Application of PMC to heavy quarkonium processes

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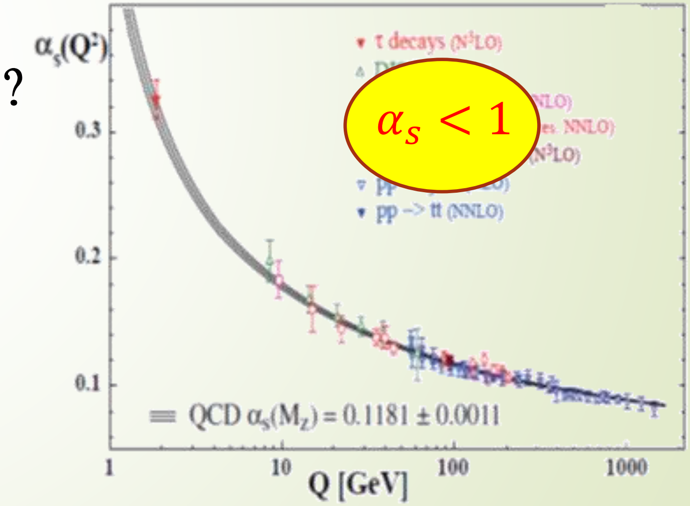
OUTLINE

- The principle of Maximum Conformality (PMC)
- Heavy quarkonium processes using the PMC
- Summary and Outlook

QCD asymptonic freedom --- 高能区域 微扰可算（第一步）
重夸克偶素过程，重夸克典型标度

Renormalization --- 消除发散，保证高阶计算的结果可靠（第二步）

如何得到准确（第三步）的固定阶微扰论预言？
1）完成更高阶计算 – Convergence
2）将已知类型大对数项求和 – Resummation
。。。。。。



$\alpha_s < 1$

若出现如下情况
不收敛（K因子大，粲夸克偶素过程大多如此）、
只能完成单圈或两圈等低阶（高阶复杂）、

α_s的准确值至关重要

The perturbative theory: A physical observable ρ could be written as the perturbative form

$$\rho = r_0 \alpha_s^{p}(\mu_R) + r_1 \alpha_s^{p+1}(\mu_R) + r_2 \alpha_s^{p+2}(\mu_R) + \ldots$$

重整化能标和重整化方案

Up to infinite order, there is no scheme- and scale-dependence, i.e., any choice of scheme/scale should result in the same prediction.

"Standard Renormalization group invariance (RGI)"

Conventional scale-setting approach

=> **"Choose"** the scale Q to be ``seemingly" **typical momentum transfer**

（消大对数项）

=> **"Vary"** in a certain range, e.g. [Q/2, 2Q], [Q/3, 3Q], to discuss its uncertainty

Main problems of conventional scale-setting

Diluted by renormalon divergence

1) Convergence **depends on α_s-power suppression**;
Once inconvergence appears, one cannot judge whether it is its intrinsic property or caused by improper choice of scale。

2) By finishing more loop-terms, the scale-dependence could be smaller, it is however caused by **large cancellation among different orders**。

Even if the prediction of guessing scale agrees with the data.

It, in fact, cannot answer why this is the case

寻找对方案和能标变化的稳定点

Optimized perturbation theory –
minimize the higher-order contributions – **PMS**

How about directly set it to satisfy the RG invariance

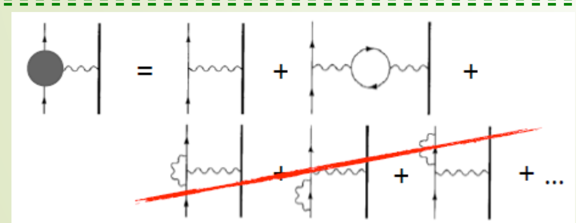
任意物理量可定义有效荷=>有效能标

Any observable <=> an effective coupling constant (idea useful)
Fastest Apparent Convergence (FAC)

How about directly cut off all higher-order-terms ?

早期解决方案=>寻找最优能标

典型
四类



BLM=> nf-term
QED极限=GM-L方案
CSR=>解决方案不确定性

消除所有真空极化图

Using RGE
Runs from μ_1 -> μ_2
e.g. using $\alpha_s(\mu_1)=\alpha_s(\mu_2)+\ldots$

降低不确定性，中心值为猜
估算部分未知高阶（有误导作用）

The reason for scale dependence at fixed-order ?

Mismatching under conventional treatment

上述几种方案均为被动寻找最优能标

目标变成

=> 或与实验一致
=> 或提高收敛性
=> 或降低能标不确定性
=> 或提高相对于能标变化的稳定性

一不小心就会被带偏方向

有没有主动方案，自动获得最优能标所具有的优点 ？

新一轮尝试：拟同时解决重整化能标及重整化方案不确定性

“最大共形原理（PMC）”

可以选择任意初始重整化能标、重整化方案完成微扰论计算；但经过PMC能标设定步骤之后，最终获得的微扰表达式与重整化能标和重整化方案的选择无关

特点：

I）可确定“正确”的动量流动值－－与重整化能标选择无关；不同方案下得到的动量流动值不一样，与共形系数相匹配＝> **Commensurate Scale Relation** －－ 保证总预言与重整化方案无关

II）可自然改善微扰收敛性（消除**Renormalon)**

III）可更好用于估算未知高阶完整贡献

核心是确定强耦合常数值，RGE至关重要

$$\beta^{\mathcal{R}} = \mu_r^2 \frac{\partial}{\partial \mu_r^2}\left(\frac{\alpha_s^{\mathcal{R}}(\mu_r)}{4\pi}\right) = -\sum_{i=0}^{\infty} \beta_i^{\mathcal{R}} \left(\frac{\alpha_s^{\mathcal{R}}(\mu_r)}{4\pi}\right)^{i+2}$$

确定并利用所有已知**RGE**－β项准确定出耦合常数值－－类似于重求和－－但与大**beta0**近似的处理根本不同

（目的：重求和后得到收敛结果）

只考虑气泡图；或是利用 $\beta_i \approx \beta_0^{i+1}$

$$\rho = \sum_{i=1}^{m} r_{i,0}^{M} a_s^{n+i-1}(Q_{M,i}) + \ldots,$$

$$\rho(Q) = \sum_{n\geq 1} \tilde{r}_{n,0}\,\alpha(Q_\star)^{n+p-1}$$

Residual scale dependence
Relatively large

$$\hat{\beta}(\hat{a}_\mu) = \mu^2 \frac{\partial \hat{a}_\mu}{\partial \mu^2} = -\frac{\beta_0 \hat{a}_\mu^2}{1 - \frac{\beta_1}{\beta_0}\hat{a}_\mu}; \quad \frac{\partial \hat{a}_\mu}{\partial C} = \hat{\beta}(\hat{a}_\mu)$$

残留的方案依赖性 – 重整化群方程本身是方案依赖的

– 引入方案无关的重整化群方程？

– C – 方案耦合常数可实现

与传统能标及方案依赖性本质不同

残留的能标依赖性 – 越是高阶，能标残留依赖越大

– 这是因为PMC能标的微扰特性

– PMC单能标方案可将其压到最低

Review

The QCD renormalization group equation and the elimination of fixed-order scheme-and-scale ambiguities using the principle of maximum conformality

Xing-Gang Wu [a,*], Jian-Ming Shen [a,b], Bo-Lun Du [a], Xu-Dong Huang [a], Sheng-Quan Wang [c], Stanley J. Brodsky [c,*]

Heavy quarkonium processes using the PMC

收敛性不好的情况下，
若重整化能标的依赖性又很大
如何判断微扰论结果的好坏 ？

$e^+e^- \to J/\psi + \eta_c$ Z. Sun, Y. Ma, XGW, SJB

Belle

mass collision energy $\sqrt{s} = 10.58$ GeV [1].

$\sigma[e^+e^- \to J/\psi + \eta_c] \times \mathcal{B}_{\geq 4} = 33^{+7}_{-6} \pm 9$ fb

$\sigma[e^+e^- \to J/\psi + \eta_c] \times \mathcal{B}_{>2} = 25.6 \pm 2.8 \pm 3.4$

BaBar $17.6 \pm 2.8^{+1.5}_{-2.1}$ fb [3]

LO-NRQCD prediction

2.3–5.5 fb

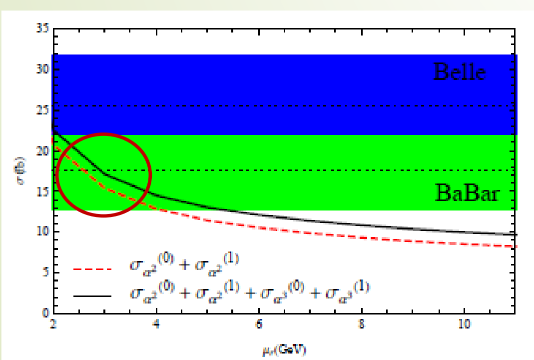
[5] E. Braaten and J. Lee, Phys. Rev. D 67, 054007 (2003).
[6] K. Y. Liu, Z. G. He, and K. T. Chao, Phys. Rev. D 77, 014002 (2008).
[7] K. Y. Liu, Z. G. He, and K. T. Chao, Phys. Lett. B 557, 45 (2003).
[8] K. Hagiwara, E. Kou, and C. F. Qiao, Phys. Lett. B 570, 39 (2003).

NLO-prediction can explain the data

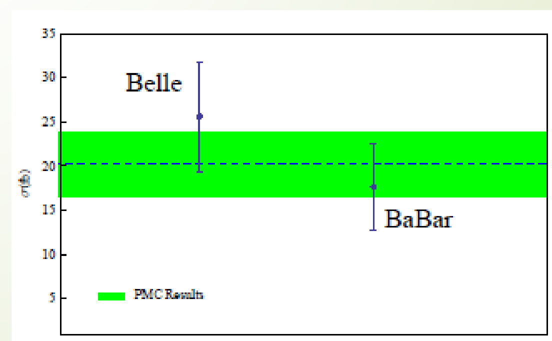
But strongly scale dependent

Guessing: $\mu_r \sim 2-3$ GeV

[14] Y. J. Zhang, Y. j. Gao and K. T. Chao, Phys. Rev. Lett. **96**, 092001 (2006).
[15] B. Gong and J. X. Wang, Phys. Rev. D **77**, 054028 (2008).



Why such scale is reasonable ?



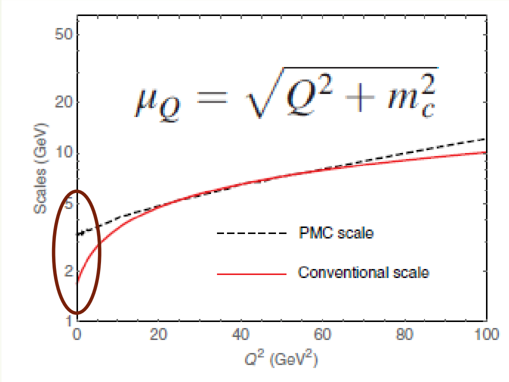
PMC scale

$Q_1 = Q_2 \equiv 2.30$ GeV

$\gamma\gamma^* \to \eta_c$ form factor puzzle

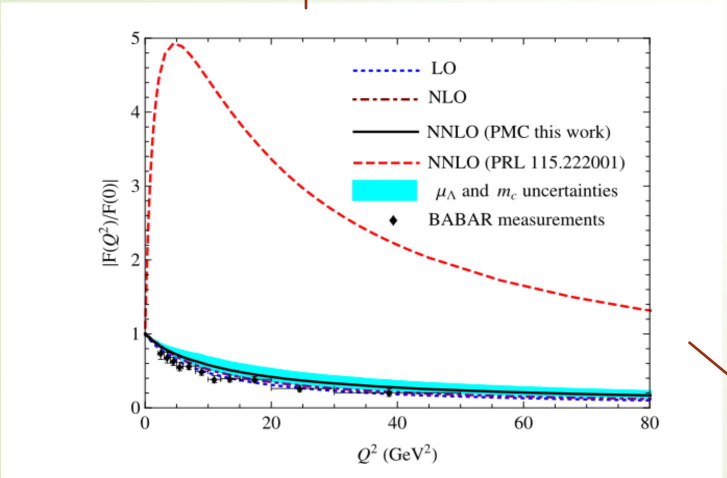
S.Q. Wang, W.L. Sang, XGW, SJB

[4] H. K. Guo, Y. Q. Ma, and K. T. Chao, Phys. Rev. D 83, 114038 (2011).

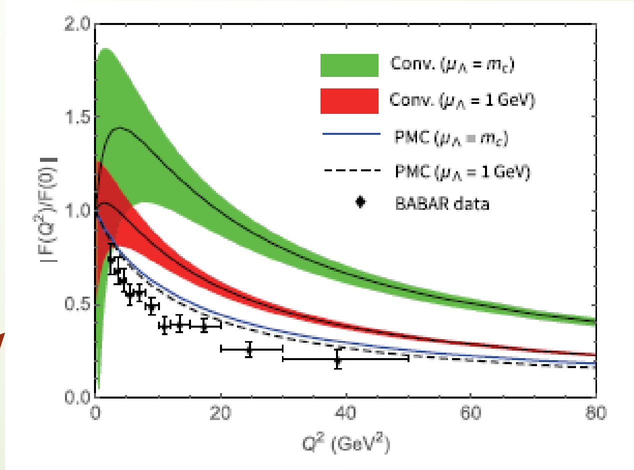


LO-roughly agrees with data
NLO-agrees better with data
NNLO-highly inconsistent with data

Special: |NNLO|>|NLO|



$|F(Q^2)/F(0)|$



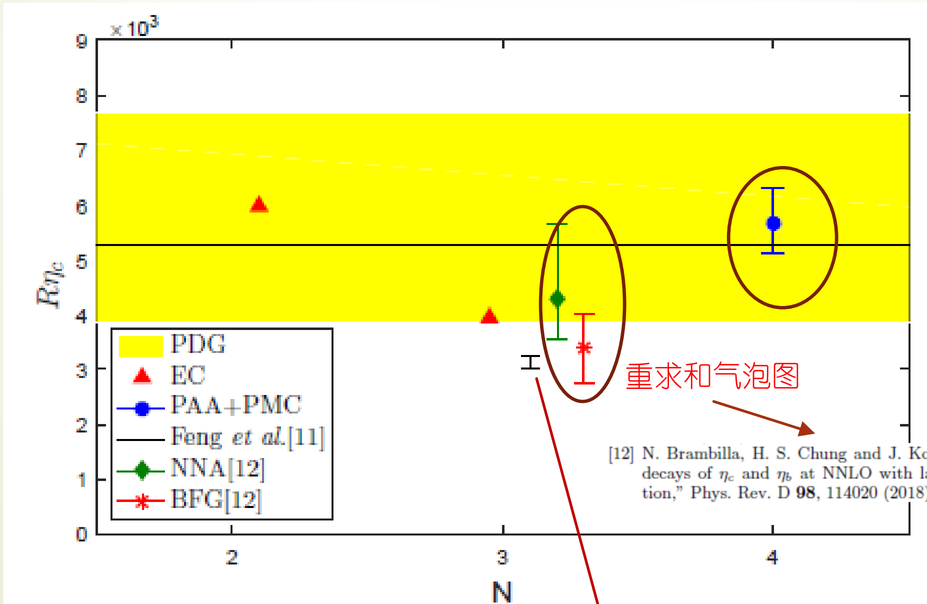
NRQCD wrong ?

PMC works

$$R = \frac{\Gamma_{\eta_Q \to LH}}{\Gamma_{\eta_Q \to \gamma\gamma}},$$

Q. Yu, XGW, J. Zeng, X.D Huang, H.M. Yu

[8] H. K. Guo, Y. Q. Ma and K. T. Chao, "$O(\alpha_s v^2)$ Corrections to Hadronic and Electromagnetic Decays of ${}^1S_0$ Heavy Quarkonium," Phys. Rev. D **83**, 114038 (2011).



$\Delta\alpha_s(M_Z^2) = \pm 0.0011$

[12] N. Brambilla, H. S. Chung and J. Komijani, "Inclusive decays of η_c and η_b at NNLO with large n_f resummation," Phys. Rev. D **98**, 114020 (2018).

Pade近似

$$\rho_n^{[N/M]}(Q) = a^p \times \frac{b_0 + b_1 a + \cdots + b_N a^N}{1 + c_1 a + \cdots + c_M a^M} = \sum_{i=1}^{n} C_i a^{p+i-1} + C_{n+1} a^{p+n} + \ldots,$$

[11] F. Feng, Y. Jia and W. L. Sang, "Next-to-Next-to-Leading-Order QCD Corrections to the Hadronic width of Pseudoscalar Quarkonium," Phys. Rev. Lett. **119**, 252001 (2017).

Summary

Up to infinite order, the predictions are scheme and scale independent, there is no scale ambiguity

At fixed-order, guessing/using typical momentum flow as the scale, one cannot get precise value for all-orders, and also for each order, becoming an important systematic error

希望即使是在有限阶，也可以找到一个普适方案确定出"最优能标"：**得到已知阶下的准确理论预言；同时通过提高收敛性得到更快更接近于真实值的理论预言；为新物理的寻找提供更好的依据**

PMC不是简单的选择“特殊/有效-能标”
而是基于-重整化群方程以及基本重整化群不变性--提供具普适性的可系统设定高能物理过程，获得有效的强耦合常数值（与重整化能标无关）的同时，自然实现“最优重整化能标”的目标

有限阶下，PMC最优能标的思想
并不违背微扰论展开到无穷阶下才能与重整化能标选择无关的基本理念

传统方案
1）收敛慢 – 收敛性纯属猜测和运气
2）计算到任意高阶也无法获得每一阶真实值—很难让人信服基于它获得的未知高阶值
3）足够高阶时才能获得与能标选择无关的预言值

采用PMC之后的微扰序列
1）自然的快速收敛
2）快速稳定，低阶下可相对快速的获得物理量的真实值
3）低阶下可与重整化能标和重整化方案选择无关，获得每一阶的准确值
4）有利于估算未知高阶贡献

注：我们已表明PMC能标与重整能标选择无关，那么真正在做时，可以直接采用通常选择的典型能标做为重整化能标（简化表达式），然后再利用标准PMC步骤处理获得预言

祝赵老师80岁生日快乐