Ab initio chiral three-body force in resonance and continuum

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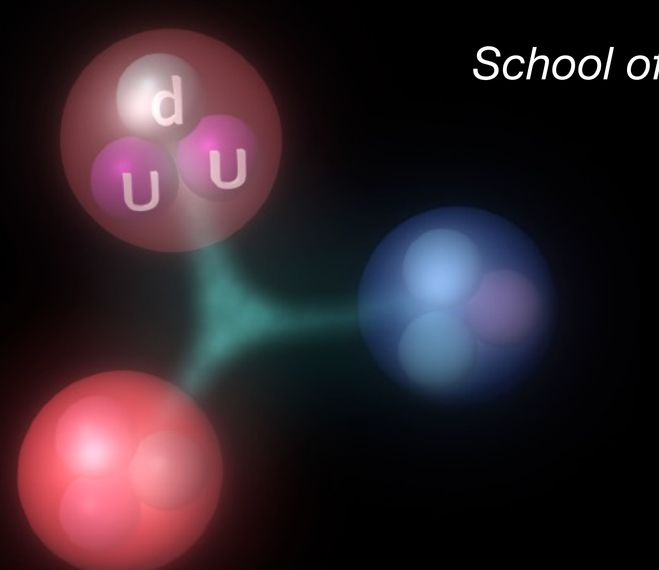

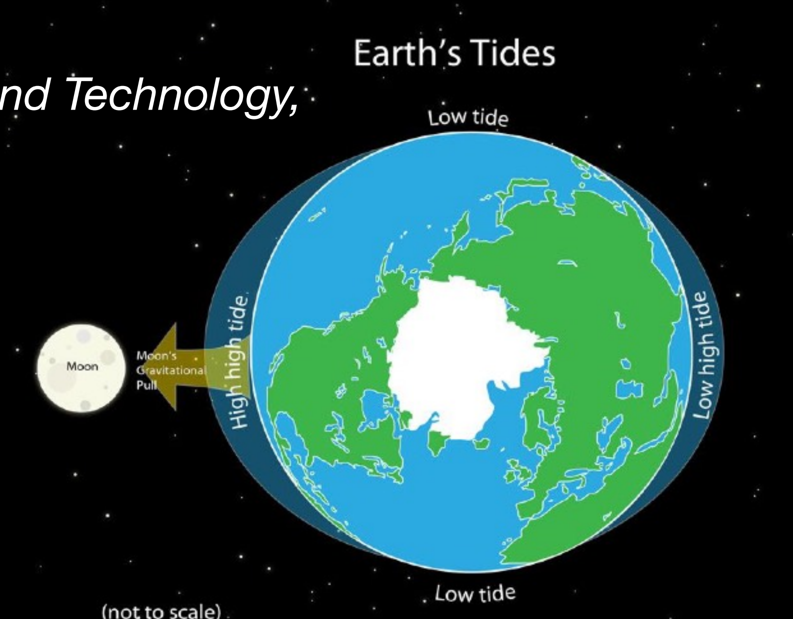

National Geography

2019-Oct-10

Acknowledgement

PKU

- F. R. Xu
- B. S. Hu
- J. G. Li
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INFN

- L. Coraggio
- T. Fukui
- A. Gargano
- L. De Angelies
- N. Itaco

IMP

N. Michel

MSU

S. M. Wang

Oak Ridge

Z. H. Sun

PKU: *School of Physics, Peking University, China*

INFN: *Istituto Nazionale di Fisica Nucleare, Sezione di Napoli, Italy*

IMP: *Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China*

MSU: *FRIB/NSCL Laboratory, Michigan State University, USA*

Oak Ridge: *Physics Division, Oak Ridge National Laboratory, Oak Ridge, TN 37831, USA*

Nuclear Force is not a Basic Force

Nuclear Force

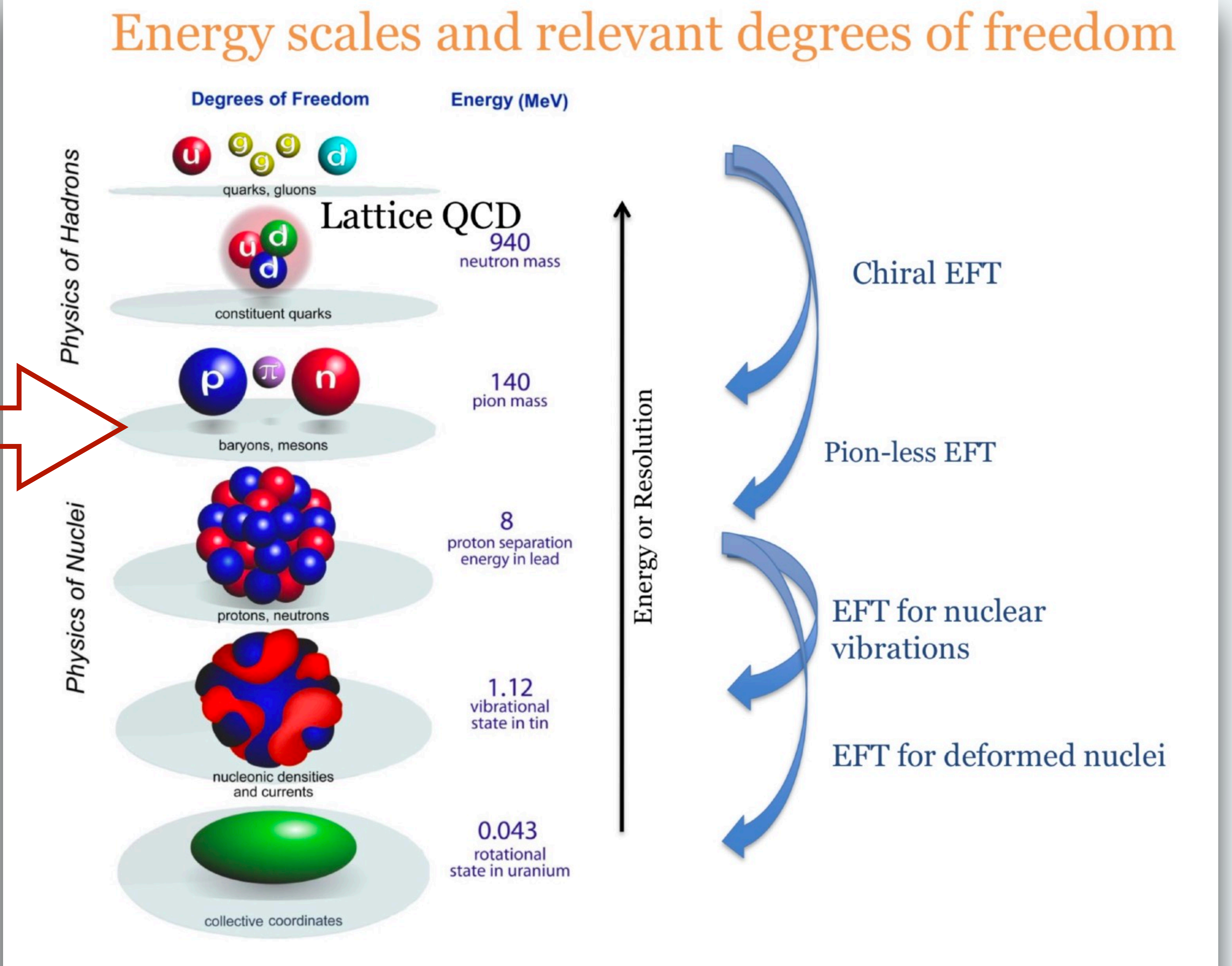

Fig.: Bertsch, Dean, Nazarewicz (2007)

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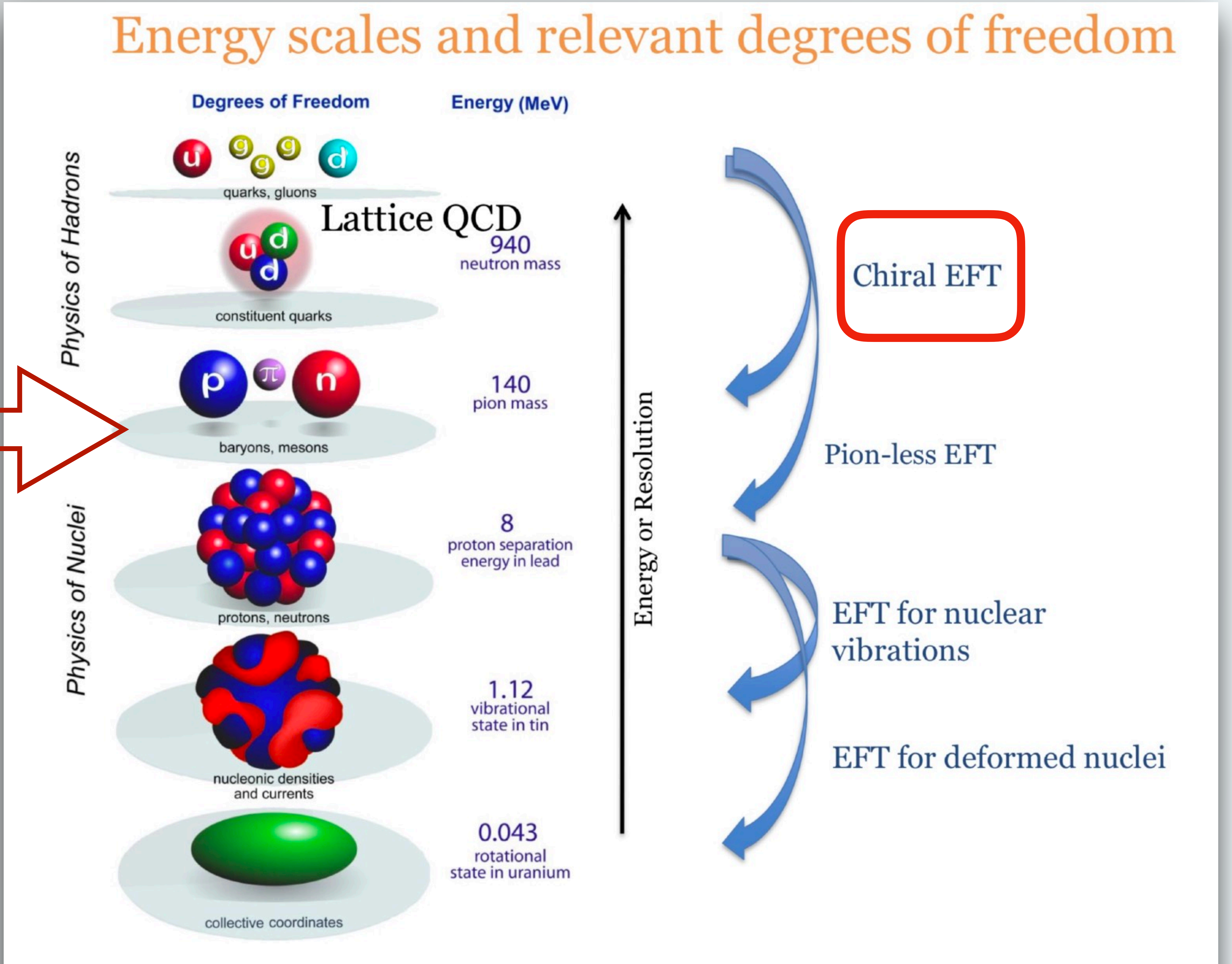


Fig.: Bertsch, Dean, Nazarewicz (2007)

3NF from Chiral EFT

QCD and **nuclear physics** can be linked by **Chiral EFT**

Effective Lagrangians (for nuclear forces)

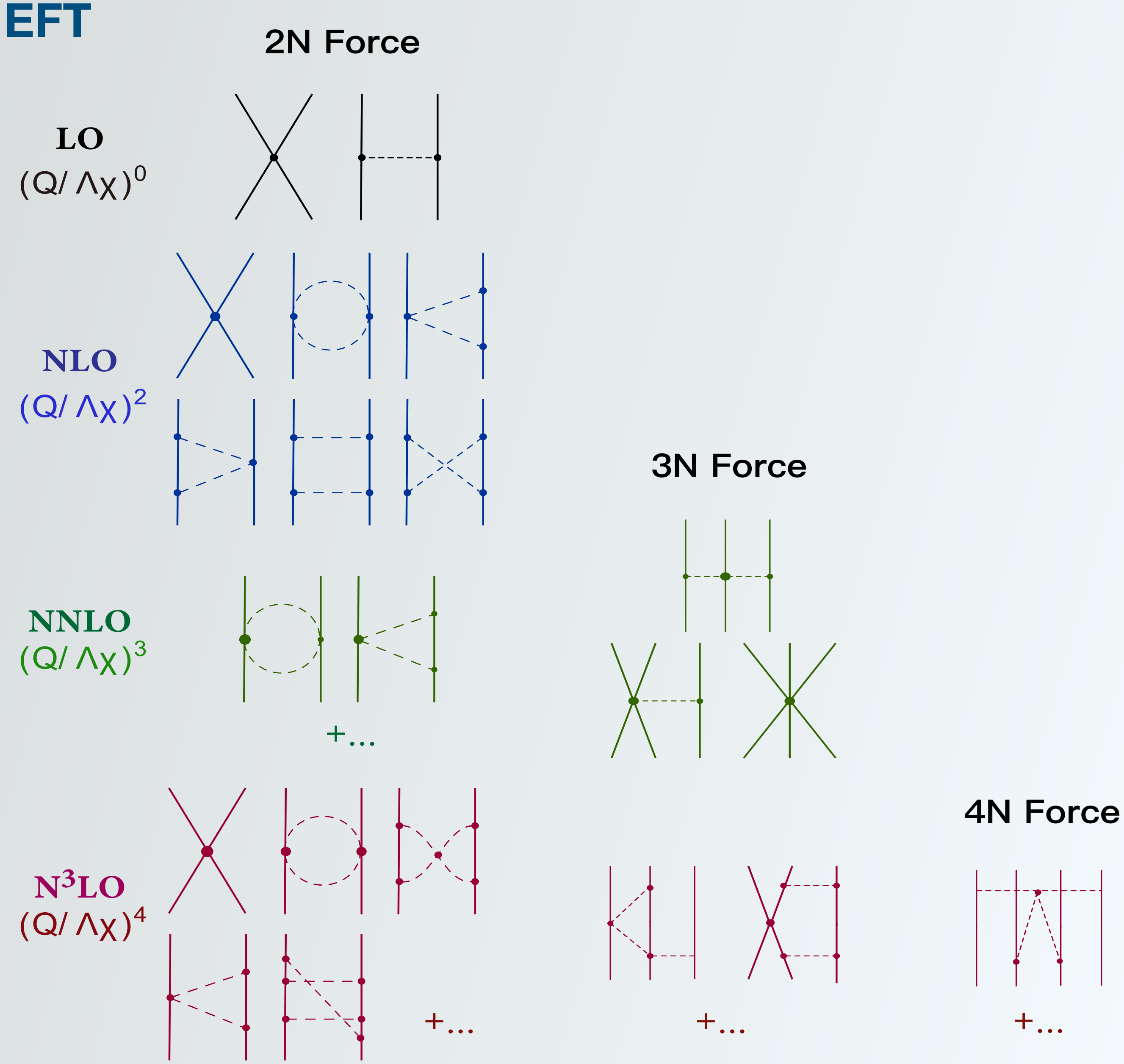
$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N} + \mathcal{L}_{NN} + \cdots$$

Infinitely many terms

A scheme to make the theory manageable and calculable

Chiral perturbation theory (ChPT)

- Degree of freedom : nucleons and pions
- Chiral symmetry
- Many body forces on equal footing



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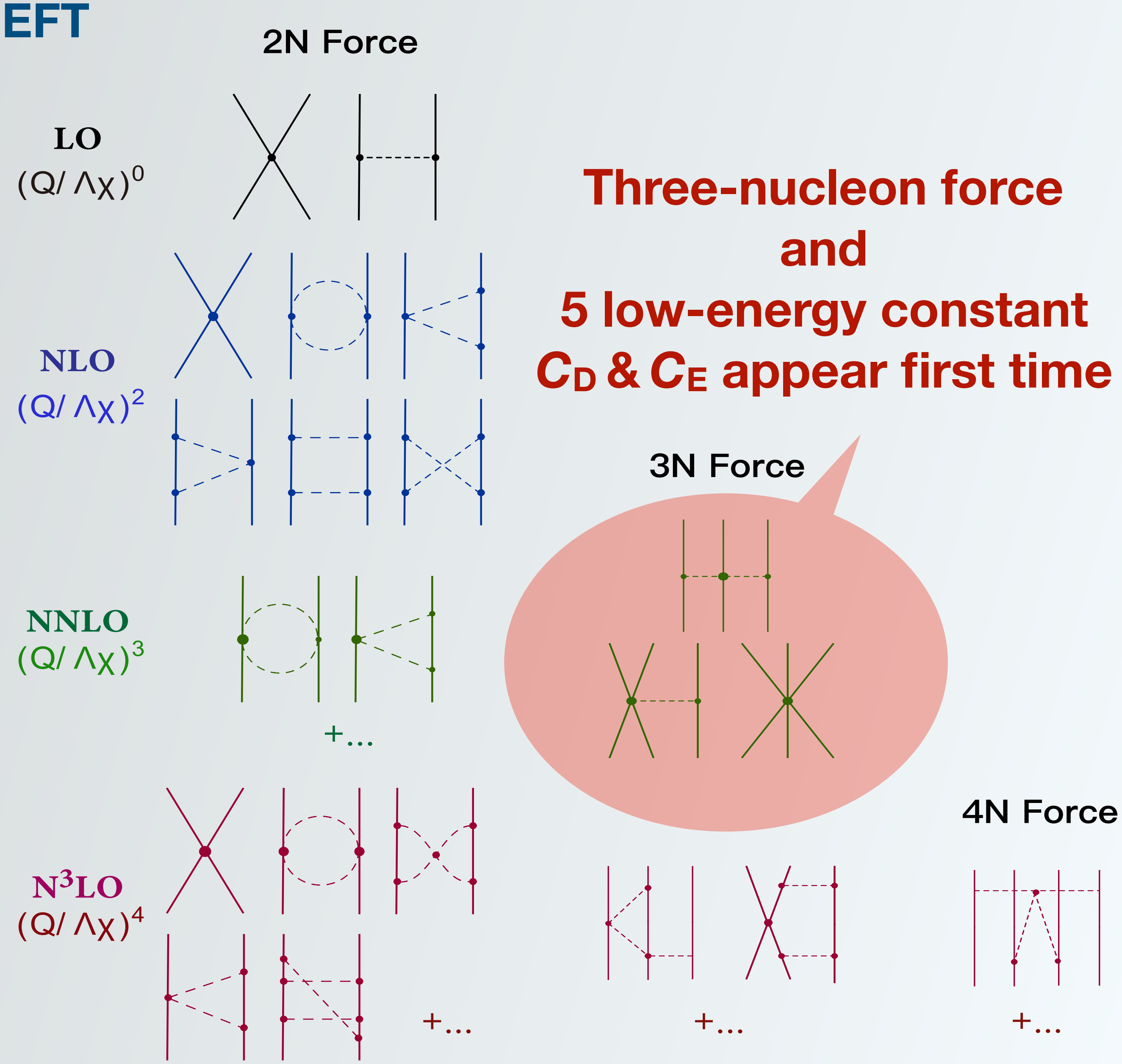
From N²LO, three-nucleon force (3NF) appears



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3NF Significance

3NF has significant contribution not only for binding energies but also for the description of excitation spectra and other observables

Ground-state energy from SRG

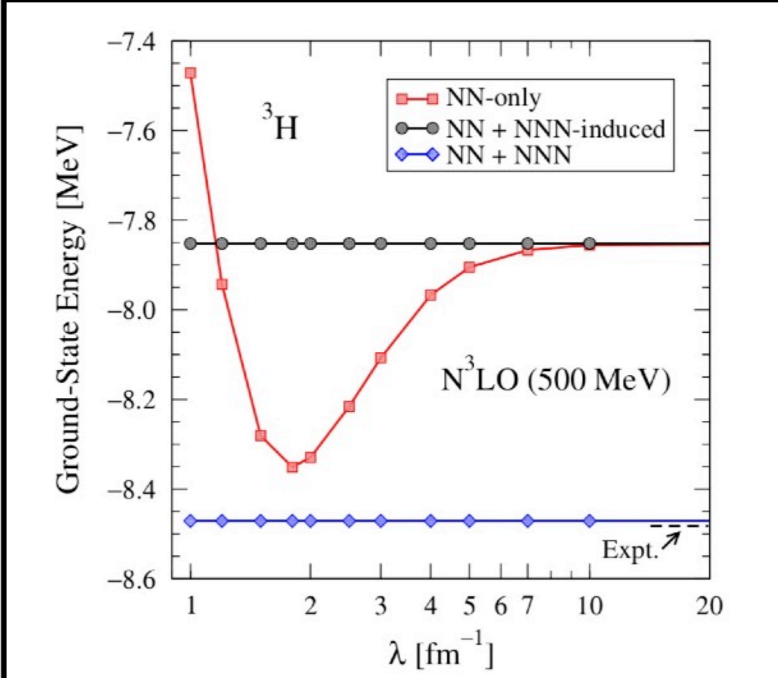

FIG. 1 (color online). Ground-state energy of ^{3}H as a function of the SRG evolution parameter, λ. See Table I for the nomenclature of the curves.

E. D. Jurgenson, P. Navrátil *et al.*,

Phys. Rev. Lett. **103**, 082501 (2009)

Saturation of nuclear matter

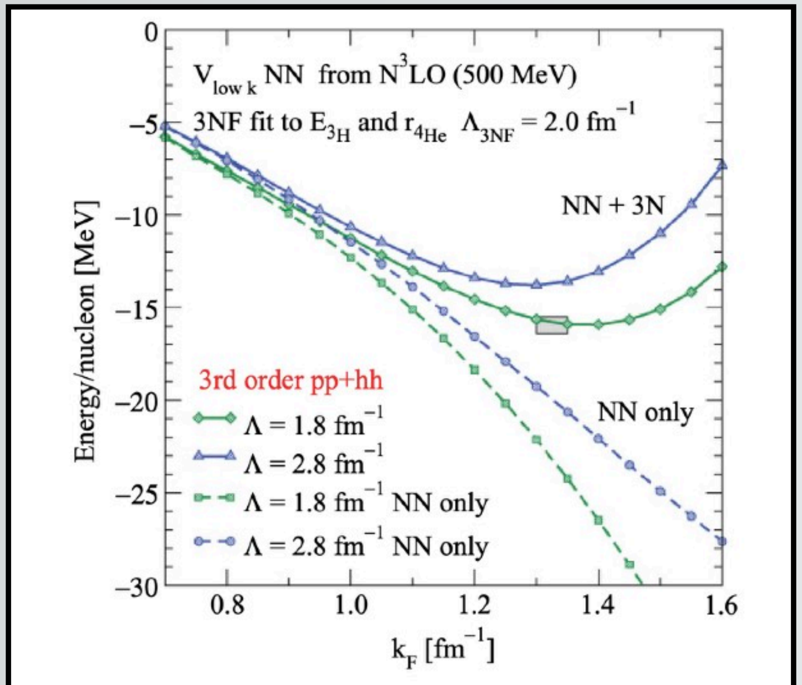

FIG. 2. (Color online) Nuclear matter energy of Fig. 1 at the third-order level compared to NN-only results for two representative NN cutoffs and a fixed 3N cutoff.

K. Hebeler, S. K. Bogner *et al.*,

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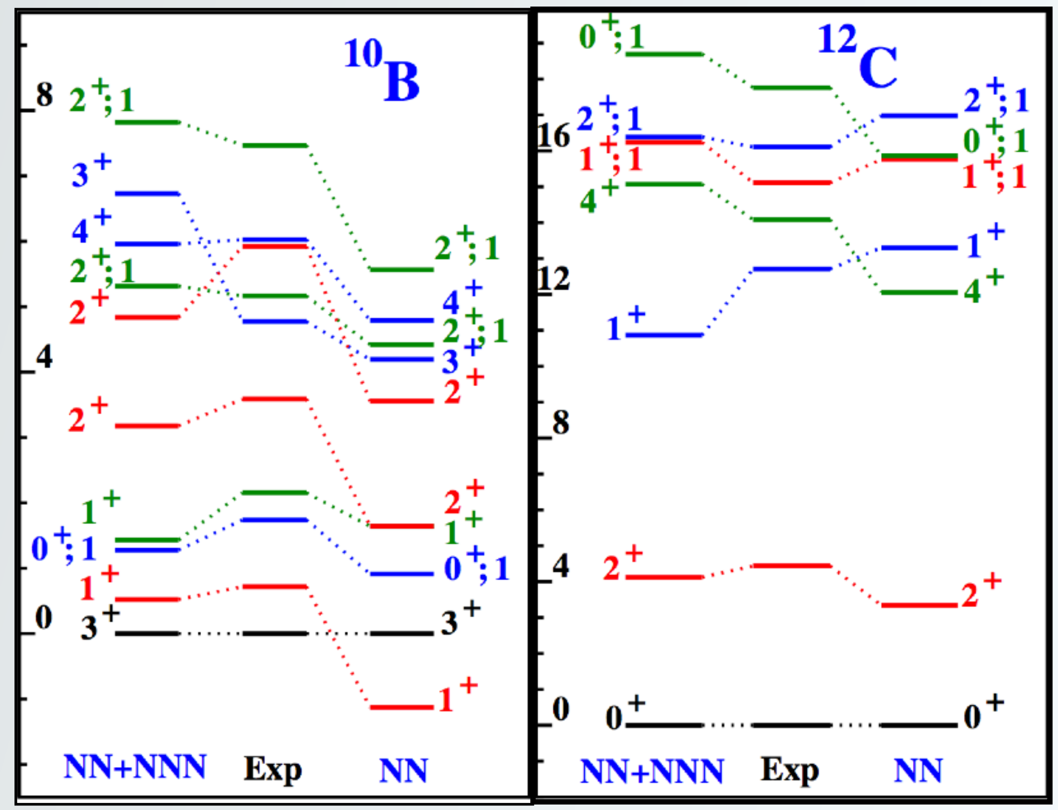
Spectra from ab-initio no-core shell model



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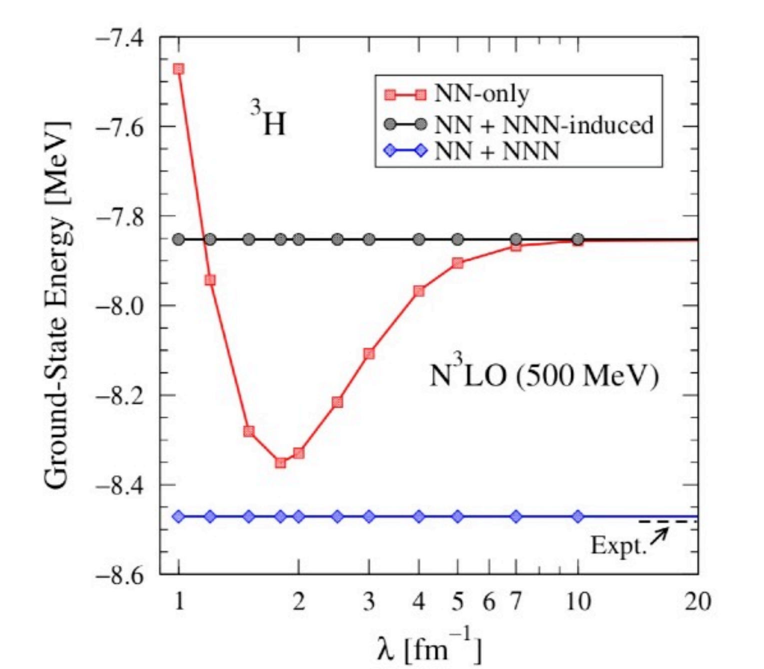

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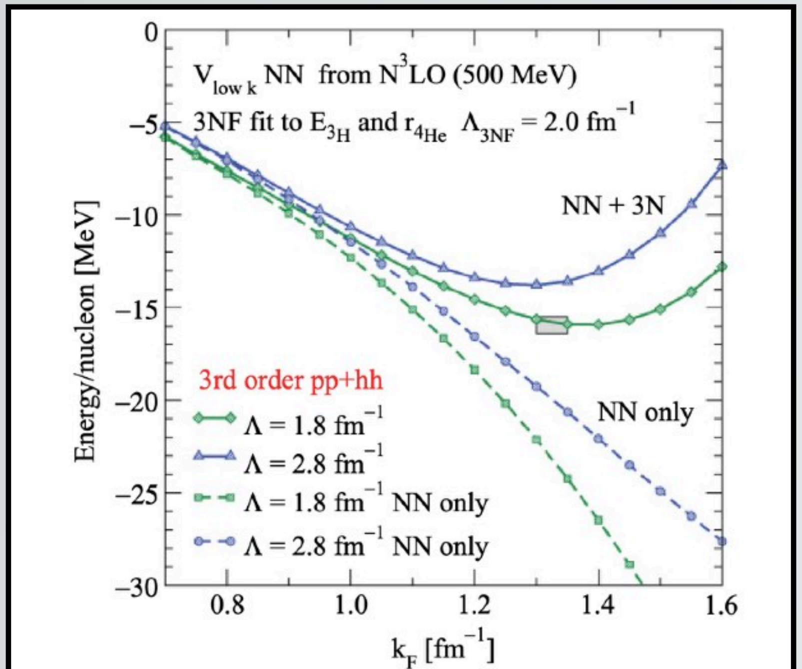

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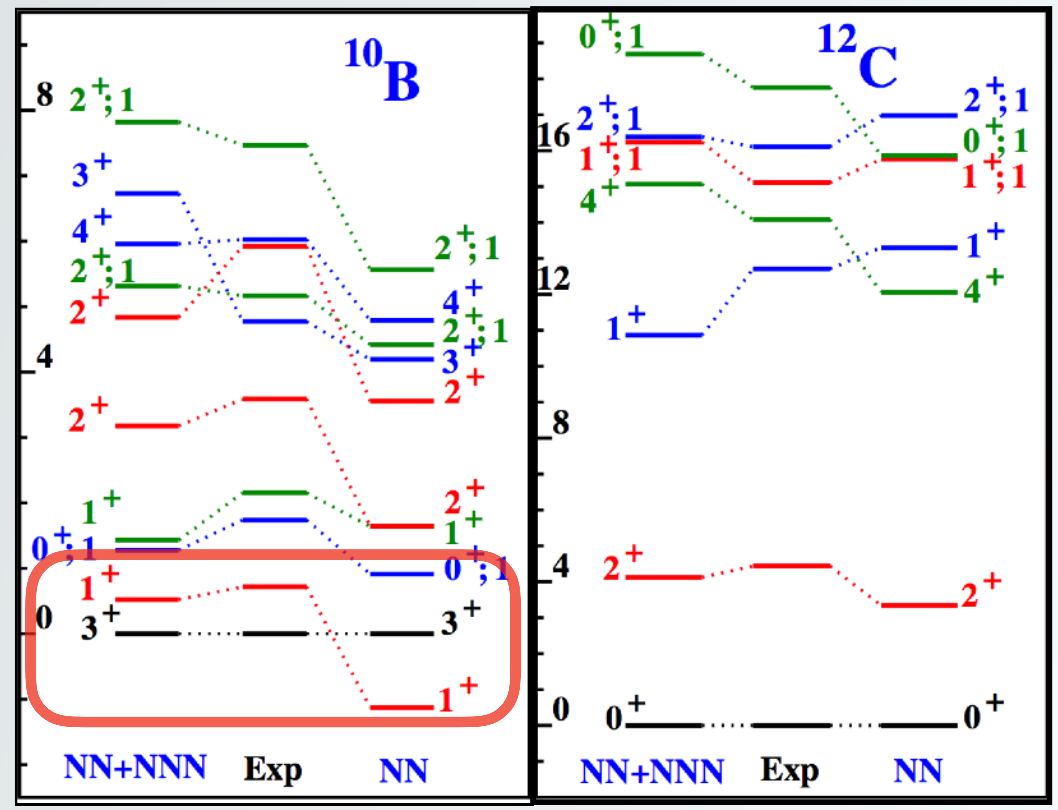

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Purpose & Procedure

Motivation

- Including 3NF based on EFT in nuclear many body calculations by means of the harmonic-oscillator basis.
- Investigating 3NF effects and the dependence of cut off (regulator), LECs, model space, etc.

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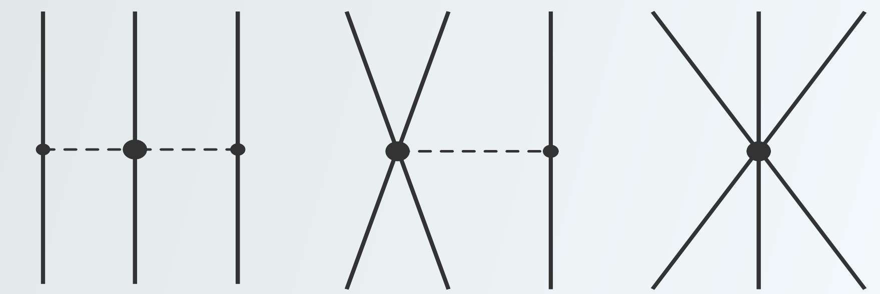
Procedure

$$ {}_{A}\langle [[\bullet\bullet]\bullet]_{JT} | V_{3N} | [[\bullet\bullet]\bullet]_{JT} \rangle_{A} $$



$V_{3N}^{2\pi-exch}$ $V_{3N}^{1\pi+contact}$ $V_{3N}^{contact}$

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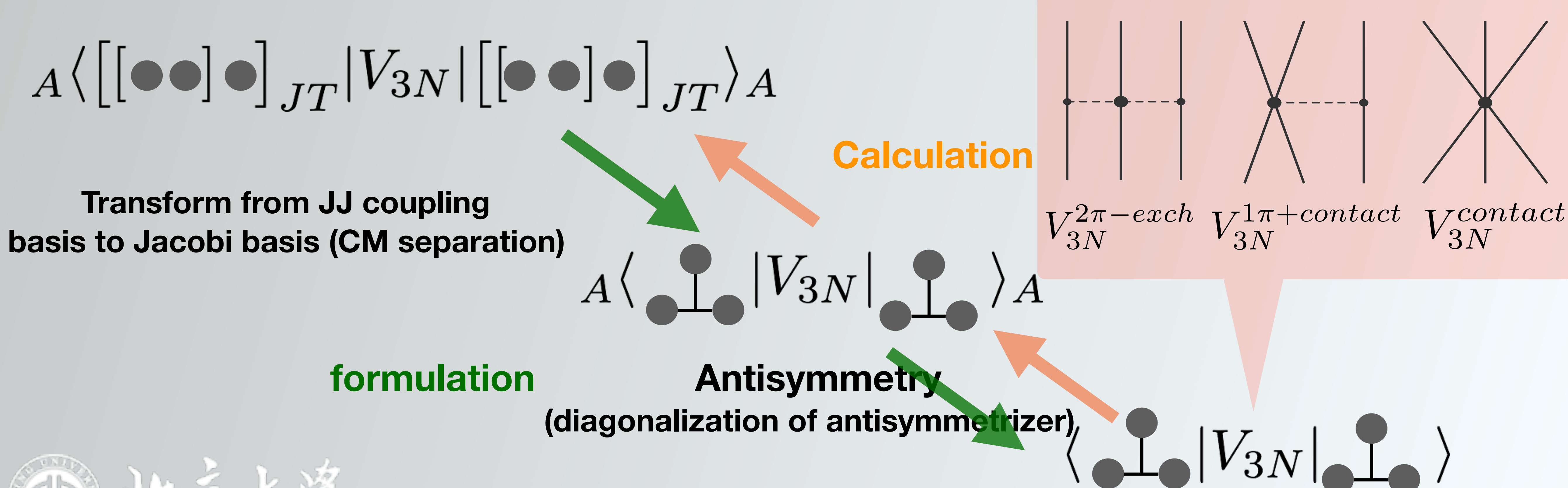

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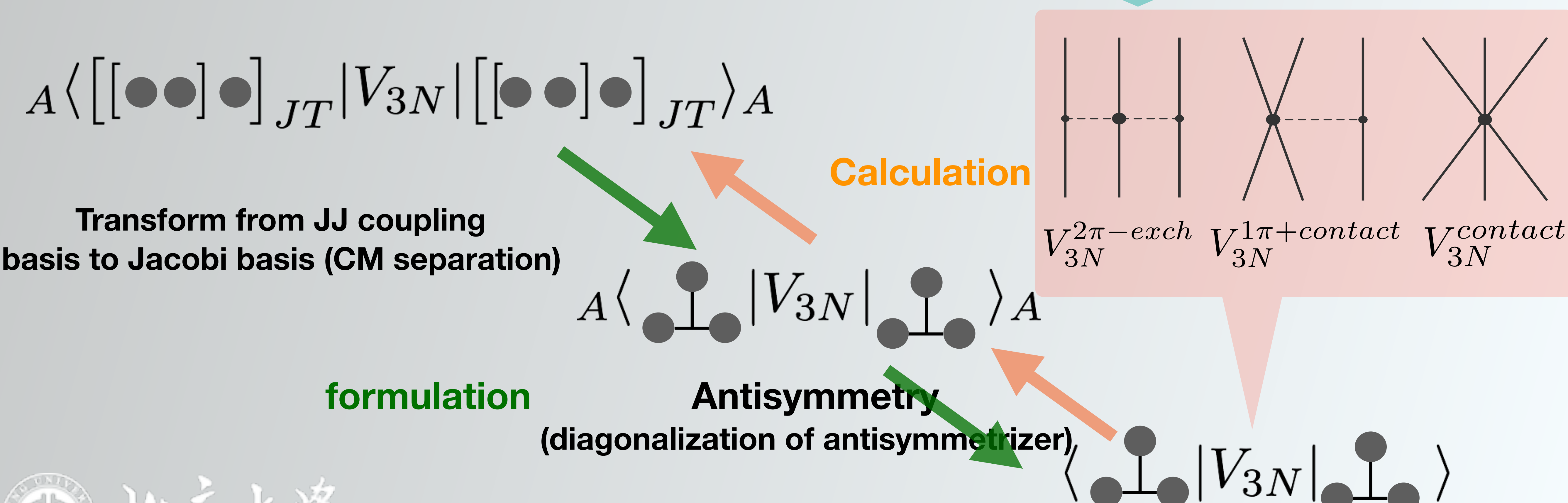
$$V_{3N}^{2\pi-exch} = \frac{g_A^2}{8f_\pi^2} \sum_{i\neq j\neq k} \frac{\sigma_i\cdot \mathbf{Q}_i}{Q_i^2+M_\pi^2}\,\frac{\sigma_j\cdot \mathbf{Q}'_j}{Q'^2_j+M_\pi^2} \times F^{\alpha\beta}_{ijk}\tau_i^\alpha\tau_j^\beta$$

$$F^{\alpha\beta}_{ijk}\tau_i^\alpha\tau_j^\beta = \delta_{\alpha\beta}[-4c_1 m_\pi^2 + 2c_3\mathbf{q}_i\cdot\mathbf{q}_j] + c_4\epsilon_{\alpha\beta\gamma}\sigma_k\cdot(\mathbf{q}_i\times\mathbf{q}_j)$$

Procedure



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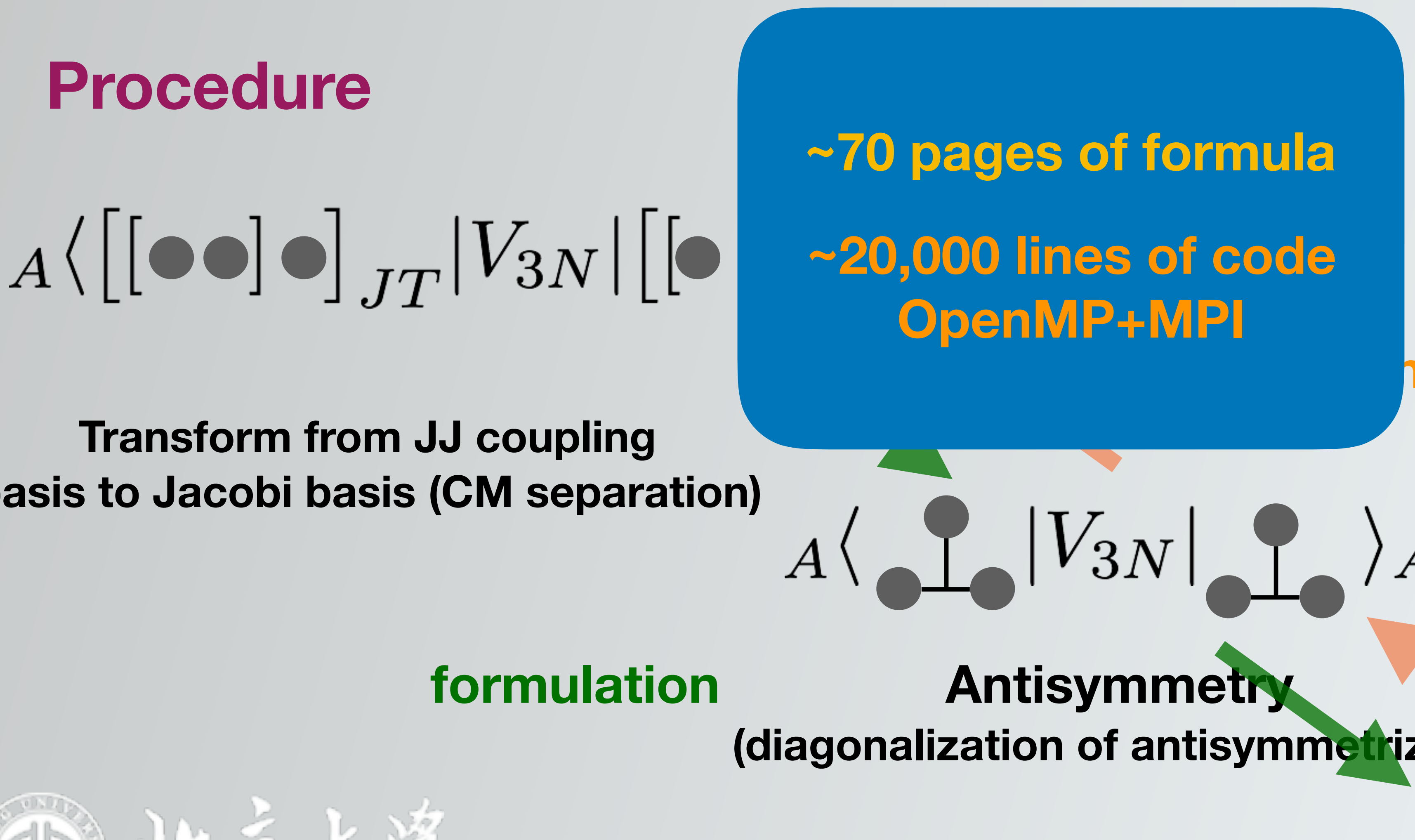
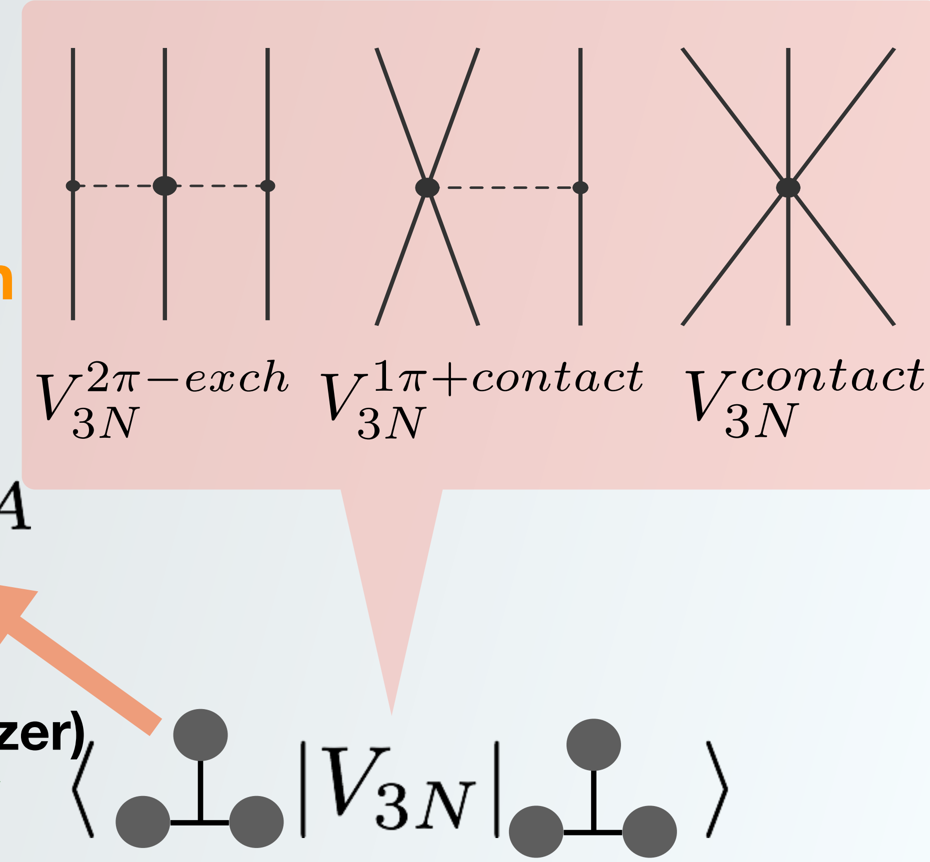
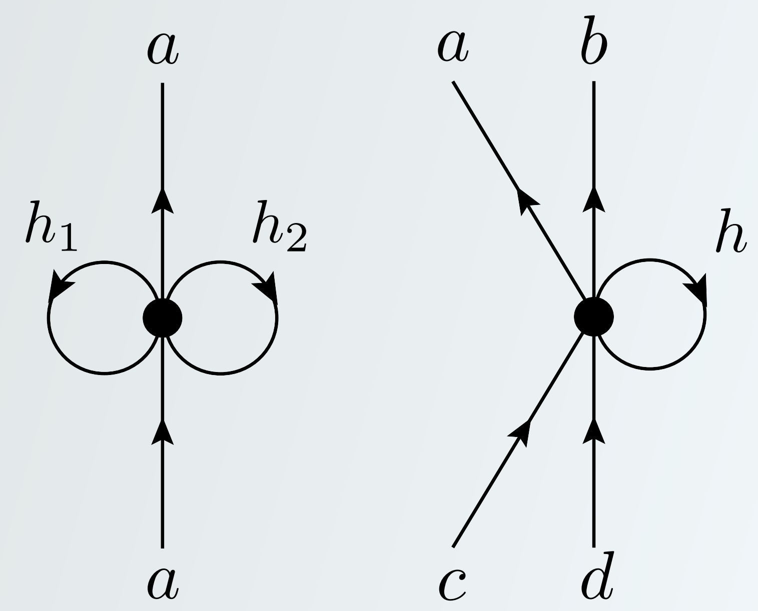




Contribution from 3NF

Hamiltonian with 3NF

$$H_{int} = (1 - \frac{1}{A}) \sum_i \frac{p_i^2}{2m} + \sum_{i<j} (V_{ij}^{NN} - \frac{p_i \cdot p_j}{mA}) + \boxed{\sum_{i<j<k} V_{ijk}^{3N}}$$



Normal ordering

Contributions from three-body forces

1. one body

$$\langle j_a | 1b_{3N} | j_a \rangle = \sum_{\substack{h_1 h_2 \\ J_{12} J}} \frac{\hat{J}^2}{2\hat{j}_a^2} \langle [(j_{h_1} j_{h_2})_{J_{12}}, j_a]_J | V_{3N} | [(j_{h_1} j_{h_2})_{J_{12}}, j_a]_J \rangle$$

2. two body

$$\langle (j_a j_b)_J | 2b_{3N} | (j_c j_d)_J \rangle = \sum_{hJ'} \frac{\hat{J}'^2}{\hat{J}^2} \langle [(j_a j_b)_J, j_h]_{J'} | V_{3N} | [(j_c j_d)_J, j_h]_{J'} \rangle$$

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Realistic Shell Model

A many-body Hamiltonian *H* defined in the full Hilbert space:

SP Num: 230 (j-scheme)
= 3542(m-scheme)

$$H = H_0 + H_1 = \sum_{i=1}^{A}(T_i + U_i) + \sum_{i<j}(V_{ij}^{NN} - U_i) + \sum_{i<j<k} V_{ijk}^{3N}$$

$C_{3000}^{8} \approx 1.6\times 10^{23}$!

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Then, introducing a similarity transformation *X* :

$$\left(\begin{array}{c|c} PHP & PHQ \\ \hline QHP & QHQ \end{array}\right) \xrightarrow[\mathcal{Q}\mathcal{H}P = 0]{\mathcal{H} = X^{-1}HX} \left(\begin{array}{c|c} P\mathcal{H}P & P\mathcal{H}Q \\ \hline 0 & Q\mathcal{H}Q \end{array}\right)$$

Suzuki & Lee : $X = e^{\omega}$ **with** $\omega = \begin{pmatrix} 0 & 0 \\ Q\omega P & 0 \end{pmatrix}$

$$H_1^{eff}(\omega) = PH_1P + PH_1Q\frac{1}{\epsilon - QHQ}QH_1P + PH_1Q\frac{1}{\epsilon - QHQ}\omega H_1{}^{eff}(\omega)$$

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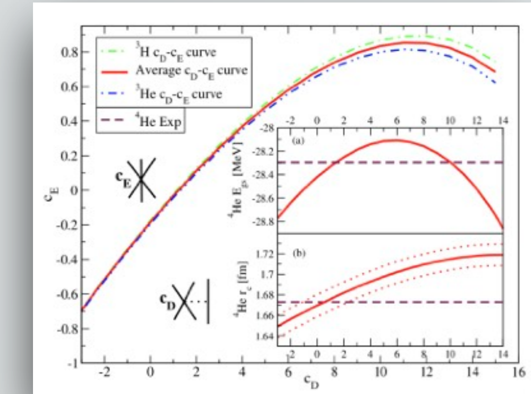
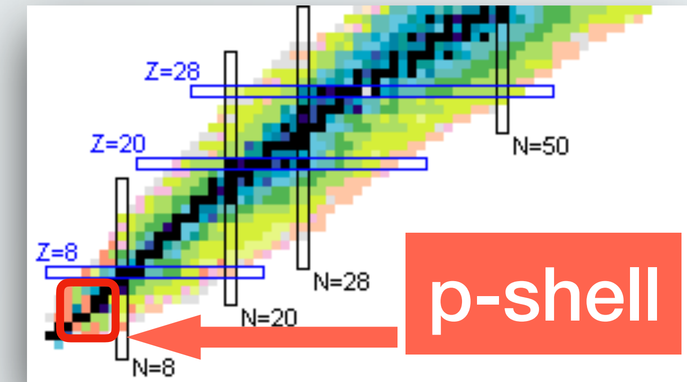
Realistic shell model calculation for p-shell nuclei

Benchmark with NCSM

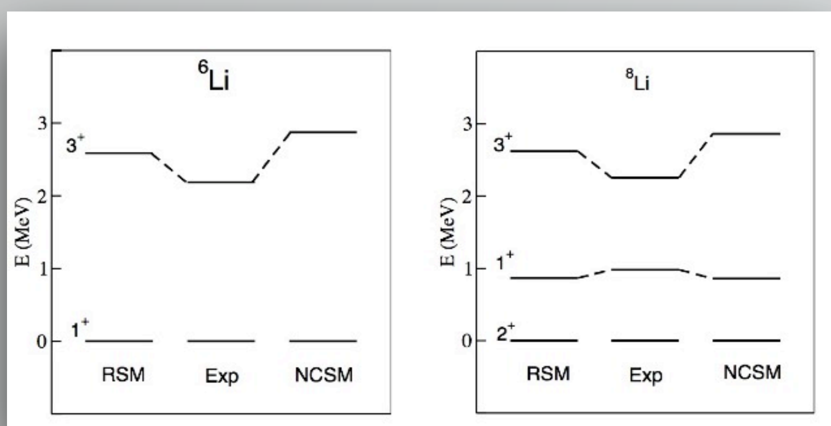
$$c_D = -1$$
$$c_E = -0.34$$

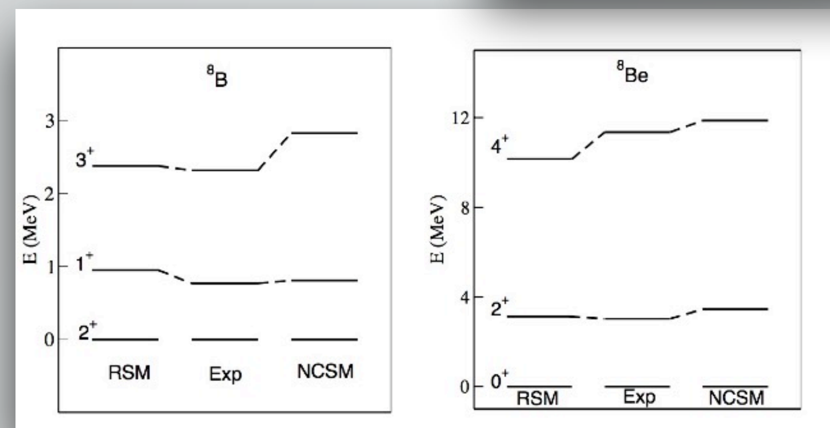


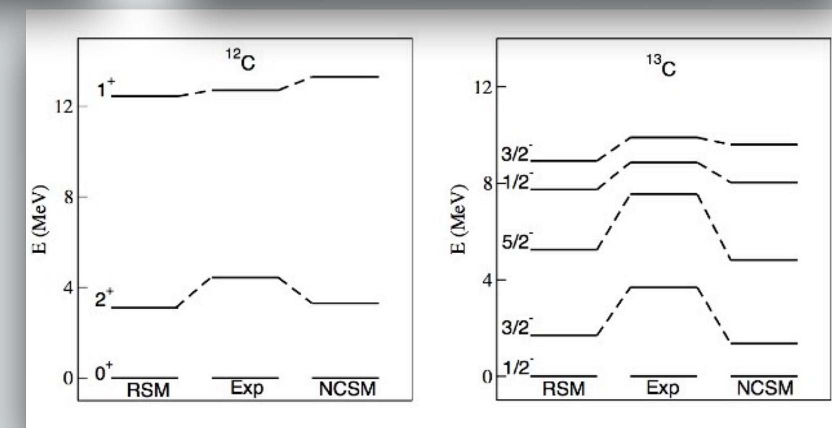


P. Navrátil, V. G. Gueorguiev, J. P. Vary *et al.*, PRL **99**, 042501 (2007)

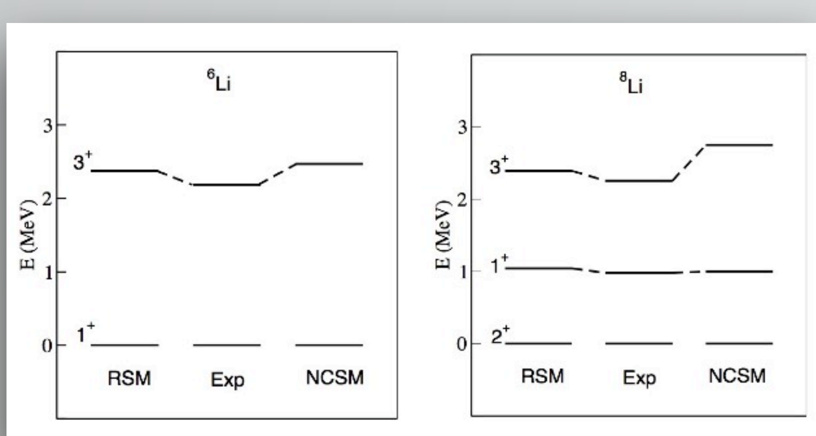
***NN* potential only**

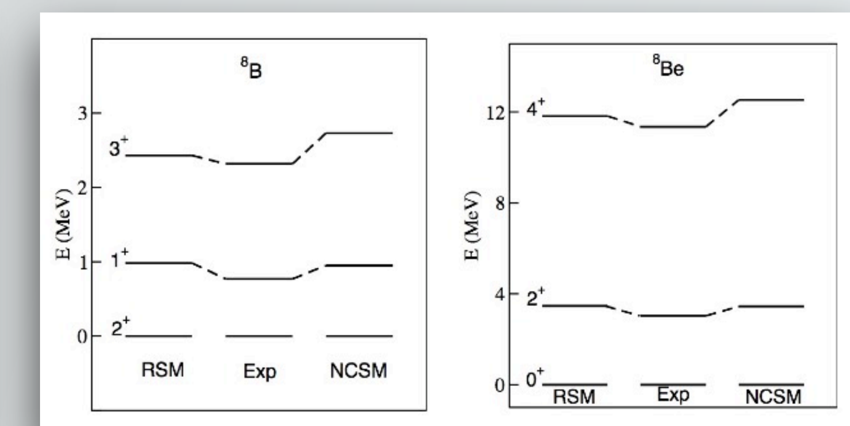


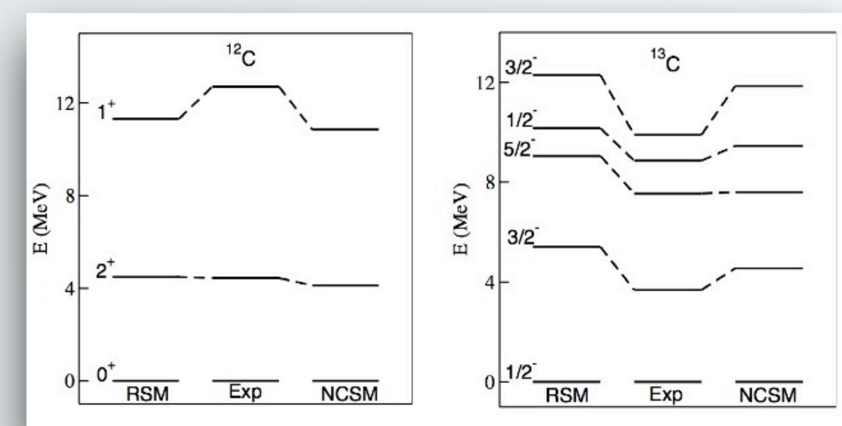




NN + 3N







RSM: T. Fukui, L. De Angelis, **Y. Z. Ma** *et al.*, PRC **98**, 044305 (2018)

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Realistic shell model calculation for p-shell nuclei

Benchmark with NCSM

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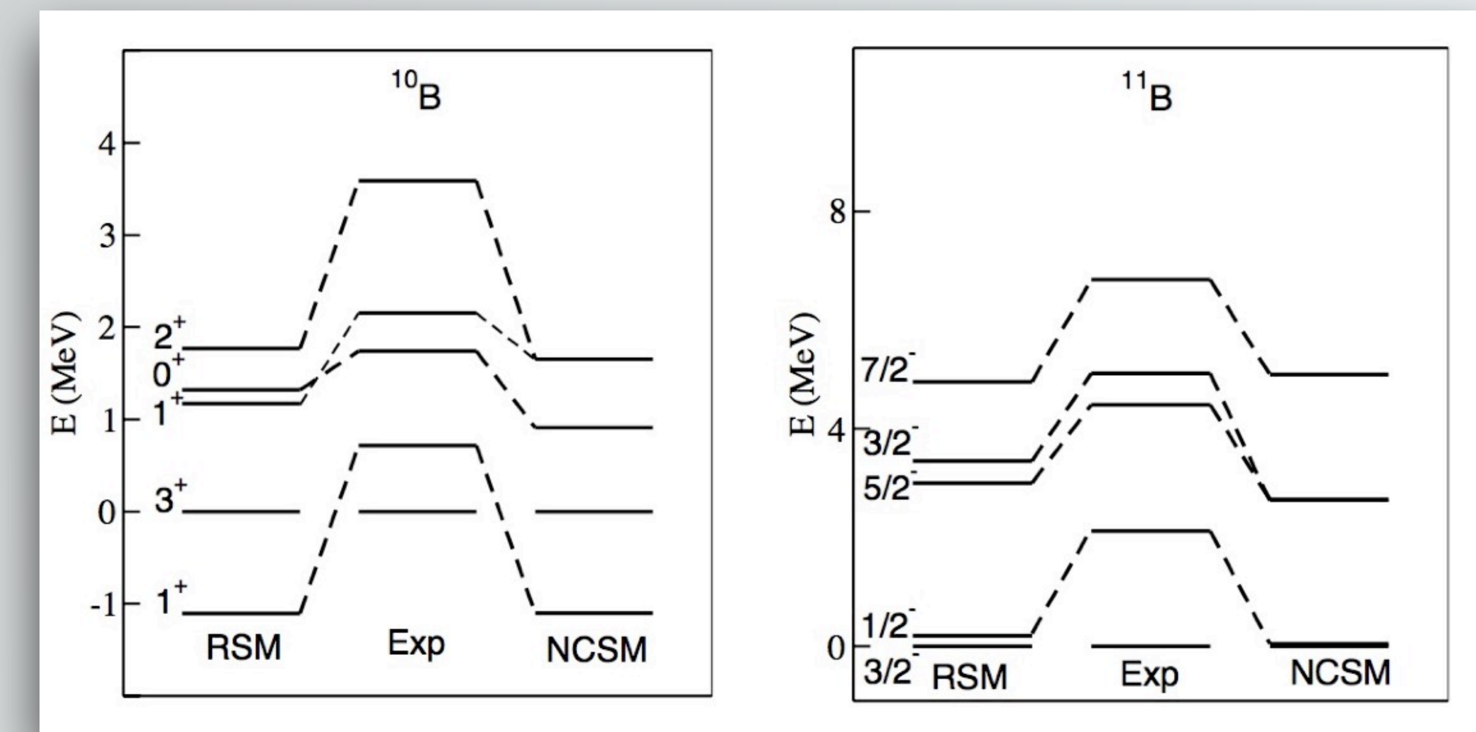

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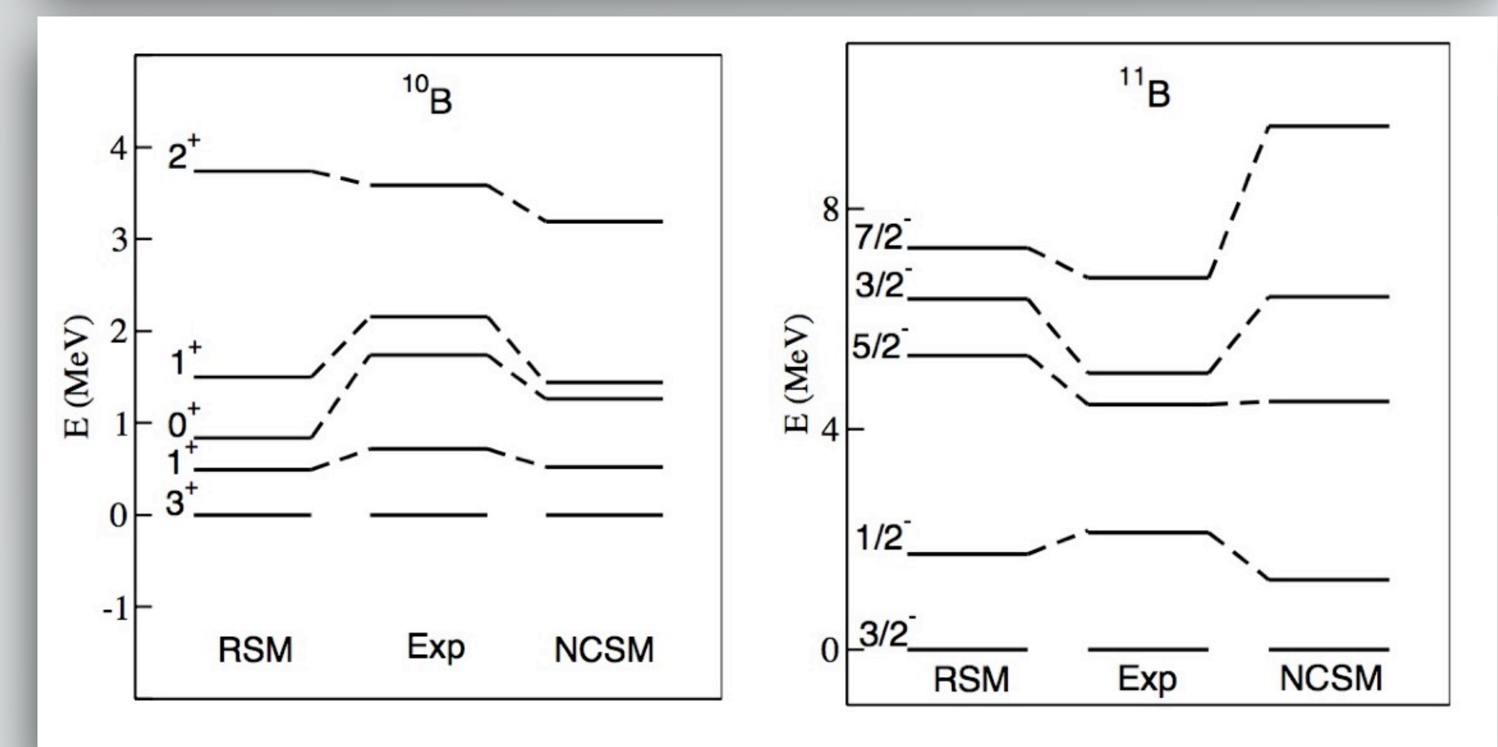

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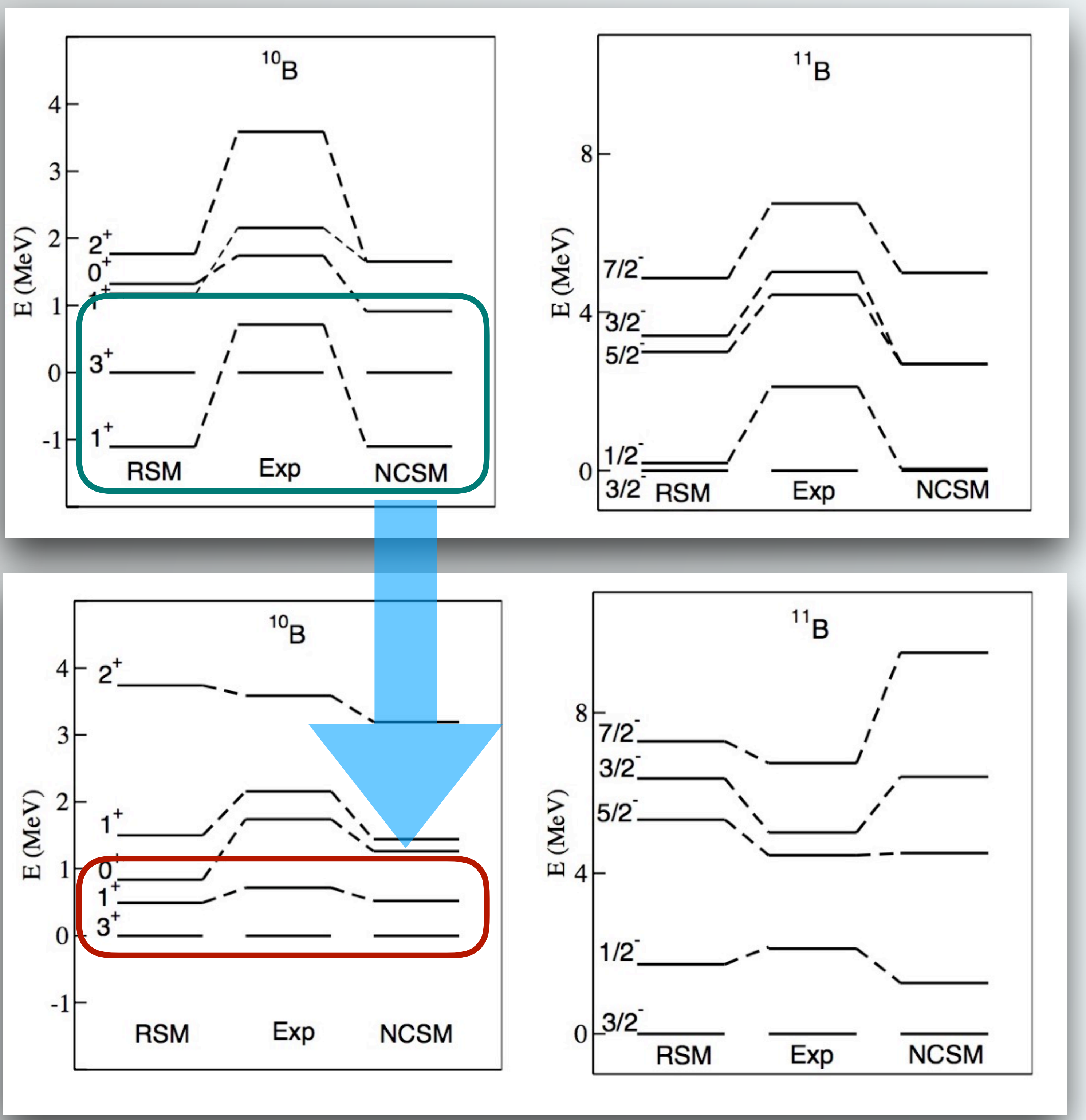

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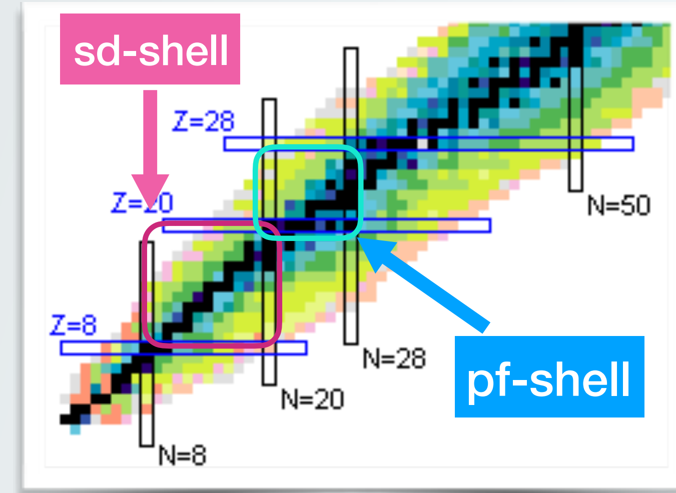
Apply 3NF to mid-mass nuclei

- Success to derive & calculate 3N matrix element from chiral NNLO
- Present the results of RSM calculation for p-shell nuclei and show that 3N force can greatly improve the agreement between our calculation and experiment observables
- Compare our calculated energy spectra with NCSM which supports the reliability of our 3N force and RSM calculations



Apply 3NF to mid-mass nuclei

fp-shell & shell evolution

↓

RSM: ^{48}Ca -> ^{56}Ni

sd-shell & neutron drip line

↓ + Gamow Basis (continuum effects)

CGSM: ^{18}O -> ^{26}O

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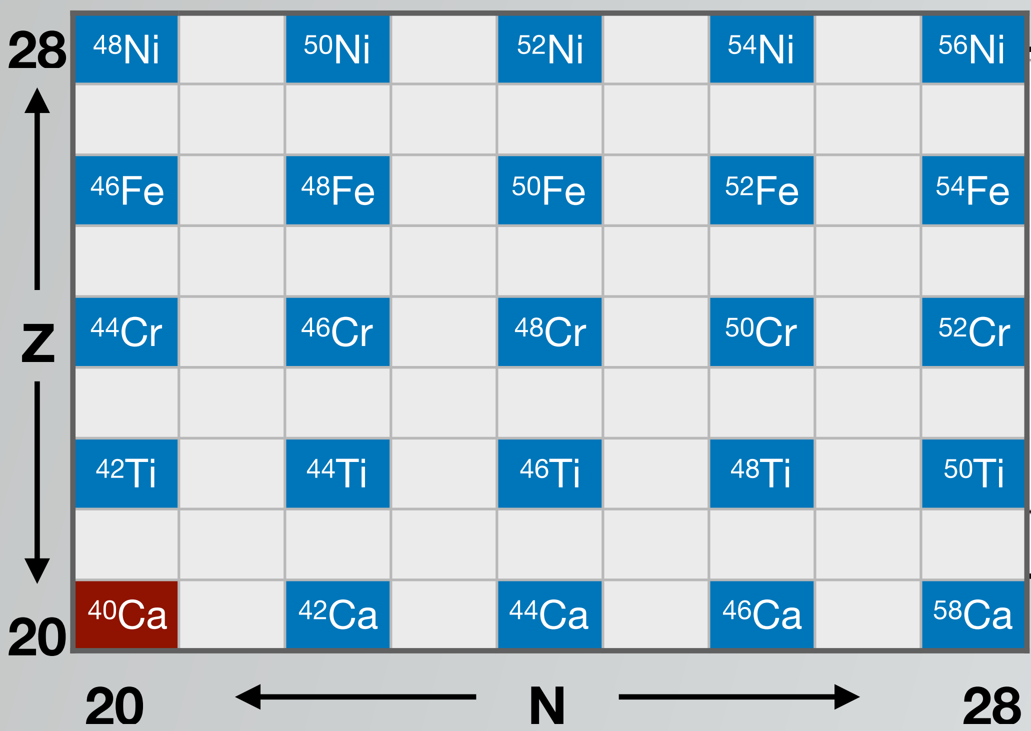
I: *fp*-shell & shell evolution (RSM)



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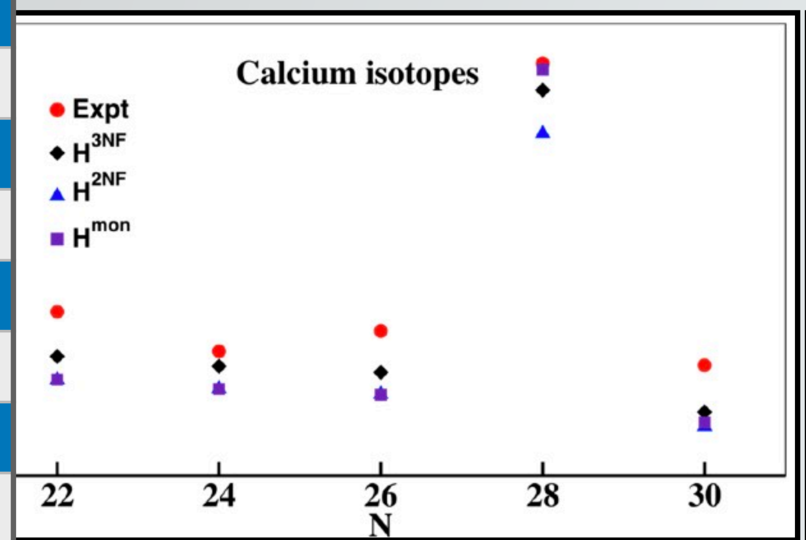

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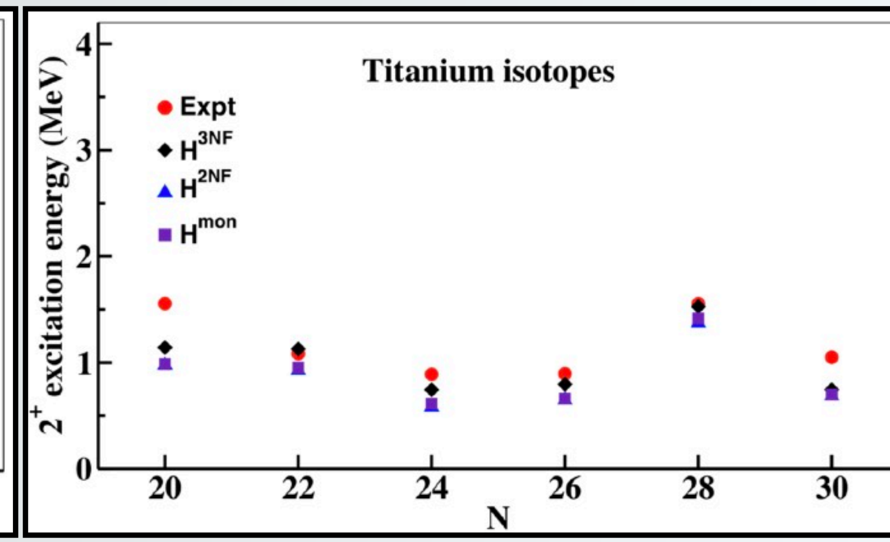

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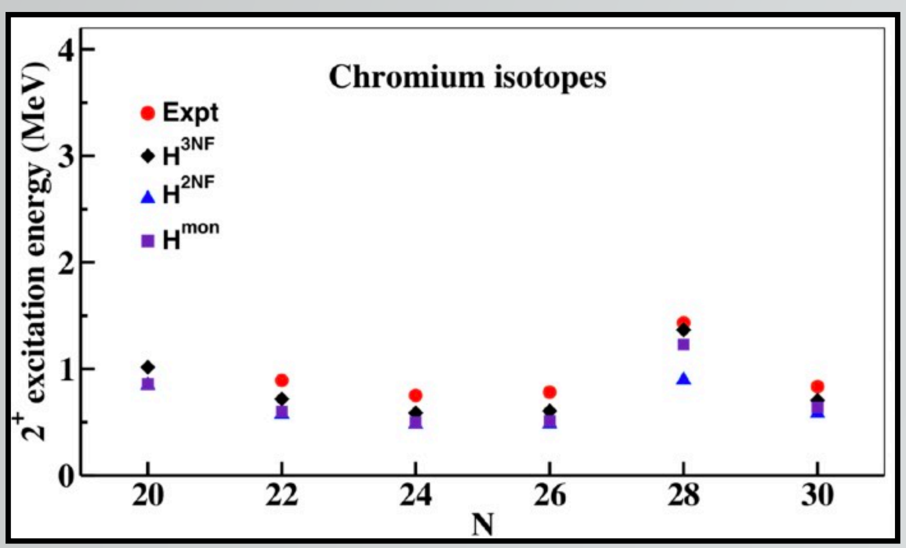

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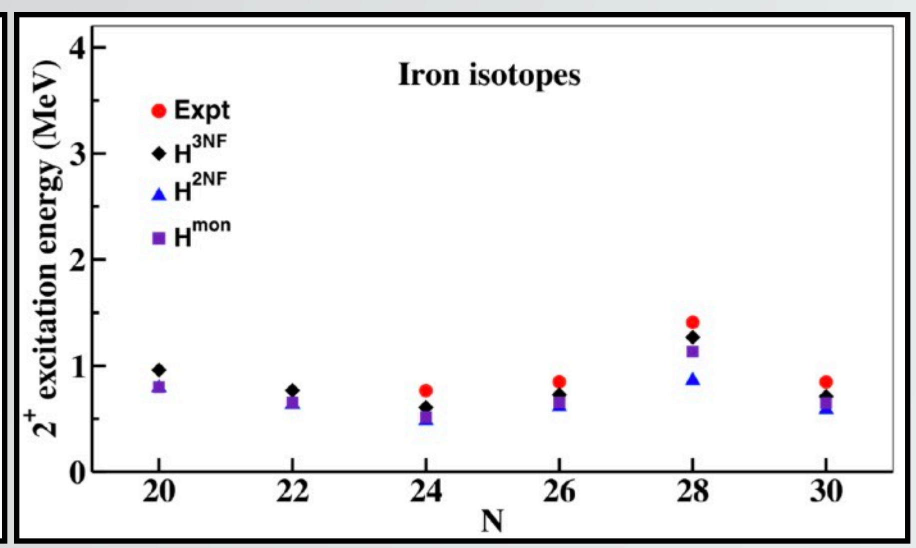

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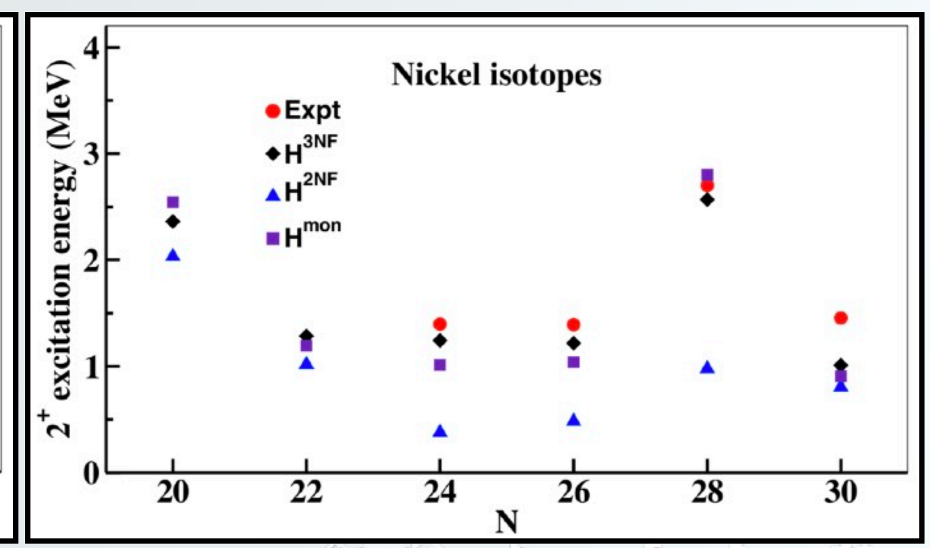

Y. Z. Ma *et al.*, PRC.100.034324(2019)

Ⅰ: *fp*-shell & shell evolution (RSM)

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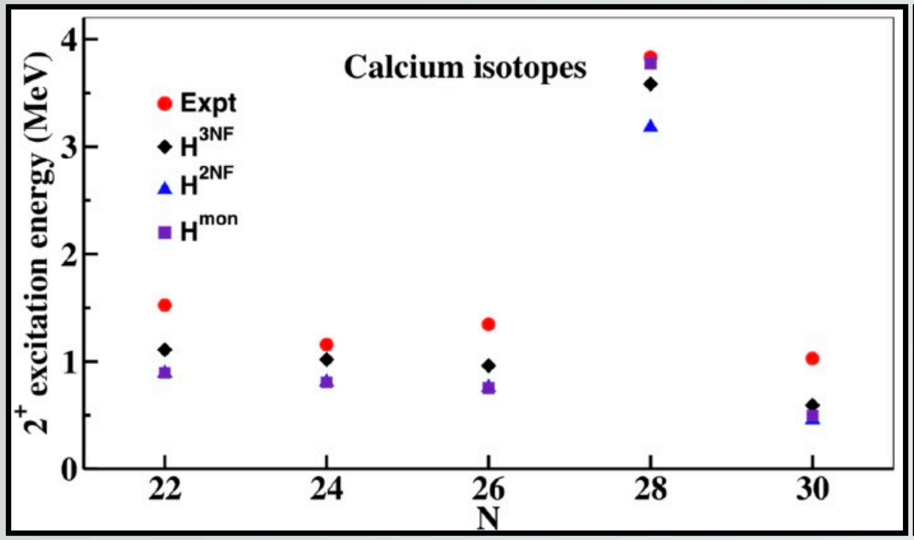

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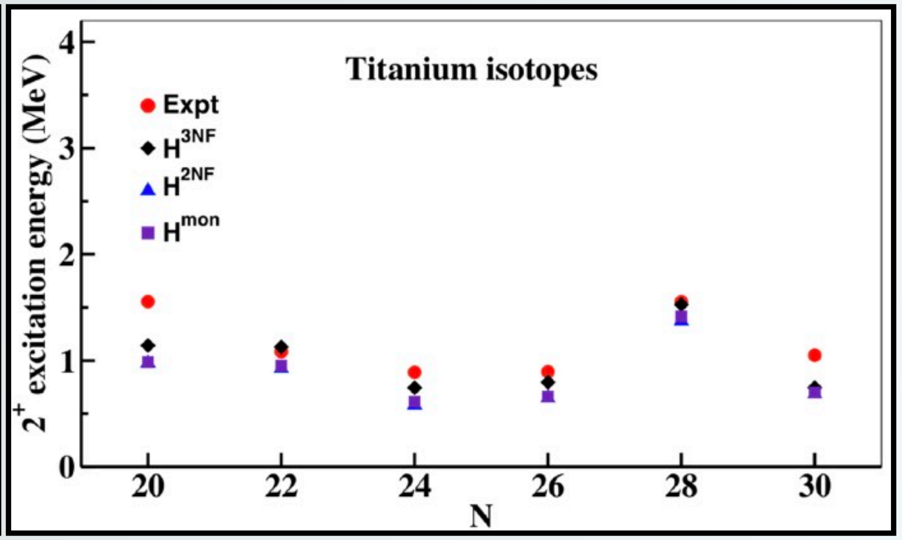

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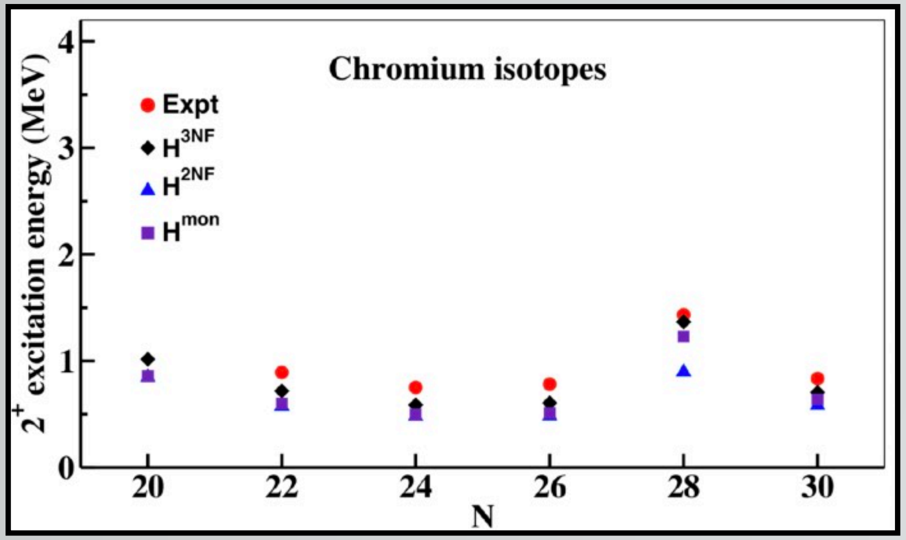

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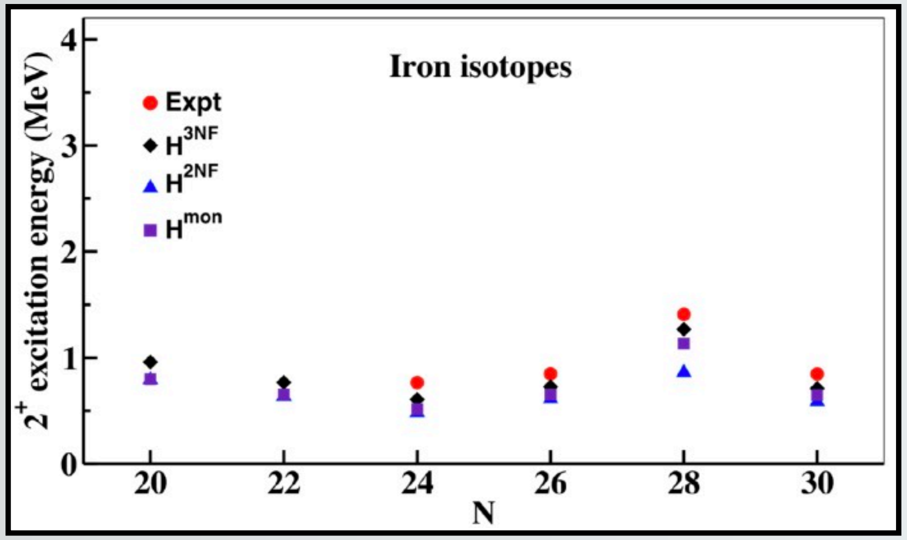

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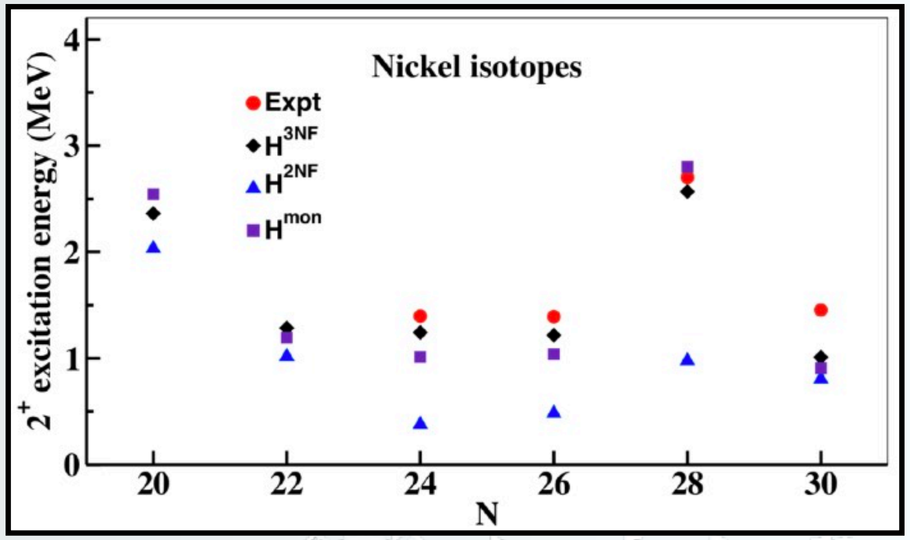
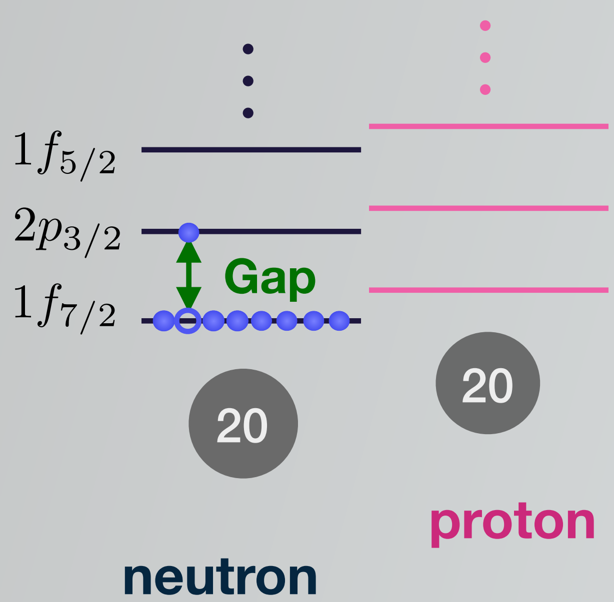


Y. Z. Ma *et al.*, PRC.100.034324(2019)

Ⅰ: *fp*-shell & shell evolution (RSM)



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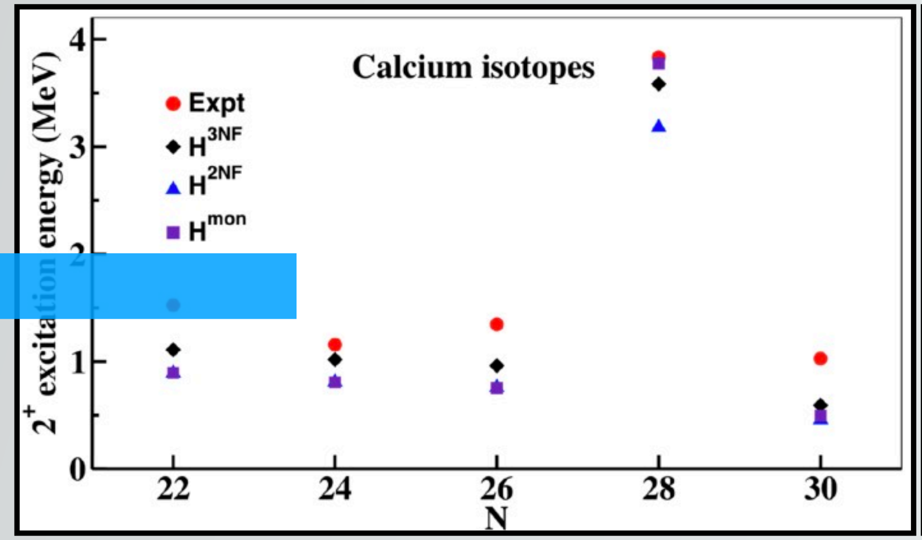

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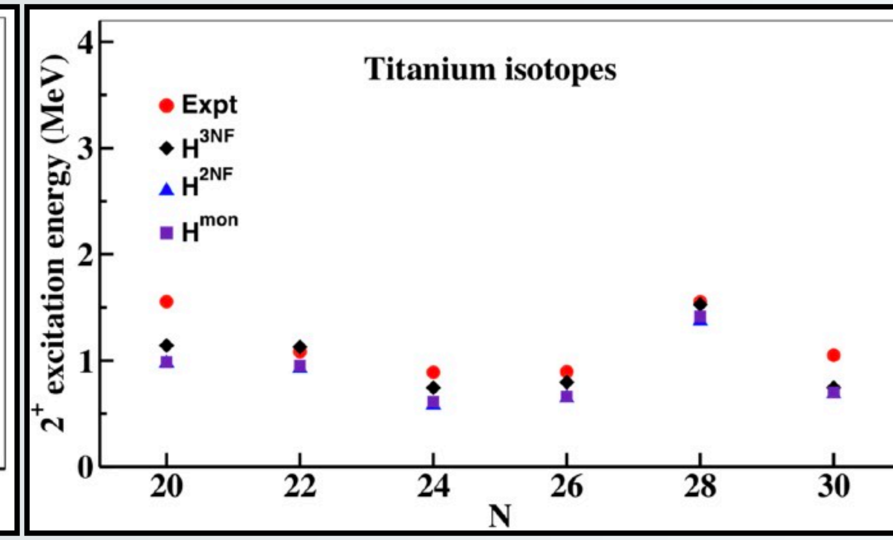

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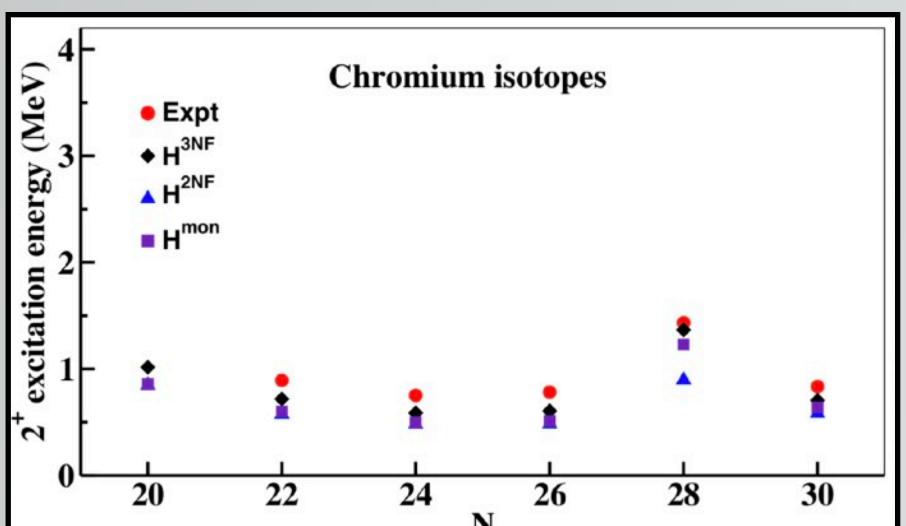

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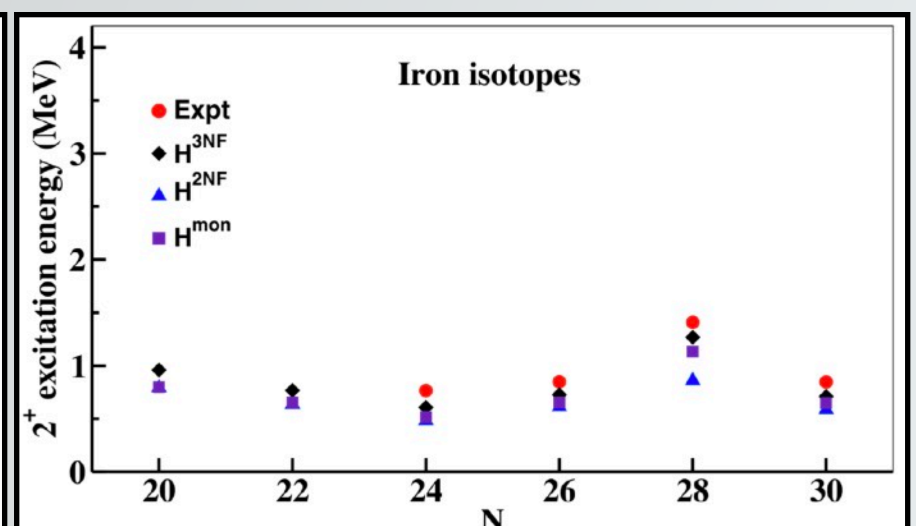

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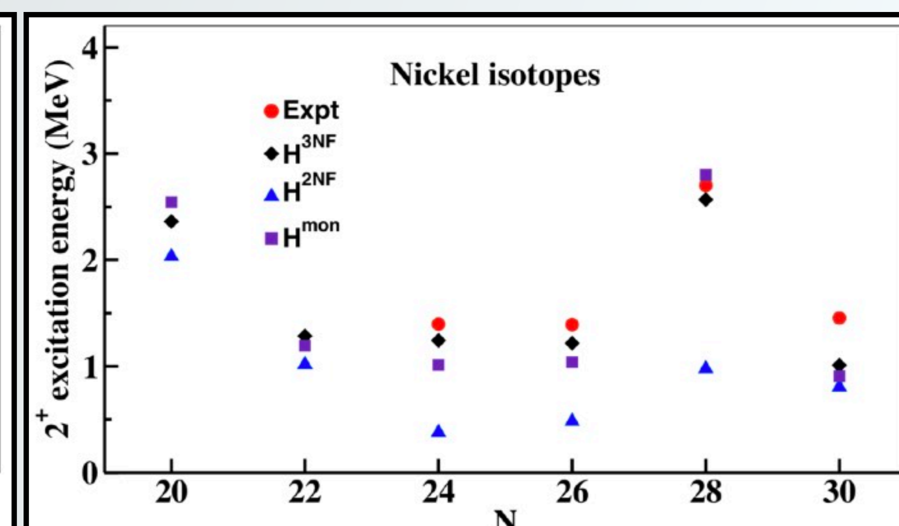

Y. Z. Ma *et al*., PRC.100.034324(2019)

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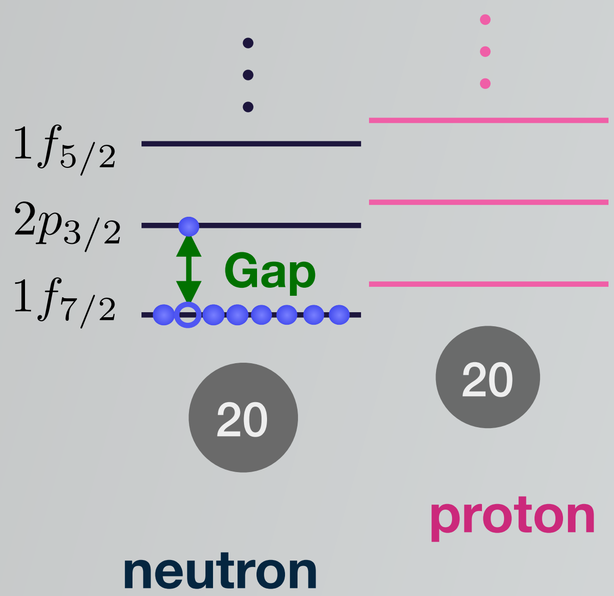

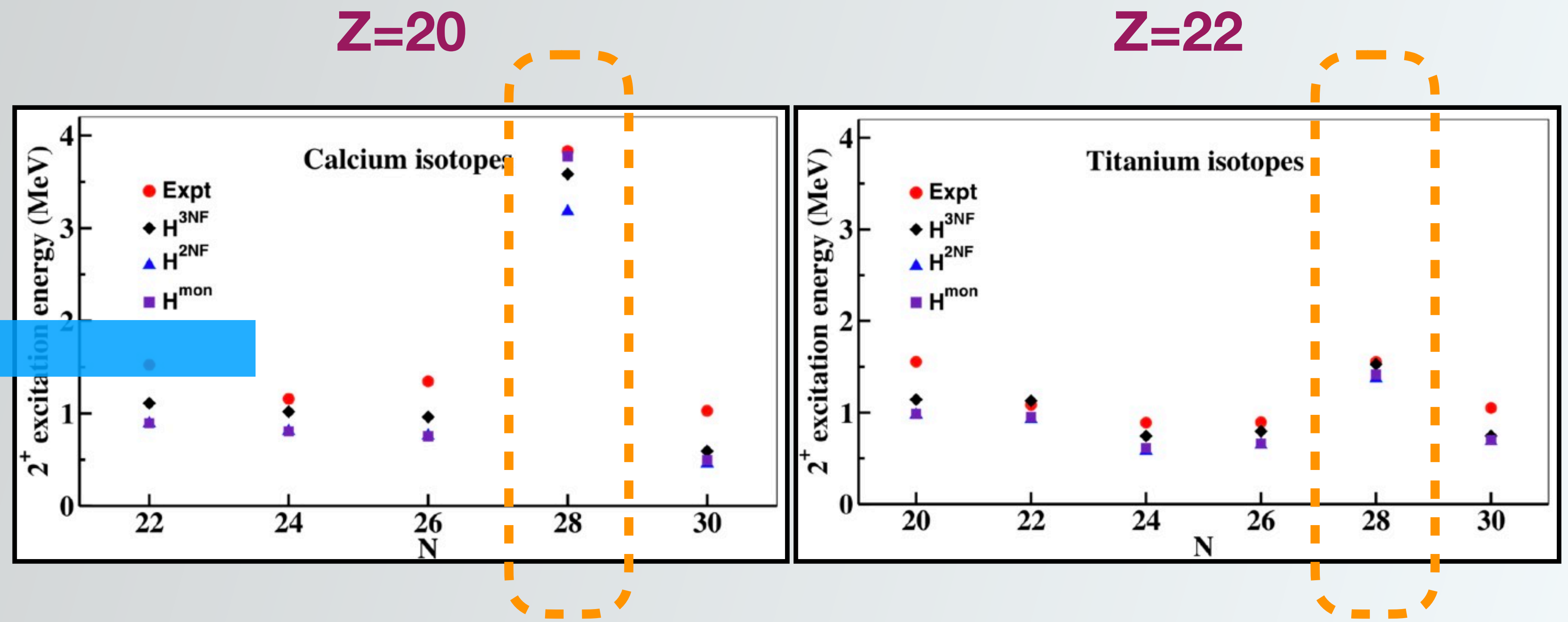

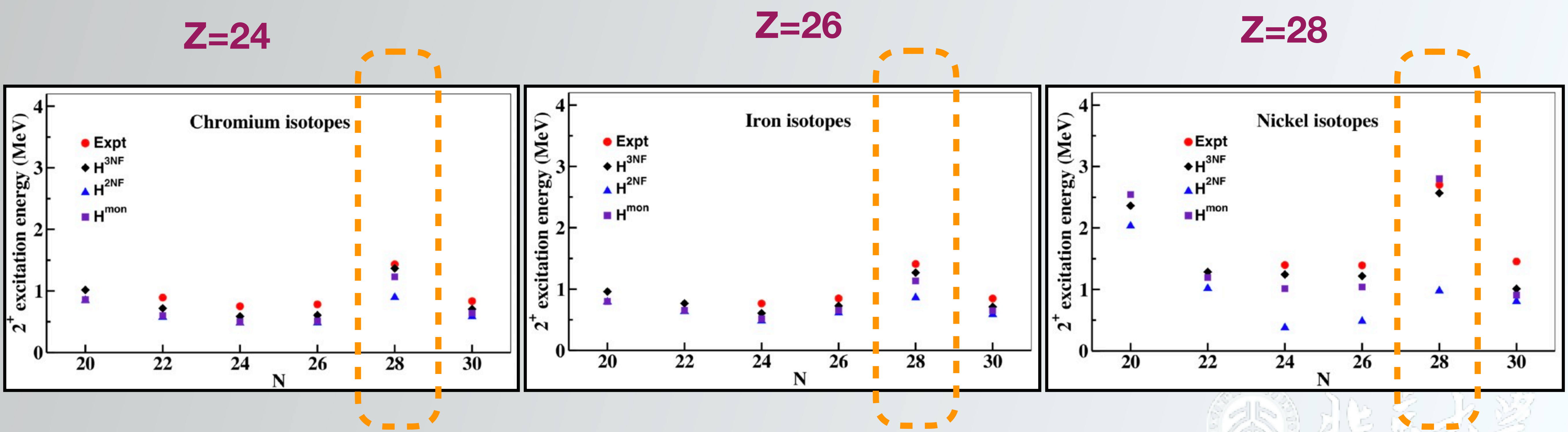

Y. Z. Ma *et al.*, PRC.100.034324(2019)

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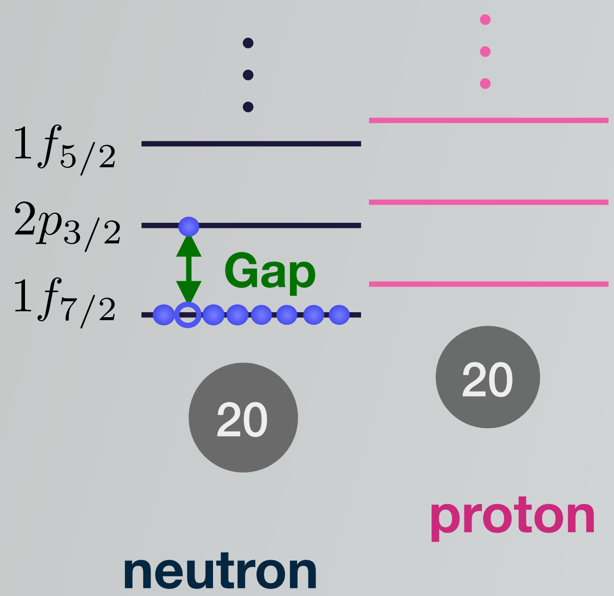

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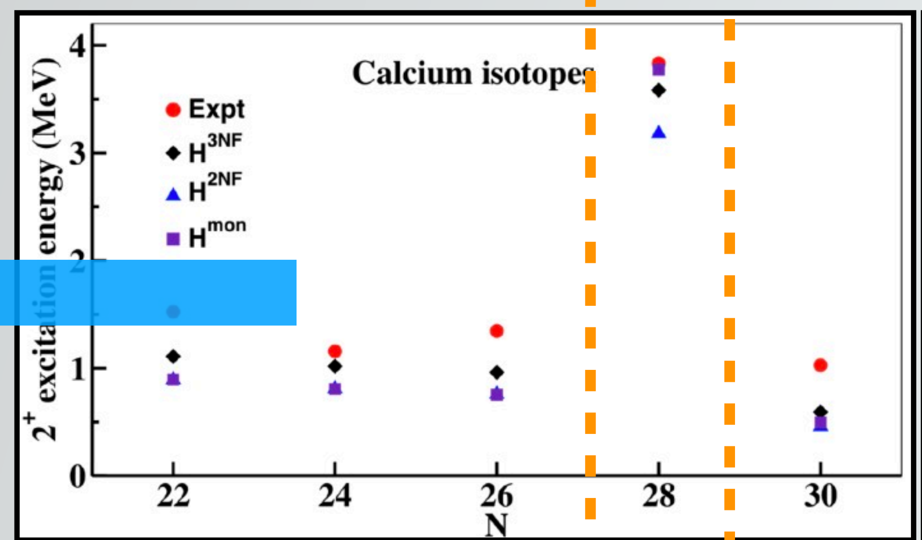
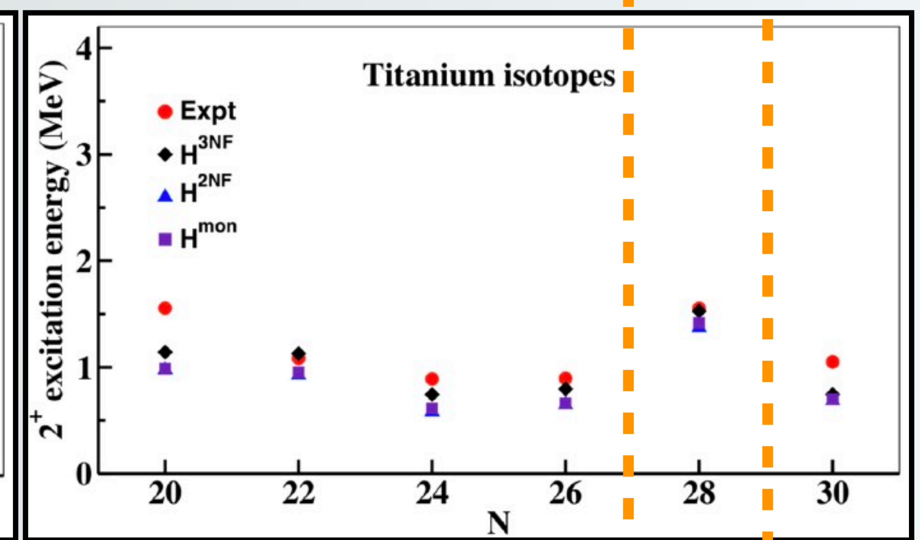
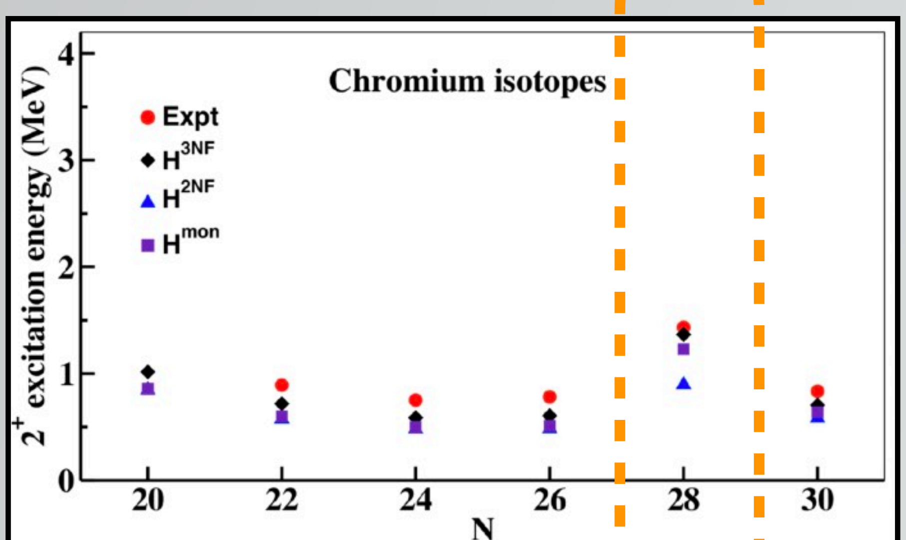


Z=22



$$\langle i,j|H^{mon}_{eff}|i,j\rangle = \epsilon_i + \epsilon_j + \frac{\sum_J (2J+1)\langle i,j|V_{eff}|i,j\rangle_J}{\sum_J (2J+1)}$$

Z=24



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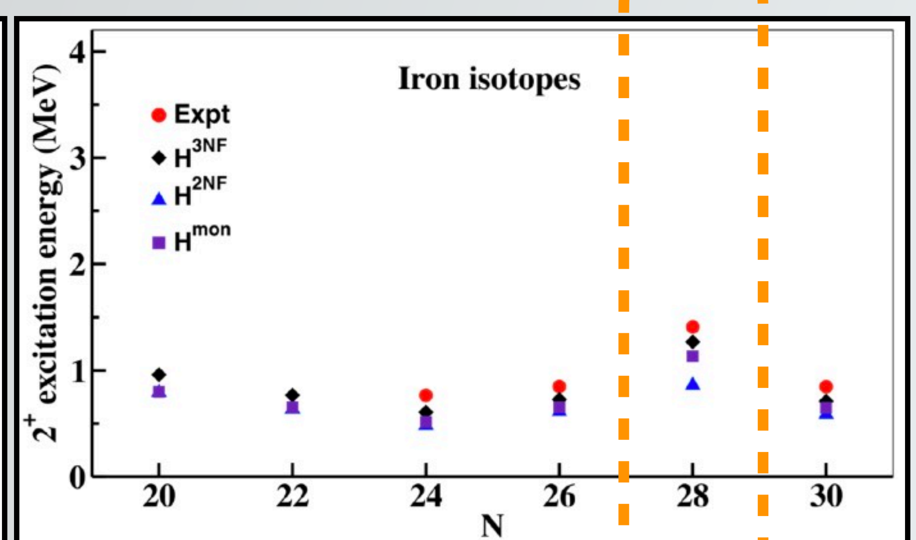
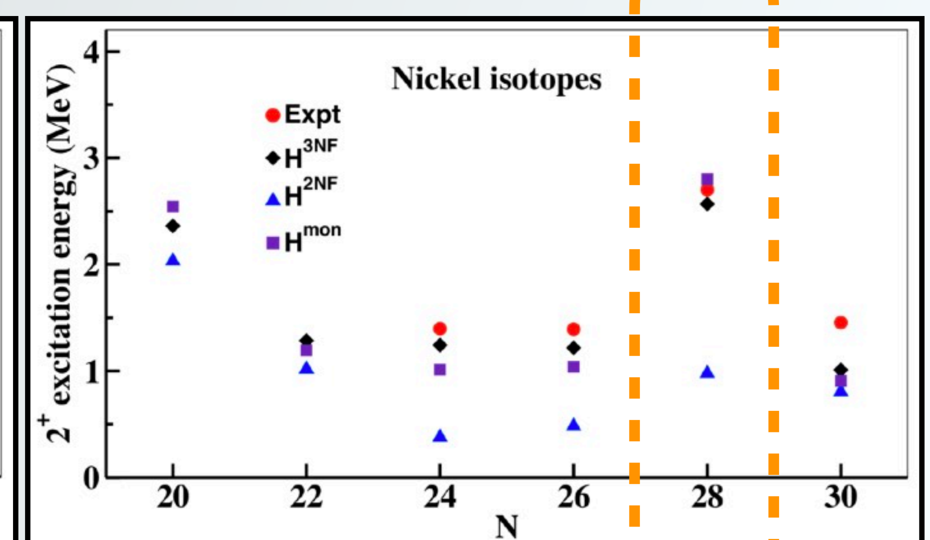


Z=28



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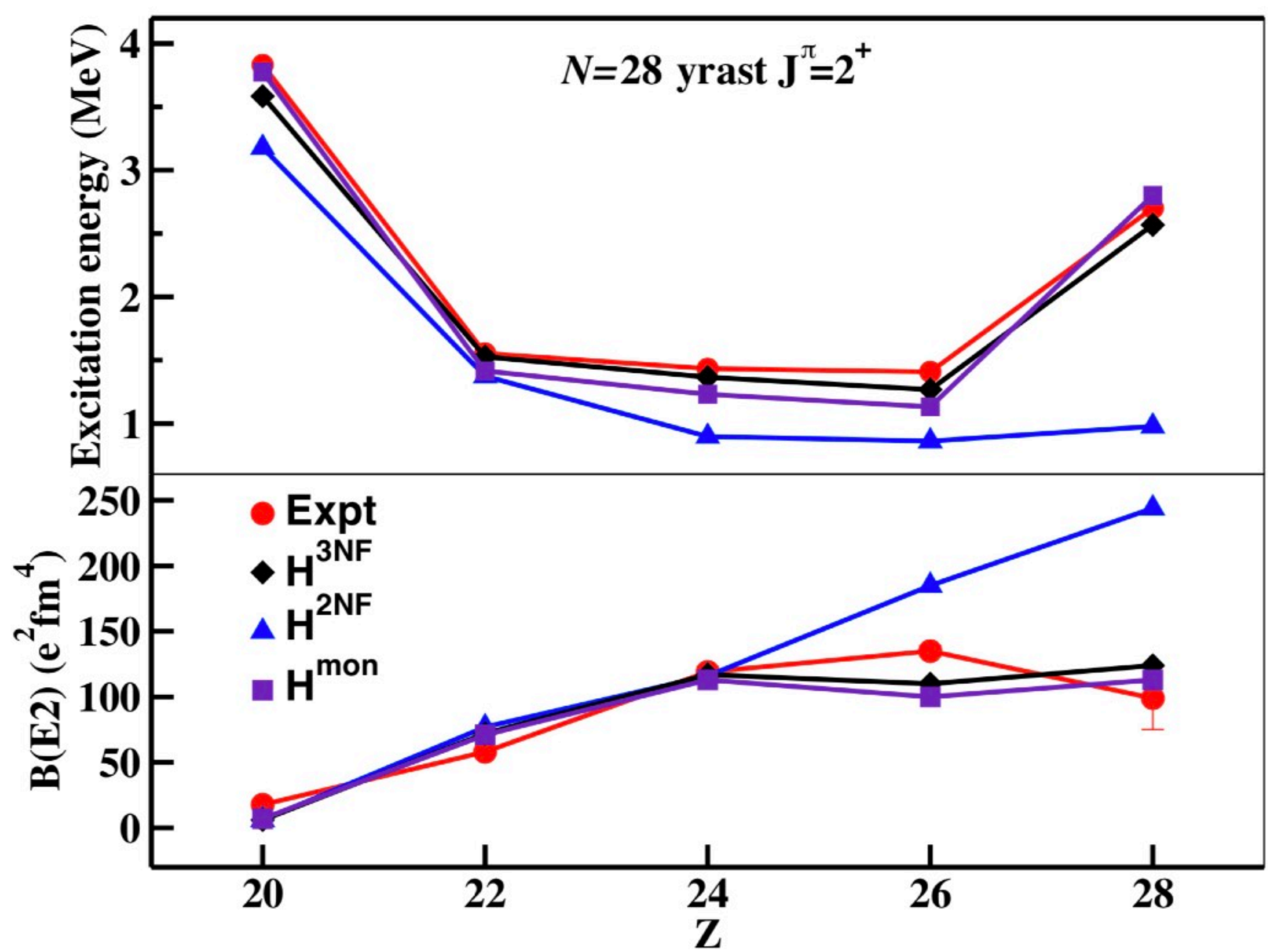

FIG. 16: Experimental and calculated excitation energies of the yrast $J^\pi = 2^+$ states and $B(E2; 2_1^+ \rightarrow 0_1^+)$ transition rates for the $N = 28$ isotones. See text for details.

Y. Z. Ma *et al*., PRC.100.034324(2019)

- **High lying 2^+ state and low B(E2) value of ^{48}Ca (Z=20), indicates a well closed shell.**
- **Only 2NF can not obtain the right 2^+ state and B(E2) value of ^{56}Ni (Z=28) against data.**
- **3NF is crucial for shell-evolution and 3NF monopole component can mainly take the responsibility for the shell closure properties.**

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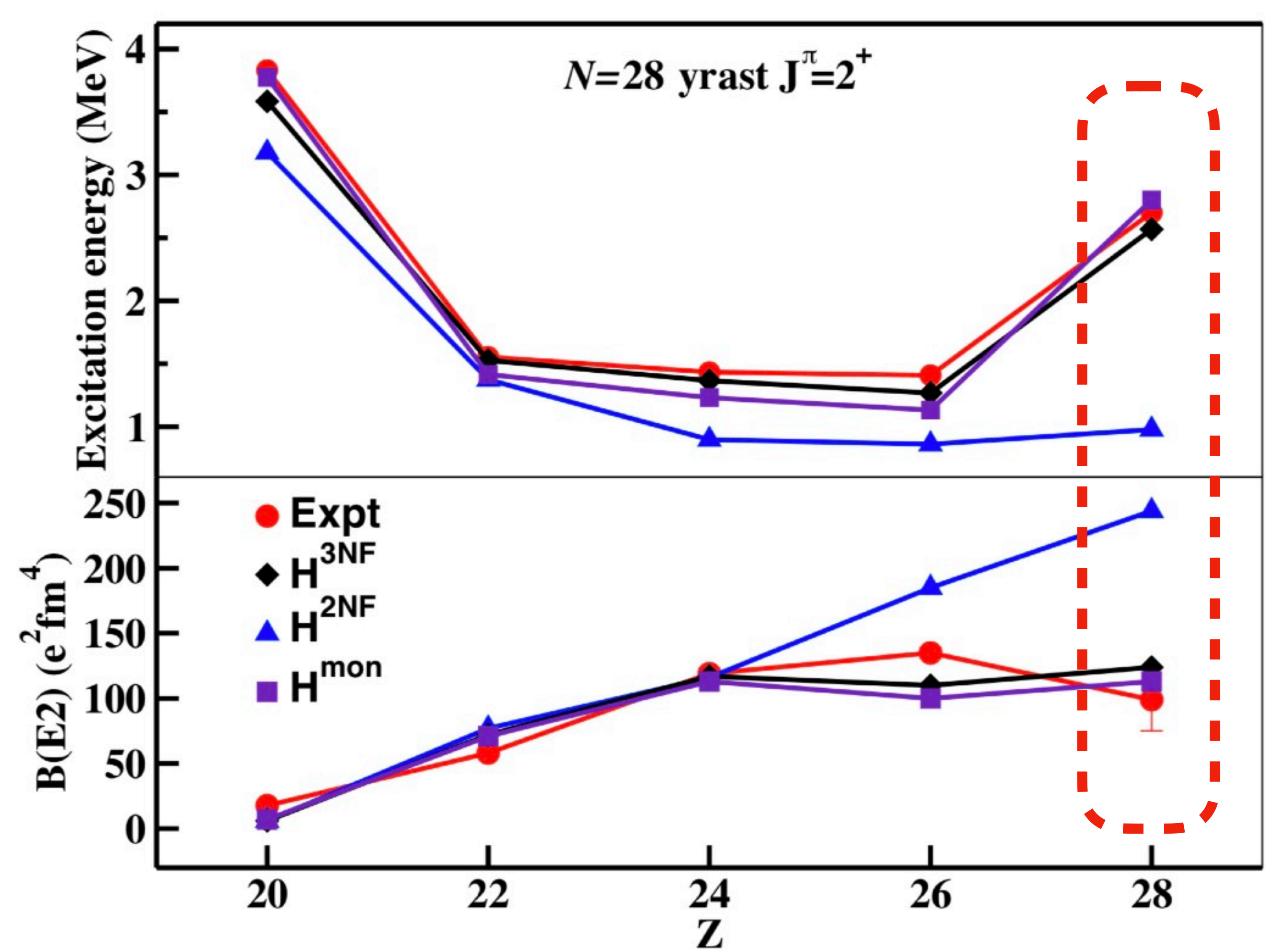

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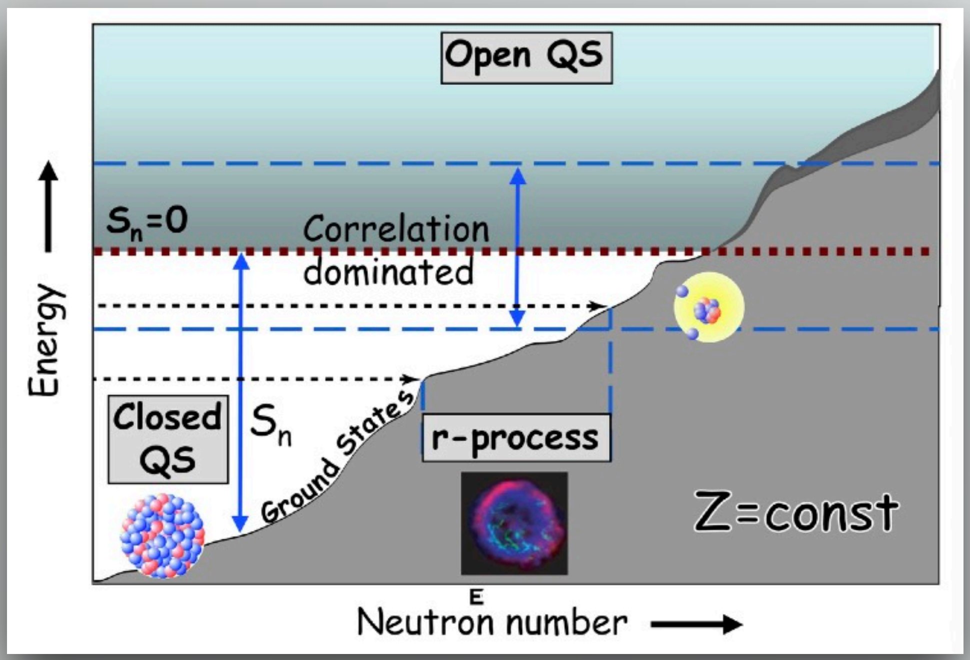
II: *sd*-shell & neutron drip line (3NF & continuum)



N Michel, W Nazarewicz, et al., J. Phys. G **36** (2009) 013101

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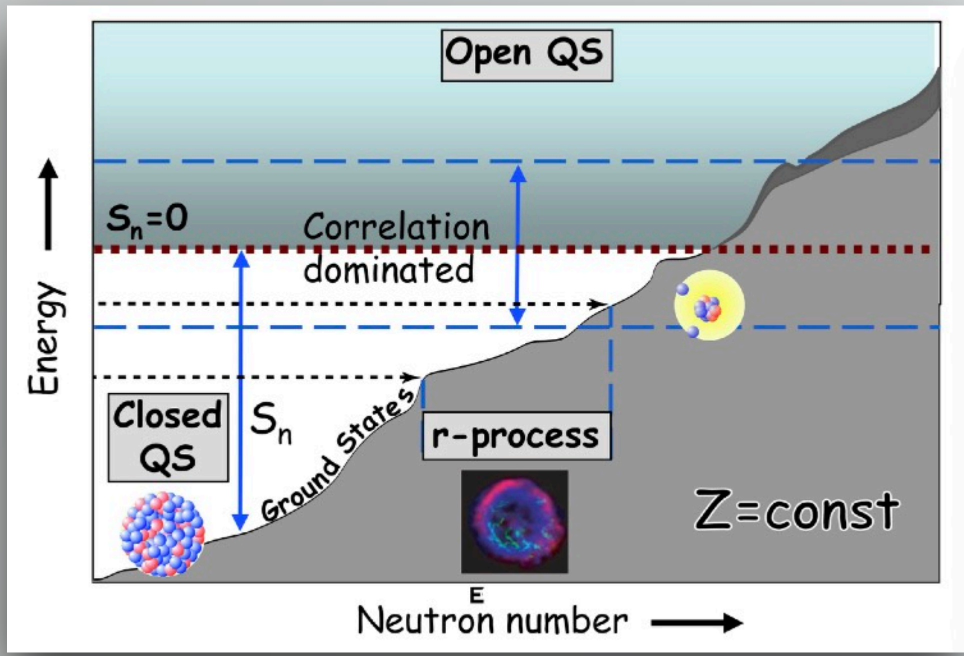

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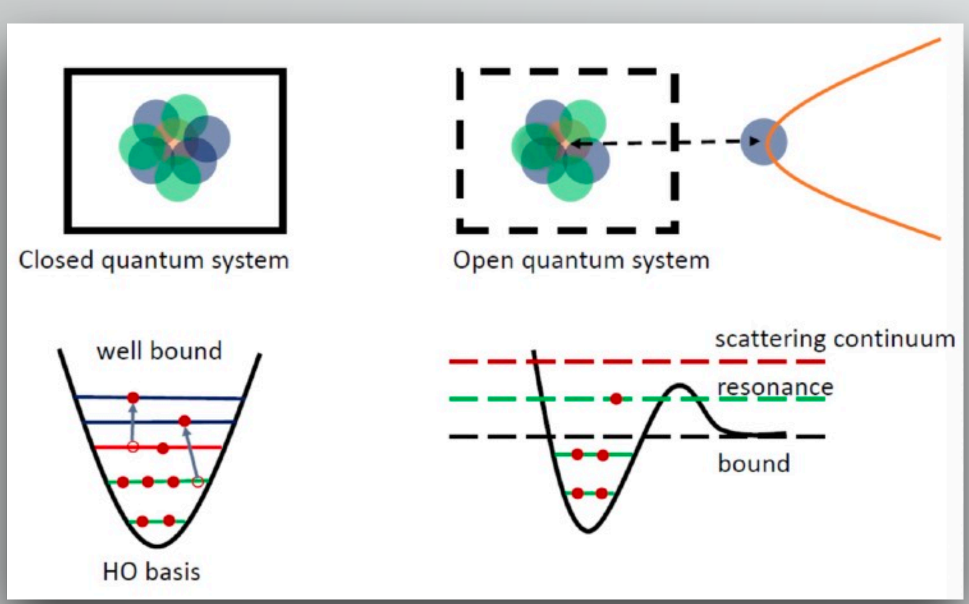

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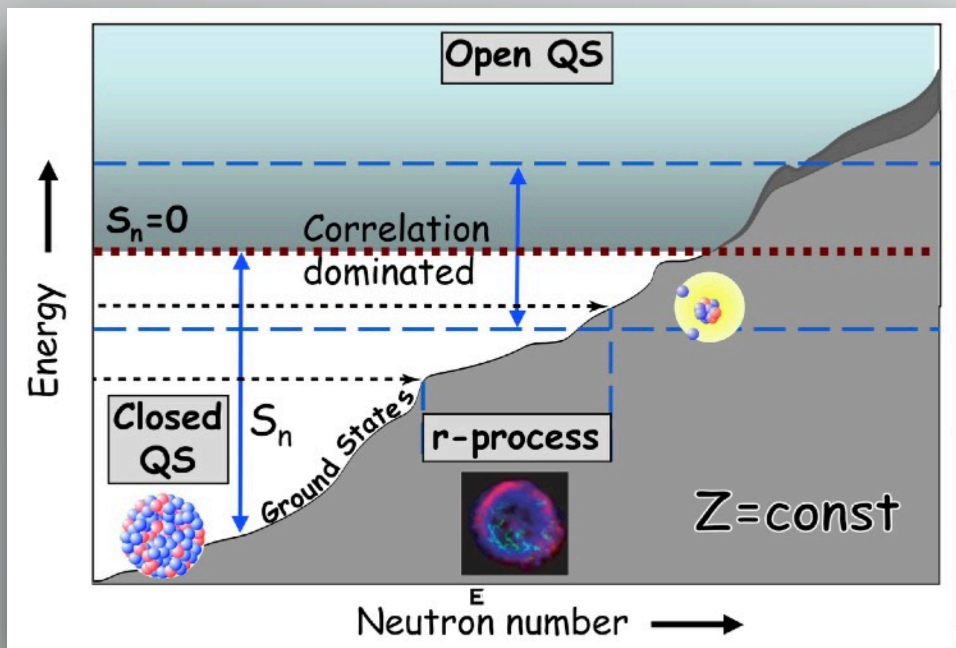

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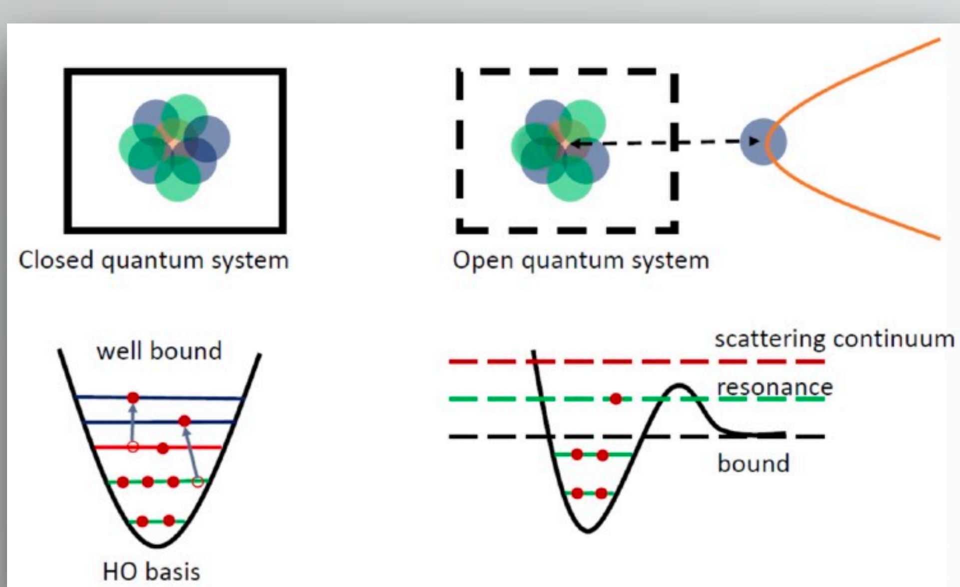
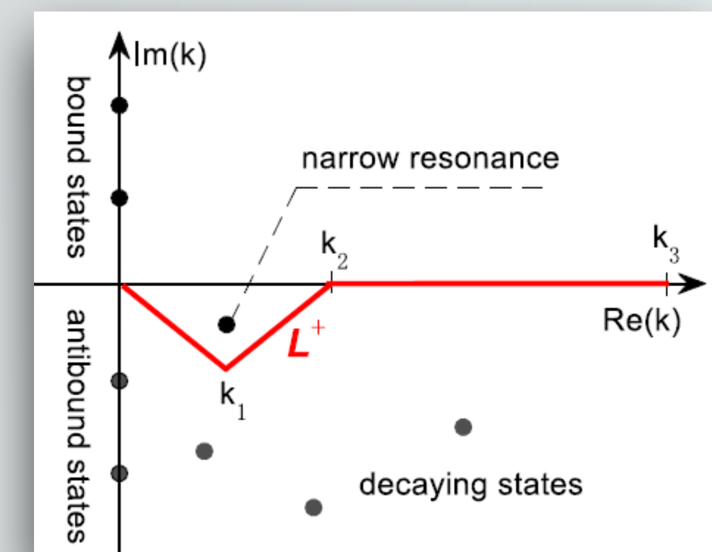


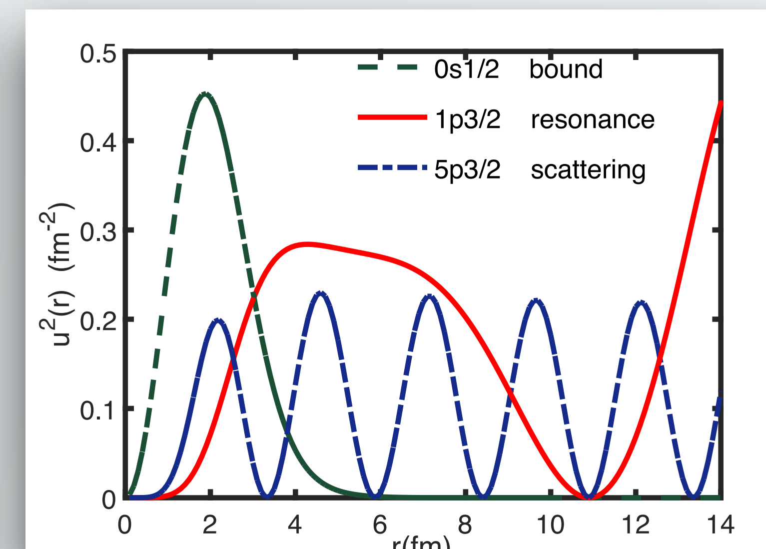
One-body problem: Berggren basis

$$\frac{d^2u(k,r)}{dr^2} = \Big[\frac{l(l+1)}{r^2} + \frac{2m}{\hbar^2}U(r) - k^2\Big]u(k,r)$$ **in complex-*k* space**

$$e = \frac{\hbar^2k^2}{2m} = e_n - i\frac{\gamma_n}{2}$$

Different wavefunction behavior of: bound, resonance, continuum states





Z. H. Sun *et al.*, PLB 769 (2017) 227–232

$$\sum_n u_n(r)u_n(r') + \int_{L+} u(k,r)u(k,r')dk = \delta(r-r')$$

T. Berggren, Nucl. Phys. A109 (1968) 265

Ⅱ: *sd*-shell & neutron drip line (3NF & continuum)

Many-body problem: Effective Hamiltonian for CGSM (core Gamow Shell Model)

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Ⅱ: *sd*-shell & neutron drip line (3NF & continuum)

Many-body problem: Effective Hamiltonian for CGSM (core Gamow Shell Model)

1. Add 3N contribution to Hamiltonian

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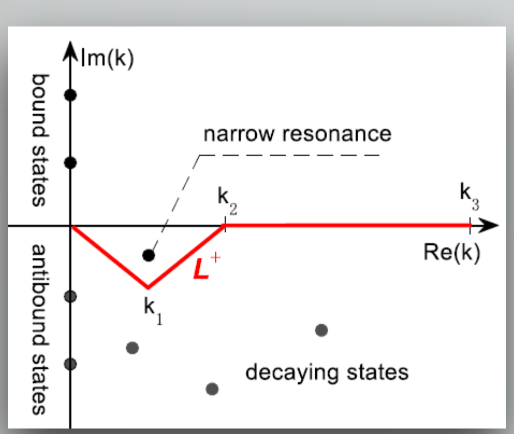
Many-body problem: Effective Hamiltonian for CGSM (core Gamow Shell Model)

1. Add 3N contribution to Hamiltonian

2. Transfer to Berggren basis



$$\langle ab|V|cd\rangle = \sum_{\alpha\leq\beta}\sum_{\gamma\leq\delta}\langle ab|\alpha\beta\rangle\langle\alpha\beta|V|\gamma\delta\rangle\langle\gamma\delta|cd\rangle$$

N_L=6+6+8

Z. H. Sun *et al.*, PLB 769 (2017) 227–232

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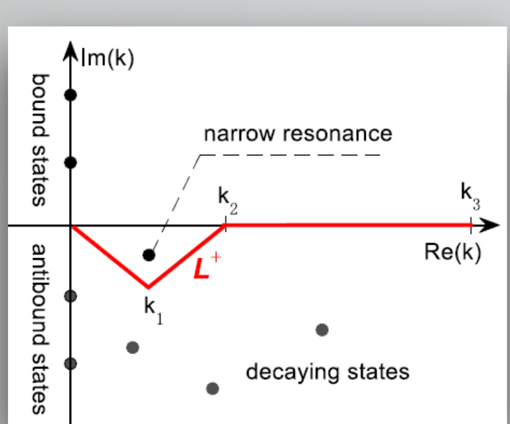

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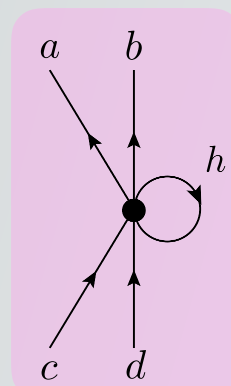
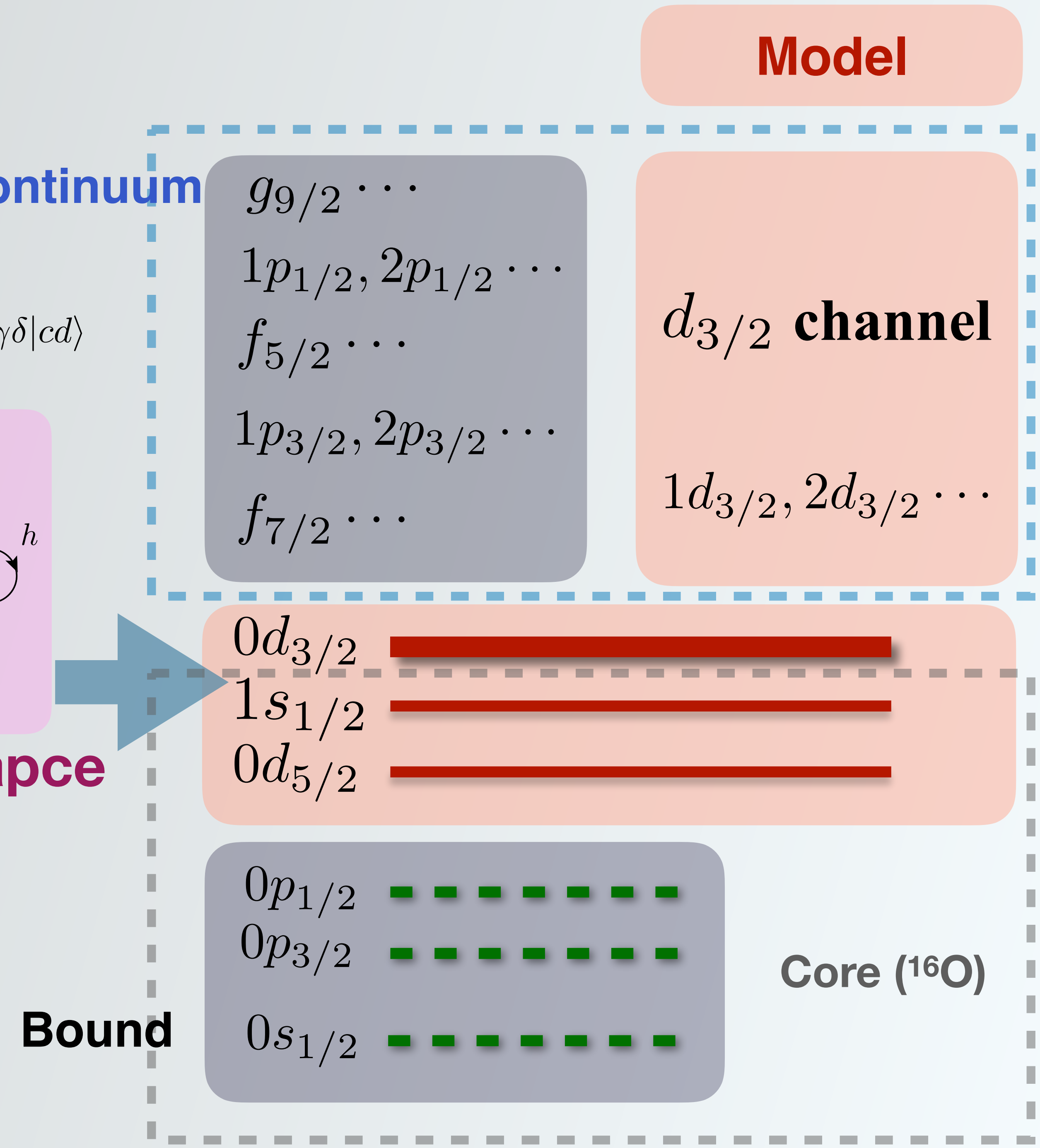


Z. H. Sun *et al.*, PLB 769 (2017) 227–232

3. Q-box folded diagrams in complex-*k* sapce

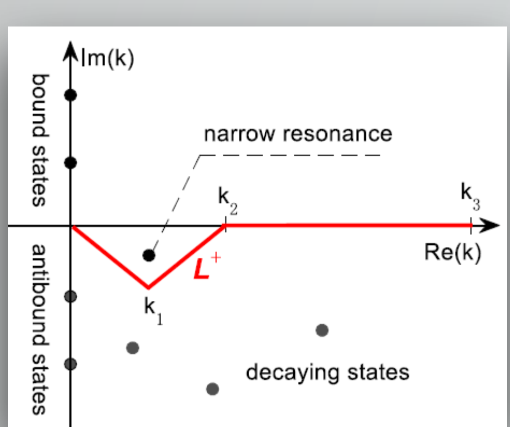
$$\hat{Q}(E) = PVP + PVQ\frac{1}{E-H}QVP$$



Many-body problem: Effective Hamiltonian for CGSM (core Gamow Shell Model)

1. Add 3N contribution to Hamiltonian

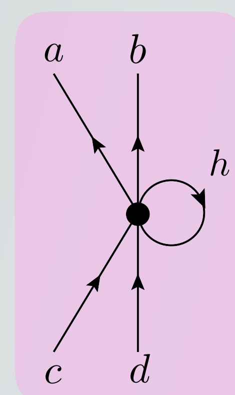
2. Transfer to Berggren basis

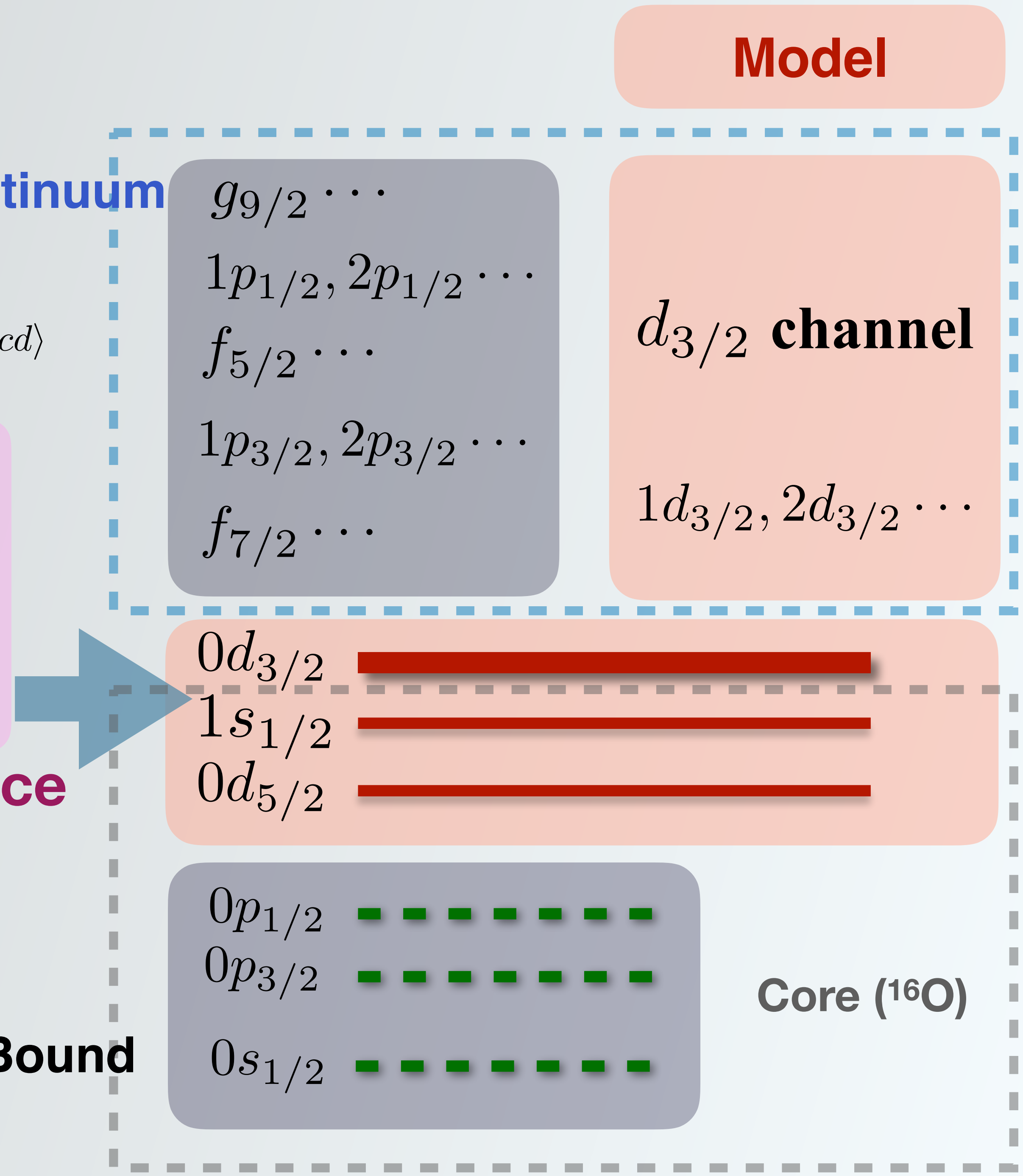


N_L=6+6+8

$$\langle ab|V|cd\rangle = \sum_{\alpha\le\beta}\sum_{\gamma\le\delta}\langle ab|\alpha\beta\rangle\langle\alpha\beta|V|\gamma\delta\rangle\langle\gamma\delta|cd\rangle$$

Z. H. Sun *et al.*, PLB 769 (2017) 227–232



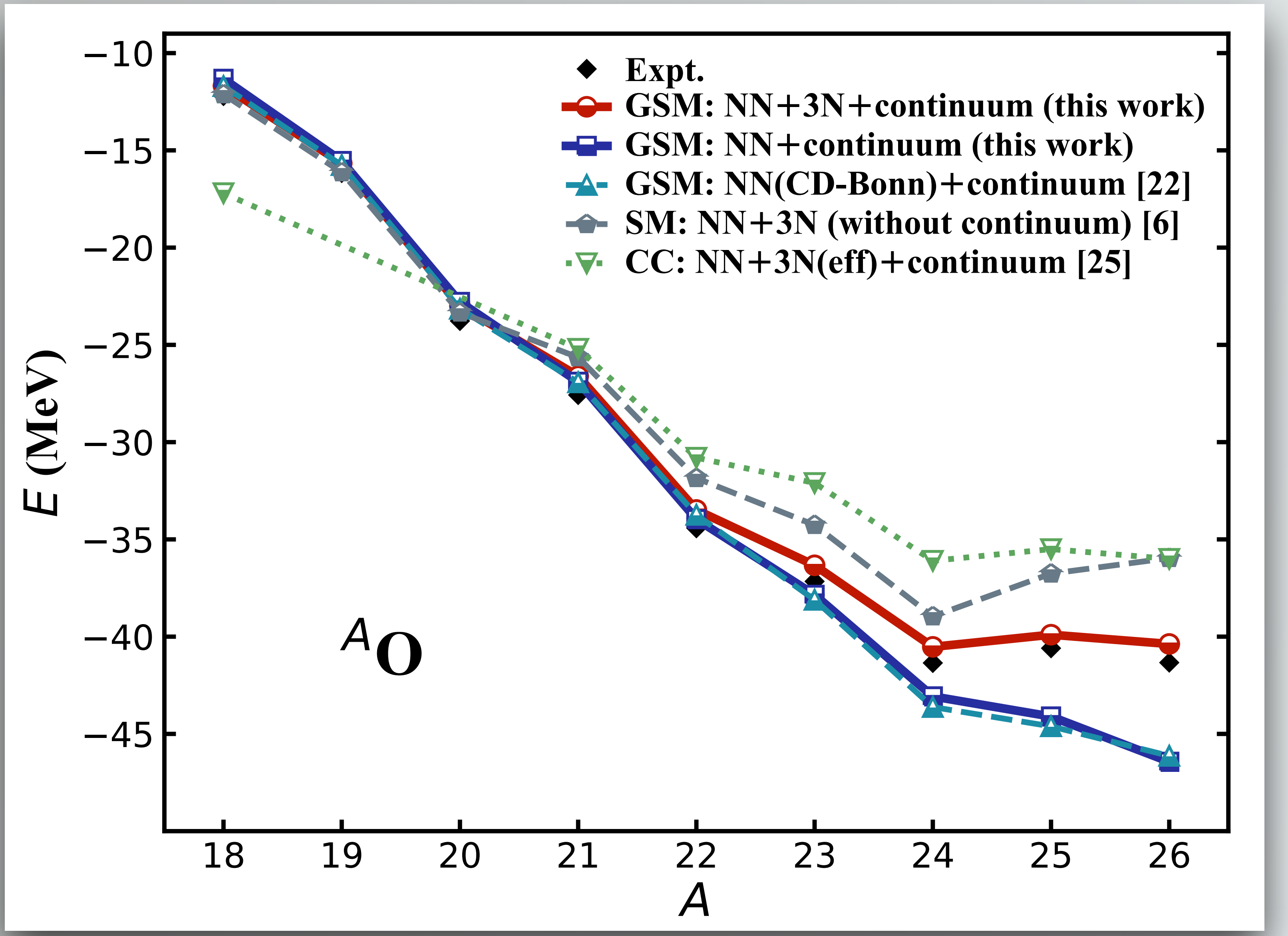


3. Q-box folded diagrams in complex-*k* sapce

$$\hat{Q}(E) = PVP + PVQ\frac{1}{E-H}QVP$$

4. Diagonalization in complex-*k* space by Davidson method (cooperation with Nicolas)

II: *sd*-shell & neutron drip line (3NF & continuum)



NN(CD-Bonn) + Continuum: Z. H. Sun *et al.,* PLB 769 (2017) 227–232
NN + 3N: T. Otsuka *et al.,* PRL 105, 032501 (2010)
NN + 3N(Effective) + Continuum: G. Hagen *et al.*, PRL 108, 242501 (2012)

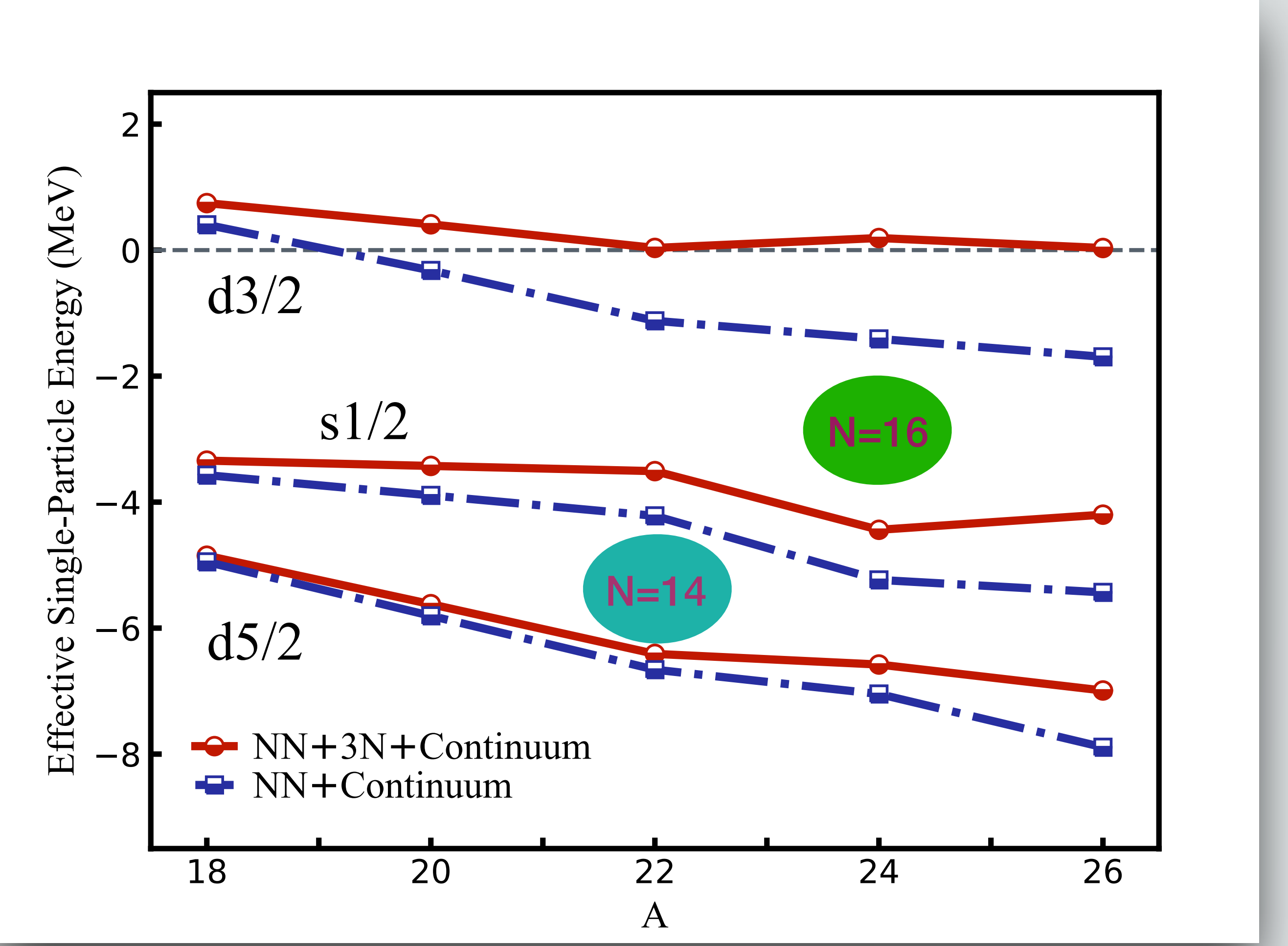
Y. Z. Ma, F. R. Xu *et al.*, in preparation

NN: N^{3}LO two-body forces
3N: N^{2}LO three-body forces
(c_D=-1.0, c_E=-0.34)

S_{2n}(MeV)	NN	NN+3N	Expt.
^{24}O	9.110	7.038	6.925
^{25}O	6.254	3.568	3.453
^{26}O	3.362	-0.150	-0.018

- **3NF & Continuum is crucial to reproduce Oxygen drip line, especially for the ground state of ^{26}O.**
- **3NF behaves repulsive effects**
- **3NF effects increase rapidly as the increasing of neutron number**

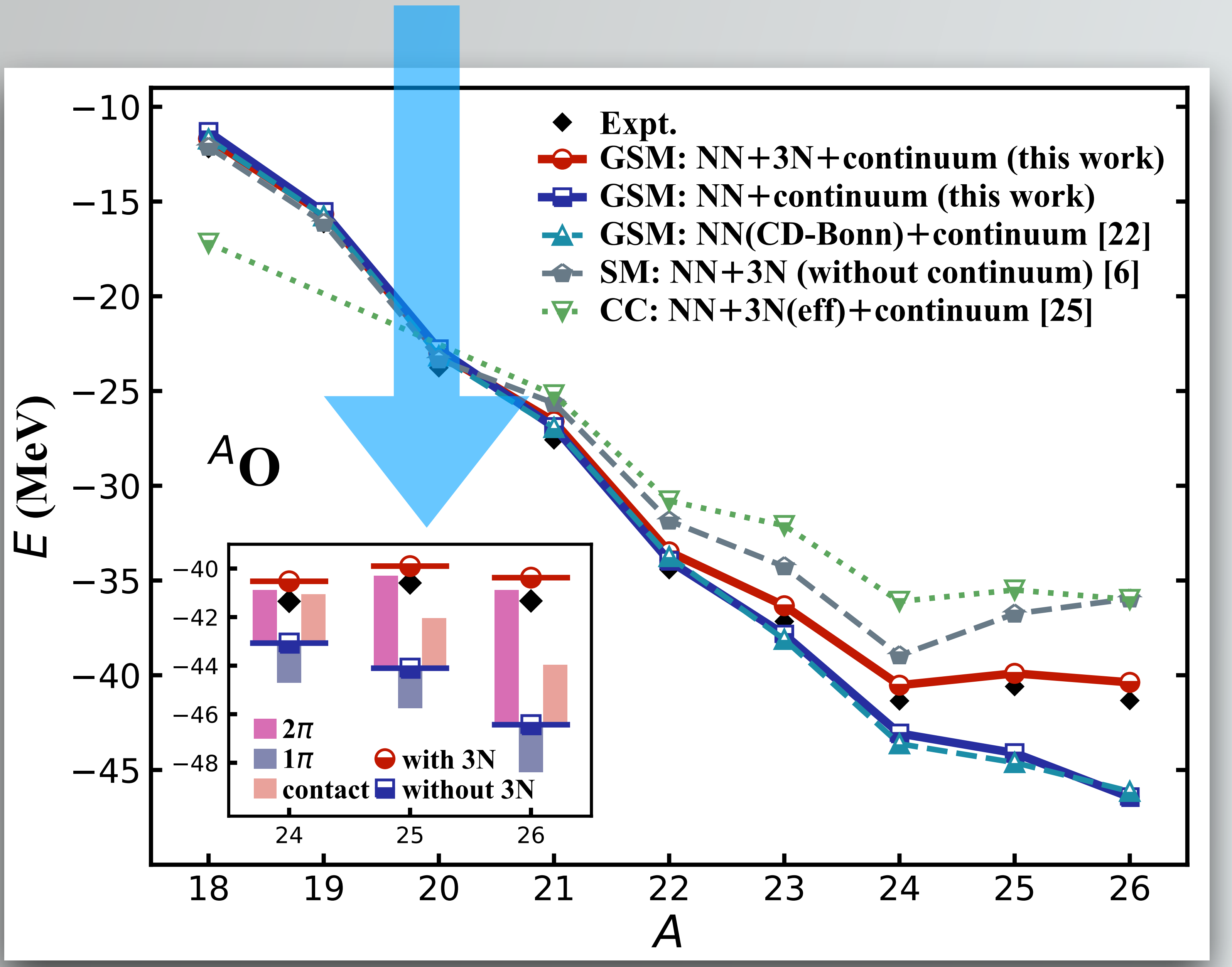
Effective single-particle energy

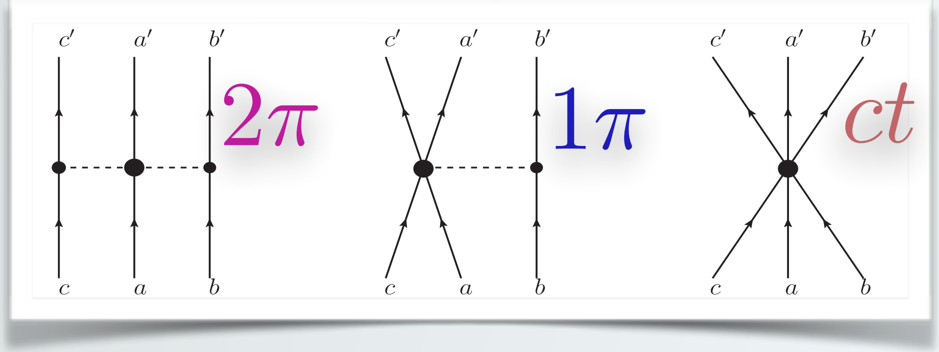


$$\mathrm{ESPE}(j) = \epsilon_j + \sum_{j'} V^{\mathrm{mon}}_{jj'} n_{j'}$$

- **3NF lifts all the three orbits especially d3/2.**
- **When A=26, d3/2 orbital is slightly above 0, which is consistent with ^{26}O narrow resonance ground state.**
- **3NF (especially monopole part) plays an important role for the magic number N=14 between d5/2 and s1/2.**
- **3NF enlarges the gap between s1/2 and d3/2 when N=16.**

Contributions from different components (2-π, 1-π and contact term) of 3NF



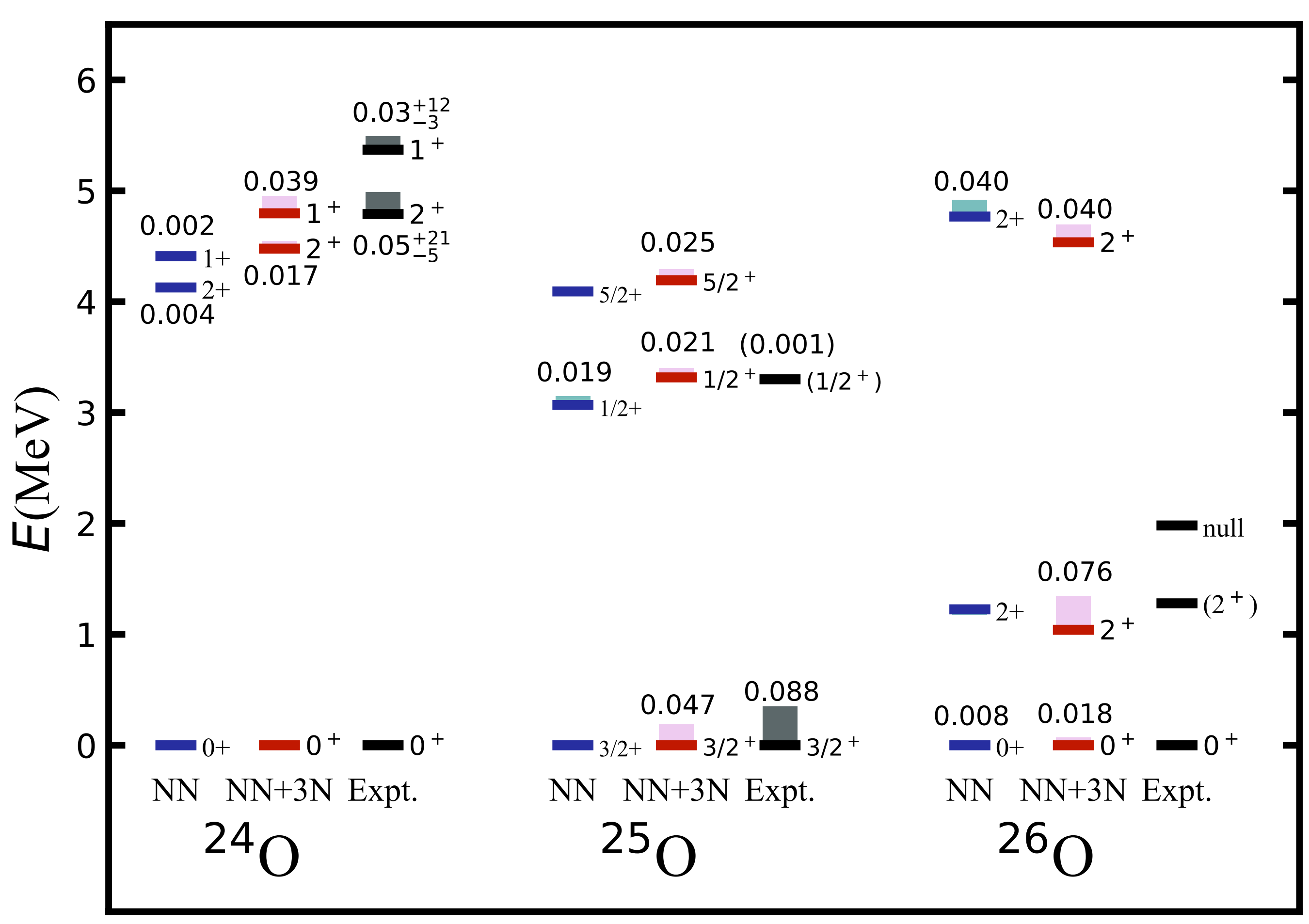


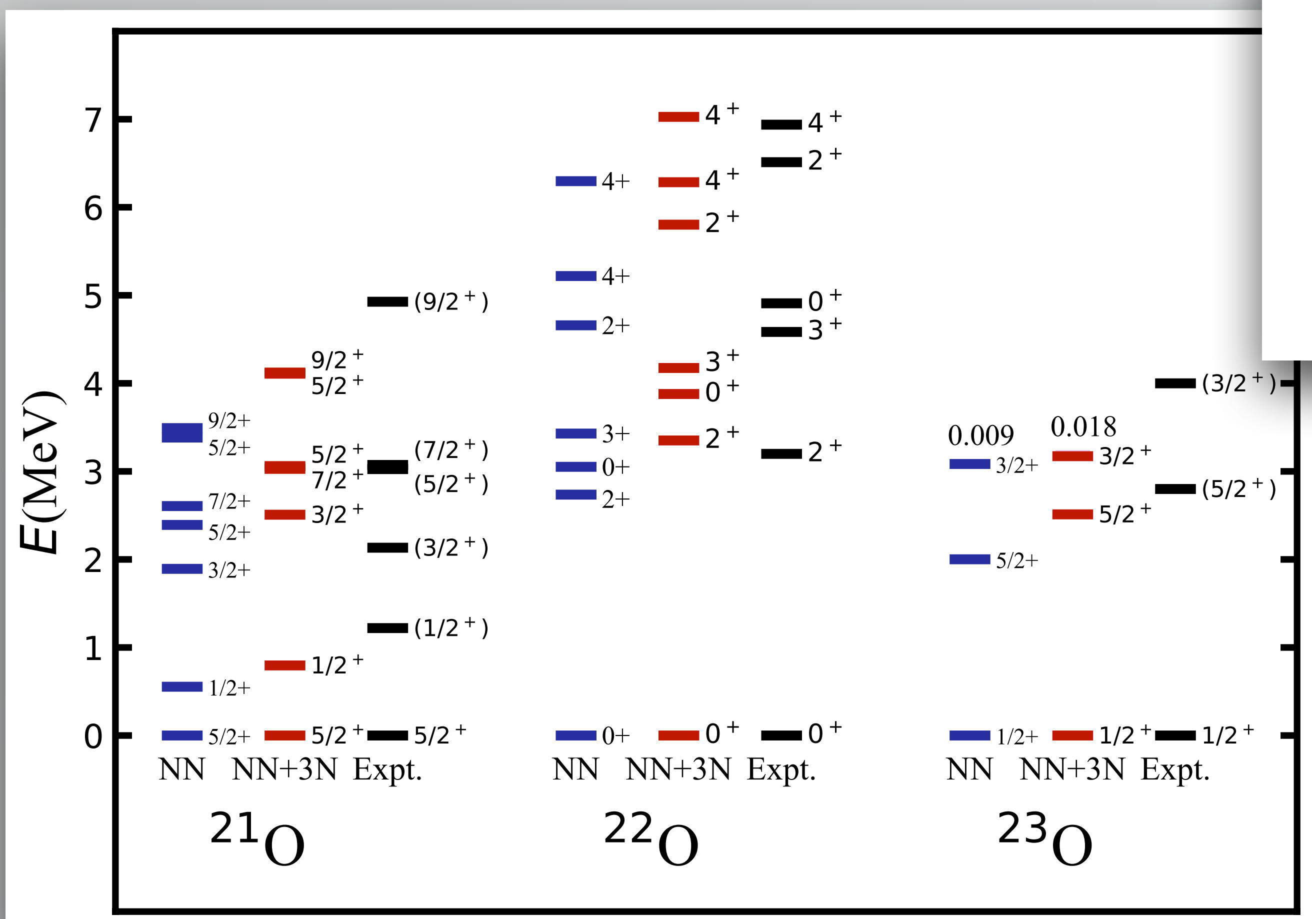
- Besides long-range 2-π exchange term, 1-π exchange & contact term also have a significant contribution.
- 2-π exchange & contact term have repulsive effects, while 1-π exchange term has an attractive contribution.
- 1-π exchange + contact term has a small contribution.
- 2-π exchange term increases faster than the other two terms with the increasing of neutron number.

Y. Z. Ma, F. R. Xu *et al.*, in preparation

3NF systematically improves the spectra of oxygen isotopes

- All the first excited states have been lift by 3NF.
- The first 2+ state of ^{22}O and ^{24}O is very close to data and the big gap above ground state indicates the sub-shell when neutron number equals 14 and 16.





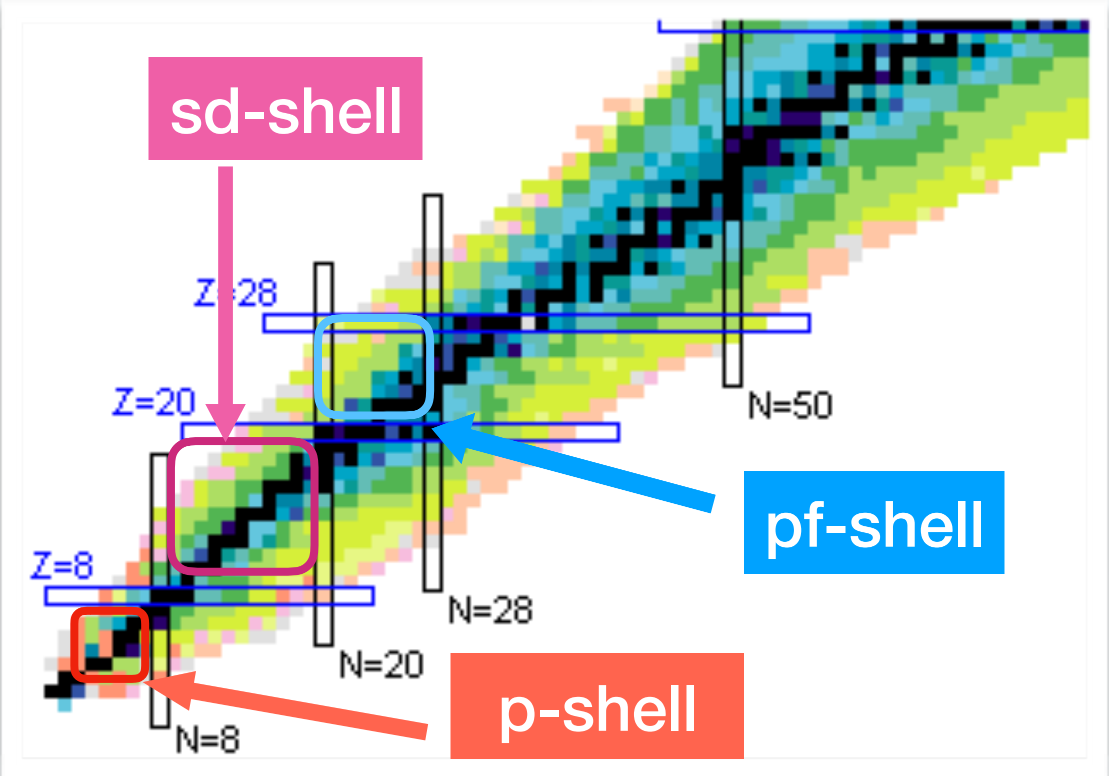
Y. Z. Ma, F. R. Xu *et al.*, in preparation

- In general, 3NF+Continuum can give out very reasonable Resonance width against data.
- In ^{24}O, the first 2+ and 1+ have a width of 17 KeV and 39 KeV.
- In ^{25}O, only NN + 3N force can reproduce ground state resonance nature and we obtain a very good result of first 1/2+ state consistent with data (PRC 96, 054322 (2017)).
- In ^{26}O, ground state has very narrow resonance.

Conclusion & Outlook

Summary:

1. Success to derive & calculate 3N matrix element from chiral NNLO
2. Test the reliability of our 3NF and many-body methods by the benchmark of *p*-shell nuclei
3. Study of *fp*-shell nuclei uncovers the role played by 3NF in the shell evolution
4. Both 3NF and Continuum are crucial to reproduce the Oxygen neutron drip line.



Our goal is:

- To adopt 3N force to nuclear many body method (RSM, Gamow-SM, Gamow-IMSRG …)
- To investigate the role of 3N force plays in heavier nuclei
- …

Challenge:

- Including high order contribution from 3N force.
- To adopt 3N force to heavier nuclei we need calculate 3N matrix element in a much larger model space which means the demand of huge computation resource and highly optimized program.

Du to the explosion of computation cost with the increasing of model space, up to now, we can not calculate the 3N matrix element in a very big model space.

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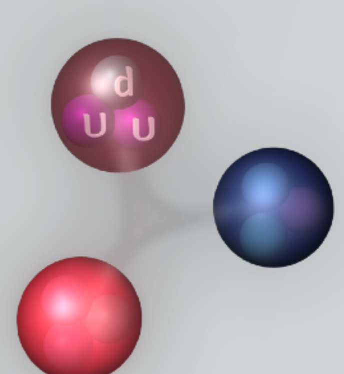
Thank you !

PKU: F. R. Xu, B. S. Hu, J. G. Li, Y. F. Geng, S. Zhang

INFN: L. Coraggio, T. Fukui, A. Gargano, L. De Angelies, N. Itaco

IMP: N. Michel **MSU:** S. M. Wang **Oak Ridge:** Z. H. Sun



PKU: *School of Physics, Peking University, China*

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IMP: *Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China*

MSU: *FRIB/NSCL Laboratory, Michigan State University, USA*

Oak Ridge: *Physics Division, Oak Ridge National Laboratory, Oak Ridge, TN 37831, USA*

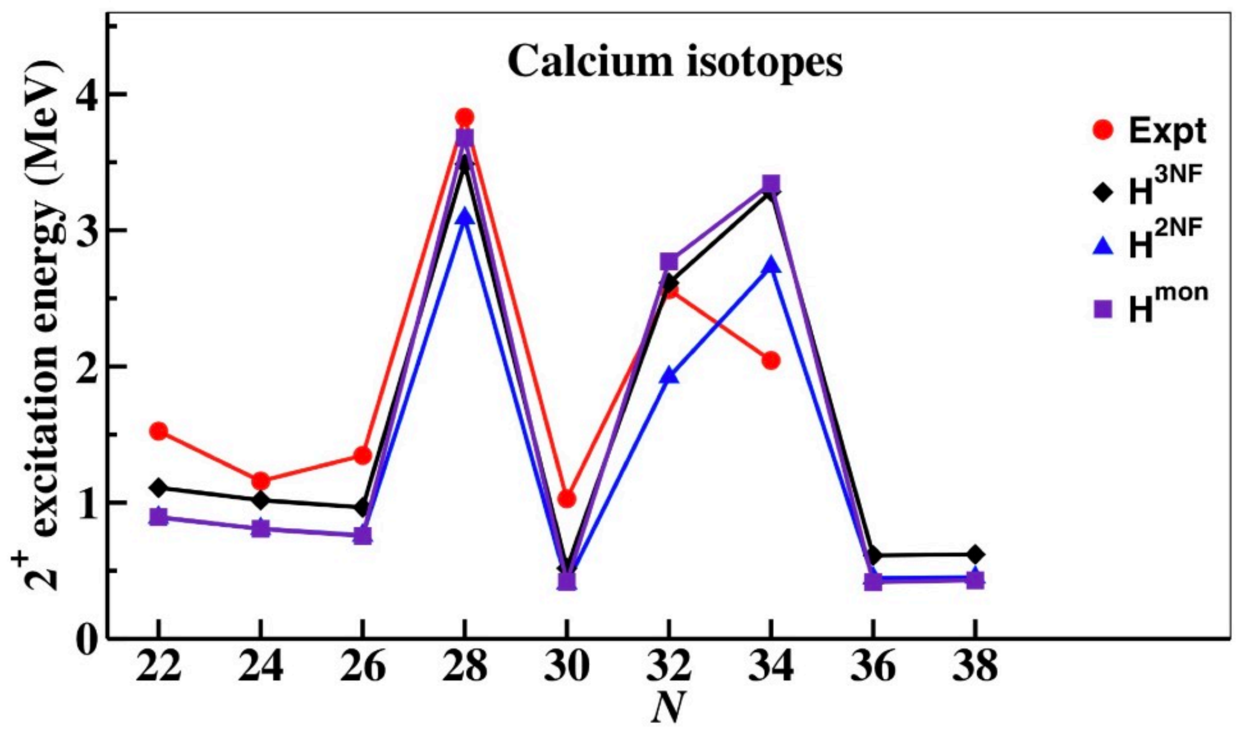


FIG. 12: Experimental and calculated excitation energies of the yrast $J^\pi = 2^+$ states for calcium isotopes from $N = 22$ to 40. See text for details.

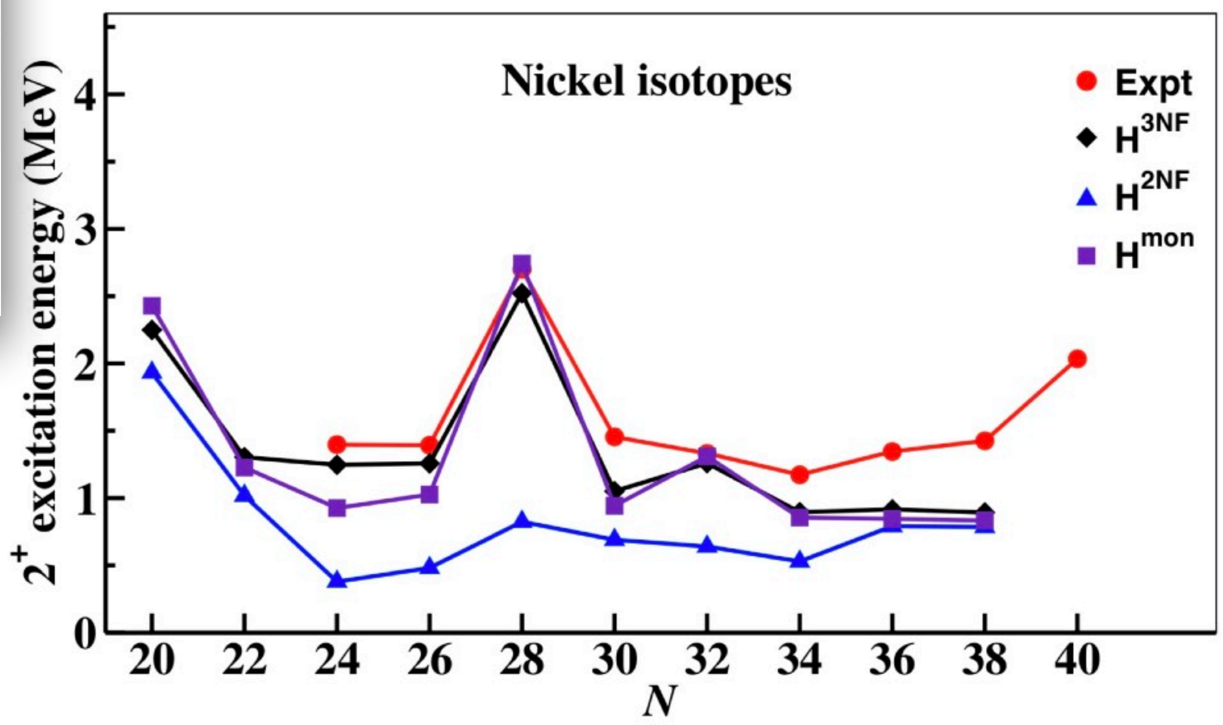
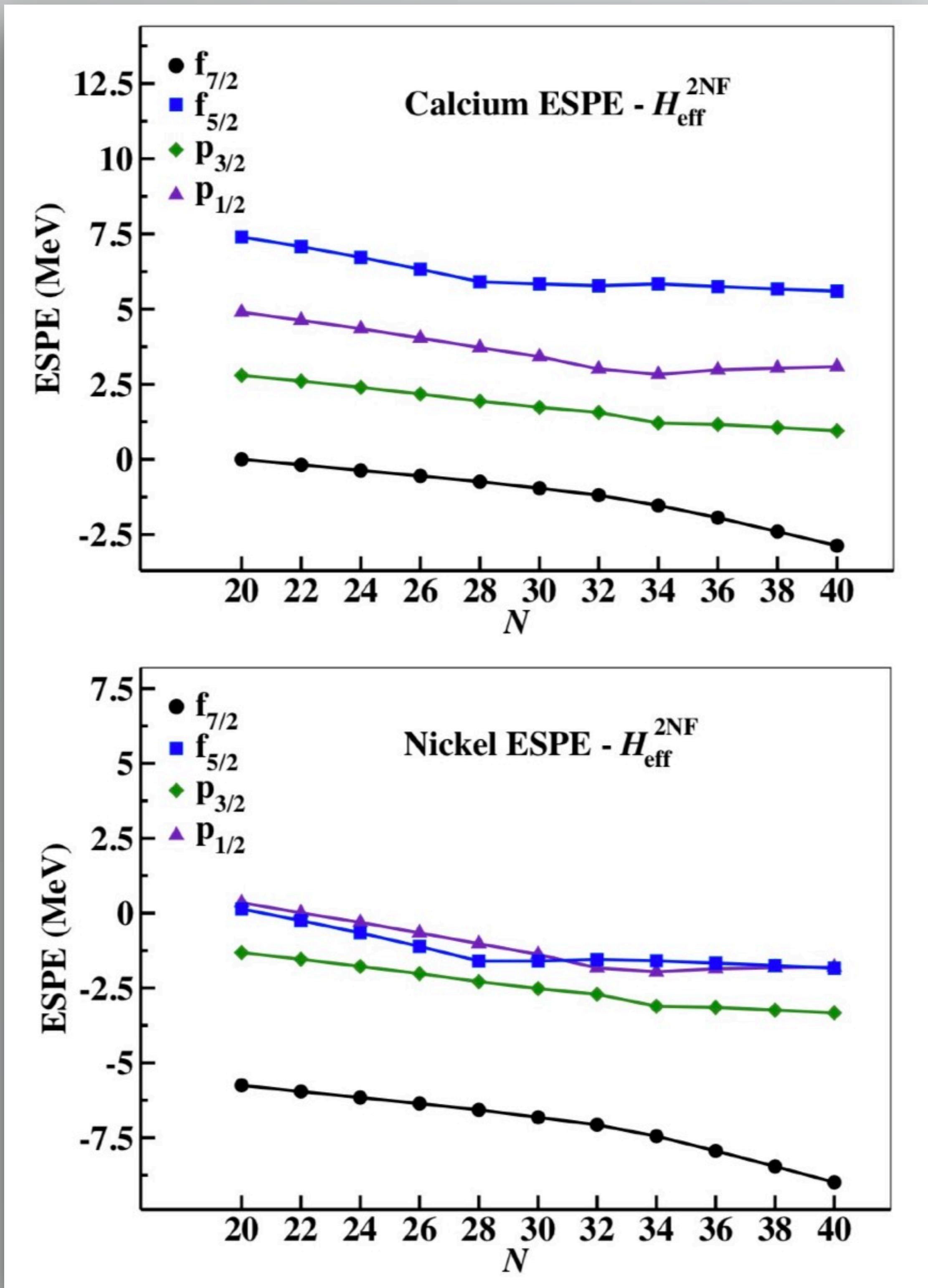
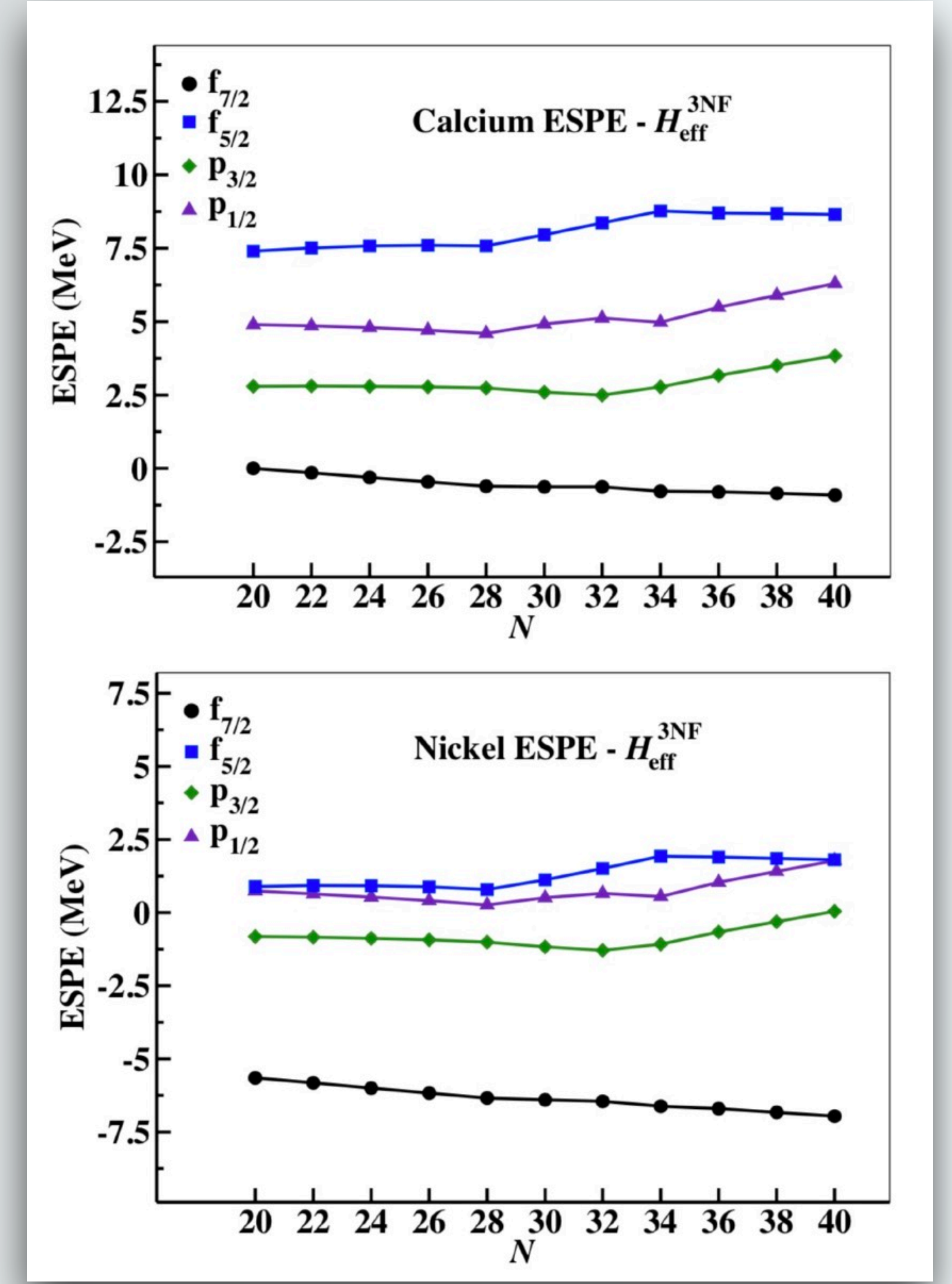
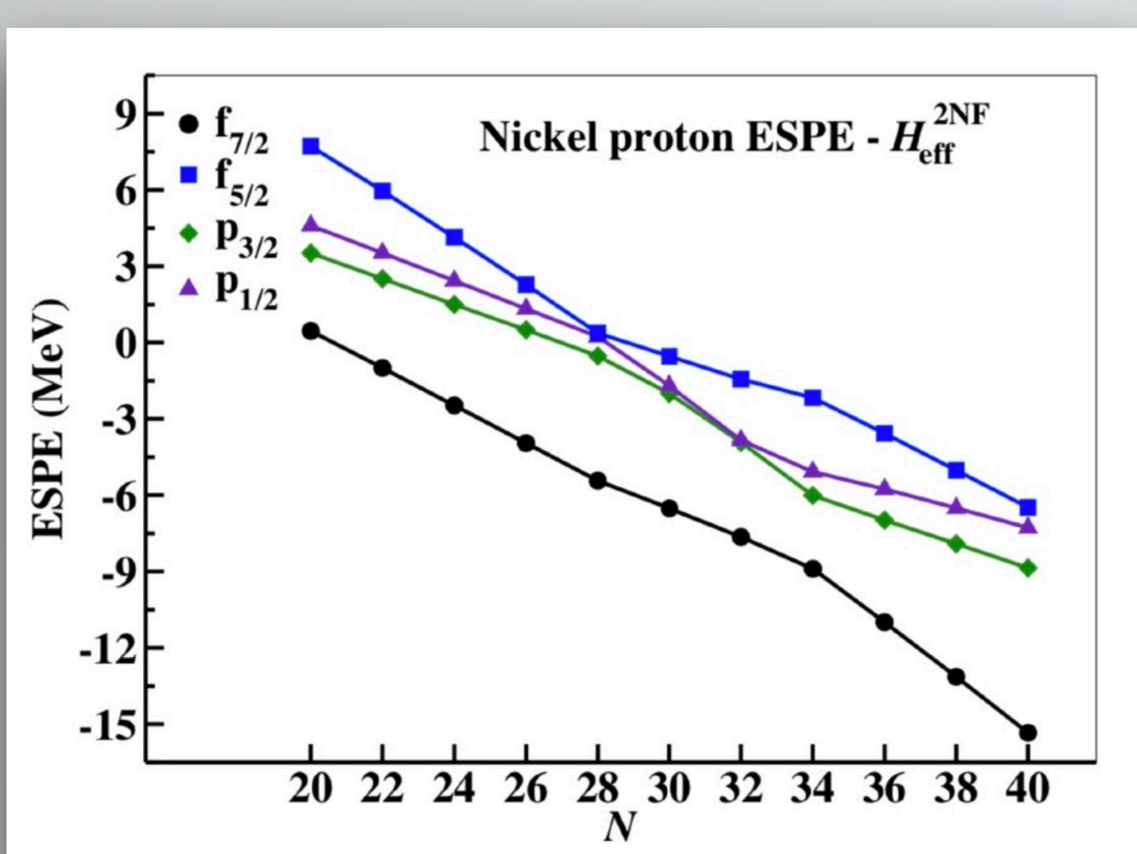


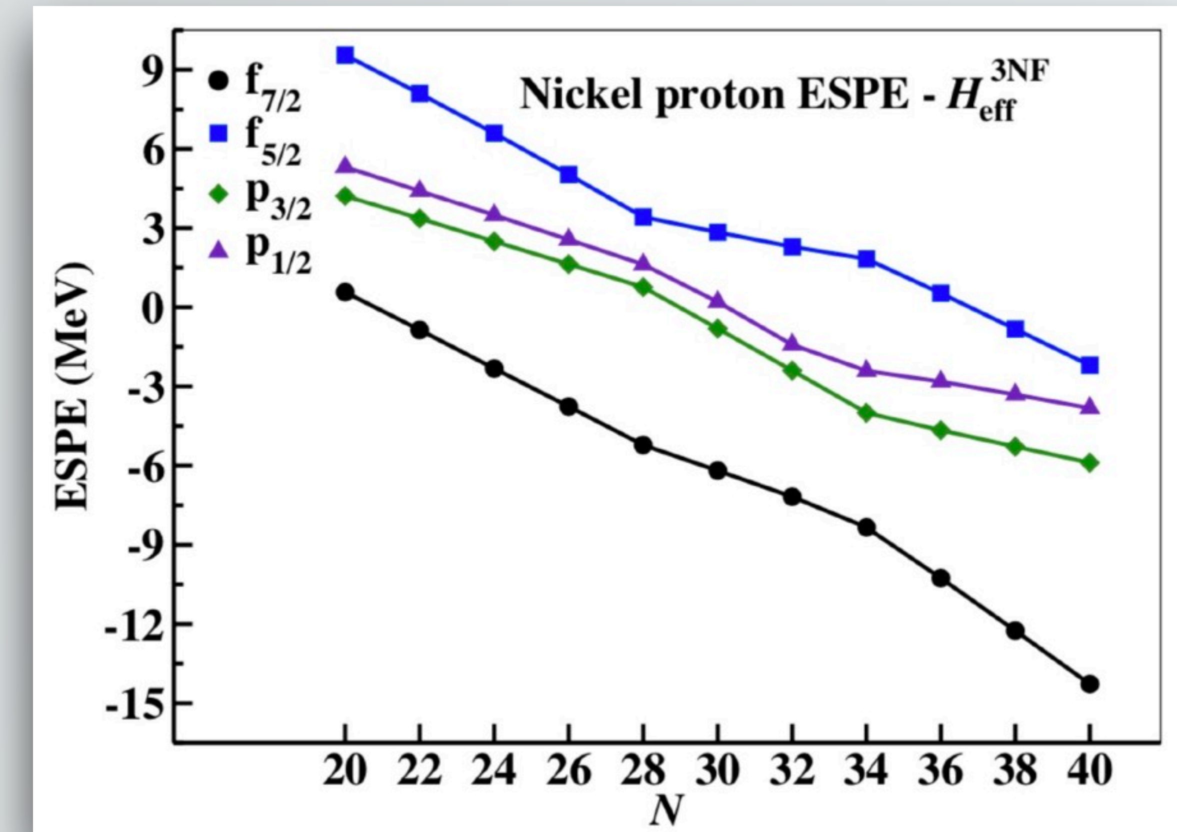
FIG. 20: Same as in Fig. 14, but for nickel isotopes. See text for details.


Calcium ESPE - H_{eff}^{2NF}
Nickel ESPE - H_{eff}^{2NF}
$f_{7/2}$
$f_{5/2}$
$p_{3/2}$
$p_{1/2}$
ESPE (MeV)
N


Calcium ESPE - H_{eff}^{3NF}
Nickel ESPE - H_{eff}^{3NF}
$f_{7/2}$
$f_{5/2}$
$p_{3/2}$
$p_{1/2}$
ESPE (MeV)
N



Nickel proton ESPE - $H_{\mathrm{eff}}^{\mathrm{2NF}}$
$f_{7/2}$
$f_{5/2}$
$p_{3/2}$
$p_{1/2}$
ESPE (MeV)
9
6
3
0
-3
-6
-9
-12
-15
20 22 24 26 28 30 32 34 36 38 40
N



Nickel proton ESPE - $H_{\mathrm{eff}}^{\mathrm{3NF}}$
$f_{7/2}$
$f_{5/2}$
$p_{3/2}$
$p_{1/2}$
ESPE (MeV)
9
6
3
0
-3
-6
-9
-12
-15
20 22 24 26 28 30 32 34 36 38 40
N

Realistic Shell Model

This recursive equation for *H*eff may be solved using iterative techniques (Krenciglowa-Kuo, Lee-Suzuki, Extended Krenciglowa-Kuo …)

$$H_{\text{eff}} = \hat{Q} - \hat{Q}' \int \hat{Q} + \hat{Q}' \int \hat{Q} \int \hat{Q} - \hat{Q}' \int \hat{Q} \int \hat{Q} \int \hat{Q} \cdots$$

with $\hat{Q}$-box vertex function:

$$\hat{Q}(\epsilon) = PH_1P + PH_1Q \frac{1}{\epsilon - QHQ} QH_1P$$

For a many-body system, exact calculation of the $\hat{Q}$-box is prohibitive, then we perform a perturbative expansion:

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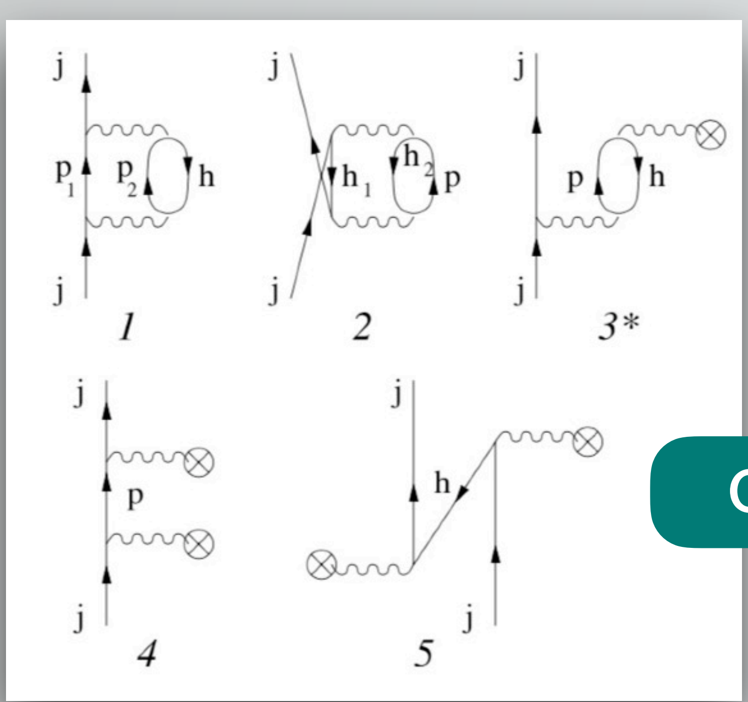

1st- & 2nd-ordrder

Two body

One body

3rd-ordrder: 126 diagrams

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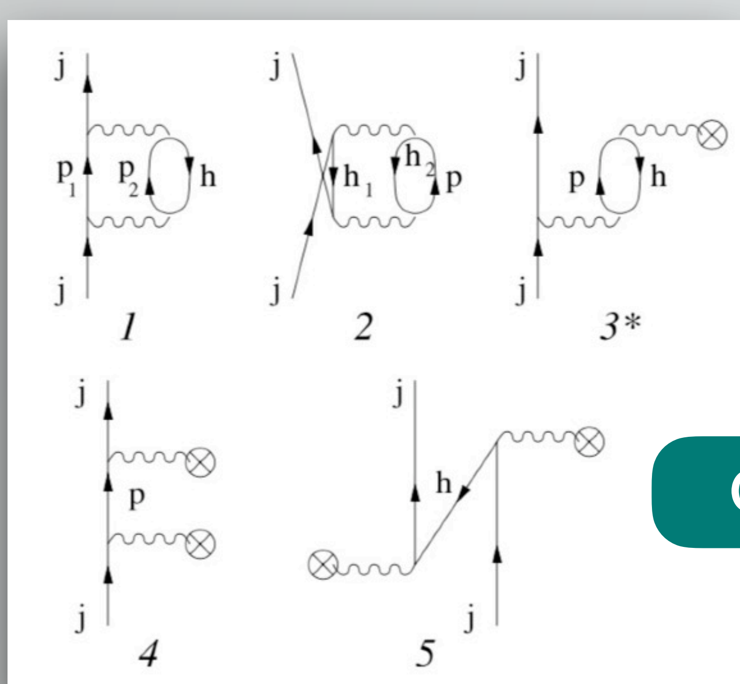

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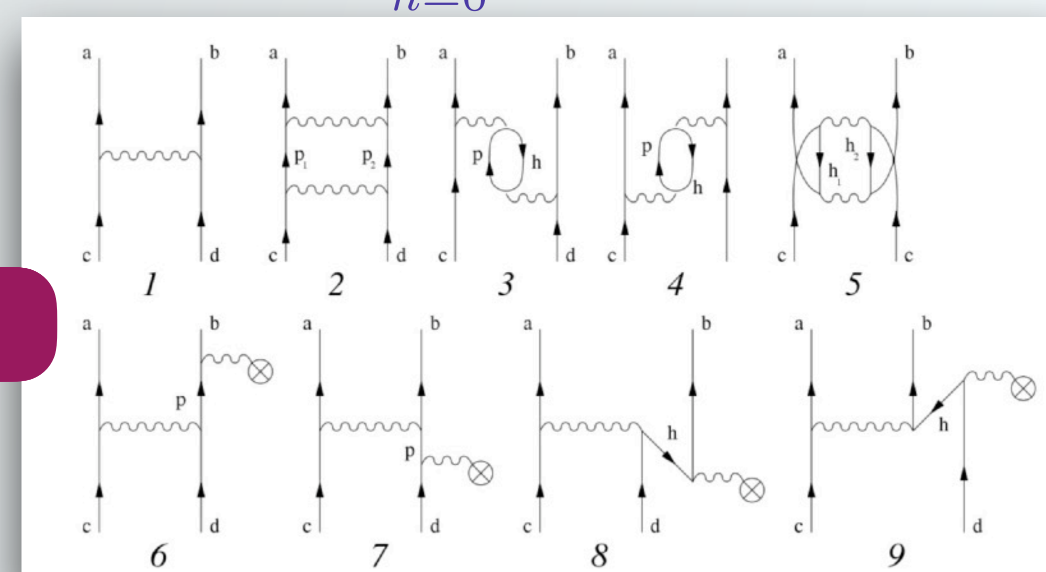

3rd-ordrder: 126 diagrams

L. Coraggio *et al.*, Annals of Physics 327 (2012)