Tau decays and some interesting applications

Lianrong Dai

Liaoning Normal University

研究目标

国内正拟建超级τ-粲工厂，但目前稀缺 τ 物理相关研究。

τ 衰变是唯一重轻子衰变，它有助于同时研究弱相互作用和强相互作用性质。

本工作提出了新颖代数方法，建立衰变振幅的解析表达式，可以进一步用来探讨tau衰变中可能的新机制和新现象。

结合手征幺正理论和三角奇异机制，首次在τ 衰变中检验了标量介子和轴矢量介子共振态的分子性质，还首次探讨了衰变末态介子的极化问题。

本工作开辟了一个全新的角度

研究强子共振态的性质

Outline

1. A novel approach on tau decays

Algebra method [LRD, Pavao, Sakai & Oset, EPJA55(2019)20]

2. First applications on tau decays

a) Scalar resonances [LRD, Yu & Oset, PRD99(2019)016021]

b) Axial-vector resonances [LRD, Roca & Oset, PRD99(2019)096003]

$\Longrightarrow$ testing the nature of these resonances within the chiral unitary approach

c) Polarization amplitude [LRD & Oset, EPJA54 (2018) 219]

1. A **novel** approach on tau decays

LRD, Pavao, Sakai & Oset, "$\tau^- \to \nu_\tau M_1 M_2$, with M_1, M_2 pseudoscalar or vector mesons", EPJA55(2019)20

Pseudoscalar (P) **Vector (V)**

Motivation

1) **Tau decays have been instrumental to learn about weak interaction as well as strong interaction.**
2) **Several modes are well measured, $\tau^- \to \nu_\tau \mathrm{PP}$ and $\tau^- \to \nu_\tau \mathrm{PV}$.**
3) **Surprisingly, there are no $\tau^- \to \nu_\tau \mathrm{VV}$ reported in the PDG**

$\Longrightarrow$ **we wonder whether there is some fundamental reason for this experimental fact ???**

a) P and V mesons **differ** only by the **spin arrangement** of the quarks
$\Longrightarrow$ **possible to relate** the rates of decay for $\tau^- \to \nu_\tau \mathrm{PP}, \mathrm{PV}, \mathrm{VV}$

b) one important issue is charge symmetry [S. Weinberg, PR112(1958) 1375]

one interesting reaction $\tau^- \to \nu_\tau \pi^- \eta(\eta')$
[Leroy & Pestieau, PLB72(1978)398] **[forbidden by *G*-parity]**

The *G*-parity plays an important role in these reactions. We offer a new perspective into this issue.

Algebra method [EPJA55(2019)20]

The derivation requires some patience, but we succeed using Racah algebra!!!

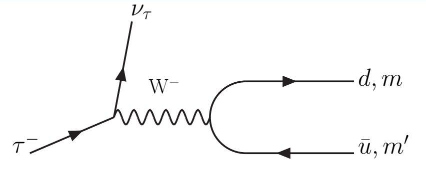
1) **no any free parameter**
2) **relate** the different processes
   a) relevant form factors would be the same
   b) Yet, the structures can be very different for the produced P or V

Finally we obtained the analytical amplitudes for each reaction

1) We evaluated the branching ratios of rates for PP, PV & VV cases $\Longrightarrow$ good agreement with experiment

2) make some predictions of invariant mass distributions and branching ratios of rates

Diagrams for elementary $\tau^- \to \nu_\tau d\bar{u}$ decay (left)
Hadronization through $\bar{q}q$ creation with vacuum quantum numbers (right)



$$M = \begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix}$$

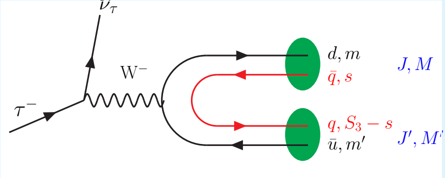
Cabibbo-favored $d\bar{u}$ production

$$d\bar{u} \to \sum_{i=1}^{3} d\,\bar{q}_i\,q_i\,\bar{u} = M_{2i}\,M_{i1} = (M \cdot M)_{21}$$

Cabibbo-suppressed $s\bar{u}$ production

$$s\bar{u} \to \sum_{i=1}^{3} s\,\bar{q}_i\,q_i\,\bar{u} = M_{3i}\,M_{i1} = (M \cdot M)_{31}$$

Hadronization of the primary $d\bar{u}$ pair to produce two mesons



For the hadronization, we use the $^3P_0$ model

1)L. Micu, Nucl. Phys. B 10, 521 (1969)
2)A. Le Yaouanc, et. al., Phys. Rev. D 8, 2223 (1973)
3)F. E. Close, An Introduction to Quark and Partons, Academic Press, 1979

The $^3P_0$ model has been widely used in the literature and recently it has been found very instrumental to address different problems in hadron physics

$$P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{\eta}{\sqrt{3}} + \frac{2\eta'}{\sqrt{6}} \end{pmatrix}$$

$$V_\mu = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}_\mu$$

Cabibbo-favored $d\bar{u}$ production

$$\begin{aligned} (P\cdot P)_{21} &= \frac{1}{\sqrt{2}}(\pi^-\pi^0 - \pi^0\pi^-) + \frac{1}{\sqrt{3}}(\pi^-\eta + \eta\pi^-) + \frac{1}{\sqrt{6}}(\pi^-\eta' + \eta'\pi^-) + K^0K^- , \\ (P\cdot V)_{21} &= \frac{1}{\sqrt{2}}(\pi^-\rho^0 + \pi^-\omega) - \frac{\pi^0\rho^-}{\sqrt{2}} + \frac{\eta\rho^-}{\sqrt{3}} + \frac{\eta'\rho^-}{\sqrt{6}} + K^0K^{*-} , \\ (V\cdot P)_{21} &= \frac{\rho^-\pi^0}{\sqrt{2}} + \frac{\rho^-\eta}{\sqrt{3}} + \frac{\rho^-\eta'}{\sqrt{6}} + \frac{1}{\sqrt{2}}(-\rho^0\pi^- + \omega\pi^-) + K^{*0}K^- , \\ (V\cdot V)_{21} &= \frac{1}{\sqrt{2}}(\rho^-\rho^0 - \rho^0\rho^-) + \frac{1}{\sqrt{2}}(\rho^-\omega - \omega\rho^-) + K^{*0}K^{*-} . \end{aligned} \tag{1}$$

Similarly, Cabibbo-suppressed $s\bar{u}$ production:

$(P\cdot P)_{31}$, $(P\cdot V)_{31}$, $(V\cdot P)_{31}$, $(V\cdot V)_{31}$

Weak matrix elements in Standard Model (SM)

$$H = \mathcal{C} L^{\alpha} Q_{\alpha}$$

where $\mathcal{C}$ containing weak interaction constants and radial matrix elements.

L^{α} is the leptonic current

$$L^{\mu} = \langle \bar{u}_{\nu} | \gamma^{\mu} - \gamma^{\mu}\gamma_5 | u_{\tau} \rangle$$

Q^{α} is the quark current

$$Q^{\mu} = \langle \bar{u}_{d} | \gamma^{\mu} - \gamma^{\mu}\gamma_5 | v_{\bar{u}} \rangle$$

For the evaluation of the matrix element Q_{μ} we assume that the quark spinors are at rest in that frame

$$u_r = \begin{pmatrix} \chi_r \\ 0 \end{pmatrix}, v_r = \begin{pmatrix} 0 \\ \chi_r \end{pmatrix}$$

$$\begin{aligned} Q_0 &= \langle \chi' | 1 | \chi \rangle \equiv M_0 \\ Q_i &= \langle \chi' | \sigma_i | \chi \rangle \equiv N_i \end{aligned}$$

Denoting for simplicity,

$$\overline{L}^{\mu\nu} = \overline{\sum} \sum L^{\mu} L^{\nu\,\dagger}$$

The amplitudes

$$\overline{\sum} \sum |t|^2 = \overline{\sum} \sum L^{\mu} L^{\nu\,\dagger} Q_{\mu} Q^{*}_{\nu}$$
$$= \overline{L}^{00} M_0\, M_0^{*} + \overline{L}^{0i} M_0\, N_i^{*} + \overline{L}^{i0} N_i\, M_0^{*} + \overline{L}^{ij} N_i\, N_j^{*}$$

where we sum over the final polarizations of the mesons produced.

$\overline{L}^{\mu\nu}$ is easily evaluated in [PRD92(2015)014031]

p-wave production

assuming $d, \bar{u}$ quarks are produced in their ground state

$\Longrightarrow$ this leads to a negative parity $q\bar{q}$ state, which makes the pair of mesons after the hadronization to be produced in p-wave.

[Later we find that p-wave production is compatible with experiment of $\tau^- \rightarrow \nu_\tau PP$ decays]

Table 1 M_0 matrix elements in p-wave

PP	$J=0, J'=0$	$M_0 = 0$
PV	$J=0, J'=1$	$M_0 = (-1)^{-M-M'} \frac{1}{\sqrt{6}}\, q\, Y_{1,-(M+M')}(\hat{\boldsymbol{q}})\, \delta_{M0}$
VP	$J=1, J'=0$	$M_0 = (-1)^{-M-M'} \frac{1}{\sqrt{6}}\, q\, Y_{1,-(M+M')}(\hat{\boldsymbol{q}})\, \delta_{M'0}$
VV	$J=1, J'=1$	$M_0 = (-1)^{-M-M'} \frac{1}{\sqrt{3}}\, \mathcal{C}(111; M, M', M+M')\, q\, Y_{1,-(M+M')}(\hat{\boldsymbol{q}})$

Table 2 N_μ matrix elements in p-wave

PP	$J=0, J'=0$	$N_\mu = \frac{1}{\sqrt{6}}\, q\, Y_{1,\mu}(\hat{\boldsymbol{q}})\, \delta_{M0}\, \delta_{M'0}$
PV	$J=0, J'=1$	$N_\mu = (-1)^{1-M'} \frac{1}{\sqrt{3}}\, q\, Y_{1,\mu-M'}(\hat{\boldsymbol{q}}) \mathcal{C}(111; M', -\mu, M'-\mu)\, \delta_{M0}$
VP	$J=1, J'=0$	$N_\mu = (-1)^{-M} \frac{1}{\sqrt{3}}\, q\, Y_{1,\mu-M}(\hat{\boldsymbol{q}}) \mathcal{C}(111; M, -\mu, M-\mu)\, \delta_{M'0}$
VV	$J=1, J'=1$	$N_\mu = \frac{1}{\sqrt{6}}\, q\, Y_{1,\mu-M-M'}(\hat{\boldsymbol{q}}) \{(-1)^{-M'} \delta_{\mu M}$
		$+2\,(-1)^{-M} \mathcal{C}(111; M, -\mu, M-\mu) \mathcal{C}(111; M', -M-M'+\mu, -M+\mu)\}$

with $\boldsymbol{q} = \boldsymbol{p}_1 - \boldsymbol{p}_2$, where $\boldsymbol{p_1}$, $\boldsymbol{p_2}$ are the momenta of the mesons produced.

It is the value of $M_0 = 0$ for PP and G-parity $\Longrightarrow$ what makes the matrix elements zero for the $\tau^- \to \nu_\tau \pi^- \eta(\eta')$ channels

We show from a different perspective that $\tau^- \to \nu_\tau \pi^- \eta(\eta')$ are forbidden by G-parity $\Longrightarrow$ in coincidence with results obtained through different methods [C. Leroy, J. Pestieau, PLB72(1978)398]

p-wave production and G-parity

Table 3 Signs resulting in the M_0 and N_μ amplitudes by permuting the order of the mesons in p-wave production

	PP	PV	VP	VV
M_0	0	$-$	$-$	$+$
N_μ	$-$	$+$	$+$	$-$

1) Taking the $\pi^-\pi^0$ channel

It comes with the combination $\pi^-\pi^0 - \pi^0\pi^-$. As a consequence N_μ adds for the two terms and we have a weight $2\frac{1}{\sqrt{2}}$ for the $\pi^-\pi^0$ channel

2) For interesting $\pi^-\eta$ channel

It comes with the combinations $\pi^-\eta + \eta\pi^-$, and then the combination of the two terms cancels $\Longrightarrow$ do not have $\pi^-\eta$ production

Table 4 Weights for the different channels after taking into account the M_1M_2 and M_2M_1 components after the hadronization

Channels	h_i (for M_0)	$\bar{h}_i$ (for N_μ)
$\pi^0\pi^-$	0	$\sqrt{2}$
$\pi^-\eta$	0	0
$\pi^-\eta'$	0	0

Table 5 Contributions of the different non-strange M_1M_2 pairs. The cross indicates non zero contribution for $\tau^- \to \nu_\tau PP$ decays

Channels	G-parity	M_0	N_μ
$\pi^-\pi^0$	$+$	0	$\times$
$\pi^-\eta$	$-$	0	0
$\pi^-\eta'$	$-$	0	0

1) Taking into account the G-parity of the mesons, we can associate a G-parity to all nonstrange M_1M_2 pairs.

2) on the other hand, the G-parity can be established from the original $d\bar{u}$ pair and the operator producing them, 1 or σ_i

The G-parity for quarks belonging to the same isospin multiplet is given

$$G = (-1)^{L+S+I}$$

$$L = 0, I = 1$$

$S = 0$ for the "1" operator

$S = 1$ for the "σ_i" operator

$\Longrightarrow$ G-parity negative for the 1 operator positive parity for the σ_i operator

s-wave production

Since the masses of these mesons are larger, the resulting momenta for the mesons are much smaller and the p-wave mechanism will lead to very small widths. Certainly, in this case, s-wave production shall be preferable.

For $\tau^- \to \nu_\tau$PV and $\tau^- \to \nu_\tau$VV production, two mesons with negative parity and s-wave have positive parity
$\Longrightarrow$ This means that the $d\bar{u}$ must be produced in an $L' = 1$ state.

[Later we find that s-wave production is compatible with experiment of $\tau^- \to \nu_\tau$PV and $\tau^- \to \nu_\tau$VV decays]

M_0 matrix elements in s-wave		
PP	$J=0, J'=0$	$M_0 = 0$
PV	$J=0, J'=1$	$M_0 = \frac{1}{\sqrt{6}}\frac{1}{4\pi}$
VP	$J=1, J'=0$	$M_0 = \frac{1}{\sqrt{6}}\frac{1}{4\pi}$
VV	$J=1, J'=1$	$M_0 = \frac{1}{\sqrt{3}}\frac{1}{4\pi}\mathcal{C}(111; M, M', M+M')$

N_μ matrix elements in s-wave		
PP	$J=0, J'=0$	$N_\mu = \frac{1}{\sqrt{6}}\frac{1}{4\pi}\,\delta_{M0}\,\delta_{M'0}\,(-1)^{-\mu}$
PV	$J=0, J'=1$	$N_\mu = -(-1)^{-\mu}\frac{1}{\sqrt{3}}\frac{1}{4\pi}\mathcal{C}(111; M', -\mu, M'-\mu)\,\delta_{M0}$
VP	$J=1, J'=0$	$N_\mu = (-1)^{-\mu}\frac{1}{\sqrt{3}}\frac{1}{4\pi}\mathcal{C}(111; M, -\mu, M-\mu)\,\delta_{M'0}$
VV	$J=1, J'=1$	$N_\mu = \frac{1}{\sqrt{6}}\frac{1}{4\pi}\Big\{\delta_{M\mu} + 2\,(-1)^{-\mu-M'}\mathcal{C}\,(111; M, -\mu, M-\mu)$ $\times\mathcal{C}\,(111; M', -M-M'+\mu, -M+\mu)\Big\}$

Signs resulting in the M_0 and N_μ amplitudes by permuting the order of the mesons in s-wave production

	PP	PV	VP	VV
M_0	$-$	$+$	$+$	$-$
N_μ	$+$	$-$	$-$	$+$

The analytical amplitudes for each reaction in s-wave production

1) PP, $J = 0, J' = 0$

$$\overline{\sum}\sum |t|^2 = \frac{1}{m_\tau m_\nu}\left(\frac{1}{4\pi}\right)^2 \left(E_\tau E_\nu - \frac{\boldsymbol{p}^2}{3}\right)\frac{1}{2}\overline{h}_i^2 \qquad \text{(2-a)}$$

2) PV, $J = 0, J' = 1$; VP, $J = 1, J' = 0$

$$\overline{\sum}\sum |t|^2 = \frac{1}{m_\tau m_\nu}\left(\frac{1}{4\pi}\right)^2 \left[\left(E_\tau E_\nu + \boldsymbol{p}^2\right)\frac{1}{2}h_i^2 + \left(E_\tau E_\nu - \frac{\boldsymbol{p}^2}{3}\right)\overline{h}_i^2\right] \text{(2-b)}$$

3) VV, $J = 1, J' = 1$

$$\overline{\sum}\sum |t|^2 = \frac{1}{m_\tau m_\nu}\left(\frac{1}{4\pi}\right)^2 \left[\left(E_\tau E_\nu + \boldsymbol{p}^2\right)h_i^2 + \frac{7}{2}\left(E_\tau E_\nu - \frac{\boldsymbol{p}^2}{3}\right)\overline{h}_i^2\right] \text{(2-c)}$$

h_i and $\bar{h}_i$ coefficient

Table 6 h_i **and** $\bar{h}_i$ **coefficient for different channels with the two final mesons in s-wave production**

channels	h_i (for M_0)	$\bar{h}_i$ (for N_μ)	
$\pi^-\rho^0$	0	$\sqrt{2}$	
$\pi^-\omega$	$\sqrt{2}$	0	
$\pi^0\rho^-$	0	$-\sqrt{2}$	
$\eta\rho^-$	$\frac{2}{\sqrt{3}}$	0	
$\eta'\rho^-$	$\frac{2}{\sqrt{6}}$	0	
$K^{*0}K^{*-}$	1	1	**Axial-vector resonances**
$\rho^-\rho^0$	$\sqrt{2}$	0	
$\rho^-\omega$	0	$\sqrt{2}$	
ηK^{*-}	0	$-\frac{2}{\sqrt{3}}\tan\theta_c$	
$\eta' K^{*-}$	$\frac{3}{\sqrt{6}}\tan\theta_c$	$\frac{1}{\sqrt{6}}\tan\theta_c$	
$K^{*0}K^-$	1	1	**Scalar resonances**
K^0K^{*-}	1	1	

The final differential mass distribution

$$\frac{d\Gamma}{dM_{\rm inv}(M_1 M_2)} = \frac{2\,m_\tau 2\,m_\nu}{(2\pi)^3} \frac{1}{4m_\tau^2}\, p_\nu \widetilde{p}_1 \overline{\sum} \sum |t|^2 \tag{3}$$

where p_ν is the neutrino momentum in the τ rest frame

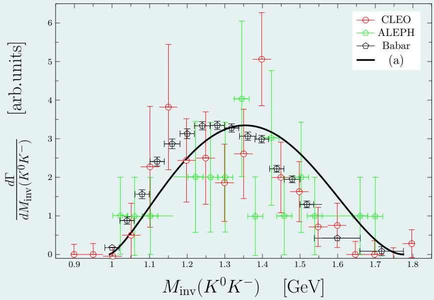
$$p_\nu = \frac{\lambda^{1/2}(m_\tau^2, m_\nu^2, M_{\rm inv}^2(M_1 M_2))}{2M_\tau}\,, \qquad \widetilde{p}_1 = \frac{\lambda^{1/2}(M_{\rm inv}^2(M_1 M_2), m_{M_1}^2, m_{M_2}^2)}{2M_{\rm inv}(M_1 M_2)}$$

and $\widetilde{p}_1$ the momentum of M_1 in the M_1, M_2 rest frame.

Then by integrating Eq. (3) **over the** $M_1 M_2$ **invariant mass, we obtain the width** [more details can be found in EPJA55(2019)20]

Comparison with invariant mass distributions

Invariant mass distribution for $\tau^- \to \nu_\tau K^0_S K^-$ decay (PP)

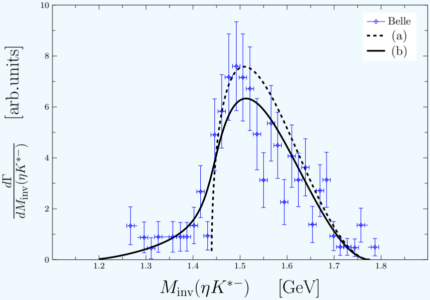


CLEO,PRD53(1996)6037;
ALEPH, EPJC4 (1998)29;

BaBar, PRD98(2018)032010 (precise spectrum)

The agreement of our results (a) with experiments is relatively good, particularly taking into account the very different shape compared to the (PV) case

Invariant mass distribution for $\tau^- \to \nu_\tau \eta K^{*-}$ decay (PV)



[Belle Collaboration],PLB672(2009)209

our results: (a) without a convolution (b) takes into account the width of the K^*

Again the agreement with experiment is good, and the shape of the distribution is very different to the one of K^-K^0 (PP) $\Longrightarrow$ This supports our conclusion that the PV pair is produced in s-wave

The obtained results

[EPJA55(2019)20]

♦ **From the comparison of our results with experiment for rates of branching ratios and invariant mass distributions finding that**

a) PP case, p-wave production

b) PV and VV cases, s-wave production

♦ **predictions for unmeasured decays**

Comparison with other theoretical approaches

our results are in line with the results of other approaches

2. Interesting applications -Part A

Final State Interaction + Triangle Singularity + The Chiral Unitary Approach

L. R. Dai, Q. X. Yu & E. Oset, "Triangle singularity in $\tau^- \to \nu_\tau \pi^- f_0(980)$ $(a_0(980))$ decays" [arXiv:1809.1100 & PRD99 (2019) 016021]

$\Longrightarrow$ **testing the nature of scalar $f_0(980)$ and $a_0(980)$ resonances**

In the chiral unitary approach that the $f_0(980)$ and $a_0(980)$ are dynamically generated resonances from the interaction of pseudoscalar mesons in coupled channels [Oller & Oset, NPA620(1997) 438 and ...].

$\tau^- \to \nu_\tau \pi^- f_0(980)\ (a_0(980))$

From the work [EPJA55(2019)20] we obtain the results for the $J = 1, J' = 0$ case, which correspond to the $\tau \to \nu_\tau K^{*0} K^-$ decay

Note that while M_0 is the same for VP and PV productions, N_i changes sign. This sign is essential for the conservation of G-parity in the reaction.

Signs resulting in the M_0 and N_μ amplitudes by permuting the order of the mesons in **s-wave production**

	PP	PV	VP	VV
M_0	$-$	$+$	$+$	$-$
N_μ	$+$	$-$	$-$	$+$

It was shown that the order in which the **vector** and **pseudoscalar** mesons are produced is essential to understand the G-parity symmetry of these reactions

G-Parity

π	$f_0(980)$	$a_0(980)$
$-$	$+$	$-$

a) $\pi^- f_0(980)$ **will proceed with the N_i amplitude**

b) **while $\pi^- a_0(980)$ proceeds with the M_0 term**

c) **there is no simultaneous contribution of the two terms in these reactions**

This we shall see analytically when evaluating explicitly the amplitudes for the next processes ...

Experimentally, the branching ratio $\mathcal{B}(\tau \to \nu_\tau K^{*0} K^-) = \frac{1}{\Gamma_\tau}\Gamma(\tau \to \nu_\tau K^{*0} K^-) = (2.1 \pm 0.4) \times 10^{-3}$

We obtain

$\frac{\mathcal{C}^2}{\Gamma_\tau} = (2.10 \pm 0.40) \times 10^{-5}\,\mathrm{MeV}^{-3}$,

where the errors come from the uncertainty of $\mathcal{B}(\tau \to \nu_\tau K^{*0} K^-)$

Diagram for the decay of $\tau^- \to \nu_\tau \pi^- \pi^+ \pi^-$

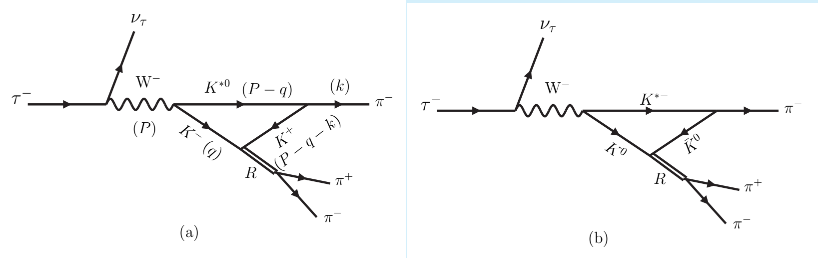
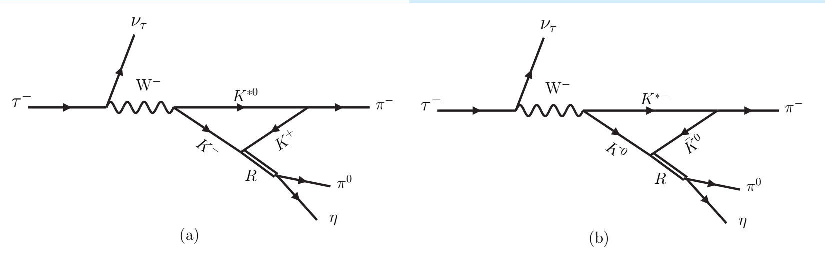


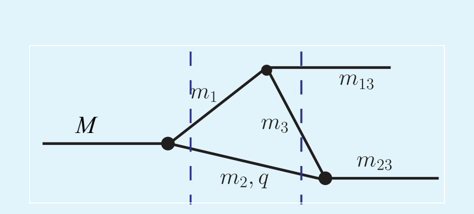
Diagram for the decay of $\tau^- \to \nu_\tau \pi^- \pi^0 \eta$



(see ZHAO Qiang's talk at 11:30, Oct.9 2019)

Triangle Singularity (TS)

if three intermediate particles are on shell and K^* and π^- are parallel $\Longrightarrow$ the mechanism generates a singularity in the amplitude for zero width of the K^*, or a peak if the width is considered



L. D. Landau, Nucl. Phys. 13 (1959) 181; Coleman, Norton, Nuovo Cim. 38 (1965)438;

More details in M. Bayar, F. Aceti, F. K. Guo & E. Oset, PRD 94, 074039 (2016)

TS at the physical boundary can be obtained by solving the equation

$$q_{\text{on}+} = q_{a-}\,, \text{with} \quad q_{\text{on}+} = \frac{1}{2M}\sqrt{\lambda(M^2, m_1^2, m_2^2)}, \;\; q_{a-} = \gamma\,(\beta\, E_2^* - p_2^*)$$

where $E_2^* = (m_{23}^2 + m_2^2 - m_3^2)/(2m_{23})$ and $p_2^* = \sqrt{\lambda(m_{23}^2, m_2^2, m_3^2)}/(2m_{23})$

TS in simulating a resonance

requires very special kinematics process dependent !!!

In explaining successfully the COMPASS "$a_1(1420)$" peak

[1] Mikhasenko, Ketzer & Sarantsev, PRD91,094015;

[2] Aceti, Dai & Oset, PRD94 (2016) 096015

"$a_1(1420)$ peak as the $\pi f_0(980)$ decay mode of the $a_1(1260)$ "

In some particular modes, the production rate is enhanced by the presence of a TS in the reaction mechanism.

Evaluation of the triangle diagram

◦ **In the chiral unitary approach**

we calculated the $K\bar{K} \to \pi^+\pi^-(\pi^0\eta)$ amplitudes

J. A. Oller and E. Oset, Nucl. Phys. A 620 (1997) 438;
W. H. Liang and E. Oset, Phys. Lett. B 737, 70 (2014);
J. J. Xie, L. R. Dai and E. Oset, Phys. Lett. B 742 (2015)363

◦ The $K^* \to K\pi$ vertex is obtained from the VPP Lagrangian

$$\mathcal{L}_{VPP} = -ig \langle V^\mu [P, \partial_\mu P] \rangle$$

with the coupling g given by $g = m_V/2f_\pi$ in the local hidden gauge approach, with $m_V = 800$ MeV and f_π=93 MeV. The equation is rather general and it can be obtained as well in massive Yang-Mills theory . From Eq. (0.6), the vertex $K^* \to K\pi$ is of the type $\epsilon[k - (-q - k)]$.

C. N. Yang and R. L. Mills, Phys. Rev. 96(1954)191;
S. Gasiorowicz and D. A. Geffen, Rev. Mod. Phys. 41(1969)531;
J. Schechter and Y. Ueda, Phys. Rev. 188(1969)2184;
U. G. Meissner, Phys. Rept. 161 (1988)213;

$$t_L = \int \frac{d^3q}{(2\pi)^3} \frac{1}{8\omega_{K^*}\omega_{K^+}\omega_{K^-}} \frac{1}{k^0 - \omega_{K^+} - \omega_{K^*} + i\frac{\Gamma_{K^*}}{2}} \left(2 + \frac{\boldsymbol{q}\cdot\boldsymbol{k}}{|\boldsymbol{k}|^2}\right) \frac{1}{P^0 + \omega_{K^-} + \omega_{K^+} - k^0}$$

$$\times \frac{1}{P^0 - \omega_{K^-} - \omega_{K^+} - k^0 + i\epsilon} \frac{2P^0\omega_{K^-} + 2k^0\omega_{K^+} - 2(\omega_{K^-} + \omega_{K^+})(\omega_{K^-} + \omega_{K^+} + \omega_{K^*})}{P^0 - \omega_{K^*} - \omega_{K^-} + i\frac{\Gamma_{K^*}}{2}} \theta(q_{\max} - q^*)$$

F. Aceti, J. M. Dias and E. Oset, Eur. Phys. J. A 51 (2015) 48;
M. Bayar, F. Aceti, F. K. Guo and E. Oset, Phys. Rev. D 94 (2016) 074039

Explicit filter of G-parity states

For the production of $\pi^- f_0(980)$ $\Longrightarrow$ **negative G-parity**

$$\overline{\sum}\sum |t|^2 = \bar{L}^{ij}\widetilde{N}_i\,\widetilde{N}_j^*\,g^2\,|\,2\,t_{K^+K^-,\pi^+\pi^-}|^2$$
$$= \frac{\mathcal{C}^2}{m_\tau m_\nu}\left(E_\tau E_\nu - \frac{1}{3}p^2\right)\frac{1}{3}\frac{1}{(4\pi)^2}\,k^2|t_L|^2\,g^2\,|\,2\,t_{K^+K^-,\pi^+\pi^-}|^2$$

For the production of $\pi^- a_0(980)$ $\Longrightarrow$ **positive G-parity**

$$\overline{\sum}\sum |t|^2 = \bar{L}^{00}\widetilde{M}_0\,\widetilde{M}_0^*\,g^2\,|\,2\,t_{K^+K^-,\pi^0\eta}|^2$$
$$= \frac{\mathcal{C}^2}{m_\tau m_\nu}\left(E_\tau E_\nu + p^2\right)\frac{1}{6}\frac{1}{(4\pi)^2}\,k^2|t_L|^2\,g^2\,|\,2\,t_{K^+K^-,\pi^0\eta}|^2$$

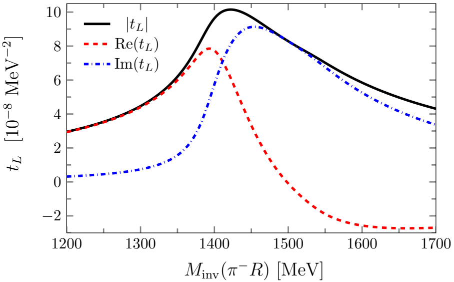
For $\tau^- \to \nu_\tau \pi^- \pi^+ \pi^-$ decay, **the double differential mass distribution** for $M_{\rm inv}(\pi^+\pi^-)$ and $M_{\rm inv}(\pi^- f_0)$ is given

$$\frac{1}{\Gamma_\tau}\frac{d^2\Gamma}{dM_{\rm inv}(\pi^- f_0)dM_{\rm inv}(\pi^+\pi^-)} = \frac{1}{(2\pi)^5}\frac{1}{\Gamma_\tau} k\, p'_\nu\, \tilde{q}_{\pi^+}\, \frac{2m_\tau 2m_\nu}{4M_\tau^2}\overline{\sum}\sum |t|^2$$

$$k = \frac{\lambda^{1/2}(M^2_{\rm inv}(\pi^- f_0), m_\pi^2, M^2_{\rm inv}(\pi^+\pi^-))}{2M_{\rm inv}(\pi^- f_0)},$$

$$p'_\nu = \frac{\lambda^{1/2}(m_\tau^2, m_\nu^2, M^2_{\rm inv}(\pi^- f_0))}{2m_\tau},$$

$$\tilde{q}_{\pi^+} = \frac{\lambda^{1/2}(M^2_{\rm inv}(\pi^+\pi^-), m_\pi^2, m_\pi^2)}{2M_{\rm inv}(\pi^+\pi^-)}.$$



◇ It can be observed that $\mathrm{Re}(t_T)$ has a peak around 1393 MeV, and $\mathrm{Im}(t_T)$ has a peak around 1454 MeV, ◇ there is a peak for $|t_T|$ around 1425 MeV.

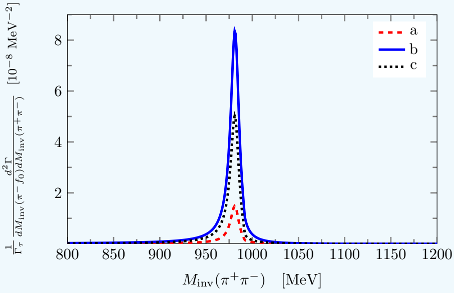
◇ The peak of the real part is related to the K^*K threshold

◇ The imaginary part, that dominates for the larger $\pi^- R$ invariant masses, to the triangle singularity

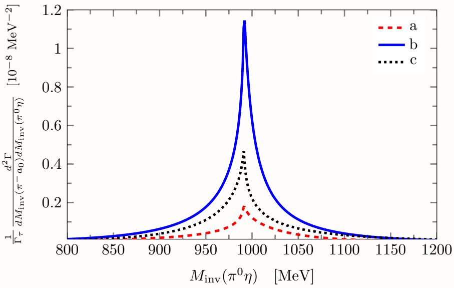
Note that around 1420 MeV and above the triangle singularity dominates the reaction

double differential mass distribution

For the $\tau^- \to \nu_\tau \pi^- \pi^+ \pi^-$ decay



For the $\tau^- \to \nu_\tau \pi^- \pi^0 \eta$ decay

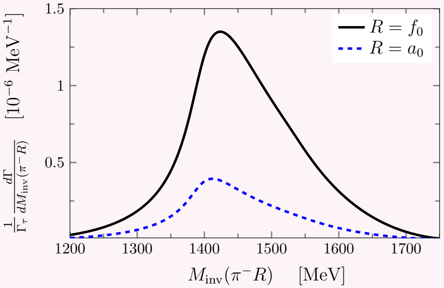


Lines a, b and c show the values at $M_{\rm inv}(\pi^- R)$ 1317 MeV, 1417 MeV, and 1517 MeV, respectively, as a function of $M_{\rm inv}(R)$

◇ The distribution with largest strength is near $M_{\rm inv}(\pi^- R)$=1417 MeV
◇ A strong peak in the $\pi^+\pi^-$ mass distribution around 980 MeV corresponding to the $f_0(980)$
◇ The distinctive cusp like $a_0(980)$ peak around 990 MeV for the $\pi^0\eta$ mass distribution

The branching ratios

In order to give a branching ratio for what an experimentalist would brand as $\pi^- f_0(980)$ or $\pi^- a_0(980)$ decay, we must integrate the strength of the double differential width.



Integrating $\frac{d\Gamma}{dM_{\rm inv}(\pi^- R)}$ over $M_{\rm inv}(\pi^- R)$ we obtain the branching fractions

$$\mathcal{B}(\tau^- \to \nu_\tau \pi^- f_0(980); \mathbf{f_0(980)} \to \pi^+\pi^-) = (2.6 \pm 0.5) \times 10^{-4}$$

$$\mathcal{B}(\tau^- \to \nu_\tau \pi^- a_0(980); \mathbf{a_0(980)} \to \pi^0\eta) = (7.1 \pm 1.4) \times 10^{-5}$$

Since the rate of $f_0 \to \pi^0\pi^0$ is one half that of $f_0 \to \pi^+\pi^-$, so

$$\mathcal{B}(\tau^- \to \nu_\tau \pi^- f_0(980)) = (3.9 \pm 0.8) \times 10^{-4}$$

The errors in these numbers count only the relative error of the branching ratio.

These numbers are within measurable range since branching ratios of 10^{-5} and smaller are quoted in the PDG for the τ decays.

2. Interesting applications -Part B

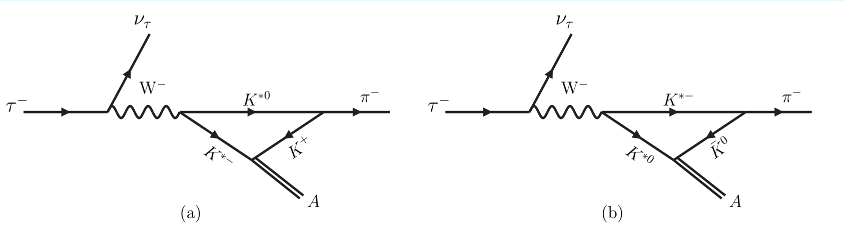
Final State Interaction + Triangle Singularity + The Chiral Unitary Approach

L. R. Dai, L. Roca & E. Oset, "τ decay into a pseudoscalar and an axial-vector meson" [arXiv:1811.06875 & PRD99 (2019) 096003]

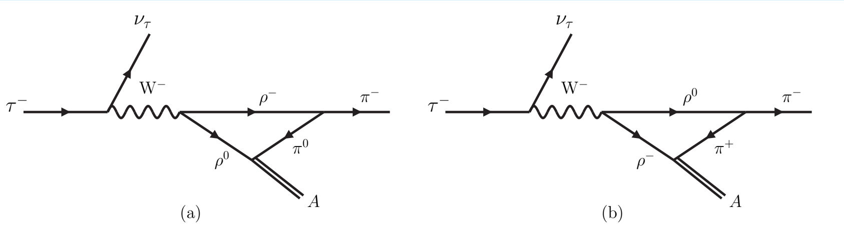
$\Longrightarrow$ **testing the nature of axial-vector resonances in the τ decay**

In the chiral unitary approach that the $f_1(1285)$, $b_1(1235)$, $h_1(1170)$, $h_1(1380)$, $a_1(1260)$ and two poles of the $K_1(1270)$ are dynamically generated axial-vector resonances from the interaction of vector-pseudoscalar (VP) mesons in coupled channels. [L. Roca, E. Oset and J. Singh, PRD72 (2005) 014002]

Diagrams for the decay of $\tau^- \to \nu_\tau \pi^- A$, with A axial vectors



The same as above but for $\tau^- \to \nu_\tau \rho^- \rho^0, \nu_\tau \rho^0 \rho^-$ decays



G-Parity		
π	$f_1(1285)$	$b_1(1235)$
$-$	$+$	$+$
$h_1(1170)$	$h_1(1385)$	$a_1(1260)$
$-$	$-$	$-$

a) $\pi^- f_1(1285)$ **and** $\pi^- b_1(1235)$ **will proceed with the** N_i **amplitude**

b) **while** $\pi^- h_1(1170)$, $\pi^- h_1(1380)$ **and** $\pi^- a_1(1260)$ **proceeds with the** M_0 **term**

c) **there is no simultaneous contribution in these reactions**

Explicit filter of G-parity states

For G-parity positive axial states:

$$\overline{\sum}\sum |t|^2 = \frac{\mathcal{C}^2}{m_\tau m_\nu}\frac{1}{(4\pi)^2}\frac{7}{6}(E_\tau E_\nu - \tfrac{1}{3}p^2)\, g^2 k^2 |g_{A,K^*\bar{K}}|^2\, |t_L(K^*\bar{K}^*)|^2$$

For G-parity negative axial states:

$$\overline{\sum}\sum |t|^2 = \frac{\mathcal{C}^2}{m_\tau m_\nu}\frac{1}{(4\pi)^2}\frac{1}{3}(E_\tau E_\nu + p^2)\, g^2\, k^2\, |(-1) g_{A,K^*\bar{K}} t_L(K^*\bar{K}^*) - 2\, D (-1) g_{A,\rho\pi} t_L(\rho\rho)|^2$$

Experimentally, the branching ratio

$$\mathcal{B}(\tau \to \nu_\tau K^{*0}K^{*-}) = \frac{1}{\Gamma_\tau}\Gamma(\tau \to \nu_\tau K^{*0}K^{*-}) = (2.1 \pm 0.5) \times 10^{-3}$$

We obtain $\frac{\mathcal{C}^2}{\Gamma_\tau} = (5.0) \times 10^{-4}\,\mathrm{MeV}^{-1}$

The final branching ratios

The branching ratios for $\tau^- \to \nu_\tau \pi^- A$

	$\mathcal{B}$
$h_1(1170)$	3.1×10^{-3}
$a_1(1260)$	1.3×10^{-3}
$b_1(1235)$	2.4×10^{-4}
$f_1(1285)$	2.4×10^{-4}
$h_1(1380)$	3.8×10^{-5}

The branching ratios for $\tau^- \to \nu_\tau K^- K_1$ decays

	$\mathcal{B}$
$K_1(1)$	2.1×10^{-5}
$K_1(2)$	4.1×10^{-6}

These numbers are within measurable range!!!

[details in PRD99 (2019) 096003]

2. Interesting applications -Part C

Polarization amplitudes

L. R. Dai & E. Oset, "Polarization amplitudes in $\tau^- \to \nu_\tau VP$ decay beyond the Standard Model"
[arXiv:1809.02510 & EPJA54 (2018) 219]

Application of the established formalism

[EPJA55(2019)20]

- $\tau^- \to \nu_\tau VP$
- **project over spin components**
- M, M' are the third components of the K^{*0} and K^-, respectively,

K^{*0}	$J = 1$	$M = 0, \pm 1$
K^-	$J' = 0$	$M' = 0$

- The **quantization axis** is taken along the **direction of the neutrino** in the τ^- rest frame.

We obtain the τ decay amplitude for different spin M components

[EPJA54(2018)219]

1) $M = 0$

$$\overline{\sum}\sum |t|^2 = \frac{1}{m_\tau m_\nu}\frac{1}{6}\frac{1}{(4\pi)^2}\left(3E_\tau E_\nu - p^2\right)$$

2) $M = 1$

$$\overline{\sum}\sum |t|^2 = \frac{1}{m_\tau m_\nu}\frac{1}{6}\frac{1}{(4\pi)^2}\left[3E_\tau E_\nu + p^2 + (3E_\nu + E_\tau)p\right]$$

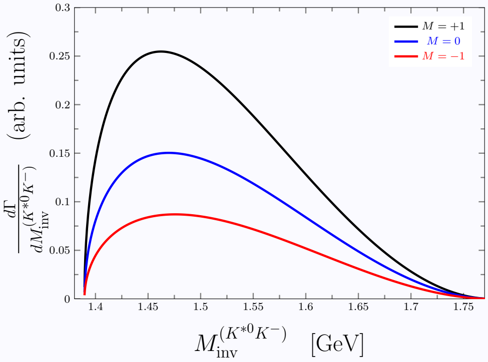
3) $M = -1$

$$\overline{\sum}\sum |t|^2 = \frac{1}{m_\tau m_\nu}\frac{1}{6}\frac{1}{(4\pi)^2}\left[3E_\tau E_\nu + p^2 - (3E_\nu + E_\tau)p\right]$$

The final differential width for each M

$$\frac{d\Gamma}{dM_{\rm inv}^{(K^{*0}K^-)}} = \frac{2\,m_\tau 2\,m_\nu}{(2\pi)^3}\,\frac{1}{4m_\tau^2}\,p_\nu \widetilde{p}_1 \overline{\sum}\sum |t|^2$$

where p_ν is the momentum of neutrino in the τ rest frame and $\widetilde{p}_1$ of K^{*0} in the $K^{*0}K^-$ rest frame



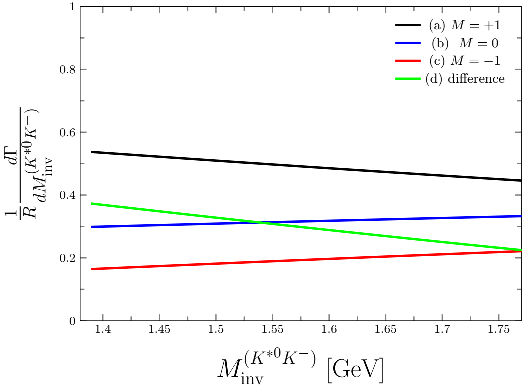
$\Longrightarrow$ We show the **individual** contributions are different for each M

Ratios divided by the total differential width R

$\Longrightarrow$ propose the measurement

$$R = \frac{d\Gamma}{dM_{\rm inv}^{(K^{*0}K^-)}}\Big|_{M=+1} + \frac{d\Gamma}{dM_{\rm inv}^{(K^{*0}K^-)}}\Big|_{M=0} + \frac{d\Gamma}{dM_{\rm inv}^{(K^{*0}K^-)}}\Big|_{M=-1}$$

the difference $\frac{1}{R}\left[\frac{d\Gamma}{dM_{\rm inv}^{(K^-K^{*0})}}\Big|_{M=+1} - \frac{d\Gamma}{dM_{\rm inv}^{(K^-K^{*0})}}\Big|_{M=-1}\right]$



different contributions for each M

line (d): the difference $\Longrightarrow$ a big sensitivity of magnitude

Consideration of right-handed quark currents

◇ The literature about models **beyond the Standard Model (BSM)** is **large** and this is not the place to discuss it

◇ Some models **BSM** have quark currents that contain the combination $\gamma^\mu + \gamma^\mu \gamma_5$ [X. G. He and G. Valencia, PRD 87(2013)014014; PLB 779(2018)52]

The above models could be accommodated with an operator

$$a(\gamma^\mu - \gamma^\mu\gamma_5) + b(\gamma^\mu + \gamma^\mu\gamma_5) \\ = (a+b)\left\{\gamma^\mu - \frac{a-b}{a+b}\gamma^\mu\gamma_5\right\} = \gamma^\mu - \alpha\gamma^\mu\gamma_5 \,.$$

we will study the distributions **for different M' as a function of α**

The new differential widths (BSM)

[EPJA54(2018)219]

1) $M = 0$

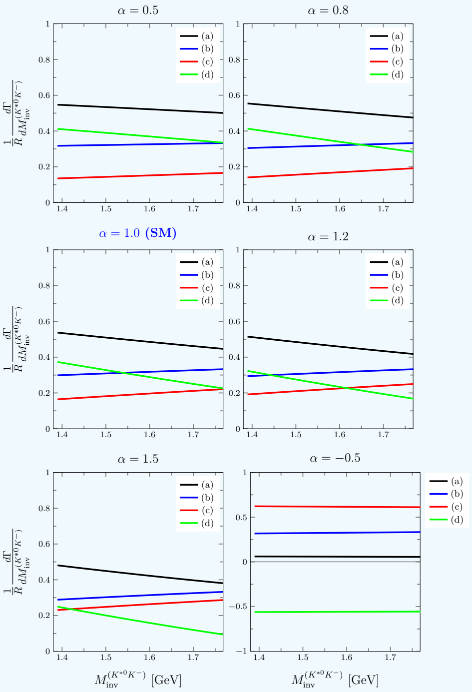
$$\overline{\sum}\sum |t|^2 = \frac{1}{m_\tau m_\nu}\frac{1}{6}\frac{1}{(4\pi)^2}\left\{\left(E_\tau E_\nu + p^2\right) + 2\alpha^2\left(E_\tau E_\nu - p^2\right)\right\}$$

2) $M = 1$

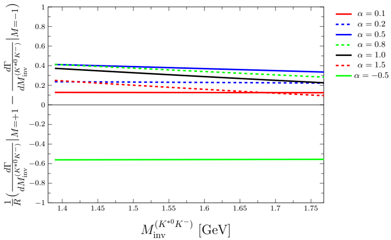
$$\overline{\sum}\sum |t|^2 = \frac{1}{m_\tau m_\nu}\frac{1}{6}\frac{1}{(4\pi)^2}\left\{(E_\tau E_\nu + p^2) + 2\alpha(E_\nu + E_\tau)p + \left[2E_\tau E_\nu + (E_\nu - E_\tau)p\right]\alpha^2\right\}$$

3) $M = -1$

$$\overline{\sum}\sum |t|^2 = \frac{1}{m_\tau m_\nu}\frac{1}{6}\frac{1}{(4\pi)^2}\left\{(E_\tau E_\nu + p^2) - 2\alpha(E_\nu + E_\tau)p + \left[2E_\tau E_\nu - (E_\nu - E_\tau)p\right]\alpha^2\right\}$$



let us see the difference



Individual contributions depend strongly on different α

case (d): very sensitive to the change of α

$\Longrightarrow$ This magnitude should be easy to differentiate experimentally

结论：到底做了什么

针对国内正拟建超级τ-粲工厂，而国际上τ 衰变研究工作很稀缺这一问题，开展了相关的工作。

a) 本工作提出了新颖代数方法，建立衰变振幅的解析表达式。

b) 本工作在τ衰变中开辟了一个全新角度，研究强子共振态性质，探讨可能的新机制和新现象。

结合手征幺正理论，本工作首次提出了在τ 衰变中研究triangle singularity机制以检验标量介子和轴矢量介子共振态的分子性质。

我们首次探讨了τ 衰变末态介子的极化问题和可能的新物理。

谢谢！