DSEs and QCD

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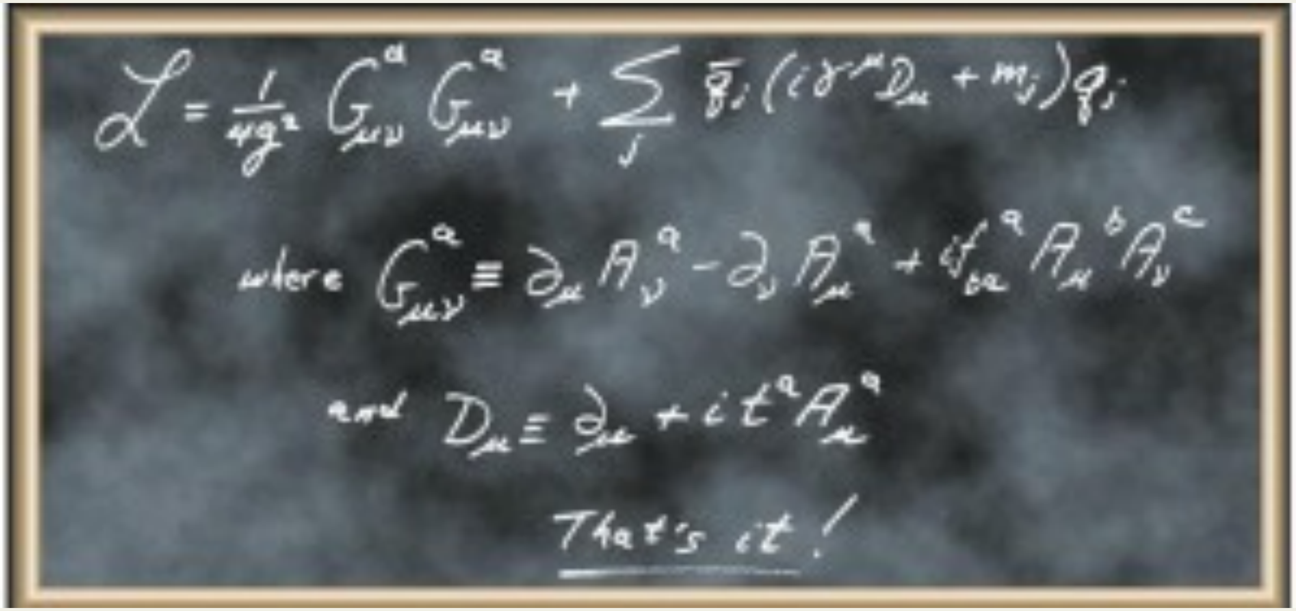
量子色动力学的未来：机遇和挑战，北京大学

什么是QCD？

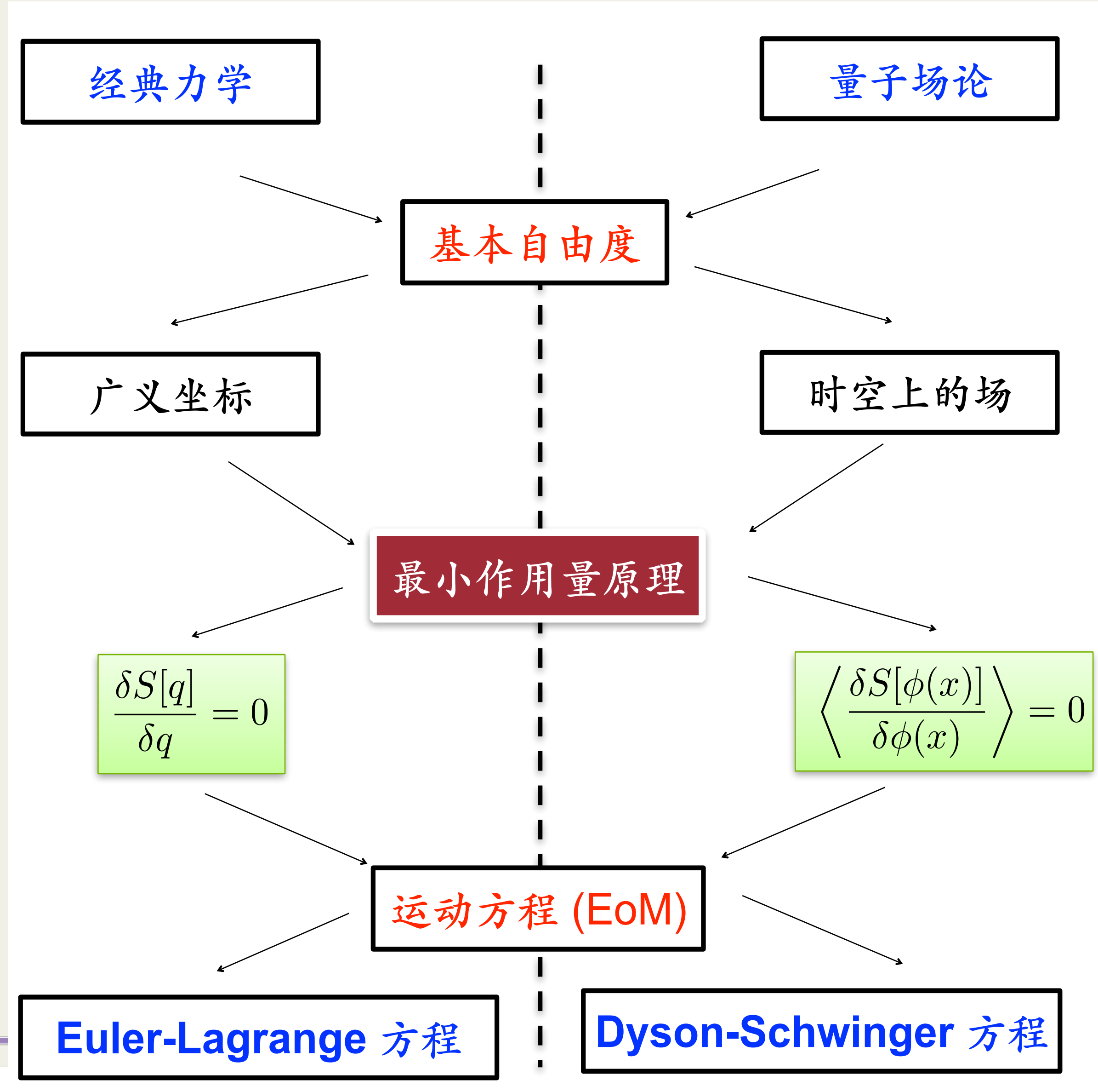


- 流夸克质量， Higgs.
- 强子物理能标– 1 GeV

– is an emergent feature of the Standard Model

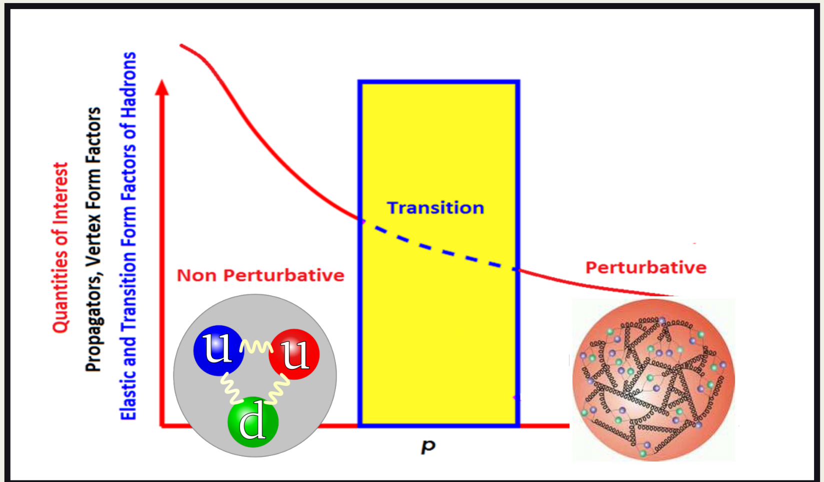
No amount of staring at L_{QCD} can reveal that scale

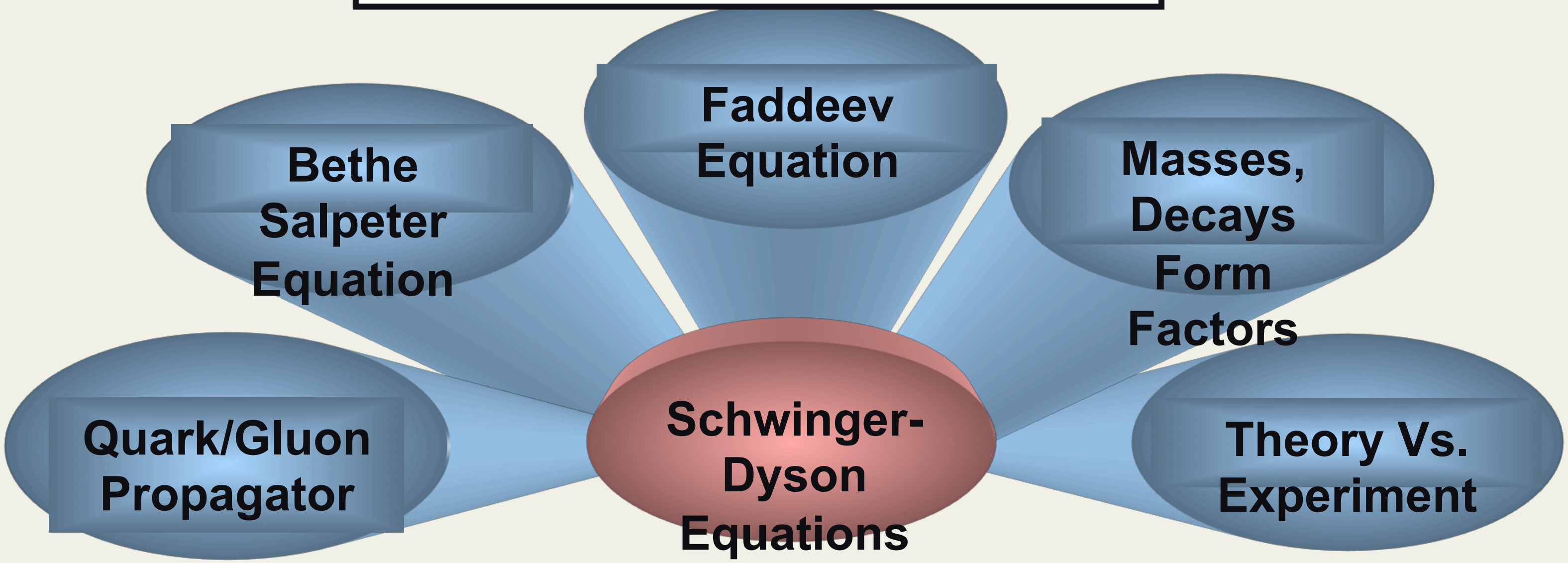
什么是Dyson-Schwinger方程？



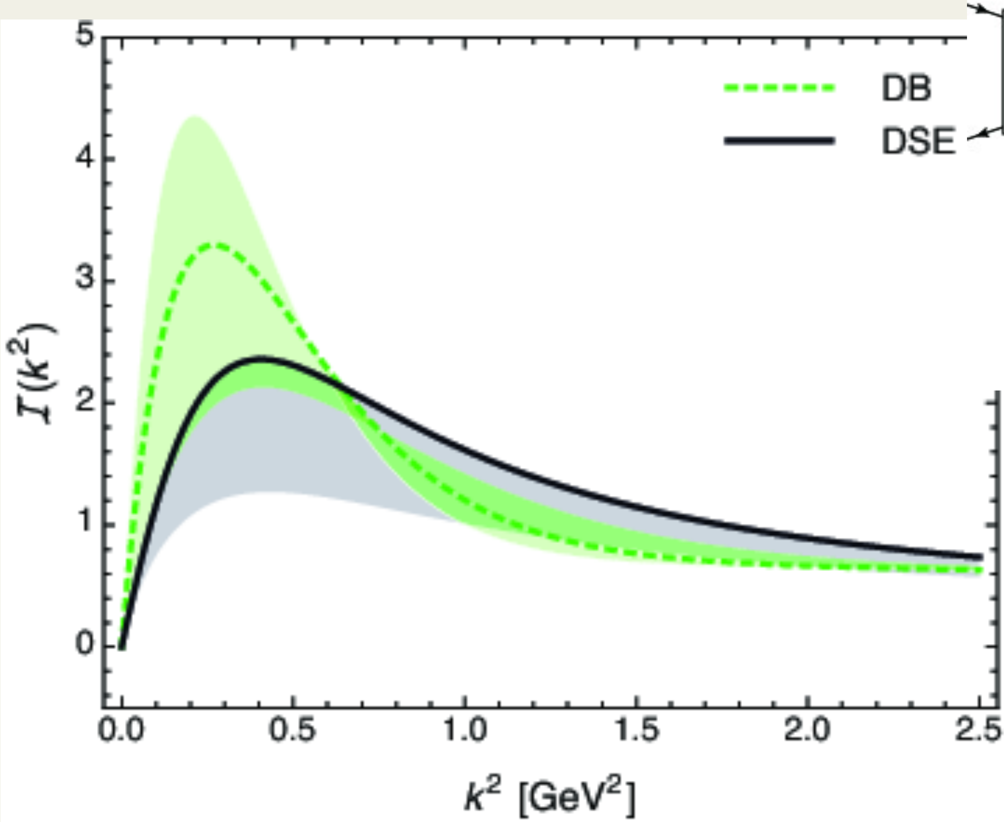
From Sixue Qin

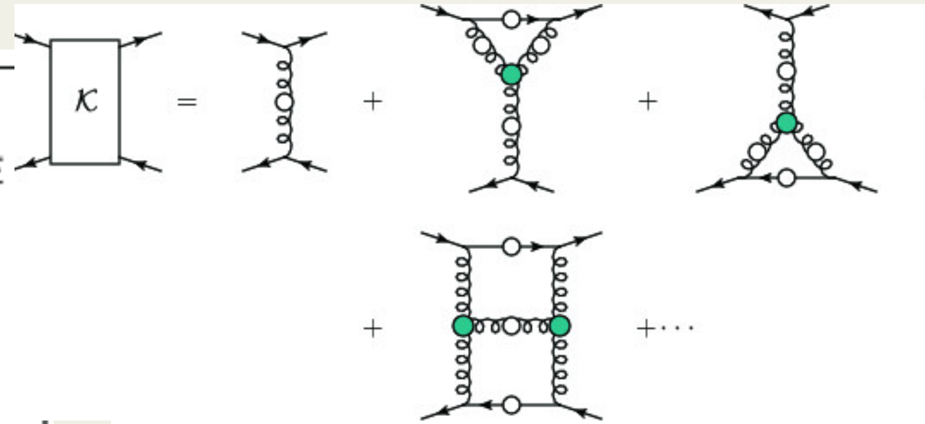
DS方程能做什么？

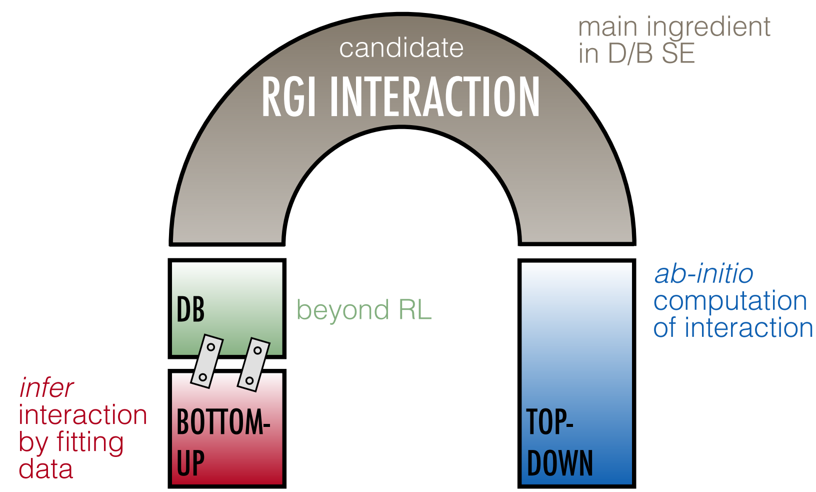

Quantities of Interest
Propagators, Vertex Form Factors
Elastic and Transition Form Factors of Hadrons
Non Perturbative
Transition
Perturbative
u
u
d
p


Faddeev Equation
Bethe Salpeter Equation
Masses, Decays Form Factors
Quark/Gluon Propagator
Schwinger-Dyson Equations
Theory Vs. Experiment

什么制约DS方程只能是一个非微扰唯象（目前）？







Lei Chang and C. D. Roberts, Phys. Rev. Lett.103 (2009) 081601;
Lei Chang, Yu-xin Liu and C. D. Roberts, Phys. Rev. Lett.106 (2011) 072001

– Interaction predicted by modern analyses of QCD's gauge sector coincides with that required to describe ground-state observables using the sophisticated DSE truncation

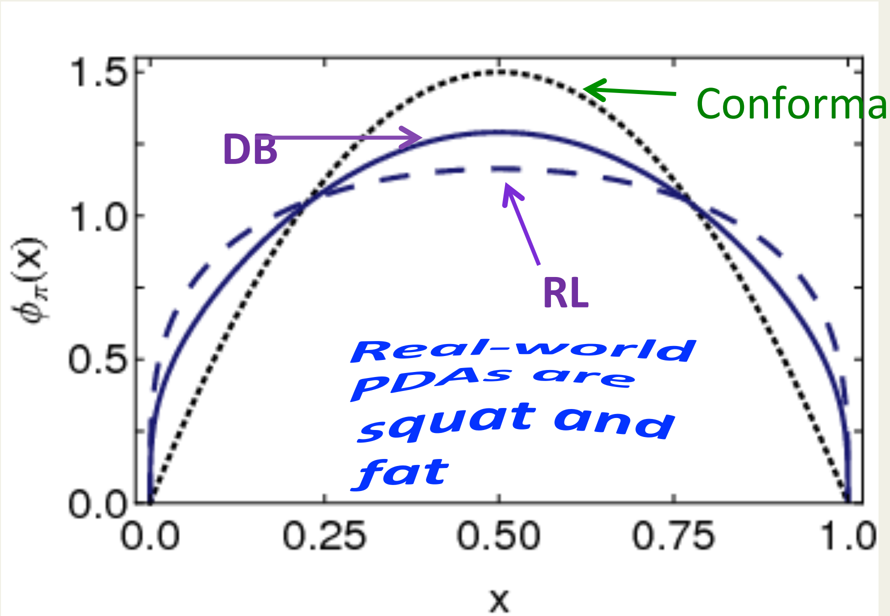
系统Power Counting的缺失，非微扰本质

一，关于QCD模型无关的陈述

二，利用能够追踪到QCD基本自由度本身、可调控的模型，阐述与模型无关的陈述

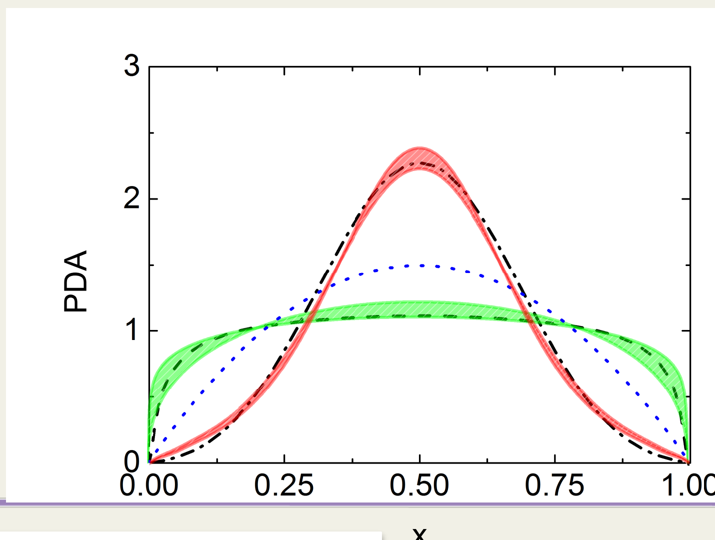
三，有一定QCD基础、但无法做到和QCD系统连接的阐述

C. D. Roberts



- Continuum-QCD prediction:
  marked broadening of $\varphi_\pi(x)$, which owes to DCSB
- Scale evolution quite slow

Imaging dynamical chiral symmetry breaking: pion wave function on the light front, Lei Chang, *et al.*,
Phys. Rev. Lett. 110 (2013) 132001



Physics Letters B 770 (2017) 551–555

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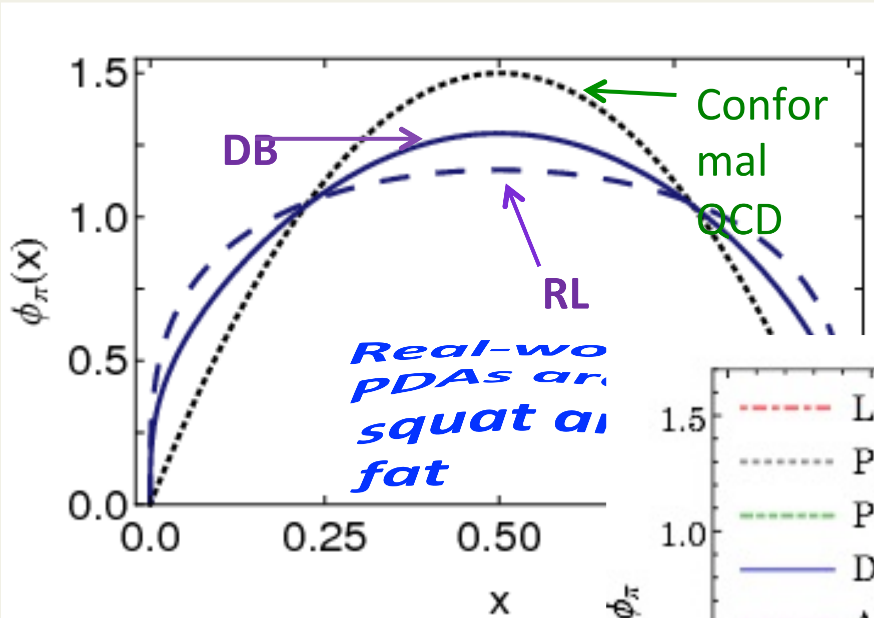
Bayesian extraction of the parton distribution amplitude from the Bethe–Salpeter wave function

Fei Gao [b,c], Lei Chang [a,*], Yu-xin Liu [b,c,d]



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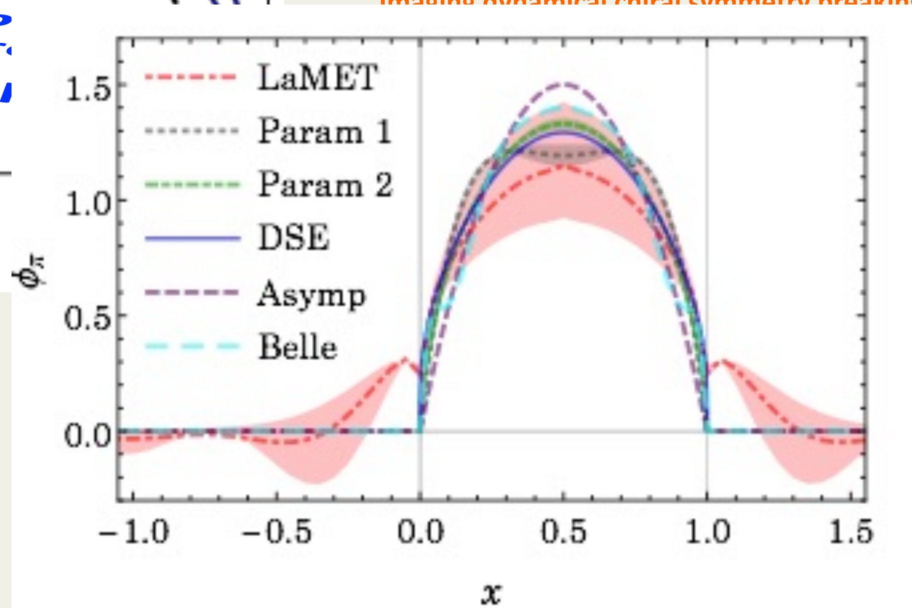
Imaging dynamical chiral symmetry breaking:

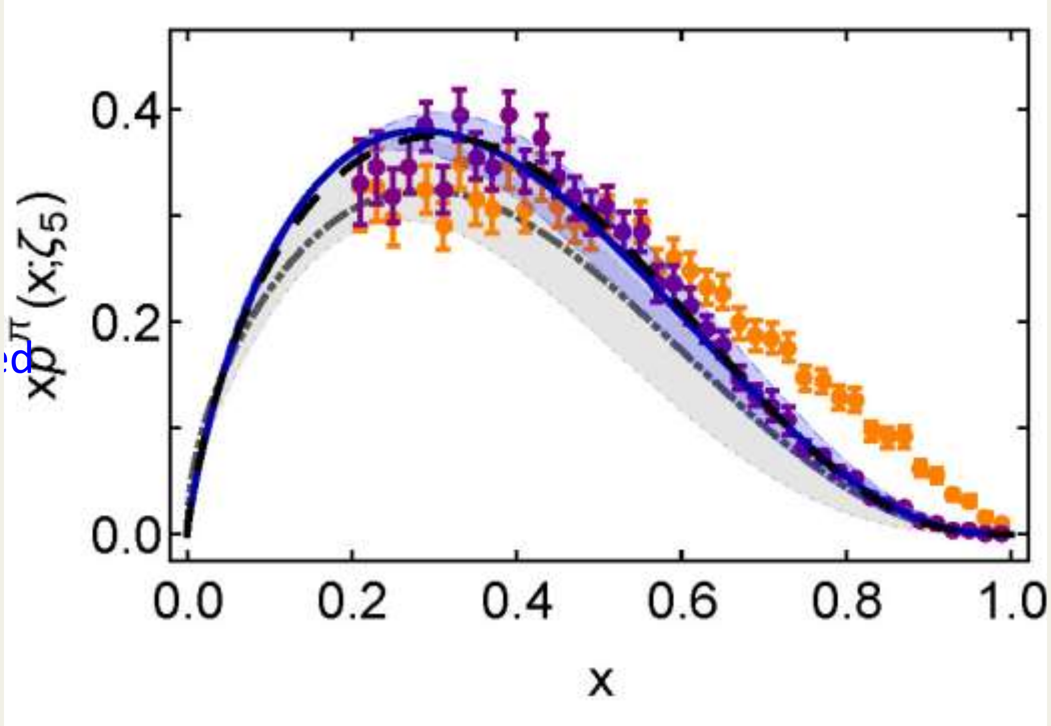


Pion Distribution Amplitude from Lattice QCD

Jian-Hui Zhang,[1,*] Jiunn-Wei Chen,[2,3,†] Xiangdong Ji,[4,5,‡] Luchang Jin,[6,§] and Huey-Wen Lin[7,8,¶]

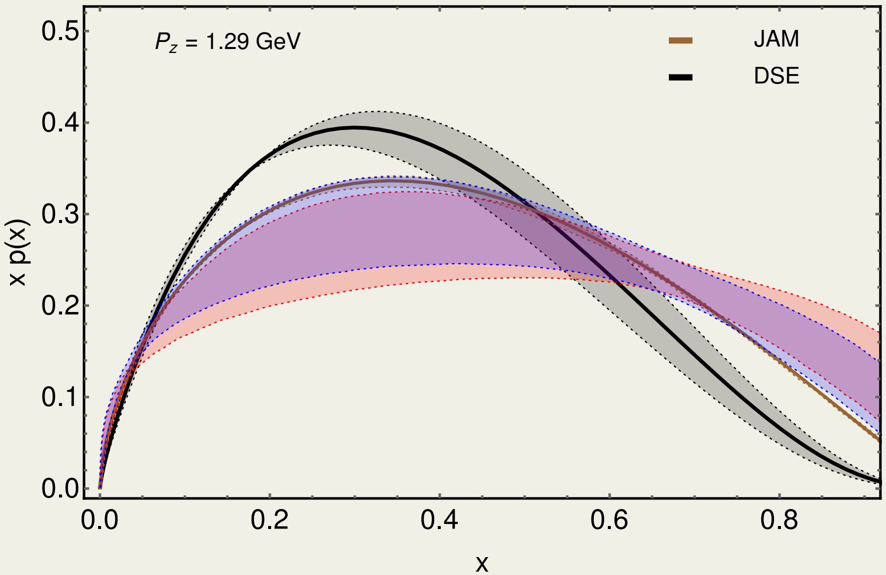
Pion PDF 30年　　进展一

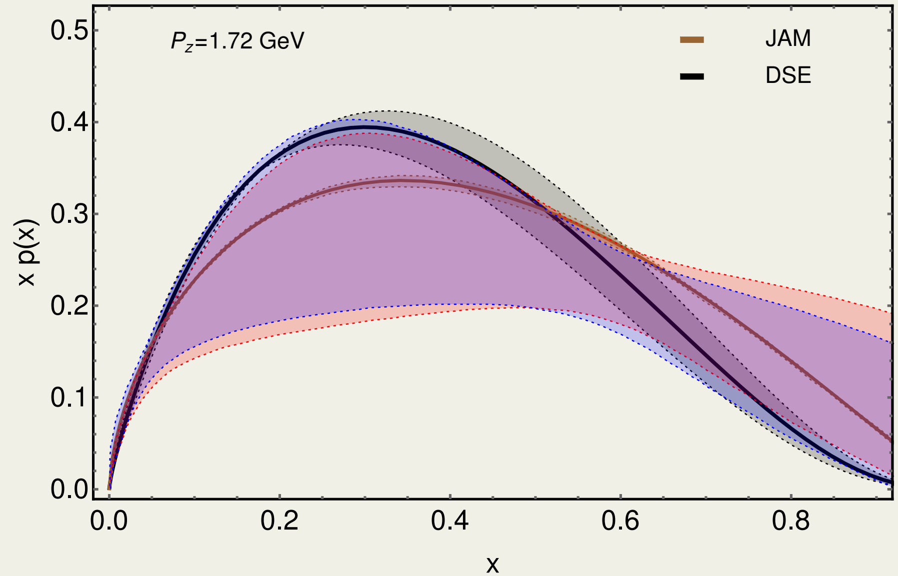
- 1989…Conway et al. Phys. Rev.D 39 (1989) 92
  Leading-order analysis of Drell-Yan data
- 2010...Aicher et al. Phys. Rev. Lett. 105 (2010) 252003
  Consistent next-to-leading order anaylsis
- lQCD, model, ...
- 2019/04...Ding, et al.
  Continuum QCD prediction
- 2019/01...Sufian, et al.
  1st exploratory lattice-QCD calculation
  Using lattice-calculated matrix element obtained through spatially separated current-current correlations in coordinate space



Sufian…extending lattice calculation on three other ensembles and the preliminary result gives an indication MORE COINCIDE.

Large Momentum Effective Theory





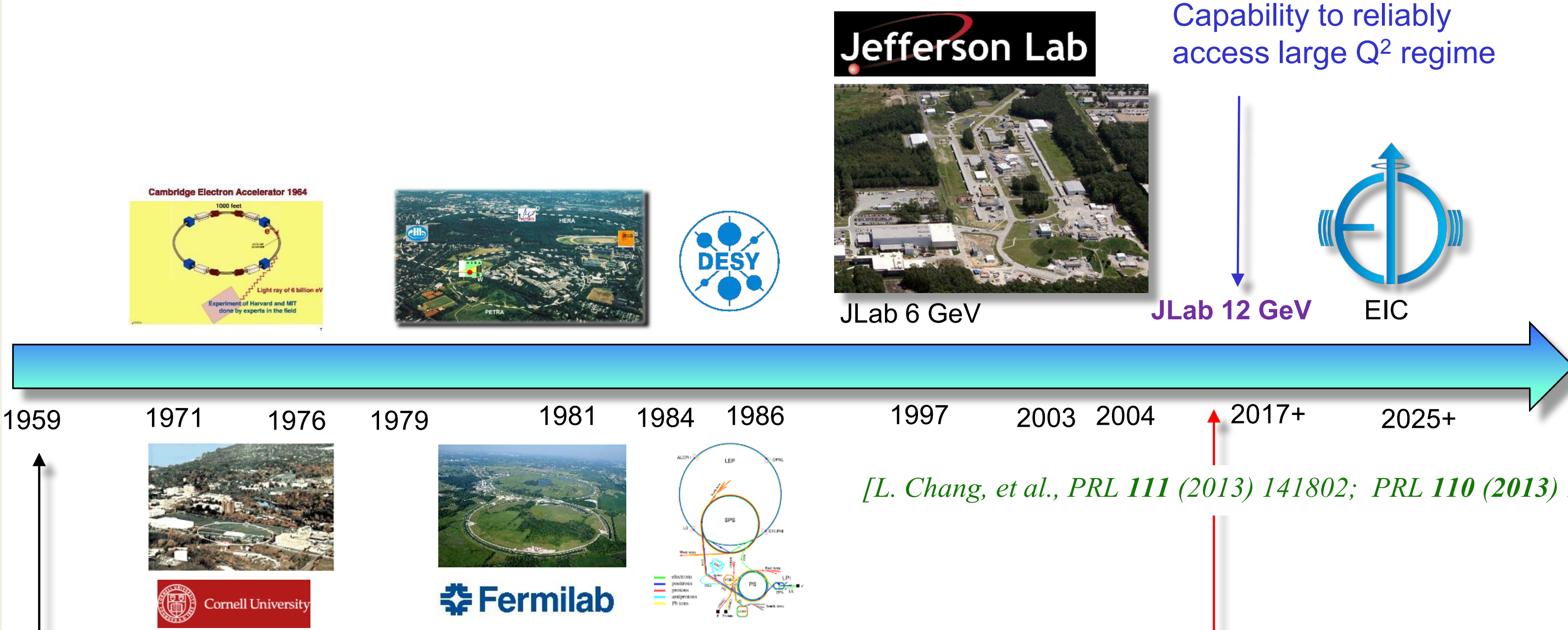
lQCD result, T. Izubuchi *et al*, arXiv:1905.06349.
Valence parton distribution function of pion from fine lattice

还不是时候

Meson Form Factor Data Evolution

Experiment

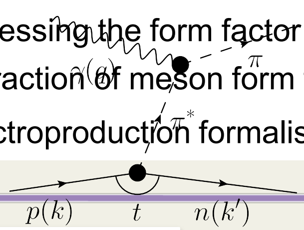
Capability to reliably access large Q^2 regime



[L. Chang, et al., PRL **111** (2013) 141802; PRL **110** (**2013**) 1322001]

Theory

- Accessing the form factor through electroproduction
- Extraction of meson form factor from data
- Electroproduction formalism

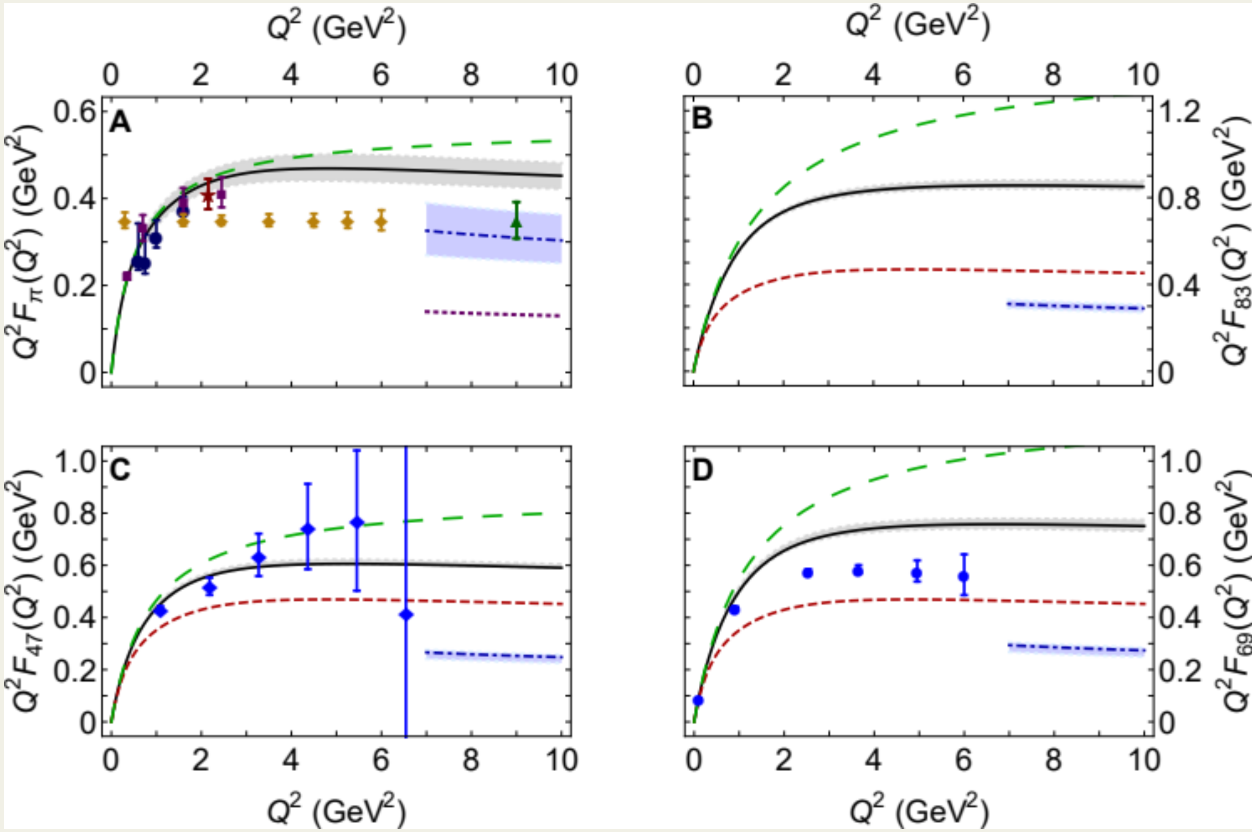


Theory

Major progress on large Q^2 behavior of meson form factor

T.Horn

- ✓ 彩虹梯子近似
- ✓ 赝标介子质量分别 m_{0-}/GeV=0.14, 0.47, 0.69, 0.83
- ✓ 电磁形状因子 $Q^2 \lesssim 10\ GeV^2$.
- ✓ 与格点结果比较



- lQCD(QCDSF/UKQCD/CSSM Collaborations) in panel C: PRD96(2017)114509
- lQCD(HPQCD Collaboration) in panel D: PRD96(2017)054501

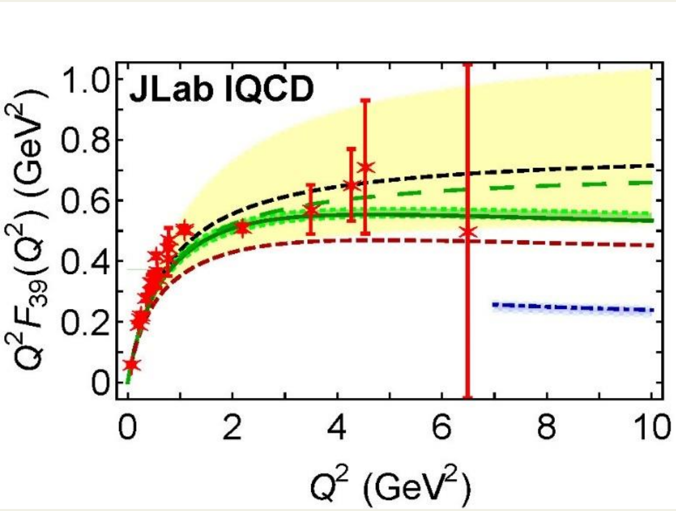


Figure 2. Elastic form factor of a pion-like pseudoscalar meson with mass m_{0^-} = 0.39 GeV. Red asterisks -- JLab lattice results drawn from the ECT presentation by D.G. Richards. Curves: Solid green curve within green bands -- our prediction, obtained using the methods described in our submission; long-dashed green curve -- single-pole vector meson dominance result obtained with vector meson mass, m_{1^-}=0.86 GeV, computed consistent with the form factor prediction; short-dashed black curve within yellow bands -- range of uncertainty in single-monopole fit to lQCD results following from the JLab-quoted error on the meson's radius; and dot-dashed blue curve within blue bands -- result from QCD hard-scattering formula, Eq. (1) in our submission, computed with the consistent meson decay constant and PDA, as predicted by the our analysis. For comparison, the dashed red curve is the DSE prediction for the physical-pion, taken from Fig. 1A in our submission.*

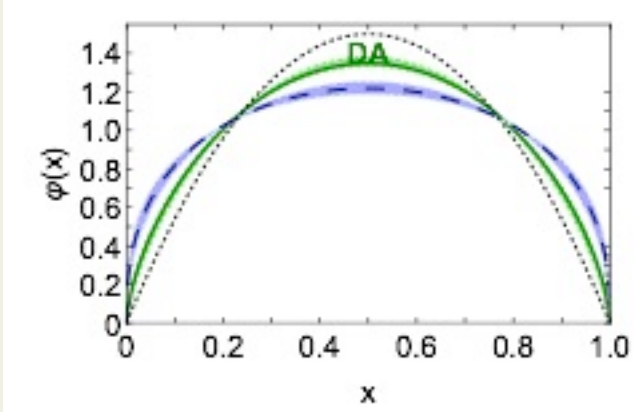


FIG. 2. Dressed-valence-quark distribution amplitude of pion-like pseudoscalar mesons. Solid green curve within green bands – prediction for m_{0^-} = 0.39 GeV; long-dashed blue curve within blue bands – prediction from Ref. [1] for the physical pion; and dotted black curve – asymptotic profile, $\varphi_\infty(x) = 6x(1-x)$.

The lattice results in Figure 2 were drawn from a presentation by D. G. Richards at the ECT* Workshop on Mapping Parton Distribution Amplitudes and Functions. That presentation is available here

https://indico.ectstar.eu/event/22/contributions/503/attachments/387/532/pion_pdf_richards.pdf

	r_{0^-}/fm	m_{1^-}/GeV	f_{0^-}/GeV
DSE	0.58(1)	$0.86^{+0.04}_{-0.02}$	0.109(1)
lQCD	0.55(10)	$0.88^{+0.19}_{-0.13}$	?

Notably, in common with the Adelaide lattice-QCD results (Fig. 1C in our submission), the JLab results are consistent with our predictions. The Glasgow results (Fig. 1D in our submission) alone appear to be inconsistent with physics expectations.

为何关注Pion/Kaon，一个小结

Maris, Roberts and Tandy, Phys. Lett. **B420**(1998) 267-273

- Pion's Bethe-Salpeter amplitude

  Solution of the Bethe-Salpeter equation

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\Big[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\,k_\mu P_\nu\, H_\pi(k;P)\Big]$$

- Dressed-quark propagator $S(p) = \dfrac{1}{i\gamma\cdot p\,A(p^2) + B(p^2)}$

- Axial-vector Ward-Takahashi identity entails(chiral limit)

$$f_\pi E(k;P|P^2=0) = B(k^2) + (k\cdot P)^2\frac{d^2B(k^2)}{d^2k^2} + \ldots$$

PHYSICAL REVIEW D VOLUME 43, NUMBER 5 1 MARCH 1991

Calculation of chiral-symmetry breaking and pion properties as a Goldstone boson

Yuan-ben Dai, Chao-shang Huang, and Dong-sheng Liu

Institute of Theoretical Physics, Academia Sinica, P. O. Box 2735, Beijing, China

(Received 19 June 1990; revised manuscript received 5 November 1990)

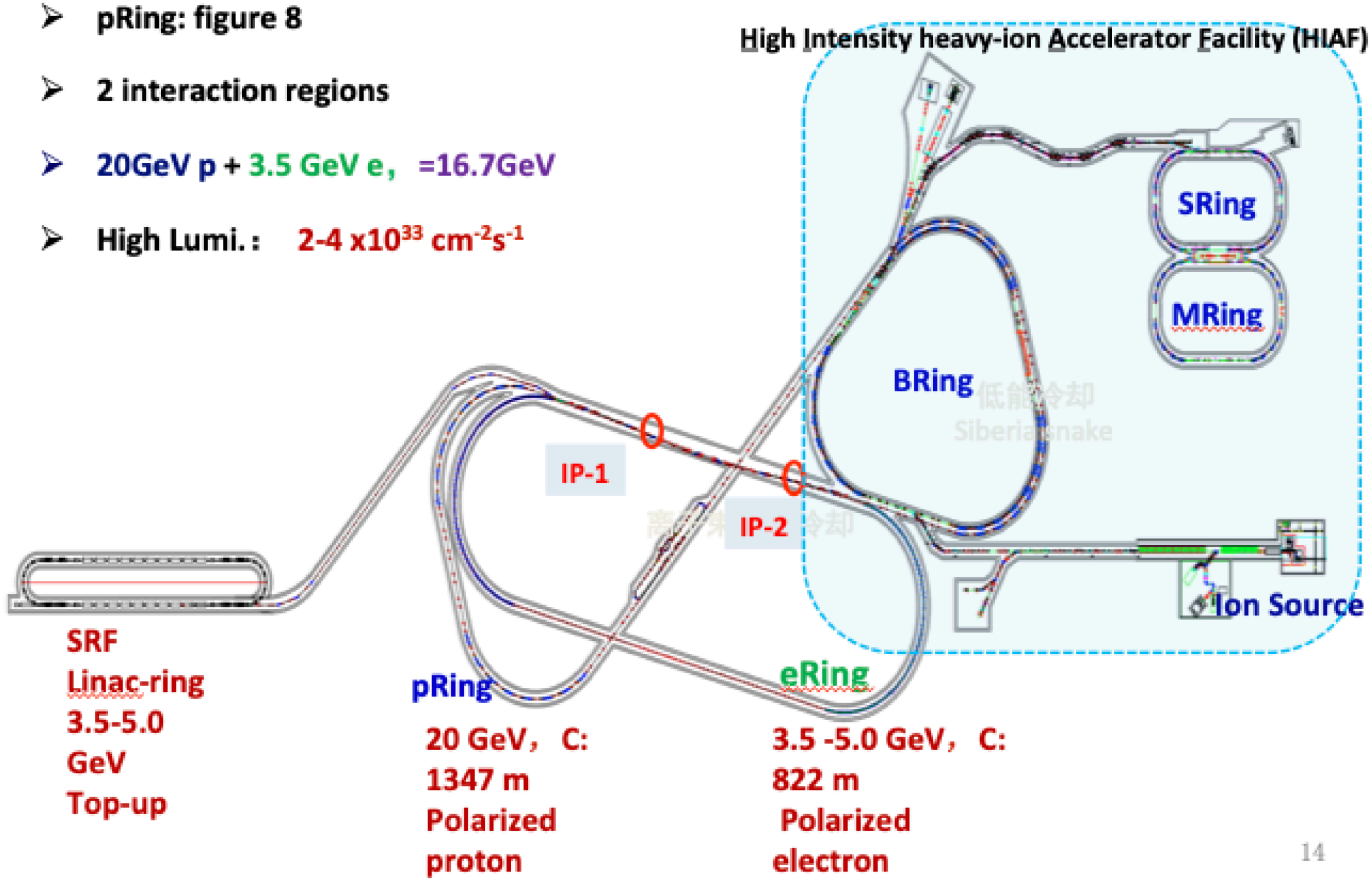
为何关注Pion，一个小结

Maris, Ro

梁羽铁

EicC accelerator complex overview

- pRing: figure 8
- 2 interaction regions
- 20GeV p + 3.5 GeV e， =16.7GeV
- High Lumi.： $2\text{-}4 \times 10^{33}\ cm^{-2}s^{-1}$



PHYS

1991

14

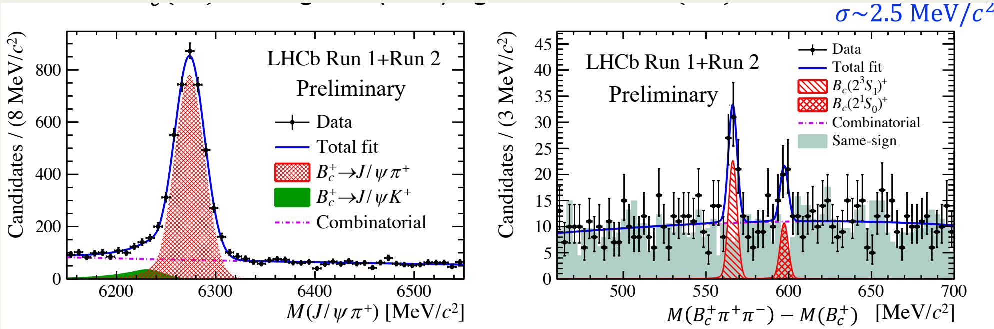
Bc谱之现状　　　　进展三

J^P	$\bar{M}^{\rm RL}_{c\bar{b}}$	$M^{\rm QM}_{c\bar{b}}$	$M^{\rm LQCD}_{c\bar{b}}$	$M^{\rm expt.}_{c\bar{b}}$
0^-	6290(15)(3)	<u>6271</u>	6276(7)	6275(1)
1^-	6343(15)(2)	<u>6326</u>	6331(7)	–
0^+	6703(15)(2)	6714	6712(19)	–
1^+_1	6745(15)(5)(8)	6757	6736(18)	–
1^+_2	6781(15)(4)(8)	6776	–	–
2^+	6793(15)(3)	6787	–	–

- RL：　陈慕阳等
- QM：钟显辉等
- LQCD：PRL121，202002

	$M_{\eta_c(2S)}$	$M_{\psi(2S)}$	$M_{\psi(2S)} - M_{\eta_c(2S)}$
here	3.606(18)	3.645(18)	0.039
expt.	3.638(1)	3.686(1)	0.048
	$M_{B_c^+(2S)}$	$M_{B_c^{*+}(2S)}$	$M_{B_c^+(2S)} - M^{\rm rec}_{B_c^{*+}(2S)}$
here	6.813(16)	6.841(18)	0.039
expt.	6.872(2)	–	0.031
	$M_{\eta_b(2S)}$	$M_{\Upsilon(2S)}$	$M_{\Upsilon(2S)} - M_{\eta_b(2S)}$
here	9.915(15)	9.941(15)	0.026
expt.	9.999(4)	10.023(1)	0.024

$\sigma \sim 2.5$ MeV/c^2



3785 ± 73 B_c^+ signals　　**51 ± 10** $B_c^*(2S)^+$; **24 ± 9** $B_c(2S)^+$

$M(B_c^*(2S)^+)_{\rm rec} = 6841.2 \pm 0.6({\rm stat}) \pm 0.1({\rm syst}) \pm 0.8(B_c^+)$ MeV/c^2

$M(B_c(2S)^+) = 6872.1 \pm 1.3({\rm stat}) \pm 0.1({\rm syst}) \pm 0.8(B_c^+)$ MeV/c^2

$M(B_c(2S)^+) - M(B_c^*(2S)^+)_{\rm rec} = 31.0 \pm 1.4({\rm stat})$ MeV/c^2

Decay constant——波函数细节

$$f_{0^-}^{fg}(P^2)P_\mu = Z_2 N_c \,\mathrm{tr}\int_{dk}^{\Lambda} \gamma_5\gamma_\mu \chi_{0^-}^{fg}(k;P),$$

$$f_{1^-}^{fg}(P^2)\sqrt{-P^2} = \frac{Z_2 N_c}{3}\,\mathrm{tr}\int_{dk}^{\Lambda} \gamma_\mu \chi_{1^-,\mu}^{fg}(k;P),$$

Ground States

$f_{\eta_c(1S)}$	$f_{\psi(1S)}$	$f_{B_c^+(1S)}$	$f_{B_c^{*+}(1S)}$	$f_{\eta_b(1S)}$	$f_{\Upsilon(1S)}$	
278(0)	304(1)	312(1)	305(5)	472(0)	442(3)	Here
247.7	230.4	306.1	307.3	456.8	457.7	NR QuarkMode
278(2)	289(4)	307(10)	298(9)	472(5)	459(22)	Lattice QCD

Excited States

$f_{\eta_c(2S)}$	$f_{\psi(2S)}$	$f_{B_c^+(2S)}$	$f_{B_c^{*+}(2S)}$	$f_{\eta_b(2S)}$	$f_{\Upsilon(2S)}$	
-0.097(2)	-0.119(6)	-0.165(10)	-0.161(7)	-0.310(5)	-0.320(6)	Here
196.9	182.0	251.4	252.0	366.8	367.3	NR QuarkMod

for $V = \Upsilon(2S)$ [26]. Comparing with $f_{\psi(2S)}^{\mathrm{expt.}} = -0.208(2)$ GeV and $f_{\Upsilon(2S)}^{\mathrm{expt.}} = -0.352(2)$ GeV, one can know that the RL truncation underestimates the decay constants about 42% for $\psi(2S)$ and 12% for $\Upsilon(2S)$. Because the interaction Eq. (3)-Eq. (4) takes into the flavor dependence, we expect the higher order corrections decrease smoothly as the meson mass increases. $f_{B_c^{*+}(2S)}$ is underestimated roughly by $(42\% + 12\%)/2 = 27\%$, while the higher order corrections to $f_{B_c^+(2S)}$ is expected to be smaller than $f_{B_c^{*+}(2S)}$.

NR QuarkModel

N. R. Soni,, B. R. Joshi, R. P. Shah, H. R. Chauhan, and J. N. Pandya, Eur. Phys. J. **C** 78, 592 (2018).

DSEs的边界。。。。。。。

TABLE II: Masses (in MeV) of the charmonium with $J^{PC} = 0^{-+}, 1^{--}, 0^{++}, 1^{+-}, 1^{++}, 2^{++}$, the normal states in the quark model. $M_{c\bar{c}}^{\rm RL}$ is our RL approximation result. $M_{c\bar{c}}^{\rm expt.}$ is the experiment value [37]. $\Delta M_{c\bar{c}}^{\rm RL} = M_{c\bar{c}}^{\rm RL} - M_{c\bar{c}}^{\rm expt.}$ is the deviation of our results from the experiment value. Three sets of parameters in Tab. I are used in our calculation.

$J^{\rm PC}$		0^{-+}	1^{--}	0^{++}	1^{+-}	1^{++}	2^{++}
$M_{c\bar{c}}^{\rm RL}$	Para-1	2984	3134	3327	3400	3417	3497
	Para-2	2984	3132	3331	3416	3426	3511
	Para-3	2984	3130	3332	3426	3431	3518
$\Delta M_{c\bar{c}}^{\rm RL}$	Para-1	0	37	-88	-125	-94	-59
	Para-2	0	35	-84	-109	-85	-45
	Para-3	0	33	-83	-99	-80	-38
$M_{c\bar{c}}^{\rm expt.}$		2984	3097	3415	3525	3511	3556

TABLE III: Masses (in MeV) of the bottomonium. The meanings of the quantities are the same as in Tab. II.

$J^{\rm PC}$		0^{-+}	1^{--}	0^{++}	1^{+-}	1^{++}	2^{++}
$M_{b\bar{b}}^{\rm RL}$	Para-1	9399	9453	9754	9793	9788	9820
	Para-2	9399	9453	9762	9805	9799	9833
	Para-3	9399	9453	9765	9810	9804	9835
$\Delta M_{b\bar{b}}^{\rm RL}$	Para-1	0	-7	-105	-106	-106	-92
	Para-2	0	-7	-97	-94	-94	-79
	Para-3	0	-7	-94	-89	-89	-77
$M_{b\bar{b}}^{\rm expt.}$		9399	9460	9859	9899	9893	9912

一个例子

注意RL误差(S-level, P-level)
可以控制吗？
可以系统控制吗？

DSEs的边界。。。。。。。

TABLE II: Masses (in MeV) of the charmonium with $J^{PC} = 0^{-+}, 1^{--}, 0^{++}, 1^{+-}, 1^{++}, 2^{++}$, the normal states in the quark model. $M^{\rm RL}_{c\bar{c}}$ is our RL approximation result. $M^{\rm expt.}_{c\bar{c}}$ is the experiment value [37]. $\Delta M^{\rm RL}_{c\bar{c}} = M^{\rm RL}_{c\bar{c}} - M^{\rm expt.}_{c\bar{c}}$ is the deviation of our results from the experiment value. Three sets of parameters in Tab. I are used in our calculation.

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	Para-2	9399	9453	9762	9805	9799	9833
	Para-3	9399	9453	9765	9810	9804	9835
$\Delta M^{\rm RL}_{b\bar{b}}$	Para-1	0	-7	-105	-106	-106	-92
	Para-2	0	-7	-97	-94	-94	-79
	Para-3	0	-7	-94	-89	-89	-77
$M^{\rm expt.}_{b\bar{b}}$		9399	9460	9859	9899	9893	9912

机遇与挑战