*第十七届全国核物理大会(2019)报告摘要(核天体物理)*

**用**‘**相对论约化R-矩阵理论’获取高精度的12C（α，γ）16O**

**的天体物理S因子和反应率**

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**（孙伟力牵头申请的自然科学基金项目，正在准备发表）**

**I. 简介**

求得到准确的（要求误差小于10%）12C（α，γ）16O 的天体物理S因子和反应率（E=0.3 MeV）,对于研究天体演化过程极为重要，曾被称为核天体物理研究的‘圣杯’。全世界50多年的努力还没有得到公认的满意的结果，目前除了我们发表的初步结果外，绝大多数的数值精度远大于10%。主要原因首先是在天体物理感兴趣的能区（E=0.3 MeV）12C（α，γ）16O的反应截面太小太小,用目前的实验装置不可能直接测量到,只能通过测量较高能区(Ecm >0.8 MeV)的截面和利用R-矩阵理论分析向低能（E=0.3 MeV）外推得到。这个R-矩阵理论外推分析也是非常复杂和困难的。

**II.实验数据状况**

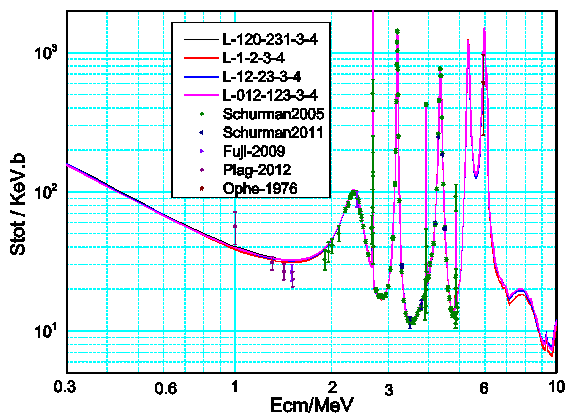
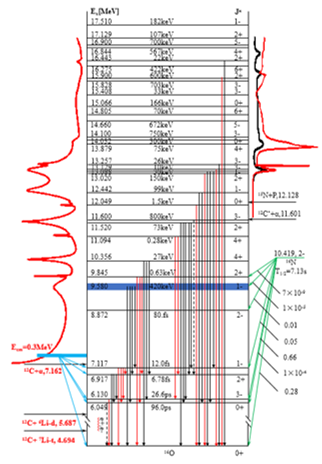


图3. 16O 核系统的能级结构及γ跃迁 图4. 12C（α，γ）16O 的天体物理S因子

图3.显示 16O 核系统的能级结构及5类12C（α，γ）16O的γ跃迁γn+16On（n=0,1,2,3,4）,有关15N+p 的反应状况，以及16N 的β衰变分支比。这些数据都要用于R-矩阵分析之中。

图4.显示 12C（α，γ）16O 的总天体物理S因子Stot的实验状况。Stot= S0+S1+S2+S3+S4。 近40年来，做了上百家各类实验，最低实验值是Ecm=1 MeV，远高于我们感兴趣的Ecm=0.3 MeV区域。可得数据比较全面，可以用来做全系统GLOBAL 分析。

**III.理论分析状况**

近40年来，有30多家有参考价值的理论分析。外国的理论分析普遍采用的‘普通R矩阵’模型，不宜做global拟合，必然导致多家分析结果误差很大和差别很大。为此，我们创立了‘相对论约化R-矩阵理论’，及配套程序RAC，先后采用5类分析方案，都得到了很好的结果。

与国外同类R-矩阵程序和同类工作相比，RAC和该工作具有以下的特点和先进性。

A. 采用相对论的能量计算公式，可以巧妙地将γn+16On（n=0,1,2,3,4）当作复合核2体反应道处理，为此可以同时分析16O系统中所有反应道的实验数据；

B. 采用多道多能级‘约化R-矩阵模型’，可以分析全能区的实验数据，为此可以同时分析16O系统中所有可以利用的实验数据；国外的R-矩阵程序还固守在使用普通的R-矩阵模型；

C. 采用严格的‘广义最小二乘’公式和‘误差传播理论’，可以严格地使用实验数据的统计误差和系统误差信息，为此可以准确地给出所有反应道评价值的协方差矩阵；国外的R-矩阵程序还固守在使用普通的最小二乘公式，只能近似使用实验数据的统计误差和系统误差信息；

D. 采用了多种精密的有特色的数值拟合方法，比如反复迭代，单参数梯度法，智能记忆路径，等等，有可能获得最佳拟合。

15年来，通过国际合作与学术交流，关于必须应用和如何应用‘约化R-矩阵理论’的共识在不断增加。

**IV. RAC 采用的5类分析方案**

这里的5类方案是基于组合采用下述3类分析方法。

1. 本工作采用的物理模型有2类，一是经典的R-矩阵理论（Lane-1958），它只可将γ粒子道当作普通的重粒子反应道处理；二是改进了的R-矩阵理论，它可将γ粒子道当作γ跃迁处理；

2. γ的本征自旋为1，γ是横波。一类方法是将γ当作普通粒子处理，宇称守恒公式是 = ≡ πα (*3*) ，其自旋投影γz=-1,0，1，这是近似的处理方法；二是将γ当作横波处理，其自旋投影γz=-1,1， 不可为0. 宇称守恒公式是 =/。这是严格的处理方法。

3. 计算γ出射微分截面采用下述两类公式；

公式1，它适用于普通的重粒子对反应道，这是近似的处理方法。

(1.1)

( (1.2)

(1.3)

公式 2，它适用于将γ当作横波处理，这是严格的处理方法。

(2L-1)/4 (2.1)

(2.2)

With the definitions

= (2.3)

=( (2.4)

(2.5)

(2.6)

[ ] = (2.7)

表1 显示5类方案的计算结果， 其中第一行‘安振东-2015’是采用近似方法得到的结果，已经发表在PRC上；第2, 3, 4行是不断改进的结果；‘孙伟力-2019’是采用准确方法得到的结果，目前正在准备发表，相对于‘安振东-2015’，Stot减少了1.2%，精度提高了0.5%。最后一行是5类研究方案的平均值。

**Table 1. Main characteristics and S factor (0.3 MeV) in the five schemes** (RR Unit=10-15 cm3mol-1s-1),

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Scheme  --Sn | Formulae for  γ channel | allαγ | /keVb | Error  /% | SE10  /keVb | SE20  /keVb | S6.05  /keVb | S6.13  /keVb | S6.92  /keVb | S7.12  /keVb | RR/  Unit |
| 安振东2015 | Lane1958-s3 | / | 162.7±7.3 | 4.49 | 98.0±7.0 | 56.0±4.1 | 4.90±1.20 | 0.20±0.10 | 3.00±0.40 | 0.60±0.20 | 7.83 |
| Lane-L3-S3 | Lane1958-s3 | 1901 | 160.38±7.16 | 4.47 | 94.71±6.40 | 55.70±4.46 | 2.25±0.61 | 0.14±0.05 | 6.96±1.60 | 0.45±0.04 | 7.79 |
| Lane-L3-S2 | Lane1958-s2 | 2087 | 159.78±8.16 | 5.11 | 91.46±6.78 | 56.70±4.58 | 4.04±0.43 | 0.17±0.06 | 6.95±1.59 | 0.45±0.04 | 7.76 |
| Lane-L2-S3 | Lane1958-s3 | 2346 | 160.79±9.99 | 6.31 | 99.83±9.52 | 53.35±9.22 | 3.80±0.34 | 0.17±0.05 | 3.05±1.43 | 0.59±0.21 | 7.83 |
| 孙伟力-2019 | Seyler1979-s2 | 2228 | 160.81±6.36 | 3.96 | 90.83±5.80 | 58.66±4.50 | 3.99±0.40 | 0.18±0.06 | 7.21±1.03 | 0.55±0.09 | 7.81 |
| Mean values | / | / | 160.5±3.8 | 2.5 | 92.3±3.0 | 57.0±2.5 | 3.8±0.4 | 0.17±0.05 | 7.0±0.4 | 0.50±0.05 | 7.78 |

**V. 目前的最佳结果（孙伟力-2019）**

R-矩阵分析是一种‘唯像’拟合和计算，以拟合值与实验值的符合程度作为判断工作质量的标准。

C:\Users\zhpchen\Desktop\Graph05.eps

**Figure 2. Results of the best *R*-matrix fit for S6.05 (A), S6.13 (B), S6.92 (C), and S7.12 (D), respectively**

在**Figure 2.**中，带误差棒的点表示实验值，各类线段表示评价值。图A, 图B, 图C 和 图D 分别表示对4类束缚态跃迁的S因子的拟合状况。总的说来 RAC-2016最接近实验值。

C:\Users\zhpchen\Desktop\Graph03.eps

**Figure 3. Results of the best *R*-matrix fits for data of (A), *α* spectra (B), (C) and the published RR (D).**

在**Figure 3.**中，带误差棒的点表示实验值，各类线段表示评价值。图A表示对基态跃迁的S因子的拟合状况, 图C表示对总的S因子的拟合状况,显然RAC-2016最接近实验值。图C表示对α谱的拟合状况，这个拟合对于确定2个关键能级的宽度起到重要作用。图D 表示9家反应率对RAC-2016 的比值。可以看出这9家反应率，有非常大的差距，评价状况不容乐观。最新的RAC评价值RAC-2019与RAC-2016极为接近，只有图示看不到的差别。RAC评价值高于9家平均值的1%。

因此说，继续开展更为接近0.3 MeV的测量还是必要的。

**12C(α,γ)16O 的**参数化的天体物理反应率RR

(5)

**Table 2. Parameters a0-a11 for analytic expression (4)**

|  |  |  |
| --- | --- | --- |
| =0.59605698D+09 | =0.56057001D+00 | =0.31559988D+02 |
| =0.40000000D+03 | =0.22658324D+17 | =0.41569810D+02 |
| =0.43745037D+02 | =0.25901325D+04 | =0.28428833D+02 |
| =0.11173825D+13 | =-0.10135299D+01 | =0.39356875D+02 |

谢谢！！

待发表的英文初稿：

**Global Evaluation for Astrophysical S Factor and**

**Reaction Rate of 12C (α, γ) 16O**

The astrophysical S factor and reaction rate of 12C (α, γ) 16O are very important basic data for cosmological models. Although to get accurate values of the total S factor (error < 10%) has been regarded as the ‘holy grail’ of nuclear astrophysics, the world’s tremendous efforts over 40 years have not obtained the [acknowledged](http://www.baidu.com/link?url=0_1504MjWSgtWteVydPsKY0X9Dg14Ar3vqGE8pYlXr1KhIWaBF958bfYBthyLeiljHYjatZ8S5lhKYTbl8KsPpP35c04OeFm8tO3MkL5vOq) result. We has developed a new systematically evaluation method RAC-CERNGEPLIS, and combine the formulae of classical *R*-matrix theory and γ transition theory, taking the 12C(α,γn)16On (n=0, 1, 2, 3, 4) as independent reaction channel, and conduct a global evaluation for 16O system, a set of accurate S factor and reaction rate have been obtained. At E=0.3 MeV the total S factor is 160.81±6.36 keVb, an error of 3.96%. At T9 = 0.2 the reaction rate is (7.81±0.31)10-15 (cm3mol-1s-1), an error of 4.1%.

During core-He burning, the 3α and 12C(α,γ)16O reactions compete to determine the helium burning time scale and the relative abundances of oxygen and carbon prior to core-carbon burning. Writing in the mid-1980s, W. A. Fowler stressed two fundamental difficulties in nuclear astrophysics: determining the ratio 12C/16O and solving the solar neutrino problem (*1*, *2*). An accuracy of 10% or better is needed for the S factor (SF) and reaction rate (RR) of 12C (α, γ) 16O, but the scientific community is still far from this goal (Table S1 and S2). The largest uncertainties arise from the deficiencies in the analysis model used to evaluate available data. The new work (*3*) given out a better result, but it did not use the γ transition theory. The present study develops an innovative global fitting procedure that yields the most reliable values for the SF and RR of 12C (α, γ) 16O ever found. Technical details are discussed in the supporting online material (SOM). In what follows, we use the notation "(SOM I.J)" to refer to "SOM chapter I.J".

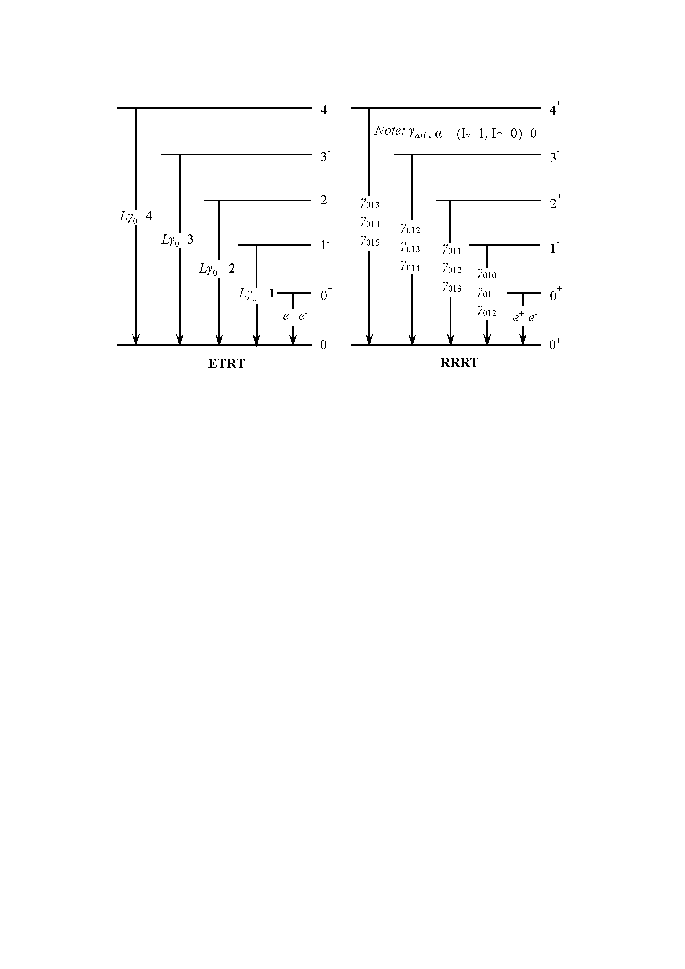
The original R-matrix theory of Lane and Thomas (*4*) used the standard R-matrix formulae to describe two-body nuclear reactions. Because these formulae are not justified for γ radiative capture (since particles are created and destroyed), a collision matrix was developed that was a sum of three parts: internal resonant, external resonant, and direct capture (*5*). References (*6*) found detailed formulae for the integrated cross section, but include adjustable parameters for practical applications. We refer to this approach as "electromagnetic transition R-matrix theory" (ETRT). Because the R-matrix model is based on a complete level-set, and the channels and levels have intimate correlation and strong interference, its analysis result can be accurate and unique only when it describes the entire database for a nuclear system (global fitting). The ETRT models do not satisfy this demand, since the angular distribution of 12C(α, γ)16O must be treated separately by first fitting them with Legendre polynomials to determine SE10 and SE20 . It is virtually impossible to get an accurate value for SE20 in such an approach. In addition, the error-propagation law cannot be applied to the entire database and the large errors of for the bound states cause serious difficulties for error estimation. These are insurmountable obstacles in ETRT. References (*7, 8*) published an *R*-matrix code AZURE, which allows simultaneous analysis of the integrated and differential data but has not yet determined the SF down to 0.3 MeV.

For these reasons, a new systematically evaluation method RAC-CERNGEPLIS has been developed to resolve the problem, this method includes：

RAC—R-matrix Analysis Code with multi-levels and multi-channels theory (3, *9*); C—Covariance statistics and Generalized Least Squares(GLS) fitting (*10*); E—Error propagation law(*10*); R—Relativistic calculation for energy; N—Normalization for relative data (Scaling factor) and absolute data (Normalized factor); G—Global database for a nuclear system; E—Elimination of channel is used to expended energy range (3); P—PPP modification is considered (*10*); L—Lett’s criteria; I—Iterative fitting procedure; S—Systematic error is updated according to the errors of fitted values, this is a new and key idea. In GLS fitting which kind of systematic error should be used finally? For a data-set (Y), in fitting procedure it is modified with normalization factor (or scaling factor) (N) to minimum 2. If the N is good enough, it means that YN is the data-set actually used. By now, what is the systematic error of YN? It should not be the original one absolutely, it should be a new one, it should be the **‘residual’** of original systematic error. In this case maybe exist follow relation: E-kσ< YN < E+kσ, 1<k<2. The E is the expectation value, and σ is the error of E. In GLS fitting the PPP is a big problem, experiences show that if the systematic error is larger than 40% of statistic error, the PPP will happen obviously. By now, the experimental paper often give out rather small statistic error and rather larger estimate value of systematic error, with this kind of data, the GLS fitting can’t be used absolutely. So at first the ‘Conventional Least-Squares’ fitting is used, after get very good fitting, the kσ is taken as systematic error, a technical criteria is let kσ much less statistic error to avoid bring out PPP . This method need a iterative procedure, in which, every experimental data play an effect to determine expectation value, and every experimental data is improved for its normalizing factor and systematical error, until all parameters, all normalization factors, all calculation value approach very stable, so the final evaluated value can be think very near the expectation value. In this way using different priors will get the same final results.

The main physical basis for creating the RRRT is the fact that the external resonant and direct capture parts of the collision matrix are very small (SOM 4.5). Owing to the large binding energy (7.2 MeV) of 16O, as compared to the α+12C threshold, its wave function and effective charge for the state decrease rapidly for radii larger than a certain value (*6*). Therefore, the external and direct contribution of E10 can be neglected. In addition, the external contribution for the E20 transition can be neglected (*12*). Reference (*13*) notes that the direct capture part of (0.3 MeV) is only about 2.5% (SOM 4.8). It is therefore reasonable that to describe the non-resonant capture of E20 with the parameters of distant levels. The five primary transitions +16On (n = 0, 1, 2, 3, 4) (Fig. S3.1) are taken as independent reaction channels and the sum of their probabilities is the total production cross section of 16O0. There is no need to consider subsequent reactions, so the problem about particles are created or destroyed is avoided. The reduced masses of the γ particle channels are represented by relativistic energy. Due to the fact that the compound resonant contribution plays a dominant function for 12C (α, γ) 16O, a global fitting for the entire 16O system can be done using the standard R-matrix formulae of (*3*).

In the ETRT model, each reaction channel *L*has only one value (figure 1). In the RRRT model, the relative orbital angular momentum, *l* , is used, allowing angular momentum coupling to be considered. The quantum states provided by RRRT are complete for every state with three sub-reaction channels or less. When max=4, the reaction channel sets are able to describe all types of experimental data to high precision. Our early work, in which only two sub-reaction channels are considered, is described in SOM 6.4 and 6.5 and (*14, 15*).



**Figure. 1.** **Transition processes to ground-state () described by ETRT and RRRT for 16O system.**

The primary wave function is expressed with , where *α* refers to particle pair and *s* the channel spin. The can be expressed with level wave functions, , which have different total angular momentum and parity, , and energy level, . The is expressed with the equation V.2. (2.6) in (*3*) as Eq. (1) as follows:

The collection of is able to describe all types of γ transitions of phenomenological significance. Furthermore, the reduced channel width is used to expand the analysis to higher energy (SOM 2). All the levels from the resonances in 12C+α up to Ex = 17.510 MeV have been adopted (*16*) and the corresponding parameters are determined by referring to the γ transition scheme of Fig. S3.1. This is crucial for the calculation of the RR of 12C(α, γ)16O from = 0.01 to 10 ( = 109 K).

To analyze the 16O system, we used our RRRT model with our powerful "R-matrix analyzing code" (RAC). The RAC was used to produce accurate ( ˂ 1%) 6Li (n, α) and 10B (n, α) cross sections, the same ones that were used to construct the International Evaluation of Neutron Cross Section Standards (*17*). When compared with the R-Matrix codes EDA and SAMMY (*17*), the RAC results are virtually identical when the same parameters are used. The calculated α-spectrum with the parameters of (*18*) is also identical with the α-spectrum of (*18*). A special version of RAC (*19*) for analyzing 16O system was written according to the formulae in (*3, 9, 10, 19*) and we refer to this as RAC2016.

According to the constraints imposed by conservation laws, there are nine reaction channels: elastic scattering (α, 12C), the ground state gamma (γ0, 16O0); the cascade gammas (γ1, 16O1), (γ2, 16O2), (γ3, 16O3) and (γ4, 16O4), representing the primary transitions to the excited states , , and , respectively; the heavy particles (ɑ1, 12C\*) and (p, 15N); and the reduced channel (α, x), the width if which, , represents the contribution of reaction channels besides those already listed. All available experimental data pertaining to the formation of the 16O system by α+12C were evaluated (SOM 6). A complete global database was formed, including 4400 data points from -1.113 MeV to 17.51 MeV (Tab. S6.3). The evaluated integrated data are consistent with each other and cover the entire energy region.

The γ is transverse wave with intrinsic spin 1, which means the γ spin projection can be -1 and +1 but not 0. In this model, satisfying "the demand that the gamma is a transverse wave" (DGTW) is a key problem. In labeling the four fitting schemes, we use ‘S2’ to indicate that DGTW was met and ‘S3’ to indicate that it was not met. Another key problem is how to deal with the conservation of parity. A real heavy-particle channel is relative to only one nuclear state and a channel parity of = ≡ πα (*3*), where is the parity of particle and the parity of residual nucleus, is used to satisfy the conservation of parity. If both a particle channel and a real heavy-particle channel is required, then = / ≡ πα is used to satisfactory the conservation of parity. If the *l* are two non-consecutive integers, we use the notation "L2". This has been used in the scheme Lane-L2-S3 and (*14*). A gamma particle channel +16On, is relative to both initial and final state; the parity of is πγ ≡/, the symbol [ ] = is used to satisfactory the conservation of parity (*9*). According to =γ+ =+, *l=*j-1, j and j+1, the *l* can be 3 consecutive integers and, in this case, we use the notation "L3". In this way a complete set of quantum states will satisfy the demand of global fitting. This is used in the schemes Lane-L3-S3, Lane-L3-S2, and RRRT-L3-S2. All four schemes are based on the same database and energy level construction. All four schemes use the same formulae of (*3*) for the heavy particle channels (2.1, 2.2 and 2,3), but use different formulae for the γ particle channels.

The scheme Lane-L3-S3 used the formulae of (*3*) as follows:

(2.1)

( (2.2)

(2.3)

Here, the channel is c ≡ (α s ι, *JM*), where refers to channel particle pair, *s* the channel spin, *J* and *M* the total angular momentum and its projection. The levels include =0, 1, and 2, while the 2+ levels include =1, 2 and 3. The fits of Lane-L3-S3 for all data sets are excellent, but the model has the drawback of not incorporating DGTW.

The scheme Lane-L3-S2 used the formulae of (*3*), but DGTW was considered. For the transition with f = 0, when the projection value of the channel spin is = 0, the γ spin projection must also be zero. Therefore, when using the formulae (2.1) to calculate the angular distributions (AD) of γ, if the loop for = 0 is ignored, then the γ spin projection will not be zero. It is fortunate that the CG coefficients are zero when = 0 in the calculation of (γ0, 16O0) and (γ1, 16O1) (Table S4.3.3.). This means that the AD of (γ0, 16O0) and (γ1, 16O1) described by equation (2.1) satisfy the DGTW, and the integrated cross section and AD of (γ0, 16O0) are consistent. For the cascade transitions (γ2, 16O2), (γ3, 16O3) and (γ4, 16O4), DGTW is satisfied by selecting specific waves. All the integrated data of 12C(α, γ)16O is obtained by numerical integration.

The scheme RRRT-L3-S2 used the formulae of reference (*9*), the electric-magnetic transition model with the statistical-tensor-efficiency-tensor approach (*20*). The expressions for the Legendre coefficients in the channel spin representation were derived with formulae (3.1 to 3.7), which satisfy the DGTW. Note that the in (3.1 to 3.7) have a different physical meaning with the γ that appear in the formula γ+γ= from reference (*9*), which is an indispensable angular momentum coupling formula for constructing the electromagnetic vector potential. The γ is an ‘unobservable’ orbital angular momentum and does not appear in the formulae used for calculation. The γ is the intrinsic spin of γ and is the total angular momentum of γ.

For the gamma channel, let *p* ≡ (*ε L*), where refer to final state and ε indicates magnetic or electric transitions. If Ωp ≡ 1, then is the normal formulae defined in the electric-magnetic transition model, and is accurate only when using the wave function for the electromagnetic vector potential. In RRRT-L3-S2 the wave function is the Coulomb wave function, and a modifying factor (2L-1)/4 must be used to get accurate fitting for the AD of γ capture. The formulae used to calculate AD of γ particle channels in RRRT-L3-S2 are

(2L-1)/4 (3.1)

(3.2)

With the definitions

=

(3.3)

=

( (3.4)

(3.5)

(3.6)

[ ] = (3.7)

Here, refers to the incident particle pair (), *s* is the incident channel spin, is the relative orbital angular momentum of the incident pair, and *J* is the total angular momentum. For the γ particle channel, the calculated transition matrix elements are obtained with the (α s ι, *JM*) representation, the include all required and , and the formula (3.7) is used to select the suitable and to satisfy the conservation laws. In this scheme, the original theoretical formula for integrated data become inconsistent with the formula used above. Therefore, the integrated data of 12C (α, γ) 16O was obtained by numerical integration of the calculated AD of 12C (α, γ) 16O.

In the 16O system formed by (+12C), s=0, Iα1=0, Iα2=0, all γ transition to the ground state are electric transitions, and so ε ≡ 1 and ε’ ≡ 1. When and k are selected for a set of (*L, L*’*, l, l’, J, J’*), there is a pair of relative and ,which have one or more corresponding , and that are used in the calculation. In this way, the calculated result satisfies the constraints of all conservation laws and DGTW.

The scheme Lane-L2-S3 does not consider DGTW and was used in (*18*). However, for the sake of completeness, we performed an improved evaluation that includes the newest experimental data (ED).

In each of the four schemes, to optimize the representation of the data, we minimize the quantity

(4)

Here is the vector of the ED, is the vector of calculated values, and is the covariance matrix. Covariance statistics and the error-propagation law (S4.1) are used in determining the expected values and the standard deviation () of the SF. Statistical biases due to normalization errors, known as the Peelle Pertinent Puzzle, were corrected by the method of (*17*)*.*

An iterative fitting procedure is used to get the optimized systematic error and the optimized parameter set (Fig. S5.2). Two ED input files are used. The first is a fixed record file containing the original ED. The second is a dynamic file, which contains the normalized values of the data used for each iteration of the fitting process. The systematic errors are updated by the improved calculated values, while the statistical errors are renewed with the original ones and corrected according to Letts’ criteria. Using this iterative process, the parameters fitting the ED are gradually optimized such that *a priori* and subjective judgments are minimized (SOM 5.2).

In order to get a unique of 12C(α,γ)16O, we adopted some special methods (SOM 6.4). The *S* 6.05 and *S*6.13 at 0.3 MeV (*21*) were studied carefully and the values *S* 6.05 = 4.36 and *S* 6.13 = 0.12 were found to be best.  We first determined the ‘reduced width amplitudes’ for the bound states from the experimental width (, ) and used them as fixed R-matrix parameters in searching for a unique optimized parameter set and database. We then used the experimental width for the bound states with the original error as the ED to search all R-matrix parameters simultaneously and freely. The constraint condition for stopping this search was│-│ < /3, where and refer to the experimental and evaluated value of a parameter, respectively, and is the absolute error of . The final results and fitting information are listed in Tab. S6.4 and Table 1. Most of the or show good fits (Table S7.8.2). By now, the experimental of andhave a relatively large error (32.0% and 33.8% respectively), the error of of is large too (5.8%). These are the main reasons for the relatively large error for SF at E=0.3 MeV, and should be addressed by future experiments.

The Lane-L3-S3 scheme yields the best fits. Although this is a purely phenomenological fitting, it verifies the reliability of the SF obtained in the other three schemes. The Lane-L2-S3 schemehas largest -allαγ (the integrated for all data directly related to gamma capture) and the fit of for E < 2.2 MeV is not acceptable (Fig. 6.4B). For the Lane-L3-S2scheme, the gamma particle channels （γ0,16O0）and（γ1,16O1）satisfy DGTW, while the cascade transitions, γ2, γ3 and γ4 partly meet DGTW. The fits of RRRT-L3-S2 are excellent (SOM App. IV.2). This scheme employed the strictest constraint for γ capture calculation and meets the DGTW. For all three of these schemes, the AD of 12C (ɑ, γ0)16Oat 66 energies have virtually perfect fits. These play a crucial role in ascertaining .

Table 1 shows the main characteristics and S factor (0.3 MeV) in the four schemes and the results of (15). The evaluated SF at 0.3 MeV in the first three schemes are very close. The RRRT-L3-S2 employed the strictest theory formulae for γ capture calculation, and the fitting of is the best for E < 2.2 MeV (Fig. S6.4. B). Therefore, its evaluation is taken as the final recommended values. In Figures 2 and 3, we briefly introduce the results of the best fit for the RRRT-L3-S2 scheme (referring to it with the computer program name RAC2016). The evaluation for the ED and the ED references can be found in (SOM 7.1 to 7.9).

The present work provides an effective method of determining the SF and RR of 12C(α, γ)16O. We assembled a careful database including 4400 data points from -1.113 MeV to 17.51 MeV. In addition, we built an innovative analysis model, starting with the R-matrix formulae of Lane and Thomas (*3*), and applying the formalism in circumstances where the external resonant and direct capture parts of the collision matrix are very small. Improvements also were made by ignoring the contribution with =0, thus satisfying the demand that the gamma is a transverse wave. The entire database was fit simultaneously and globally using an iterative procedure making use of covariance statistics and error-propagation. In this way, we have gained the most reliable values for SF and RR of 12C(α, γ)16O yet found. At E=0.3 MeV the total S factor is 160.81±6.36 keVb, an error of 3.96%. At T9 = 0.2 the reaction rate is (7.81±0.31)10-15 (cm3mol-1s-1), an error of 4.1%. These results offer a reliable constraint for models of stellar evolution and, therefore, cosmology.

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**Figure 2. Results of the best *R*-matrix fit for S6.05 (A), S6.13 (B), S6.92 (C), and S7.12 (D), respectively**.

For comparison, the results of (*11*)and (*22*) are also shown. The experimental data at low energy region (<2.2 MeV) strongly constrain S6.05, S6.13, S6.92 and S7.12 at 0.3 MeV. The S6.05, S6.13, S6.92 and S7.12 show constructive interference of the direct and resonance capture (SOM 4.5), and they favor values of about 3.8±0.4, 0.17±0.05, 7.0±0.4 and 0.5±0.05 keVb at 0.3 MeV, respectively. The total cascade transition Scas favors a value of about 11.5±0.8 keVb (SOM 7.4). Table S1 shows there exist larger difference for evaluated S6.92,it should be pay strong attention in future experiments.

**Figure 3. A.** Results of the best *R*-matrix fits for data of . For comparison, the results of (*11*) and (*23-29*) are also shown. In the region *E*=2.4±0.2 MeV there are five groups of absolute ED for σγ0 and their weighted average value is σγ0 = 47±3nb. These data, along with the fitting of AD of 12C(α, γ0)16O play a key role in determining. Most of data have been described perfectly up to 6.5 MeV. This is the first time that all data from 0.8 to 6.5 MeV have been used simultaneously. The excellent fitting for the AD of 12C (α, α) 12C at 84 energies and three groups of normalized 16N α-spectra (SOM 7.2)play a central role in determining the width of (α, α). The good fits for the data of 12C(α, α1)12C and 12C(α,p)15N help reduce the uncertainty due to distant levels (SOM App. II). The Sg.s favors value of 149.5±0.8 keVb at 0.3 MeV (SOM 7.3).

**Figure. 3. B.** Results of the best *R*-matrix fits for *α* spectra, together with the decomposition into *p*- and *f* -wave contributions, the contribution of 3− states perfectly compensates the negative interference of 1− states. The *α* spectrum give a better confirmation of the reduced *α* width amplitude of and (SOM 7.2).

**Figure. 3. C.** Results of the best *R*-matrix fit for the data of . For comparison, the results of (*11, 22-24*) are also shown. These data of place a strong constraint on the extrapolation and normalization of the ground transition (*11*). The for E<2.2 MeV are the most sensitive data for determining the SF at 0.3 MeV. For E<2.2 MeV region, the evaluated of (*11* and *22*) are higher than the relative data, and the evaluated of (*23*, *24*) are too low, the fitting of RAC2016 looks be the best one. The fit of RAC2016 is good across the entire energy region; every peak and most of data are described up to 5.5 MeV perfectly. In sum, the of RAC2016, compared to all previous works, have the most reliable experimental basis and the best fits. The evaluated value (4%) is reasonable and reliable (SOM 8.2). The favors value of 160.5±1.0 keVb at 0.3 MeV (SOM 7.1). This analysis demonstrate that the data bellow 1 MeV is absolute necessary for try to improve the at 0.3 MeV. RAC can deal with inverse reaction data directly, the new data bellow 1 MeV detected with γ as incident particle will be very helpful.

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**Figure 3. Results of the best *R*-matrix fits for data of (A), *α* spectra (B), (C) and the published RR (D).**

**Figure 3. D.** **Comparisons of the RR of 12C(α,γ)16O from (*15, 22, 24, 26-29*) which are normalized to RAC2016, the number in parenthesis refer to Refs**. The upper and lower limit of RAC2016 are shown as the black-dashed lines. The information about RR at =0.2 refer to Table S2, the RR of RAC2016 is (7.81±0.31)10-15 (cm3mol-1s-1), a relative error 4.1%. In(*22*) potential model was adopted as the primary tool for extrapolation of the SF to 0.3 MeV, the agreement between predictions and data is not good, the RRis too low for < 1 and too high for > 2.5. (*26*) Is a potential model calculation and the agreement between its predicted values and ED is not good. (*24*) Is an R-matrix analysis work, the RR is close to RAC2016 for T9<1, but is too low for 1.2 < T9 < 2, and is too high for T9>4. At =0.2 the RR of (*24*) is (7.58±0.31)10-15 (cm3mol-1s-1), a relative error 32.7%. The RR of (*27*) are too high for < 0.3 and too low for T9>0.7. The RR of (*28, 29*) have very large relative error (>70%). (*15*) is an early work of our research group and its RR is systematically higher for 0.2 <T9<3 (SOM 6.5). In sum, the RR of RAC2016 has the most reliable experimental basis and the best accuracy comparing to all previous works (SOM 8.2). The parameters a0 to a11 used in equation (5) for are listed in Table 2. With these parameters, any can be calculated in the range (0.01<<10) (SOM 8.3; SOM App. V).

**Table 1. Main characteristics and S factor (0.3 MeV) in the five schemes** (RR Unit=10-15 cm3mol-1s-1),

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Scheme  --Sn | Formulae for  γ channel | allαγ | /keVb | Error  /% | SE10  /keVb | SE20  /keVb | S6.05  /keVb | S6.13  /keVb | S6.92  /keVb | S7.12  /keVb | RR/  Unit |
| Lane-L3-S3 | Lane1958-s3 | 1901 | 160.38±7.16 | 4.47 | 94.71±6.40 | 55.70±4.46 | 2.25±0.61 | 0.14±0.05 | 6.96±1.60 | 0.45±0.04 | 7.79 |
| Lane-L3-S2 | Lane1958-s2 | 2087 | 159.78±8.16 | 5.11 | 91.46±6.78 | 56.70±4.58 | 4.04±0.43 | 0.17±0.06 | 6.95±1.59 | 0.45±0.04 | 7.76 |
| RAC2016 | Seyler1979-s2 | 2228 | 160.81±6.36 | 3.96 | 90.83±5.80 | 58.66±4.50 | 3.99±0.40 | 0.18±0.06 | 7.21±1.03 | 0.55±0.09 | 7.81 |
| Lane-L2-S3 | Lane1958-s3 | 2346 | 160.79±9.99 | 6.31 | 99.83±9.52 | 53.35±9.22 | 3.80±0.34 | 0.17±0.05 | 3.05±1.43 | 0.59±0.21 | 7.83 |
| An2015(*14*) | Lane1958-s3 | / | 162.7±7.3 | 4.49 | 98.0±7.0 | 56.0±4.1 | 4.90±1.20 | 0.20±0.10 | 3.00±0.40 | 0.60±0.20 | 7.83 |
| Mean values | / | / | 160.5±3.8 | 2.5 | 92.3±3.0 | 57.0±2.5 | 3.8±0.4 | 0.17±0.05 | 7.0±0.4 | 0.50±0.05 | 7.78 |

(5)

**Table 2. Parameters a0-a11 for analytic expression (4)**

|  |  |  |
| --- | --- | --- |
| =0.59605698D+09 | =0.56057001D+00 | =0.31559988D+02 |
| =0.40000000D+03 | =0.22658324D+17 | =0.41569810D+02 |
| =0.43745037D+02 | =0.25901325D+04 | =0.28428833D+02 |
| =0.11173825D+13 | =-0.10135299D+01 | =0.39356875D+02 |

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