The QCD Calculation for hadronic B decays

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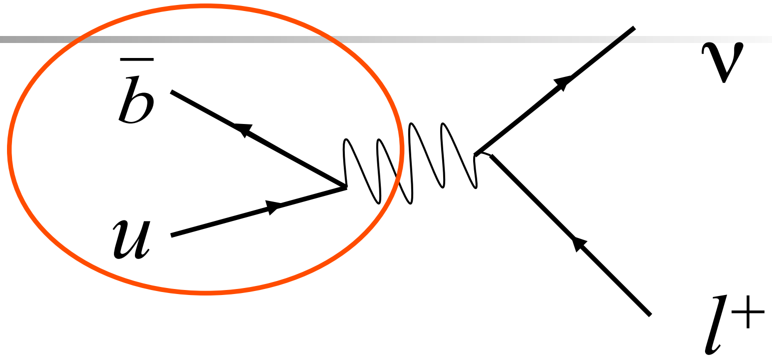
Pure leptonic decays

$$\langle P(p)|\bar{q}\gamma^{\mu}Lq'|0\rangle = if_{P}p^{\mu}.$$



- The decay constant is the normalization of the meson wave function i.e. the zero point of wave function
- The experimental measurement of pure leptonic decay can provide the product of decay constant and CKM matrix element.
- Theoretically decay constant can be calculated by QCD sum rule or Lattice QCD

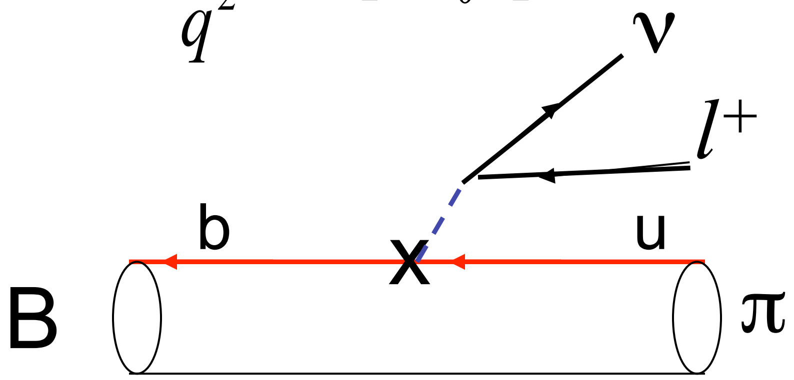
We have two hadrons in semi-leptonic decays. It is described by form factors

$$\langle \pi | \bar{u}\gamma^{\mu} b | B \rangle = p_B{}^{\mu} f_1 + p_{\pi}{}^{\mu} f_2$$

$$q = p_B - p_{\pi}$$

$$= \left[(p_B + p_{\pi})^{\mu} - \frac{m_B{}^2 - m_{\pi}{}^2}{q^2} q^{\mu} \right] F_1(q^2) + \frac{m_B{}^2 - m_{\pi}{}^2}{q^2} q^{\mu} F_0(q^2)$$

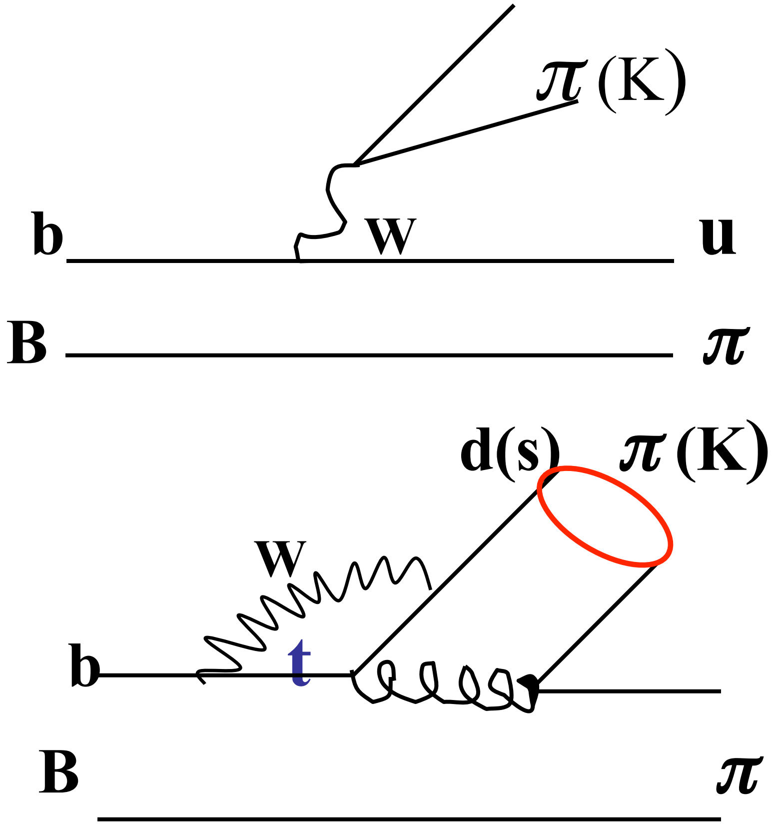
Form factors can be calculated by lattice QCD, QCD sum rules, light cone sum rules etc.



In the quark model, it is calculated by the overlap of two meson wave functions.

Rich physics in hadronic B decays

CP violation, FCNC, sensitive to new physics contribution…

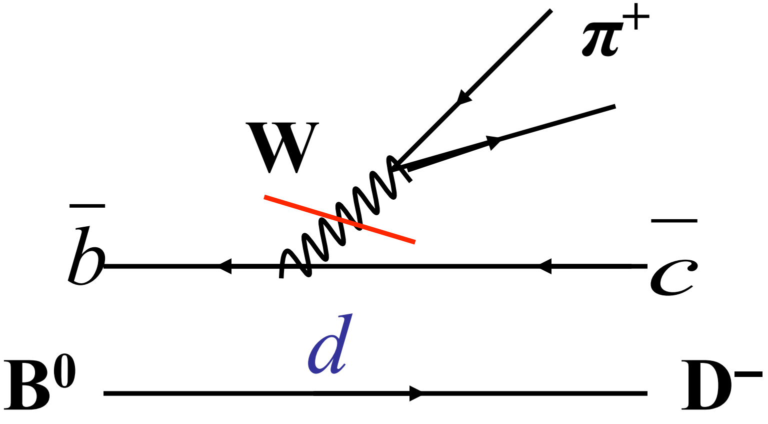


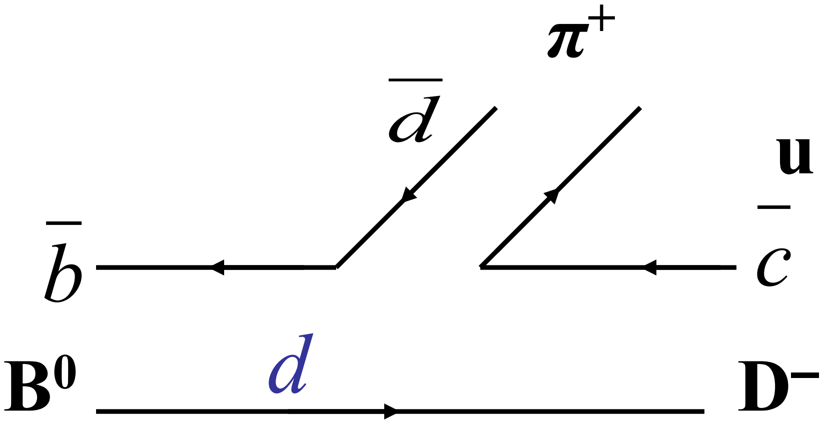
The standard model describes interactions amongst quarks and leptons

In experiments, we can only observe hadrons

How can we test the standard model without solving QCD?

Naïve Factorization (BSW model)





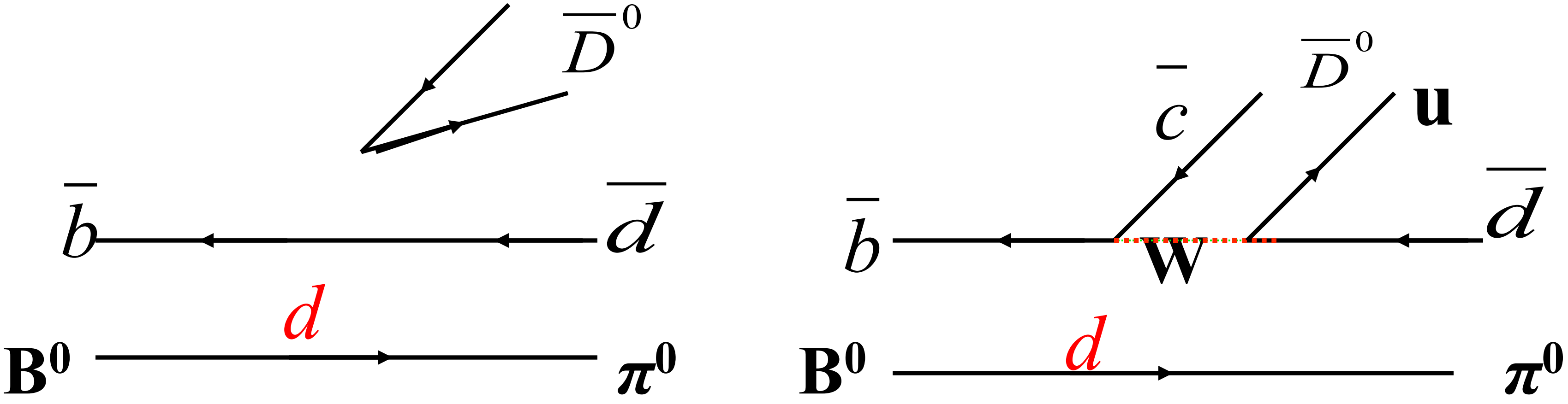
Bauer, Stech, Wirbel, Z. Phys. C29, 637 (1985); ibid 34, 103 (1987)

Hadronic parameters: Form factor and decay constant

$$< \pi^+ D^- | H_{eff} | B > = a_1 \left\langle \pi \middle| \bar{u}\gamma^\mu L d \middle| 0 \right\rangle \left\langle D \middle| \bar{b}\gamma_\mu L c \middle| B \right\rangle$$

Generalized Factorization Approach

Ali, Kramer, Lu, Phys. Rev. D58, 094009 (1998)



$$C_1 \sim -0.2 \qquad \sim \qquad C_2(1/3 + s_8) \equiv C_2/N_c \sim +1/3$$

$$\langle \pi^0 \overline{D}^0 | H_{eff} | B^0 \rangle = (C_1 + C_2/N_c)\, f_D\, F_0^{B\to\pi}$$

Non-factorizable contribution should be larger than expected, characterized by effective N_C

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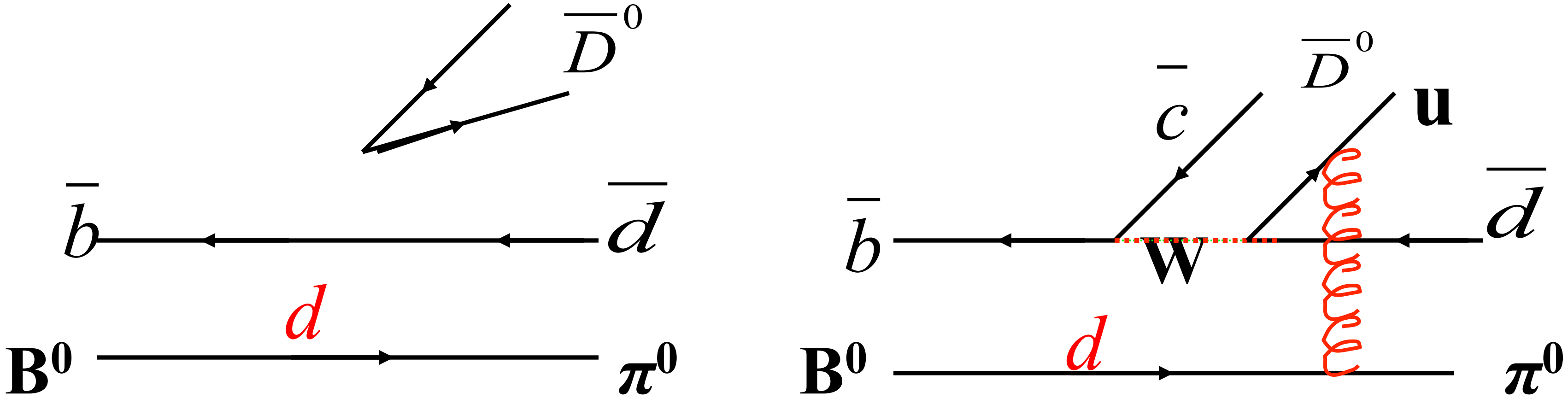

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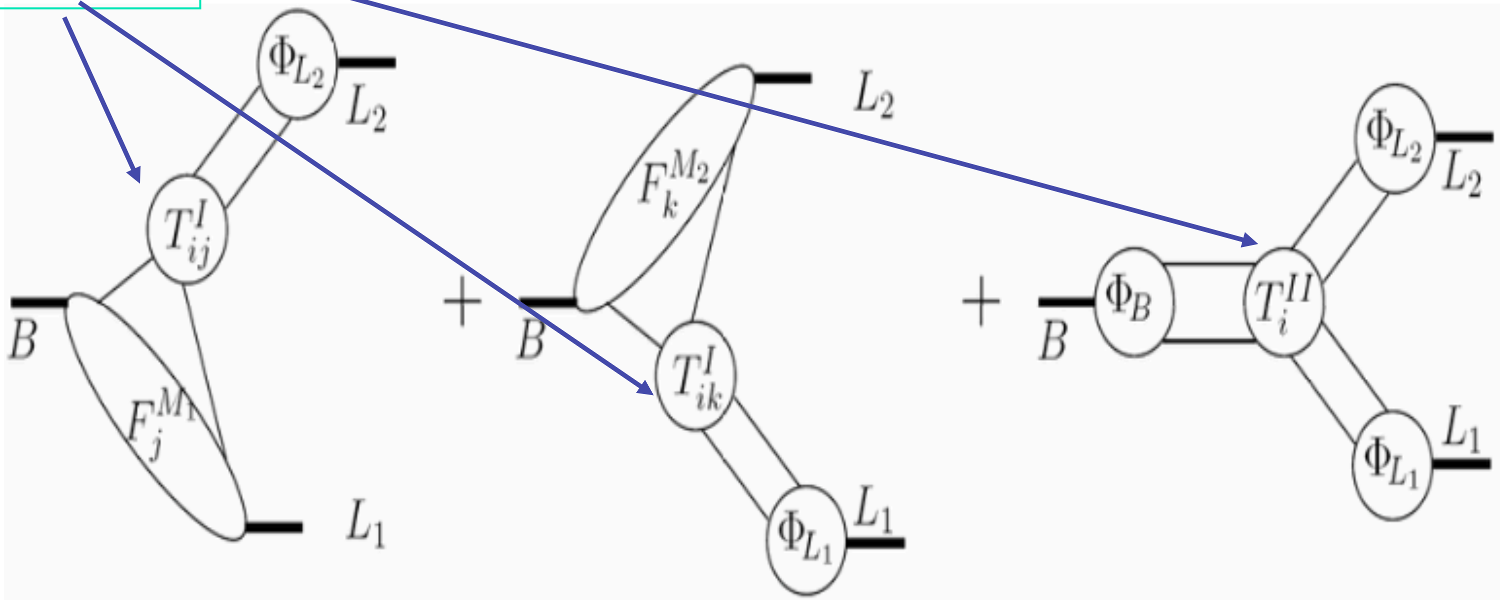
Non-factorizable contribution should be larger than expected, characterized by effective N_C

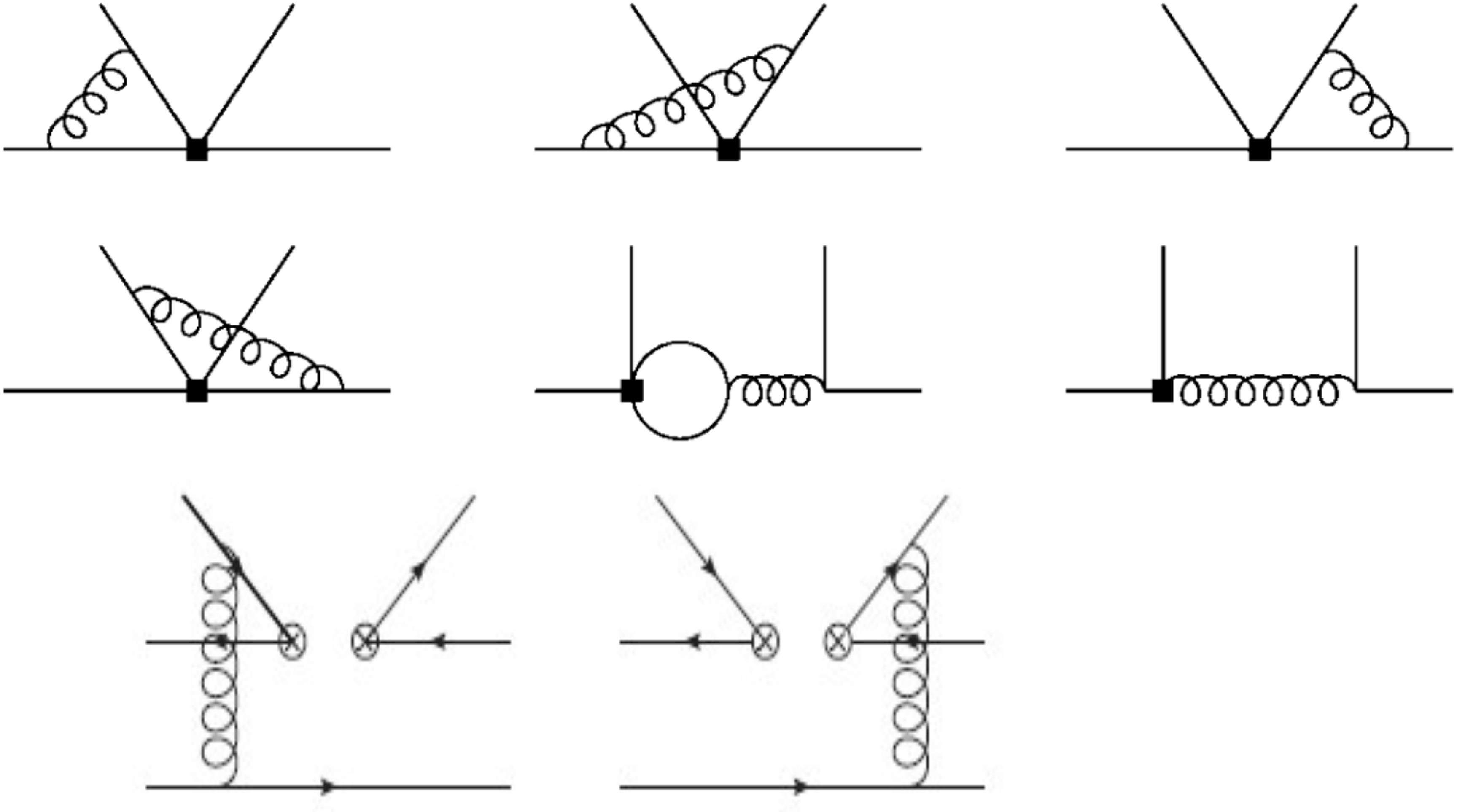
QCD factorization by BBNS: **PRL 83 (1999) 1914; NPB591 (2000) 313**

$$
\begin{aligned}
-\langle L_1 L_2 | Q_i | \bar{B} \rangle \;=\; & \sum_j F_j^{B\to L_1}(m_2^2) \int_0^1 du\, T_{ij}^I(u)\, \Phi_{L_2}(u) \\
& + \sum_k F_k^{B\to L_2}(m_1^2) \int_0^1 dv\, T_{ik}^I(v)\, \Phi_{L_1}(v), \\
& + \int_0^1 d\xi du dv\, T_i^{II}(\xi, u, v)\, \Phi_B(\xi)\, \Phi_{L_1}(v)\, \Phi_{L_2}(u)
\end{aligned}
$$

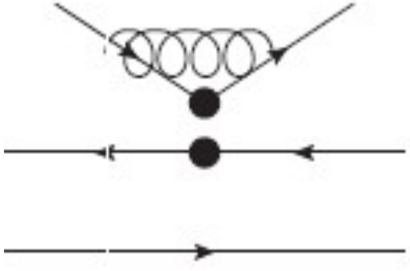
hard part



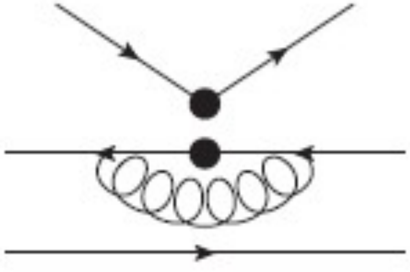
α_s corrections to the hard part T



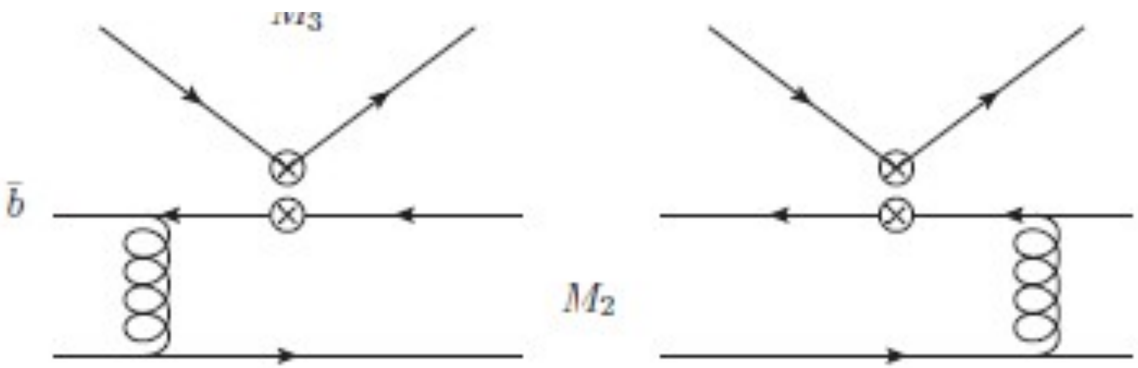
The missing diagrams, which contribute to the renormalization of decay constant or form factors



(e)

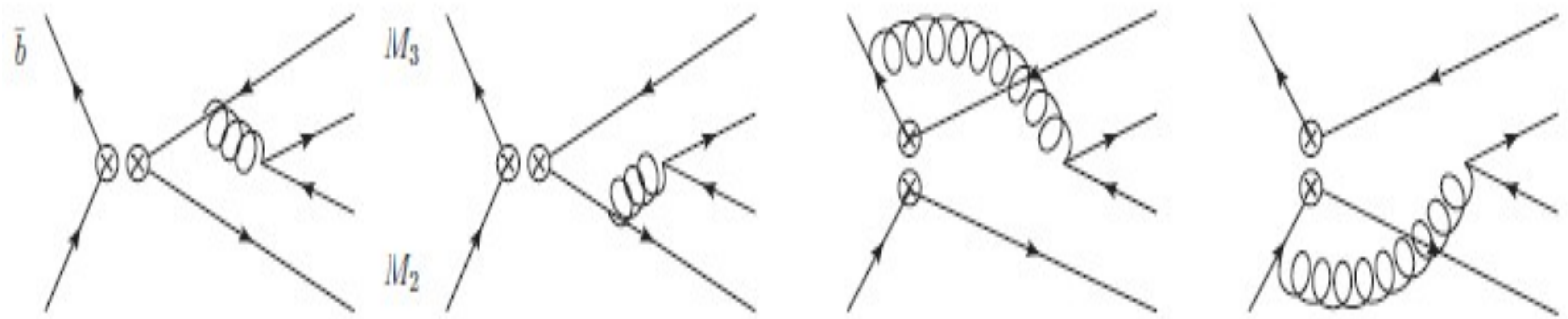


(f)



Endpoint divergence appears in these calculations

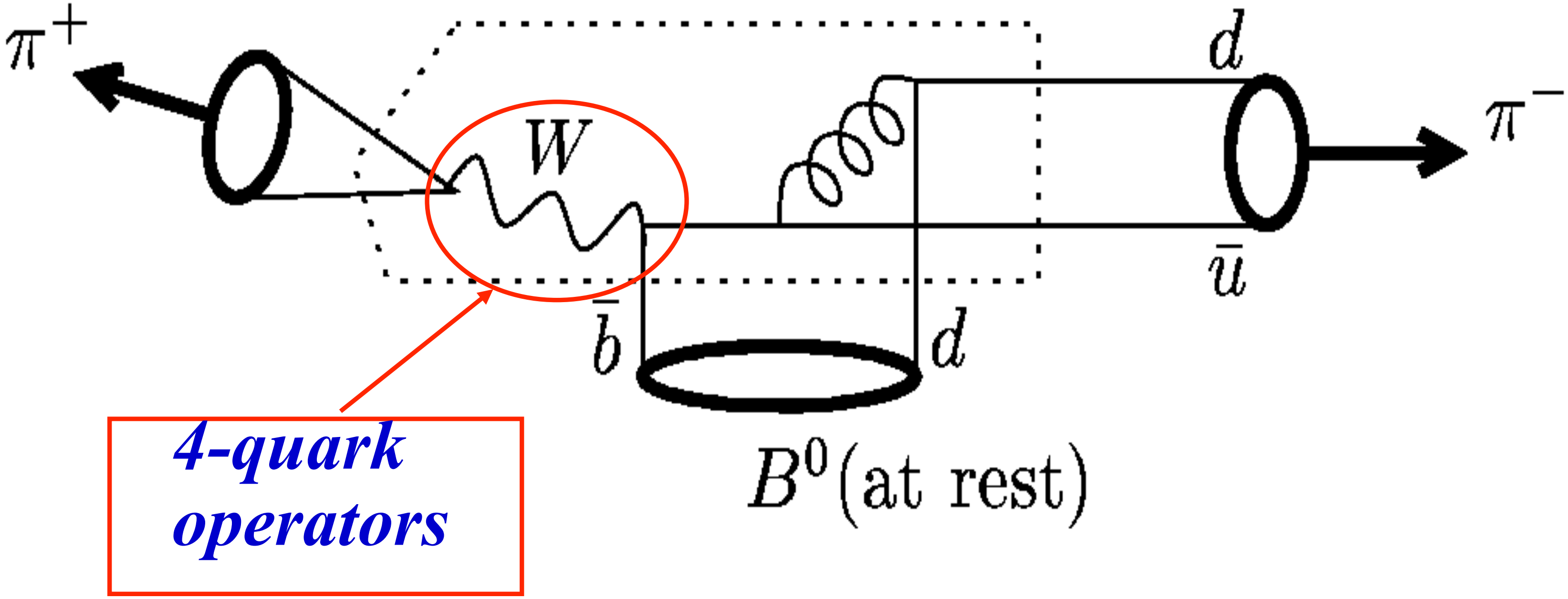
The annihilation type diagrams are important to the source of strong phases



- **However, these diagrams are similar to the form factor diagrams, which have endpoint singularity, not perturbatively calculable.**
- **These divergences are not physical, can only be treated in QCDF as free parameters, which makes CP asymmetry not predictable:**

$$\int_0^1 \frac{dy}{y} \to X_A^{M_1}, \qquad \int_0^1 dy \frac{\ln y}{y} \to -\frac{1}{2}(X_A^{M_1})^2$$

Picture of PQCD Approach

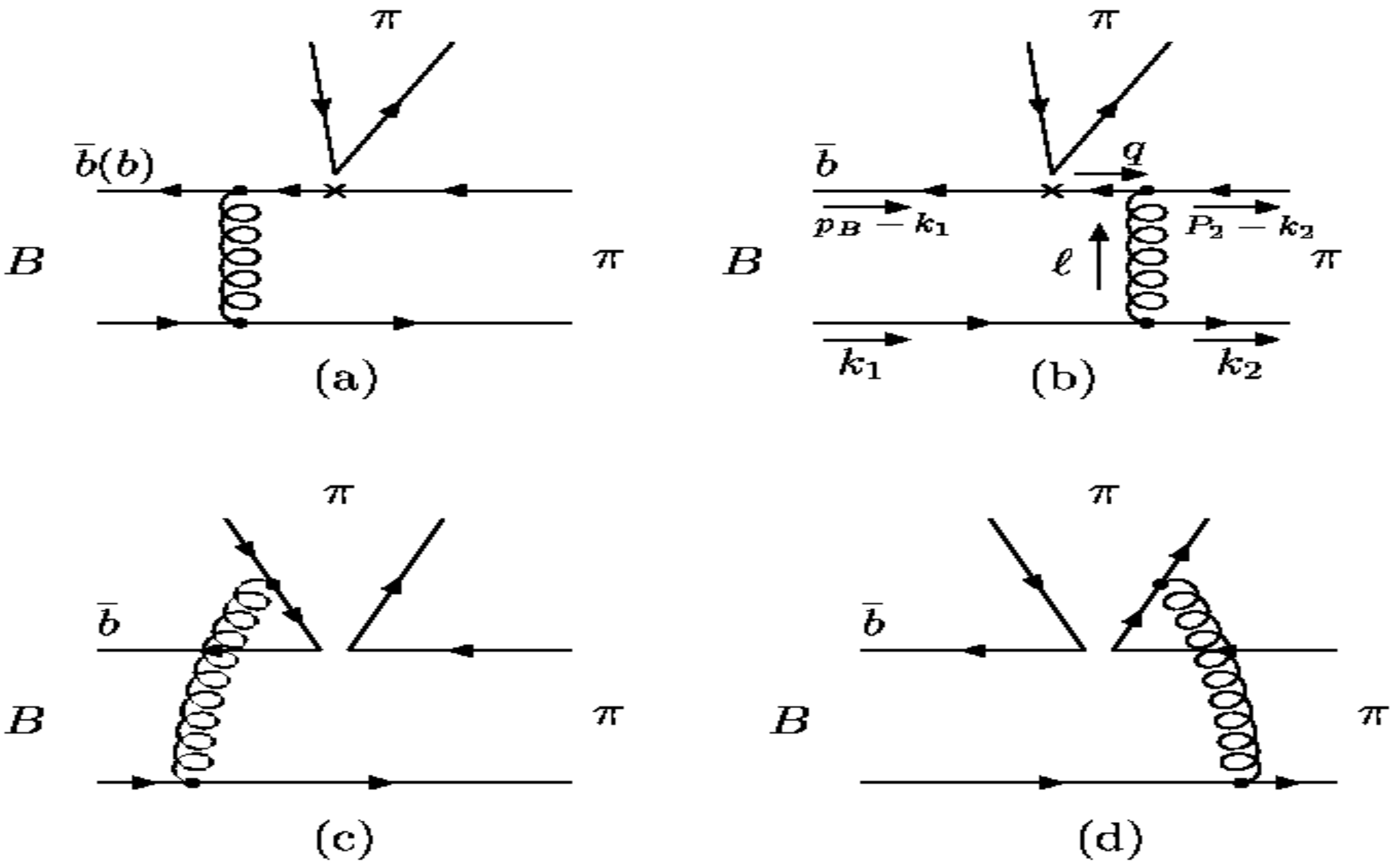


4-quark operators

Keum, Li, Sanda, **Phys.Rev. D63 (2001) 054008;**

Lu, Ukai, Yang, **Phys.Rev. D63 (2001) 074009**

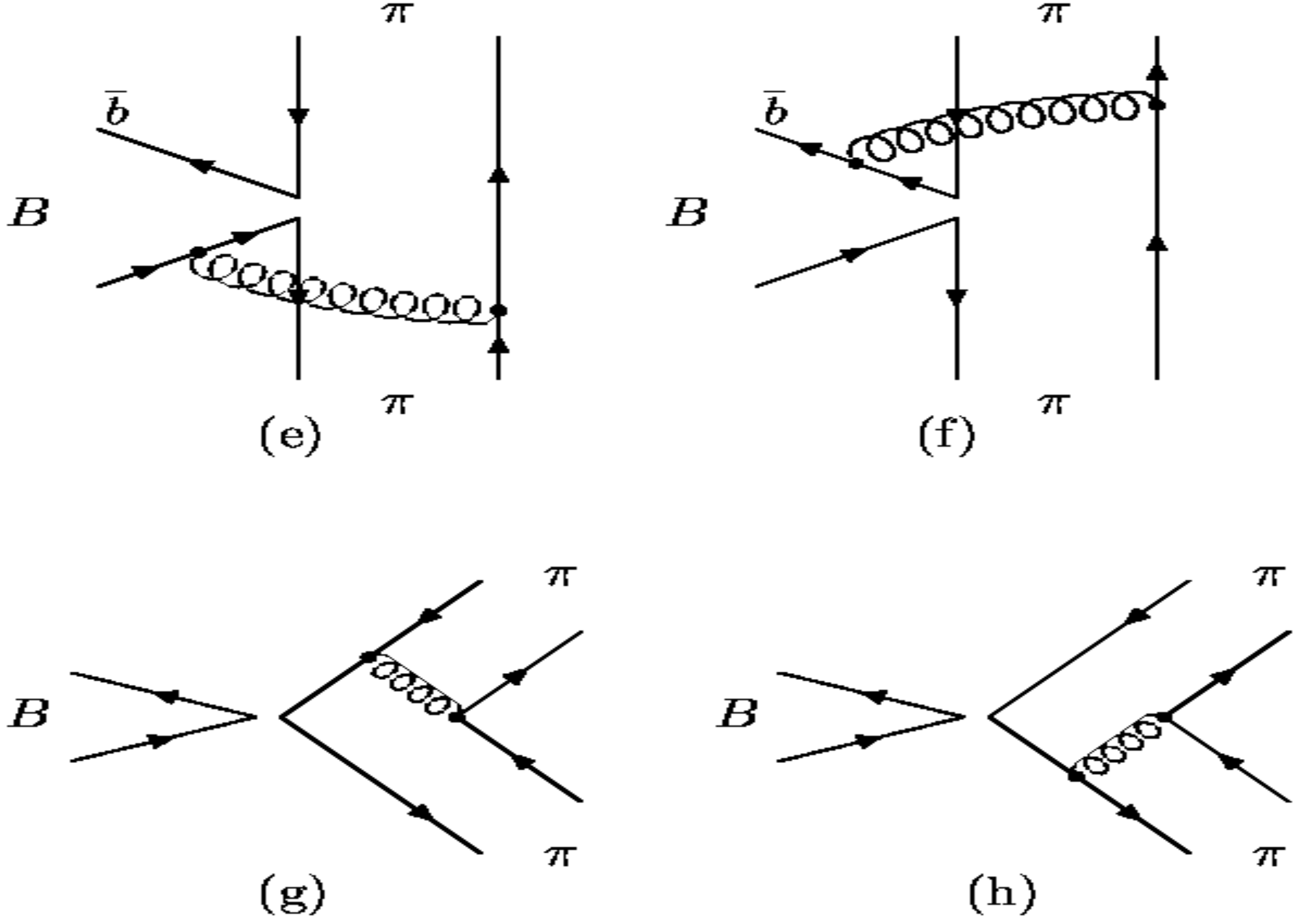
The leading order emission Feynman diagram in PQCD approach



Form factor diagram

Hard scattering diagram

The leading order **Annihilation type** Feynman diagram in PQCD approach



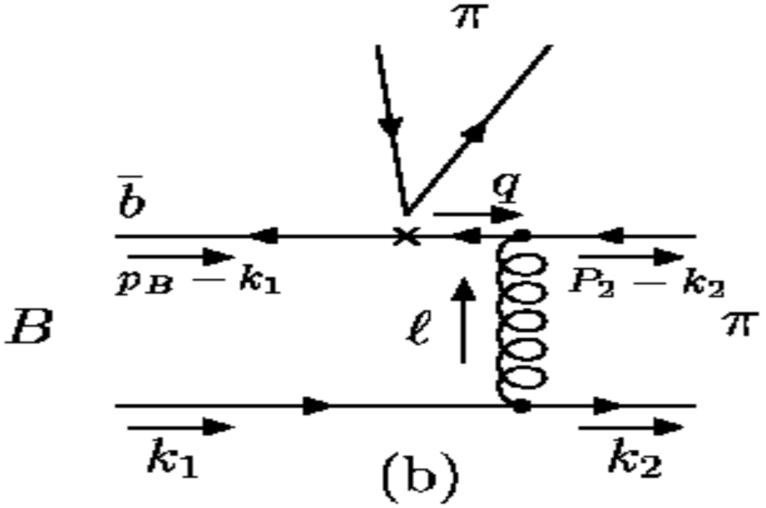
(e) (f)

(g) (h)

Endpoint singularity

- Gluon propagator

$$\frac{i}{(k_1-k_2)^2}=\frac{i}{-2xym_B^2}$$

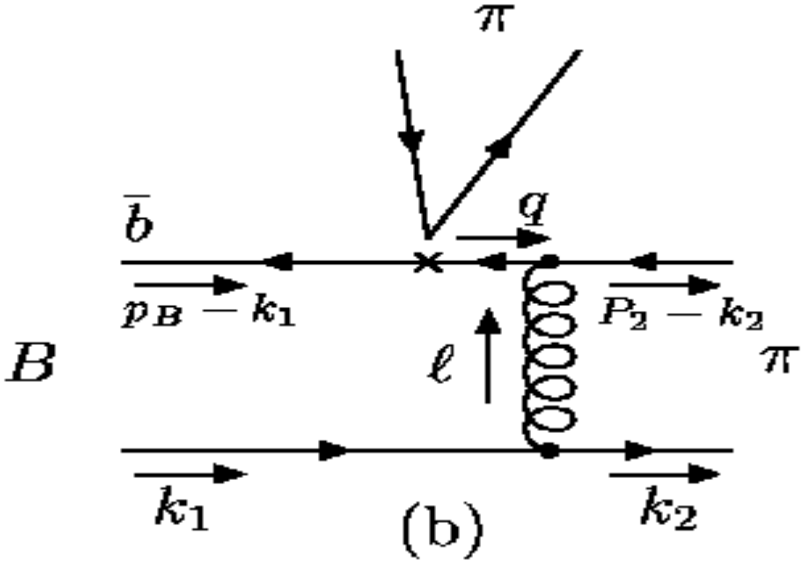


- x,y Integrate from 0→1, that is endpoint singularity
- The reason is that, one neglects the transverse momentum of quarks, which is not applicable at endpoint.
- If we pick back the transverse momentum, the divergence disappears

$$\frac{i}{(k_1-k_2)^2}=\frac{i}{-2xym_B^2-(k_1^T-k_2^T)^2}$$

Endpoint singularity

- It is similar for the quark propagator



$$\int_0^1 \frac{1}{x} dx = \ln\frac{1}{\varepsilon}$$

$$\int_0^1 \frac{1}{x+k} dx dk = \int dk \left[\ln(x+k)\right]_0^1 = \int dk\left[\ln(1+k) - \ln k\right]$$

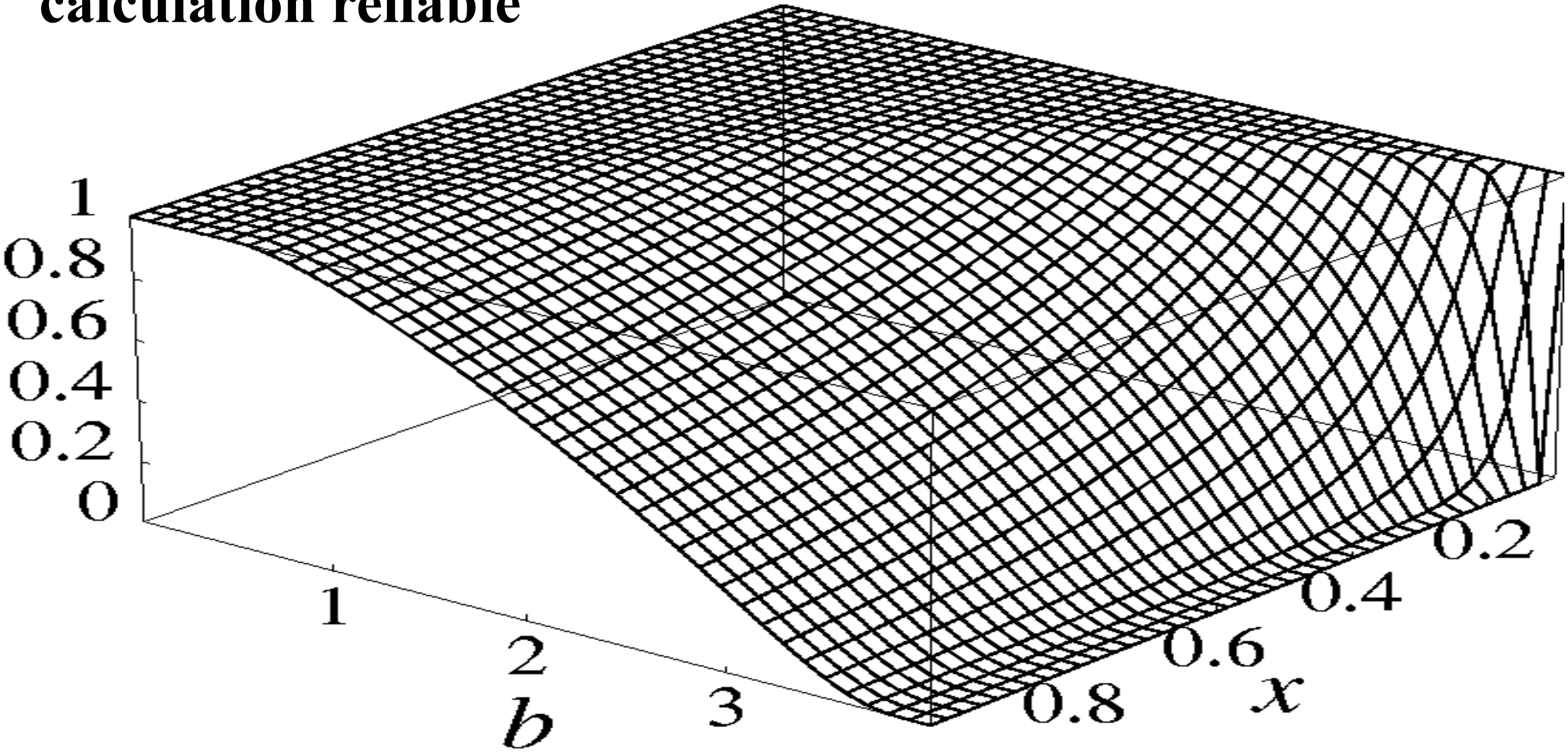
The logarithm divergence disappear if one has an extra dimension

However, with transverse momentum, means one extra energy scale

The overlap of Soft and collinear divergence will give double logarithm ln^2Pb, which is too big to spoil the perturbative expansion. We have to use renormalization group equation to resum all of the logs to give the so called Sudakov Form factor

Sudakov Form factor $\exp\{-S(x,b)\}$

This factor exponentially suppresses the contribution at the endpoint (small k_T), makes our perturbative calculation reliable



CP Violation in $B \to \pi\pi (K)$
(real prediction before exp.)

CP(%)	FA	BBNS	PQCD *(2001)*	Exp *(2004)*
$\pi^+ K^-$	+9±3	+5±9	–17±5	–11.5±1.8
$\pi^0 K^+$	+8 ± 2	7 ±9	–13 ±4	+4 ± 4
$\pi^+ K^0$	1. 7± 0. 1	1 ±1	–1. 0±0.5	–2 ±4
$\pi^+ \pi^-$	–5±3	–6±12	+30±10	+37±10

CP Violation in $B \rightarrow \pi\pi(K)$

Including large annihilation fixed from exp.

CP(%)	FA	Cheng,HY	PQCD (2001)	Exp
π^+K^-	+9±3	-7.4 ± 5.0	−17±5	−9.7±1.2
π^0K^+	+8 ± 2	0.28±0.10	−13 ±4	4.7 ± 2.6
π^+K^0	1.7± 0.1	4.9 ± 5.9	−1.0±0.5	0.9 ±2.5
$\pi^+\pi^-$	−5±3	17 ± 1.3	+30±10	+38±7

QCD-methods based on factorization work well for the leading power of $1/m_b$ expansion

collinear QCD Factorization approach
[Beneke, Buchalla, Neubert, Sachrajda, 99’]

Perturbative QCD approach based on k_T factorization
[Keum, Li, Sanda, 00’; Lu, Ukai, Yang, 00’]

Soft-Collinear Effective Theory
Bauer, Fleming, **Pirjol, Stewart, Phys.Rev. D63 (2001) 114020**

- **Work well for most of charmless B decays, except for ππ, πK puzzle etc.**

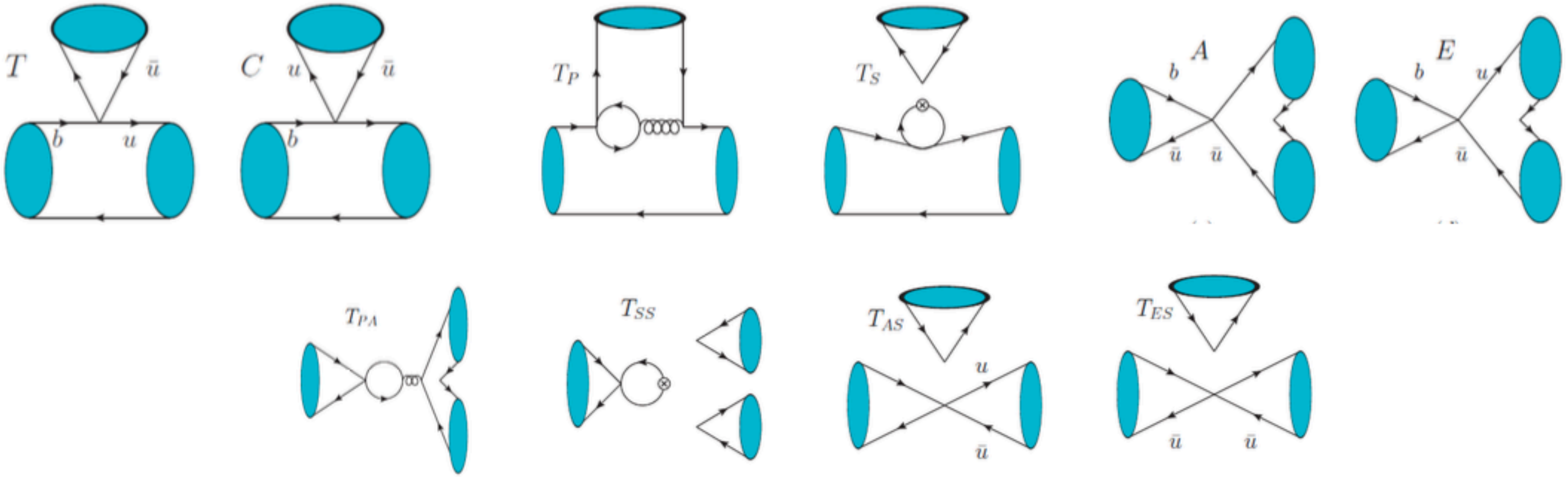
Factorization can only be proved in power expansion by operator product expansion. To achieve that, we need a hard scale Q

- In the certain order of 1/Q expansion, the hard dynamics characterized by Q factorize from the soft dynamics
- Hard dynamics is process-dependent, but calculable
- Soft dynamics are universal (process-independent) ⇨ predictive power of factorization theorem
- Factorization theorem holds up to all orders in α_s, but to certain power in 1/Q

The prove of factorization of QCD from electroweak is not needed

- Flavour SU(3) irreducible matrix elements
- Topological amplitudes (often with flavour SU(3) or SU(2))

$$T, C, P, P_{\mathrm{EW}}, S, E, A, \ldots$$

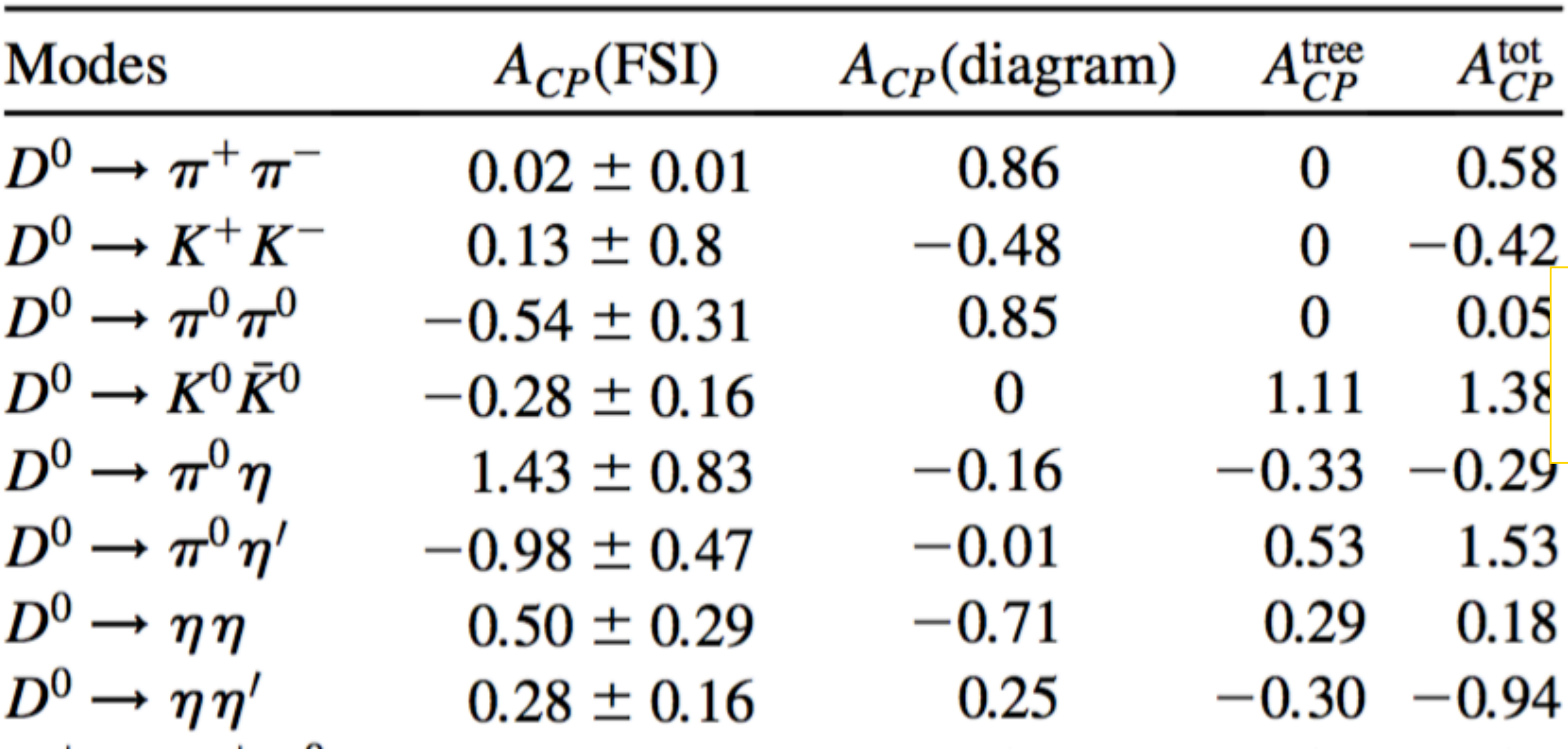


SU(3) breaking effect was lost. Limited precision!

Factorization assisted topological diagram approach first applied in hadronic D decays

[Li, Lu, Yu, PRD86 (2012) 036012]   **[FAT]**

Predictions of Direct CP asymmetries



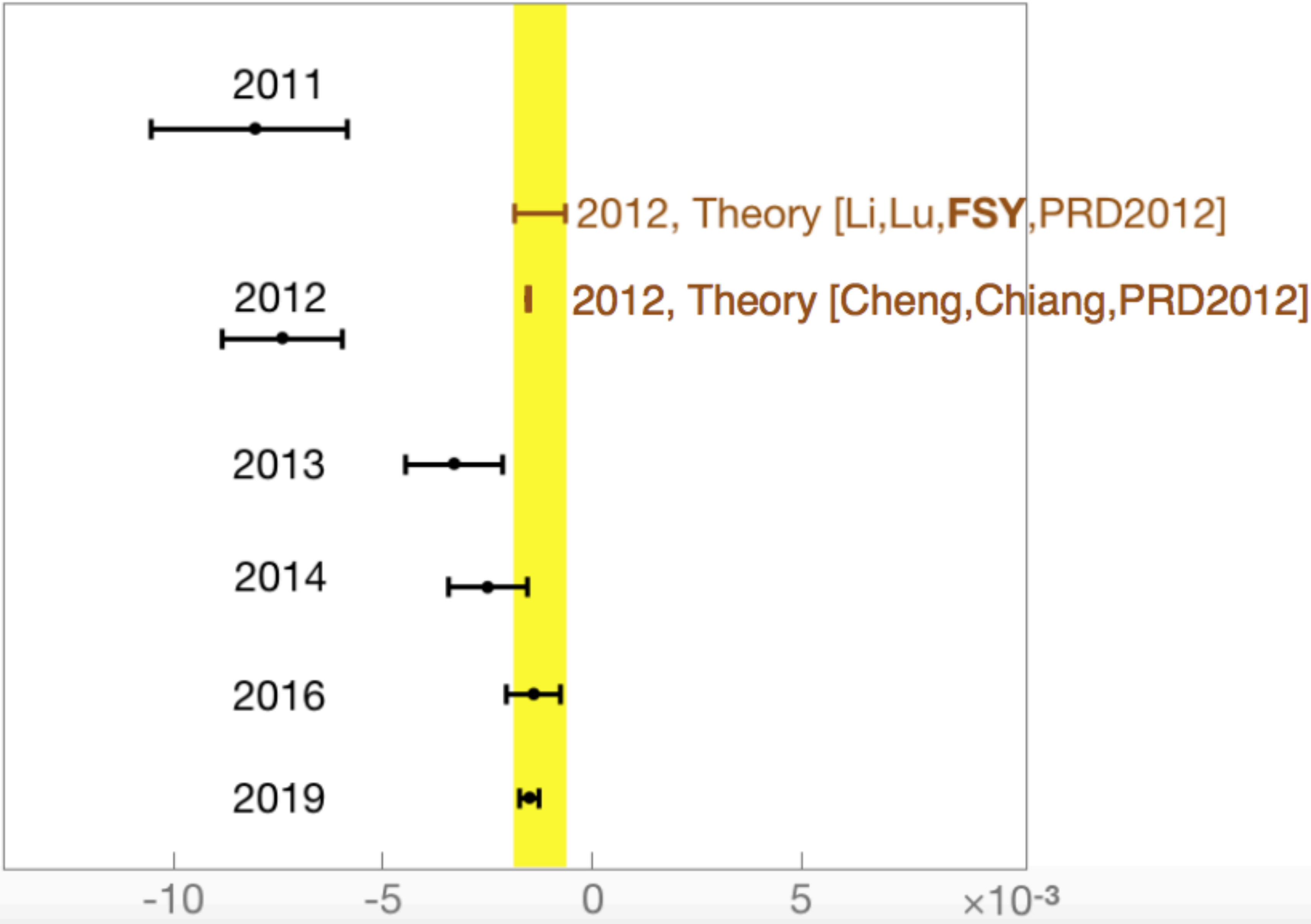
Modes	A_{CP}(FSI)	A_{CP}(diagram)	A_{CP}^{tree}	A_{CP}^{tot}
$D^0 \to \pi^+\pi^-$	0.02 ± 0.01	0.86	0	0.58
$D^0 \to K^+K^-$	0.13 ± 0.8	-0.48	0	-0.42
$D^0 \to \pi^0\pi^0$	-0.54 ± 0.31	0.85	0	0.05
$D^0 \to K^0\bar{K}^0$	-0.28 ± 0.16	0	1.11	1.38
$D^0 \to \pi^0\eta$	1.43 ± 0.83	-0.16	-0.33	-0.29
$D^0 \to \pi^0\eta'$	-0.98 ± 0.47	-0.01	0.53	1.53
$D^0 \to \eta\eta$	0.50 ± 0.29	-0.71	0.29	0.18
$D^0 \to \eta\eta'$	0.28 ± 0.16	0.25	-0.30	-0.94

⬅

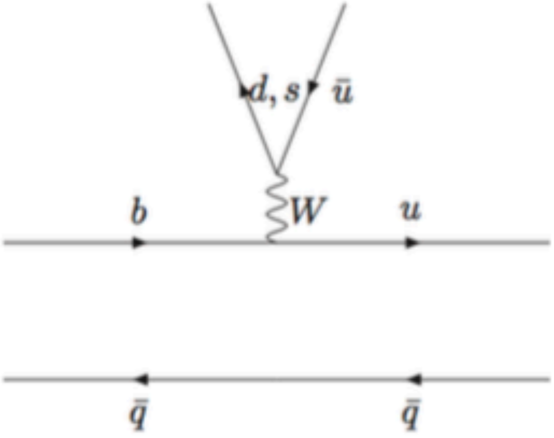
⬅

$\Delta_{CP} = -1 \times 10^{-3}$

Exp Averages



Tree topology diagram contributing to Charmless B decays



(a) T

For the color favored diagram (T), it is proved factorization to all order of α_s expansion in soft-collinear effective theory,

The decay amplitudes is just the decay constants and form factors times Wilson coeficients of four quark operators. The SU(3) breaking effect is automatically kept

$$T^{P_1P_2}=i\frac{G_F}{\sqrt{2}}V_{ub}V_{uq'}a_1(\mu)f_{p_2}(m_B^2-m_{p_1}^2)F_0^{BP_1}(m_{p_2}^2),$$

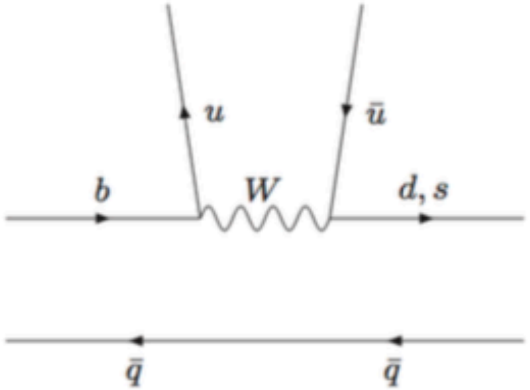
$$T^{PV}=\sqrt{2}G_FV_{ub}V_{uq'}a_1(\mu)f_Vm_VF_1^{B-P}(m_V^2)(\varepsilon_V^*\cdot p_B),$$

$$T^{VP}=\sqrt{2}G_FV_{ub}V_{uq'}a_1(\mu)f_Pm_VA_0^{B-V}(m_P^2)(\varepsilon_V^*\cdot p_B),$$

No free parameter

For other diagrams, we extract the amplitude and strong phase from experimental data by χ^2 fit

We factorize out the decay constants and form factor to keep the SU(3) breaking effect



(b) C

For the color suppressed tree diagram (C), we have two kinds of contributions

$$C^{P_1P_2} = i\frac{G_F}{\sqrt{2}} V_{ub} V_{uq'} \chi^C e^{i\phi^C} f_{P_2} (m_B^2 - m_{p_1}^2) F_0^{BP_1}(m_{p_2}^2),$$

$$C^{PV} = \sqrt{2} G_F V_{ub} V_{uq'} \chi^{C'} e^{i\phi^{C'}} f_V m_V F_1^{B-P}(m_V^2)(\varepsilon_V^* \cdot p_B),$$

$$C^{VP} = \sqrt{2} G_F V_{ub} V_{uq'} \chi^C e^{i\phi^C} f_P m_V A_0^{B-V}(m_P^2)(\varepsilon_V^* \cdot p_B),$$

Global Fit for all B→PP, VP and PV decays

with $\chi^2/\mathrm{d.o.f} = 45.2/34 = 1.3.$

35 branching Ratios and 11 CP violation observations data are used for the fit

$$\begin{aligned}
\chi^{C} &= 0.48 \pm 0.06, & \phi^{C} &= -1.58 \pm 0.08,\\
\chi^{C'} &= 0.42 \pm 0.16, & \phi^{C'} &= 1.59 \pm 0.17,\\
\chi^{E} &= 0.057 \pm 0.005, & \phi^{E} &= 2.71 \pm 0.13,\\
\chi^{P} &= 0.10 \pm 0.02, & \phi^{P} &= -0.61 \pm 0.02.\\
\chi^{P_C} &= 0.048 \pm 0.003, & \phi^{P_C} &= 1.56 \pm 0.08,\\
\chi^{P'_C} &= 0.039 \pm 0.003, & \phi^{P'_C} &= 0.68 \pm 0.08,\\
\chi^{P_A} &= 0.0059 \pm 0.0008, & \phi^{P_A} &= 1.51 \pm 0.09,
\end{aligned}$$

$$\chi^2 = \sum_{i=1}^{n} \left(\frac{x_i^{\mathrm{th}} - x_i}{\Delta x_i} \right)^2.$$

Large strong phase

Zhou, Zhang, Lyu and Lü,

EPJC (2017) 77: 125

Comparison of different contributions from FAT and QCDF

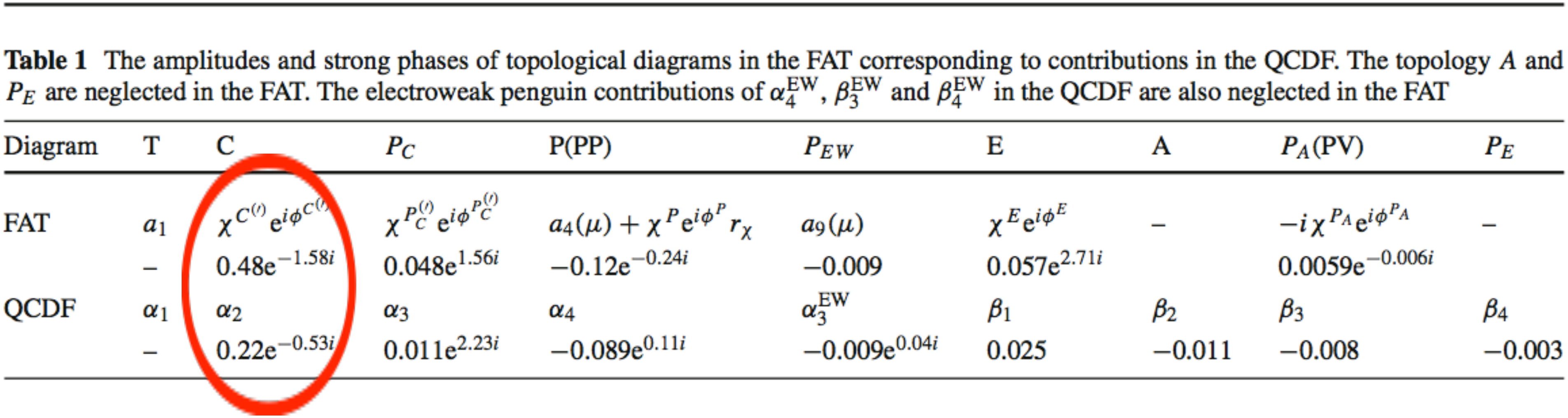


Table 1 The amplitudes and strong phases of topological diagrams in the FAT corresponding to contributions in the QCDF. The topology A and P_E are neglected in the FAT. The electroweak penguin contributions of α_4^{EW}, β_3^{EW} and β_4^{EW} in the QCDF are also neglected in the FAT

Diagram	T	C	P_C	P(PP)	P_{EW}	E	A	P_A(PV)	P_E
FAT	a_1	$\chi^{C^{(\prime)}} \mathrm{e}^{i\phi^{C^{(\prime)}}}$	$\chi^{P_C^{(\prime)}} \mathrm{e}^{i\phi^{P_C^{(\prime)}}}$	$a_4(\mu) + \chi^P \mathrm{e}^{i\phi^P} r_\chi$	$a_9(\mu)$	$\chi^E \mathrm{e}^{i\phi^E}$	–	$-i\chi^{P_A} \mathrm{e}^{i\phi^{P_A}}$	–
	–	$0.48\mathrm{e}^{-1.58i}$	$0.048\mathrm{e}^{1.56i}$	$-0.12\mathrm{e}^{-0.24i}$	-0.009	$0.057\mathrm{e}^{2.71i}$		$0.0059\mathrm{e}^{-0.006i}$	
QCDF	α_1	α_2	α_3	α_4	α_3^{EW}	β_1	β_2	β_3	β_4
	–	$0.22\mathrm{e}^{-0.53i}$	$0.011\mathrm{e}^{2.23i}$	$-0.089\mathrm{e}^{0.11i}$	$-0.009\mathrm{e}^{0.04i}$	0.025	-0.011	-0.008	-0.003

CKM angle gamma extraction

All the tree amplitudes in charmless B decays are proportional to $V_{ub}V_{ud,s}^{*}$; while the penguin amplitudes are proportional to $V_{tb}V_{td,s}^{*} = -(V_{ub}V_{ud,s}^{*} + V_{cb}V_{cd,s}^{*})$.

Except $V_{ub} \equiv |V_{ub}|e^{-i\gamma}$, all other CKM matrix elements are approximately real numbers without electroweak phase.

So after input the magnitudes of the following CKM matrix elements,

$$
\begin{aligned}
&|V_{ud}| = 0.97420 \pm 0.00021\,, \quad &&|V_{us}| = 0.2243 \pm 0.0005\,, \quad &&|V_{ub}| = 0.00394 \pm 0.00036\,, \\
&|V_{cd}| = 0.218 \pm 0.004\,, &&|V_{cs}| = 0.997 \pm 0.017\,, &&|V_{cb}| = 0.0422 \pm 0.0008\,.
\end{aligned}
$$

We can extract the CKM angle gamma by global fit all the charmless B decays

Global Fit for all B→PP, VP and PV decays with gamma as free parameter

with $\chi^2/\mathrm{d.o.f} = 45.4/33 = 1.4.$

We use 37 branching ratios and 11 CP violation observations of all B → P P, P V decays from the current experimental data

$$\boxed{\gamma = (69.8 \pm 2.1)^\circ}$$

$$\begin{aligned}
\chi^C &= 0.41 \pm 0.06, & \phi^C &= -1.74 \pm 0.09, \\
\chi^{C'} &= 0.40 \pm 0.17, & \phi^{C'} &= 1.78 \pm 0.10, \\
\chi^E &= 0.06 \pm 0.006, & \phi^E &= 2.76 \pm 0.13, \\
\chi^P &= 0.09 \pm 0.003, & \phi^P &= 2.55 \pm 0.03 \\
\chi^{P_C} &= 0.045 \pm 0.003, & \phi^{P_C} &= 1.53 \pm 0.08, \\
\chi^{P_C'} &= 0.037 \pm 0.003, & \phi^{P_C'} &= 0.67 \pm 0.08, \\
\chi^{P_A} &= 0.006 \pm 0.0008, & \phi^{P_A} &= 1.49 \pm 0.09,
\end{aligned}$$

Global Fit for all B→PP, VP and PV decays with gamma as free parameter

with $\chi^2/\text{d.o.f} = 45.4/33 = 1.4.$

$$\gamma = (69.8 \pm 2.1 \pm 0.9)^\circ$$

We use 37 branching ratios and 11 CP violation observations of all $B \to PP, PV$ decays from the current experimental data

$$\gamma = (69.8 \pm 2.1)^\circ$$

Uncertainty from input parameters

$$\begin{aligned}
\chi^{C} &= 0.41 \pm 0.06, & \phi^{C} &= -1.74 \pm 0.09,\\
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\end{aligned}$$

Comparison of gamma measurement

$$\gamma = (69.8 \pm 2.1 \pm 0.9)^\circ$$

HFLAV Collaboration $\gamma = (71.1^{+4.6}_{-5.3})^\circ$

CKMfit Collaboration $\gamma = (73.5^{+4.2}_{-5.1})^\circ$

Less uncertainty than others

UTfit Collaboration $\gamma = (70.0 \pm 4.2)^\circ$

Recent LHCb result $\gamma = (74.0^{+5.0}_{-5.8})^\circ$

Zhou and Lu.
arXiv: 1910.03160

Summary/Challenges

- Hadronic B Decays are important in the test of standard model and search for signals of new physics.
- A great progress has been made in both theoretical and experimental sides
- Next-to-leading order perturbative calculations and power corrections in QCD is needed to explain the more and more precise experimental data

祝赵老师生日快乐 🎂

2006

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2011

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2012

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2012

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