Precise Cross Section Measurement of $e^+e^- \to \pi^+\pi^-\psi(3686)$ with XYZ Data

Xuhong Li[1,2], Xiaorong Zhou[1,2], Peilian Li[1,2], Zhentian Sun[3], Haiping Peng[1,2]

[1] University of Science and Technology of China
[2] State Key Laborator of Particle Detection and Electronics
[3] Institute of High Energy Physics
lixuhong@mail.ustc.edu.cn

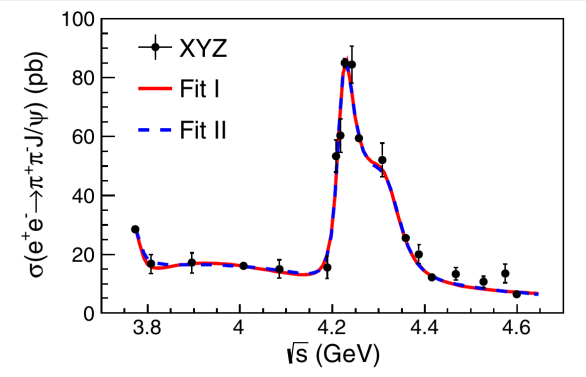
May 21, 2019

Overview

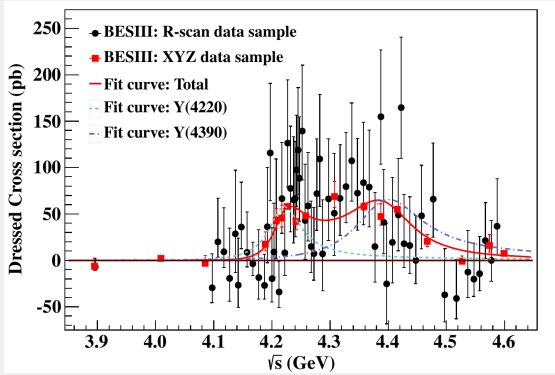
Motivation

- Two resonances are observed in three processes with very close mass and width
- With new XYZ data and old XYZ data, more precise measurement can be done
- More precise results will help to confirm the nature of these resonances

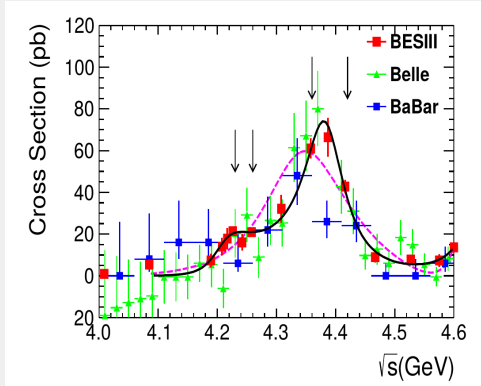
$e^+e^- \to \pi^+\pi^- J/\psi$
PRL.118.092001 (2017)



$e^+e^- \to \pi^+\pi^- h_c$
PRL.118.092002 (2017)



$e^+e^- \to \pi^+\pi^-\psi(3686)$
PRL.96.032004 (2017)



Data Set

Data Sets	E_{cm}(GeV)	$\mathcal{L}(pb^{-1})$
4009	4007.62	482.0
4090	4085.45	52.86
4180	4178.37	3161
4190	4188.59	43.33
4190	4188.8	522.5
4200	4198.9	524.6
4210	4207.73	54.95
4210	4209.2	518.1
4220	4217.13	54.60
4220	4218.7	514.3
4230	4226.26	1100.
4237	4235.7	530.6
4245	4241.66	55.88
4246	4243.8	537.4
4260	4257.97	828.4
4270	4266.8	529.7
4280	4277.7	175.5
4310	4307.89	45.08
4360	4358.26	543.9
4390	4387.40	55.57
4420	4415.58	1090.
4470	4467.06	111.0
4530	4527.14	112.1
4575	4574.50	48.93
4600	4599.53	586.9

- Decay channel:
  - $e^+e^- \to \pi^+\pi^-\psi(3686), \psi(3686) \to \pi^+\pi^- J/\psi$
  - $e^+e^- \to \pi^+\pi^-\psi(3686), \psi(3686) \to neutrals\ J/\psi$
- Boss version: 703
- Signal MC: 200K for each decay channel at each energy point
- Generator: KKMC+BesEvtGen

Event Selection for charged mode

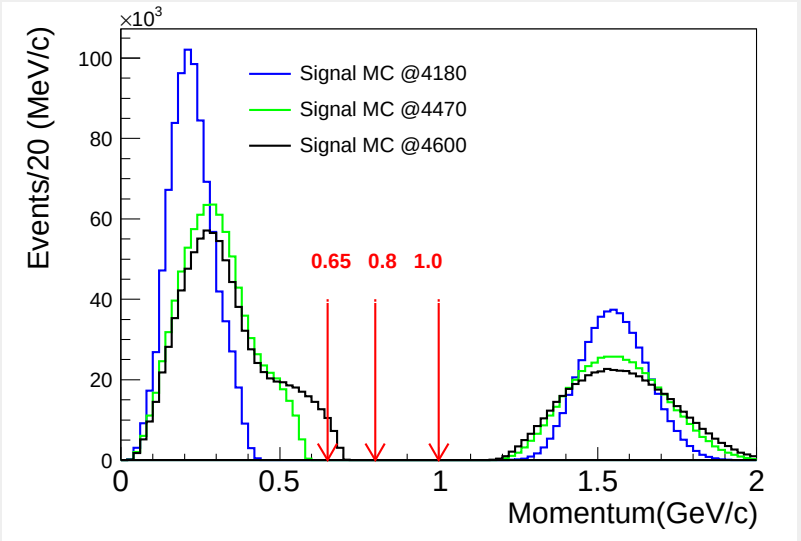
Charged mode: $\psi(3686) \to \pi^+\pi^- J/\psi, J/\psi \to l^+l^-$
($\psi(3686)$ signal is reconstructed by $\pi^+\pi^- l^+ l^-$)

Good Charged tracks

- Vertex cuts: $|V_{xy}| < 1.0cm$, $|V_z| < 10cm$
- Polar angle in MDC: $|cos\theta| \leq 0.93$

PID

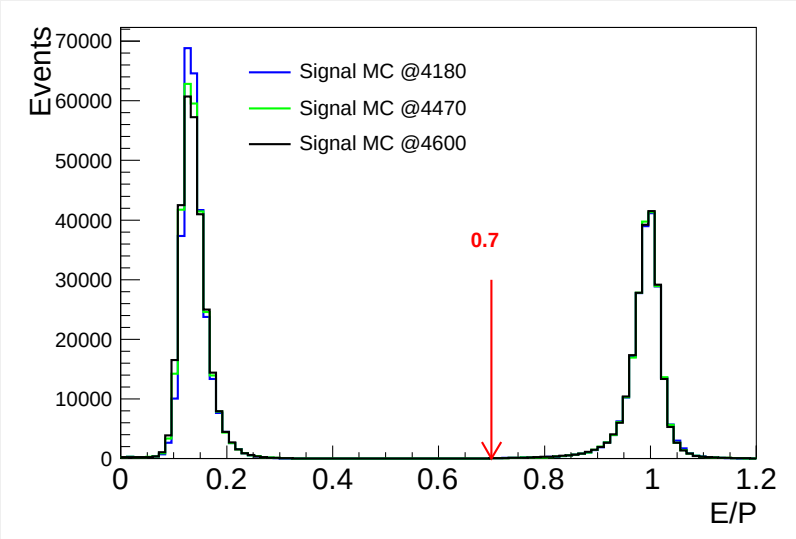
- One pair of lepton: $p > 1.0GeV$
- Two pairs of pion: three or four tracks
  - $p < 0.65\ GeV$ for data sets with $\sqrt{s} < 4.465\ GeV$
  - $p < 0.8\ GeV$ for data sets with $\sqrt{s} > 4.465\ GeV$

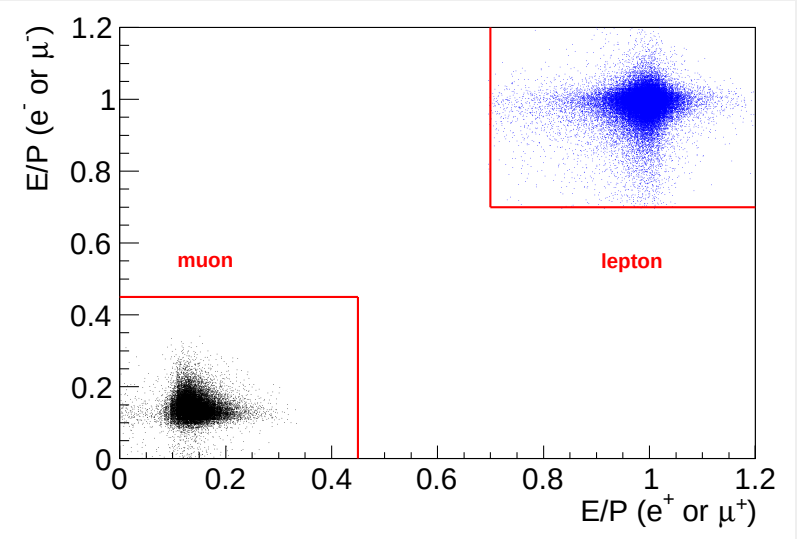


Event Selection for charged mode

PID

- Momentum of lepton: $p > 1.0 GeV$
- Electron: $E/p > 0.7$ for both tracks
- Muon: $E < 0.45 GeV$ for both tracks

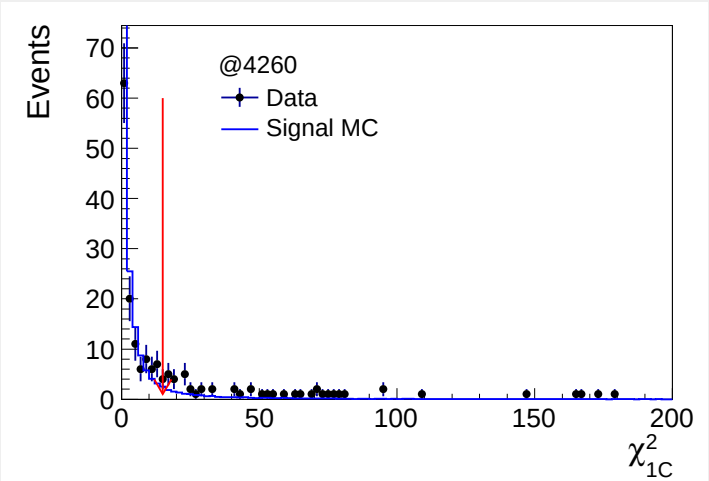


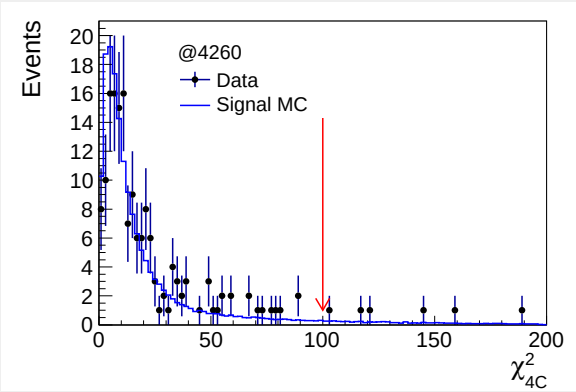


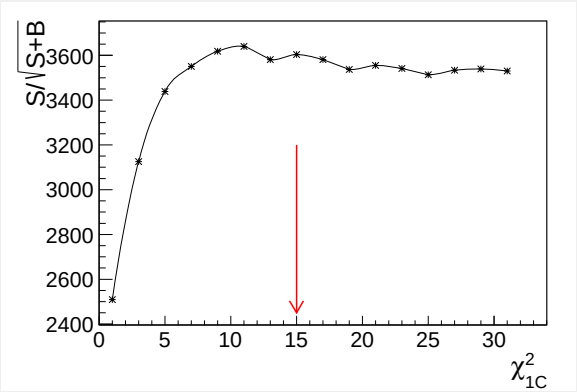
Event Selection for charged mode

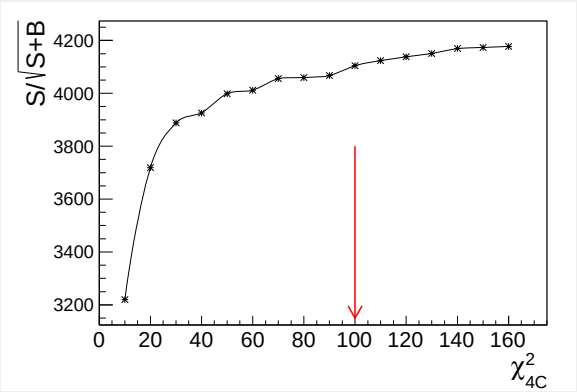
Kinematic Fit

- 6 tracks
  - 4C: $\chi^2_{4C} < 100$
  - 5C: no χ^2_{5C} cut
- 5 tracks (one missing pion)
  - 1C: $\chi^2_{1C} < 15$
  - 2C: no χ^2_{2C} cut



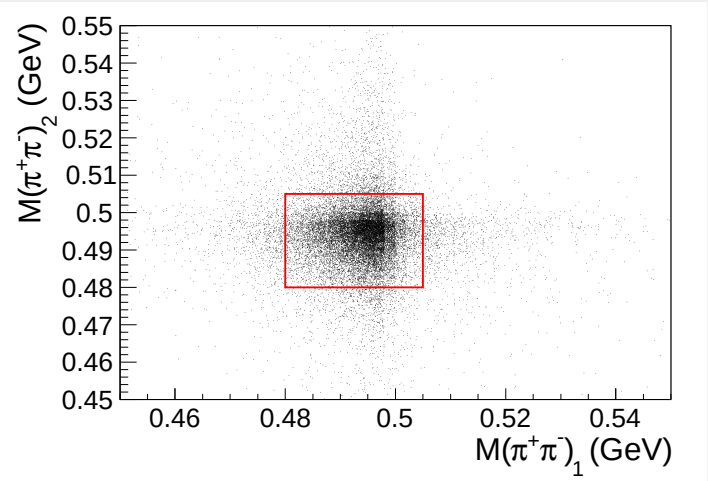


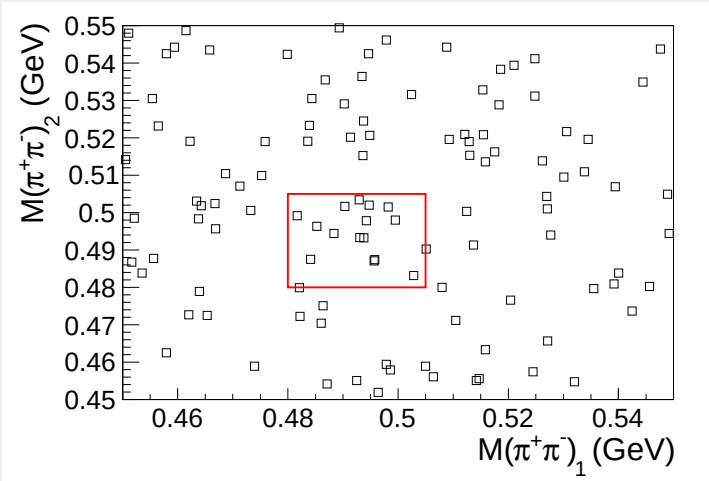




Background analysis

No.	decay chain	nEvt	nTot
0	$Y(4260) \to \pi^+\pi^-\psi', \psi' \to J/\psi\pi^+\pi^-, J/\psi \to \mu^+\mu^-$	296	296
1	$Y(4260) \to \pi^+\pi^-\psi', \psi' \to J/\psi\pi^+\pi^-, J/\psi \to e^+e^-$	178	471
2	$Y(4260) \to K_SK_SJ/\psi, K_S \to \pi^+\pi^-, K_S \to \pi^+\pi^-, J/\psi \to \mu^+\mu^-$	18	492
3	$Y(4260) \to \pi^+\pi^-\psi', \psi' \to J/\psi\pi^+\pi^-, J/\psi \to e^+e^-\gamma_{FSR}$	18	510
4	$Y(4260) \to K_SK_SJ/\psi, K_S \to \pi^+\pi^-, K_S \to \pi^+\pi^-, J/\psi \to e^+e^-$	12	522
5	$Y(4260) \to \pi^+\pi^-\psi', \psi' \to J/\psi\pi^+\pi^-, J/\psi \to \mu^+\mu^-\gamma_{FSR}$	9	531

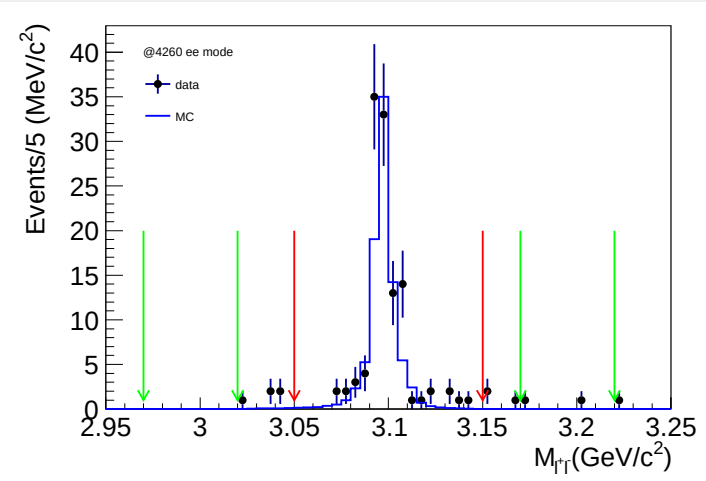


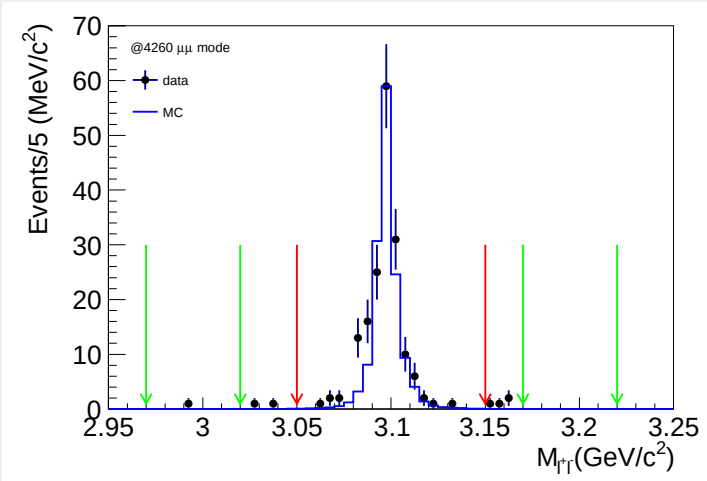


- The main background events are from process $Y(4260) \to K_SK_SJ/\psi$
- No obvious enhancement is observed from data

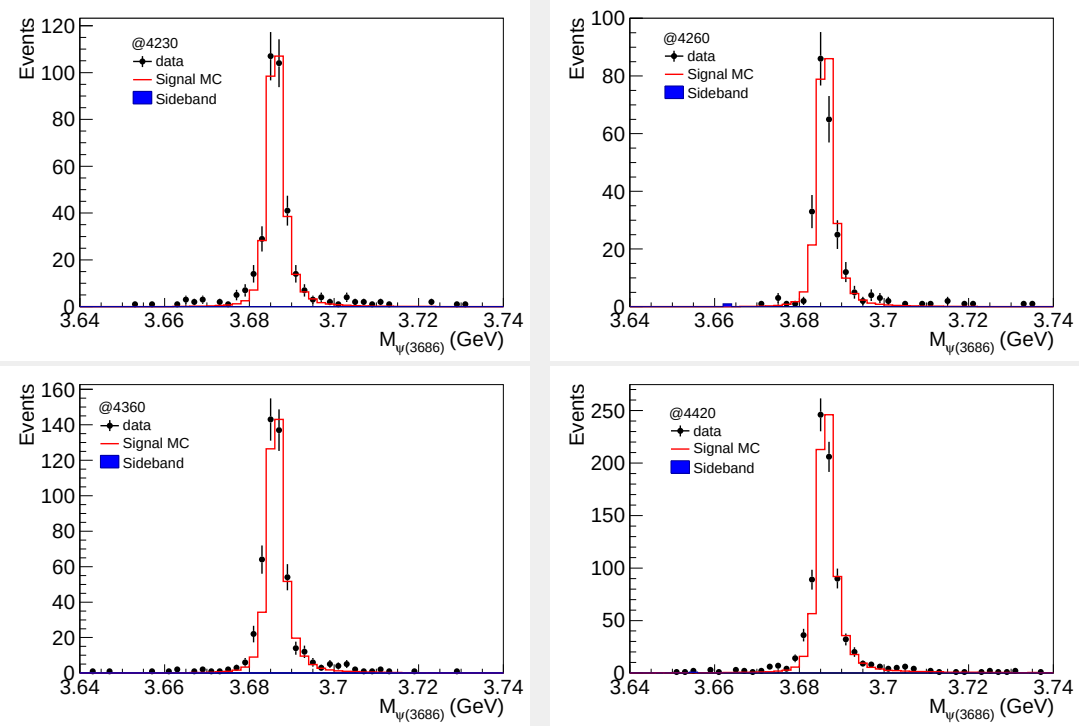
J/ψ mass window

- Signal: [3.05, 3.15] GeV
- Sideband: [2.97, 3.02] GeV, [3.17,3.22] GeV





Signal candidates



- The combination of $\pi^+\pi^- l^+ l^-$ with mass closest to $M_{\psi(3686)}$ is used to reconstruct $\psi(3686)$ signal
- The distributions of $\psi(3686)$ invariant mass at different energy points
- Clear $\psi(3686)$ signal and very low backgrounds

Cross section measurement

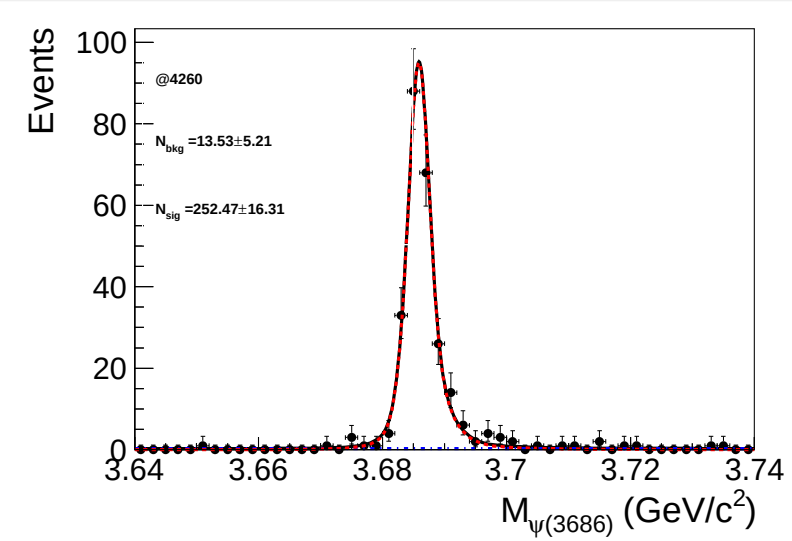
$$\sigma^B = \frac{N^{obs}}{\mathcal{L}_{int}(1+\delta^r)(1+\delta^v)\epsilon\mathcal{B}}$$

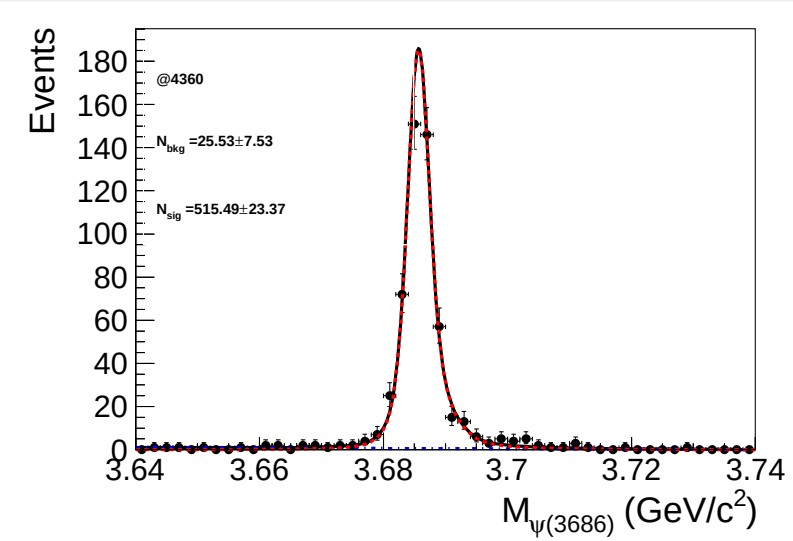
- N^{obs} is the number of observed events from data
- ϵ is detection efficiency
  - efficiencies for 4180 $\sim$ 4280, 4360 and 4420 are obtained with PWA MC
  - efficiencies for other energy points are obtained with PHSP MC
- $\mathcal{B}$ is the branch ration of $\psi(3686) \to \pi^+\pi^- J/\psi \to \pi^+\pi^- l^+ l^-$
- $(1+\delta^r)$ is the radiative correction factor
- $(1+\delta^v)$ is vacuum polarization factor taken from QED

Signal Extraction

Fit method

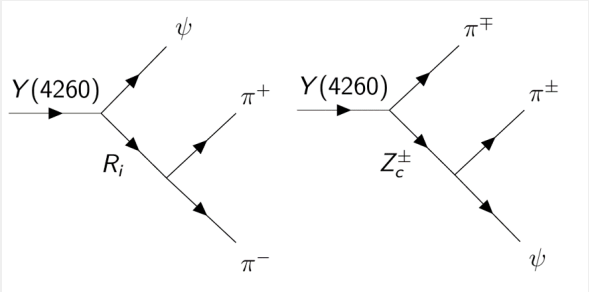
- Signal: MC shape $\otimes$ Gaussian function
- Background: $1st$ polynomial





PWA for efficiencies

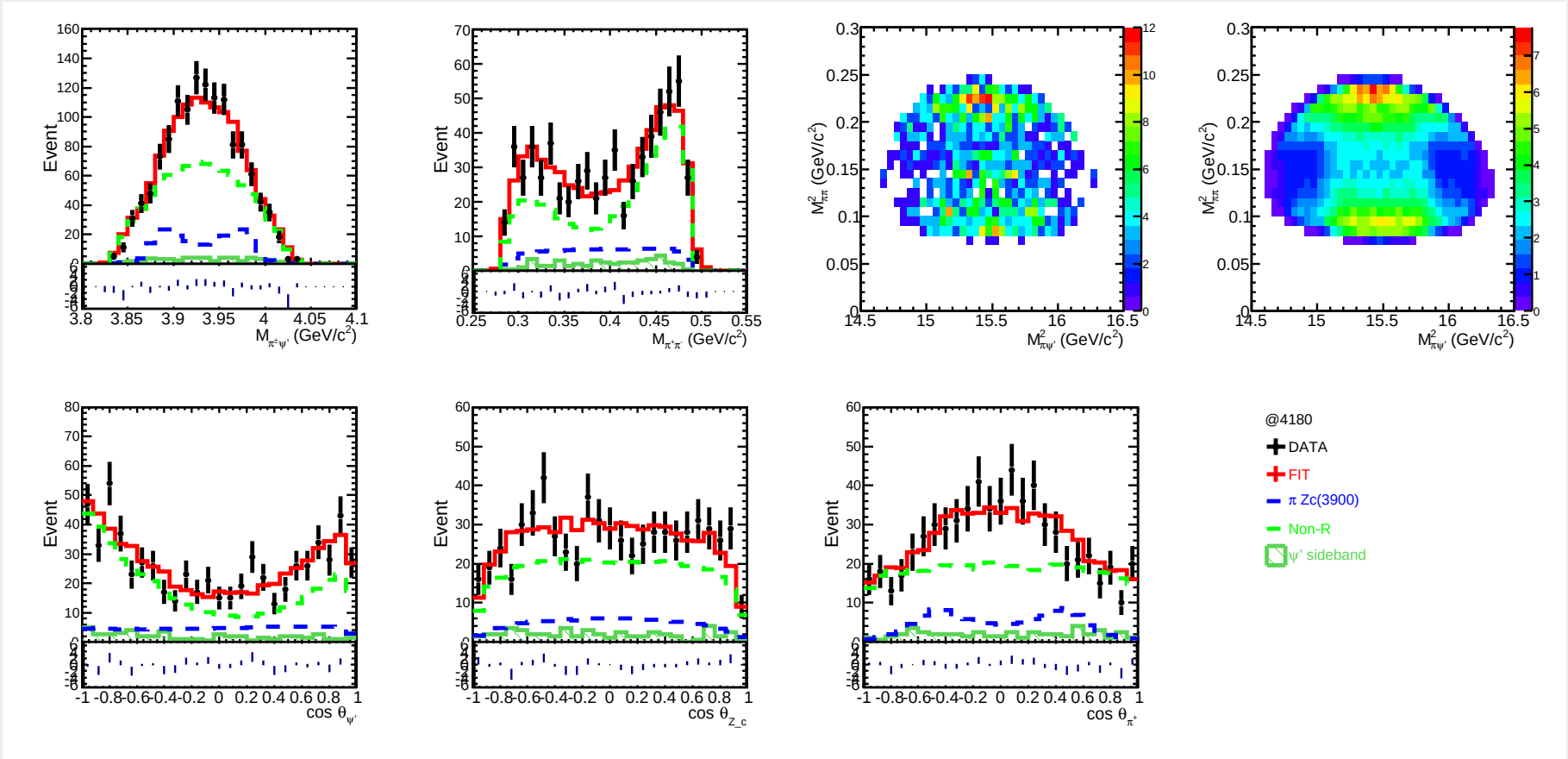
- Partial Wave Analysis is used to get efficiencies for data set 4180 $\sim$ 4280, 4360 and 4420
- Amplitude construction(helicity method)



  - $\mathcal{A}^{R}_{\lambda_Y\lambda_\psi} = \sum_{\lambda_R} F^{J_Y}_{\lambda_R,\lambda_\psi} D^{J_Y *}_{\lambda_Y,\lambda_R-\lambda_\psi}(\Omega_R) F^{J_R}_{0,0} D^{J_R *}_{\lambda_R,0}(\Omega_{\pi^+})\, T_R$
  - $\mathcal{A}^{Z_c}_{\lambda_Y\lambda_\psi} = \sum_{\lambda_{Z_c}} F^{J_Y}_{\lambda_{Z_c},0} D^{J_Y *}_{\lambda_Y,\lambda_{Z_c}}(\Omega_{Z_c}) F^{J_{Z_C}}_{\lambda_\psi,0} D^{J_{Z_c} *}_{\lambda_{Z_c},\lambda_\psi}(\Omega_\psi)\, T_{Z_c}$
- Total invariant amplitude:
  - $\sum_{\lambda_Y,\lambda_\psi} |\mathcal{M}_{\lambda_Y\lambda_\psi}|^2 = \sum_{\lambda_Y,\lambda_\psi} |\mathcal{A}^{R}_{\lambda_Y\lambda_\psi} + \sum_{\lambda'_\psi} d^{J_\psi}_{\lambda'_\psi,\lambda_\psi}(\theta_p) \mathcal{A}^{Z_c}_{\lambda_Y\lambda_\psi}|^2$

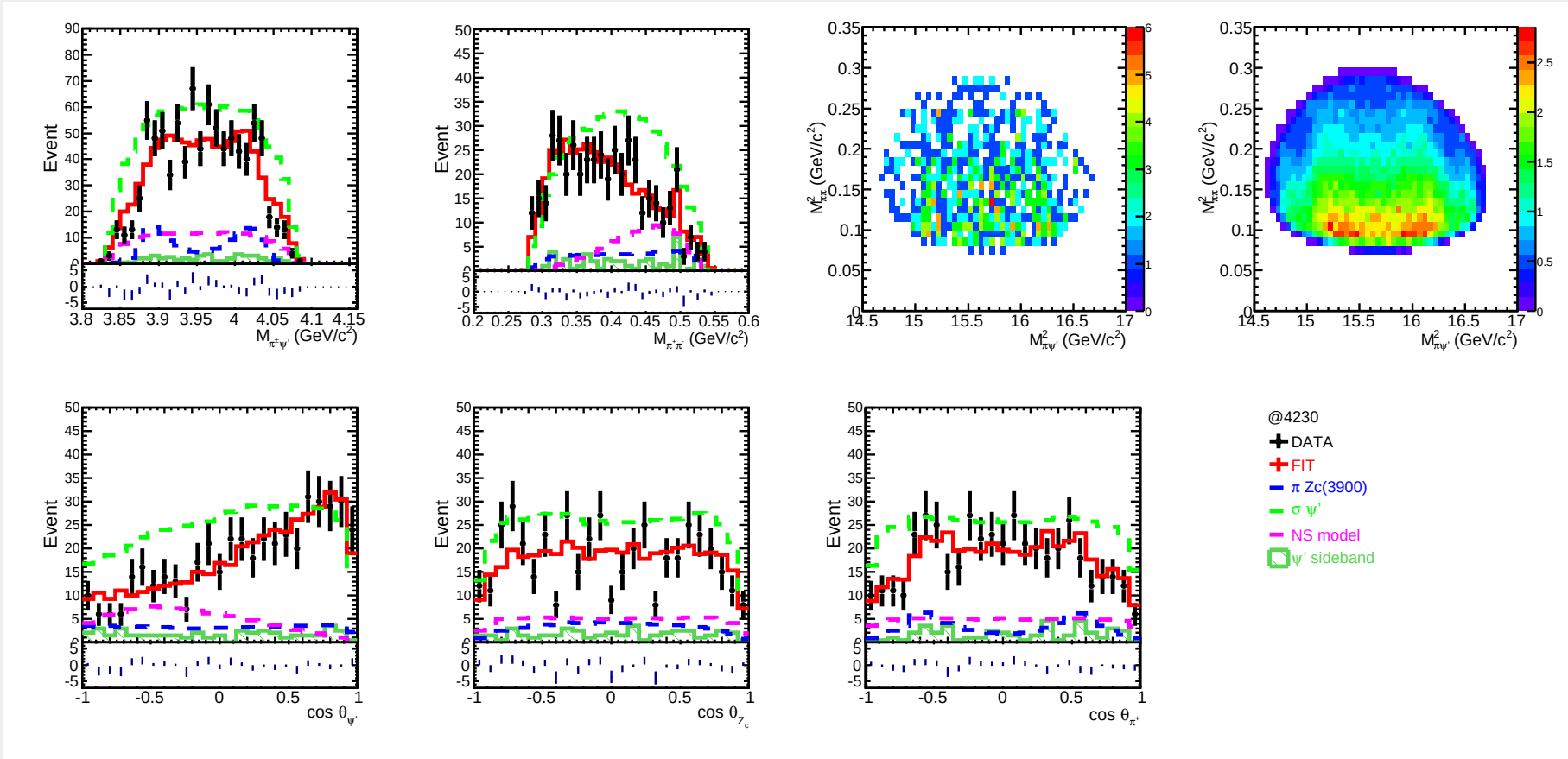
PWA for efficiencies

- 4180-4220 energy points
  - $Z_c(3900)$ contribution (PhysRevLett.119.072001)
  - Non-resonance contribution(PhysRevD.91.072003)



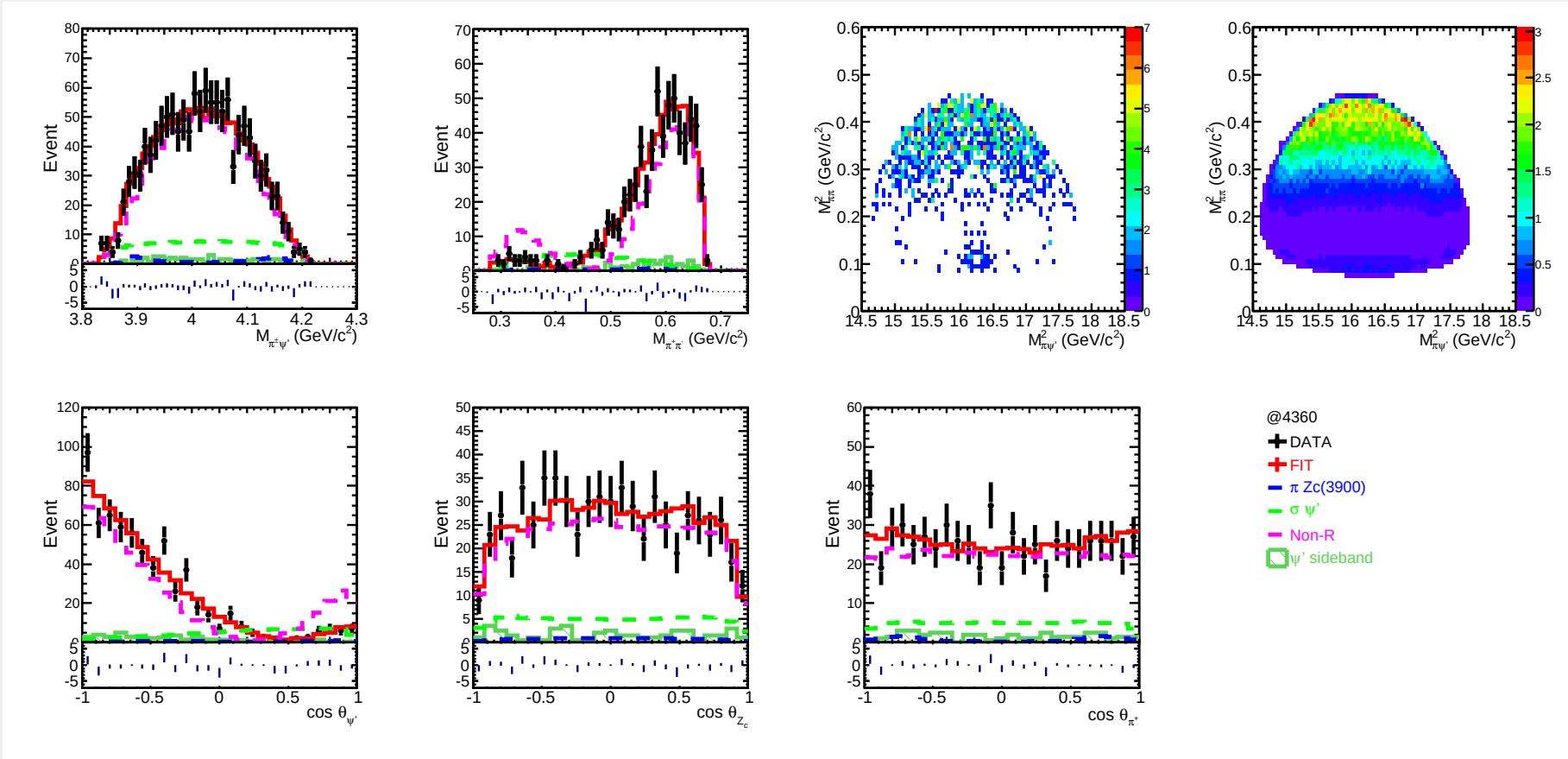
PWA for efficiencies

- 4230-4280 energy points
  - $Z_c(3900)$ contribution
  - $\sigma(600)$ contribution
  - NS model contribution



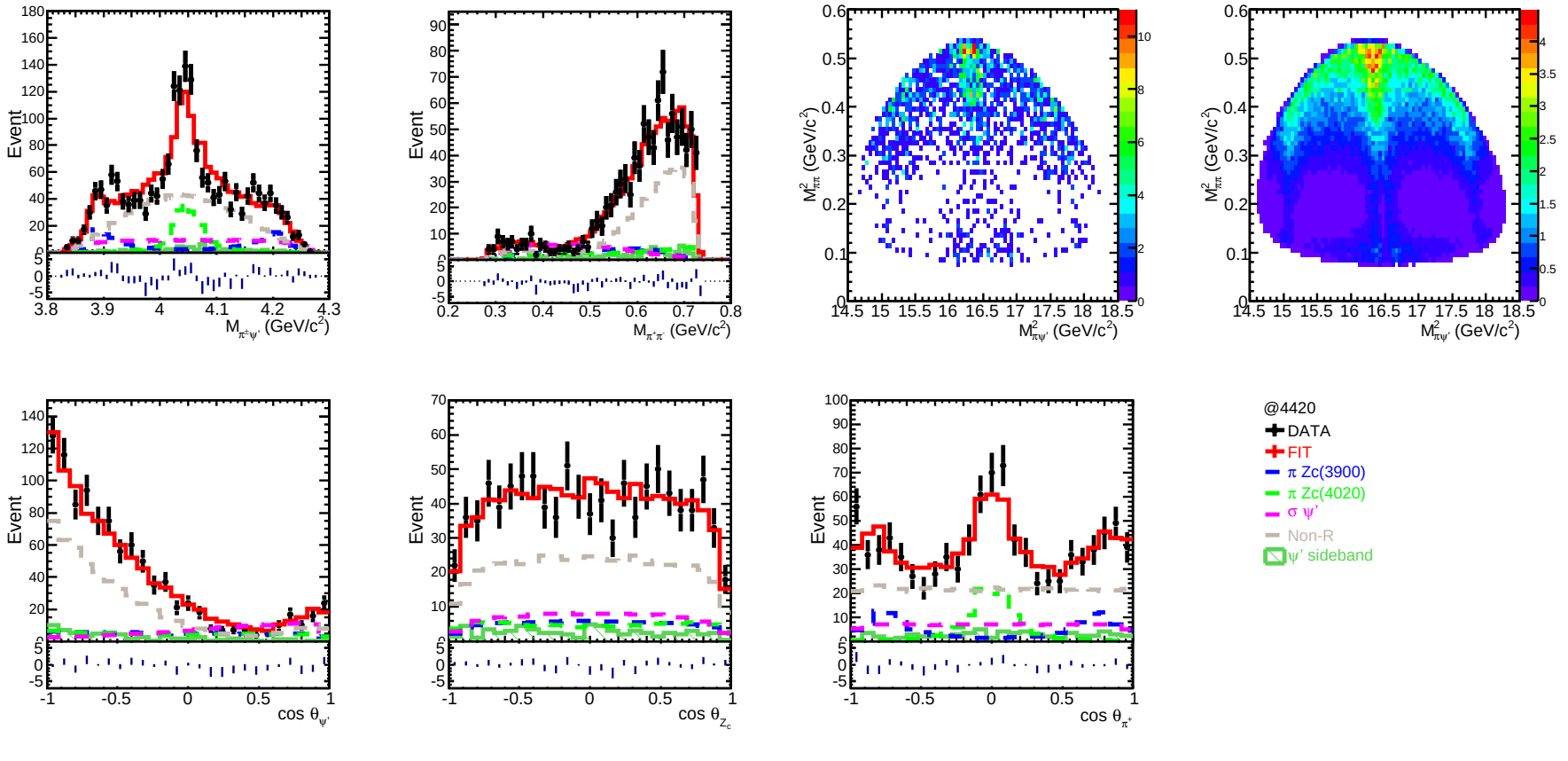
PWA for efficiencies

- 4360 energy point
  - $Z_c(3900)$ contribution
  - $\sigma(600)$ contribution
  - Non-resonance contribution



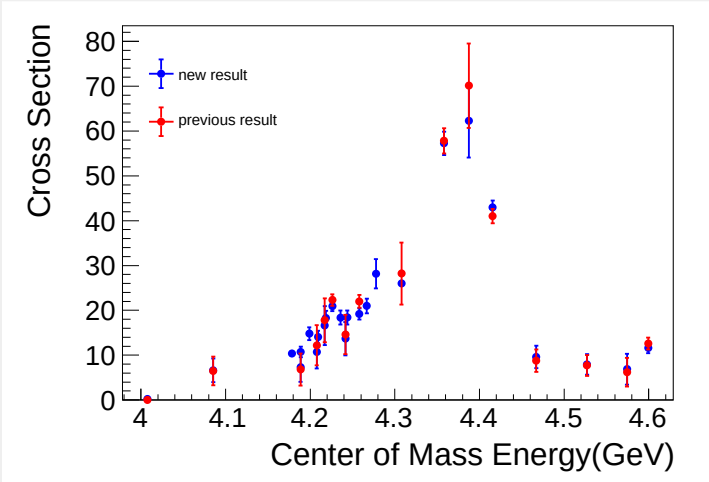
PWA for efficiencies

- 4420 energy point
  - $Z_c(3900)$ contribution
  - $Z_c(4020)$ contribution (described by Breit-Wigner function)
  - $\sigma(600)$ contribution
  - Non-resonance contribution



Cross section measurement

energy point	E_{cm}(GeV)	N_{obs}	ϵ	$(1+\delta)$	$\frac{1}{\lvert 1-\Pi\rvert^2}$	σ^B(pb)
4.009	4007.62	1.0 ± 0.0	30.72%	0.699	1.044	0.2 ± 0.0
4.090	4085.45	5.0 ± 2.0	43.56%	0.753	1.051	6.6 ± 2.6
4.180	4178.37	501.9 ± 23.3	46.26%	0.758	1.055	10.4 ± 0.5
4.190	4188.59	4.9 ± 2.2	46.84%	0.758	1.056	7.3 ± 3.3
4.190	4188.8	85.3 ± 9.5	46.07%	0.758	1.056	10.7 ± 1.2
4.200	4198.9	120.2 ± 11.3	46.80%	0.758	1.056	14.8 ± 1.4
4.210	4207.73	9.2 ± 3.2	47.31%	0.758	1.056	10.7 ± 3.7
4.210	4209.2	110.4 ± 10.9	45.82%	0.758	1.056	14.0 ± 1.4
4.220	4217.13	14.0 ± 3.6	46.77%	0.758	1.055	16.6 ± 4.3
4.220	4218.7	140.7 ± 12.4	45.27%	0.757	1.055	18.3 ± 1.6
4.230	4226.26	356.3 ± 19.6	46.74%	0.757	1.055	21.0 ± 1.2
4.237	4235.7	151.7 ± 12.9	47.12%	0.756	1.055	18.4 ± 1.6
4.245	4241.66	12.0 ± 3.3	47.63%	0.755	1.055	13.7 ± 3.7
4.246	4243.8	153.4 ± 13.0	47.00%	0.755	1.056	18.4 ± 1.6
4.260	4257.97	252.5 ± 16.3	48.40%	0.753	1.052	19.2 ± 1.2
4.270	4266.8	173.6 ± 13.6	47.84%	0.751	1.051	21.0 ± 1.6
4.280	4277.7	76.4 ± 8.9	47.35%	0.750	1.052	28.2 ± 3.3
4.310	4307.89	18.2 ± 0.0	47.90%	0.747	1.051	26.0 ± 0.0
4.360	4358.26	515.5 ± 23.4	48.17%	0.791	1.050	57.3 ± 2.6
4.390	4387.40	59.5 ± 7.9	45.46%	0.869	1.051	62.3 ± 8.2
4.420	4415.58	817.9 ± 29.8	42.03%	0.954	1.053	42.9 ± 1.6
4.470	4467.06	19.1 ± 4.9	37.18%	1.101	1.054	9.6 ± 2.5
4.530	4527.14	15.9 ± 4.6	32.94%	1.244	1.054	7.9 ± 2.3
4.575	4574.50	5.9 ± 3.0	32.62%	1.236	1.054	6.9 ± 3.4
4.600	4599.53	115.9 ± 11.6	36.22%	1.078	1.054	11.6 ± 1.2



- Cross section measurement results for charged mode
- The results are consistent with previous work

Event Selection for neutral mode

Neutral mode: $\psi(3686) \to neutrals\ J/\psi, J/\psi \to l^+l^-$
($\psi(3686)$ signal is reconstructed by recoiling $\pi^+\pi^-$)

Decay Channels	Branch Fraction(%)
$\psi(3686) \to \pi^0\pi^0 J/\psi, J/\psi \to l^+l^-$	2.124
$\psi(3686) \to \eta J/\psi, J/\psi \to l^+l^-$	0.290
$\psi(3686) \to \pi^0 J/\psi, J/\psi \to l^+l^-$	0.015
$\psi(3686) \to \gamma\chi_{c0}, \chi_{c0} \to \gamma J/\psi, J/\psi \to l^+l^-$	0.0164
$\psi(3686) \to \gamma\chi_{c1}, \chi_{c1} \to \gamma J/\psi, J/\psi \to l^+l^-$	0.399
$\psi(3686) \to \gamma\chi_{c2}, \chi_{c2} \to \gamma J/\psi, J/\psi \to l^+l^-$	0.216

- The main decay processes considered for signal MC

Event Selection for neutral mode

- Preliminary Selection:

Good Charged tracks

- Vertex cuts: $|V_{xy}| < 1.0cm$, $|V_z| < 10cm$
- Polar angle in MDC: $|cos\theta| \leq 0.93$
- 4 good charged tracks with 0 net charge

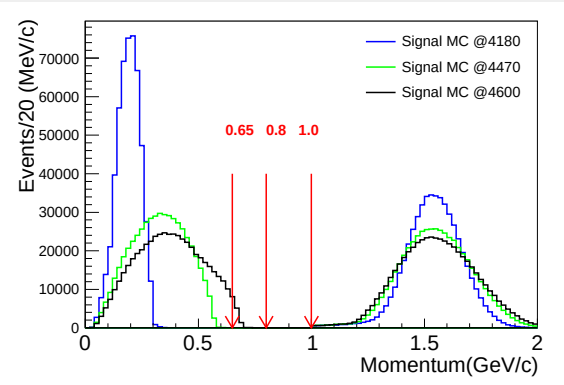
Good photons

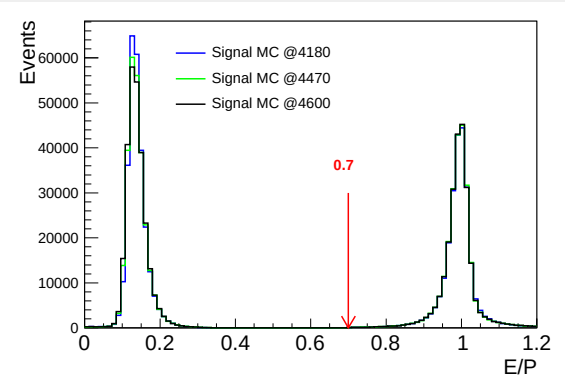
- Threshold energy: $E_{barrel} > 25\ MeV, E_{endcap} > 50\ MeV$
- Angle between γ and charged tracks: $\Delta_{Ang} > 10^{\circ}$
- EMC times: $0 < TDC < 14(50\ ns)$
- at least 2 good photon candidates

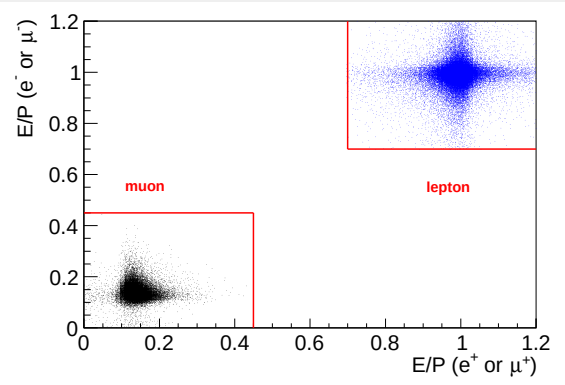
Event Selection for neutral mode

PID

- One pair of pion:
  - $p < 0.65\ GeV$ for data sets with $\sqrt{s} < 4.465\ GeV$
  - $p < 0.8\ GeV$ for data sets with $\sqrt{s} > 4.465\ GeV$
- One pair of lepton: $p > 1.0\ GeV$
  - Momentum of lepton: $p > 1.0 GeV$
  - Electron: $E/p > 0.7$ for both tracks
  - Muon: $E < 0.45 GeV$ for both tracks (more than 5 MUC layers for 2 muons)







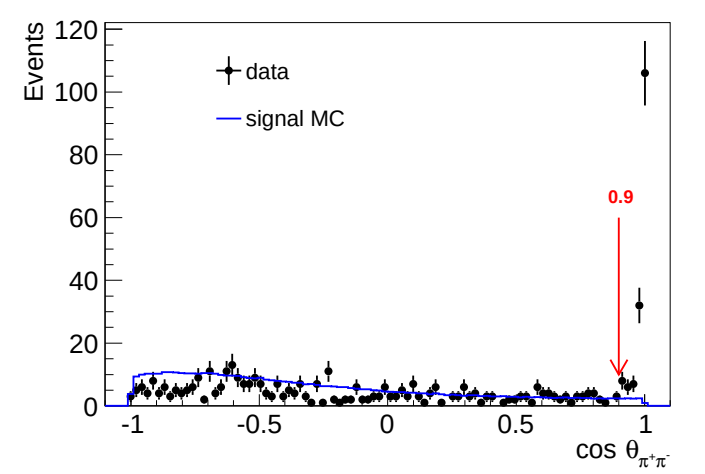
Background analysis

Gamma conversion background

- The backgrounds come from $e^+e^- \to \gamma e^+e^- or\, \gamma\mu^+\mu^-$ with $\gamma \to e^+e^-$ misidentified as $\pi^+\pi^-$

$e^+e^- \to \pi^+\pi^- J/\psi$ with fake photons or radiative photons

- There should be at least 2 good photons with deposited energy larger than 80 MeV

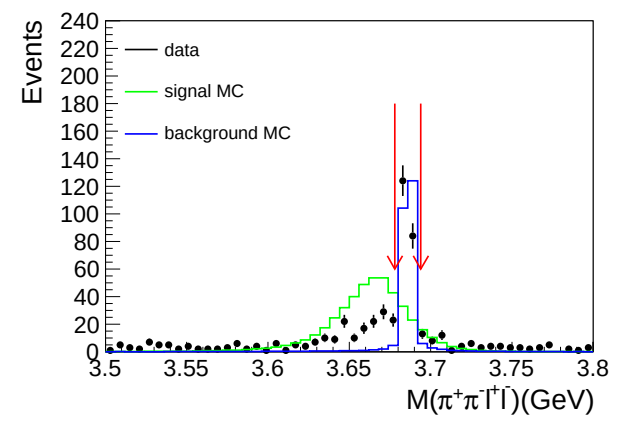


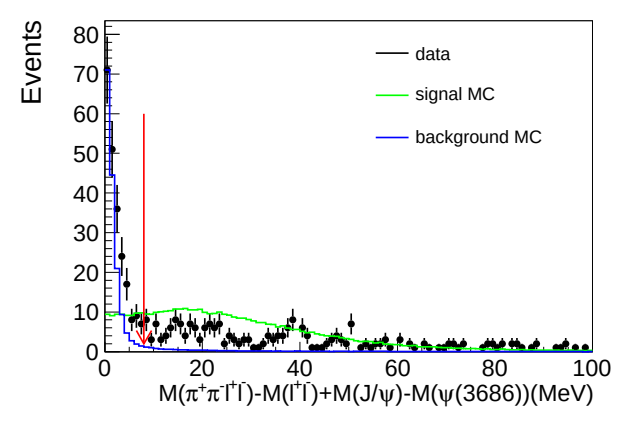
Background analysis

Main background

- The main background events are from $Y(4260) \to neutrals\ \psi(3686)$ with $\psi(3686) \to \pi^+\pi^- J/\psi$
- $|M_{\pi+\pi-l+l-} - M_{l+l-} + M_{J/\psi} - M_{\psi(3686)}| > 8$ MeV

Decay Channels
$Y(4260) \to \pi^0\pi^0\psi(3686), \psi(3686) \to \pi^+\pi^- J/\psi$
$Y(4260) \to \pi^0\psi(3686), \psi(3686) \to \pi^+\pi^- J/\psi$
$Y(4260) \to \eta\psi(3686), \psi(3686) \to \pi^+\pi^- J/\psi$
$Y(4260) \to \gamma_{ISR}\psi(3686), \psi(3686) \to \pi^+\pi^- J/\psi$
$Y(4260) \to unknown\ \psi(3686), \psi(3686) \to \pi^+\pi^- J/\psi$

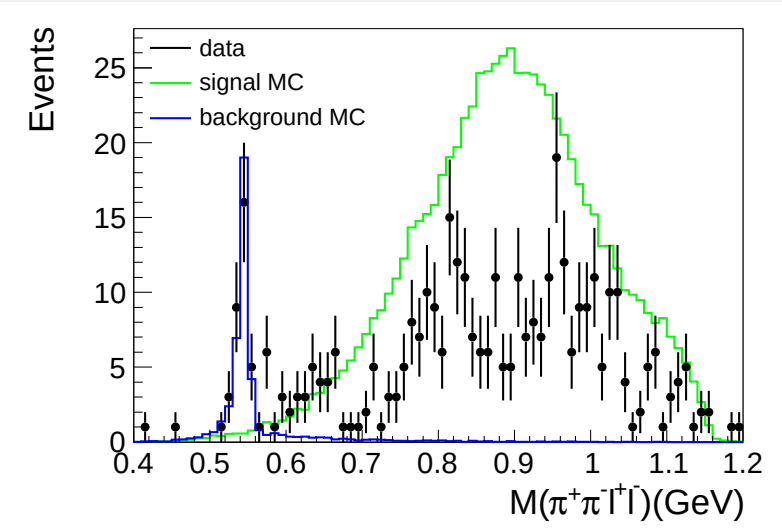


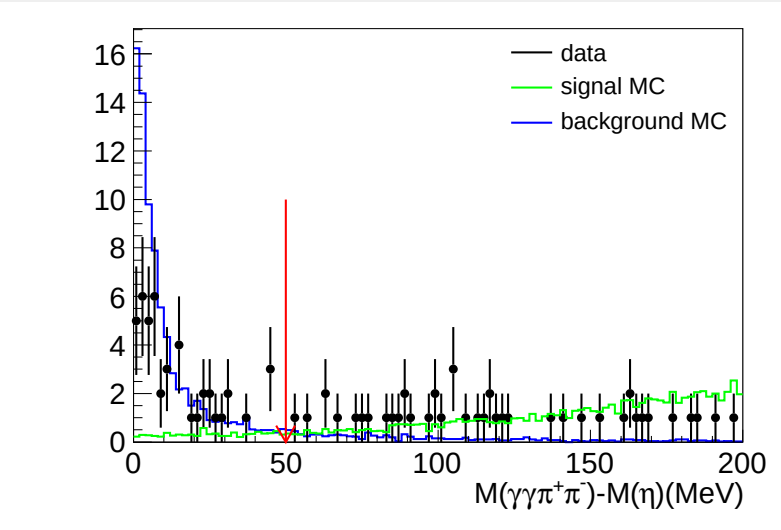


Background analysis

Backgrounds with $\eta \to \pi^+\pi^-\pi^0$

- Possible decay sequences:
  - $Y(4260) \to \eta J/\psi, \eta \to \pi^+\pi^-\pi^0$
  - $Y(4260) \to \gamma_{ISR}\psi 3686, \psi(3686) \to \eta J/\psi, \eta \to \pi^+\pi^-\pi^0$
- $|M_{\gamma\gamma\pi^+\pi^-} - M_\eta| > 50$ MeV

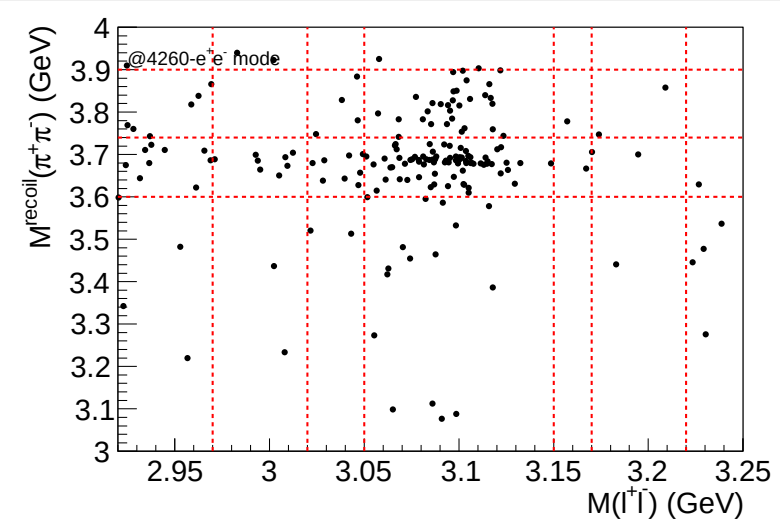


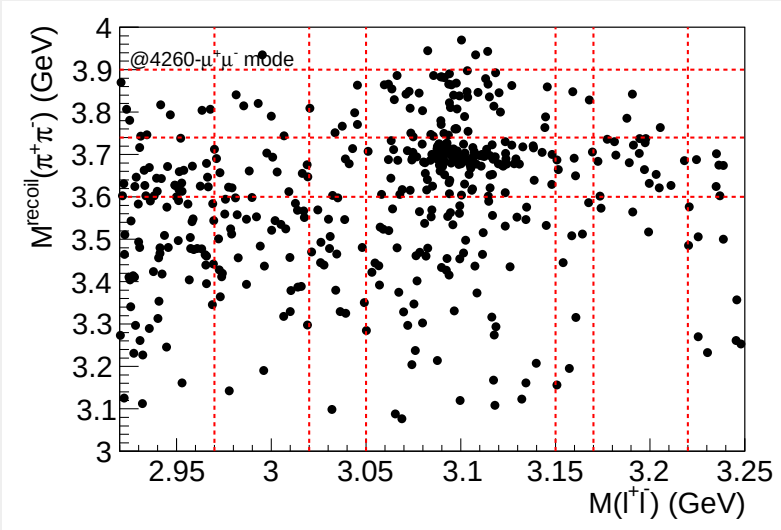


Signal candidates

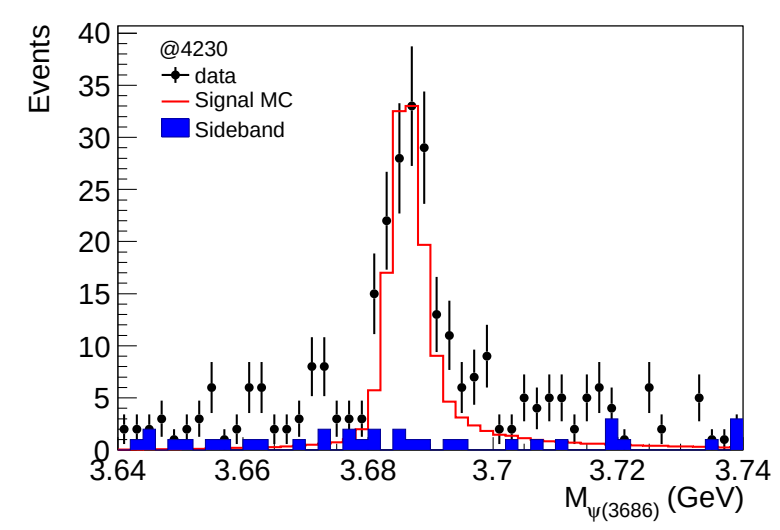
J/ψ mass window

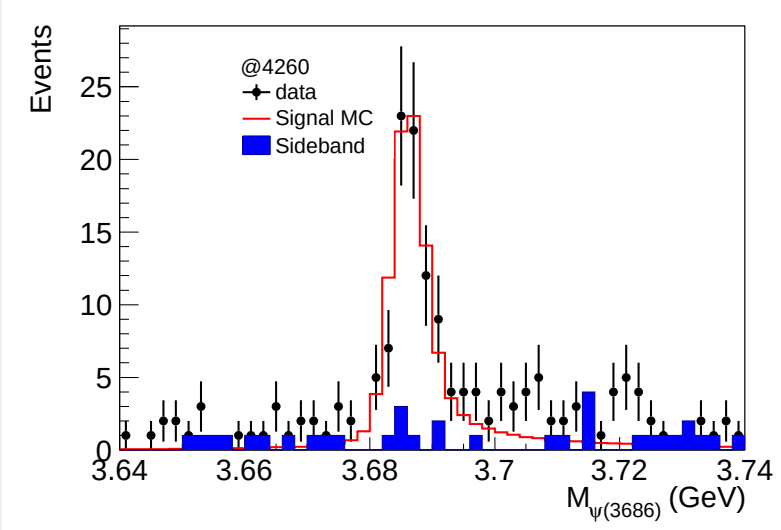
- Signal: [3.05, 3.15] GeV
- Sideband: [2.97, 3.02] GeV, [3.17, 3.22] GeV

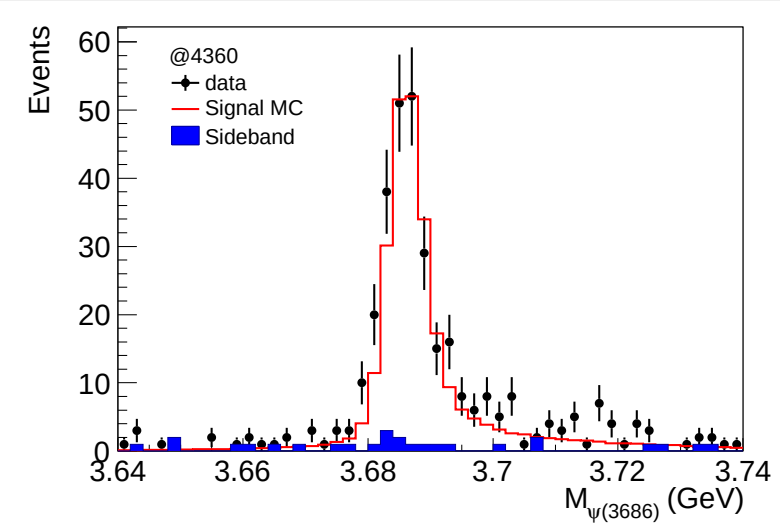


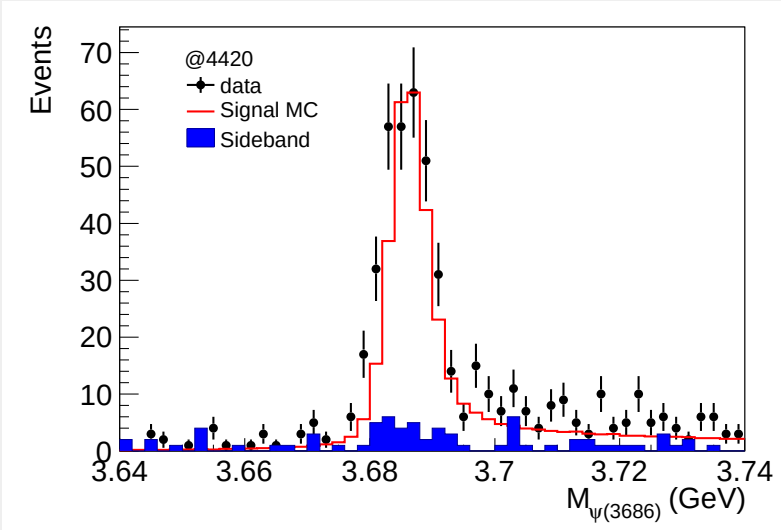


Signal candidates





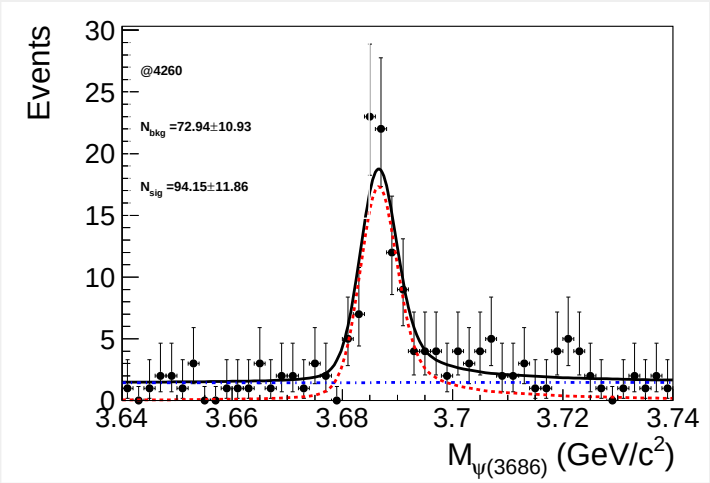


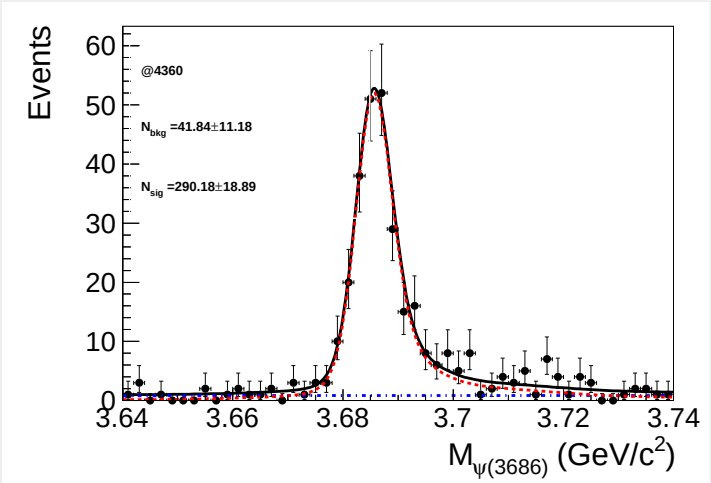


- The distributions of $\psi(3686)$ invariant mass at different energy points

Cross section measurement

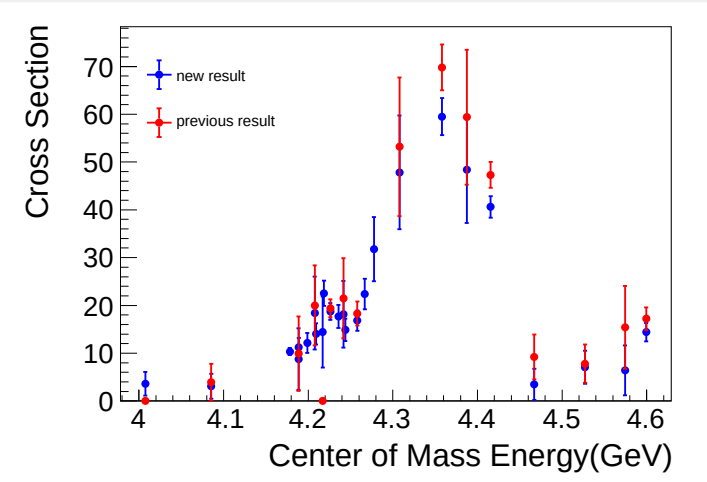
- Efficiencies are obtained in the same way as charged mode
- Signals are extracted by the same fitting method as charged mode





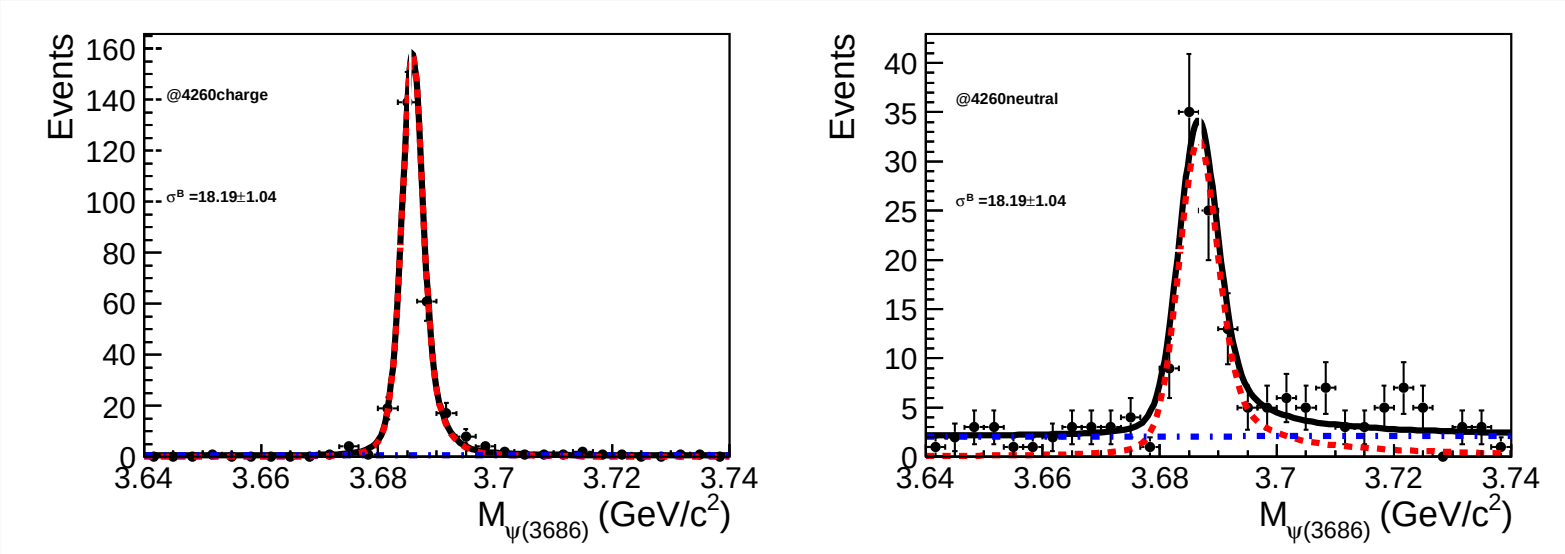
Cross section measurement

energy point	$E_{cm}(GeV)$	N_{obs}	ϵ	$(1+\delta)$	$\frac{1}{\lvert 1-\Pi\rvert^2}$	σ^B(pb)
4.009	4007.62	2.9 ± 2.0	7.58%	0.699	1.044	3.6 ± 2.8
4.090	4085.45	1.0 ± 0.9	25.91%	0.744	1.051	3.0 ± 3.0
4.180	4178.37	253.9 ± 18.7	32.55%	0.758	1.055	10.1 ± 0.7
4.190	4188.59	3.0 ± 2.2	32.60%	0.758	1.056	8.8 ± 6.4
4.190	4188.8	45.5 ± 7.8	31.97%	0.758	1.056	11.2 ± 1.9
4.200	4198.9	50.0 ± 8.6	30.60%	0.758	1.056	12.8 ± 2.2
4.210	4207.73	8.1 ± 3.4	30.65%	0.759	1.056	19.8 ± 8.2
4.210	4209.2	56.6 ± 8.9	29.78%	0.758	1.056	15.0 ± 2.4
4.220	4217.13	6.4 ± 3.3	30.97%	0.757	1.055	11.5 ± 6.7
4.220	4218.7	90.7 ± 10.7	30.12%	0.758	1.055	23.4 ± 2.8
4.230	4226.26	165.1 ± 15.6	31.07%	0.756	1.055	19.0 ± 1.8
4.237	4235.7	73.8 ± 10.0	32.33%	0.755	1.055	17.2 ± 2.4
4.245	4241.66	8.0 ± 3.1	32.29%	0.755	1.055	17.5 ± 6.9
4.246	4243.8	61.2 ± 9.6	31.63%	0.755	1.056	14.1 ± 2.3
4.260	4257.97	94.2 ± 11.9	30.81%	0.753	1.052	14.9 ± 1.9
4.270	4266.8	70.2 ± 10.1	25.66%	0.751	1.051	20.9 ± 3.0
4.280	4277.7	30.5 ± 6.5	23.09%	0.750	1.052	30.3 ± 6.6
4.310	4307.89	16.0 ± 4.0	30.97%	0.747	1.051	47.7 ± 11.9
4.360	4358.26	290.2 ± 18.9	37.93%	0.790	1.050	55.3 ± 3.6
4.390	4387.40	26.1 ± 6.0	34.69%	0.868	1.051	48.3 ± 11.2
4.420	4415.58	443.1 ± 24.8	34.32%	0.955	1.053	38.4 ± 2.2
4.470	4467.06	3.9 ± 3.6	28.16%	1.100	1.054	3.4 ± 3.2
4.530	4527.14	7.8 ± 3.8	24.55%	1.244	1.054	7.1 ± 3.5
4.575	4574.50	3.0 ± 2.4	23.98%	1.237	1.054	6.4 ± 5.2
4.600	4599.53	77.7 ± 10.2	26.47%	1.078	1.054	14.4 ± 1.9



- Cross section measurement results for neutral mode

Combination of two decay modes



- Combination of two decay modes:
  - simultaneous fit to two channels is applied to obtain the cross sections
  - signal shape is described by MC shape convolved with a Gaussian function
  - background shape is described by first order polynomial
  - cross section is shared for two modes during the fitting process

Combination of two decay modes

energy point	$E_{cm}(GeV)$	$L(pb^{-1})$	charged mode			neutral mode			σ^B(pb)
			ϵ	$(1+\delta)$	$\frac{1}{\lvert 1-\Pi\rvert^2}$	ϵ	$(1+\delta)$	$\frac{1}{\lvert 1-\Pi\rvert^2}$	
4.009	4007.62	482.0	30.72%	0.699	1.044	7.58%	0.699	1.044	0.4 ± 0.0
4.090	4085.45	52.86	43.56%	0.753	1.051	25.91%	0.744	1.051	5.6 ± 2.0
4.180	4178.37	3161	46.26%	0.758	1.055	32.55%	0.758	1.055	10.3 ± 0.4
4.190	4188.59	43.33	46.84%	0.758	1.056	32.60%	0.758	1.056	7.6 ± 3.0
4.190	4188.8	522.5	46.07%	0.758	1.056	31.97%	0.758	1.056	10.8 ± 1.0
4.200	4198.9	524.6	46.80%	0.758	1.056	30.60%	0.758	1.056	14.2 ± 1.2
4.210	4207.73	54.95	47.31%	0.758	1.056	30.65%	0.759	1.056	13.2 ± 3.6
4.210	4209.2	518.1	45.82%	0.758	1.056	29.78%	0.758	1.056	14.3 ± 1.2
4.220	4217.13	54.60	46.77%	0.758	1.055	30.97%	0.757	1.055	16.3 ± 3.8
4.220	4218.7	514.3	45.27%	0.757	1.055	30.12%	0.758	1.055	19.9 ± 1.4
4.230	4226.26	1100.9	46.74%	0.757	1.055	31.07%	0.756	1.055	20.6 ± 1.0
4.237	4235.7	530.6	47.12%	0.756	1.055	32.33%	0.755	1.055	18.2 ± 1.3
4.245	4241.66	55.88	47.63%	0.755	1.055	32.29%	0.755	1.055	15.0 ± 3.5
4.246	4243.8	537.4	47.00%	0.755	1.056	31.63%	0.755	1.056	17.4 ± 1.3
4.260	4257.97	828.4	48.40%	0.753	1.052	30.81%	0.753	1.052	18.2 ± 1.0
4.270	4266.8	529.7	47.84%	0.751	1.051	25.66%	0.751	1.051	21.1 ± 1.5
4.280	4277.7	175.5	47.35%	0.750	1.052	23.09%	0.750	1.052	28.8 ± 2.9
4.310	4307.89	45.08	47.90%	0.747	1.051	30.97%	0.747	1.051	31.6 ± 5.8
4.360	4358.26	543.9	48.17%	0.791	1.050	37.93%	0.790	1.050	56.6 ± 2.1
4.390	4387.40	55.57	45.46%	0.869	1.051	34.69%	0.868	1.051	58.0 ± 6.6
4.420	4415.58	1090.7	42.03%	0.954	1.053	34.32%	0.955	1.053	41.5 ± 1.3
4.470	4467.06	111.09	37.18%	1.101	1.054	28.16%	1.100	1.054	8.3 ± 2.0
4.530	4527.14	112.12	32.94%	1.244	1.054	24.55%	1.244	1.054	7.7 ± 1.9
4.575	4574.50	48.93	32.62%	1.236	1.054	23.98%	1.237	1.054	6.8 ± 2.9
4.600	4599.53	586.9	36.22%	1.078	1.054	26.47%	1.078	1.054	12.4 ± 1.0

- Cross section measurement results of process $e^+e^- \to \pi^+\pi^-\psi(3686)$

Systematic uncertainties

- **Luminosity** Common value as 1.0% using BhaBha events
- **The Branch fraction** Taken from PDG, the total uncertainty is 1.7%
- **Radiative correction** Replacing in-put lineshape of this work to generate MC, iterating this process until the result is stable. The cross section difference of last two times is taken as systematic uncertainty.
- **The vacuum polarization factor** 0.5% taken from QED calculation
- **Tracking efficiency of leptons** 1.0% per track and 2.0% is assigned in total
- **Photon efficiency** 1.0% per photon and change the neutral mode efficiencies by 2%. The cross section difference between original results and results after changing efficiencies is taken as systematic uncertainty
- **Tracking efficiency of pions** 1.0% per track, change the charged mode efficiencies by 3% and change the neutral mode efficiencies by 2%. The cross section difference before and after changing efficiencies is taken as systematic uncertainties

Systematic uncertainties

- **Hits number in MUC** The cross section difference between results with and without MUC hit number requirement is considered as systematic uncertainty
- **The J/ψ mass window cut** Fit $M_{J/\psi}$ spectrum of data with MC shape convolved with a Gaussian function and first polynomial. Smear MC J/ψ mass with the Gaussian and get new efficiency. The cross section difference before and after smearing is taken as systematic uncertainty
- **The $\pi^+\pi^-$ open angle θ requirement** Varying $cos\theta_{\pi^+\pi^-}$ requirement from 0.9 to 0.8, the cross section difference is taken as systematic uncertainty
- **The $M_{\pi^+\pi^- l^+ l^-}$ mass requirement** Varying the requirement $M_{\pi^+\pi^- l^+ l^-} - M_{l^+ l^-} + M_{J/\psi} - M_{\psi(3686)}$ from 8 MeV to 10 MeV, the cross section difference is taken as systematic uncertainty
- **The $M_{\gamma\gamma\pi^+\pi^-}$ mass requirement** Varying the requirement $M_{\gamma\gamma\pi^+\pi^-}$ from 50 MeV to 60 MeV, the difference of cross section is taken as systematic uncertainty

Systematic uncertainties

- **Kinematic fit** The PULL distribution of charged tracks parameters are corrected to make MC samples agree well with data. The cross section difference before and after correction is taken as systematic uncertainty
- **Fitting range** Varying the fitting range of $M_{\psi(3686)}$ from [3.64, 3.74] GeV to [3.62, 3.76] GeV and the cross section difference is taken as systematic uncertainty
- **Background shape** In the simultaneous fitting, replacing the first order polynomial to second order polynomial, and the cross section difference is taken as systematic uncertainty
- **Signal shape** The signal shape is described by MC convolved with Gaussian function. Changing the width of Gaussian function, the cross section difference is taken as systematic uncertainty

Systematic uncertainties

- **MC models** For the energy points using PHSP MC to obtain efficiencies, their nearest large statistics data points are used to estimate the systematic errors. 4009 and 4090 use the point 4180, 4310 $\sim$ 4600 use point 4420. The cross section differences between results calculated with PHSP and PWA MC are taken as systematic uncertainty of these energy points. For the energy points using PWA MC to obtain efficiencies, the parameters of PWA results are assumed to obey multi-variable Gaussian function. The parameters are changed within errors randomly for 100 times to get efficiencies by generating new MC. Use a Gaussian function to fit the efficiency distribution and the fitting width is seen as error. Varying PWA efficiencies with this error, the cross section difference before and after this process is taken as systematic uncertainty
- **Others** 1.0% is taken

Systematic uncertainties

Table: Summary of the systematic uncertainties(%) at different energy points. The sources with star markers are the common relative systematic errors for different energy points.

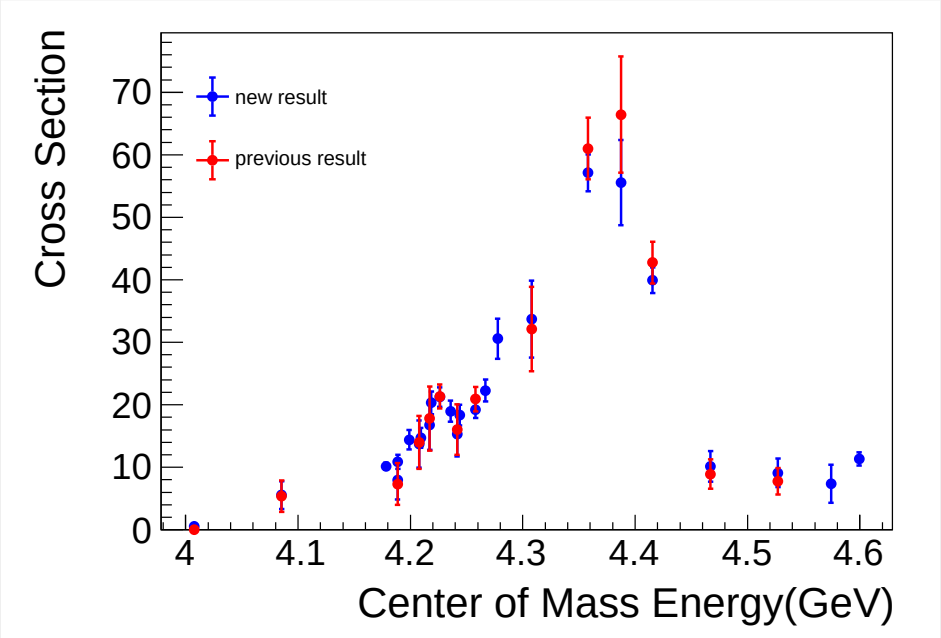
Data set	4009	4090	4180	4190	4190	4200	4210	4210	4220	4220	4230	4237
$E_{cm}(GeV)$	4.00762	4.08545	4.17837	4.18859	4.1888	4.1989	4.20773	4.2092	4.21713	4.2187	4.22626	4.2357
Luminosity*	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Branch fraction*	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7
$(1+\delta^v)$ factor*	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
Radiative correction	0.7	0.7	0.7	0.4	0.4	0.5	0.8	0.8	0.0	0.0	0.3	0.3
Tracking(l^+l^-)	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
Tracking($\pi^+\pi^-$)	2.6	2.6	2.6	2.6	2.6	2.6	2.7	2.7	2.7	2.7	2.7	2.6
Photon detection	0.6	0.6	0.6	0.6	0.6	0.5	0.6	0.6	0.5	0.5	0.5	0.6
MUC hit number	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.6	0.5
J/ψ mass window	0.2	0.2	0.2	0.3	0.3	0.7	0.2	0.2	0.1	0.1	0.2	0.1
$\pi^+\pi^-$ open angle cut	2.0	2.0	2.0	2.2	2.2	3.4	3.5	3.5	2.4	2.4	2.2	2.1
$M_{\pi^+\pi^-l^+l^-}$ cut	0.0	0.0	0.0	0.0	0.0	3.4	3.5	3.5	2.4	2.4	2.2	2.1
$M_{\gamma\gamma\pi^+\pi^-}$ cut	0.2	0.2	0.2	0.2	0.2	3.4	3.5	3.5	2.4	2.4	2.2	2.1
Kinematic fit	1.6	1.6	1.6	2.0	2.0	1.8	2.1	2.1	2.1	2.1	2.1	1.9
Background shape	0.1	0.1	0.1	0.3	0.3	0.3	0.1	0.1	0.0	0.0	0.0	0.0
Signal shape	0.2	0.2	0.2	0.2	0.2	0.1	0.0	0.0	0.6	0.6	0.6	0.4
Fitting range	1.1	1.1	1.1	0.6	0.6	0.4	0.4	0.4	0.3	0.3	2.1	2.0
MC models	2.6	2.6	0.7	0.9	0.9	0.9	0.7	1.0	1.0	1.0	0.8	0.8
Others*	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
total	4.3	4.3	4.0	4.2	4.2	6.9	7.0	7.0	5.6	5.5	5.7	5.4

Systematic uncertainties

Table: Summary of the systematic uncertainties(%) at different energy points. The sources with star markers are the common relative systematic errors for different energy points.

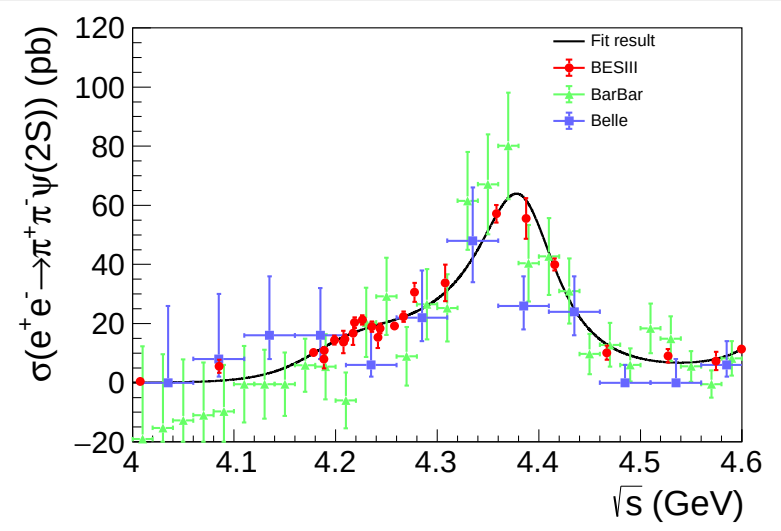
Data set	4245	4246	4260	4270	4280	4310	4360	4390	4420	4470	4530	4575	4600
$E_{cm}(GeV)$	4.24166	4.2438	4.25797	4.2668	4.2777	4.30789	4.35826	4.38740	4.41558	4.46706	4.52714	4.57450	4.59953
Luminosity*	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Branch fraction*	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7
$(1+\delta^v)$ factor*	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
Radiative correction	0.3	0.3	0.1	0.1	0.4	0.4	0.5	0.6	0.0	0.0	0.7	0.7	0.7
Tracking(l^+l^-)	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
Tracking($\pi^+\pi^-$)	2.7	2.7	2.7	2.7	2.7	2.7	2.6	2.6	2.6	2.6	2.6	2.6	2.6
Photon detection	0.5	0.5	0.5	0.4	0.4	0.4	0.6	0.6	0.7	0.7	0.6	0.6	0.6
MUC hit number	0.5	0.5	0.4	0.4	0.4	0.4	0.6	0.5	0.6	0.6	0.6	0.6	0.6
J/ψ mass window	0.2	0.2	0.0	0.3	0.1	0.1	0.2	1.5	0.4	0.4	1.9	1.9	1.9
$\pi^+\pi^-$ open angle cut	1.5	1.5	0.5	0.4	0.2	0.2	0.5	1.4	0.8	0.8	1.2	1.2	1.2
$M_{\pi^+\pi^-l^+l^-}$ cut	1.5	1.5	0.5	0.4	0.2	0.2	0.5	1.4	0.8	0.8	1.2	1.2	1.2
$M_{\gamma\gamma\pi^+\pi^-}$ cut	1.5	1.5	0.5	0.4	0.2	0.2	0.5	1.4	0.8	0.8	1.2	1.2	1.2
Kinematic fit	1.7	1.7	1.9	2.2	2.1	2.1	1.8	2.2	2.2	2.2	2.1	2.1	2.1
Background shape	0.5	0.5	0.4	0.1	0.0	0.0	0.3	0.6	0.9	0.9	0.2	0.2	0.2
Signal shape	0.8	0.8	0.3	0.4	0.3	0.3	0.2	0.3	0.2	0.2	1.2	1.2	1.2
Fitting range	2.6	2.6	1.1	1.2	0.3	0.3	0.7	0.3	0.9	0.9	0.6	0.6	0.6
MC models	0.8	0.8	0.6	0.7	0.9	2.2	0.8	2.2	0.7	2.2	2.2	2.2	2.2
Others*	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
total	5.1	5.1	3.7	3.9	3.6	3.6	3.6	4.6	4.1	4.1	4.7	4.7	4.7

Fit the Cross Section Lineshape



- A binned χ^2 fit is applied to the cross section
- Parameterization: $\sigma_{fit}(\sqrt{s}) = |BW_1(s)e^{i\phi_1} + BW_2(s) + BW_3(s)e^{i\phi_2}|^2$
  - $BW_1(s)$: the resonance $Y(4220)$(with float parameters)
  - $BW_2(s)$: the resonance $Y(4390)$(with float parameters)
  - $BW_3(s)$: the resonance $Y(4660)$(with Belle results)

Fit the Cross Section Lineshape

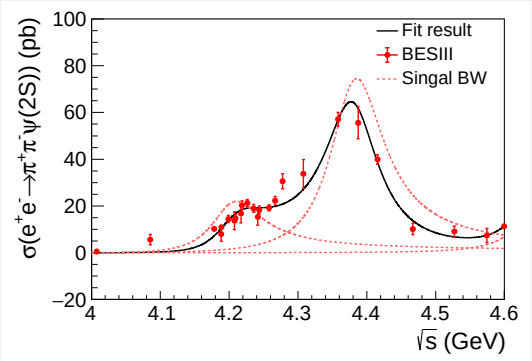


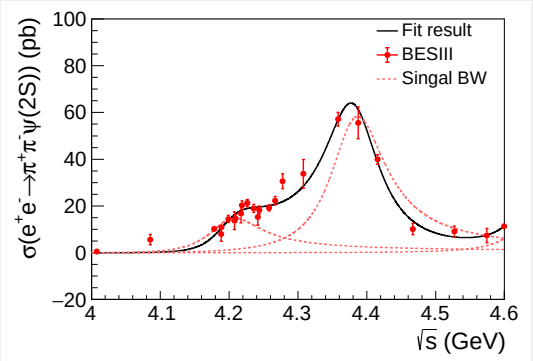
Parameters	Solution1	Solution2
M(Y4220) (MeV/c^2)	4188.0 ± 8.5	
$\Gamma_{tot}(Y4220)$ (MeV)	152.0 ± 43.0	
$B\Gamma_{ee}(Y4220)$ (eV)	2.66 ± 1.93	2.05 ± 1.53
M(Y4390) (MeV/c^2)	4384.6 ± 4.7	
$\Gamma_{tot}(Y4390)$ (MeV)	97.2 ± 13.9	
$B\Gamma_{ee}(Y4390)$ (eV)	12.24 ± 7.78	6.87 ± 5.22
$B\Gamma_{ee}(Y4660)$ (eV)	3.48 ± 2.45	2.18 ± 2.43
ϕ_1 (rad)	2.81 ± 0.09	3.46 ± 0.09
ϕ_2 (rad)	4.26 ± 0.11	1.63 ± 0.10

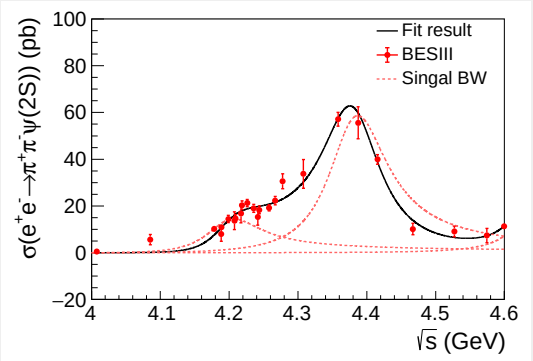
Summary

- Precise cross section measurement of $e^+e^- \to \pi^+\pi^-\psi(3686)$ is performed
- The cross section lineshape is fitted with coherent summation of three resonance structures
- The parameters of Y(4220) are $M = 4188.0 \pm 8.5$ MeV and $\Gamma = 152.0 \pm 43.0$ MeV
- The parameters of Y(4390) are $M = 4384.6 \pm 4.7$ MeV and $\Gamma = 97.2 \pm 13.9$ MeV
- Memo is ready

Backup







Parameters	Solution1	Solution2	Solution3
M(Y4220) (MeV/c^2)	4202.0 ± 12.0	4198.8 ± 4.96	4196.3 ± 5.5
$\Gamma_{tot}(Y4220)$ (MeV)	89.2 ± 18.4	92.7 ± 19.5	97.3 ± 26.6
M(Y4390) (MeV/c^2)	4379.9 ± 6.8	4381.0 ± 4.2	4380.1 ± 5.4
$\Gamma_{tot}(Y4390)$ (MeV)	95.3 ± 13.7	98.0 ± 13.6	105.0 ± 15.4

- Solution1: fit with 21 energy points(without 4180, 4270 and 4280)
- Solution2: fit with 22 energy points(without 4180 and 4280)
- Solution3: fit with 23 energy points(without 4180)