Nuclear Science Computing Center at CCNU

QCD phase structure from Lattice QCD

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Central China Normal University (华中师范大学)

- 简介
- 研究现状
- 未来机遇与挑战

量子色动力学的未来：机遇与挑战
2019年11月10-11日@北京大学

NuclearScience
Computing CenteratCCNU

华中师范大学
核科学计算中心

N: Nuclear **S**: Science **C³**: Color 3 -> QCD

“道生一，一生二，二生三，三生万物。” — 《道德经》老子 600 BC

”Tao gives birth to One, One gives birth to Two, Two gives birth to Three, Three gives birth to everything.“- Lao Tzu

18 computing nodes
(144 V100 GPUs)
Peak performance:
1 PFlops/s
(每秒1千万亿次浮点运算)
Storage:
1 PB

June, 2018
TOP 500 The List.
The GREEN 500

160 more GPUs
to arrive next week

Lattice年会：格点场论领域最高级别会议

第27届, 2009年7月26-31日@北京

第37届, 2019年6月16-22日@武汉

Lattice 2019: 340余人参会，其中280余名国际参会者

主办单位：华中师范大学

协办单位：北京大学,清华大学, 南开大学, 浙江大学, 四川大学, 湖南师范大学, 江苏大学, 西安工业大学，台湾交通大学, 中科院高能物理研究所, 中科院理论物理研究所，中科院近代物理研究所

第九届中国物理学会高能物理分会@华中师范大学, 2014.4.19-22

第九届中国物理学会高能物理分会@华中师范大学, 2014.4.19-22

2019.11.9@北大农园

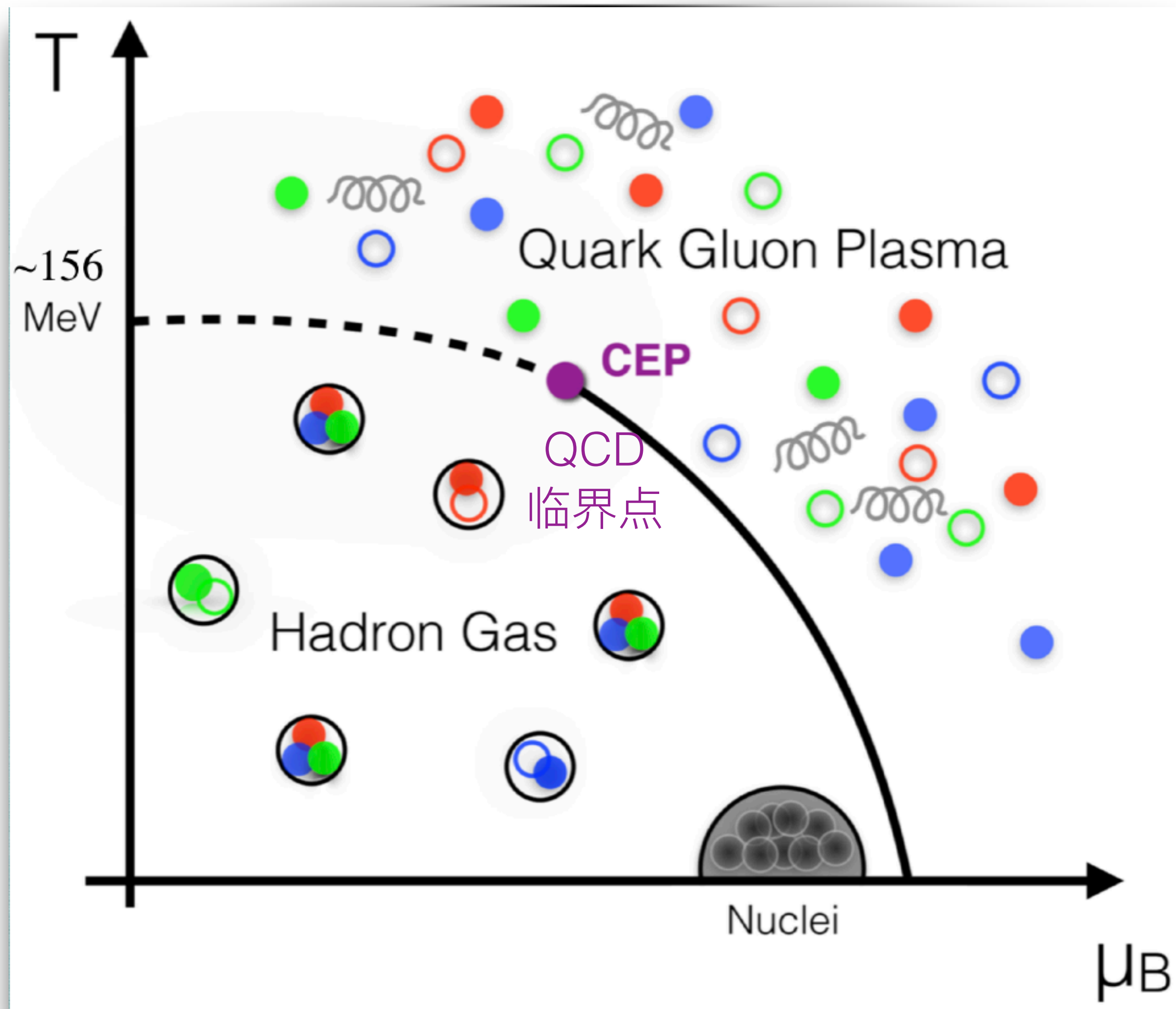
第九届中国物理学会高能物理分会@华中师范大学, 2014.4.19-22

2019.11.9@北大农园

贺赵老师步入八零后抒怀

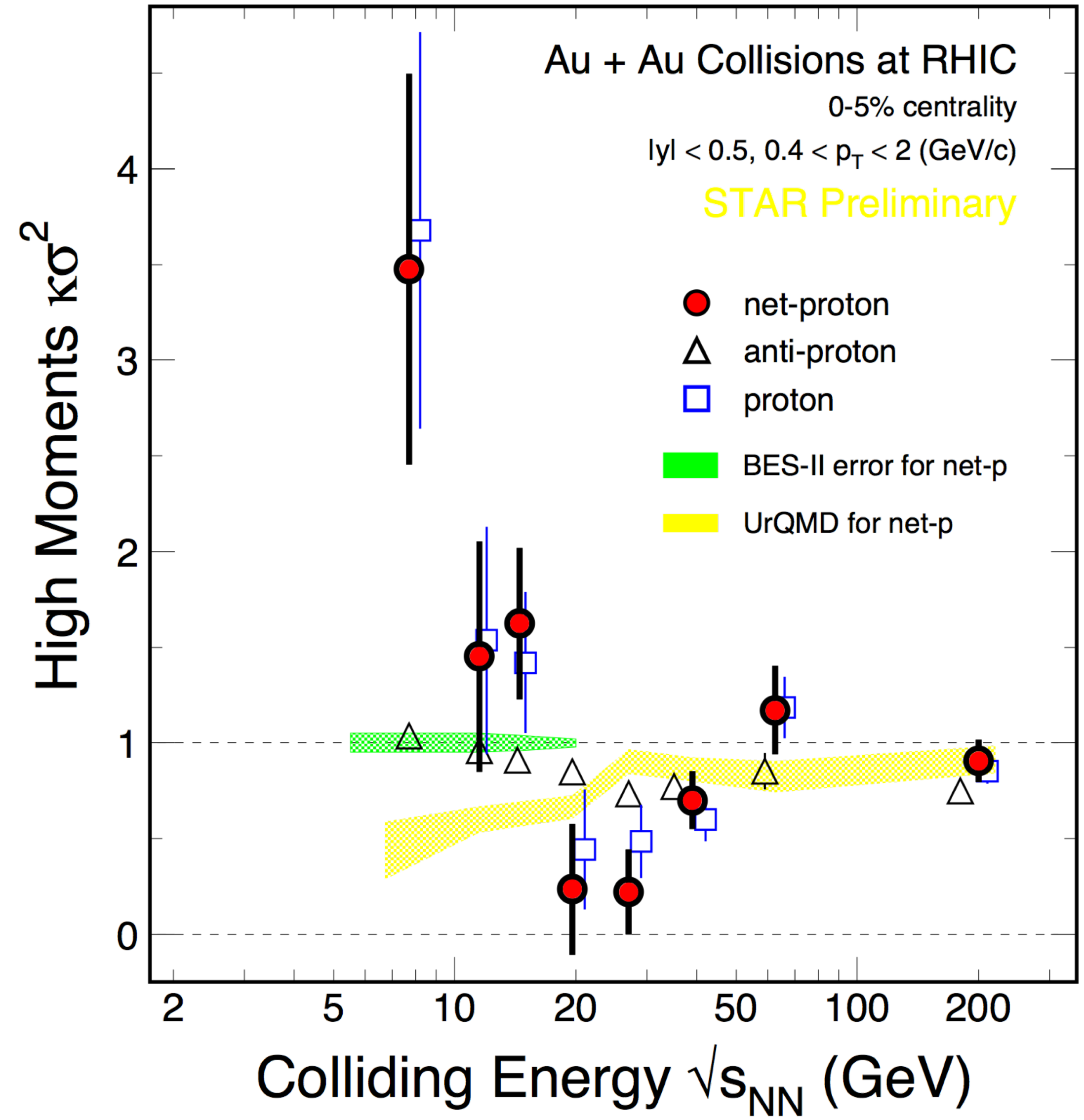
高能强子斩棘中，光达九州力学通。
我沐春风行大步，竿头一点揽星空。

Search for QCD criticality: critical end point?



See review paper:
HTD, F. Karsch, S. Mukherjee,
"Thermodynamics of strong-interaction matter from Lattice QCD"
Int.J.Mod.Phys. E24 (2015) no.10, 1530007

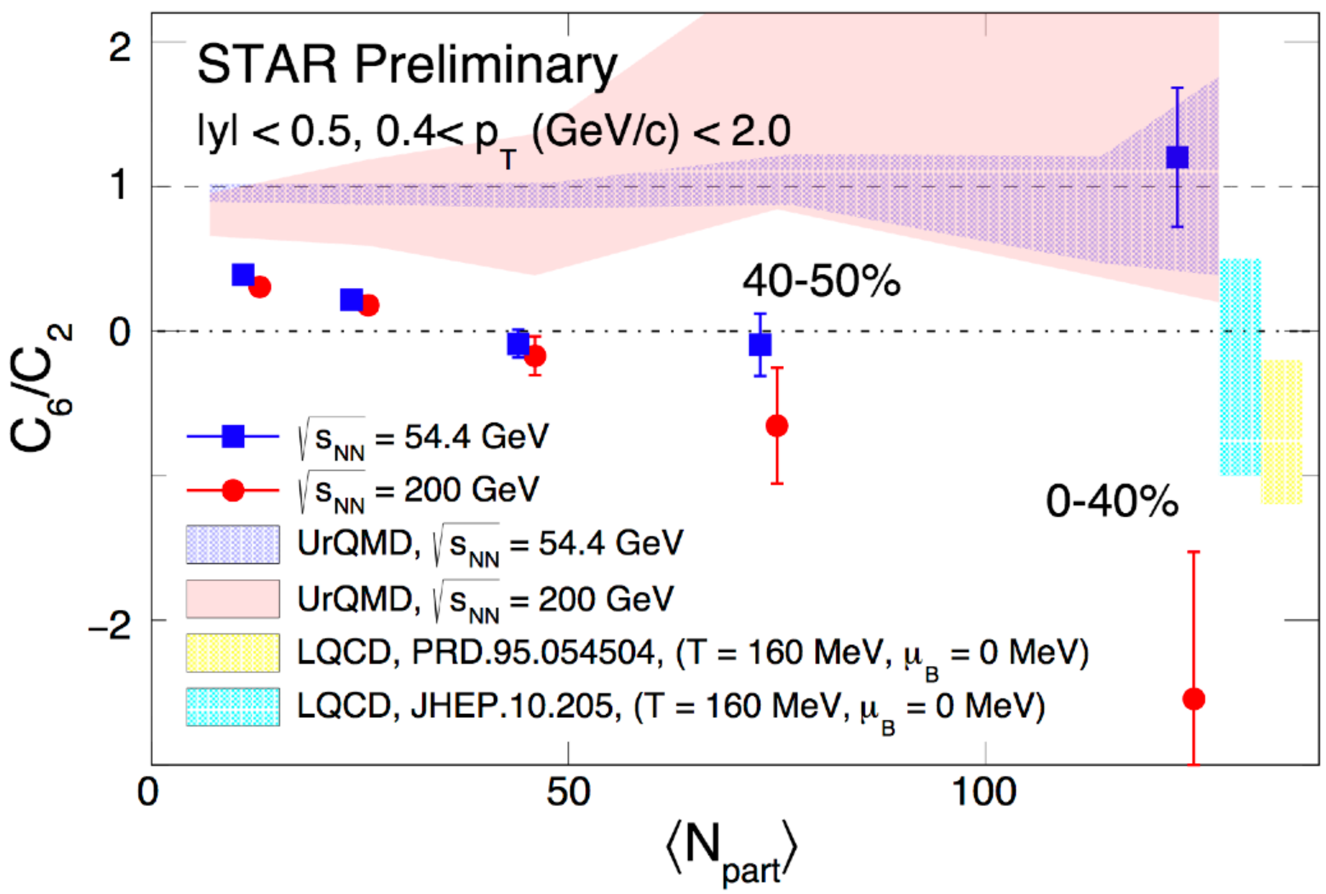
4th to 2nd order proton number fluctuations



See review paper:
X. F. Luo and N. Xu,
Nucl.Sci.Tech. 28 (2017) no.8, 112

Search for QCD criticality: critical end point?

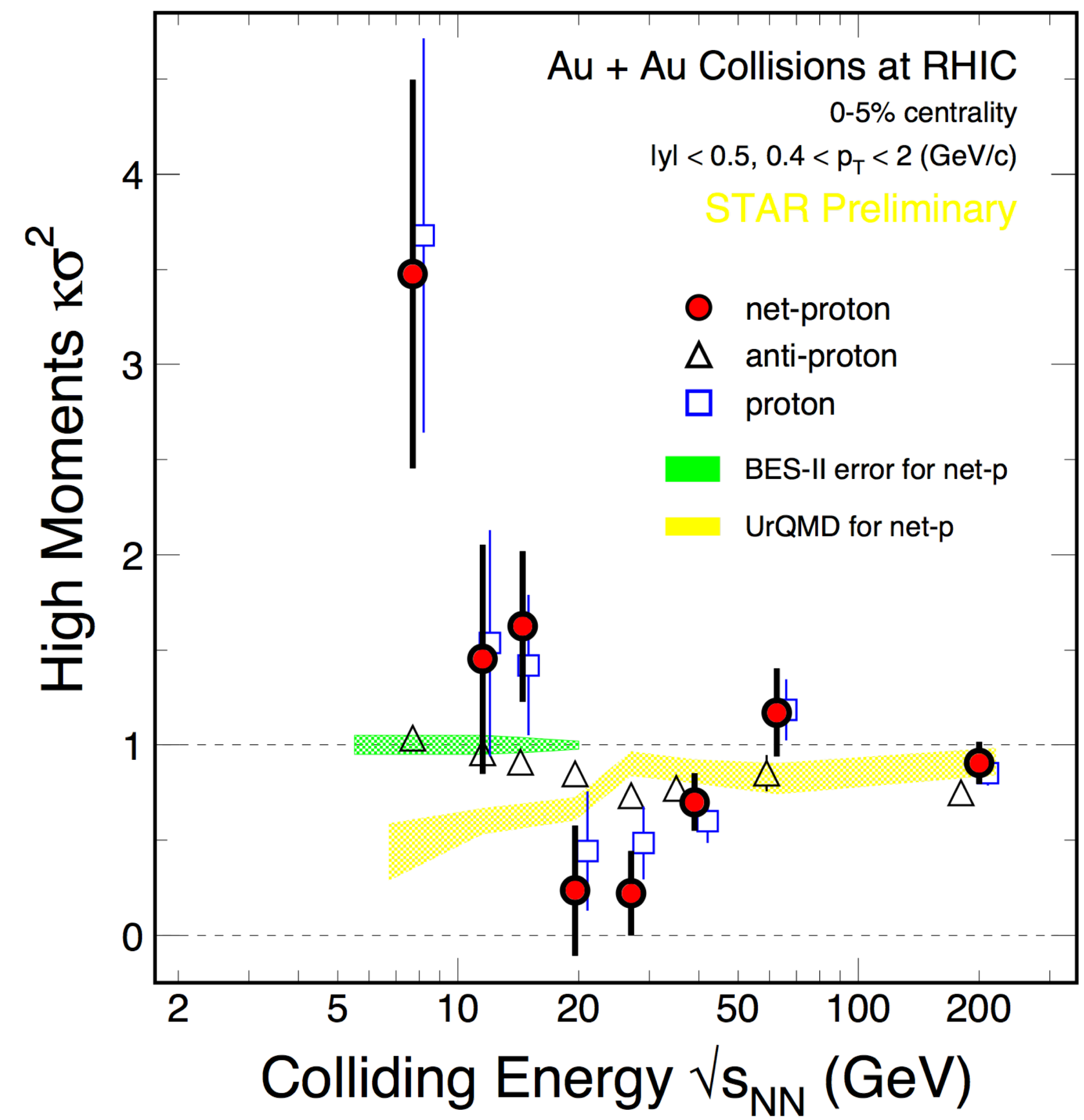
6th to 2nd order proton number fluctuations



Toshihiro Nonaka, Quark Matter 2019

Ashish Pandav, Quark Matter 2019

4th to 2nd order proton number fluctuations



See review paper:
X. F. Luo and N. Xu,
Nucl.Sci.Tech. 28 (2017) no.8, 112

Symmetries of QCD in the vacuum

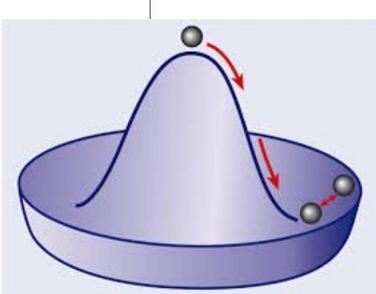
$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \sum_{q=u,d,s,c,b,t} \bar{q}\Big[i\gamma^{\mu}(\partial_{\mu} - igA_{\mu}) - m_q\Big]q$$

Classical QCD symmetry (m_q=0)

$$SU(N_f)_L \times SU(N_f)_R \times U(1)_V \times U(1)_A$$

Quantum QCD vacuum (m_q=0)

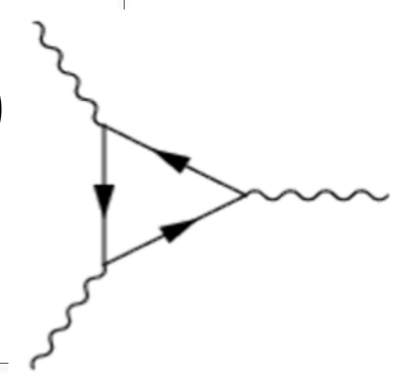
Chiral condensate:
spontaneous mass generation



$$\langle \bar{q}_R q_L \rangle \neq 0$$

Axial anomaly:
quantum violation of $U(1)_A$

$$\partial_{\mu} j_5^{\mu} = \frac{g^2 N_f}{16\pi^2} tr(\tilde{F}_{\mu\nu}F^{\mu\nu})$$



$$SU(N_f)_V \times U(1)_V$$

Landau functional of QCD

Pisarski & Wilczek (1984)

Symmetry: $SU(N_f)_L \times SU(N_f)_R \times U(1)_V \times U(1)_A$

Chiral field: $\Phi_{ij} \sim \frac{1}{2}\bar{q}^j(1-\gamma_5)q^i = \bar{q}^j_R q^i_L$

Chiral transformation: $\Phi \to e^{-2i\alpha_A} V_L \, \Phi \, V_R^\dagger$

$$\mathcal{L}_{eff} = \frac{1}{2}\,\mathrm{tr}\,\partial\Phi^\dagger\partial\Phi + \frac{a}{2}\,\mathrm{tr}\,\Phi^\dagger\Phi + \frac{b_1}{4!}\left(\mathrm{tr}\,\Phi^\dagger\Phi\right)^2 + \frac{b_2}{4!}\,\mathrm{tr}\,\left(\Phi^\dagger\Phi\right)^2$$

→ $SU(N_f)_L \times SU(N_f)_R \times U(1)_A$

$$-\frac{c}{2}\left(\mathrm{det}\Phi + \mathrm{det}\Phi^\dagger\right)$$

→ $SU(N_f)_L \times SU(N_f)_R$

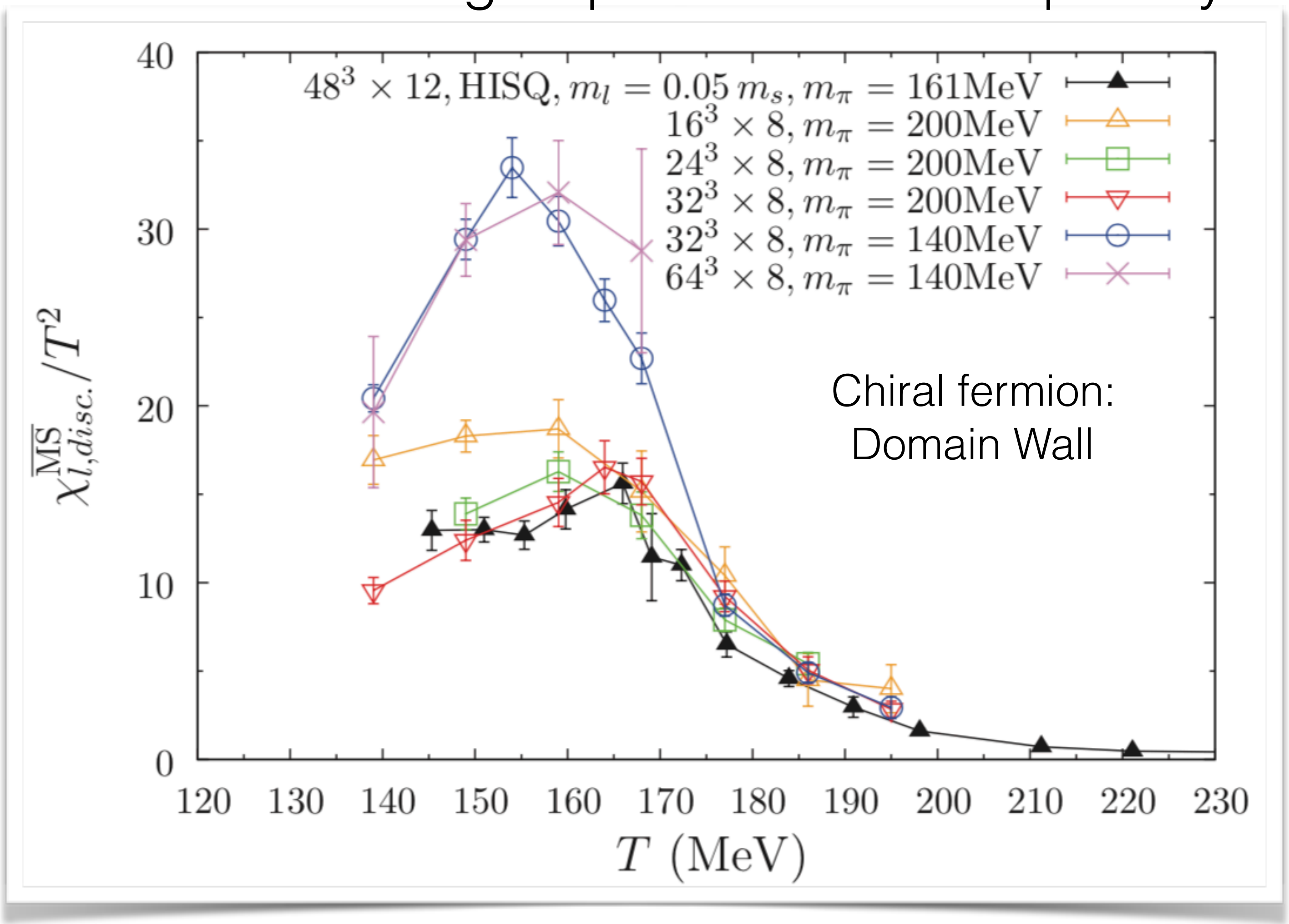
$$-\frac{d}{2}\mathrm{tr}\,h\left(\Phi + \Phi^\dagger\right).$$

→ Quark mass term

Results on phase transitions should be eventually checked by Lattice QCD

QCD crossover(平滑过渡) with m_π =140 MeV from Hadron phase to Quark Gluon Plasma phase

disconnected light quark chiral susceptibility



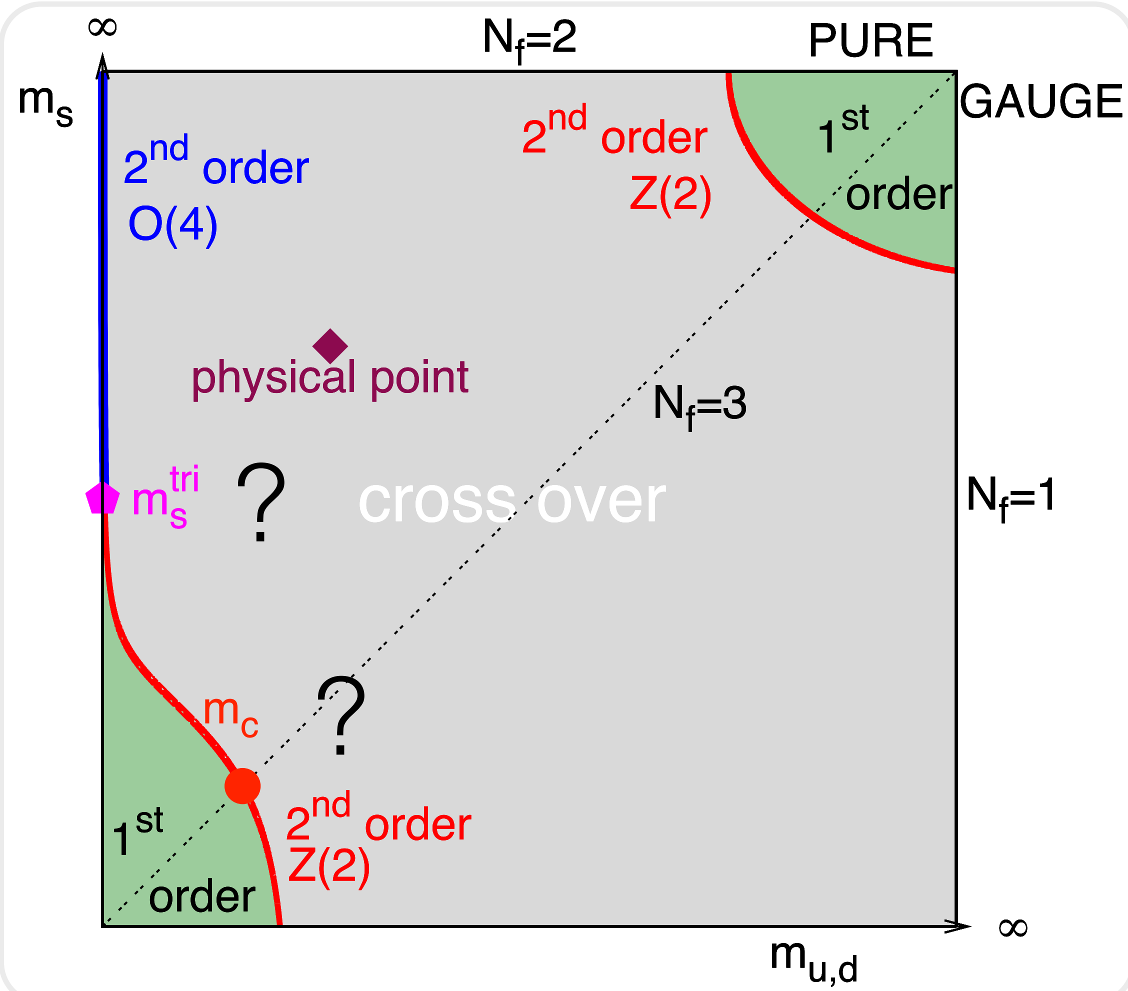
Bhattacharya, Buchoff, Christ, HTD et al.[HotQCD], Phys. Rev. Lett.,113(2014)082001

No Criticality:
Rapid crossover!

Ambiguities in the definition of **Pseudo-critical** temperature

QCD criticality relevant to the real world

Columbia plot:
QCD phase diagram in quark mass plane



- At physical point: crossover type transition

  [HotQCD] PRL,113(2014)082001, PRD 85(2012)054503
  [Wuppertal-Budapest], JHEP 1009 (2010) 073

- Critical lines of second order transition

  N_f=2: O(4) universality class — Pisarski & Wilczek PRD '84, Kogut & Sinclair, PRD '06

  N_f=3: Ising universality class — Karsch, Laermann, Schmidt PLB '04,...

- Chiral T_c^0: possible upper bound of transition T at the critical end point

  M. A. Halasz et al, PRD 58 (1998) 096007
  Y. Hatta & T. Ikeda, PRD67 (2003) 014028
  Hegde & HTD, PoS LATTICE2015 (2016) 1
  O. Kaczmarek et al., PRD83 (2011) 014504

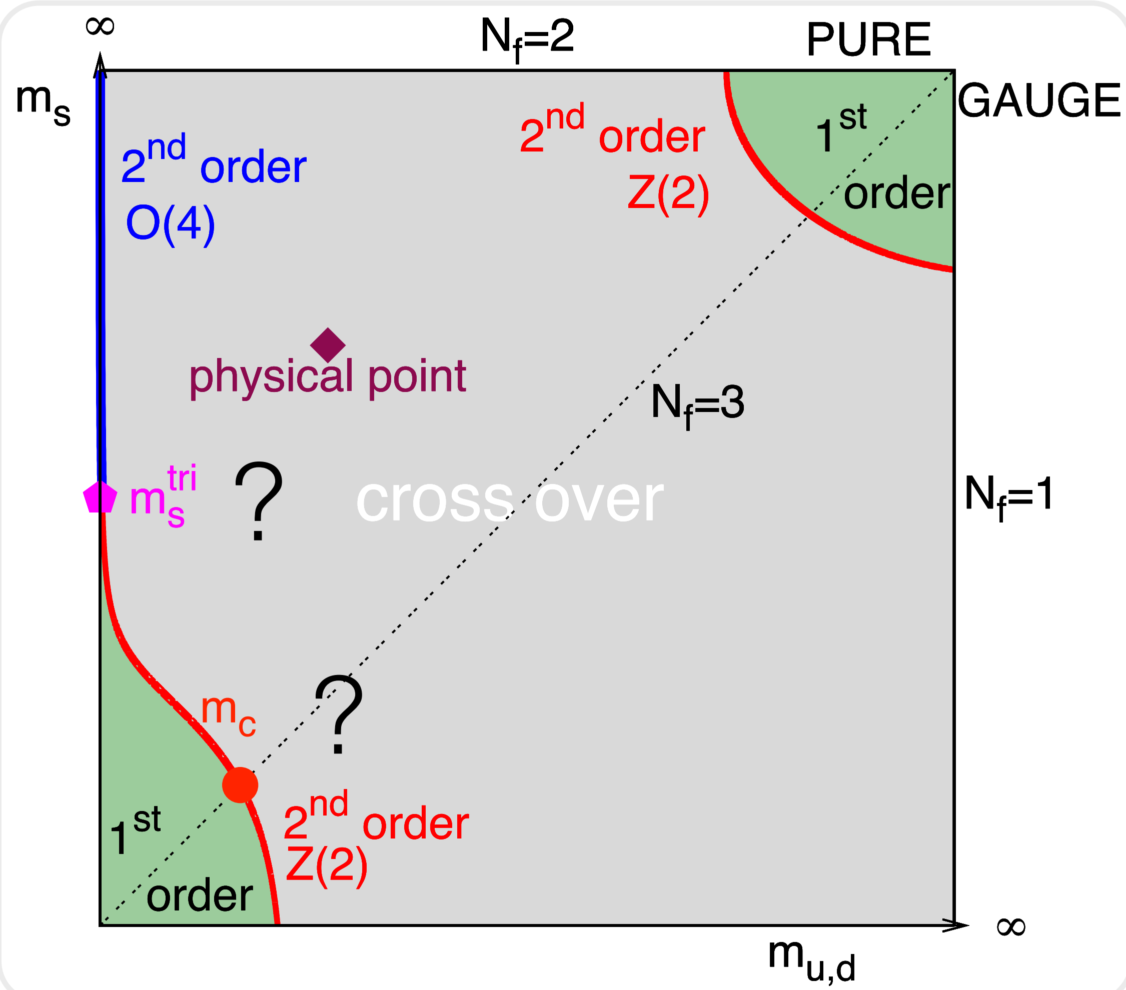
- N_f=2+1 QCD: fate of axial U(1) symmetry ?

- Criticalities that are relevant to QCD thermodynamics at the physical point ?

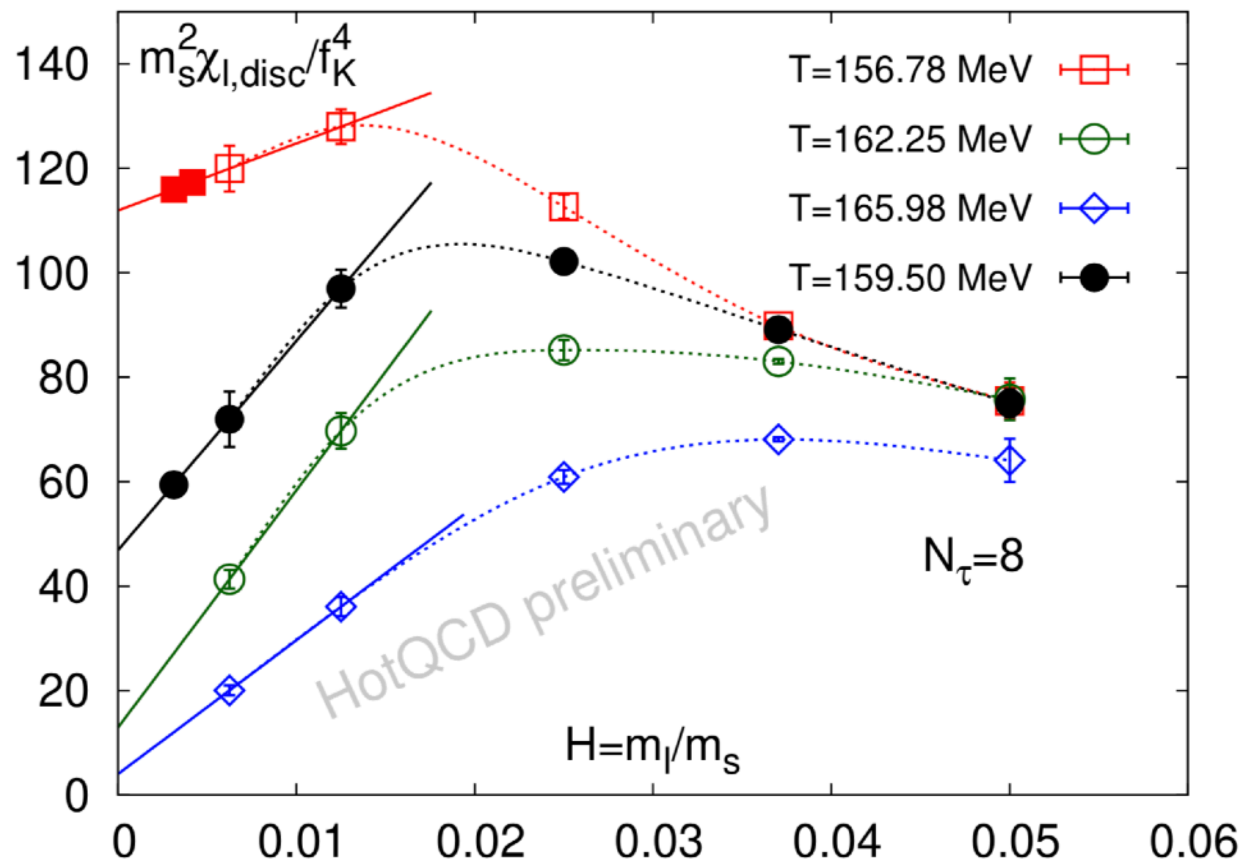
- Fundamental scale of QCD: chiral T_c^0 ?

QCD criticality relevant to the real world

Columbia plot:
QCD phase diagram in quark mass plane



disconnected chiral susceptibility

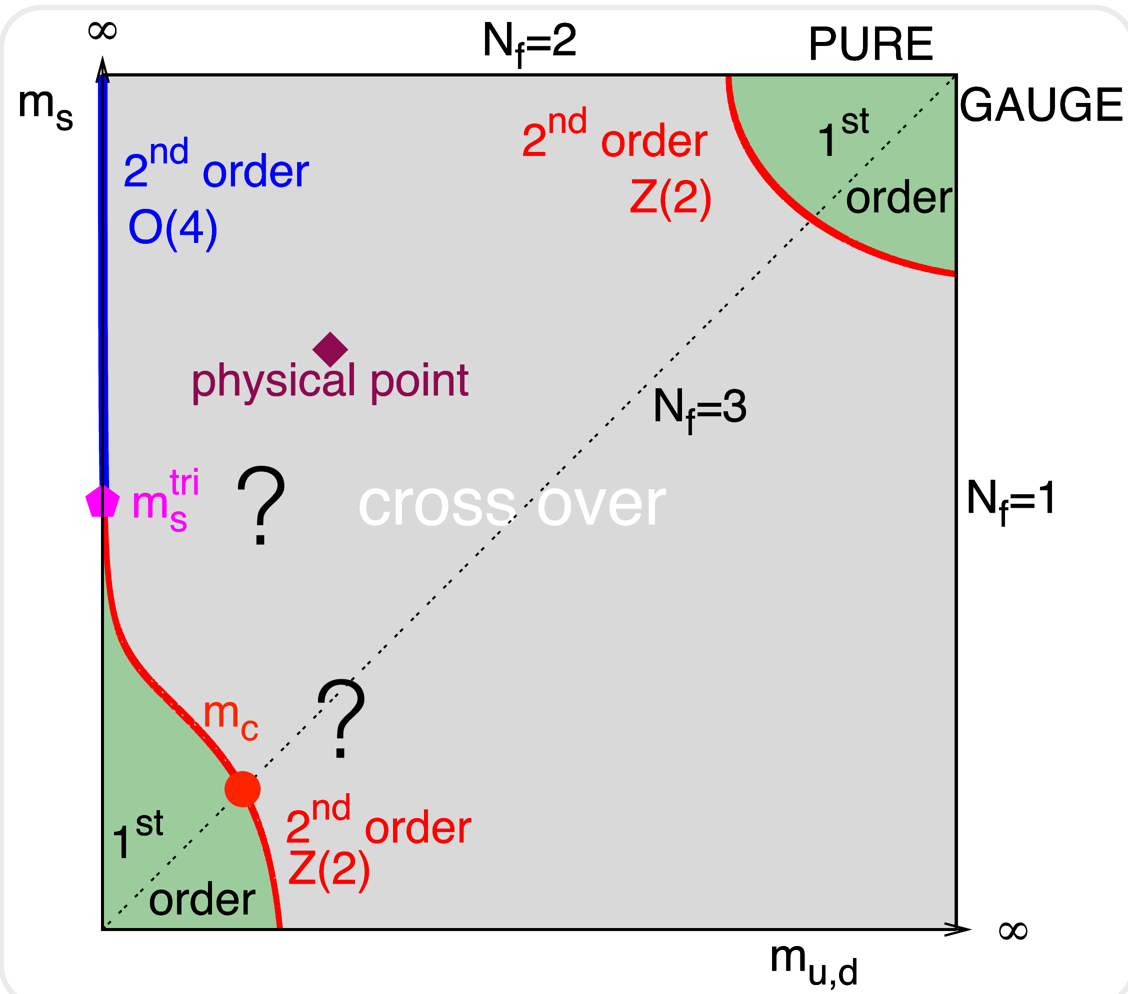


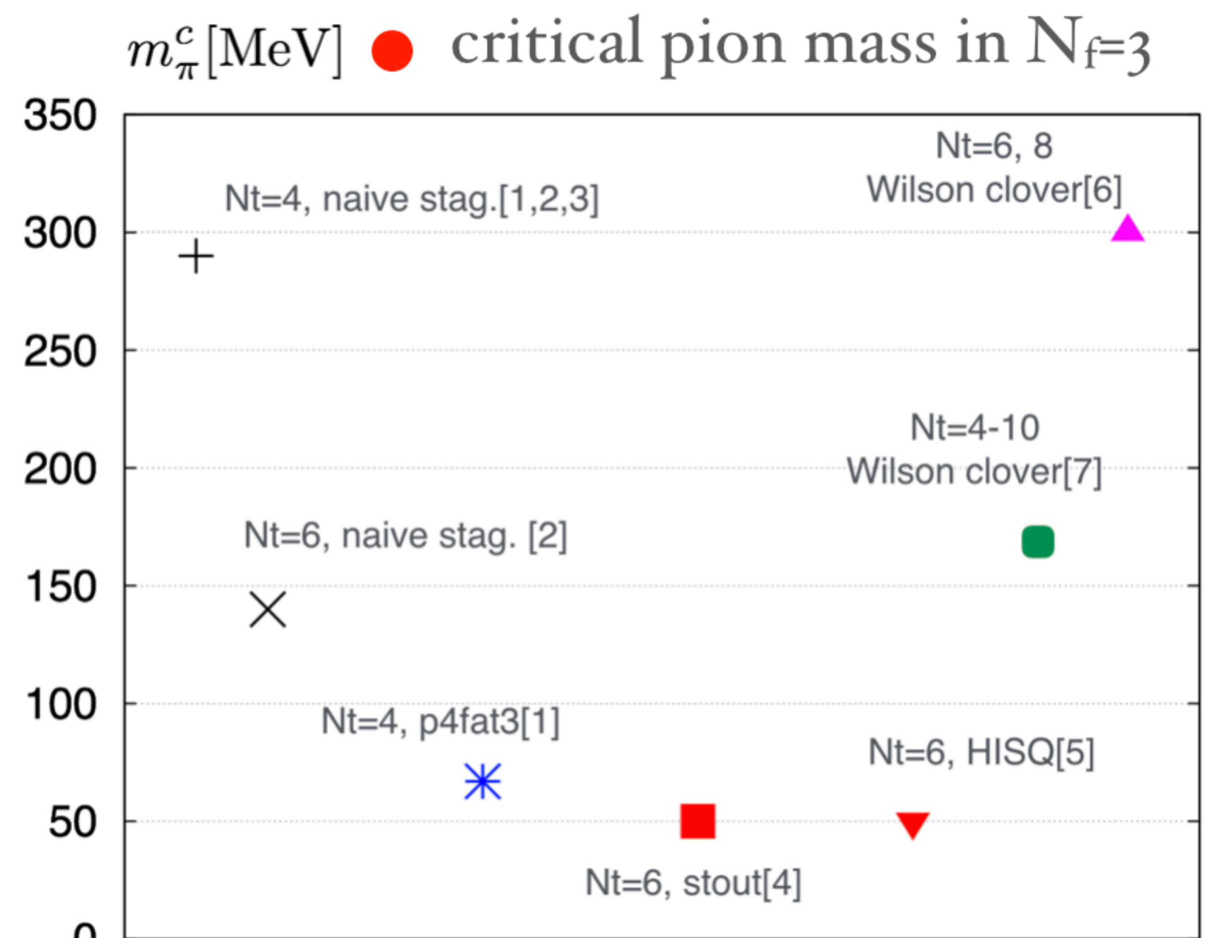
HTD, Quark Matter 2019 Plenary

- Axial U(1) symmetry remains broken at $T_{\chi SB}$ as $\chi_{l,disc} \neq 0$
- In the chiral limit of Nf=2 QCD: 2nd order O(4) phase transition

QCD criticality relevant to the real world

Columbia plot:
QCD phase diagram in quark mass plane



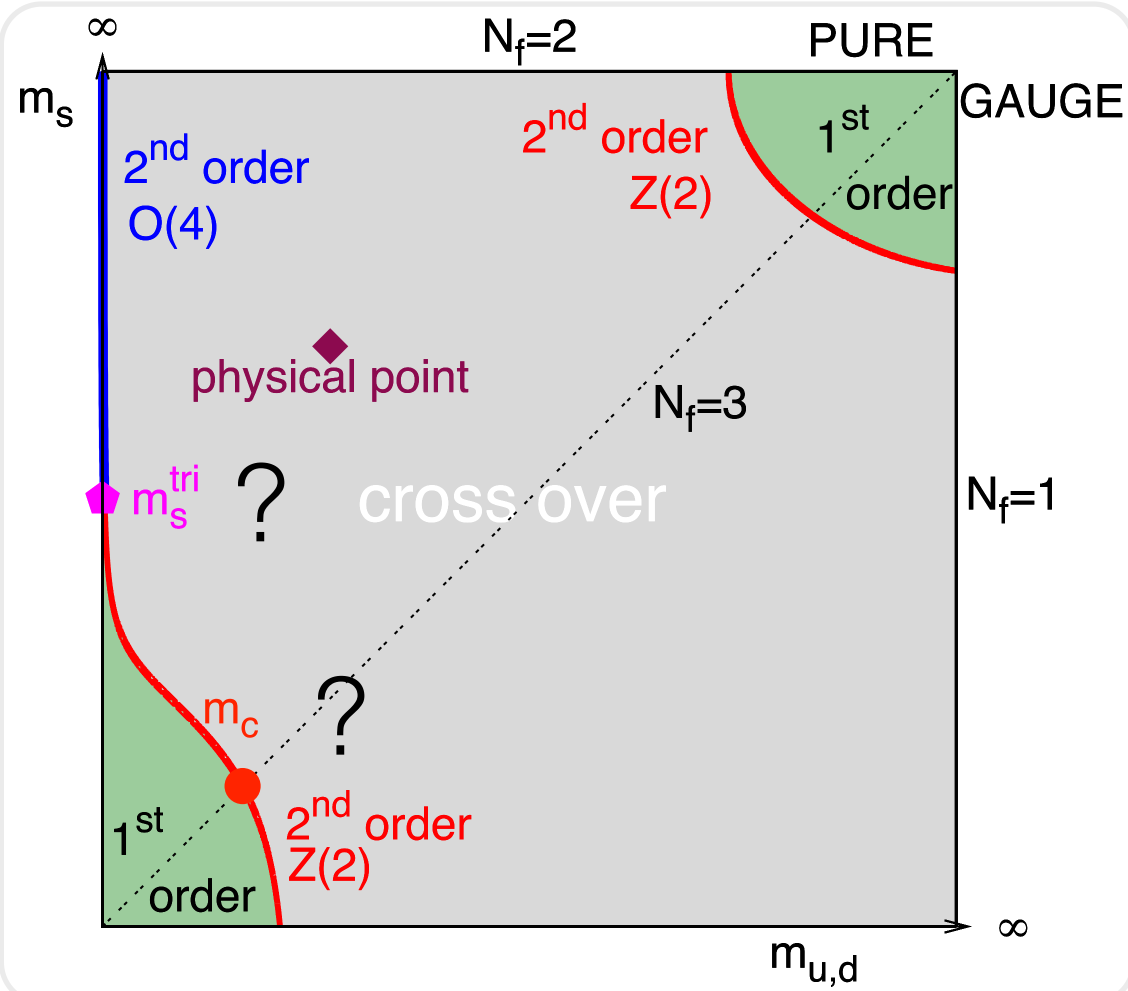


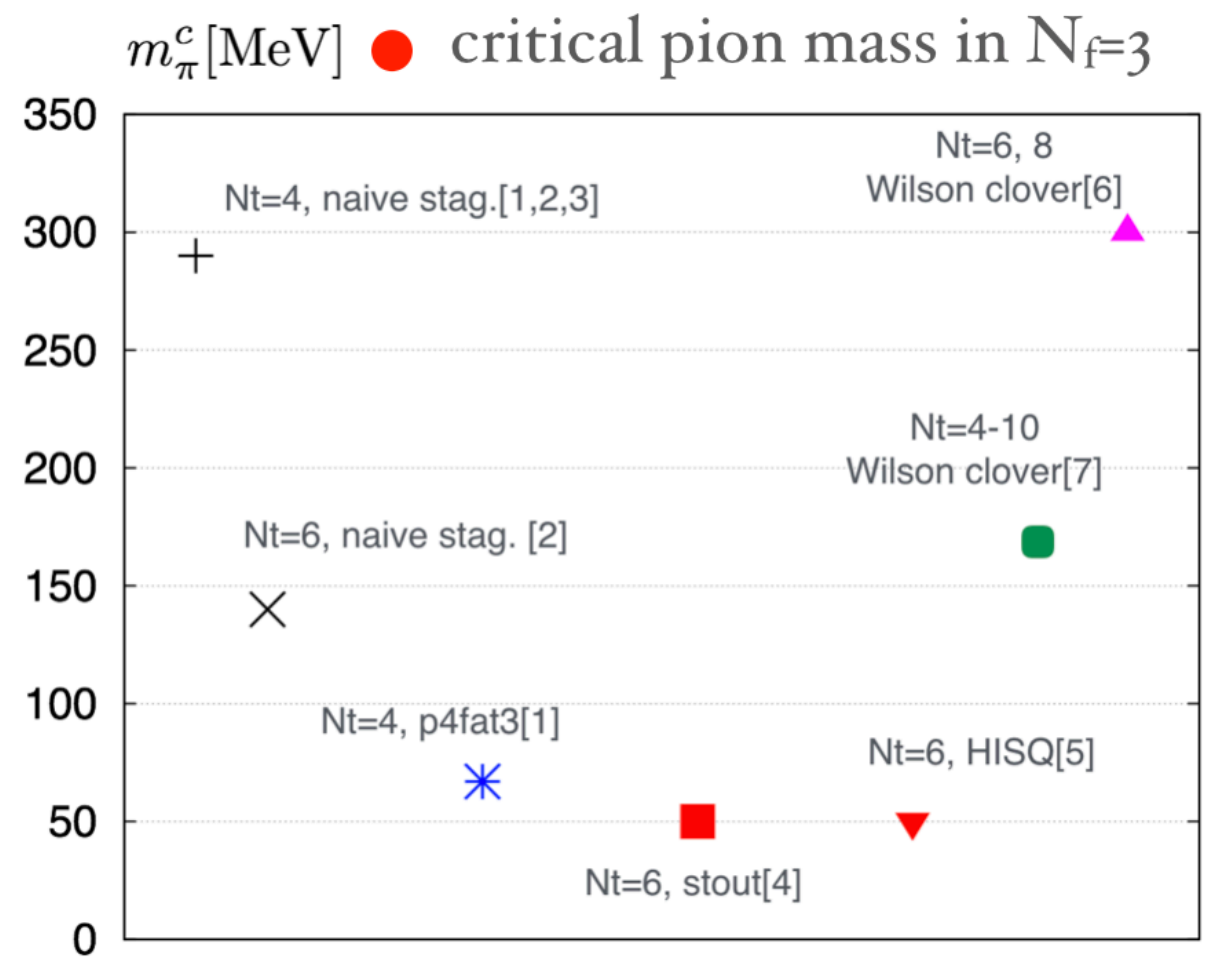
HTD, Quark Matter 2019 Plenary

Not relevant: 1st order chiral phase transition region
as it becomes small and is away from the physical point

QCD criticality relevant to the real world

Columbia plot:
QCD phase diagram in quark mass plane





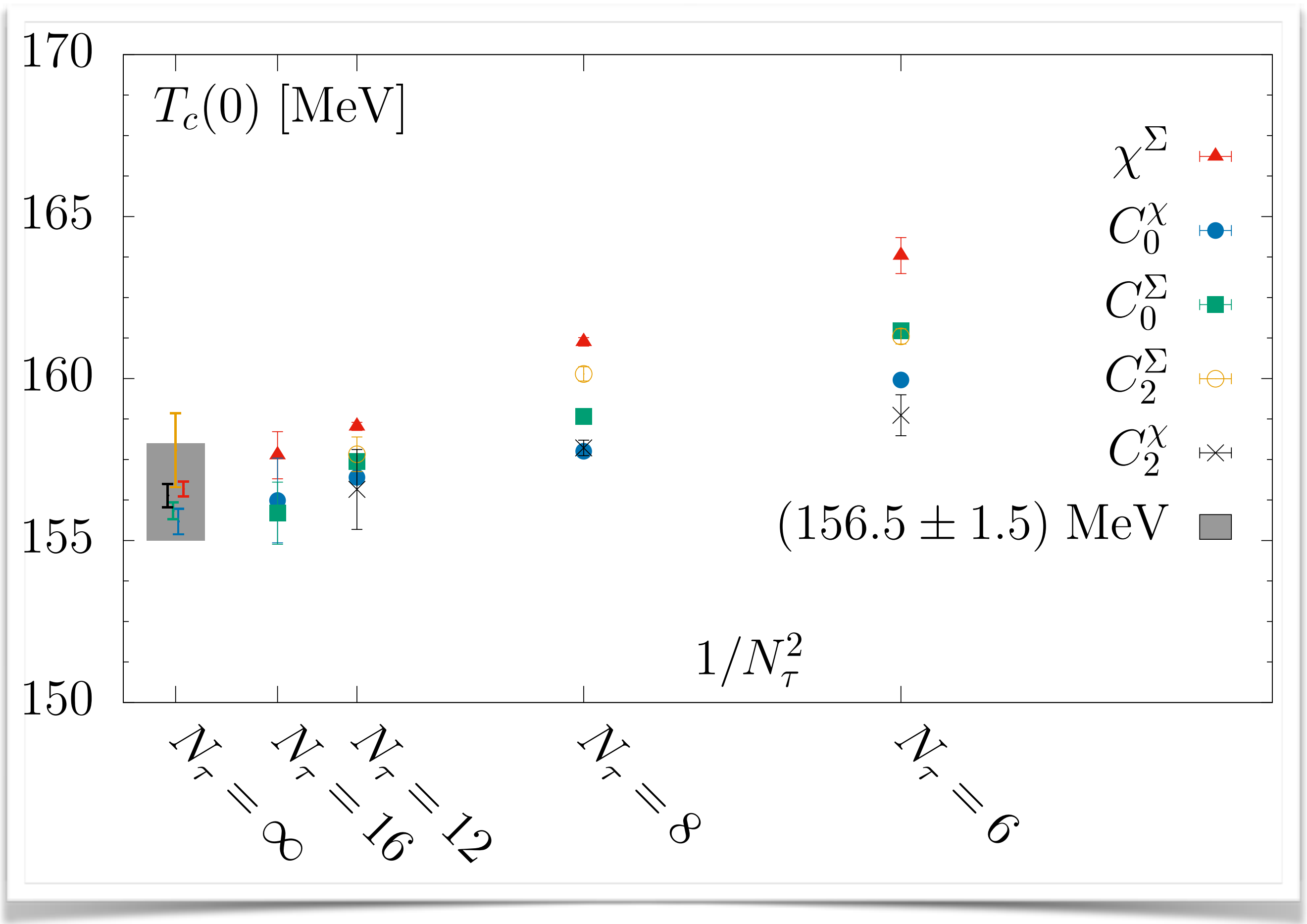
HTD, Quark Matter 2019 Plenary

Not relevant: 1st order chiral phase transition region
as it becomes small and is away from the physical point

Relevant: 2nd order O(4) chiral phase transition
as $U_A(1)$ symmetry is not effectively restored at the critical temperature

QCD chiral crossover temperature with m_π =140 MeV

New from



A. Bazavov, HTD, P. Hegde et al. [HotQCD], Phys. Lett. B795 (2019) 15

Rigorous definition based on the O(4) critical behavior of QCD in the chiral limit

$$\partial_T \chi^\Sigma(T),\quad \partial_T C_0^\chi(T),\quad C_2^\chi(T) \quad \sim \quad m^{1/\delta-1-1/\beta\delta} f'_\chi(z)$$

$$\partial_T^2 C_0^\Sigma(T),\quad \partial_T C_2^\Sigma(T) \quad \sim \quad m^{1/\delta-2/\beta\delta} f''_G(z)$$

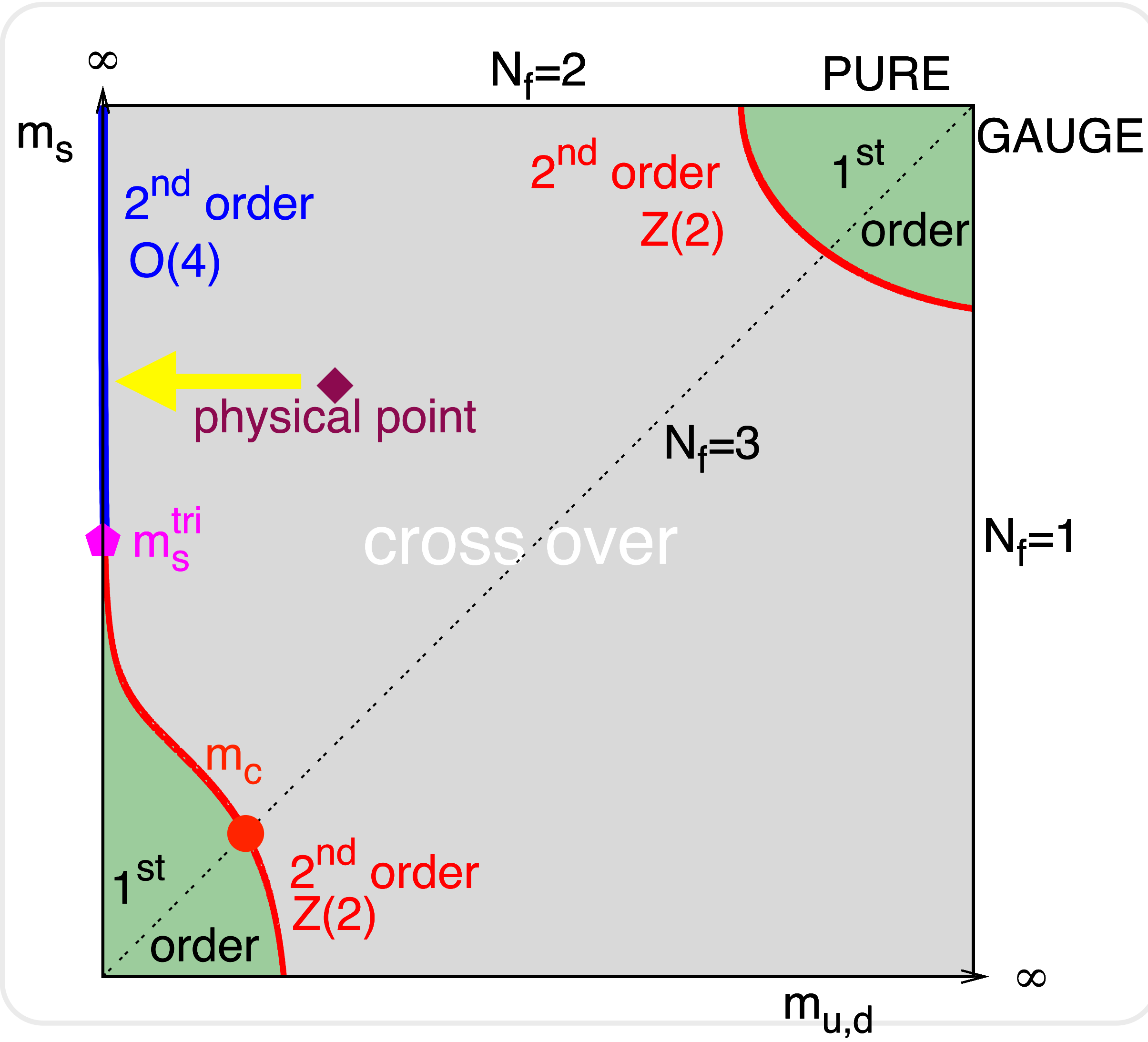
Chiral crossover temperature in the continuum limit:

T_{pc} =156.5(1.5)MeV

Chiral phase transition temperature

New from NSC³

Possible upper bound the transition T at CEP



HTD, P. Hegde, O. Kaczmarek et al.[HotQCD], Phys.Rev. Lett. 123 (2019) 062002

- HISQ/tree action
- **N_f=2+1:**
  - ☑ Nt=6,8,12
  - ☑ $m_s^{phy}/\mathbf{m_l}$ = 20, 27, 40, 60, 80

    $m_\pi \approx$160,140,110,90,80,55 MeV
  - ☑ $7 \geq N_s/N_t \geq 4 \;\Leftrightarrow\; 5 \gtrsim m_\pi L \gtrsim 3$

This allows us to perform
infinite volume, continuum and then chiral extrapolation!

A novel approach to estimate T_c^0

- Pseudo-critical temperature at H

$$T_{pc}(H) = T_c^0 \left(1 + \frac{z_p}{z_0} H^{\frac{1}{\beta\delta}}\right)$$

$$z = \frac{1}{t_0}\frac{T - T_c^0}{T_c^0}\left(\frac{H}{h_0}\right)^{-1/\beta\delta} = z_0 \frac{T - T_c^0}{T_c^0} H^{-1/\beta\delta}$$

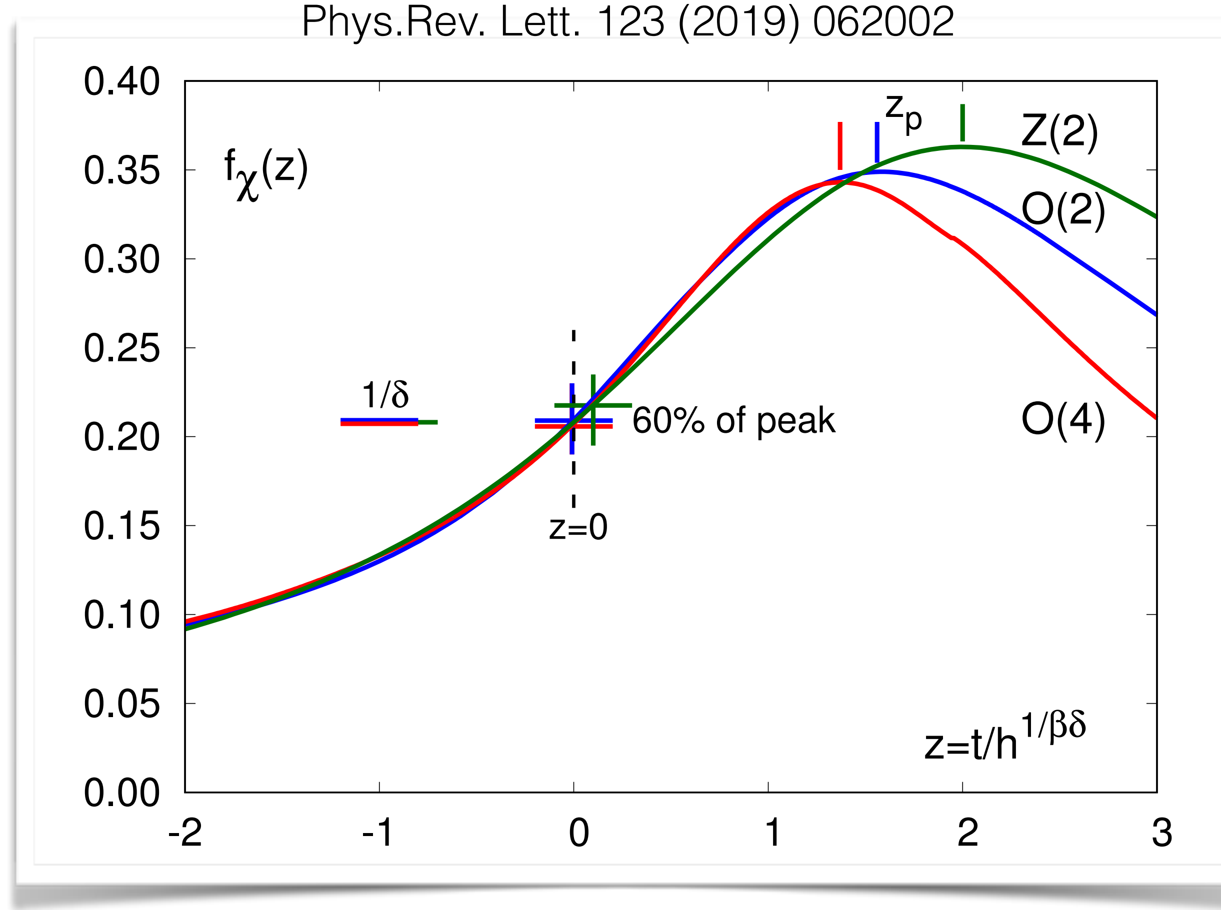
- Estimate of the chiral transition T_c^0

$$\frac{H\chi_M(T_\delta, H, L)}{M(T_\delta, H, L)} = \frac{1}{\delta} \Longleftrightarrow z(T_\delta)=0$$

$$\chi_M(T_{60}, H) = 0.6\chi_M^{max} \Longleftrightarrow z(T_{60})\simeq 0$$

- ☑ small quark mass dependence
- ☑ small variations among universality classes

HTD, P. Hegde, O. Kaczmarek et al.[HotQCD], Phys.Rev. Lett. 123 (2019) 062002

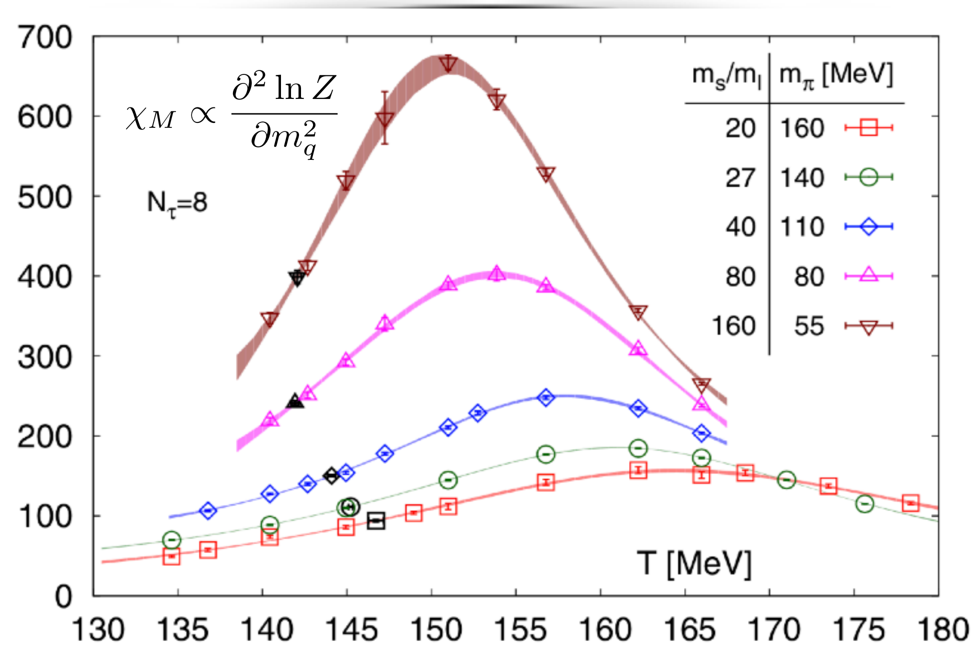


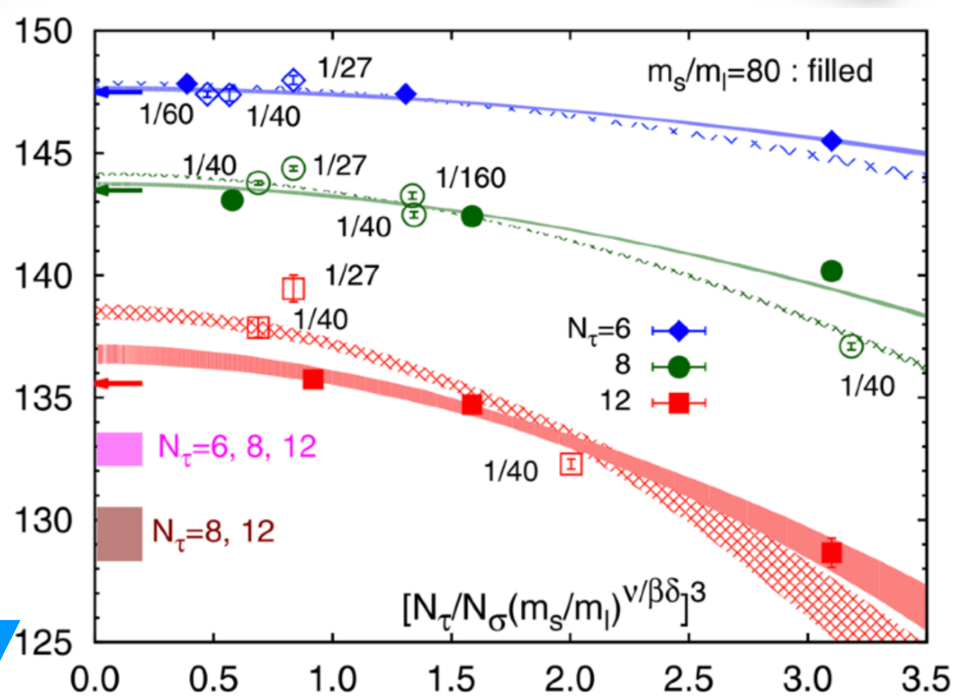
z_p: peak location of the susceptibility

z_{60}: location of 60% of peak height from left

	δ	z_p	z_{60}^-
Z(2)	4.805	2.00(5)	0.10(1)
O(2)	4.780	1.58(4)	-0.005(9)
O(4)	4.824	1.37(3)	-0.013(7)

Order parameter susceptibility

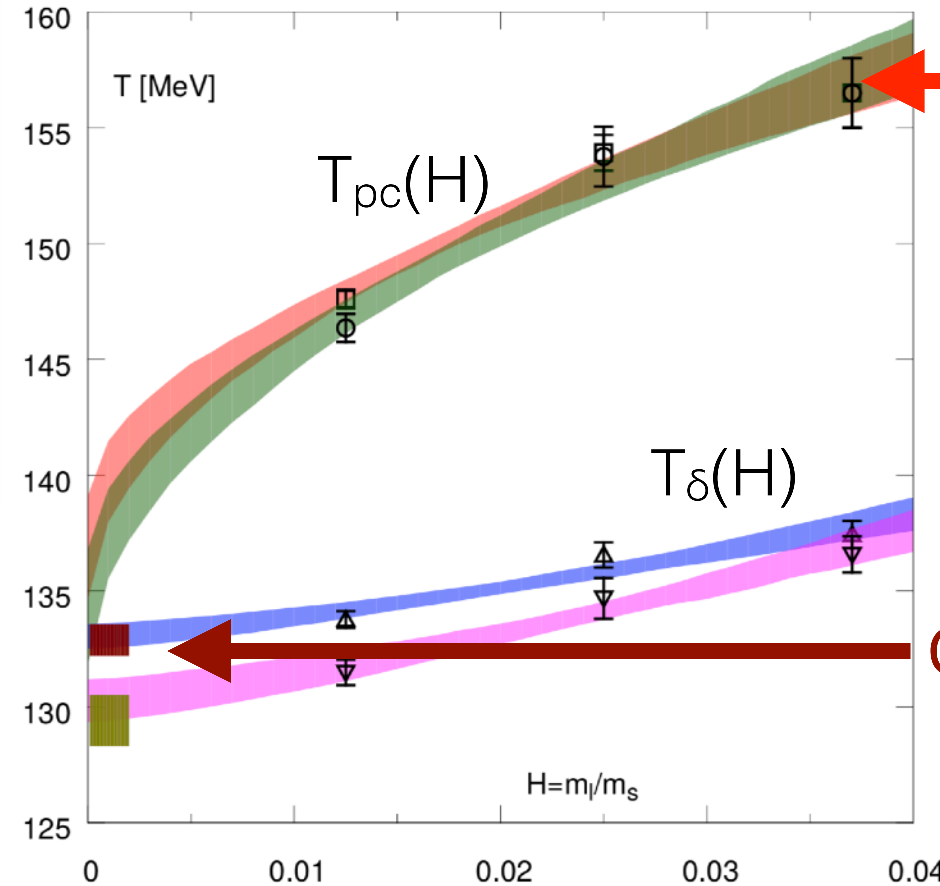




Continuum extrapolation

Infinite volume & chiral extrapolation

O(4) scaling analyses: Nf=2+1, Nt=6,8 & 12 lattices, M_π down to 55 MeV thermodynamic, continuum & chiral extrapolated



Chiral crossover
T_{pc}^{phy}= 156.5(1.5) MeV

$$T_{pc}(H) = T_c^0 \left(1 + \frac{z_p}{z_0} H^{\frac{1}{\beta\delta}}\right)$$

Chiral phase transition T

$$T_c^0 = 132^{+3}_{-6} \text{ MeV}$$

HTD, P. Hegde, O. Kaczmarek et al.[HotQCD], Phys.Rev. Lett. 123 (2019) 062002

T_c^0 is ~ 25 MeV smaller than the chiral crossover T!

See QCD-inspired model calculations:
e.g. J. Berges, D. U. Jungnickel and C. Wetterich, Phys. Rev.D59, 034010 (1999)
J. Braun, B. Klein, H.-J. Pirner and A. H. Rezaeian,Phys. Rev. D73, 074010 (2006)

Explore the QCD phase diagram through fluctuations of conserved charges x=B,Q,S

HIC

mean: M_x

variance: σ_x^2

skewness: S_x

kurtosis: κ_x

hyper-skewness: S_x^h

hyper-kurtosis: κ_x^h

$$\frac{M_x(\sqrt{s})}{\sigma_x^2(\sqrt{s})} = \frac{\langle N_x \rangle}{\langle (\delta N_x)^2 \rangle} = \frac{\chi_1^x(T,\mu_B)}{\chi_2^x(T,\mu_B)} = R_{12}^x(T,\mu_B)$$

$$\frac{S_x(\sqrt{s})\,\sigma_x^3(\sqrt{s})}{M_x(\sqrt{s})} = \frac{\langle (\delta N_x)^3 \rangle}{\langle N_x \rangle} = \frac{\chi_3^x(T,\mu_B)}{\chi_1^x(T,\mu_B)} = R_{31}^x(T,\mu_B)$$

$$\kappa_x(\sqrt{s})\,\sigma_x^2(\sqrt{s}) = \frac{\langle (\delta N_x)^4 \rangle}{\langle (\delta N_x)^2 \rangle} = \frac{\chi_4^x(T,\mu_B)}{\chi_2^x(T,\mu_B)} = R_{42}^x(T,\mu_B)$$

$$\frac{S_x^h(\sqrt{s})\,\sigma_x^5(\sqrt{s})}{M_x(\sqrt{s})} = \frac{\langle (\delta N_x)^5 \rangle}{\langle N_x \rangle} = \frac{\chi_5^x(T,\mu_B)}{\chi_1^x(T,\mu_B)} = R_{51}^x(T,\mu_B)$$

$$\kappa_x^h(\sqrt{s})\,\sigma_x^4(\sqrt{s}) = \frac{\langle (\delta N_x)^6 \rangle}{\langle (\delta N_x)^2 \rangle} = \frac{\chi_6^x(T,\mu_B)}{\chi_2^x(T,\mu_B)} = R_{62}^x(T,\mu_B)$$

LQCD

generalized susceptibilities

$$\chi_n^x(T,\mu_B) = \frac{1}{VT^3}\frac{\partial^n \ln Z(T,\vec{\mu})}{\partial(\mu_x/T)^n}$$

This makes the comparison between Exp. and LQCD possible

Change in Degree of freedom

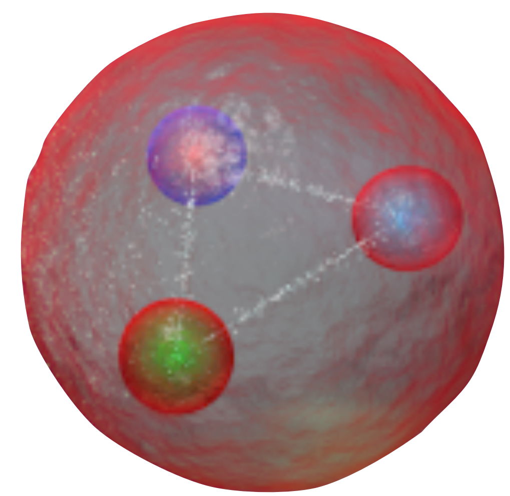
Baryon number —> B

Baryon number fluctuation —> B^2

Bound state of quarks:

B=1, -1

$B^2=1$

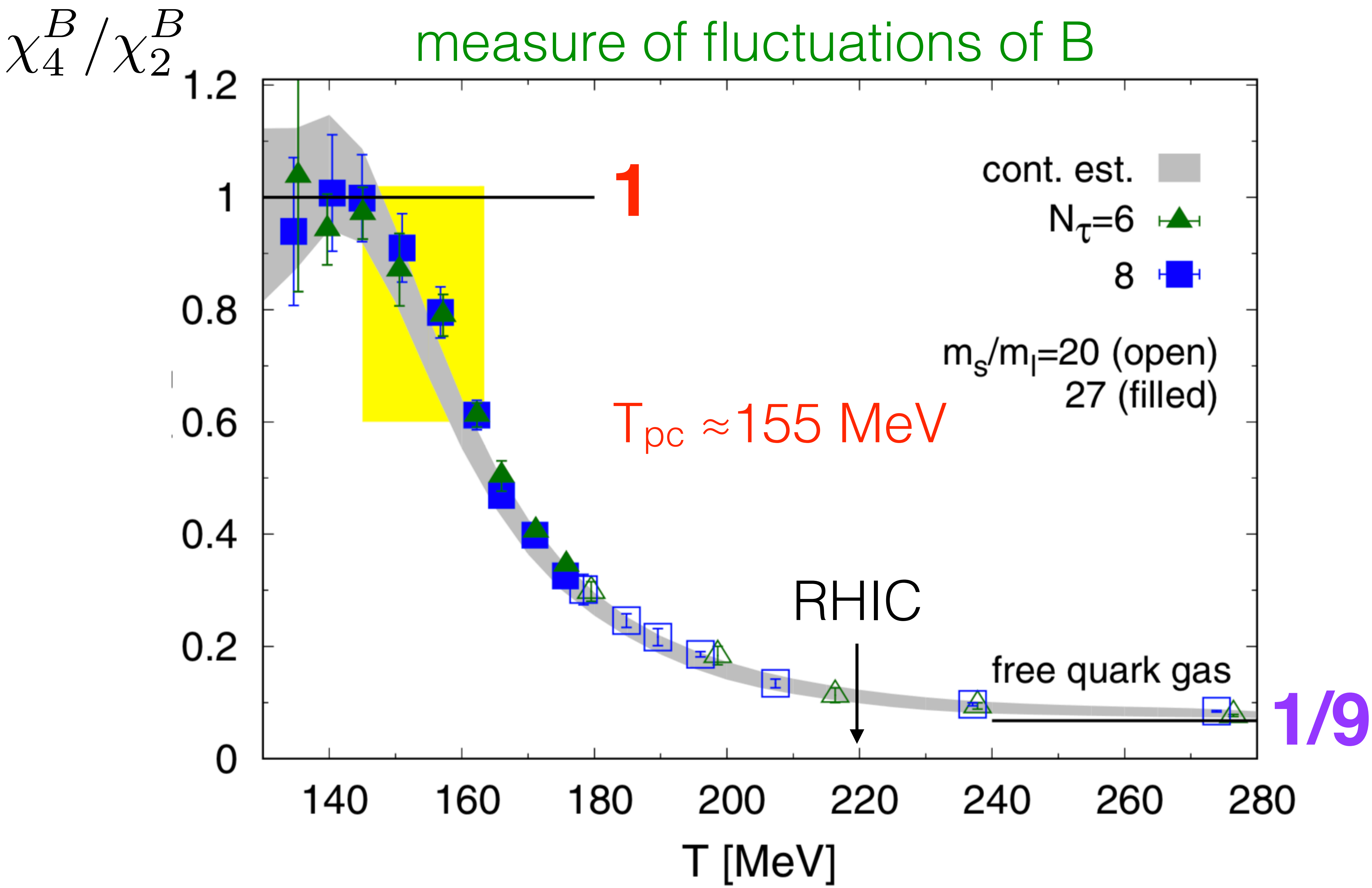


Unbounded quarks:

B=1/3, -1/3

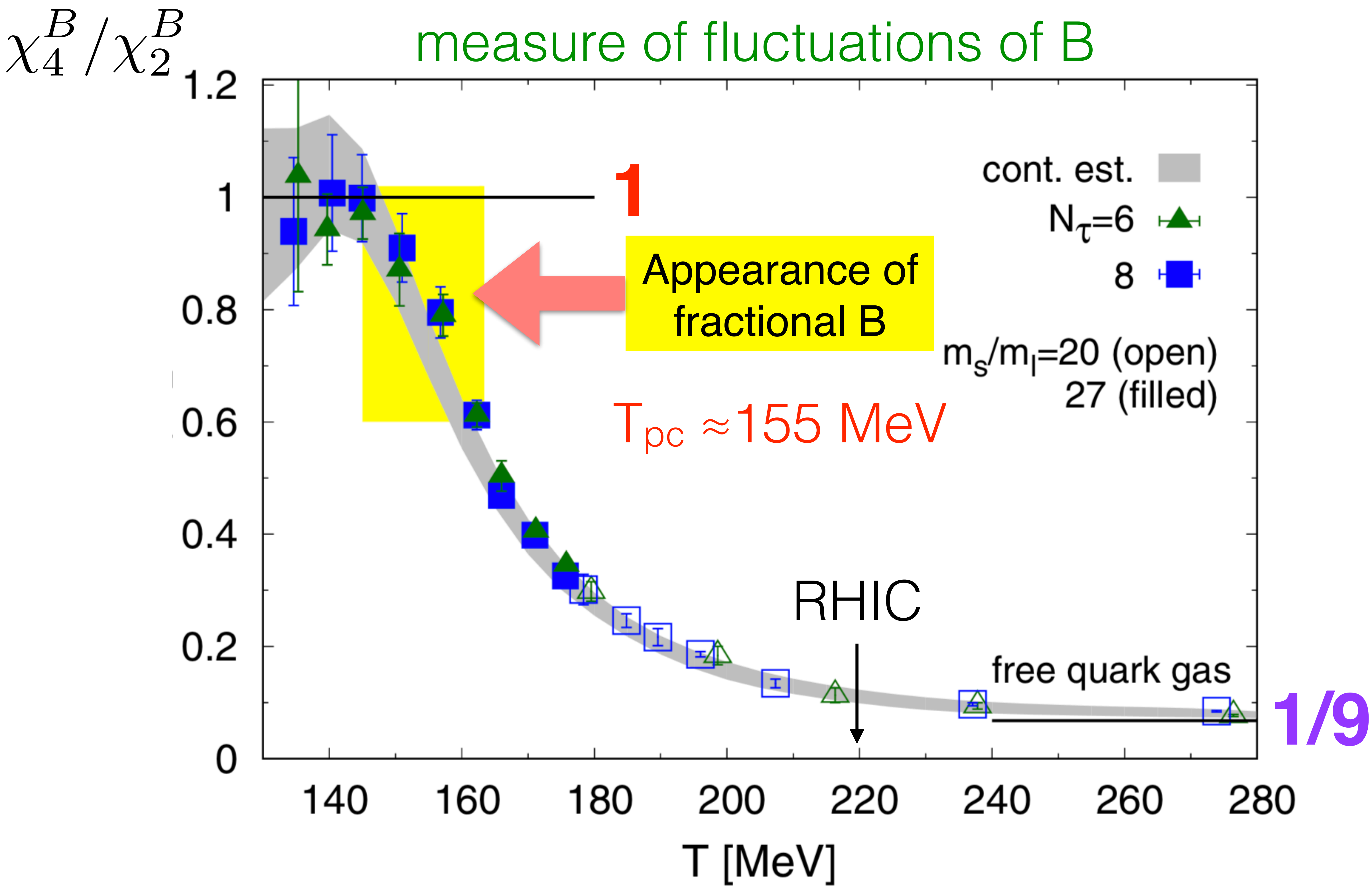
$B^2=1/9$

Change in Degree of freedom



Bielefeld-BNL-CCNU: PRL 111(2013) 082301, PLB 737(2014) 210

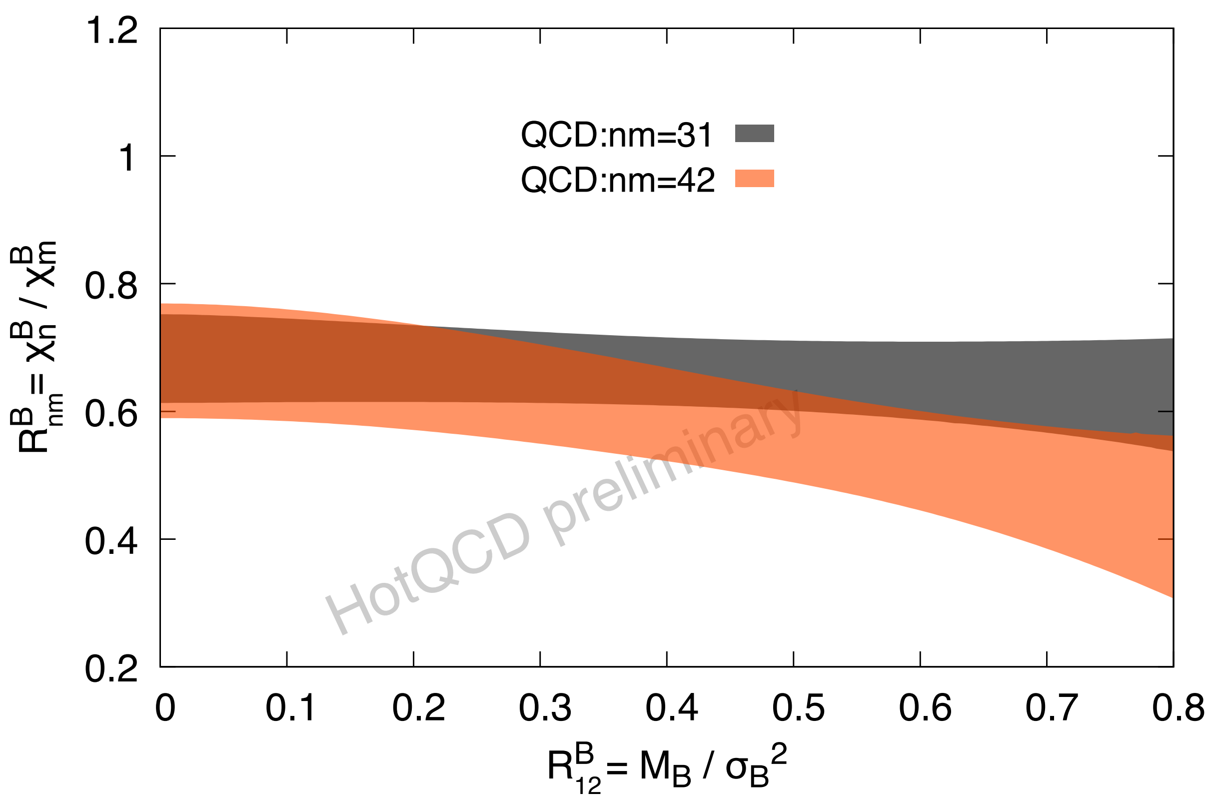
Change in Degree of freedom

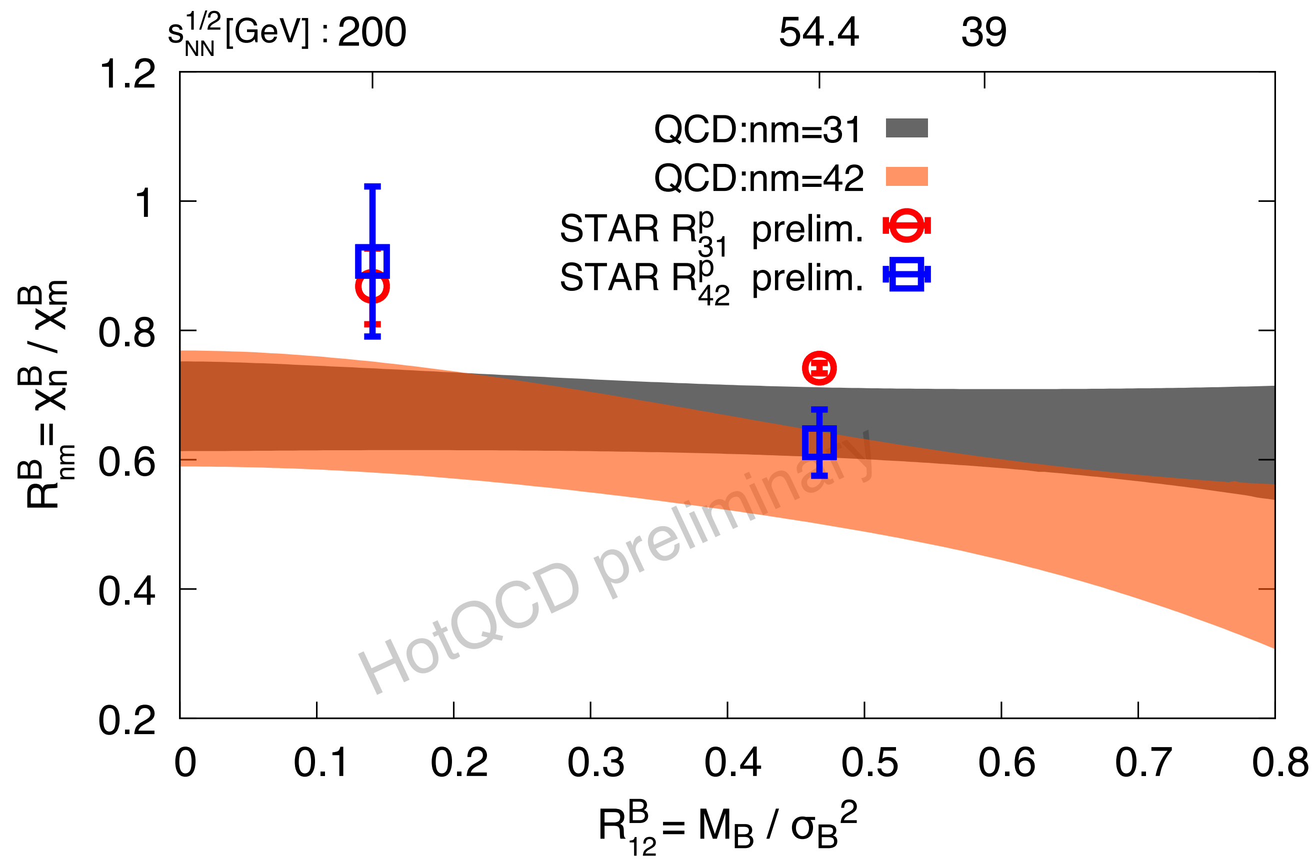


Bielefeld-BNL-CCNU: PRL 111(2013) 082301, PLB 737(2014) 210

QCD v.s. Experimental data: skewness (R_{31}) & kurtosis (R_{42}) ratios

HTD, Quark Matter 2019 Plenary





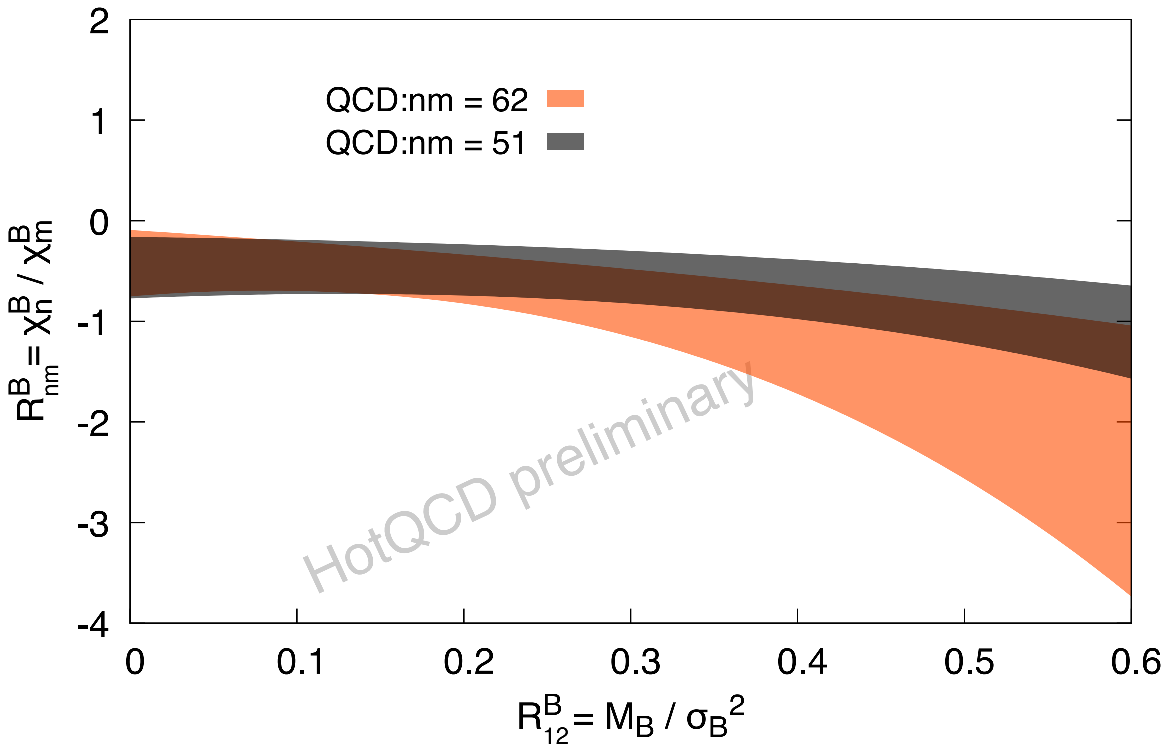
R_{31} is almost flat while R_{42} decreases faster with R_{12}

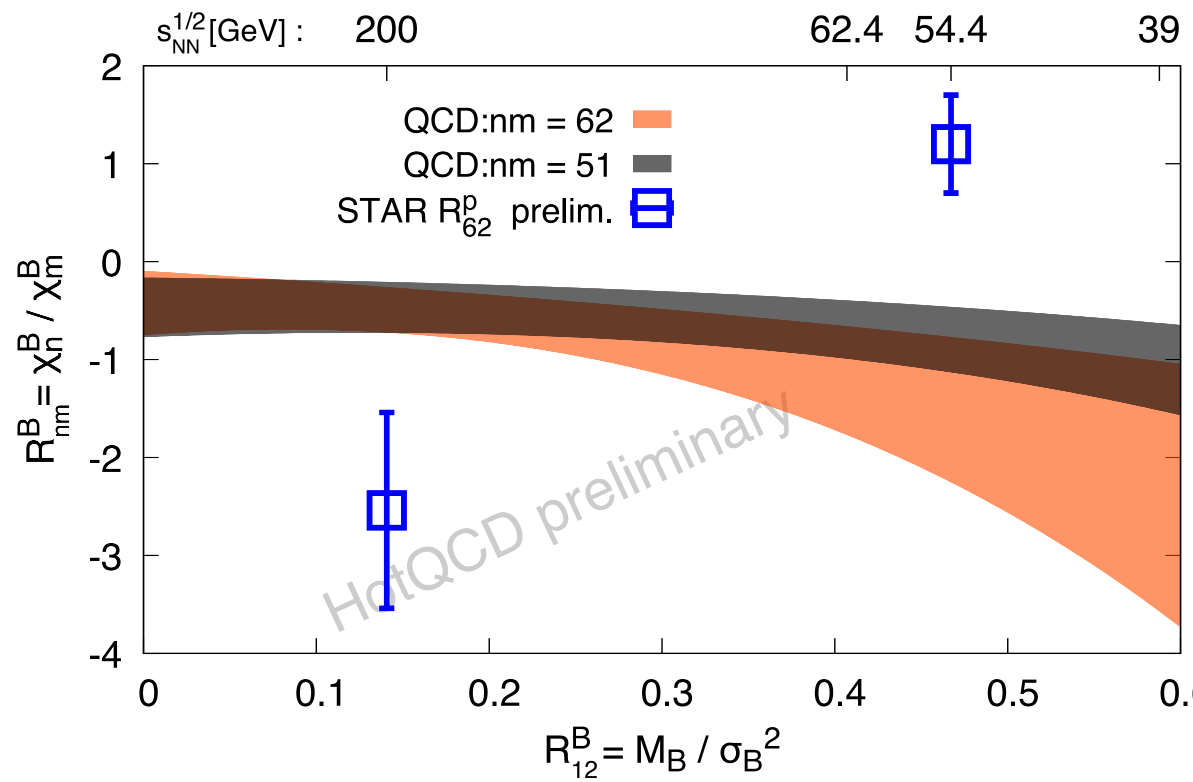
Nice consistency between QCD and Exp. data at $\sqrt{s_{NN}}$=54.4 GeV

QCD v.s. Experimental data: hyper-skewness (R_{51}) & hyper-kurtosis (R_{62}) ratios

HTD, Quark Matter 2019 Plenary

$$R^B_{62}(\mu_B/T) = \frac{\chi^B_6(\mu_B/T)}{\chi^B_2(\mu_B/T)} = \frac{\chi^B_6(0)}{\chi^B_2(0)} + \frac{1}{2}\left(\frac{\mu_B}{T}\right)^2 \left(\frac{\chi^B_8(0)}{\chi^B_2(0)} - \frac{\chi^B_6(0)}{\chi^B_2(0)}\frac{\chi^B_4(0)}{\chi^B_2(0)}\right) + \mathcal{O}\left(\frac{\mu_B}{T}\right)^4$$





R_{51} and R_{62}: Statistics-hungry quantities

LQCD: Both hyper-skewness and hyper-kurtosis are negative down to $\sqrt{s_{NN}}$=39 GeV

Nice consistency seen in skewness & kurtosis at $\sqrt{s_{NN}}$=54.4 GeV with QCD while deviations seen at both $\sqrt{s_{NN}}$=54.4 & 200 GeV

Summary

- Chiral crossover transition temperature is determined:

$$T_{pc} = 156.5(1.5)\ \mathrm{MeV}$$

A. Bazavov, HTD, P. Hegde et al. [HotQCD], Phys. Lett. B795 (2019) 15

- The possible location of critical end point is

HTD, P. Hegde, O. Kaczmarek et al.[HotQCD], Phys.Rev. Lett. 123 (2019) 062002

$$T_c^{CEP} \lesssim T_c^0 = 132^{+3}_{-6}\ \mathrm{MeV}$$

- hyper-skewness and hyper-kurtosis ratios are obtained in NLO in μ_B

HotQCD, in preparation

$\sqrt{s_{NN}}$=200 GeV: $R_{51}^B = -0.5(3)$, $R_{62}^B = -0.7(3)$

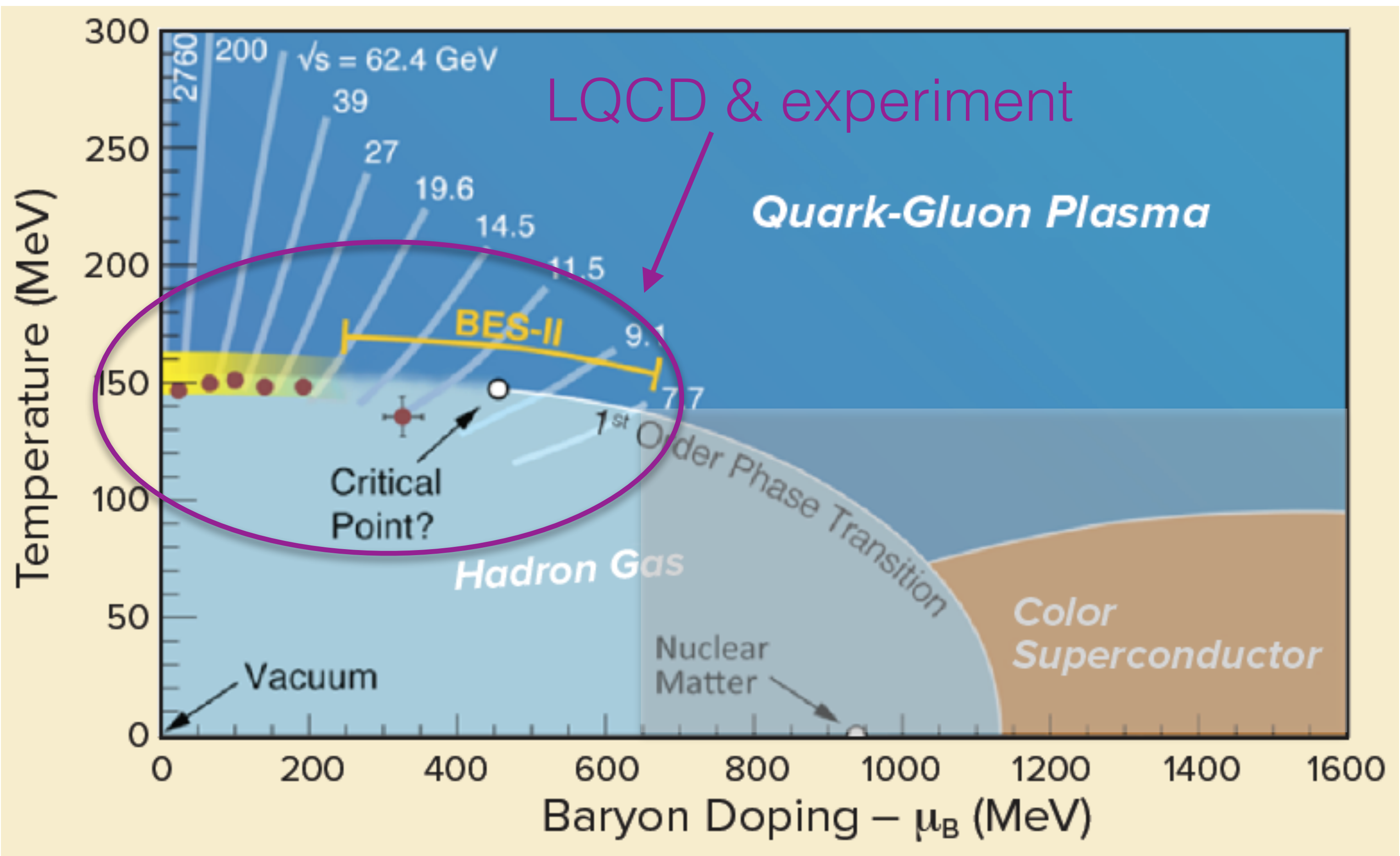
$\sqrt{s_{NN}}$=54.4 GeV: $R_{51}^B = -0.7(4)$, $R_{62}^B = -2(1)$

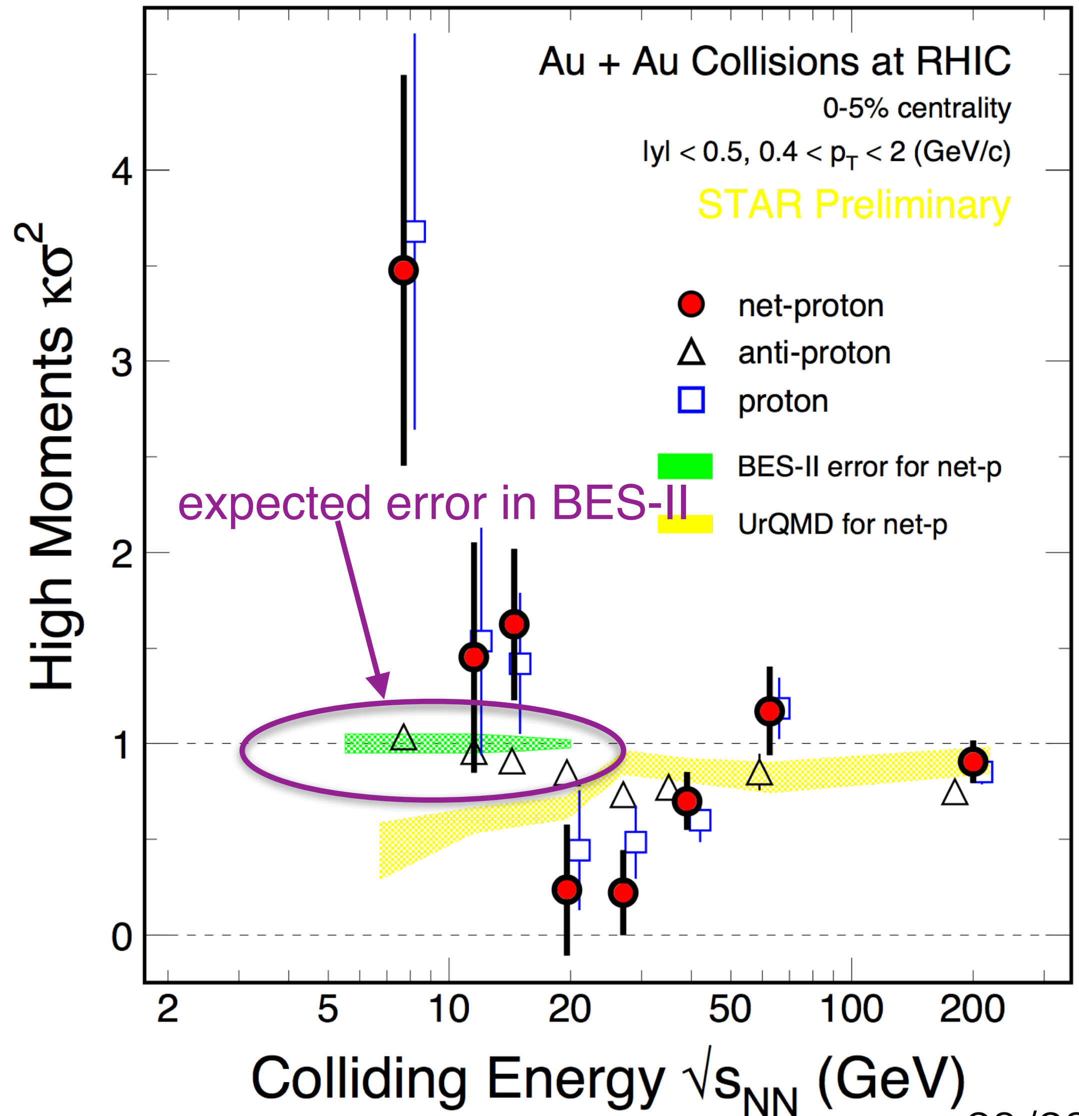
展望：寻找QCD临界点

RHIC Beam Energy Scan，Phase II (BES-II)

2019-2020: at least 10 times more statistics for each $\sqrt{s_{NN}}$

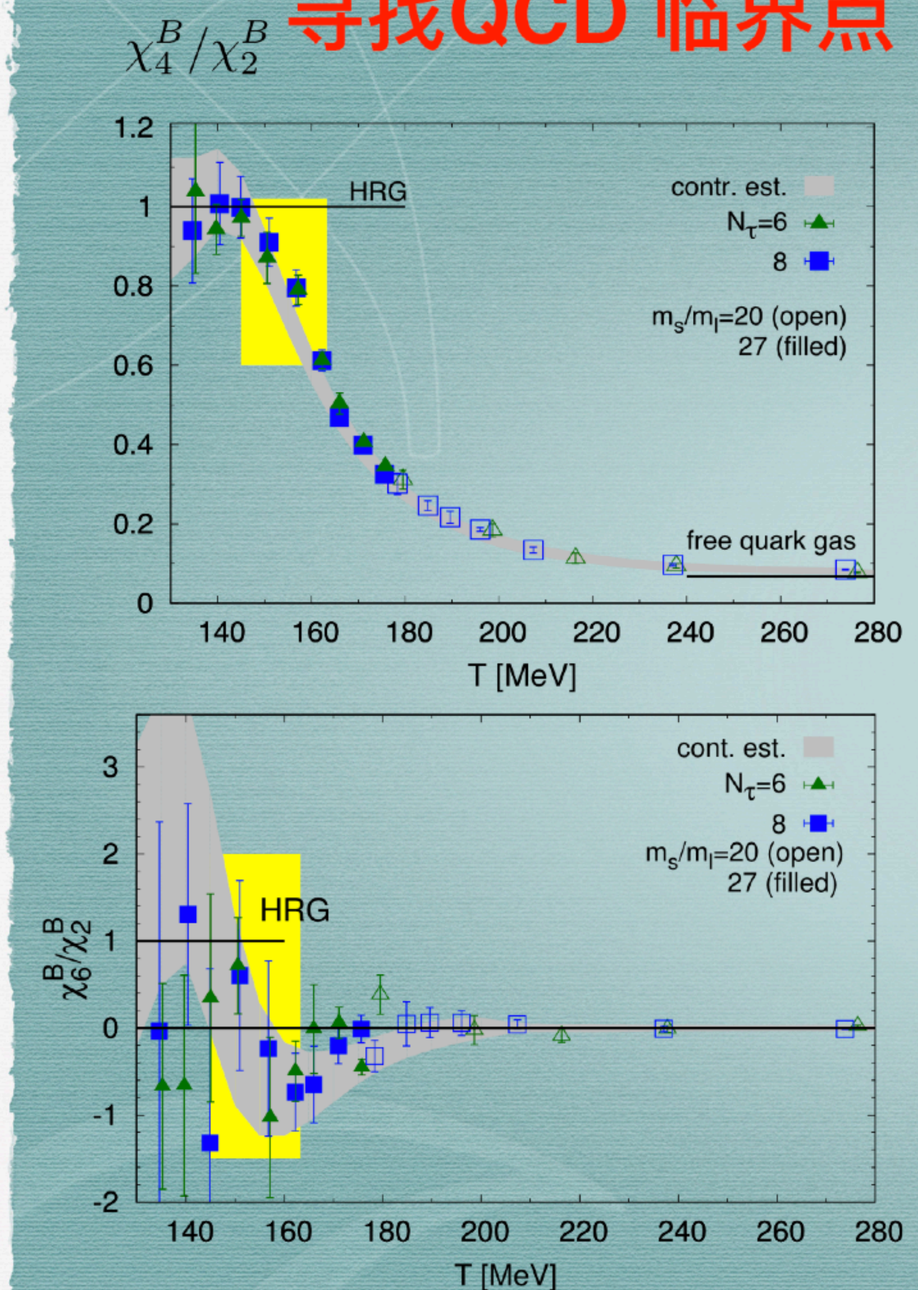
LQCD: higher accuracy for the 6th & 8th or even higher order Taylor expansion coefficients





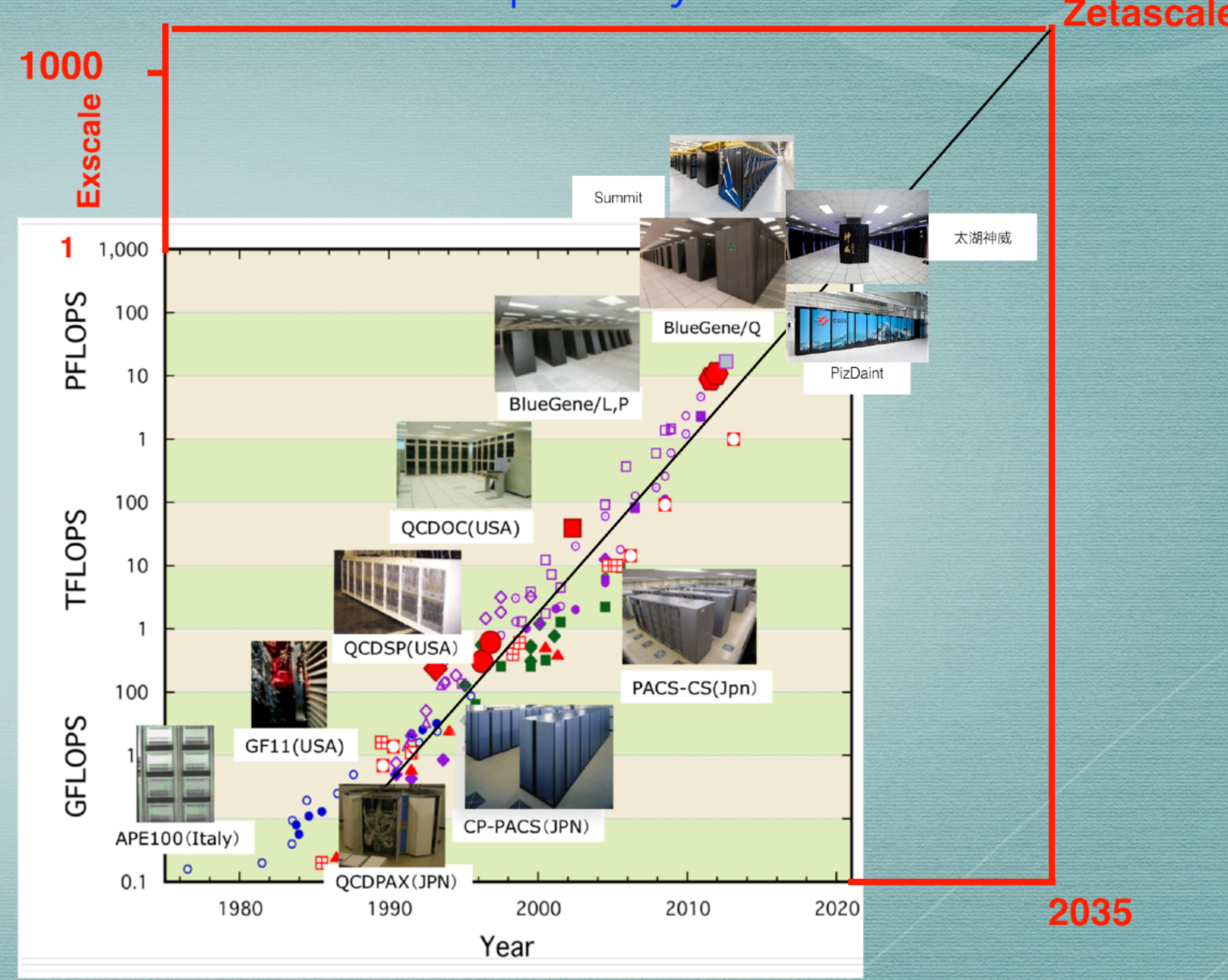
展望： 寻找QCD临界点

寻找QCD 临界点



[Bielefeld-BNL-CCNU],PRD95 (2017)no 5, 054504

LQCD: Upto 10th order coefficients known soon!
Next step in 5 years



based on A. Ukawa, 2013 HPC summer school

祝赵老师

寿比南山，福如东海！