

$Z \rightarrow \pi\pi, KK$, a Perturbative QCD calculation at future Z factory

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Overview

Motivation

Form factor and decay width

Numerics

Outlook

Motivation

- † Tera-Z factory like CEPC and FCC-ee, Higgs and Z properties, hadron spectroscopy [A. Ali et.al., 1805.02535] as well as QCD.
- † Distribution amplitudes (momentum distribution)
Form factor (redistribution) , Factorization approaches.
- † Form factor can be calculated in three different ways,
PQCD is the most powerful approach.
- † PQCD: developed in two-body nonleptonic B decays,
QCD and CPV, SM and NP.
- † The controversy in small-x region,
 k_T factorization and PQCD VS other approaches,
Resummed to a Sudakov factor VS leading power of $1/m_b$.

A touchstone of PQCD approach

- † In Z decay, energetic final states, leading power is enough, a clean channel with the total uncertainty $\in 10\%$.
- † Why $\pi\pi$, KK channel ? flavour conserving, tree level.

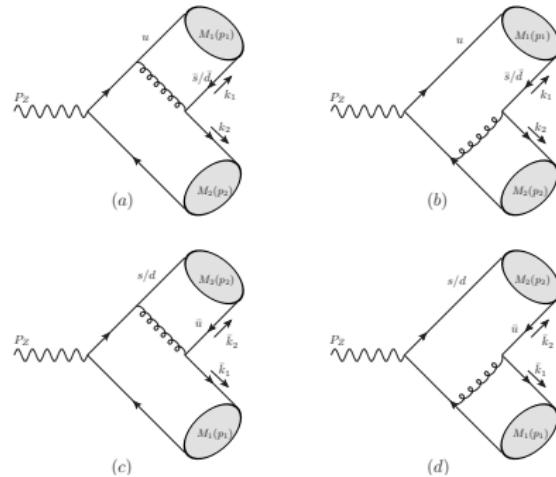
Decay width of $Z \rightarrow \pi\pi$

- $J_\mu^{(Z)} = \frac{g}{2 \cos \theta_w} \sum_q [(T_q - 2Q_q \sin^2 \theta_w) \bar{q} \gamma_\mu q - T_q \bar{q} \gamma_\mu \gamma_5 q]$,
SU(2) gauge coupling, weak angle, hypercharge, electrocharge.
- Decay amplitude $i\mathcal{M}(Z \rightarrow \pi^+ \pi^-) = \epsilon_Z^\mu \langle \pi^+ \pi^- | J_\mu^{(Z)} | 0 \rangle$,
 $\langle \pi^+ \pi^- | \bar{u} \gamma^\mu u | 0 \rangle = (p_1^\mu - p_2^\mu) \mathcal{G}(Q^2)$,
 $\langle \pi^+ \pi^- | \bar{d} \gamma^\mu d | 0 \rangle = -(p_1^\mu - p_2^\mu) \mathcal{G}(Q^2)$.
- Spin-averaged decay width

$$\Gamma(Z \rightarrow \pi^+ \pi^-) = \frac{1}{48\pi M_Z} \sum_s |\mathcal{M}|^2 = \frac{M_Z}{48\pi} (g_V^u - g_V^d)^2 |\mathcal{G}(M_Z^2)|^2.$$

- $Z\bar{q}q$ coupling $g_V^q = g/(2 \cos \theta_w) \times (T_q - 2Q_q \sin^2 \theta_w)$.
- ~~The mass (high twist) effect ($\mathcal{O}(m_\pi^2, m_K^2)$)~~.
- At leading power, $Z \rightarrow \pi^0 \pi^0, \bar{K}^0 K^0$ are forbidden by $(g_V^{q_1} - g_V^{q_2})^2$.
 $\mathcal{B}[Z \rightarrow \pi^0 \pi^0] < 1.52 \times 10^{-5}$. [PDG, 2018]

Kinematic



- $M_1 = \pi^+, K^+$, $M_2 = \pi^-, K^-$,
- $p_Z = \frac{m_Z}{\sqrt{2}}(1, 1, 0)$, $p_1 = \frac{m_Z}{\sqrt{2}}(1, 0, 0)$, $p_2 = \frac{m_Z}{\sqrt{2}}(0, 1, 0)$,
- $k_1 = (x_1 \frac{m_Z}{\sqrt{2}}, 0, k_{1T})$, $k_2 = (0, x_2 \frac{m_Z}{\sqrt{2}}, k_{2T})$, $\bar{k}_1 = p_1 - k_1$, $\bar{k}_2 = p_2 - k_2$,
- $\alpha_1 \equiv x_2 Q^2$, $\alpha_2 \equiv x_1 Q^2$, $\beta \equiv x_1 x_2 Q^2$.

Form factor at LO

$$\begin{aligned} \mathcal{G}_{II}(q^2)_{\text{LO}} &= -16\pi C_F Q^2 \int_0^1 dx_1 dx_2 \int db_1 db_2 b_1 b_2 \alpha_s(\mu) x_2 \phi_\pi(x_1) \phi_\pi(x_2) \\ &\quad \times h_{II}(x_1, b_1, x_2, b_2) \text{Exp}[-S_{II}(x_1, b_1, x_2, b_2, \mu)] , \end{aligned} \quad (1)$$

- $\mu = \text{Max}[\sqrt{\alpha_i}, 1/b_i]$, $\phi(x) = f_\pi 6x(1-x)/(2\sqrt{2N_C})$.
- Sudakov factor S_{II} , double and single log in the vertex correction + single log in the quark self correction. [J. Botts, eta, 1989 ; H.N. Li, eta, 1992]
- Hard kernel h_{II} :

$$\begin{aligned} \int \frac{k_{1T}^2}{(2\pi)^2} \frac{k_{2T}^2}{(2\pi)^2} \frac{1}{[\beta - (k_T^2) + i\epsilon]} \frac{1}{[\alpha_1 - k_{1T}^2 + i\epsilon]} &\longrightarrow \int db_1 db_2 b_1 b_2 h_{II}(x_i, b_i) \\ h_{II}(x_i, b_i) &= K_0(i\sqrt{\beta} b_2) \left[K_0(i\sqrt{\alpha_1} b_1) J_0(i\sqrt{\alpha_1 b_2}) \Theta(b_1 - b_2) + \{b_1 \leftrightarrow b_2\} \right] \end{aligned} \quad (2)$$

- † Bessel function of the first kind and the modified Bessel function.
- † The integral in Eq (1) holds well in the moderate region, $1 - 50 \text{ GeV}^2$.
- † Oscillates violently when Q^2 goes more higher.
- † No physical meaning, large hierarchy between Q^2 and k_T^2 .
In B decays, the integral converges quickly.

The hard function is not "hard"

- The integral of the hard function in Eq (1) is not stable.
- How to evade the oscillation ?
- Hierarchy of $x_i Q^2 \gg x_1 x_2 Q^2 \sim k_T^2$ in PQCD @ NLO.
 - † Drop the sub-leading term in the quark propagator, retain it in the gluon, **when Q^2 is enough large.**
 - † Form factor is mainly determined by the hard gluon [H.C. Hu, etc, 2013 ; S. Cheng, etc, 2015].
 - † The hard function reduces to

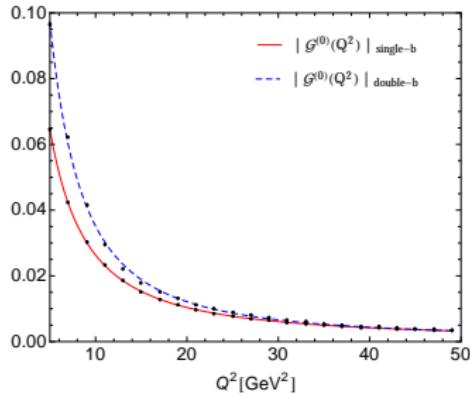
$$\frac{1}{\alpha_1} \int \frac{k_T^2}{(2\pi)^2} \frac{1}{[\beta - (k_T^2) + i\epsilon]} \rightarrow \frac{1}{\alpha_1} \int dbb h_l(x_i, b_i)$$
$$h_l(x_i, b) = 1/\alpha_1 \times K_0(i\sqrt{\beta}b_2) \quad (2)$$

- The modified form factor at LO, $b = b_1 = b_2$,

$$\mathcal{G}_l(q^2)_{\text{LO}} = 16\pi C_F \int_0^1 dx_1 dx_2 \int dbb \alpha_s(\mu) \phi_\pi(x_1) \phi_\pi(x_2) K_0(i\sqrt{\beta}b) \text{Exp}[-S_{ll}(x_i, b, \mu)], \quad (3)$$

- Extend the valid Q^2 from dozens- to thousands GeV².

Form factor at LO



- We suggest a parameterization, reciprocal of square polynomial

$$Abs[\mathcal{G}(Q^2)] = \frac{A + Q^2 B}{Q^4 + Q^2 C + A}, \quad [A. Khodjamirian, 1999] \quad (4)$$

† @ LO, $A^{(0)} = 0.0879$, $B^{(0)} = 46.1$, $C^{(0)} = 10.9$.

The NLO correction

- The NLO hard correction is available for single-b convoluted formula

$$\begin{aligned} \mathcal{G}_I(q^2)_{\text{NLO}} = & 16\pi C_F \int_0^1 dx_1 dx_2 \int db b \alpha_s(\mu) \phi_\pi(x_1) \phi_\pi(x_2) \text{Exp}[-S_{II}(x_i, b, \mu)], \\ & \times \frac{\alpha_s C_F}{4\pi} \left[\tilde{h}(x_i, b, Q, \mu) K_0(\sqrt{i\beta}b) + \frac{i\pi}{2} H_0^{(1)''}(i\sqrt{\beta}b) \right], \end{aligned} \quad (5)$$

† $\tilde{h}(x_i, b, Q)$ in [H.C. Hu, etc, 2013]

† For $H_0^{(1)''}(x) \equiv \left[d^2 H_\alpha^{(1)}(x) / d^2 \alpha \right]_{\alpha=0}$, we parameterize,

$\text{Re}[H_0^{(1)''}(x)]$

= Which[$x \geq 10$,

$$\begin{aligned} & \frac{\alpha_s C_F}{4\pi} \left[0.798 + 0.454x - 0.0603x^2 + 0.00590x^3 - 0.00021x^4 - 1.35 \log x \right. \\ & \quad + J_0(x) \left(-0.581 + 1.48 \log x - 0.497 \log^2 x \right) \\ & \quad \left. + Y_0(x) \left(-3.62 - 0.194x + 0.665 \log x + 0.331 \log^2 x \right) \right], \end{aligned}$$

$x < 10$,

...],

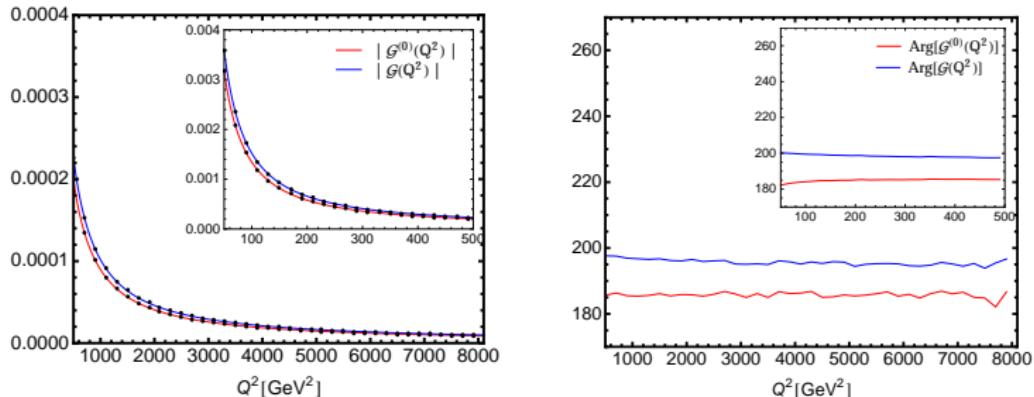
$\text{Im}[H_0^{(1)''}(x)] \dots$

(6)

(7)

† For the small and large argument, reproduce the result obtained by the asymptotic expansion of Hankel function.

NLO correction



- The NLO correction mostly comes from \tilde{h} term,
 $\sim 11\%$ for Abs in the whole energy region,
 $< 8\%$ for Arg deviated from the LO π .
- @ LO, $A^{(0)} = 0.0879$, $B^{(0)} = 46.1$, $C^{(0)} = 10.9$.
@ NLO, $A^{(1)} = 0.0996$, $B^{(1)} = 48.2$, $C^{(1)} = 12.6$.

$$Abs[\mathcal{G}(Q^2)] = \frac{A + Q^2 B}{Q^4 + Q^2 C + A}, \quad [A. Khodjamirian, 1999] \quad (8)$$

Branching ratio of $Z \rightarrow \pi\pi$

- $\sin^2 \theta_W(\overline{\text{MS}}) = 0.231$, $\alpha_s(m_Z^2) = 0.1182$, $\alpha(m_Z)^{-1} = 127.950$.
- $\Lambda_{\overline{\text{MS}}}^{(5)} = 0.2327 \Leftarrow \text{two-loop } \alpha_s(m_Z)$,
 $f_\pi = 130.2 \text{ MeV}$, $f_K = 155.6 \text{ MeV}$,
- Eqs (3, 5)
$$\Gamma(Z \rightarrow \pi^+ \pi^-) = \frac{1}{48\pi M_Z} \sum_s |\mathcal{M}|^2 = \frac{M_Z}{48\pi} (g_V^u - g_V^d)^2 |\mathcal{G}(M_Z^2)|^2.$$

$$\begin{aligned} \mathcal{G}(M_Z^2) \times 10^6 &= (-8.29 - i0.771)|_{LO} \\ &\quad + (-0.975 - i1.59)|_{NLO1} \\ &\quad + (0.211 + i0.00760)|_{NLO2}, \end{aligned} \tag{7}$$

- Branching ratios

$$\begin{aligned} \mathcal{B}(Z \rightarrow \pi^+ \pi^-) &= 0.83 \pm 0.02 \pm 0.02 \pm 0.04 \times 10^{-12}, \\ \mathcal{B}(Z \rightarrow K^+ K^-) &= 1.74 \pm 0.04 \pm 0.04 \pm 0.02 \times 10^{-12}, \end{aligned}$$

factorizable scale, strong coupling and decay constant.

Conclusion

- CEPC, $7 \times 10^{11} Z^0$ in 2 years', $32 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$;
[CEPC study group, 1809.00285, 1811.10545]

FCC-ee, $56 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$, quadruple at least.
[TLEP Design study Working Group, 1308.6176]
- Quite hopeful to be measured at a Tera-Z factory,
the first exclusive Z decay measurement.
- More events can be expected if the "small-x" region are dominant,
A good touchstone of PQCD.

Outlook

- $Z \rightarrow B_{(c)} B_{(c)}, D_{(s)} D_{(s)}$
 - † Factorization (m_b and/or m_c to λ_{QCD}),
 - † Wave function of fast moving $B_{(c)}$ (spin effect, HQET ?).
 - † Sudakov factor of B_c (quark mass effect).
- FCNC, $Z \rightarrow BK$ (penguin and new loop effect, mass effect, CPV).
- Data inspired channels:
 - $\mathcal{B}[Z \rightarrow 4l] = (4.45 \pm 0.32) \times 10^{-6}$;
[PDG, 2018 ⇐ CMS 2012, 2016, 2018 and ATLAS 2014, 2016]
 - $\mathcal{B}[Z \rightarrow J/\psi ll]/\mathcal{B}[Z \rightarrow 4l] = 0.67 \pm 0.18 \pm 0.05$; [CMS, 2018]
 - $\mathcal{B}[Z \rightarrow J/\psi \gamma, \Upsilon(1S, 2S, 3S)\gamma] < 0.67 \times 10^{-6}, (3.4, 6.5, 5.4) \times 10^{-6}$;
[ATLAS 2015, CMS 2016]
 - $\mathcal{B}[Z \rightarrow \phi(\rho)\gamma] < 0.9(2.5) \times 10^{-6}$; [ATLAS 2016, 2018]
 - $\mathcal{B}[Z \rightarrow \pi\pi\pi] < 1.01 \times 10^{-6}$; [CMS 2019]

The End, Thank You.