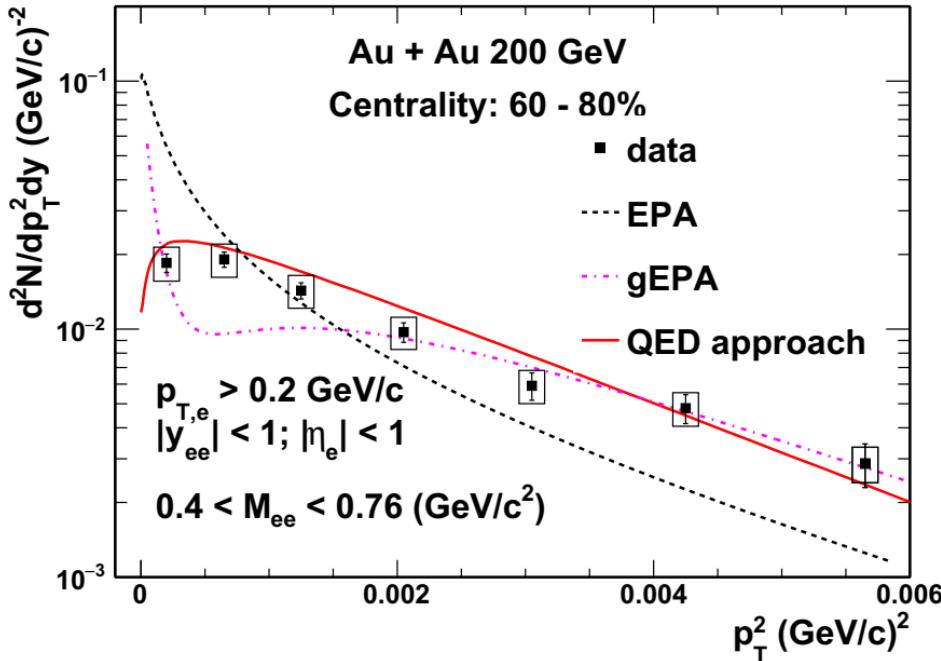
- Fail to reproduce data at very low p_T !
- Strange dip structure!

The initial impact parameter dependence

$$\sum_s |M|^2 = (Z\alpha)^4 \frac{4}{\beta^2} \int d^2\Delta q_1 d^2q_1 [N_0 N_1 N_3 N_4]^{-1} \exp(i\Delta \vec{q}_1 \cdot \vec{b}) \quad \text{QED approach}$$

$$\times \text{Tr} \left\{ (\not{p}_- + m) \left[N_{2D}^{-1} \psi^{(1)} (\not{p}_- - \not{q}_1 + m) \psi^{(2)} + N_{2X}^{-1} \psi^{(2)} (\not{q}_1 - \not{p}_+ + m) \psi^{(1)} \right] \right.$$

$$\left. \times (\not{p}_+ - m) \left[N_{5D}^{-1} \psi^{(2)} (\not{p}_- - \not{q}'_1 + m) \psi^{(1)} + N_{5X}^{-1} \psi^{(1)} (\not{q}'_1 - \not{p}_+ + m) \psi^{(2)} \right] \right\}$$

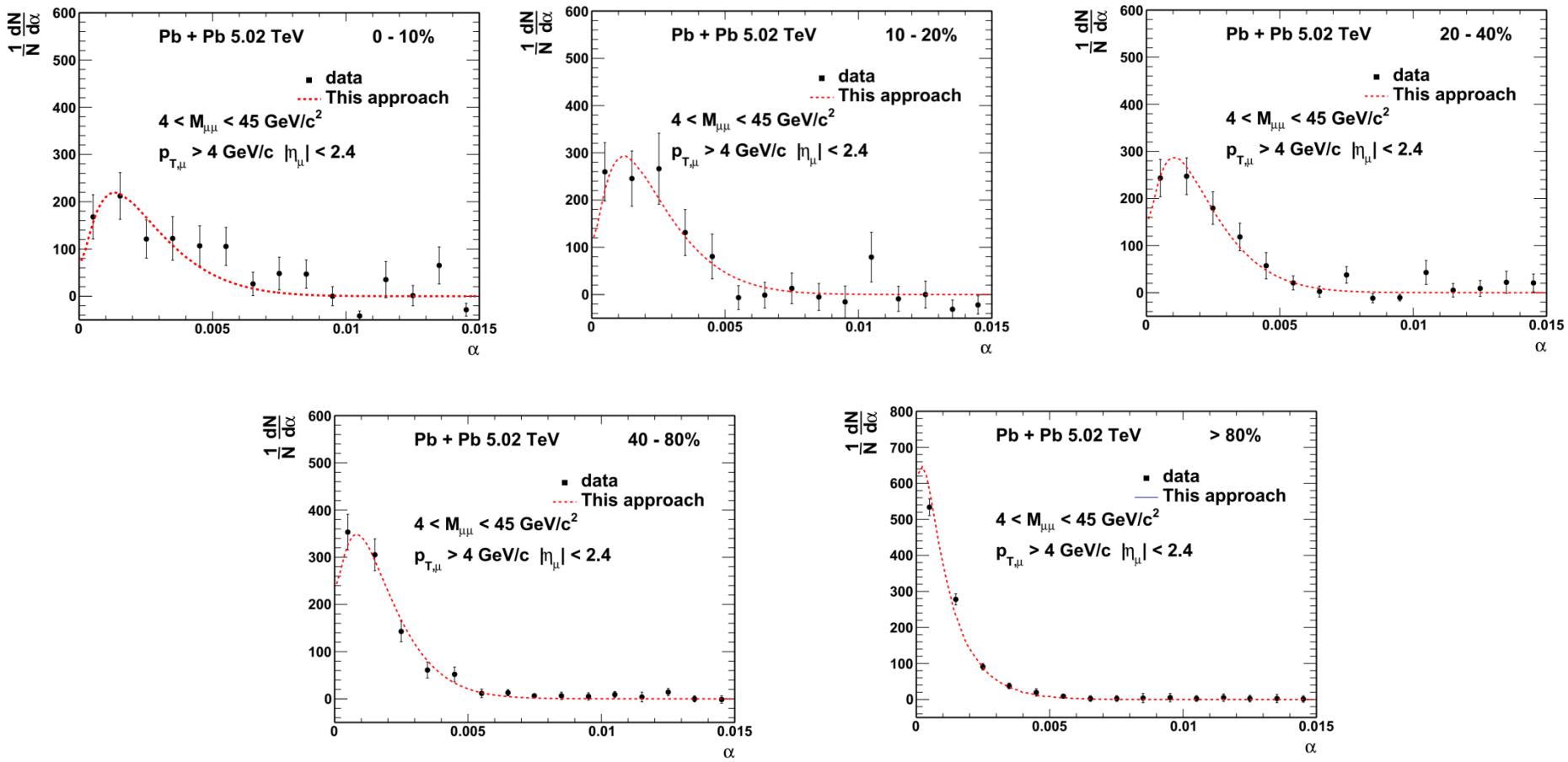


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- Reasonably describe the p_T spectrum.

W. Zha et al., arXiv: 1812.02820

The initial impact parameter dependence

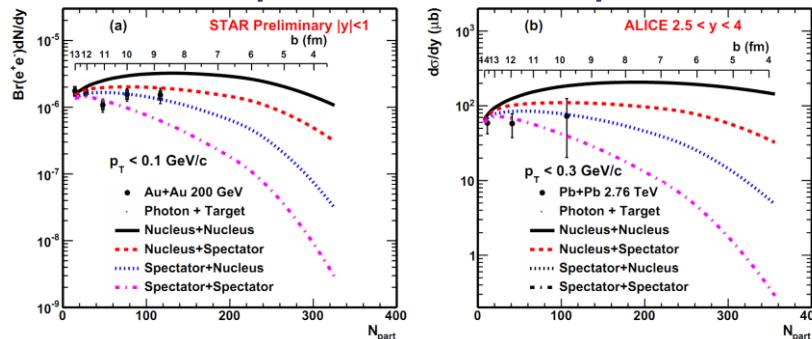
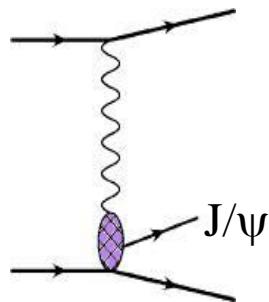


Successfully reproduce the centrality dependence of acoplanarity

Summary

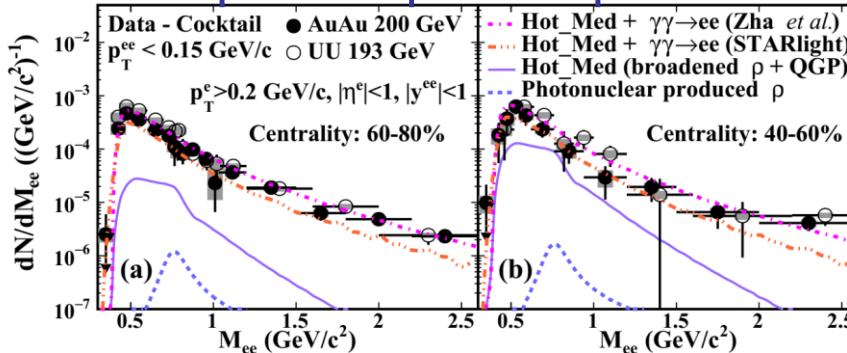
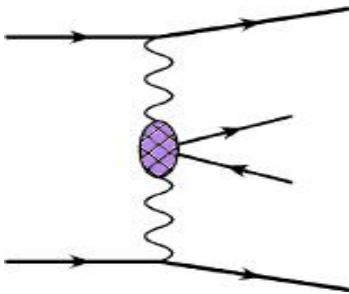
● Excess of J/ψ at very low p_T !

✓ Consistent with coherent photonuclear production in HHIC!



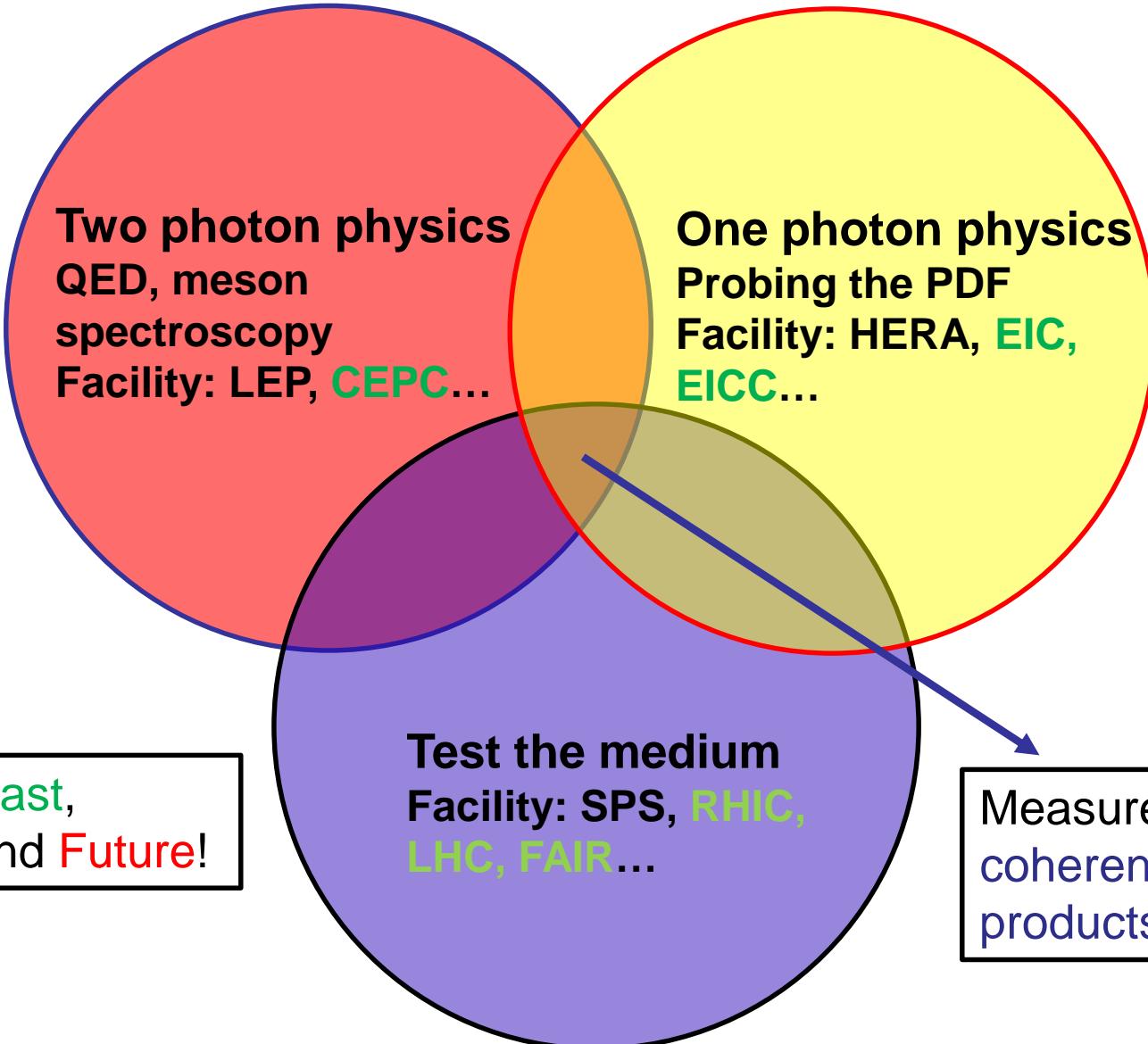
● Excess of dielectron at very low p_T !

✓ Consistent with coherent photon-photon production in HHIC!



● More coherent photon products are promising in HHIC!

Outlook



Recover the impact parameter dependence: gEPA

$$A_1^\mu(k_1, b) = -2\pi Z_1 e e^{ik_1^\tau b_\tau} \delta(k_1^\nu u_{1\nu}) \frac{F(-k_1^\rho k_{1\rho})}{k_1^\sigma k_{1\sigma}} u_1^\mu$$

$$A_2^\mu(k_2, 0) = -2\pi Z_2 e \delta(k_2^\rho u_{2\rho}) \frac{F(-k_2^\rho k_{2\rho})}{k_2^\sigma k_{2\sigma}} u_2^\mu$$

$$S(P\alpha, \mathbf{b}) = \int \frac{d^4 k_1}{(2\pi)^4} \int \frac{d^4 k_2}{(2\pi)^4} [A_1^\mu(k_1, \mathbf{b}) \Gamma_{\mu\nu}(k_1 k_2; P\alpha) A_2^\nu(k_2, 0)] (2\pi)^4 \delta^4(k_1 + k_2 - P)$$

$$\sigma = \int d^2 b \int |S(P\alpha; \mathbf{b})|^2 \frac{d^4 P}{(2\pi)^4} d\alpha$$

Impact parameter dependence!

$$\sigma = 16 \frac{Z^4 e^4}{(4\pi)^2} \int d^2 b \int \frac{d\omega_1}{\omega_1} \int \frac{d\omega_2}{\omega_2} \int \frac{d^2 k_{1\perp}}{(2\pi)^2} \int \frac{d^2 k_{2\perp}}{(2\pi)^2} \int \frac{d^2 q_\perp}{(2\pi)^2} e^{-i\mathbf{b}\cdot\mathbf{q}_\perp}$$

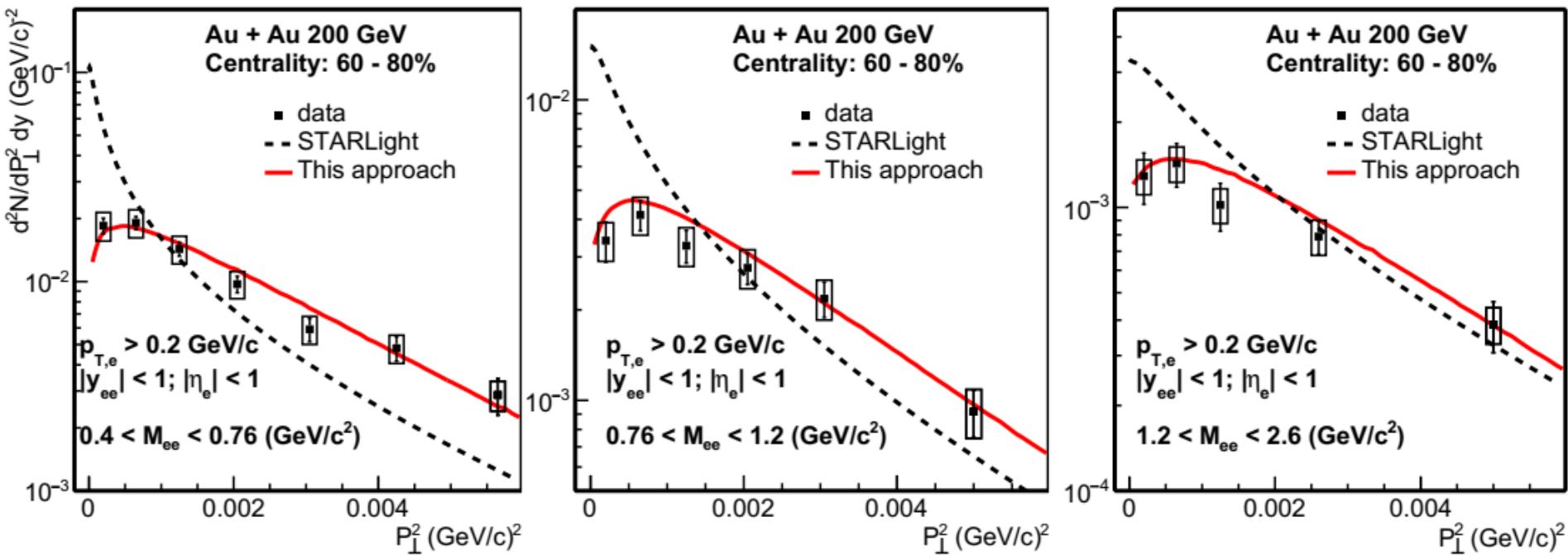
$$\begin{aligned} & \times \mathcal{F}_1(\mathbf{k}_{1\perp}, \omega_1) \mathcal{F}_2(\mathbf{k}_{2\perp}, \omega_2) \mathcal{F}_1^*(\mathbf{k}_{1\perp} - \mathbf{q}_\perp, \omega_1) \mathcal{F}_2^*(\mathbf{k}_{2\perp} + \mathbf{q}_\perp, \omega_2) \\ & \times \{ (\mathbf{k}_{1\perp} \cdot \mathbf{k}_{2\perp}) ((\mathbf{k}_{1\perp} - \mathbf{q}_\perp) \cdot (\mathbf{k}_{2\perp} + \mathbf{q}_\perp)) \sigma_s(\omega_1, \omega_2) \\ & + (\mathbf{k}_{1\perp} \times \mathbf{k}_{2\perp}) \cdot ((\mathbf{k}_{1\perp} - \mathbf{q}_\perp) \times (\mathbf{k}_{2\perp} + \mathbf{q}_\perp)) \sigma_{ps}(\omega_1, \omega_2) \} . \end{aligned}$$

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The dilepton cross section from two real photons

The results

The approach **describes** the data reasonably well!

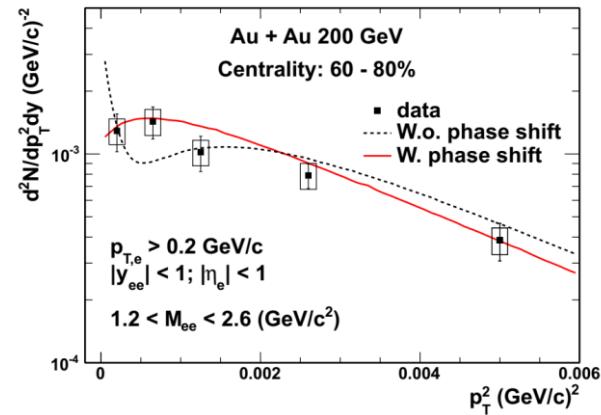
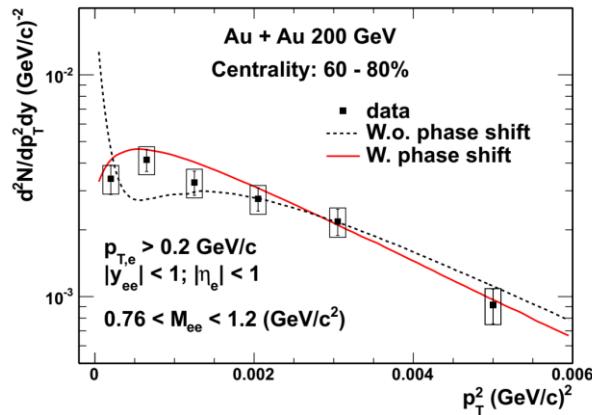
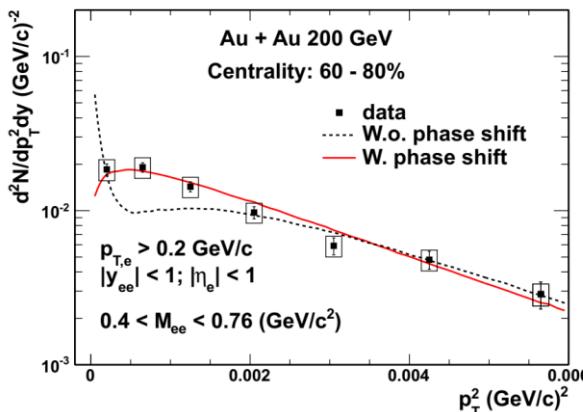


However, in the code implementation, an additional phase shift has been added:

$$\sigma = 16 \frac{Z^4 e^4}{(4\pi)^2} \int d^2 b \int \frac{d\omega_1}{\omega_1} \int \frac{d\omega_2}{\omega_2} \int \frac{d^2 k_{1\perp}}{(2\pi)^2} \int \frac{d^2 k_{2\perp}}{(2\pi)^2} \int \frac{d^2 q_\perp}{(2\pi)^2} e^{-i\mathbf{b}\cdot\mathbf{q}_\perp} \times e^{-i(\alpha_1 + \alpha_2)} \quad \text{phase shift}$$

F(k) e^{iφ} $\alpha_1 = \mathbf{k}_1 \cdot \mathbf{Angle}(\mathbf{k}_1 - \mathbf{q}) \times \mathcal{F}_1(\mathbf{k}_{1\perp}, \omega_1) \mathcal{F}_2(\mathbf{k}_{2\perp}, \omega_2) \mathcal{F}_1^*(\mathbf{k}_{1\perp} - \mathbf{q}_\perp, \omega_1) \mathcal{F}_2^*(\mathbf{k}_{2\perp} + \mathbf{q}_\perp, \omega_2)$
 ϕ is the
 direction of \mathbf{k}_T $\alpha_2 = \mathbf{k}_2 \cdot \mathbf{Angle}(\mathbf{k}_2 + \mathbf{q}) \times \{ (\mathbf{k}_{1\perp} \cdot \mathbf{k}_{2\perp}) ((\mathbf{k}_{1\perp} - \mathbf{q}_\perp) \cdot (\mathbf{k}_{2\perp} + \mathbf{q}_\perp)) \sigma_s(\omega_1, \omega_2)$
 $+ (\mathbf{k}_{1\perp} \times \mathbf{k}_{2\perp}) \cdot ((\mathbf{k}_{1\perp} - \mathbf{q}_\perp) \times (\mathbf{k}_{2\perp} + \mathbf{q}_\perp)) \sigma_{ps}(\omega_1, \omega_2) \} \quad .$

The phase shift matters

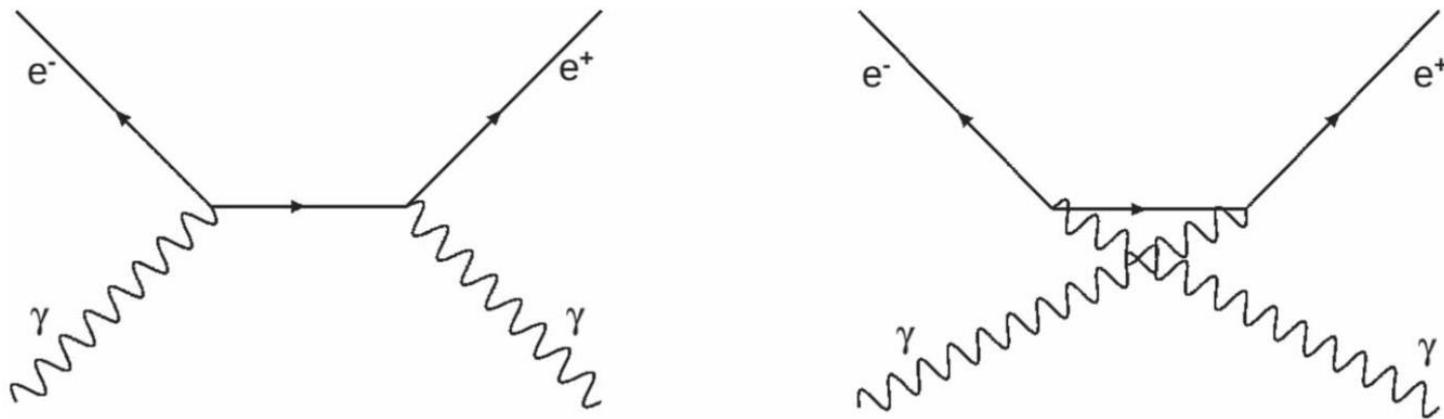


Without the phase shift, the approach can not describe the data!
However, we can not find the physical meaning for the additional phase shift.

One check:

Replace the cross section of dilepton production in the gEPA equation with the lowest order feynman diagram

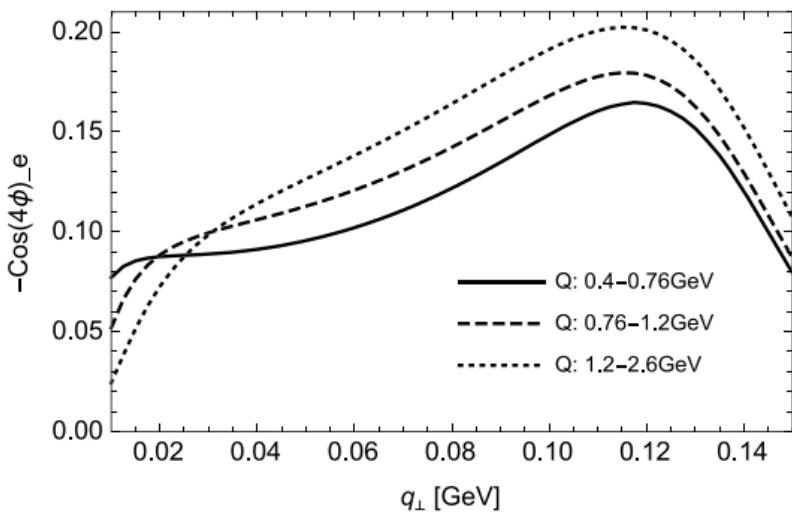
The lowest order Feynman diagram



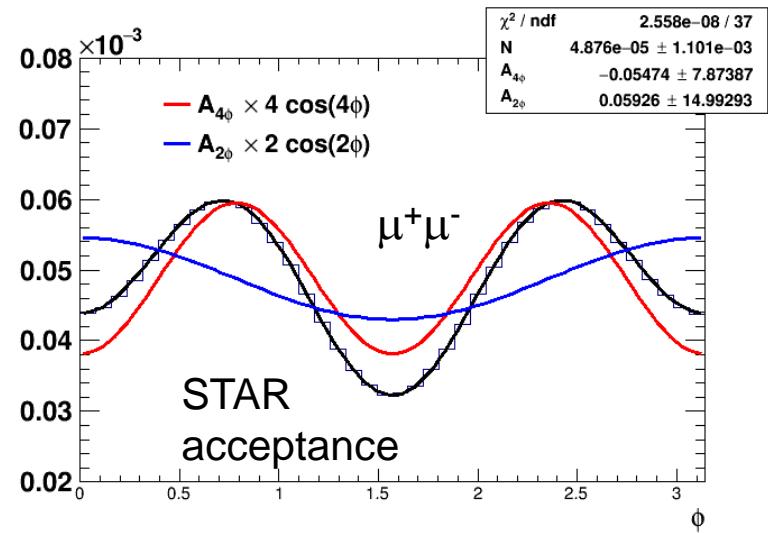
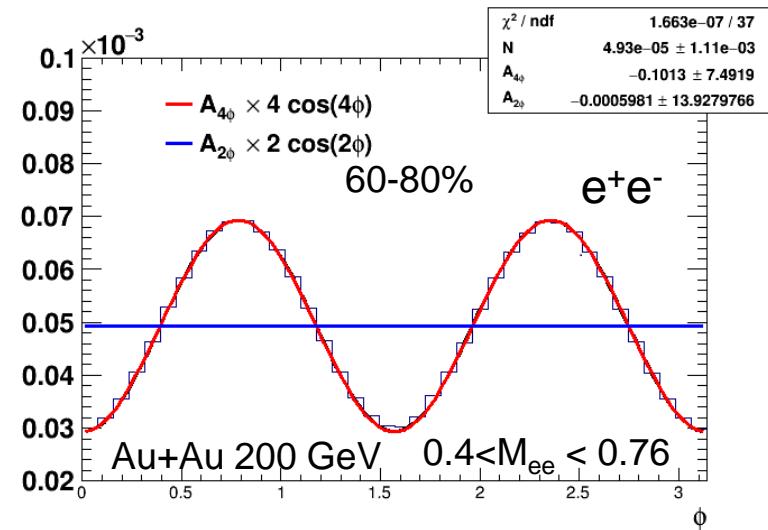
$$\sum_s |M|^2 = (Z\alpha)^4 \frac{4}{\beta^2} \int d^2 \Delta q_1 d^2 q_1 [N_0 N_1 N_3 N_4]^{-1} \exp(i \Delta \vec{q}_1 \cdot \vec{b}) \\ \times \text{Tr} \left\{ (\not{p}_- + m) \left[N_{2D}^{-1} \not{\psi}^{(1)} (\not{p}_- - \not{q}_1 + m) \not{\psi}^{(2)} + N_{2X}^{-1} \not{\psi}^{(2)} (\not{q}_1 - \not{p}_+ + m) \not{\psi}^{(1)} \right] \right. \\ \left. \times (\not{p}_+ - m) \left[N_{5D}^{-1} \not{\psi}^{(2)} (\not{p}_- - \not{q}'_1 + m) \not{\psi}^{(1)} + N_{5X}^{-1} \not{\psi}^{(1)} (\not{q}'_1 - \not{p}_+ + m) \not{\psi}^{(2)} \right] \right\}$$

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Linear polarization for photons



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- ✓ The linear polarization of photons leads to the $\cos 4\phi$ and $\cos 2\phi$ terms.