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Future progress with EW precision calculations

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Outline

- motivations: precision tests of the Standard Model and searches for New Physics signals
- discussion about the MW and the $\sin^2\theta_{eff}$ determinations at a future e+e- collider

- evolution of the precision measurement problem, from LEP to LHC to future e+e- colliders

- measurement: comparison of "a" model against the data
  which model? which Pseudo-Observables?
  which simulation code? EW input scheme?
  → methodological challenges

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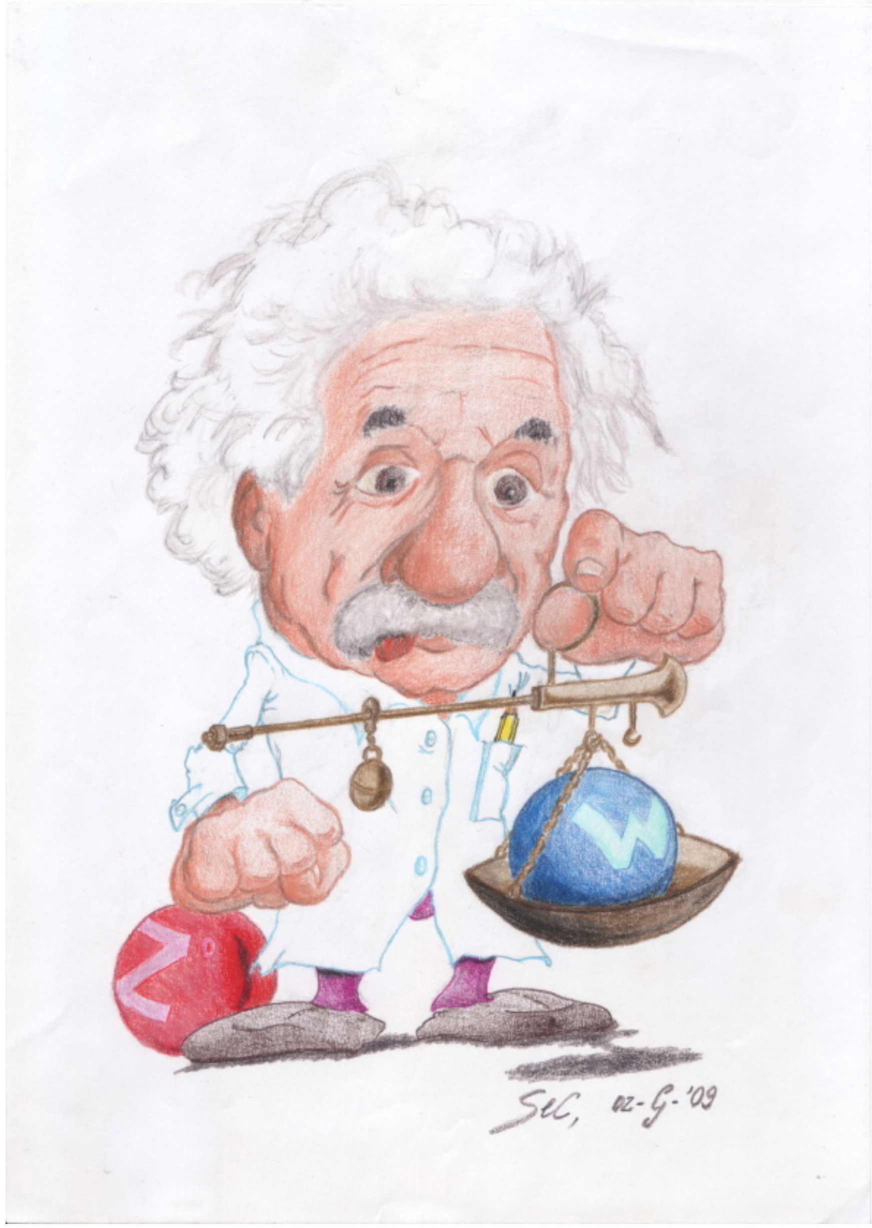
our weight scales and rulers!



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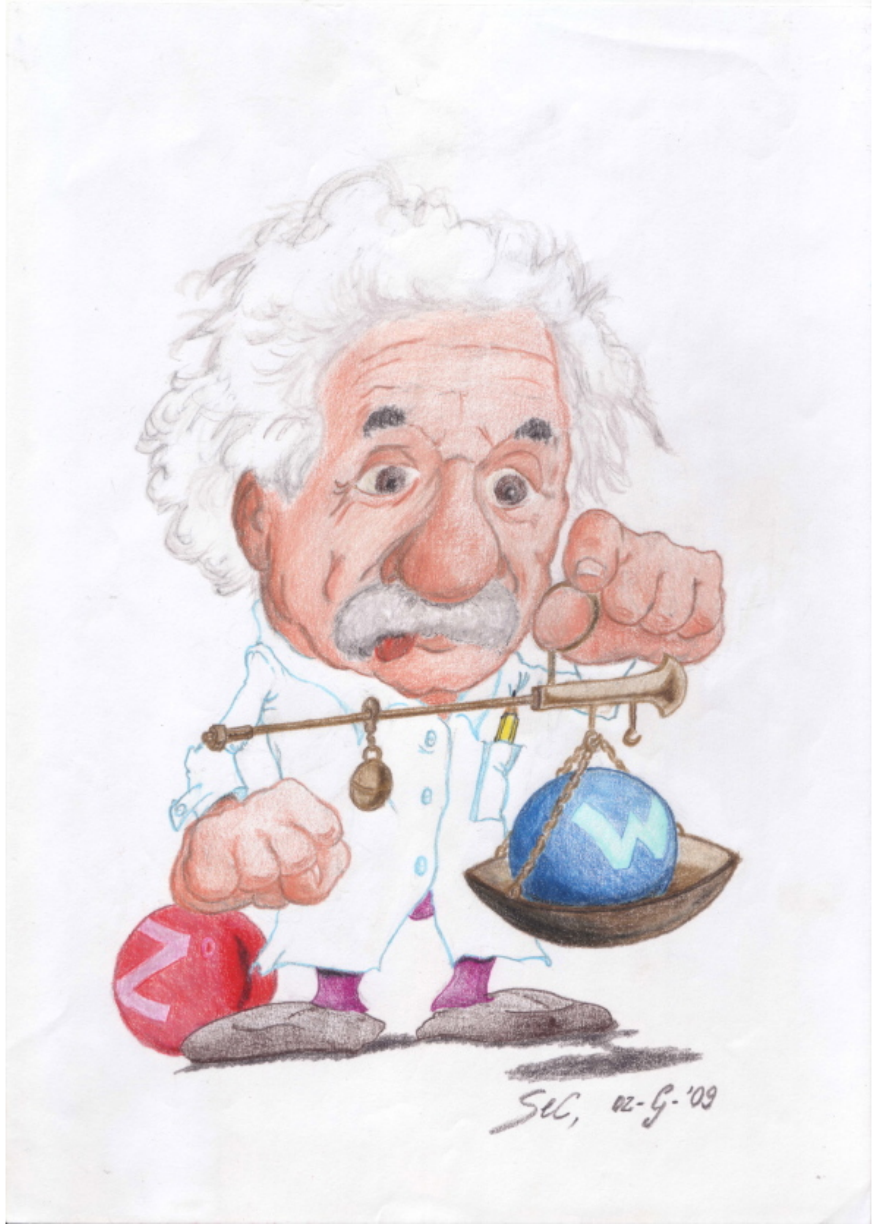

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  of matching procedures of fixed- and all-order results
  → technical challenges

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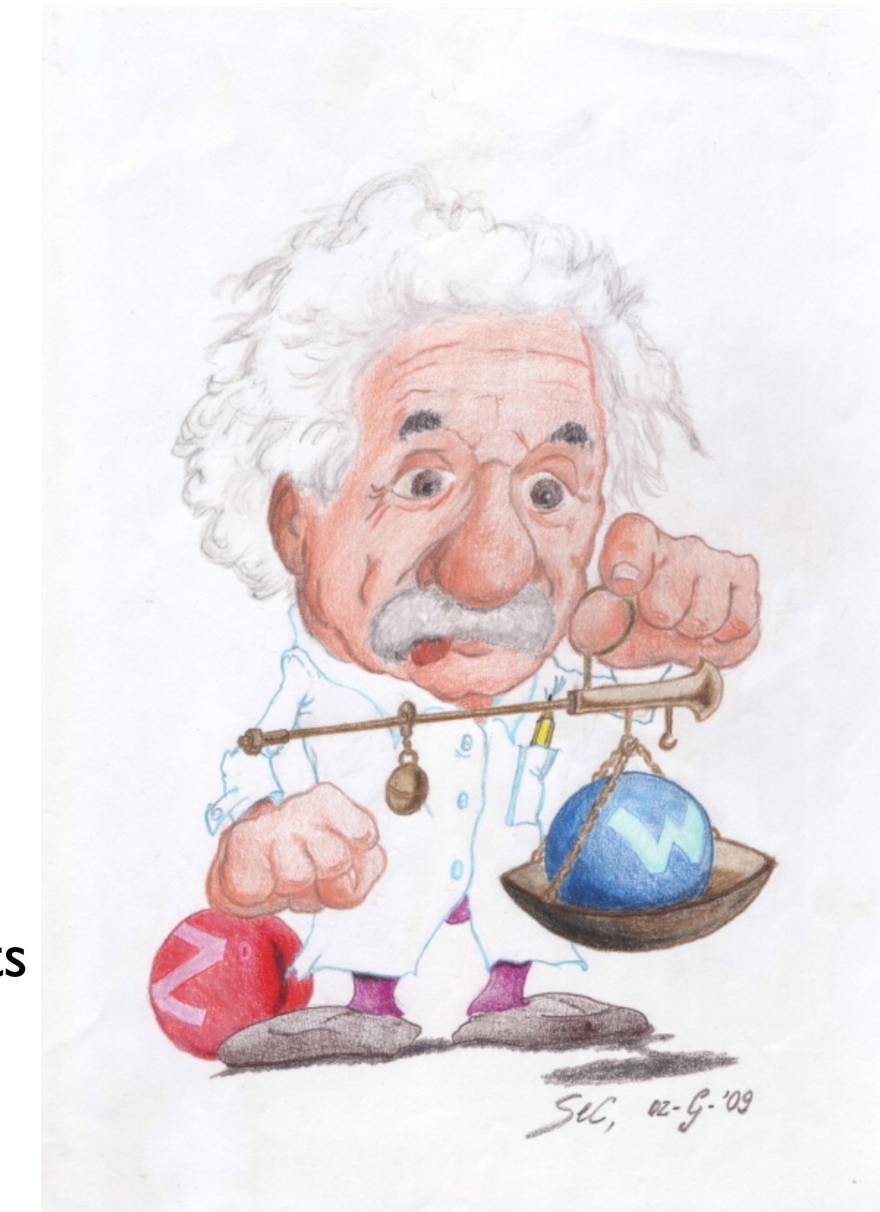


- development of new fixed-order calculations
  of matching procedures of fixed- and all-order results
  → technical challenges

how can we exploit the great expected experimental precision
and determine with corresponding precision some of the fundamental parameters of Nature?

Disclaimer

- an important activity has taken place in the last 3 years, focussing on the theoretical issues relevant for the precision physics program at future e+e- colliders, CEPC and FCC-ee and is documented in several reports, where complete lists of references can be found
  - arXiv:1703.01626 Physics beyond precision
  - arXiv: 1809.01830 Standard Model Theory for the FCC Tera-Z stage
  - arXiv:1811.10545 CEPC Conceptual Design Report
  - arXiv:1905.05078 Theory report of the 11th FCC-ee workshop
  - arXiv:1906.05739 Theoretical uncertainties to electroweak and Higgs-boson precision measurement at FCC-ee
  - arXiv:1909.12245 Polarization and Centre-of-mass energy calibration at FCC-ee

- I am indebted with all the colleagues of the LHC EW-WG and my colleagues from Pavia for continuous discussions on the MW and $\sin^2\theta_{eff}$ determination at hadron colliders

Motivations

from the Fermi theory to the current best predictions of MW and $\sin^2\theta$

From the Fermi theory of weak interactions to the discovery of W and Z

Fermi theory of β decay

muon decay $\mu^- \to \nu_\mu e^- \bar{\nu}_e$ $$\frac{1}{\tau_\mu} \to \Gamma_\mu \to G_\mu$$

QED corrections to Γ_μ necessary for precise determination of G_μ
computable in the Fermi theory (Kinoshita, Sirlin, 1959)

The independence of the QED corrections of the underlying model (Fermi theory vs SM) allows

- to define G_μ and to measure its value with high precision

$$G_\mu = 1.1663787(6)\ 10^{-5}\ \ \mathrm{GeV}^{-2}$$

- to establish a relation between G_μ and the SM parameters

$$\frac{G_\mu}{\sqrt{2}} = \frac{g^2}{8 m_W^2}(1+\Delta r)$$

The properties of physics at the EW scale
with sensitivity to the full SM and possibly to BSM via virtual corrections (Δr)
are related to a very well measured low-energy constant

From the Fermi theory of weak interactions to the discovery of W and Z

The SM predicts the existence of a new neutral current, different than the electromagnetic one
(Glashow 1961, Weinberg 1967, Salam 1968)

The observation of weak neutral current immediately allowed the estimate of the value of the weak mixing angle in the correct range
GARGAMELLE, Phys.Lett. 46B (1973) 138-140

From the basic relation among the EW parameters it was immediately possible to estimate the order of magnitude of the mass of the weak bosons, in the 80 GeV range
(Antonelli, Maiani, 1981)

The discovery at the CERN SPPS of the W and Z bosons and the first determination of their masses allowed the planning of a new phase of precision studies accomplished with the construction of two e^+e^- colliders (SLC and LEP) running at the Z resonance

The precise determination of MZ and of the couplings of the Z boson to fermions
and in particular the value of the effective weak mixing angle
allowed to establish a framework for a test of the SM at the level of its quantum corrections

There is evidence of EW corrections beyond QED with 26 σ significance!
Full 1-loop and leading 2-loop radiative corrections are needed to describe the data
(indirect evidence of bosonic quantum effects)

The renormalisation of the SM and a framework for precision tests

- The Standard Model is a renormalizable gauge theory based on $SU(3) \times SU(2)_L \times U(1)_Y$
- The gauge sector of the SM lagrangian is assigned specifying (g, g', v, λ) in terms of 4 measurable inputs
- More observables can be computed and expressed in terms of the input parameters, including the available radiative corrections, at any order in perturbation theory
- The validity of the SM can be tested comparing these predictions with the corresponding experimental results

- The input choice $(g, g', v, \lambda) \leftrightarrow (\alpha, G_\mu, MZ, MH)$ minimises the parametric uncertainty of the predictions

$$
\begin{aligned}
\alpha(0) &= 1/137.035999139(31) \\
G_\mu &= 1.1663787(6) \times 10^{-5}\ \mathrm{GeV}^{-2} \\
m_Z &= 91.1876(21)\ \mathrm{GeV}/c^2 \\
m_H &= 125.09(24)\ \mathrm{GeV}/c^2
\end{aligned}
$$

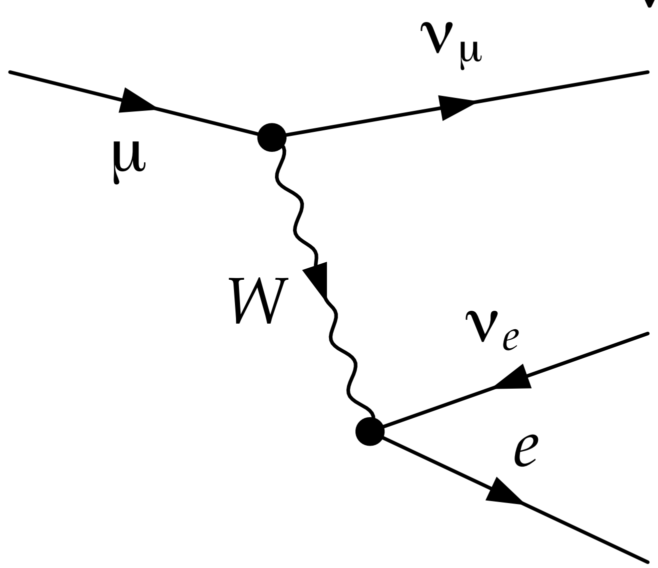
- with these inputs, MW and the weak mixing angle are predictions of the SM, to be tested against the experimental data

The W boson mass: theoretical prediction

$$\mathcal{L}_{SM} = \mathcal{L}_{SM}(\alpha,\, G_\mu,\, m_Z; m_H; m_f; CKM)$$

$\rightarrow$ we can compute m_W

$$\frac{G_\mu}{\sqrt{2}} = \frac{g^2}{8m_W^2}\,(1+\Delta r)$$



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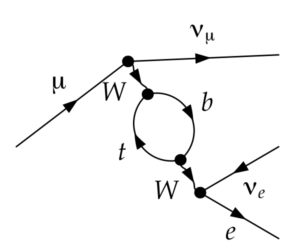

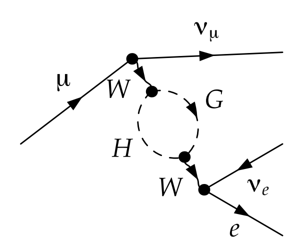

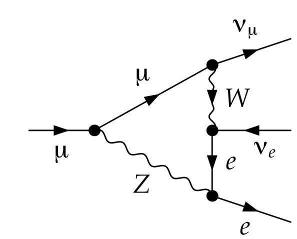

+ ....

$$\Delta r = \Delta r(\alpha, G_\mu, m_Z, m_H; m_f; CKM)$$

$$m_W^2 = \frac{m_Z^2}{2}\left(1+\sqrt{1-\frac{4\pi\alpha}{G_\mu\sqrt{2}m_Z^2}(1+\Delta r)}\right)$$

The W boson mass: theoretical prediction

Sirlin, 1980, 1984; Marciano, Sirlin, 1980, 1981;
van der Bij, Veltman, 1984; Barbieri, Ciafaloni, Strumia 1993;
Djouadi, Verzegnassi 1987; Consoli, Hollik, Jegerlehner, 1989;
Chetyrkin, Kühn, Steinhauser, 1995;
Barbieri, Beccaria, Ciafaloni, Curci, Viceré, 1992, 1993; Fleischer, Tarasov, Jegerlehner, 1993;
Degrassi, Gambino, AV, 1996; Degrassi, Gambino, Sirlin, 1997;
Freitas, Hollik, Walter, Weiglein, 2000, 2003;
Awramik, Czakon, 2002; Awramik, Czakon, Onishchenko, Veretin, 2003; Onishchenko, Veretin, 2003

The best available prediction includes
the full 2-loop EW result, higher-order QCD corrections, resummation of reducible terms

$$m_W = w_0 + w_1 dH + w_2 dH^2 + w_3 dh + w_4 dt + w_5 dH dt + w_6 da_s + w_7 da^{(5)}$$

$$dt = [(M_t/173.34\,\mathrm{GeV})^2 - 1]$$

$$da^{(5)} = [\Delta\alpha^{(5)}_{\mathrm{had}}(m_Z^2)/0.02750 - 1]$$

$$dH = \ln\left(\frac{m_H}{125.15\,\mathrm{GeV}}\right)$$

$$dh = [(m_H/125.15\ \mathrm{GeV})^2 - 1].$$

$$da_s = \left(\frac{\alpha_s(m_Z)}{0.1184} - 1\right)$$

	$124.42 \le m_H \le 125.87$ GeV	$50 \le m_H \le 450$ GeV
w_0	80.35712	80.35714
w_1	-0.06017	-0.06094
w_2	0.0	-0.00971
w_3	0.0	0.00028
w_4	0.52749	0.52655
w_5	-0.00613	-0.00646
w_6	-0.08178	-0.08199
w_7	-0.50530	-0.50259

G.Degrassi, P.Gambino, P.Giardino, arXiv:1411.7040

The weak mixing angle(s): theoretical prediction(s)

- the prediction of the weak mixing angle can be computed in different renormalisation schemes differing for the systematic inclusion of large higher-order corrections

- on-shell definition:

  Sirlin, 1980

$$\sin^2\theta_{OS} = 1 - \frac{m_W^2}{m_Z^2}$$

definition valid to all orders

- MSbar definition:

  Marciano, Sirlin, 1980; Degrassi, Sirlin, 1991

$$\frac{G_\mu}{\sqrt{2}} = \frac{g_0^2}{8m_{W,0}^2} \longrightarrow \hat{s}^2\hat{c}^2 = \frac{\pi\alpha}{\sqrt{2}G_\mu m_Z^2(1-\Delta\hat{r})} \qquad \hat{s}^2 \equiv \sin^2\hat{\theta}$$

weak dependence on top-quark corrections

Awramik, Czakon, Freitas, hep-ph/0608099

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weak dependence on top-quark corrections

- the effective leptonic weak mixing angle enters in the definition of the effective Z-f-fbar vertex at the Z resonance

$$\mathcal{M}^{eff}_{Zl^+l^-} = \bar{u}_l\gamma_\alpha\left[\mathcal{G}_v^f(m_Z^2) - \mathcal{G}_a^f(m_Z^2)\gamma_5\right]v_l\varepsilon_Z^\alpha \qquad 4|Q_f|\sin^2\theta^f_{eff} = 1 - \frac{g_V^f}{g_A^f}$$

and can be computed in the SM (or in other models) in different renormalisation schemes

$$\sin^2\theta^{lep}_{eff} = \kappa(m_Z^2)\sin^2\theta_{OS} = \hat{\kappa}(m_Z^2)\sin^2\hat{\theta}$$

Awramik, Czakon, Freitas, hep-ph/0608099

The effective leptonic weak mixing angle: theoretical prediction

parameterization of the full two-loop EW calculation + different sets of 3- and 4-loop corrections

I.Dubovyk, A.Freitas, J.Gluza, T.Riemann, J.Usovitsch, arXiv:1906.08815

$$\sin^2\theta^f_{\rm eff} = s_0 + d_1 L_H + d_2 L_H^2 + d_3 L_H^4 + d_4 \Delta_\alpha + d_5 \Delta_{\rm t} + d_6 \Delta_{\rm t}^2 + d_7 \Delta_{\rm t} L_H + d_8 \Delta_{\alpha_{\rm s}} + d_9 \Delta_{\alpha_{\rm s}} \Delta_{\rm t} + d_{10} \Delta_{\rm Z}$$

$$L_{\rm H} = \log \frac{M_{\rm H}}{125.7\,{\rm GeV}}, \qquad \Delta_{\rm t} = \left(\frac{m_{\rm t}}{173.2\,{\rm GeV}}\right)^2 - 1,$$

$$\Delta_{\alpha_{\rm s}} = \frac{\alpha_{\rm s}(M_{\rm Z})}{0.1184} - 1, \qquad \Delta_\alpha = \frac{\Delta\alpha}{0.059} - 1, \qquad \Delta_{\rm Z} = \frac{M_{\rm Z}}{91.1876\,{\rm GeV}} - 1$$

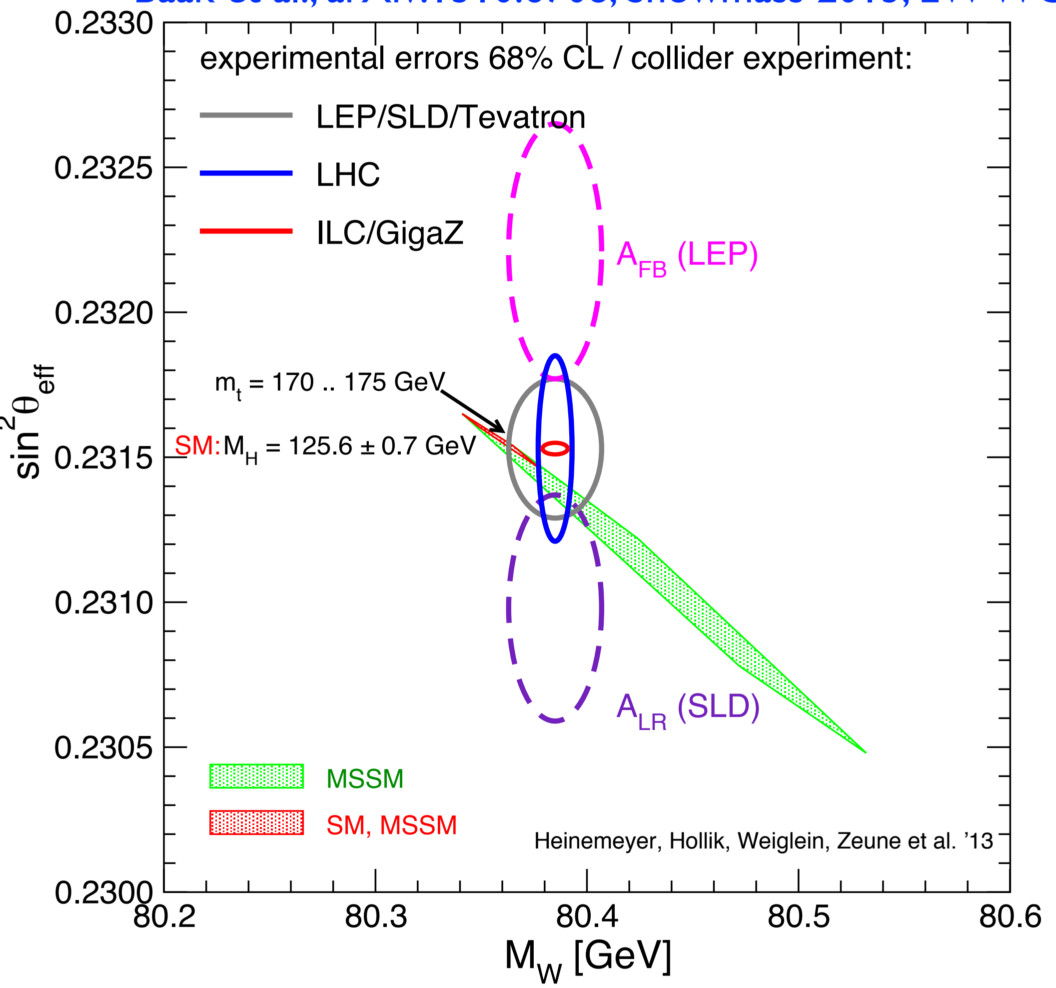
Observable	s_0	d_1	d_2	d_3	d_4	d_5
$\sin^2\theta^\ell_{\rm eff} \times 10^4$	2314.64	4.616	0.539	-0.0737	206	-25.71
$\sin^2\theta^b_{\rm eff} \times 10^4$	2327.04	4.638	0.558	-0.0700	207	-9.554

Observable	d_6	d_7	d_8	d_9	d_{10}	max. dev.
$\sin^2\theta^\ell_{\rm eff} \times 10^4$	4.00	0.288	3.88	-6.49	-6560	< 0.056
$\sin^2\theta^b_{\rm eff} \times 10^4$	3.83	0.179	2.41	-8.24	-6630	< 0.025

Awramik, Czakon, Freitas, hep-ph/0608099

Relevance of new high-precision measurement of EW parameters

Baak et al., arXiv:1310.6708, Snowmass 2013, EW WG



The precision measurement of MW and $\sin^2\theta_{eff}$

with an error of 0.7 MeV and 0.000004

(5 MeV and 0.000100 at a hadron collider)

(formidable challenges!)

would offer a very stringent test of the SM likelihood

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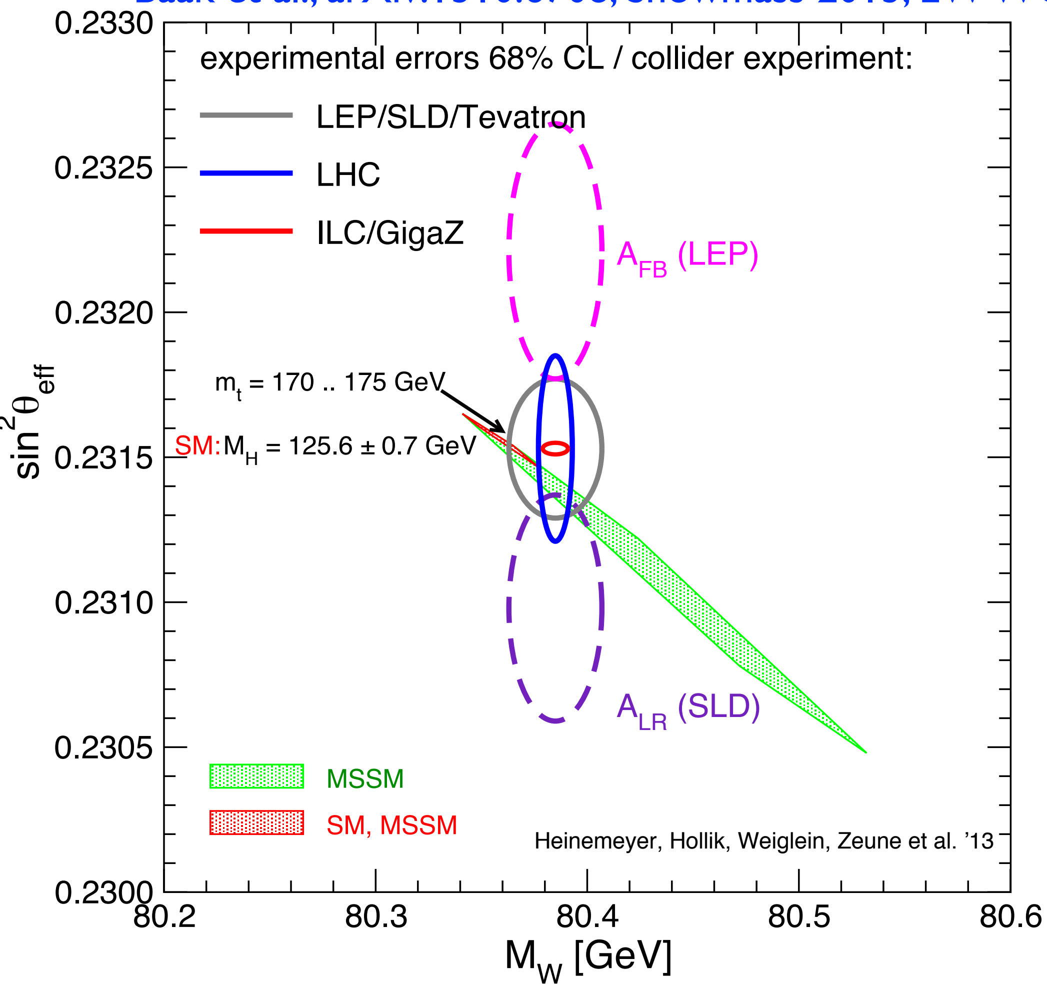

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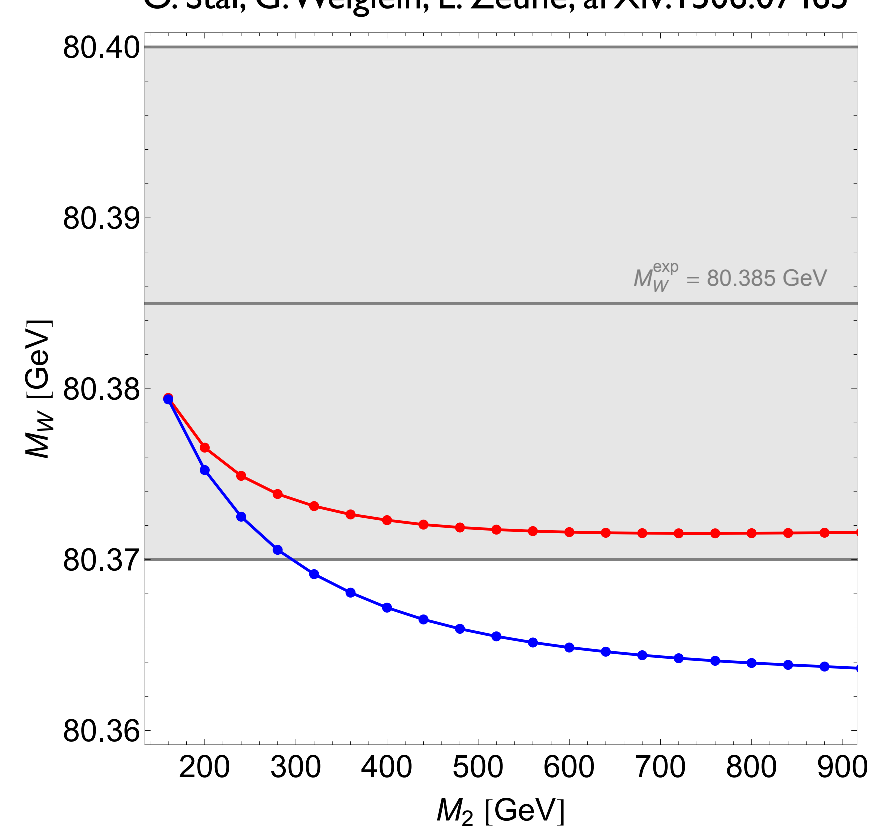
(formidable challenges!)
would offer a very stringent test of the SM likelihood

In the case a BSM particle had been discovered
a very precise MW value would offer
a strongly discriminating tool about the mass spectra
in BSM models

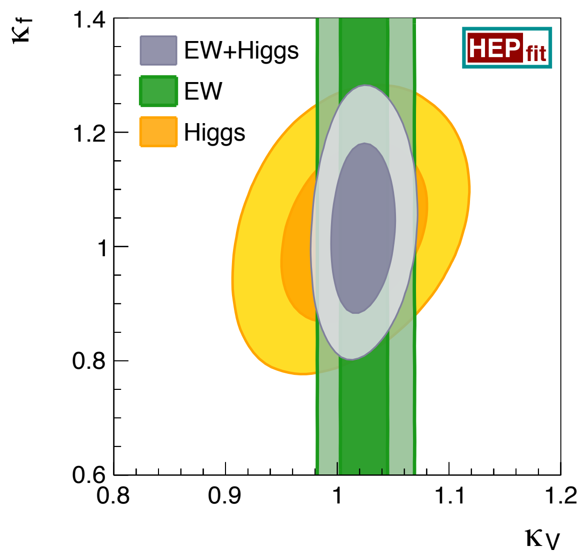
different dependence on the neutralino mass M_2
of the MW prediction in the MSSM and NMSSM

O. Stål, G. Weiglein, L. Zeune, arXiv:1506.07465



Relevance of new high-precision measurement of EW parameters

de Blas et al, arXiv:1608.01509

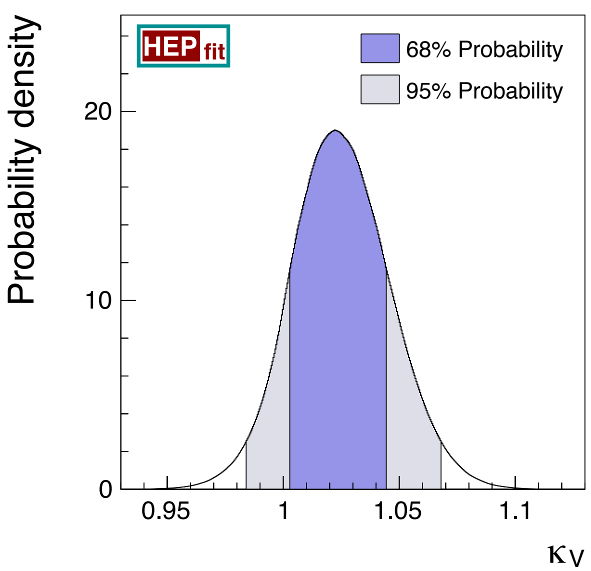


$$\mathcal{L}_{\rm Eff} = \sum_{d=4}^{\infty} \frac{1}{\Lambda^{d-4}} \mathcal{L}_d = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda}\mathcal{L}_5 + \frac{1}{\Lambda^2}\mathcal{L}_6 + \cdots$$

$$\mathcal{L}_d = \sum_i C_i^d \mathcal{O}_i \qquad [\mathcal{O}_i] = d \longrightarrow \left(\frac{q}{\Lambda}\right)^{d-4}$$

Effects suppressed by $q = v, E < \Lambda$

Λ: Cut-off of the EFT



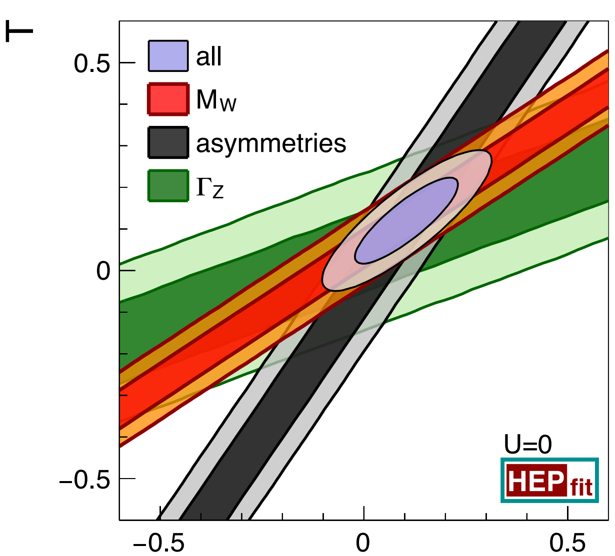
$$\mathcal{O}_{\phi WB} = \phi^\dagger \sigma_a \phi B^{\mu\nu} W^a_{\mu\nu}$$

EWSB →

$v^2 B^{\mu\nu} W^3_{\mu\nu}$ gauge boson masses

$vh B^{\mu\nu} W^3_{\mu\nu}$ $h \to ZZ, \gamma\gamma$

$$M_W^2 = M_Z^2 c^2 \left[1 - \frac{c^2}{c^2 - s^2} \left(\frac{1}{2} C_{\phi D} + 2\frac{s}{c} C_{\phi WB} + \frac{s^2}{c^2} \Delta_{G_\mu} \right) \frac{v^2}{\Lambda^2} \right]$$



A precise measurement of MW and of $\sin^2\theta_{\rm eff}$ constrains several dim-6 operators contributing to Higgs and gauge interaction vertices.

Today still one of the strongest constraints

High-precision measurements

MW and $\sin^2\theta$ determination at colliders

Projections based on the expected statistics and systematics

Observable	LEP precision	CEPC precision	CEPC runs	CEPC $\int \mathcal{L}dt$
m_Z	2.1 MeV	0.5 MeV	Z pole	8 ab^{-1}
Γ_Z	2.3 MeV	0.5 MeV	Z pole	8 ab^{-1}
$A_{FB}^{0,b}$	0.0016	0.0001	Z pole	8 ab^{-1}
$A_{FB}^{0,\mu}$	0.0013	0.00005	Z pole	8 ab^{-1}
$A_{FB}^{0,e}$	0.0025	0.00008	Z pole	8 ab^{-1}
$\sin^2\theta_W^{\text{eff}}$	0.00016	0.00001	Z pole	8 ab^{-1}
R_b^0	0.00066	0.00004	Z pole	8 ab^{-1}
R_μ^0	0.025	0.002	Z pole	8 ab^{-1}
m_W	33 MeV	1 MeV	WW threshold	2.6 ab^{-1}
m_W	33 MeV	2–3 MeV	ZH run	5.6 ab^{-1}
N_ν	1.7%	0.05%	ZH run	5.6 ab^{-1}

How can we keep the theoretical systematics under control?

$\sin^2\theta_{eff}$ determination at future colliders: which strategies?

Which is our primary goal?

· Consistency check of the SM or · Indirect search for New Physics signals

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Consistency check of the SM

· Fit of the data in the SM and comparison with the SM theoretical prediction

An EW scheme with $\sin^2\theta_{eff}$ as input parameter
allows to avoid the introduction of pseudo observables
in favour of a direct fit of the observables in the SM

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Indirect search for New Physics signals

- The pseudo observable decomposition of the Z resonance

  it allows to establish a direct link to New Physics entering via the oblique corrections

- Fit of the data in the SMEFT, with $\sin^2\theta_{eff}$ and all the Wilson coefficients as input parameters

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Pro's and con's

Z resonance pseudo observable decomposition → highly developed since LEP, most mature approach

Fit in full (SM or SMEFT) with $\sin^2\theta_{eff}$ as input → direct relation of observables and parameters

clear estimate of the systematic uncertainties 😊

more development needed beyond NLO-EW 😟

Vocabulary

Observables quantities accessible via counting experiments
cross sections and asymmetries

Pseudo-Observables quantities that are functions of the cross section and asymmetries
require a model to be properly defined
- the Z boson mass at LEP as the pole of the Breit-Wigner resonance factor
- any cross section subtracted of QED/QCD universal corrections
- the Z boson decay widths

Template fit
- several histograms describing a differential distribution, computed in a given model, with the highest available theoretical accuracy and degree of realism in the detector simulation letting the fit parameter (e.g. MW) vary in a range
- the histogram that best describes the data selects the preferred, i.e. measured, MW value
- the result of the fit depends
  1) on the chosen model
  2) on the hypotheses used to compute the templates (→ theoretical systematic errors)
- accurate calculations, properly implemented in Monte Carlo event generators are needed to reduce this systematic error

Model dependency
- new physics might affect the kinematical distributions via virtual corrections (whose impact depends on the specific formulation of the event generator)

how different is the result for MW with MSSM templates vs SM templates ?

Fit of observables, parameter determination and EW input schemes

Fit of observables, parameter determination and EW input schemes

Parameter determination:

The templates are theoretical predictions, functions only of the lagrangian input parameter

e.g. in the SM $\mathscr{T} = \mathscr{T}(g, g', v; \lambda; m_f; CKM)$

We choose a set of experimental quantities (EW inputs) to express the lagrangian couplings.
All the other pseudoobservables and parameters are predictions,
which can be tested but not used as fit parameters.

examples: at LEP1 the choice (α, G_μ, MZ, MH) as inputs allowed to determine MZ,

at LEP2 for the MW determination introduction of the (G_μ, MW, MZ, MH) scheme

(no-one would have used (α, G_μ, MZ, MH) as input scheme to fit MW)

in these two schemes $\sin^2\theta_{eff}$ is a prediction and can not be used as a fit parameter!

3 criteria for the choice of an input scheme

i) we have to fit one of the input param's

ii) the input param's guarantee the minimal parametric uncertainty

iii) the input param's reabsorb, already in lowest order, some classes of large radiative corrections

Fit of observables, parameter determination and EW input schemes

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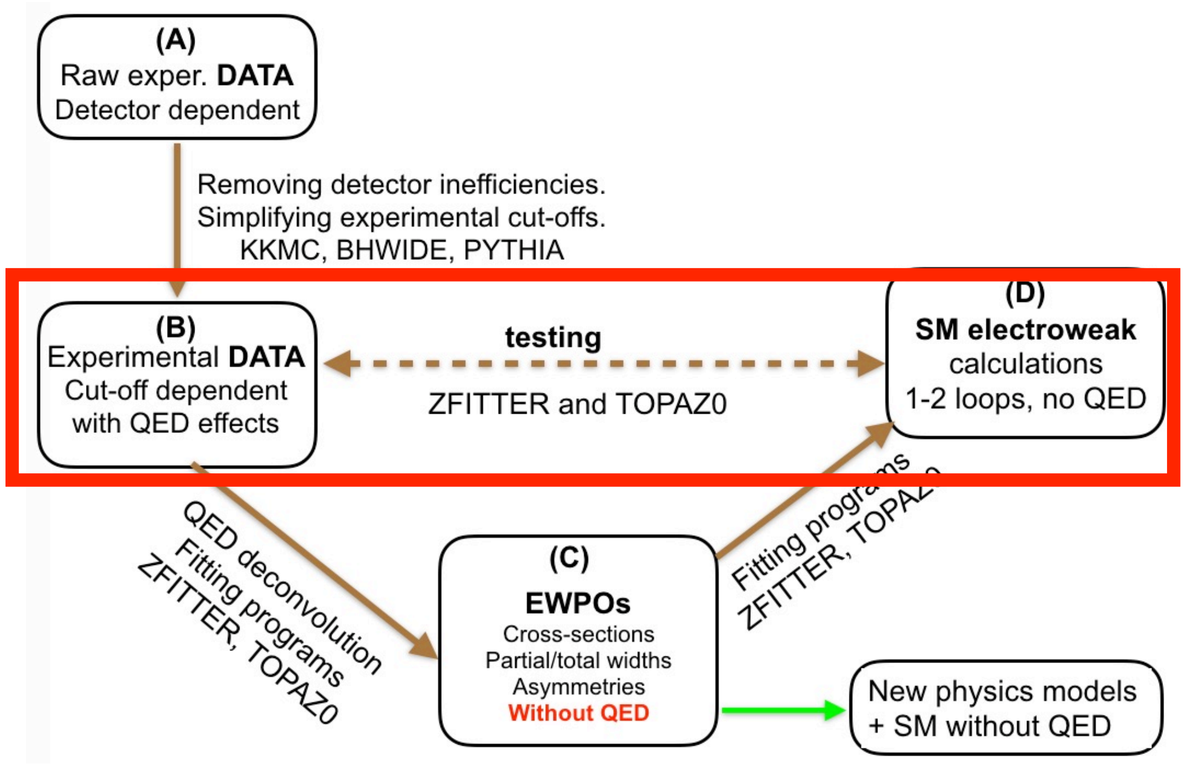
The LEP/SLD legacy: $\sin^2\theta_{eff}$ determination; two distinct approaches (1)

- SM prediction of cross sections and asymmetries and comparison with data (SM test)



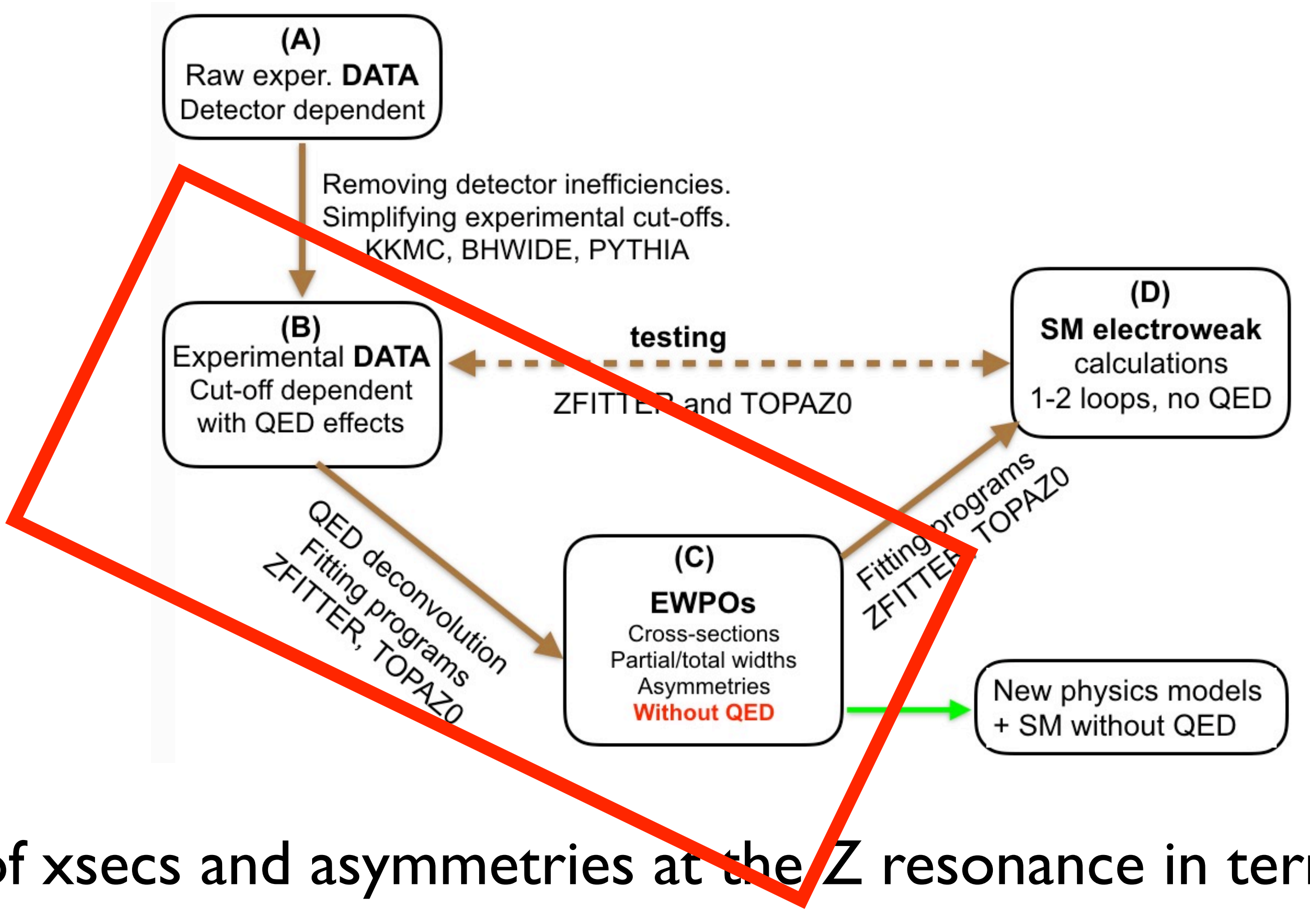
- SM prediction of xsecs and asymmetries computed as a function of (α, Gmu, MZ; mt, MH)

- mt and MH fit to the data to maximise the agreement

- $\sin^2\theta_{eff}$ has then been computed in the SM using Zfitter/TOPAZ0 with best mt and MH values and compared with the pseudo observable determination (next slide)

The LEP/SLD legacy: $\sin^2\theta_{eff}$ determination; two distinct approaches (2)

· Extraction of $\sin^2\theta_{eff}$ from pseudo-observables introduced to describe the Z resonance



· parameterisation of xsecs and asymmetries at the Z resonance in terms of pseudoobservables (≠SM)

$$m_Z, \Gamma_Z, \sigma^0_{had}, R^0_e, R^0_\mu, R^0_\tau, A^{0,e}_{FB}, A^{0,\mu}_{FB}, A^{0,\tau}_{FB}$$

· fit of the Z-resonance model to the data → experimental values of the pseudoobservables

· tree-level relation between the experimental Z decay widths (subtracted of QED/QCD effects) and the ratio gv/ga

→ algebraic solution for $\sin^2\theta_{eff}$ → effective angle

The LEP/SLD legacy: $\sin^2\theta_{eff}$ determination; two distinct approaches (2bis)

The $\sin^2\theta_{eff}$ determination from pseudo-observables at LEP depended on:

- high precision in the measurement of the xsec e+e- → hadrons
- separation of individual flavours
- deconvolution of large universal QED/QCD corrections (Zfitter/TOPAZ0)
- subtraction of SM non-factorisable contributions (Zfitter/TOPAZ0)

  checked to be small, weakly dependent on $\sin^2\theta_{eff}$

  and precise compared to the LEP/SLD precision target

→ factorised expression (initial)x(final) form factors

$$A_{FB}^{exp}(m_Z^2) - \mathscr{A}_{nonfact} = \frac{3}{4}\mathscr{A}_e\mathscr{A}_f$$

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- The LEP precision justified the above assumptions
- The model used to describe the Z resonance in terms of factorised pseudo observable (≠SM) contains the effective weak mixing angle as free parameter
- The analysis was to a large extent model independent, for all those New Physics effects appearing in the oblique corrections

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At future e+e- colliders we (still) have to demonstrate that all the above hypotheses hold
we possibly need 3-loop calculation to control the subtraction terms
and to define the pseudoobservables

An electroweak scheme with (Gmu, MZ, $\sin^2\theta_{eff}$) as inputs

M.Chiesa, F.Piccinini, AV, arXiv:1906.11569

Alternative EW scheme, using (Gmu, MZ, $\sin^2\theta_{eff}$) as inputs of the gauge sector

it has been first developed in the framework of the LHC analyses

(need to consider extended lepton-pair invariant mass intervals

where non-factorisable corrections can be much more important than at $q^2=MZ^2$)

it can be immediately applied to any e+e- collider study

it allows to express any observable as

$$\mathcal{O} = \mathcal{O}(G_\mu, m_Z, \sin^2\theta_{eff}^{lep})$$

so that templates as a function of $\sin^2\theta_{eff}$ can be easily generated

An electroweak scheme with (Gmu, MZ, $\sin^2\theta_{eff}$) as inputs

M.Chiesa, F.Piccinini, AV, arXiv:1906.11569

The weak mixing angle is related to the left- and right-handed (vector and axial-vector) couplings of the Z boson to fermions

$$\sin^2\theta^l_{eff} = \frac{I^l_3}{2Q_l}\left(1-\frac{g^l_V}{g^l_A}\right) = \frac{I^l_3}{Q_l}\left(\frac{-g^l_R}{g^l_L-g^l_R}\right)$$

The radiative corrections (expressed with bare constants) yield left- and right-handed form factors; we focus on the scale $q^2=MZ^2$

$$\sin^2\theta_0 = \frac{I^f_3}{Q_f}\,\mathrm{Re}\left(\frac{-\mathcal{G}^f_R(M_Z^2)}{\mathcal{G}^f_L(M_Z^2)-\mathcal{G}^f_R(M_Z^2)}\right)\Bigg|_0$$

We introduce the counterterms and collect their effects together with the one of the diagrams in $\delta g_{L,R}$

$$\sin^2\theta^\ell_{eff}+\delta\sin^2\theta^\ell_{eff} = \frac{I^l_3}{Q_l}\,\mathrm{Re}\left(\frac{-g^l_R-\delta g^l_R}{g^l_L-g^l_R+\delta g^l_L-\delta g^l_R}\right)$$

The request that the tree-level relation holds to all orders fixes the counterterm for $\sin^2\theta_{eff}$

$$\delta\sin^2\theta^\ell_{eff} = -\frac{1}{2}\frac{g^\ell_L g^\ell_R}{(g^\ell_L-g^\ell_R)^2}\,\mathrm{Re}\left(\frac{\delta g^\ell_L}{g^\ell_L}-\frac{\delta g^\ell_R}{g^\ell_R}\right)$$

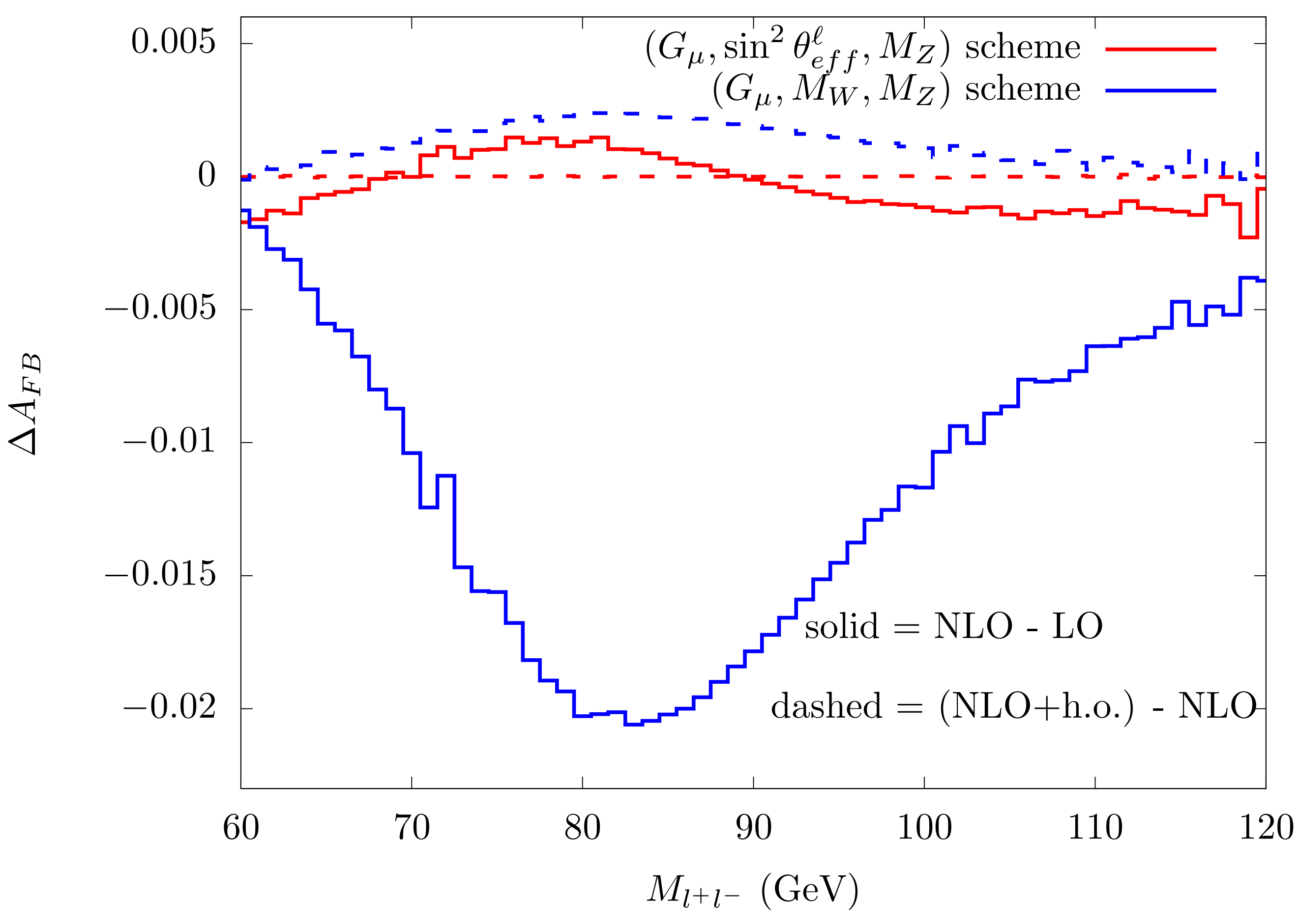
The renormalised angle is identified with the LEP leptonic effective weak mixing angle

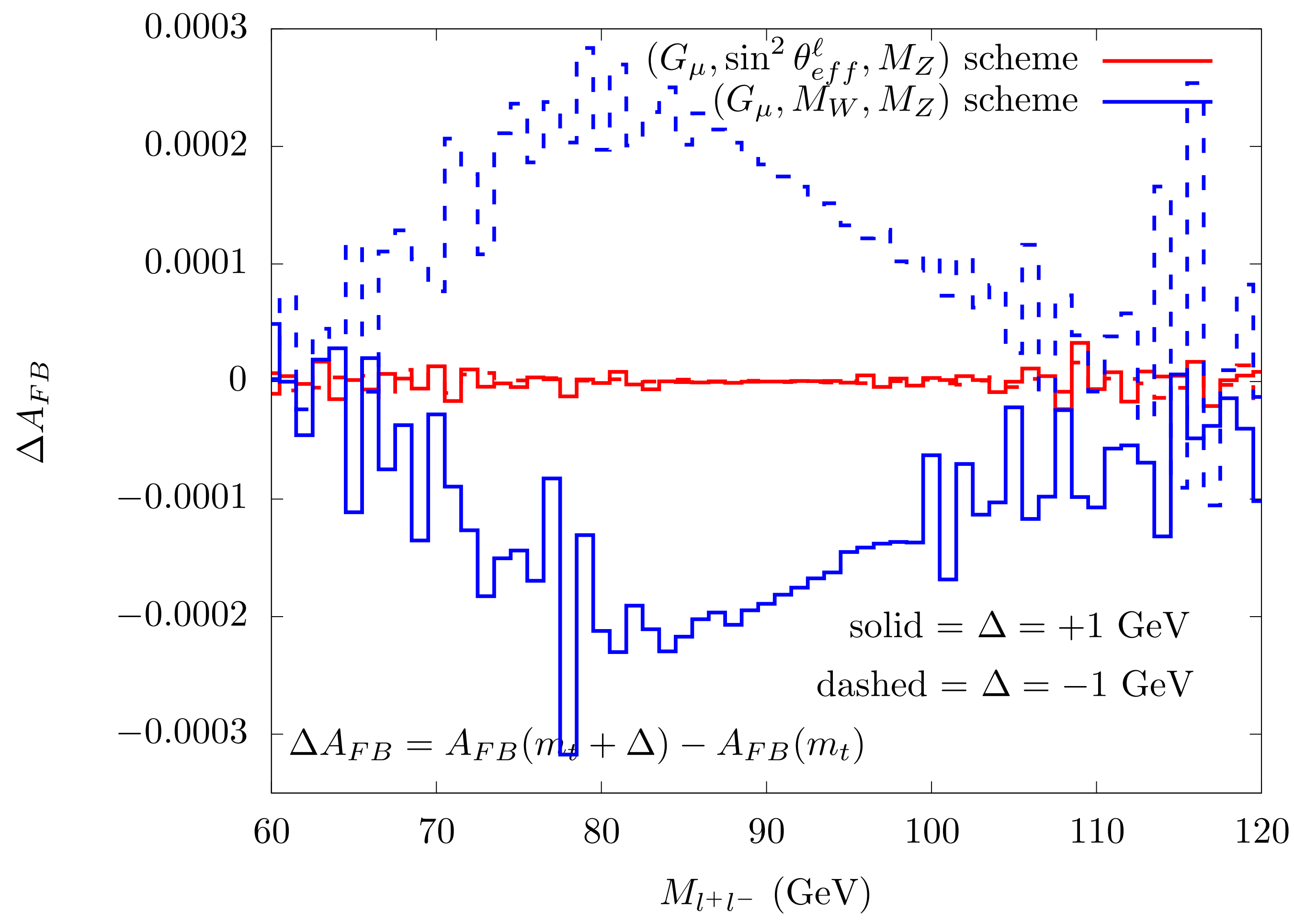
The Z mass is defined in the complex mass scheme.

Δr is evaluated with $\sin^2\theta_{eff}$ as input and differs from the usual (α,MW,MZ) expression

AFB mtop parametric uncertainties and perturbative convergence

M.Chiesa, F.Piccinini, AV, arXiv:1906.11569





prediction for AFB at the LHC in the (Gmu, MZ, $\sin^2\theta_{eff}$) input scheme (red),
comparison with (Gmu,MW,MZ) (blue)

faster perturbative convergence
very weak parametric mtop dependence → good control over the systematic uncertainties of the templates used to fit the data

An electroweak scheme with (Gmu, MZ, $\sin^2\theta_{eff}$) as inputs

M.Chiesa, F.Piccinini, AV, arXiv:1906.11569

Pro's

Every observable (e.g. Z lineshape, A_FB) expressed in terms of the quantity to be measured →
→ templates are easily generated.

The determination of $\sin^2\theta_{eff}$ can be discussed directly at the level of the complete observables, systematically including all the terms which potentially break the validity of the factorised Ansatz (e.g. QED IFI terms)

Theoretical systematic errors can be directly estimated within the same template fit approach

The choice of input parameters defined at the Z resonance minimises the size of additional radiative corrections, accelerating the perturbative convergence

See also D.C.Kennedy, B.W.Lynn,Nucl.Phys.B322, 1; F.M.Renard, C.Verzegnassi, Phys.Rev.D52,1369;A.Ferroglia, G.Ossola, A.Sirlin,Phys.Lett.B507,147;

To be done

Higher-order (2-loop and higher) universal corrections have to be worked out again to achieve a consistent formulation with competitive precision (in progress the 2-loop study)

Addendum

The definition of MZ in the complex mass scheme
has to be systematically adopted in the simulation codes,
to avoid biases in the very high precision mass determination

Template evaluation and estimate of the associated theoretical systematic error in the fit

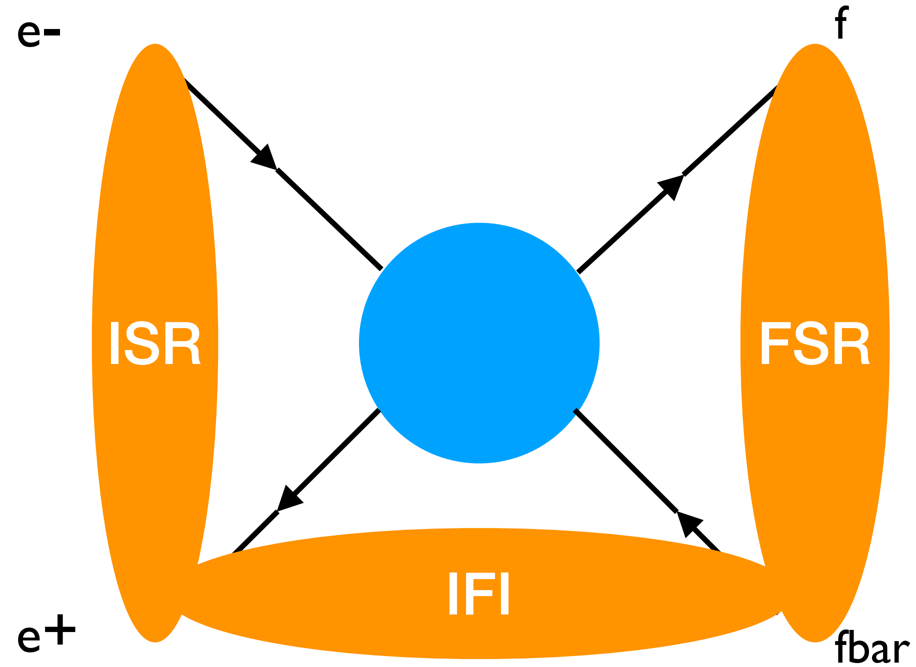
The theoretical systematic error depends on the perturbative and logarithmic accuracy of the templates

in the pseudo observable approach
the main goal is the systematic removal (deconvolution) of QED and QCD effects from the data
in order to obtain new purely weak manipulated "data", detector independent, ready for the fit

in the template fit case
the main goal is the systematic inclusion of QED (and QCD) higher corrections to all-orders
matched with the rest of the EW contributions
in order to stabilise the template shapes

the issues with the precision of the QED formulation are the same in both cases

QED factorisation in the radiative corrections to e+e- → f fbar



The largest QED corrections are associated to soft and/or collinear emissions:

$L=\log(s/m_e^2)\sim 24$, $\ell=(\delta E/E)$

Factorisation properties of the soft and/or collinear amplitudes allow to separate the bulk of the QED corrections from the hard scattering process

Different approaches to the evaluation to all orders of QED corrections and for the matching with fixed-order calculations:

1) flux functions (ZFITTER)
2) QED Parton Shower solution of DGLAP equations matched at NLO-EW (BabaYaga/HORACE)
3) CEEX

Matching schemes in the EW sector

ZFITTER flux functions, radiator functions

The complete scattering is described (LEP approach in ZFITTER) as
the convolution of a hard scattering cross section with flux functions

$$\sigma(s) = \int ds' \, \frac{1}{s} \, \rho(\frac{s'}{s}) \, \sigma(s') \qquad \rho = \rho_{ISR} + \rho_{FSR} + \rho_{IFI}$$

The flux functions encode the angular dependence of the final state recoiling against radiation.
have been computed at exact $O(\alpha)$ with soft photon exponentiation,
for ISR/FSR/IFI, inclusive or with cuts

The formulation naturally arises in the construction and dressing of a Born-improved approximation
→ Are the best available flux functions sufficiently precise and flexible?

Matching schemes in the EW sector

HORACE / BabaYaga matching scheme

$$d\sigma^{\infty}_{matched} = \Pi_S(Q^2) F_{SV} \sum_{n=0}^{\infty} d\hat{\sigma}_0 \frac{1}{n!} \prod_{i=0}^{n} \left(\frac{\alpha}{2\pi} P(x_i)\, I(k_i)\, dx_i\, d\cos\theta_i\, F_{H,i} \right)$$

$$F_{SV} = 1 + \frac{d\sigma^{\alpha,ex}_{SV} - d\sigma^{\alpha,PS}_{SV}}{d\sigma_0}$$

$$F_{H,i} = 1 + \frac{d\sigma^{\alpha,ex}_{H,i} - d\sigma^{\alpha,PS}_{H,i}}{d\sigma^{\alpha,PS}_{H,i}}$$

Monte Carlo event generators for ffbar → f'f'bar production with EW corrections

multiple photon radiation implemented via QED Parton Shower algorithm

resummation to all orders of leading logarithms of collinear and soft origin

matching with exact $O(\alpha)$ matrix elements;

matrix element corrections applied to all emitted photons (improvement towards $O(\alpha^2)$ accuracy)

→ is it possible to formulate a matching at NNLO level ?

Matching schemes in the EW sector

CEEX (Coherent Exclusive EXponentiation)

$$\sigma^{(r)} = \sum_{n=0}^{\infty} \frac{1}{n!} \int \mathrm{d}\tau_n(p_1 + p_2;\; p_3, p_4,\; k_1, \ldots, k_n)\; \mathrm{e}^{2\alpha \Re B_4(p_a,\ldots,p_d)} \frac{1}{4} \sum_{\mathrm{spin}} \left| \mathfrak{M}_n^{(r)}\left(p, k_1, k_2, \ldots k_n\right) \right|^2$$

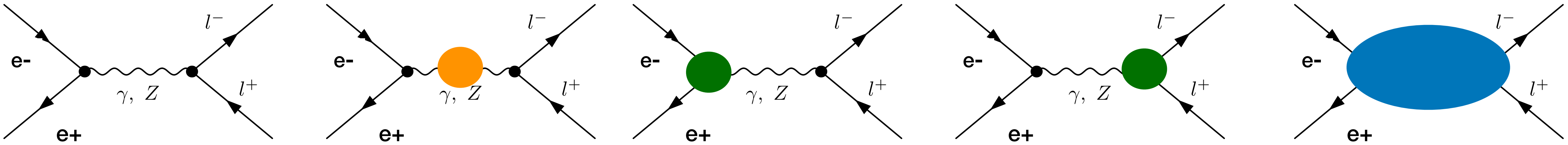
$$\mathfrak{M}_n^{(r)}(p, k_1, k_2, k_3, \ldots, k_n) = \prod_{s=1}^{n} \mathfrak{s}(k_s) \left\{ \hat{\beta}_0^{(r)}(p) + \sum_{j=1}^{n} \frac{\hat{\beta}_1^{(r)}(p, k_j)}{\mathfrak{s}(k_j)} + \sum_{j_1<j_2} \frac{\hat{\beta}_2^{(r)}(p, k_{j_1}, k_{j_2})}{\mathfrak{s}(k_{j_1})\mathfrak{s}(k_{j_2})} + \cdots \right\}$$

- soft photon contributions exponentiated on top of any amplitude
- collinear contributions and hard process dependent corrections are systematically included order by order in perturbation theory
- resummation of ISR mass logarithms not possible in this formalism

KKMC Monte Carlo code for the simulation of fermion-pair production in e+e- annihilation
it includes the full O(α) EW, from DIZET (2→2 process)
exact matrix elements for one- and two-photon emissions in QED,
properly matched with soft-photon exponentiation à la YFS

- Recent developments for the electron mass dependence of second order corrections arXiv:1910.05759
- Discussion about the matching in a full EW calculation (determination of $\hat{\beta}_n^{(r)}$ coefficients)

Higher-order corrections to e+e- → f fbar



basic building blocks

1-loop full NLO-EW results (2→2 process), in analytic form, including lepton mass dependence

2-loop exact expression for $O(\alpha\alpha_s)$ 2-loop self-energy corrections at arbitrary q^2 (massive quarks)
general massive case (arbitrary q^2) in terms of elliptic functions or via numerical methods
complete calculation of vertex corrections at $q^2 = MZ^2$ via numerical methods,
general analytic q^2 dependence in progress
box corrections with massless internal lines in analytic form; (see later for the massive cases)
complete renormalisation program (ρ, Δr, $\Delta\kappa$, δe)

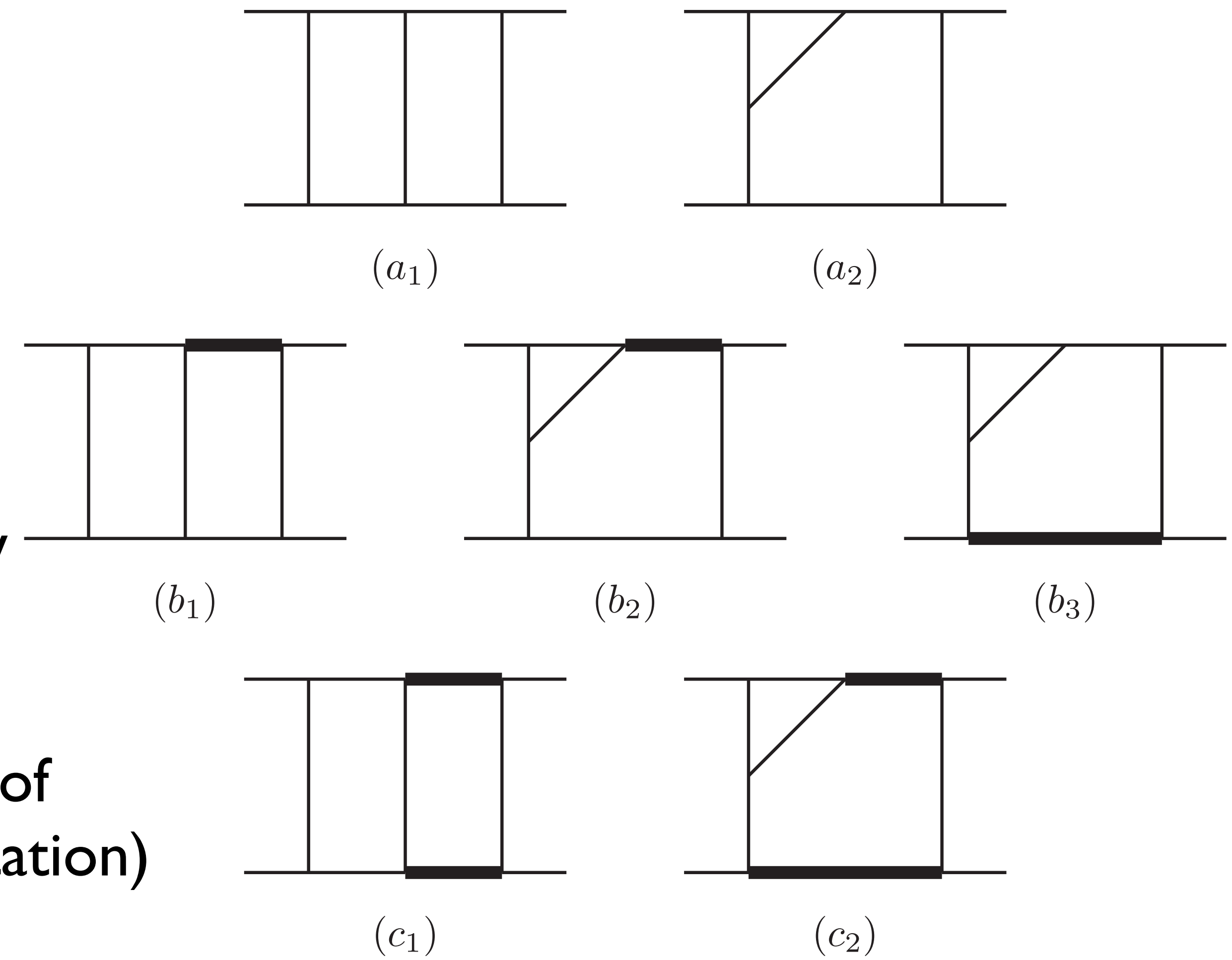
3-loop results for self-energies from QCD studies (massless); numerical approaches (massive cases)

Analytical results allow a full explicit control over the expressions needed in the matching with QED

Alternative approaches, based on numerical techniques, for the fully massive (IR finite) integrals

Analytic progress: Master Integrals for DY processes at $O(\alpha\alpha_s)$

R. Bonciani, S. Di Vita, P. Mastrolia, U. Schubert, arXiv:1604.08581



thin lines massless
thick lines massive
topologies **b** and **c** were not known

2 masses topologies evaluated with the same mass

SM results, where both W and Z appear,
can be evaluated with an expansion in ΔM=MZ-MW

49 MI identified (8 massless, 24 1-mass, 17 2-masses)
solution of differential equations expressed in terms of
iterated integrals (mixed Chen-Goncharov representation)

M.Heller, A.von Manteuffel, R.Schabinger arXiv:1907.00491

same class of diagrams expressed in terms of multiple polylogarithms

The Master Integrals are solved with the Differential Equation technique
Main issues related to number of energy scales (s, t, MW, MZ, Mmu)
at mathematical level → appearance of elliptic kernels and evaluation of boundary conditions

Recent important analytical developments for H+jet in full SM (massive quarks)
Important development in the fully numerical evaluations

Open questions: mass renormalisation scheme at 2-loop EW

resonances require the treatment of the particle decay-width

pole expansions (Laurent expansion of the amplitude) are valid only in the vicinity of the resonances

the complex-mass renormalisation scheme A. Denner, S.Dittmaier, arXiv:hep-ph/0605312
provides a general, gauge invariant, definition of mass:
a complex quantity identified as the pole of the propagator in the complex q^2 plane

$$\mu_W^2 = M_W^2 - iM_W\Gamma_W \qquad \mu_Z^2 = M_Z^2 - iM_Z\Gamma_Z$$

$$\delta\mu_V^2 = \Sigma_{VV}(\mu_V^2) \qquad \delta\mathcal{Z}_V = -\Sigma'_{VV}(\mu_V^2)$$

it is formally proven in general (Ward identities satisfied by the Green's functions)
but it requires a careful handling
of all the imaginary parts of the amplitudes and of the renormalised parameters
(e.g. evaluation of the self-energies at complex q^2
avoid double counting of self-energy and vertex terms already present in the complex mass)

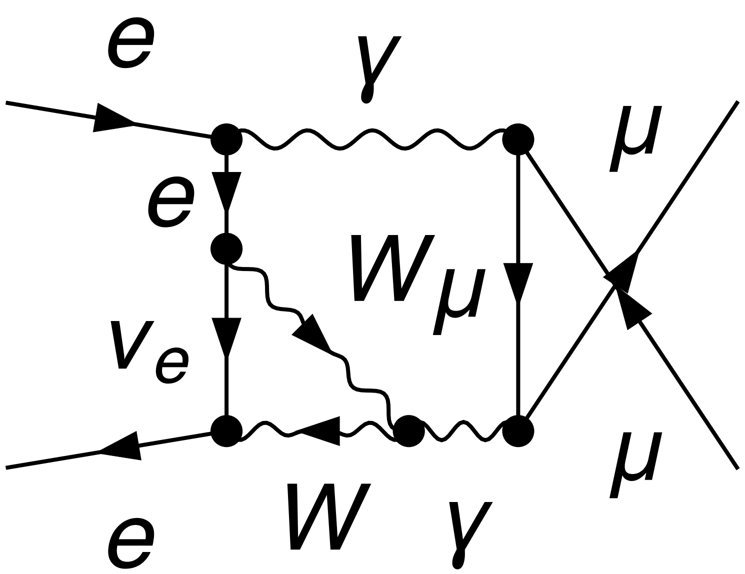
not yet systematically explored beyond NLO-EW

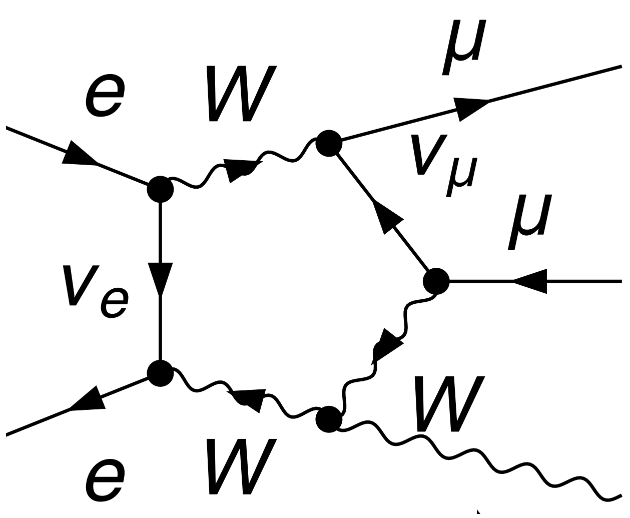
need to evaluate the remaining theoretical ambiguities in the mass definition

Open questions: matching NNLO-EW with QED resummation

in the CEEX matching approach, we need to
identify the matching coefficients $\hat{\beta}_n^{(r)}$ between the full calculation and the soft-exponentiated xsec
→ identification of the relevant gauge invariant subsets of the amplitude

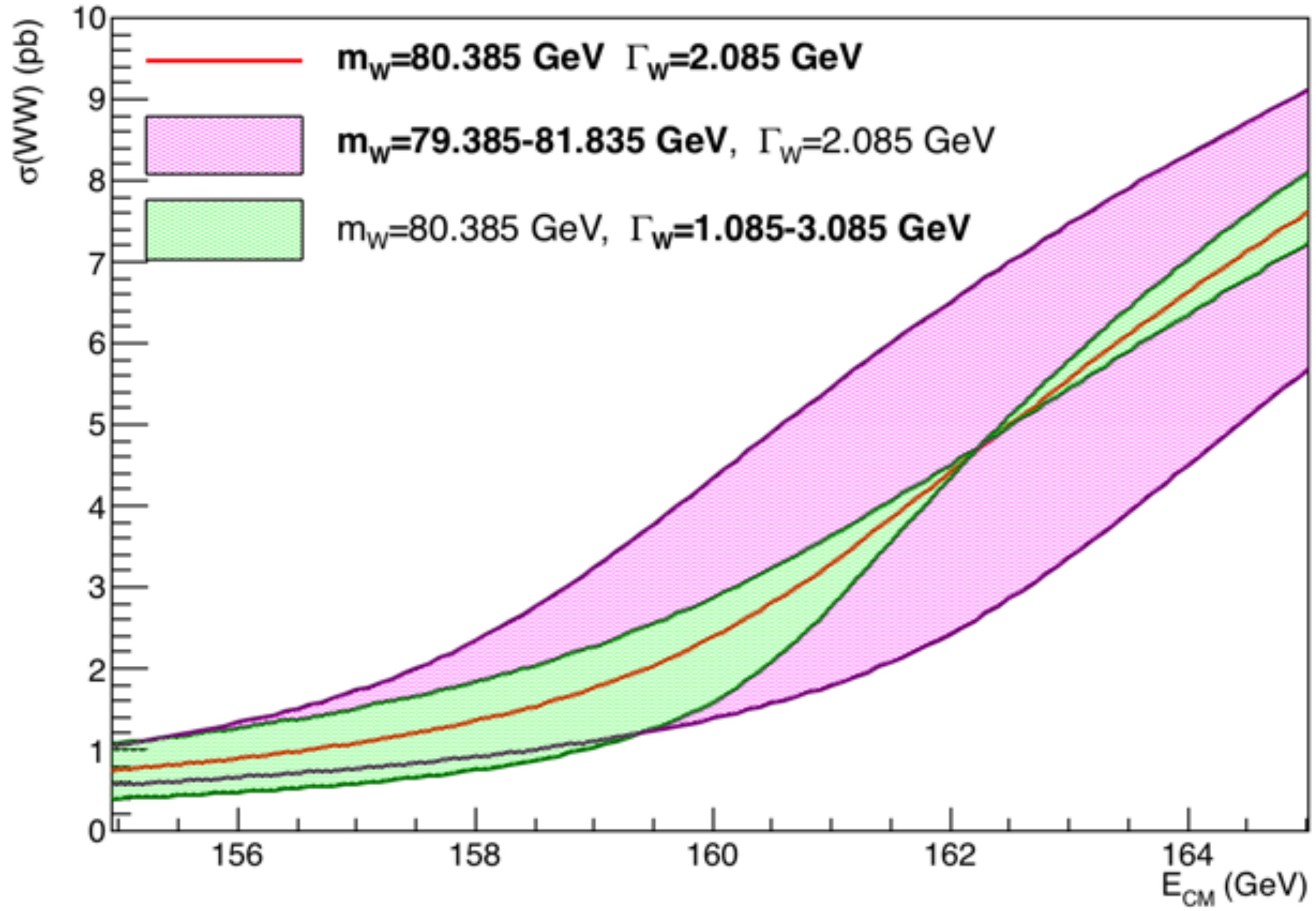
The coupling of photons and Ws must be handled with care
(respect gauge invariance and avoid double counting of imaginary parts
when the virtual corrections are included)





recipes devised at NLO-EW level must be extended at NNLO-EW level, in the complex mass scheme

MW determination from the WW threshold scan



As the cross section at the WW production threshold is very sensitive to the MW value
it is natural to compute the theoretical cross sections in the (Gmu, MW, MZ) input scheme

At threshold in lowest order $$\sigma_0(s) \approx \frac{\pi\alpha^2}{s}\frac{1}{4s_W^4}4\beta + O(\beta^3)$$

As long as $\beta \ll 1$, with low-precision requests,
MW can be determined in model independent way, based on kinematics alone

For a determination at the sub-MeV level, many details have to be considered,
with the preparation of precise SM templates

The MW determination from the WW threshold scan

see arXiv:1903.09895, 1906.05379

With a single point measurement it possible to translate the precision on the xsec into a ΔMW value

$$\Delta\sigma = 0.1\,\% \quad \longrightarrow \quad \Delta M_W = 1.5\ \mathrm{MeV}$$

An experimental precision at the $\Delta\sigma = 0.02\,\%$ is foreseen

Theoretical goal: precision of the theoretical prediction $\Delta\sigma = 0.01\,\%$

The current tools available for these analyses allow the simulation of $e^+e^- \to W^+W^- \to 4f$
at full NLO-EW + higher order Coulomb effects computed in EFT
yielding an uncertainty estimated to be $\Delta M_W \sim 3\ \mathrm{MeV}$

A reduction of $\Delta\sigma$ by one order of magnitude will require
the full NNLO-EW calculation (2→4 process!) matched with 3-loop Coulomb enhanced terms
computable in the EFT contribution

3-loop contributions without enhancement factors are estimated to be negligible

Full 2-loop QCD corrections to hadronic final states will be needed

The mass definition in the CMS and a gauge invariant handling of the imaginary parts at NNLO-EW
will be theoretical / technical points to be discussed
Matching with soft QED exponentiation at NNLO level should also be discussed

Conclusions and outlook

the experimental precision available at future e+e- colliders poses
fascinating extreme challenges to theory for the correct interpretation of data
templates, the fitting tools, must not introduce systematic errors spoiling the exp precision

basic QFT definitions, like e.g. the mass of unstable particles,
need to be clarified and implemented in simulation code at NNLO-EW level

keeping $\sin^2\theta_{eff}$ among the input parameters offers a direct way to estimate this coupling
(it would be great to reanalyse LEP data in this framework)
the same approach, formulated in the SMEFT, would offer a clean strategy to NP searches

The descriptions at NNLO-EW level of the Z lineshape and of the WW threshold
require the completion of Master Integrals calculations, with arbitrary kinematics,
at the frontier of our current ability

At the LHC EW precision physics studies are currently ongoing
https://twiki.cern.ch/twiki/bin/view/LHCPhysics/LHCEW
big challenges (hadronic environment) → development of new tools and strategies

A collaboration and cross-talk of both hadron and e+e- collider communities, in both directions,
will be necessary to achieve these ambitious goals