Single-spin asymmetry at two loops

Hsiang-nan Li

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In collaboration with S. Benic, Y. Hatta, D. Yang

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完全表頭

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主旨: Re: XS4

Dear Hsiang-nan,

Thank you, Pauchy, and Xiao-Gang very much again for your kind invitation.

日期: Tue, 8 Mar 2005 11:48:00 +0800

完全表頭

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主旨: Re: a question

Dear Hsiang-nan,

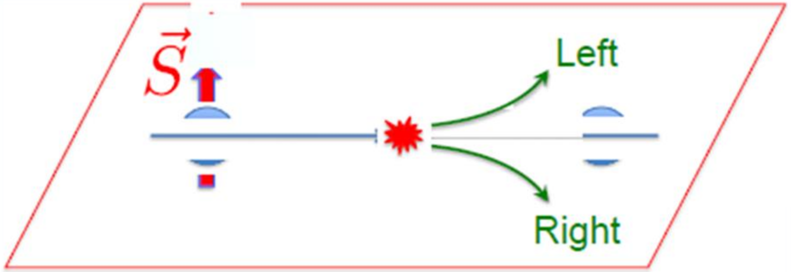
Nice to hear from you.

The meaning of "n" is different in PDG and conventional quantum mechanics.

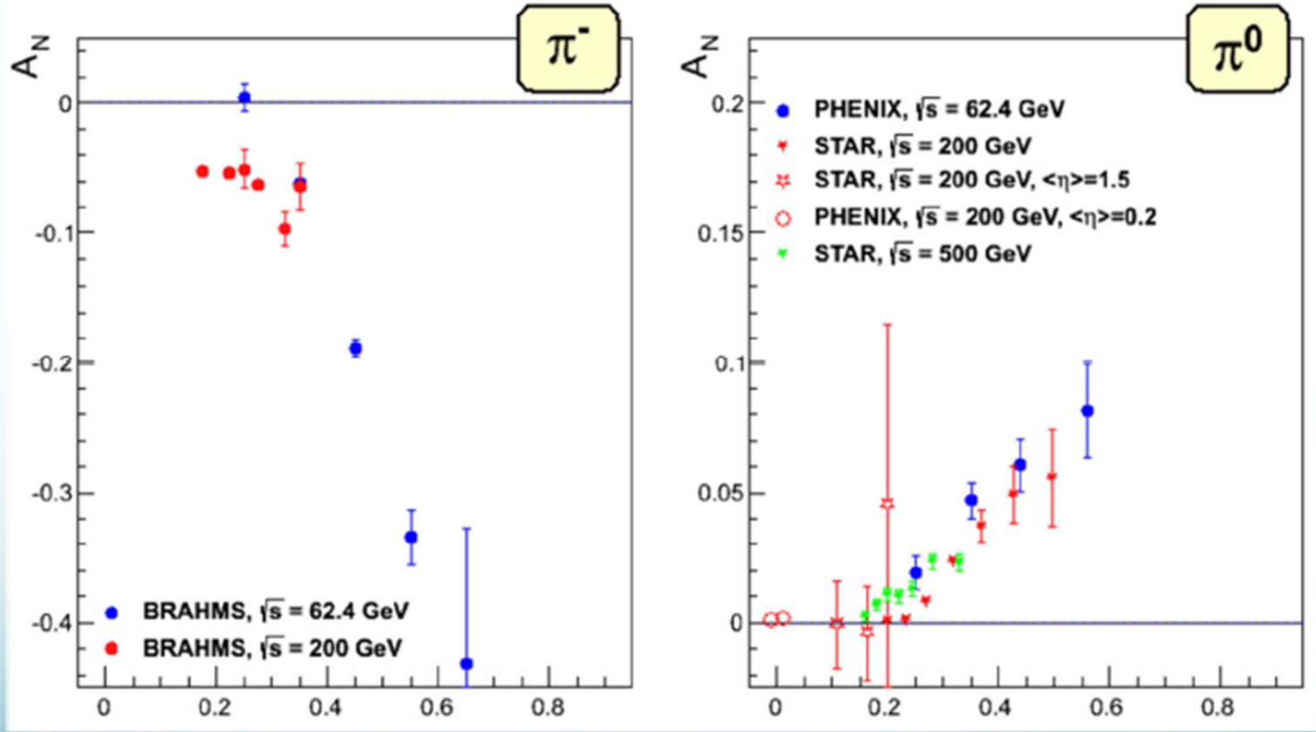
In PDG, all states having radial wave functions without any nodes are called n=1 states (1S for J/psi; 1P for chi_{cJ} (J=0,1,2); 1D for psi"(3770) (^3D_1),...). The three n=1 chi_{cJ} (J=0,1,2) states have the same P-wave (L=1) radial wave functions in the nonrelativistic limit, and by the spin (S=1)-orbit (L=1) coupling one gets three states with J=0,1,2.

Single transverse spin asymmetry (SSA)

- Consider a transversely polarized proton scatter off an unpolarized proton or electron



$$A_N \equiv \frac{L-R}{L+R} = \frac{\sigma^{\uparrow} - \sigma^{\downarrow}}{\sigma^{\uparrow} + \sigma^{\downarrow}}$$



$x_F \sim 2P_z/\sqrt{s}$ scaled longitudinal momentum

Mechanism

- There exists correlation proportional to

$$\varepsilon_{\mu\nu\rho\lambda} S_T^{\mu} p_{hT}^{\nu} \cdots$$

- To generate such term in Feynman diagram, need

$$tr\left[\gamma_5 \not{S}_T \not{p}_{hT} \cdots\right] = i\varepsilon_{\mu\nu\rho\lambda} S_T^{\mu} p_{hT}^{\nu} \cdots$$

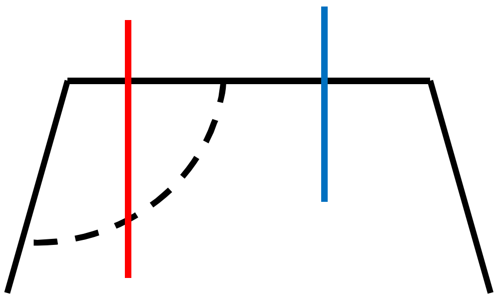
- Projector for polarized proton $\left(\not{p} + m\right)\gamma_5 \not{S}_T$
- Projector for produced hadron $\not{p}_h + m_h$
- But need strong phase to make cross section real

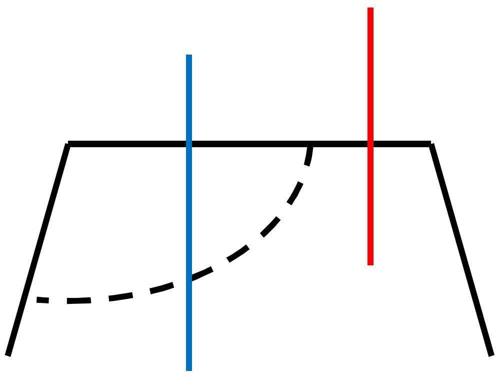
Where is phase?

- Phase comes from on-shell internal particles

$$\frac{1}{k^2 + i\varepsilon} = \frac{P}{k^2} - i\pi\delta(k^2)$$

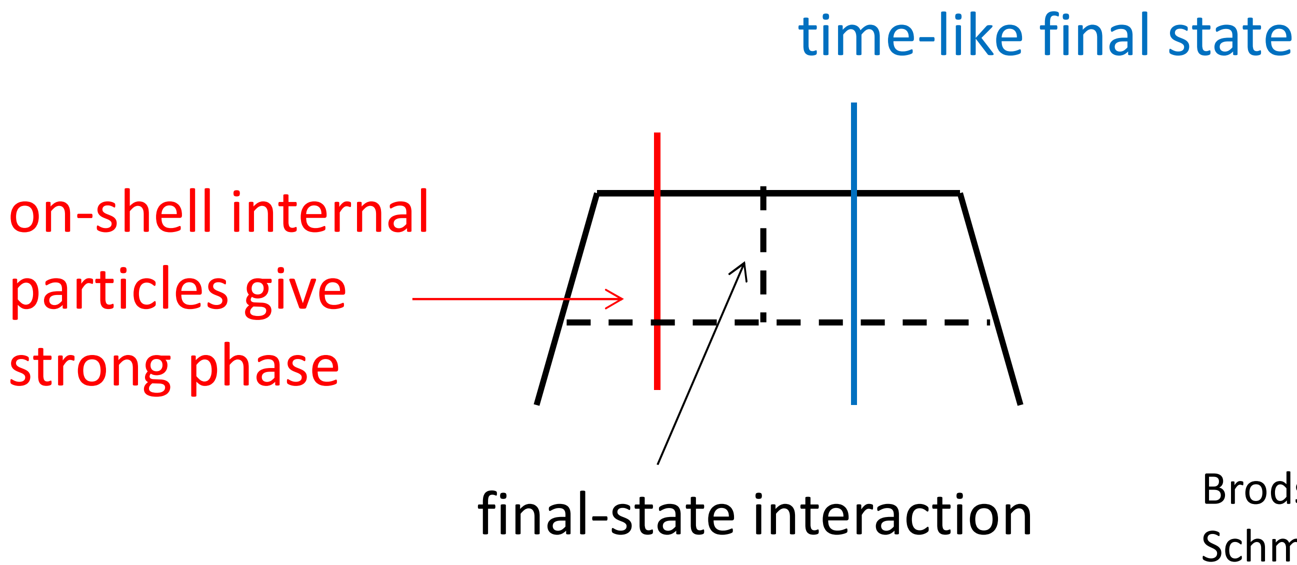
- Need time-like final states with FSI
- No phase at LO and one loop





Phase at two loops

- Need two final-state particles with one gluon exchange (FSI) between them
- Nonvanishing phase appears at two loops, and comes from box diagram



Brodsky, Hwang, Schmidt 2002

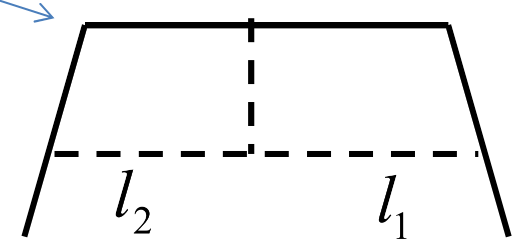
Kinematics for phase

$$q = p_2 - p_1, \quad p_2 = (p_2^+, p_2^-, 0_T) \qquad p_1^+, p_2^- >> p_2^+ >> \Lambda_{QCD}$$

$$p_2 - l_2 \qquad p_2 - l_1 \qquad l_{1T} \leftrightarrow p_{hT}$$

$p_2^2 > 0$

time-like



$$p_1 = (p_1^+, 0, 0_T)$$

$$l_1^- = l_{1T}^2/(2l_1^+)$$

$$l_2^- = l_{2T}^2/(2l_2^+)$$

$$l_1^+ = \frac{1}{2}\left(p_2^+ \pm \sqrt{p_2^{+2} - 2\frac{p_2^+}{p_2^-} l_{1T}^2}\right)$$

$$l_2^+ = \frac{1}{2}\left(p_2^+ \pm \sqrt{p_2^{+2} - 2\frac{p_2^+}{p_2^-} l_{2T}^2}\right)$$

Collinear to initial state

- Picking up plus signs, ie., (l1=+,l2=+), gluons collimate to polarized proton

$$l_{1,2}^{+} \sim O(p_2^{+}) >> l_{1T,2T} >> l_{1,2}^{-}$$

$$p_1 - l_2 \approx p_1^{+} - p_2^{+}$$

collinear

$$p_2 - l_1 \approx p_2 - l_2 \approx p_2^{-}$$

- Phase goes into Sivers function
- FSI gluon is soft

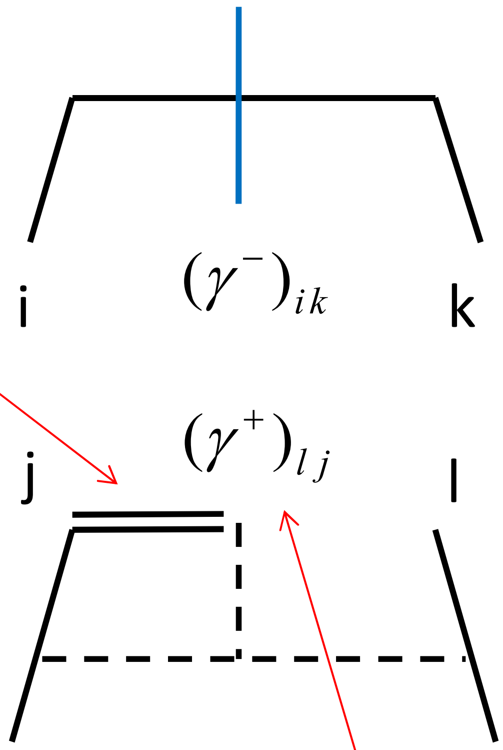
Sivers function

Sivers 1990

- Eikonalize outgoing quark and insert Fierz identity

$$I_{ij}I_{lk} = \frac{1}{4}I_{ik}I_{lj} + \frac{1}{4}(\gamma^\alpha)_{ik}(\gamma_\alpha)_{lj}$$
$$+ \frac{1}{4}(\gamma^5\gamma^\alpha)_{ik}(\gamma_\alpha\gamma^5)_{lj} + \frac{1}{4}(\gamma^5)_{ik}(\gamma^5)_{lj}$$
$$+ \frac{1}{8}(\gamma^5\sigma^{\alpha\beta})_{ik}(\sigma_{\alpha\beta}\gamma^5)_{lj}$$

give dominant (twist-2) contribution



pick up $\not p = p^+\gamma^-$

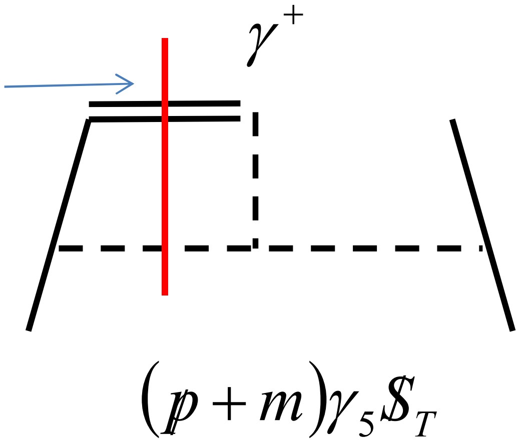
Parton transverse momentum

- Sivers function demands inclusion of parton transverse momentum

compensated by phase here

$$tr\left[\gamma_5 \not{S}_T \not{k}_T \gamma^+ \gamma^- \cdots\right] = i\varepsilon_{\mu\nu+-} S_T^{\mu} k_T^{\nu} \cdots$$

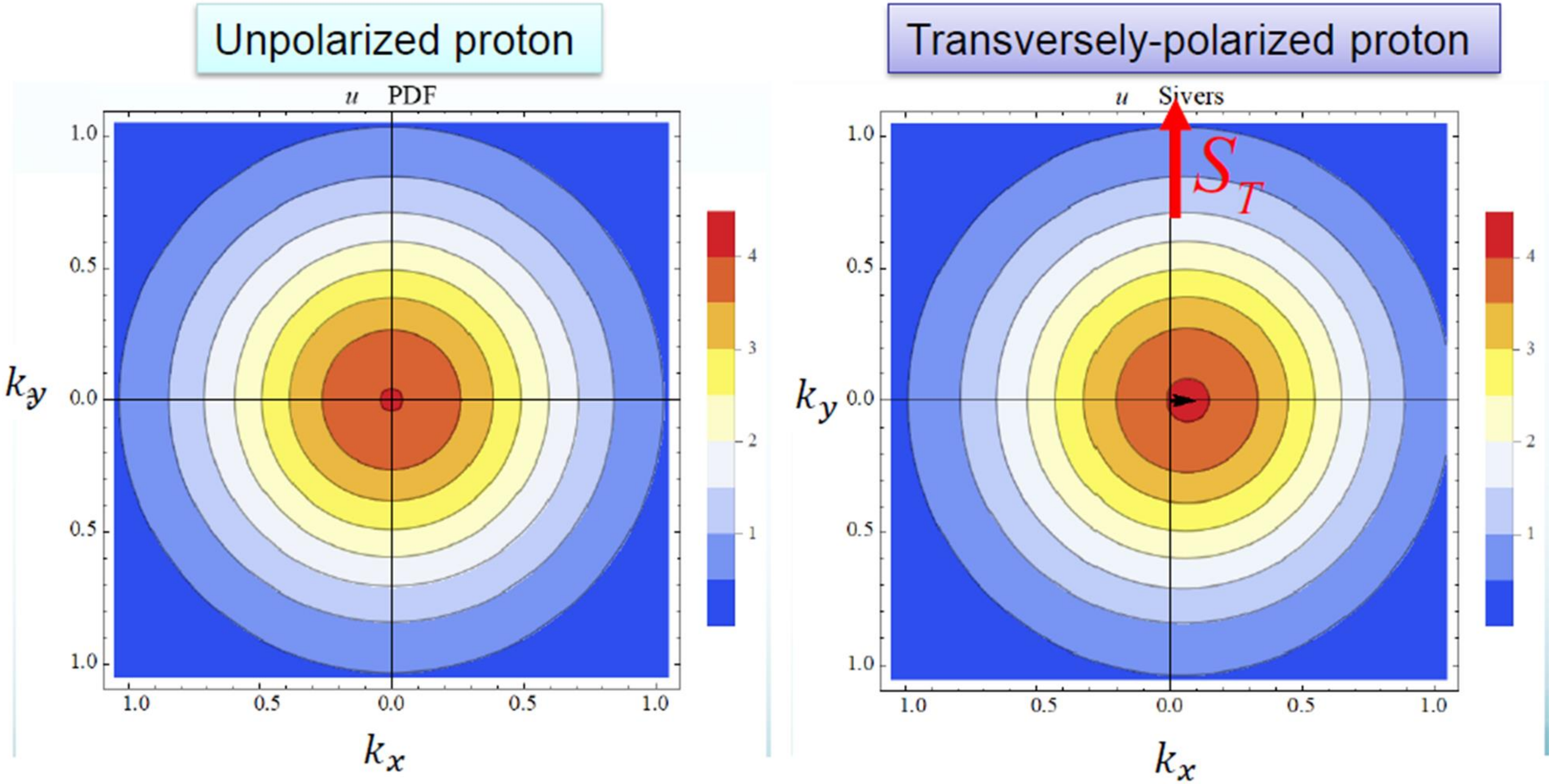
$l_{1T,2T}$



- This correlation determines preferred direction of k_T for polarized proton , which then propagates into p_h

Spin-transverse-momentum correlation

$$f_{q/p\uparrow}(x,k_T,\overrightarrow{S_T}) = f_{q/p}(x,k_T) - \frac{1}{M} f_{1T}^{\perp q}(x,k_T)\overrightarrow{S_T}\cdot(\hat{p}_h \times k_T)$$



produced hadron tends to move to right

Collinear to final state

- Picking up minus signs, ie., (-,-), gluons collimate to produced hadron

$$l_{1,2}^{-} \sim O(p_2^{-}) >> l_{1T,2T} >> l_{1,2}^{+}$$

$$p_2 - l_1 \sim O(p_2^{-}), \quad p_2 - l_2 \sim O(p_2^{-})$$

collinear

$p_1 - l_2$ highly off-shell

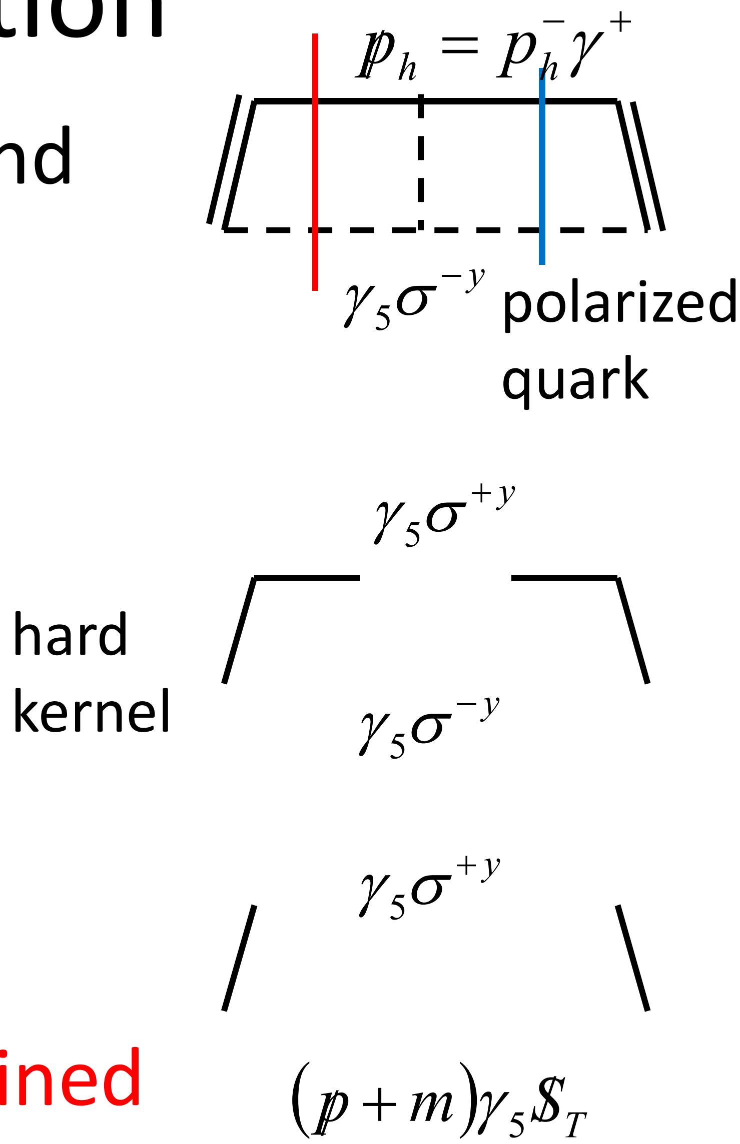
- Phase goes into Collins fragmentation function

Collins 1993

Collins function

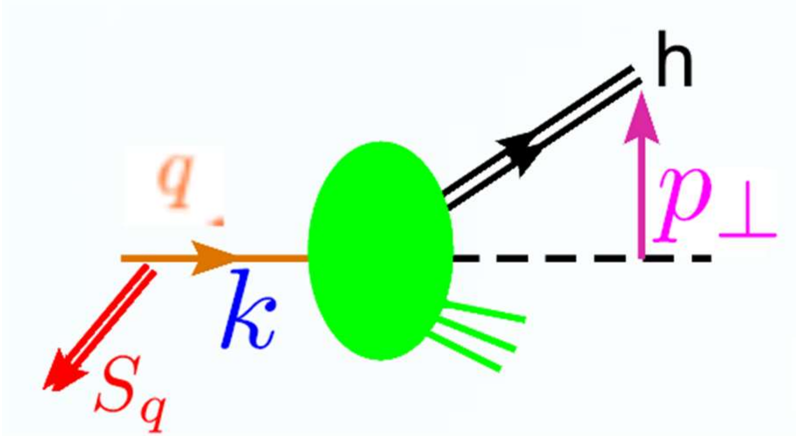
- Eikonalize incoming quark and insert Fierz identity
- $\gamma_5\sigma^{-y}$ dominates
- Collins function demands inclusion of parton k_T
- LO hard kernel demands projector for initial state

- Transversity distribution defined



Mechanism

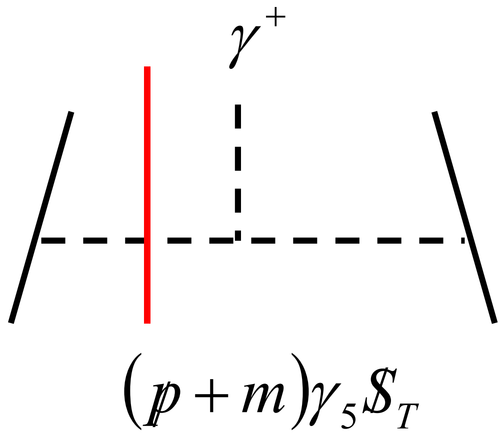
- Transversity distribution for polarized proton determines preferred direction of quark spin
- This polarized quark scattered into final state
- Collins function then determines direction of polarized quark (produced hadron) preferred by correlation
- Without preferred direction of quark spin from initial state, Collins function cannot work



Factorization of transverse gluon

- As soft gluon carries transverse polarization, outgoing quark line cannot be eikonalized
- Collinear divergence in (+,+) combination goes into three-parton TMD, whose collinear version is Efremov-Teryaev-Qiu-Sterman (ETQS) function Efremov, Teryaev 1982; Qiu, Sterman, 1991
- Similar construction for three-parton fragmentation functions

Kang, Yuan, Zhou, 2010



Twist-2 TMDs

$$\Phi^{[\gamma^+]} = f_1 - \frac{\epsilon_T^{\rho\sigma} p_{T\rho} S_{T\sigma}}{M} f_{1T}^{\perp},$$

Sivers function

$$\Phi^{[\gamma^+\gamma_5]} = S_L\, g_{1L} - \frac{p_T \cdot S_T}{M} g_{1T}\,,$$

$$\Phi^{[i\sigma^{\alpha+}\gamma_5]} = S_T^{\alpha} h_1 + S_L \frac{p_T^{\alpha}}{M} h_{1L}^{\perp} - \frac{p_T^{\alpha} p_T^{\rho} - \frac{1}{2} p_T^2 g_T^{\alpha\rho}}{M^2} S_{T\rho}\, h_{1T}^{\perp} - \frac{\epsilon_T^{\alpha\rho} p_{T\rho}}{M} h_1^{\perp}$$

transversity function

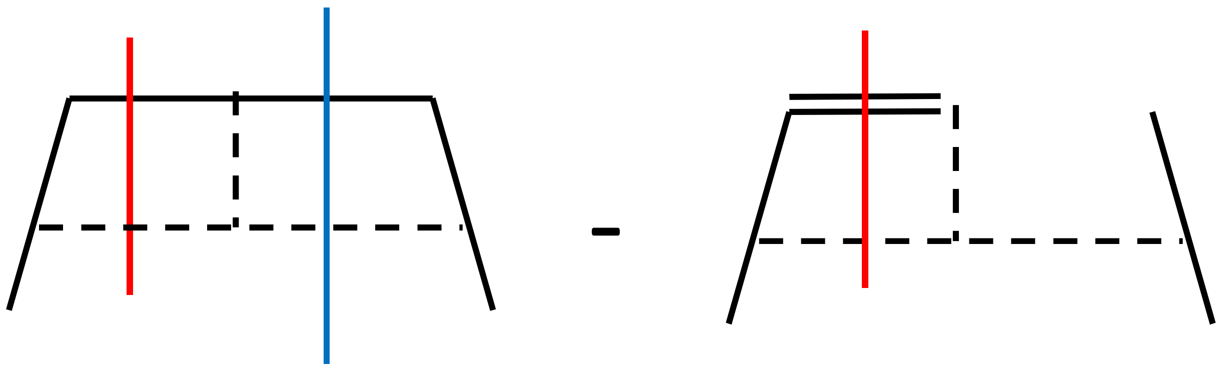
in the case of FFs, it is Collins function

Boer, Mulders 1997
Goeke, Meta, Schlegel 2005
Bacchetta et al., 2007

Phase in hard kernel

- For other sign combinations, or arbitrary transverse momenta
- phase appears in hard kernel

$H^{(2)} =$



- How to extract this phase?
- Use $\gamma_5 \gamma^{\perp}$
- A new contribution to SSA

2-parton twist-3 TMDs

$$\Phi^{[i\gamma_5]} = \frac{M}{P^+}\left[S_L\, e_L - \frac{p_T\cdot S_T}{M}\, e_T\right], \qquad \Phi^{[1]} = \frac{M}{P^+}\left[e - \frac{\epsilon_T^{\rho\sigma}\, p_{T\rho} S_{T\sigma}}{M}\, e_T^\perp\right]$$

$$\Phi^{[\gamma^\alpha]} = \frac{M}{P^+}\Bigg[-\epsilon_T^{\alpha\rho} S_{T\rho}\, f_T - S_L\,\frac{\epsilon_T^{\alpha\rho} p_{T\rho}}{M}\, f_L^\perp - \frac{p_T^\alpha p_T^\rho - \frac{1}{2}\, p_T^2\, g_T^{\alpha\rho}}{M^2}\,\epsilon_{T\rho\sigma} S_T^\sigma\, f_T^\perp + \frac{p_T^\alpha}{M} f^\perp\Bigg]$$

$$\Phi^{[\gamma^\alpha\gamma_5]} = \frac{M}{P^+}\Bigg[S_T^\alpha\, g_T + S_L\,\frac{p_T^\alpha}{M}\, g_L^\perp - \frac{p_T^\alpha p_T^\rho - \frac{1}{2}\, p_T^2\, g_T^{\alpha\rho}}{M^2}\, S_{T\rho}\, g_T^\perp - \frac{\epsilon_T^{\alpha\rho} p_{T\rho}}{M}\, g^\perp\Bigg]$$

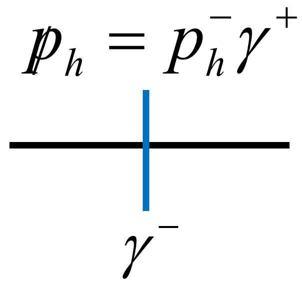
$$\Phi^{[i\sigma^{\alpha\beta}\gamma_5]} = \frac{M}{P^+}\left[\frac{S_T^\alpha p_T^\beta - p_T^\alpha S_T^\beta}{M}\, h_T^\perp - \epsilon_T^{\alpha\beta}\, h\right],$$

$$\Phi^{[i\sigma^{+-}\gamma_5]} = \frac{M}{P^+}\left[S_L\, h_L - \frac{p_T\cdot S_T}{M}\, h_T\right],$$

Boer, Mulders 1997
Goeke, Meta, and Schlegel 2005
Bacchetta et al., 2007

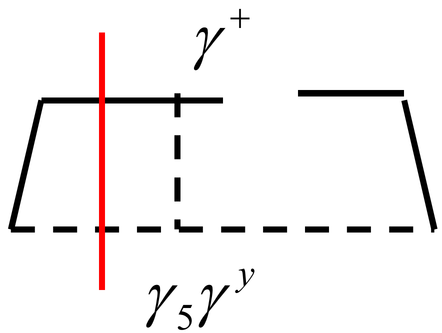
Factorization of new contribution

- Unpolarized twist-2 fragmentation function

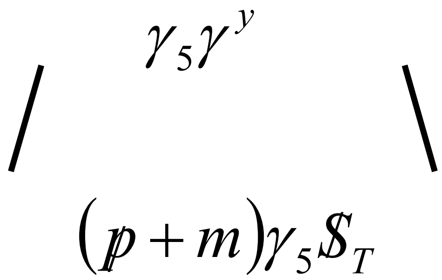


- Polarized quark scattered into direction preferred by correlation

$$tr\left[\gamma_5 \gamma^y \not{p}_{hT} \gamma^+ \gamma^- \cdots\right] = i\varepsilon_{yx+-} p_{hT}^x \cdots$$



- 2-parton twist-3 TMD g_T defined for polarized proton

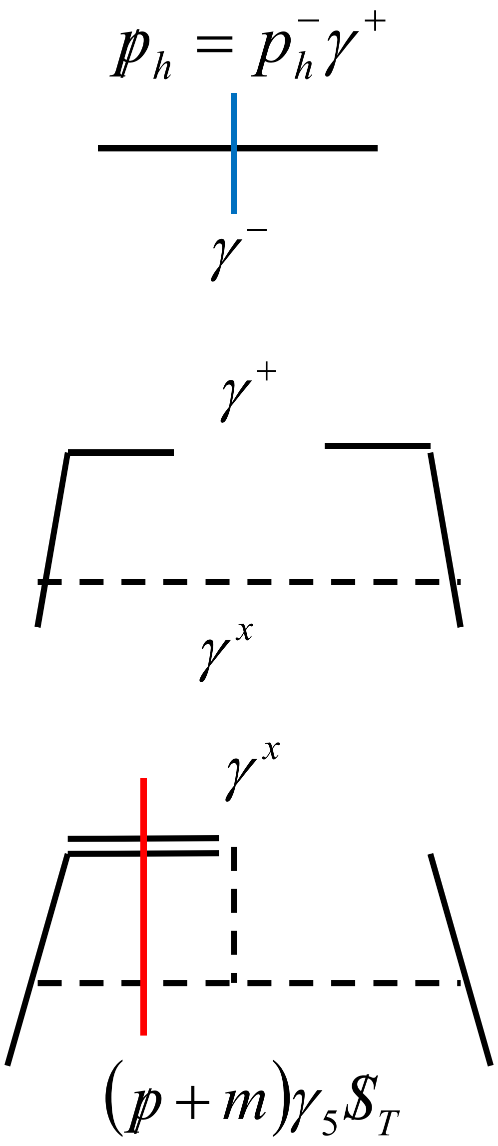


Lesson learned

- Sivers, ETQS, Collins functions all have same origin, resulting from different factorization
- Their contributions start from LO hard kernel
- If allowed to go to higher orders of hard kernel, other projectors can be used
- Though higher orders, data with Q ~ few GeV (such as COMPASS), hard kernel effect may be sizable
- Hard kernel is process-dependent
- Rich phenomenology!

At 3 loops

- At 3 loops, we can have 2-loop TMD for polarized proton and 1-loop hard kernel
- In addition to Sivers function, can use γ^x to extract phase in initial state in this case
- 2-parton twist-3 TMD f_T defined
- Another new contribution



2-parton twist-3 TMDs

$$\Phi^{[i\gamma_5]} = \frac{M}{P^+}\left[S_L\, e_L - \frac{p_T\cdot S_T}{M}\, e_T\right], \qquad \Phi^{[1]} = \frac{M}{P^+}\left[e - \frac{\epsilon_T^{\rho\sigma}\, p_{T\rho}\, S_{T\sigma}}{M}\, e_T^\perp\right]$$

$$\Phi^{[\gamma^\alpha]} = \frac{M}{P^+}\left[-\,\epsilon_T^{\alpha\rho}\, S_{T\rho}\, f_T - S_L\, \frac{\epsilon_T^{\alpha\rho}\, p_{T\rho}}{M}\, f_L^\perp - \frac{p_T^\alpha p_T^\rho - \frac{1}{2}\, p_T^2\, g_T^{\alpha\rho}}{M^2}\, \epsilon_{T\rho\sigma}\, S_T^\sigma\, f_T^\perp + \frac{p_T^\alpha}{M} f^\perp\right]$$

$$\Phi^{[\gamma^\alpha\gamma_5]} = \frac{M}{P^+}\left[S_T^\alpha\, g_T + S_L\, \frac{p_T^\alpha}{M}\, g_L^\perp - \frac{p_T^\alpha p_T^\rho - \frac{1}{2}\, p_T^2\, g_T^{\alpha\rho}}{M^2}\, S_{T\rho}\, g_T^\perp - \frac{\epsilon_T^{\alpha\rho}\, p_{T\rho}}{M}\, g^\perp\right]$$

$$\Phi^{[i\sigma^{\alpha\beta}\gamma_5]} = \frac{M}{P^+}\left[\frac{S_T^\alpha p_T^\beta - p_T^\alpha S_T^\beta}{M}\, h_T^\perp - \epsilon_T^{\alpha\beta}\, h\right],$$

$$\Phi^{[i\sigma^{+-}\gamma_5]} = \frac{M}{P^+}\left[S_L\, h_L - \frac{p_T\cdot S_T}{M}\, h_T\right],$$

Up to twist-3 NNLO

- Up to 2-parton twist-3 in TMD and FF, 2-loop in hard kernel, SSA is given by

$$
\begin{aligned}
d\sigma \;=\; & f_{1T}^{\perp} \otimes H^{(0)}_{\gamma^-,\gamma^+} \otimes D_1 + f_{1T}^{\perp} \otimes H^{(1)}_{\gamma^-,\gamma^x} \otimes D^{\perp} + f_{1T}^{\perp} \otimes H^{(2)}_{\gamma^-,\gamma_5\gamma^x} \otimes G^{\perp} \\
& + g_{1T} \otimes H^{(2)}_{\gamma_5\gamma^-,\gamma^+} \otimes D_1 + g_{1T} \otimes H^{(1)}_{\gamma_5\gamma^-,\gamma_5\gamma^y} \otimes G^{\perp} + g_{1T} \otimes H^{(2)}_{\gamma_5\gamma^-,\gamma^y} \otimes D^{\perp} \\
& + h_1 \otimes H^{(0)}_{\gamma_5\sigma^{y-},\gamma_5\sigma^{y+}} \otimes H_1^{\perp} + h_1 \otimes H^{(1)}_{\gamma_5\sigma^{y-},\gamma_5\sigma^{yx}} \otimes H^{*} + h_1 \otimes H^{(2)}_{\gamma_5\sigma^{y-},I} \otimes E^{*} \\
& + e_T \otimes H^{(1)}_{\gamma_5,\gamma_5\sigma^{y+}} \otimes H_1^{\perp} + e_T^{\perp} \otimes H^{(2)}_{I,\gamma_5\sigma^{y+}} \otimes H_1^{\perp} \\
& + f_T \otimes H^{(1)}_{\gamma^y,\gamma^+} \otimes D_1 + g_T \otimes H^{(2)}_{\gamma_5\gamma^y,\gamma^+} \otimes D_1 \\
& + h_T^{\perp} \otimes H^{(1)}_{\gamma_5\sigma^{yx},\gamma_5\sigma^{y+}} \otimes H_1^{\perp} + h_T \otimes H^{(1)}_{\gamma_5\sigma^{-+},\gamma_5\sigma^{y+}} \otimes H_1^{\perp},
\end{aligned}
$$

- Similar discussion on jet production by Song, Gao, Liang, Wang, 2011, 2014

gT contribution

- Have given comprehensive picture for all known SSA sources
- Only gT term survives in collinear limit (with kT integrated out) Ji, 1993
- Calculation is similar to Ma and Sang's quark target model at two loops; they focused on IR cancellation for known sources. Ma, Sang, 2009
- Why did they not notice this contribution?
- Because gT contribution IR finite at this order?

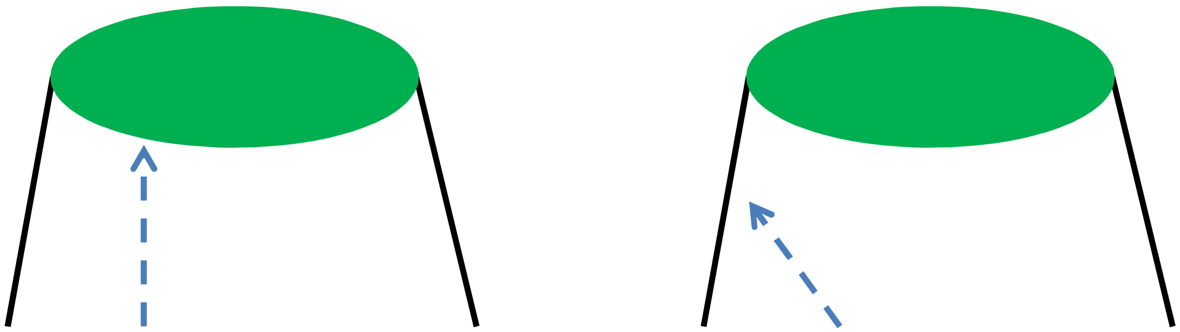
QED Gauge invariance (GI)

- Three-parton contribution is not QED gauge invariant: no complete photon contractions
- Need two-parton reducible diagram

- Boer, Qiu (2001) used special propagator to include it, but did not notice it is gT
- Jaffe, Ji (1992) noticed it, but proved GI at LO

QED and QCD GI

- We proved QED and QCD gauge invariance for combination of gT and three-parton contributions to all orders (see 1909.10864)
- For QCD, three partons do not contain complete contractions of valence gluon, so not gauge invariant by themselves

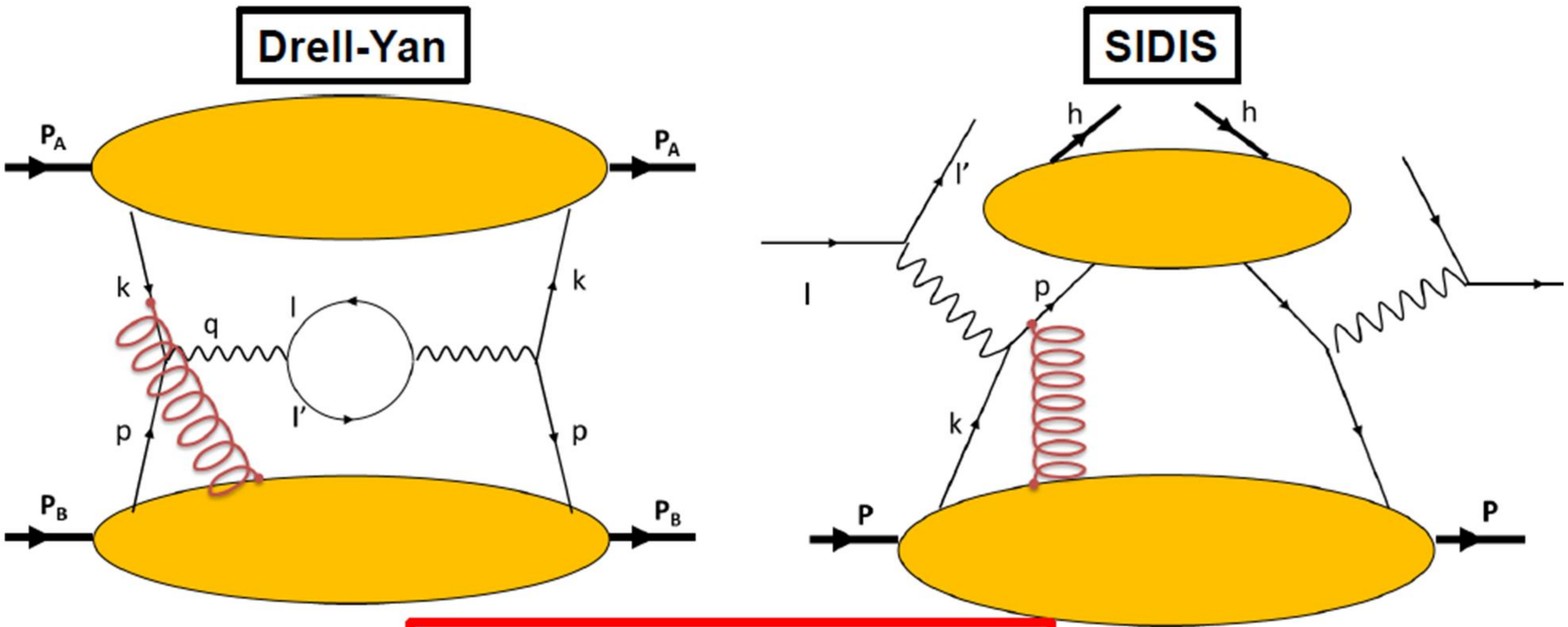


Summary and outlook

- There are many sources to SSA
- Have derived complete 2-parton twist-3 contributions to SSA up to 2 loop hard kernel
- Only gT survives in collinear factorization, needed for QED and QCD gauge invariance of 3-parton contributions
- Rich phenomenology is expected, eg., impact on extraction of Sivers, Collins functions?
- Will study subleading contributions to understand all data and identify origin of SSA

Back-up slides

Sign change of Sivers function



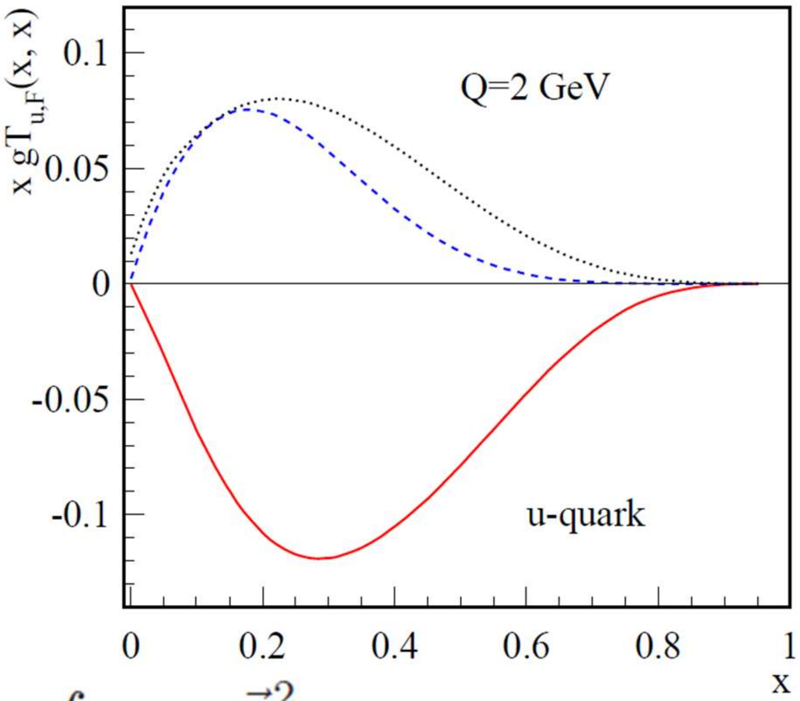
$$\text{Sivers}\,|_{DY} = -1 * \text{Sivers}\,|_{SIDIS}$$

Sign-mismatch problem

- No sign flip seen in $p^{\uparrow}p \to \pi + X$

Kang, Qiu, Vogelsang, Yuan 2011

correlation function for polarized proton, assumed to dominate



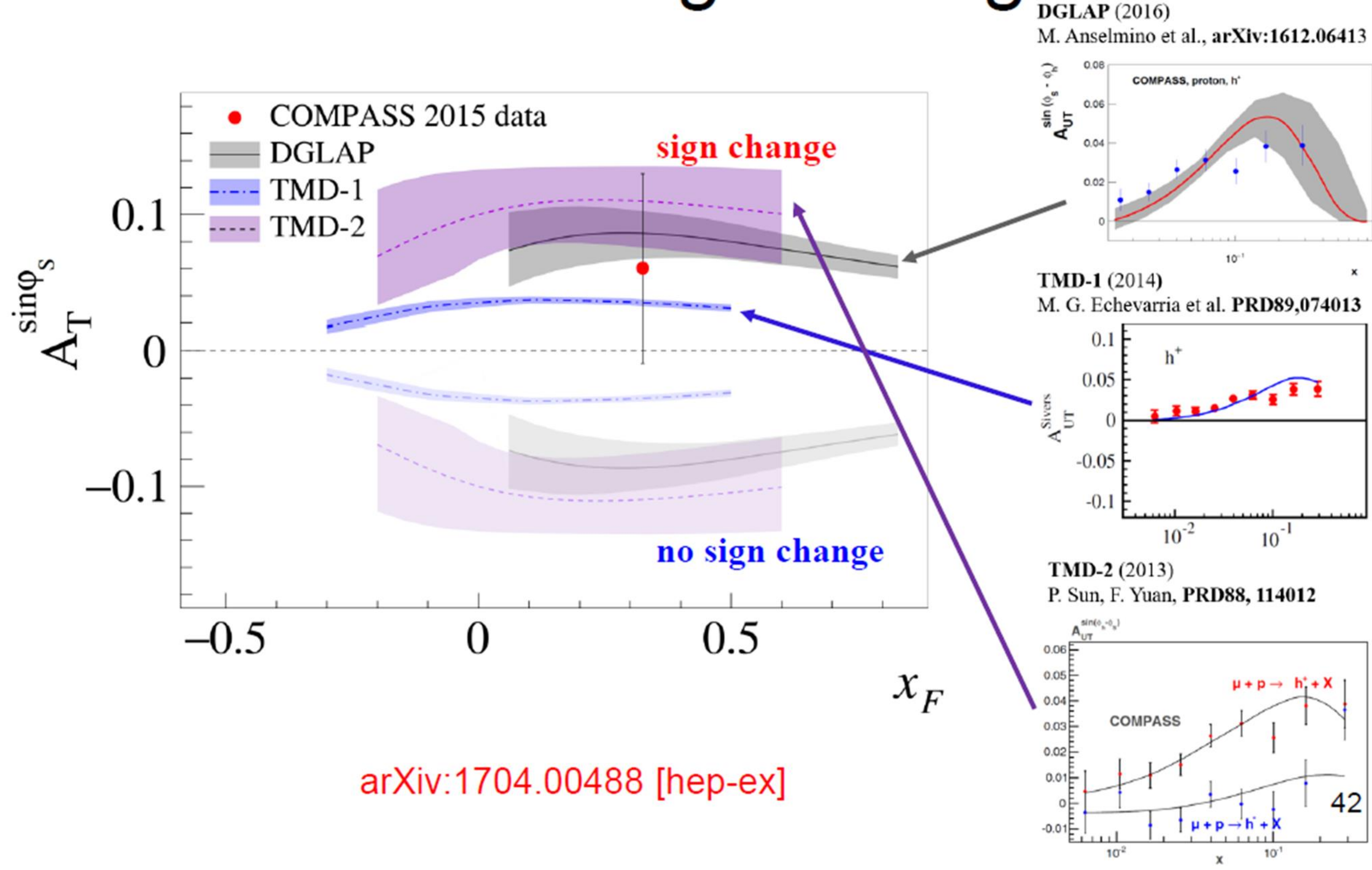
expectation from SIDIS data (HERMES, COMPASS) under sign flip

result extracted from data (E704, STAR, PHENIX, BRAHMS)

$$T_F^q(x,x) = -\int d^2\vec{p}_\perp \frac{\vec{p}_\perp^{\,2}}{M} f_{1T}^{\perp q}(x,\vec{p}_\perp^{\,2})|_{\mathrm{SIDIS}}$$

- Now there are other twist-3 contributions…

Sivers Asymmetry in Drell-Yan: Hint of Sign Change!

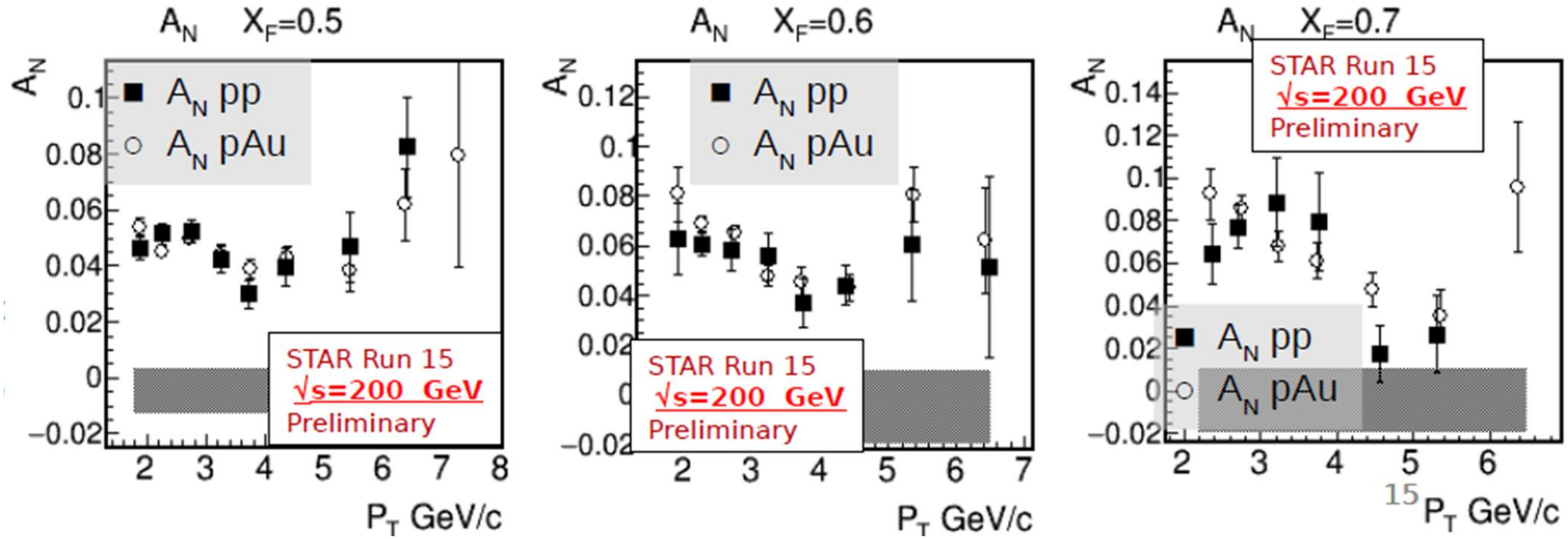


arXiv:1704.00488 [hep-ex]

Origin of SSA?

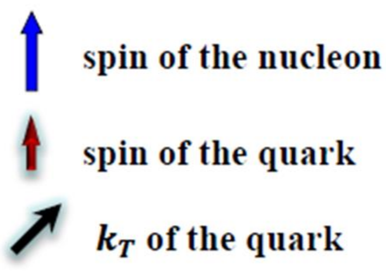
- Large twist-3 fragmentation function?
  Kanazawa, Koike, Metz, Pitonyak, 2014
- kT factorization does not apply to Drell-Yan
- Twist-3 FF implies $A_N \propto A^{-1/3}$ in pA collision?
  Hatta, Xiao, Yoshidac, Yuan 2016

STAR, DIS 2016 showed $A_N \sim A^0$



Need to study complete subleading contributions to understand all data and identify origin of SSA!

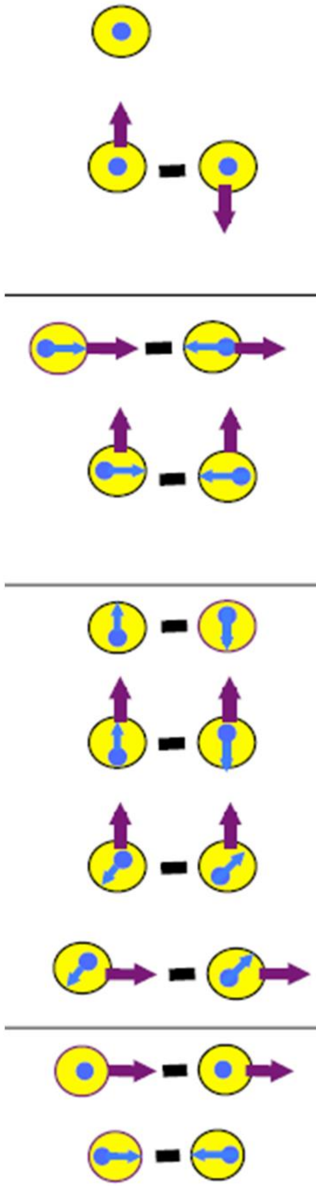
Twist-2 TMDs



Nucleon \ Quark	U	L	T
U	number density $f_1^q(x, k_T^2)$		Boer-Mulders $h_1^{q\perp}(x, k_T^2)$
L		Helicity $g_1^q(x, k_T^2)$	worm-gear L $h_{1L}^{q\perp}(x, k_T^2)$
T	Sivers $f_{1T}^{q\perp}(x, k_T^2)$	Kotzinian-Mulders worm-gear T $g_{1T}^{q\perp}(x, k_T^2)$	Transversity $h_1^q(x, k_T^2)$ Pretzelosity $h_{1T}^{q\perp}(x, k_T^2)$

kT can be integrated
out in transverse
distribution

Twist-3 TMDs



$e(x,k_\perp)$, $f^\perp(x,k_\perp)$	number density
$e_T^\perp(x,k_\perp)$, $f_T^{\perp 1}(x,k_\perp)$, $f_T^{\perp 2}(x,k_\perp)$	Sivers function
$e_L(x,k_\perp)$, $g_L^\perp(x,k_\perp)$	helicity distribution
$e_T(x,k_\perp)$, $g_T(x,k_\perp)$, $g_T^\perp(x,k_\perp)$	Worm gear: trans-helicity
$h(x,k_\perp)$	Boer-Mulders function
$h_T^\perp(x,k_\perp)$	transversity distribution
$h_T(x,k_\perp)$	pretzelocity
$h_L(x,k_\perp)$	Worm gear: longi-transversity
$f_L^\perp(x,k_\perp)$, $g^\perp(x,k_\perp)$	