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High precision determination of $\alpha_s(M_Z)$ from a global fits and lessons for CEPC

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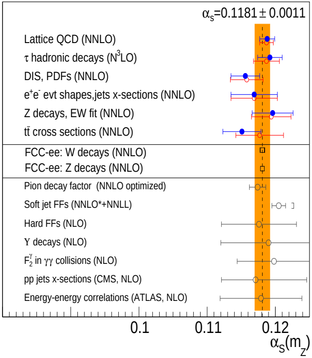
The 2019 International Workshop on the High Energy Circular Electron Positron Collider, Beijing, China, 18-21 November 2019

α_s e^+e^-: motivation in the past and in the future



Source: arXiv:1806.06156v1

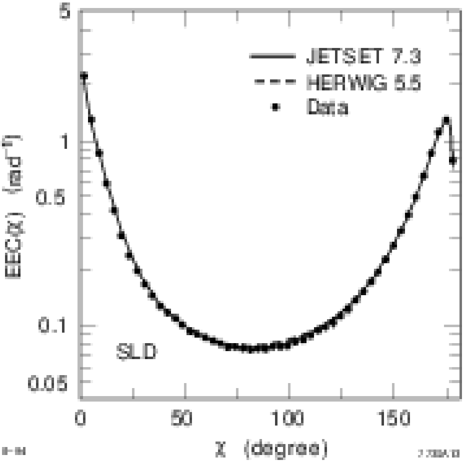
- As of 2018 α_S is known with precision of 1% if calculated from measurements with at least NNLO precision
- However, there is a large spread between measurements
- More measurements is better

+ measurements with new approached/data are important on themselves

PART I: Precise determination of $\alpha_S(M_Z)$ from a global fit of energy-energy correlation to NNLO+NNLL predictions, arXiv:1804.09146, Eur. Phys. J. C **78** (2018) no.6, 498

The energy-energy correlations

$\frac{\mathbf{dEEC}(\chi)}{\mathbf{d}\chi} = \sum_{\mathbf{i}}^{\mathbf{N}} \sum_{\mathbf{j}}^{\mathbf{N}} \frac{\mathbf{E_i E_i}}{\mathbf{E_{vis}^2}} \delta(\cos\chi - \cos\chi_{\mathbf{ij}})$, with $E_{vis} = \sum_i^N E_i$, where E_i is particle energy and χ_{ij} is angle between particles i and j.



- Used multiple times in a **distant** past for α_S extraction
- Inclusive
- Not sensitive to schemes of combinations
- Resummed NNLL predictions became available in **2017**

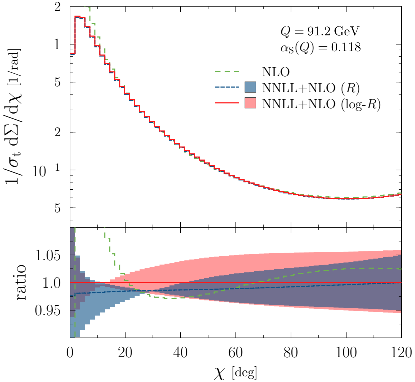
← **Looks like this[1]**

Analysis components

- Perturbative and resummed predictions Z. Tulipánt, A. Kardos and G. Somogyi, "Energy-energy correlation in electron-positron annihilation at NNLL + NNLO accuracy," Eur. Phys. J. C **77** (2017) no.11, 749 + some b mass corrections.
- Data: LEP, PEP, PETRA, SLC and TRISTAN
- Non-perturbative corrections: NLO MC by Sherpa (Lund and cluster hadronization: S^L and S^C) and Herwig7 (cluster hadronization, H^M), analytic hadronization. See details in backups.

Predictions: fixed order, matching, etc.

e^+e^- predictions in NNLO exist for some time, however



$+b$ mass corrections at NLO from ZBB4 program [2].

ColorFulNNLO, V. Del Duca et al., "Jet production in the CoLoRFulNNLO method: event shapes in electron-positron collisions," Phys. Rev. D **94** (2016) no.7, 074019 has unique features

- precision
- extendable approach

Resummation and matching have appeared recently:

Z. Tulipánt, A. Kardos and G. Somogyi, "Energy-energy correlation in electron–positron annihilation at NNLL + NNLO accuracy," Eur. Phys. J. C **77** (2017) no.11, 749

Available data

The available data covers wide range of energy: $\sqrt{s} = 14 - 91\,\text{GeV}$.

Experiment	Data $\sqrt{s}$(average)	MC $\sqrt{s}$	Data events
SLD [1]	91.2(91.2)	91.2	60000
OPAL [3]	91.2(91.2)	91.2	336247
OPAL [4]	91.2(91.2)	91.2	128032
L3 [5]	91.2(91.2)	91.2	169700
DELPHI [6]	91.2(91.2)	91.2	120600
TOPAZ [7]	59.0 — 60.0(59.5)	59.5	540
TOPAZ [7]	52.0 — 55.0(53.3)	53.3	745
TASSO [8]	38.4 — 46.8(43.5)	43.5	6434
TASSO [8]	32.0 — 35.2(34.0)	34.0	52118
PLUTO [9]	34.6(34.6)	34.0	6964
JADE [10]	29.0 — 36.0(34.0)	34.0	12719
CELLO [11]	34.0(34.0)	34.0	2600
MARKII [12]	29.0(29.0)	29.0	5024
MARKII [12]	29.0(29.0)	29.0	13829
MAC [13]	29.0(29.0)	29.0	65000
TASSO [8]	21.0 — 23.0(22.0)	22.0	1913
JADE [10]	22.0(22.0)	22.0	1399
CELLO [11]	22.0(22.0)	22.0	2000
TASSO [8]	12.4 — 14.4(14.0)	14.0	2704
JADE [10]	14.0(14.0)	14.0	2112

Data qualification criteria

- Corrected to charged and neutral final state
- Corrected for ISR
- Full χ range measured
- No overlap with other samples
- Sufficient precision
- Sufficient information on data available

Huge data-sets available for combined analysis:
20 data-sets from 11 collaborations.

Non-perturbative models

Two approaches are available on the market: analytic and MC based. **We use both.**

Analytic approach

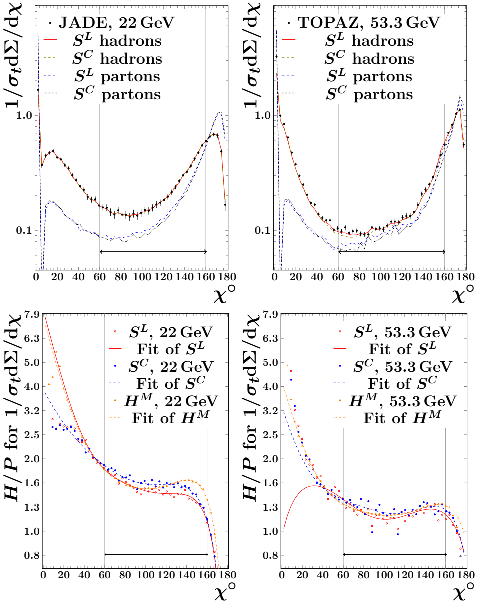
- Calculations with

  Y. L. Dokshitzer, G. Marchesini and B. R. Webber, "Nonperturbative effects in the energy energy correlation," JHEP **9907** (1999) 012
- Involves α_S moments at low scales, which are free parameters.

MC-based

- NLO MC events by particle level generators to extract with point-by-point multiplicative correction factors
- Systematics from multiple hadronization models
- Simultaneously allows to extract missing correlations of data points

MC based approach



- Good description of data.
- Event-by-event reweighting to match the data: $\log W_{event} = \sum_{bin=1}^{Nbins} k_{bin} EEC_{event}(bin)$.

- Hadr. corrections = hadron level/parton level.
- + parametrized with smooth functions.

Fits: technique

Fits are performed with `MINUIT2` [14] minimising

$$\chi^2 = \sum_{\text{data sets}} \chi^2(\alpha_\mathrm{s})_{\text{data set}},$$

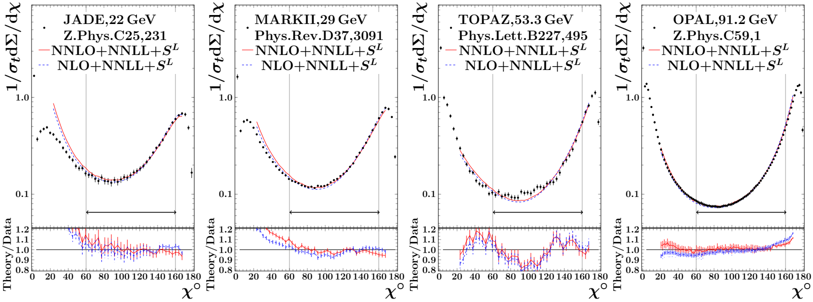
where $\chi^2(\alpha_\mathrm{s})$ was calculated for each data set as

$$\chi^2(\alpha_\mathrm{s}) = \vec{r}\, V^{-1}\, \vec{r}^{\,T}, \qquad \vec{r} \equiv (\vec{D} - \vec{P}(\alpha_\mathrm{s})), \tag{1}$$

- $\vec{D}$ vector of data points
- $\vec{P}(\alpha_\mathrm{s})$ vector of fixed order (or resummed) predictions corrected for non-perturbative effects
- V is the covariance matrix for $\vec{D}$

Result for NNLO+NNLL:
$\alpha_S(M_Z) = 0.11750 \pm 0.00018$, $\chi^2/n.d.f = 1022/623 = 1.64$

Fits: distributions

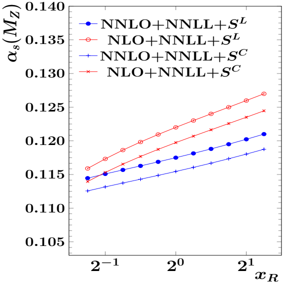


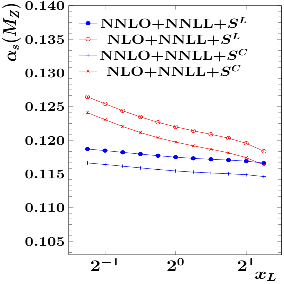
- The fits are done in different ranges.
- Criteria for central result: validity of NNLO, hadronization corrections and resummation.
- Results are insensitive to ±5° changes of fit ranges.

Ranges:

- $117 - 177^\circ$
- $117 - 165^\circ$
- $60 - 165^\circ$
- $60 - 160^\circ$ (central)

Systematics and uncertainties (see also backups)





The uncertainties that were estimated:

- Variation of renormalization scale by $2^{\pm 1}$: (*res.*)
- Variation of resummation scale by $2^{\pm 2}$: (*ren.*)
- Variation of matching power 1 or 2: neglected
- Variation of hadronization model S^L or S^C: (*hadr.*)
- Fit uncertainty is $\chi^2 + 1$ criterion from MINUIT: (*exp.*)

Conclusions for part I

Extraction of $\alpha_S(M_Z)$ from energy-energy correlations in e^+e^- collisions has been performed with NNLO+NNLL precision **for the first time** using data-sets in wide range of centre-of-mass energies. The results are

$\alpha_S(M_Z) = 0.12200 \pm 0.00023(exp.) \pm 0.00113(hadr.) \pm 0.00433(ren.) \pm 0.00293(res.)$

for NLO+NNLL(logR) scheme and

$\alpha_S(M_Z) = 0.11750 \pm 0.00018(exp.) \pm 0.00102(hadr.) \pm 0.00257(ren.) \pm 0.00078(res.)$

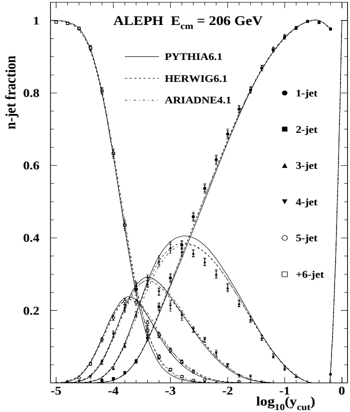
for NNLO+NNLL(logR) scheme.

The analysis can be re-done with N^3LL in the future

PART II: High precision determination of $\alpha_S(M_Z)$ from a global fit of jet rates, arXiv:1902.08158, JHEP **1908** (2019) 129

Durham jet rates

Durham jet algorithm is a sequential jet algorithm with distance. $d_{ij} = 2\min(E_i^2, E_j^2)(1-\cos\theta_{ij})$, where E_i is particle energy and θ_{ij} is angle between particles i and j. Jet rates R_n – fraction of n-jet events for $y = d_{min}/E_{vis}^2$.



- R_3 was used multiple times in past for α_S extraction
- R_2 and R_3 can be naturally combined for the first time
- Resumed NNLL predictions became available in for R_2 in **2016**

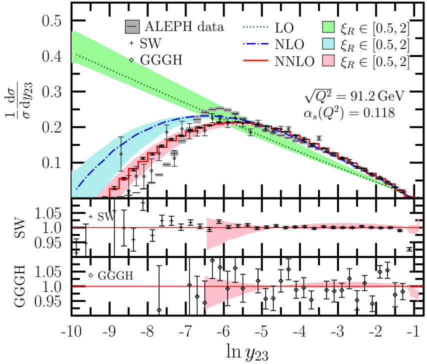
← **Looks like this[15]**

Analysis components

- Perturbative predictions V. Del Duca et al., "Jet production in the CoLoRFulNNLO method: event shapes in electron-positron collisions," Phys. Rev. D **94** (2016) no.7, 074019 + some b mass corrections
- Resummation A. Banfi et al., "The two-jet rate in e^+e^- at next-to-next-to-leading-logarithmic order," Phys. Rev. Lett. **117** (2016) no.17, 172001; (S. Catani et al., "New clustering algorithm for multi - jet cross-sections in e^+e^- annihilation," Phys. Lett. B **269** (1991) 432 for tests)
- Data: LEP and PETRA **(YES!)**. New OPAL measurements used to build correlation model for older measurements.
- Non-perturbative corrections: NLO MC by Sherpa and Herwig7 with Lund and cluster hadronization models, i.e. S^L, S^C, H^L, H^C.

Predictions: fixed order, matching, etc.

e^+e^- predictions in NNLO exist for some time, however



CoLoRFulNNLO, V. Del Duca et al., "Jet production in the CoLoRFulNNLO method: event shapes in electron-positron collisions," Phys. Rev. D **94** (2016) no.7, 074019:

- precision
- extendable approach

NLL resummation/matching is well known for R_3, for NNLL R_2 have appeared recently: A. Banfi et al., "The two-jet rate in e^+e^- at next-to-next-to-leading-logarithmic order," Phys. Rev. Lett. **117** (2016) no.17, 172001

$+b$-mass corrections

Main focus on α_S^3+NNLL for R_2

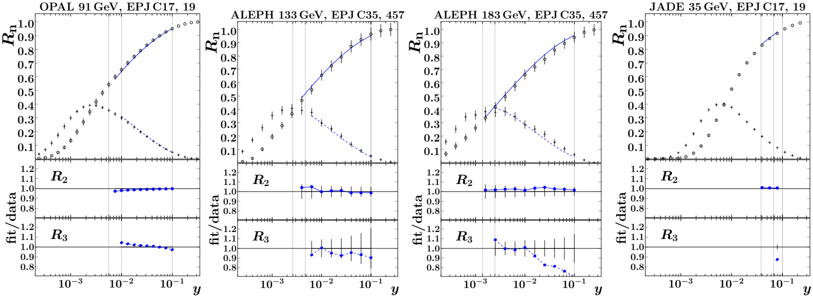
Available data, MC corrections, fits

Data qualification criteria, MC settings and fits procedure are similar to the EEC analysis.

Experiment	Data $\sqrt{s}$(average)	MC $\sqrt{s}$	Data events
OPAL [16]	91.2(91.2)	91.2	1508031
OPAL [16]	189.0(189.0)	189	3300
OPAL [16]	183.0(183.0)	183	1082
OPAL [16]	172.0(172.0)	172	224
OPAL [16]	161.0(161.0)	161	281
OPAL [16]	$130.0 - 136.0$(133.0)	133	630
L3 [17]	$201.5 - 209.1$(206.2)	206	4146
L3 [17]	$199.2 - 203.8$(200.2)	200	2456
L3 [17]	$191.4 - 196.0$(194.4)	194	2403
L3 [17]	$188.4 - 189.9$(188.6)	189	4479
L3 [17]	$180.8 - 184.2$(182.8)	183	1500
L3 [17]	$161.2 - 164.7$(161.3)	161	424
L3 [17]	$135.9 - 140.1$(136.1)	136	414
L3 [17]	$129.9 - 130.4$(130.1)	130	556
JADE [16]	$43.4 - 44.3$(43.7)	44	4110
JADE [16]	$34.5 - 35.5$(34.9)	35	29514
ALEPH [15]	91.2(91.2)	91.2	3600000
ALEPH [15]	206.0(206.0)	206	3578
ALEPH [15]	189.0(189.0)	189	3578
ALEPH [15]	183.0(183.0)	183	1319
ALEPH [15]	172.0(172.0)	172	257
ALEPH [15]	161.0(161.0)	161	319
ALEPH [15]	133.0(133.0)	133	806

- The data covers wide range of energy: $\sqrt{s} = 35 - 207\,\text{GeV}$. **Huge datasets: 20+ datasets from 4 collaborations.**
- Hadronisation correction procedure preserves overall normalization, see backups.

Fits: distributions

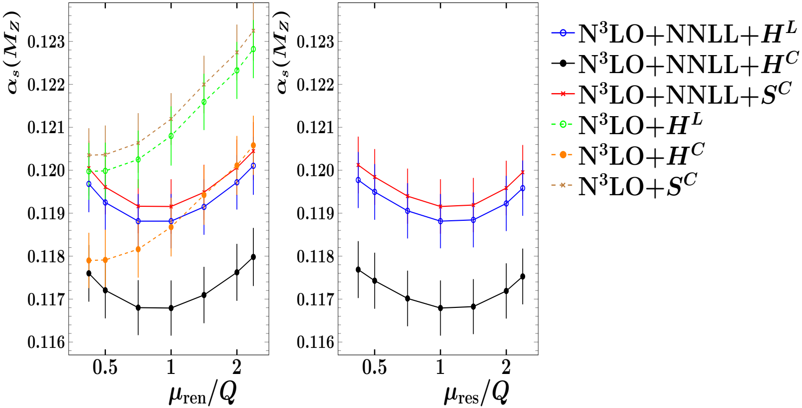


Central result and fit range selection

- Validity of N^{3}LO and resummation.
- Validity of reference hadronization model H^L(Herwig7 with Lund)

- Smallest $\chi^2/ndof$, low sensitivity to fit range.
- Q^2 dependent fit range $[-2.25+\mathcal{L},-1]$, $\mathcal{L}=\log\frac{M_Z^2}{Q^2}$
- Separate ranges for R_2 and R_3 (if used).

Systematics and uncertainties (see also backups)



The uncertainties that were estimated:

- Variation of renormalization scale by $2^{\pm 1}$: (*res.*)
- Variation of resummation scale by $2^{\pm 2}$: (*ren.*)
- Variation of hadronization model H^C instead of H^L: (*hadr.*)
- Fit uncertainty is $\chi^2 + 1$ criterion from MINUIT: (*exp.*)

Conclusions for part II

Extraction of $\alpha_S(M_Z)$ from jet rates in e^+e^- collisions has been performed with N^{3}LO+NNLL precision for the first time from R_2 in wide range of centre-of-mass energies.
The obtained value is
$\alpha_S(M_Z) = 0.11881 \pm 0.00063(exp.) \pm 0.00101(hadr.) \pm 0.00045(ren.) \pm 0.00034(res.)$
for α_S^3+NNLL(R_2) scheme.
Simultaneiuos R_2+R_3 fit is kind of more precise, but more unstable with fit range variations, see backups. The result of such fit is:
$\alpha_S(M_Z) = 0.11989 \pm 0.00045(exp.) \pm 0.00098(hadr.) \pm 0.00046(ren.) \pm 0.00017(res.)$

PART III: What one can learn from the analyses? What are the problems, implications? where to look for an improvement?

Future of more precise $\alpha_{\mathbf{S}}(\mathbf{M_Z})$ from e^+e^- data, I

Accelerators: the low-energy data could be very important.

- 10^7 events for $20 \leq \sqrt{s} \leq 80\,\mathrm{GeV}$ will superpass all previous datasets combined. Runing time of tera-Zs [18, 19] is years→ 10^7 events @ $60 GeV$ in 1 day? Use ISR?
  Understanding/modelling of hadronization → need simple data, e.g. $\sqrt{s} < M_Z$. **Don't run from hadronization, but study it!**
  $+\ R_{b,c,had}$ and $m_{b,c}$ is easier with lower energies.

Physics: new ideas are needed

- Otherwise we stuck with: 1) brute force number crunching with higher fixed&log orders, 2) need of more data or 3) NN/ML "magic".
  Can we use jet grooming [20] or some new observables instead?

Future of more precise $\alpha_S(M_Z)$ from e^+e^- data, II

Data analysis: quality is even more important than before

- Future data: Result + uncertainty + systematics + correlations + cross-sample correlations?
  Sad, but even LEP data don't have correlations.
- Compare $\alpha_S(M_Z)$ extracted from the same data using different (correlated) observables.
- Precise α_s from total vs diff. cross-section vs. EW fits?

Simulation: some hard work is needed

- LHC-era MCEGs should be made more precise for for e^+e^-.
- Implementation of higher fixed/log orders (showers) in MCEGs.

Future of more precise $\alpha_S(M_Z)$ from e^+e^- data, III

Theory: some hard work is needed

- Quark mass effects for the $\mathcal{O}(\alpha_S^3)$ predictions. Why: most important data have high r_b or m_b^2/s.
- Higher order resummation, analytic calculations.
- Obtaining $\mathcal{O}(\alpha_S^3) \times \mathcal{O}(\alpha_{EM}^n)$ predictions.

PART IV: Final conclusions + Advertisement

Final

- The presented recent extractions of α_S from data collected at LEP/PEP/TRISTAN/PETRA give us more understanding what is needed to get even better results from the future CEPC data.
- The following values of $\alpha_S(M_Z)$ were obtained in analyses:
  - $\alpha_\mathbf{S}(\mathbf{M_Z}) = \mathbf{0.11750} \pm \mathbf{0.00018(exp.)} \pm \mathbf{0.00102(hadr.)} \pm \mathbf{0.00257(ren.)} \pm \mathbf{0.00078(res.)}$
  - $\alpha_\mathbf{S}(\mathbf{M_Z}) = \mathbf{0.11881} \pm \mathbf{0.00063(exp.)} \pm \mathbf{0.00101(hadr.)} \pm \mathbf{0.00045(ren.)} \pm \mathbf{0.00034(res.)}$
  - The presented results are precise, most precise in their subclass, see backups.

- The JADE and OPAL data is preserved in MPP, so in case you have good ideas what you would like to extract from the real data, we are open for the collaboration.

Backups

Backup slides part I:MC based approach: MC setups

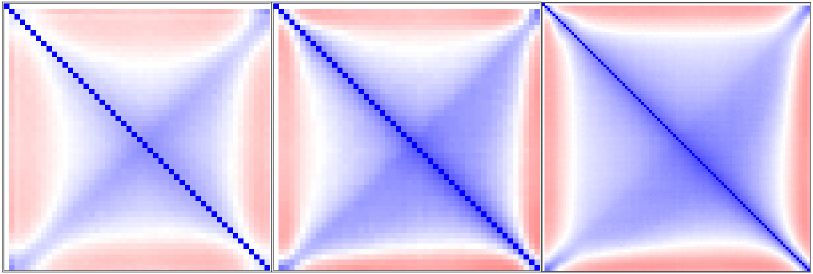
$e^+e^- \to jjjjj$ merged samples with massive b quarks and 2-jet final state in NLO precision.

- **Default setup "S^L": Sherpa2.2.4+ (Comix, Amegic, GoSam ME libraries and OLPs) + Lund (Pythia6) hadronization**
- Setup for hadronization systematics: "S^C": Sherpa2.2.4+ (Comix, Amegic, GoSam ME libraries and OLPs) + Ahadic cluster hadronization
- Setup for cross-check: "H^M": Herwig7.1.1 (Herwig, Madgraph, GoSam ME libraries and OLPs) + Herwig cluster hadronization

Merging scale was chosen to minimise its size impact on parton level in fit range.

Backup slides part I: MC based approach: correlations

JADE, $\sqrt{s} = 22\,\text{GeV}$	TOPAZ, $\sqrt{s} = 59\,\text{GeV}$	OPAL, $\sqrt{s} = 91\,\text{GeV}$

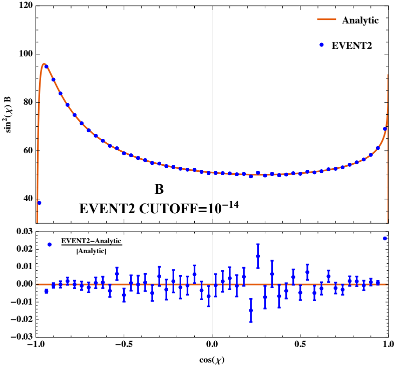


Color to correlation coefficient	-100	-80	-60	-40	-20	0 or N/A	20	40	60	80	100

- All measurements are provided without correlations.
- MC samples are used to model correlations between points, see original Fisher papers [21].

Backup slides part I: Theory work in progress?

- N^3LL resummation under study with recently SCET calculations [22].
- NLO analytic results available [23].



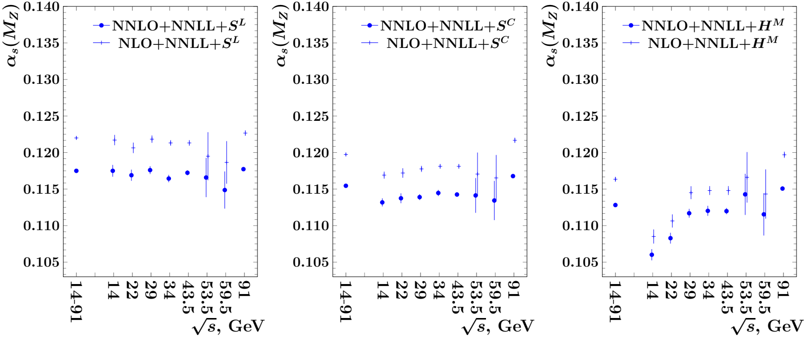
Backup slides part I: fits

Fit range,° Hadronization	NLO+NNLL $\chi^2/ndof$	NNLO+NNLL $\chi^2/ndof$
117 − 165° S^L	0.12042 ± 0.00025 765/298 = 2.57	0.11760 ± 0.00020 513/298 = 1.72
60 − 165° S^L	0.12134 ± 0.00022 1720/664 = 2.59	0.11746 ± 0.00018 1211/664 = 1.82
60 − 160° S^L	0.12200 ± 0.00023 1417/623 = 2.27	0.11750 ± 0.00018 1022/623 = 1.64
117 − 165° S^C	0.11796 ± 0.00022 631/298 = 2.12	0.11521 ± 0.00017 395/298 = 1.32
60 − 165° S^C	0.11900 ± 0.00021 1557/664 = 2.34	0.11530 ± 0.00015 951/664 = 1.43
60 − 160° S^C	0.11973 ± 0.00022 1321/623 = 2.12	0.11545 ± 0.00016 845/623 = 1.36
117 − 165° H^M	0.11272 ± 0.00037 1842/298 = 6.18	0.11044 ± 0.00029 1201/298 = 4.03
60 − 165° H^M	0.11472 ± 0.00033 3845/664 = 5.79	0.11180 ± 0.00023 2203/664 = 3.32
60 − 160° H^M	0.11634 ± 0.00033 3091/623 = 4.96	0.11281 ± 0.00023 1738/623 = 2.79
117 − 165° $An.^{DMW}$	0.12154 ± 0.00045 730/295 = 2.48	0.11781 ± 0.00037 558/295 = 1.89
60 − 165° $An.^{DMW}$	0.13555 ± 0.00052 7525/661 = 11.38	0.12937 ± 0.00039 4896/661 = 7.41
60 − 160° $An.^{DMW}$	0.13606 ± 0.00061 7364/620 = 11.88	0.12950 ± 0.00044 4827/620 = 7.78

Table: Results of the fits of the matched predictions at NLO+NNLL and NNLO+NNLL accuracy to experimental data. The given uncertainty is fit uncertainty scaled by $\sqrt{\chi^2/ndof}$.

Backup slides part I: More checks

- Analytic hadronization
- Fit range variation (see backups)
- Power in resummation expressions
- Herwig7 for hadronization
- Stability across $\sqrt{s}$ (see below)
- Scheme of b mass treatment
- NLO only fits



Backup slides part II: MC-based non-pert. corrections

Challenge: simultaneous correction of R_2 and R_3.

Introduce ξ_1, ξ_2, so

$$R_{2,parton} = \cos^2 \xi_1,$$

$$R_{3,parton} = \sin^2 \xi_1 \cos^2 \xi_2$$

and

$$R_{2,hadron} = \cos^2(\xi_1 + \delta\xi_1),$$

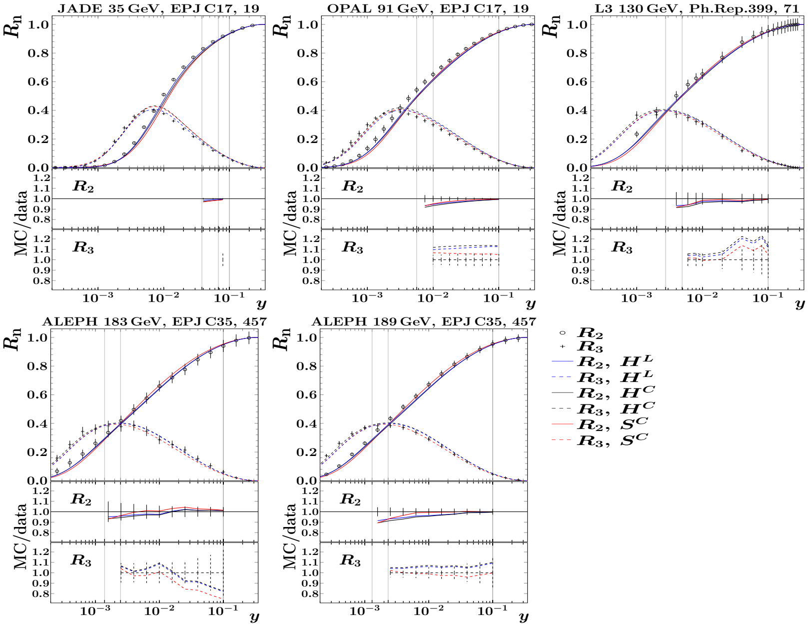
$$R_{3,hadron} = \sin^2(\xi_1+\delta\xi_1) \cos^2(\xi_2+\delta\xi_2).$$

Setup: $e^+e^- \to jjjjj$ merged samples with massive b
Differences to EEC:

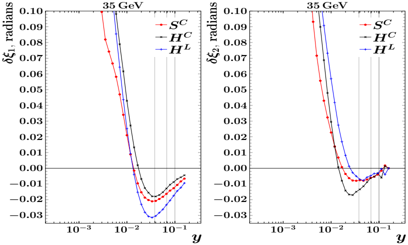
- OpenLoops [24] instead of GoSam as OLP
- SHERPA2.2.6
- Herwig7.1.4, also 3-jet FS in NLO.
- No reweighting
- Herwig7+Lund (H^L) is taken for central result

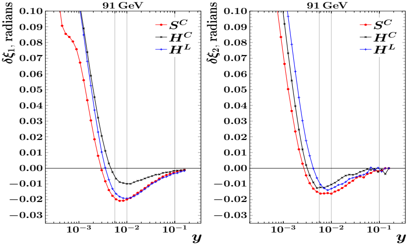
Approach preserves normalisation. $\delta\xi_1(y)$ and $\delta\xi_2(y)$ are corrections to be extracted, see backups.

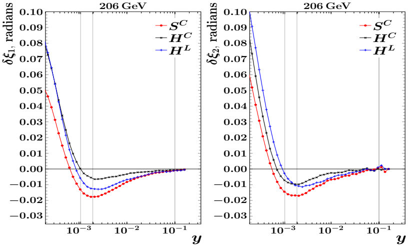
Backup slides part II: Hadron level distributions



Backup slides part II: Hadronization corrections

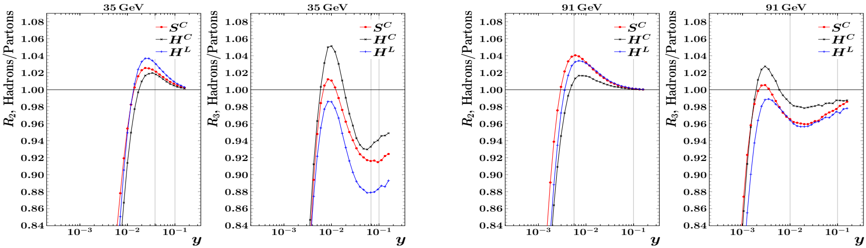


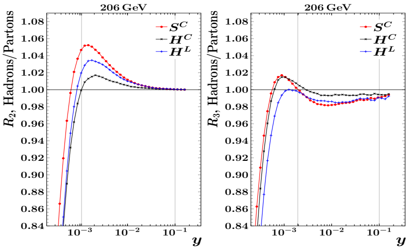




- To avoid binning effects the hadronization corrections are parametrised with smooth functions.

Backup slides part II: hadron to parton level ratios





- To avoid binning effects the hadronization corrections are parametrised with smooth functions.

Backup slides part II: R_2 fits

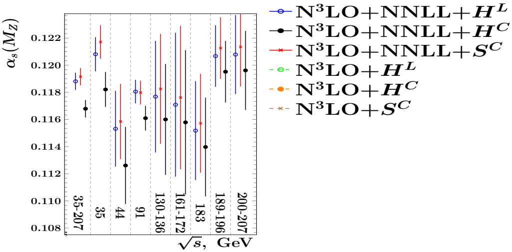
Fit ranges, log y Hadronization	$\mathrm{N}^3\mathrm{LO}$ $\chi^2/ndof$	$\mathrm{N}^3\mathrm{LO}+\mathrm{NNLL}$ $\chi^2/ndof$
$[-1.75+\mathcal{L},-1]$ S^C	0.12121 ± 0.00095 $20/86 = 0.24$	0.11849 ± 0.00092 $20/86 = 0.24$
$[-2+\mathcal{L},-1]$ S^C	0.12114 ± 0.00081 $26/100 = 0.26$	0.11864 ± 0.00075 $26/100 = 0.26$
$[-2.25+\mathcal{L},-1]$ S^C	0.12119 ± 0.00060 $44/150 = 0.29$	0.11916 ± 0.00063 $44/150 = 0.29$
$[-2.5+\mathcal{L},-1]$ S^C	0.12217 ± 0.00052 $89/180 = 0.50$	0.12075 ± 0.00055 $107/180 = 0.59$
$[-1.75+\mathcal{L},-1]$ H^C	0.11957 ± 0.00098 $22/86 = 0.26$	0.11698 ± 0.00093 $22/86 = 0.25$
$[-2+\mathcal{L},-1]$ H^C	0.11923 ± 0.00079 $29/100 = 0.29$	0.11687 ± 0.00076 $28/100 = 0.28$
$[-2.25+\mathcal{L},-1]$ H^C	0.11868 ± 0.00068 $43/150 = 0.28$	0.11679 ± 0.00064 $40/150 = 0.27$
$[-2.5+\mathcal{L},-1]$ H^C	0.11849 ± 0.00050 $58/180 = 0.32$	0.11723 ± 0.00053 $58/180 = 0.32$
$[-1.75+\mathcal{L},-1]$ H^L	0.12171 ± 0.00109 $21/86 = 0.25$	0.11897 ± 0.00092 $21/86 = 0.24$
$[-2+\mathcal{L},-1]$ H^L	0.12144 ± 0.00078 $28/100 = 0.28$	0.11893 ± 0.00075 $26/100 = 0.26$
$[-2.25+\mathcal{L},-1]$ H^L	0.12080 ± 0.00069 $43/150 = 0.28$	0.11881 ± 0.00063 $39/150 = 0.26$
$[-2.5+\mathcal{L},-1]$ H^L	0.12024 ± 0.00051 $57/180 = 0.32$	0.11897 ± 0.00053 $52/180 = 0.29$

Table: Fit of $\alpha_s(M_Z)$ from experimental data for R_2 obtained using $\mathrm{N}^3\mathrm{LO}$ and $\mathrm{N}^3\mathrm{LO}+\mathrm{NNLL}$ predictions for R_2. The reported uncertainty comes from `MINUIT2`.

Backup slides part II: more checks

- Simultaneiuos R_2+R_3 fit (see below)
- Separate R_3 fit
- Variation of χ^2 definition
- Changes of fit ranges

- Multiplicative hadronization corrections
- Sherpa MC hadronization S^C
- Stability across $\sqrt{s}$ (see below)
- Exclusion of data $\sqrt{s} < M_Z$



Simultaneiuos R_2+R_3 fit is not more precise, but much more unstable with fit variations, see backups. The result of such fit is:

$\alpha_S(M_Z) = 0.11989 \pm 0.00045(exp.) \pm 0.00098(hadr.) \pm 0.00046(ren.) \pm 0.00017(res.)$

Backup slides: Final results

- The following values of $\alpha_S(M_Z)$ were obtained in analyses:
  - $\alpha_\mathbf{S}(\mathbf{M_Z}) = \mathbf{0.11750} \pm \mathbf{0.00018(exp.)} \pm \mathbf{0.00102(hadr.)} \pm \mathbf{0.00257(ren.)} \pm \mathbf{0.00078(res.)}$
  - $\alpha_\mathbf{S}(\mathbf{M_Z}) = \mathbf{0.11881} \pm \mathbf{0.00063(exp.)} \pm \mathbf{0.00101(hadr.)} \pm \mathbf{0.00045(ren.)} \pm \mathbf{0.00034(res.)}$
- The presented results are precise, most precise in their subclass.

Determination	Data and procedure	Reference
0.1175 ± 0.0025	ALEPH 3-jet rate (NNLO+MChad)	[25]
0.1199 ± 0.0059	JADE 3-jet rate (NNLO+NLL+MChad)	[26]
0.1224 ± 0.0039	ALEPH event shapes (NNLO+NLL+MChad)	[27]
0.1172 ± 0.0051	JADE event shapes (NNLO+NLL+MChad)	[28]
0.1189 ± 0.0041	OPAL event shapes (NNLO+NLL+MChad)	[29]
$0.1164\,^{+0.0028}_{-0.0026}$	Thrust (NNLO+NLL+anlhad)	[30]
$0.1134\,^{+0.0031}_{-0.0025}$	Thrust (NNLO+NNLL+anlhad)	[31]
0.1135 ± 0.0011	Thrust (SCET NNLO+N^3LL+anlhad)	[32]
0.1123 ± 0.0015	C-parameter (SCET NNLO+N^3LL+anlhad)	[33]

Table: Determinations of the strong coupling from jet rates and event shapes in e^+e^- collisions. The uncertainties are added in quadratures. Typically the statustical uncertainty is negligible. Source: arXiv:1712.05165v2

Backup slides part II: R_2+R_3 fits

Fit ranges, $\log y$ Hadronization	N^{3}LO, NNLO $\chi^2/ndof$	N^{3}LO+NNLL, NNLO $\chi^2/ndof$
$[-1.75+\mathcal{L},-1][-1.5+\mathcal{L},-1]$ S^C	0.12195 ± 0.00072 $120/143 = 0.84$	0.12078 ± 0.00066 $140/143 = 0.98$
$[-2+\mathcal{L},-1][-1.75+\mathcal{L},-1]$ S^C	0.12163 ± 0.00061 $153/187 = 0.82$	0.12065 ± 0.00056 $176/187 = 0.94$
$[-2.25+\mathcal{L},-1][-2+\mathcal{L},-1]$ S^C	0.12075 ± 0.00044 $208/251 = 0.83$	0.11994 ± 0.00041 $222/251 = 0.88$
$[-2.5+\mathcal{L},-1][-2.25+\mathcal{L},-1]$ S^C	0.12143 ± 0.00043 $321/331 = 0.97$	0.12089 ± 0.00044 $336/331 = 1.01$
$[-1.75+\mathcal{L},-1][-1.5+\mathcal{L},-1]$ H^C	0.12068 ± 0.00073 $126/143 = 0.88$	0.11956 ± 0.00066 $147/143 = 1.03$
$[-2+\mathcal{L},-1][-1.75+\mathcal{L},-1]$ H^C	0.12006 ± 0.00061 $163/187 = 0.87$	0.11913 ± 0.00054 $188/187 = 1.01$
$[-2.25+\mathcal{L},-1][-2+\mathcal{L},-1]$ H^C	0.11869 ± 0.00043 $221/251 = 0.88$	0.11793 ± 0.00043 $238/251 = 0.95$
$[-2.5+\mathcal{L},-1][-2.25+\mathcal{L},-1]$ H^C	0.11845 ± 0.00045 $302/331 = 0.91$	0.11799 ± 0.00047 $310/331 = 0.94$
$[-1.75+\mathcal{L},-1][-1.5+\mathcal{L},-1]$ H^L	0.12248 ± 0.00068 $121/143 = 0.85$	0.12129 ± 0.00063 $141/143 = 0.99$
$[-2+\mathcal{L},-1][-1.75+\mathcal{L},-1]$ H^L	0.12211 ± 0.00057 $155/187 = 0.83$	0.12110 ± 0.00053 $180/187 = 0.96$
$[-2.25+\mathcal{L},-1][-2+\mathcal{L},-1]$ H^L	0.12071 ± 0.00044 $209/251 = 0.83$	0.11989 ± 0.00045 $227/251 = 0.90$
$[-2.5+\mathcal{L},-1][-2.25+\mathcal{L},-1]$ H^L	0.12041 ± 0.00044 $266/331 = 0.80$	0.11990 ± 0.00044 $278/331 = 0.84$

Table: Simultaneous fit of $\alpha_s(M_Z)$ from experimental data for R_2 and R_3 obtained using N^{3}LO and N^{3}LO+NNLL predictions for R_2 and NNLO predictions for R_3. The reported uncertainty comes from `MINUIT2`.

Backup slides part II: Future of more precise $\alpha_S(M_Z)$ from e^+e^- data

Some points for discussion:

- *" Since the analysis in Ref. [391] only uses data at or below the Z pole, it is expected that future data from CEPC at 250 GeV can significantly reduce the hadronization uncertainty. "* [18]

  Despite the "hadronization corrections" decrease with the energy, the related uncertainty on the $\alpha_S(M_Z)$ is defined with the **uncertainty** of "hadronization corrections". To reduce the **uncertainty**, more studies of hadronization are needed, which means more data from environments where the hadronization effects are prominent, i.e. energies below Z.

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