$H \to b\bar{b}$ at N3LO accuracy

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RM, Matthew Schiavi, Ciaran Williams, JHEP 1906 (2019) 079
RM and Ciaran Williams, JHEP 1906 (2019) 120

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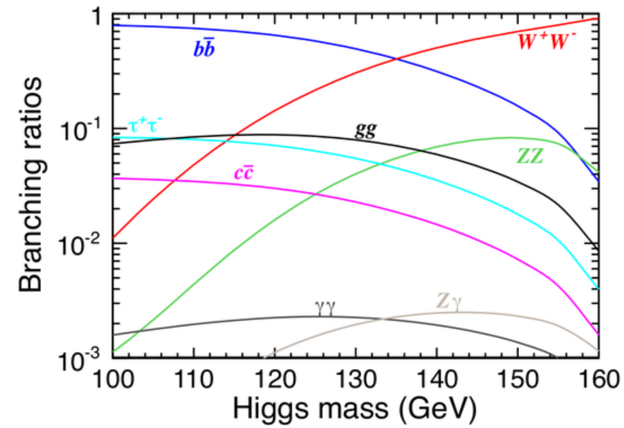
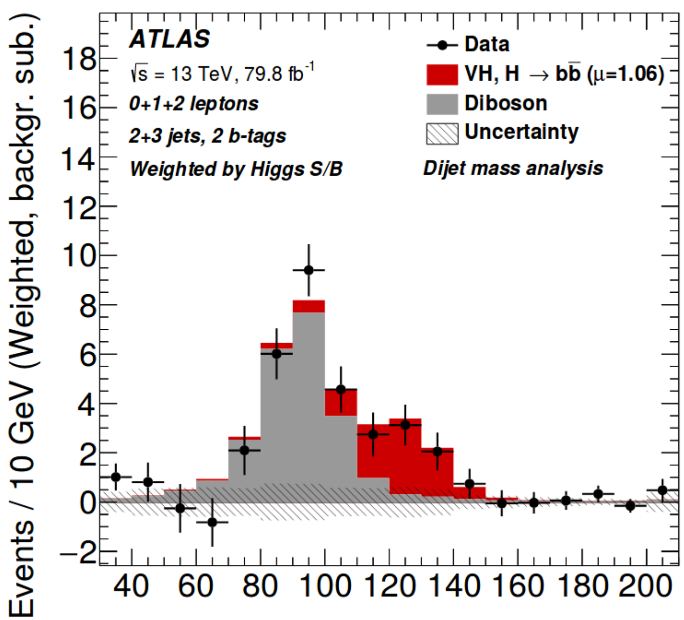
Motivation

The H→b$\bar{b}$ decay channel has the largest BR for the 125-GeV Higgs

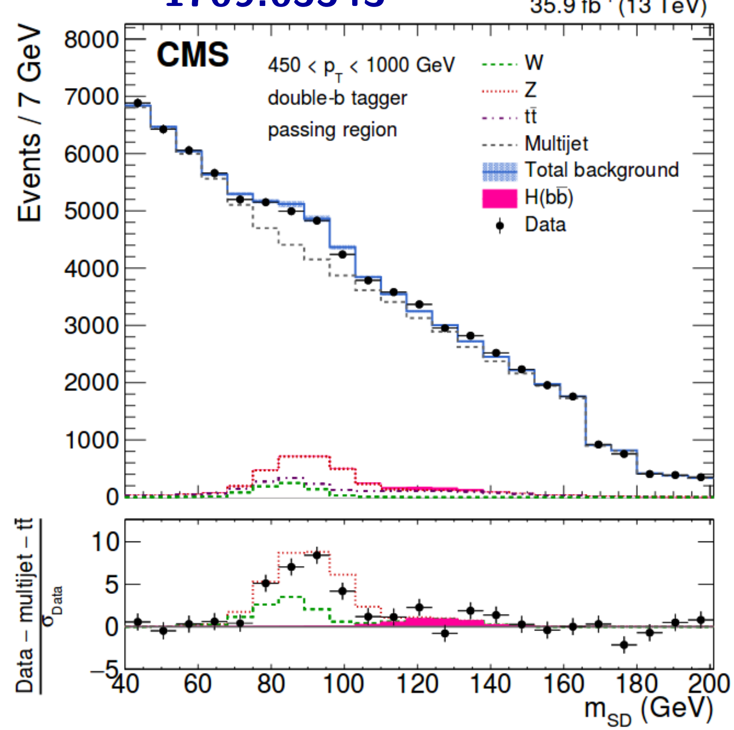
Can be accessed at the LHC through associated (VH) production or gluon-fusion at high transverse momentum





1808.08238

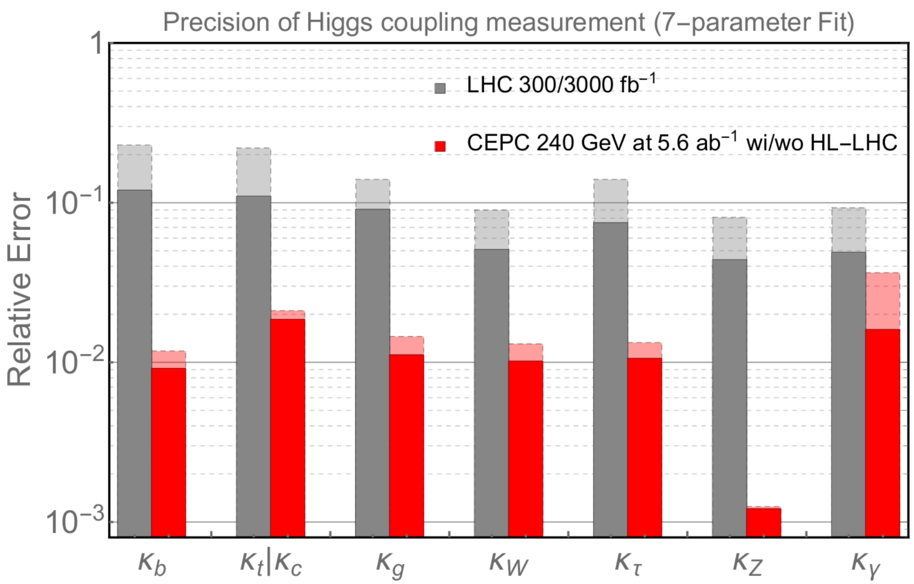
1709.05543



Motivation

At future lepton colliders such as the CEPC, most Higgs couplings will be measured at the 1% level

The increasing experimental precision mandates a similar increase in the precision of the corresponding theoretical predictions



Property	Estimated Precision
m_H	5.9 MeV
Γ_H	3.1%
$\sigma(ZH)$	0.5%
$\sigma(\nu\bar{\nu}H)$	3.2%

Decay mode	$\sigma(ZH) \times$ BR	BR
$H \to b\bar{b}$	0.27%	0.56%
$H \to c\bar{c}$	3.3%	3.3%
$H \to gg$	1.3%	1.4%
$H \to WW^*$	1.0%	1.1%
$H \to ZZ^*$	5.1%	5.1%
$H \to \gamma\gamma$	6.8%	6.9%
$H \to Z\gamma$	15%	15%
$H \to \tau^+\tau^-$	0.8%	1.0%
$H \to \mu^+\mu^-$	17%	17%
$H \to$ inv	–	$< 0.30\%$

CEPC CDR Oct 18

Overview of the calculation

$$\Gamma_{H\to b\bar{b}} = \Gamma^{\mathrm{LO}}_{H\to b\bar{b}} + \Delta\Gamma^{\mathrm{NLO}}_{H\to b\bar{b}} + \Delta\Gamma^{\mathrm{NNLO}}_{H\to b\bar{b}} + \Delta\Gamma^{\mathrm{N3LO}}_{H\to b\bar{b}} + \Delta\Gamma^{\mathrm{N4LO}}_{H\to b\bar{b}} + \ldots$$

Inclusively known up to:

- N4LO QCD [Baikov, Chetyrkin, Kuhn hep-ph/0511063]
- NLO EW [Dabelstein, Hollik (1992); Kataev hep-ph/9708292]
- Mixed QCDxEW [Kataev hep-ph/9708292; Mihaila, Schmidt, Steinhauser 1509.02294] (also QCDxEW master integrals for Htt coupling [Chaubey, Weinzierl 1904.00382])

Differentially:

- NNLO QCD [Anastasiou, Herzog, Lazopoulos 1110.2368; Del Duca, Duhr, Somogyi, Tramontano, Trócsányi 1501.07226; Bernreuther, Cheng, Si 1805.06658]
- Interfaced to VH production at NNLO QCD [Ferrera, Somogyi, Tramontano 1705.10304; Caola, Luisoni, Melnikov, Röntsch 1712.06954; Gauld, Gehrmann-De Ridder, Glover, Huss, Majer 1907.05836]

Aim: provide fully-differential predictions at N3LO QCD accuracy

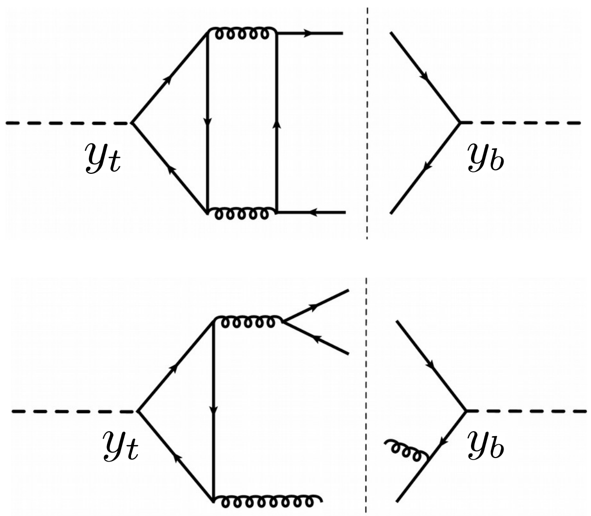
Overview of the calculation

- Treat the bottom quark as **massless**
- Focus on y_b^2 terms

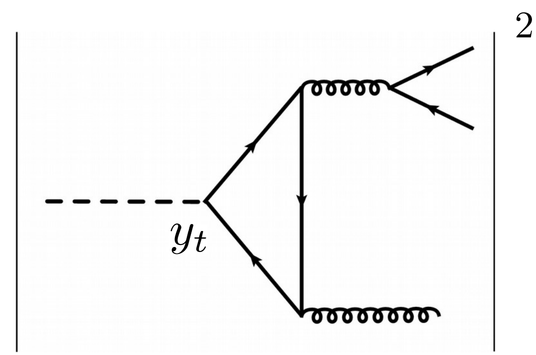
$$\Gamma^{\mathrm{N3LO}}_{H\to b\bar{b}} = y_b^2 A_b + \alpha_s y_b^2 B_b + \alpha_s^2 \left(y_b^2 C_b + y_b y_t C_{bt}\right) + \alpha_s^3 \left(y_b^2 D_b + y_b y_t D_{bt} + y_t^2 D_t\right)$$

in the full theory
[Primo, Sasso, Somogyi, Tramontano 1812.07811]

$y_b y_t C_{bt}$



$y_t^2 D_t$



+ $\mathcal{O}(\alpha_s)$ corrections

Ongoing work to include neglected terms (as well as EW and QCDxEW)

Overview of the calculation

Differential N3LO coefficient:

$$
\begin{aligned}
\frac{d\,\Delta\Gamma^{\mathrm{N3LO}}_{H\to b\bar{b}}}{d\,\mathcal{O}_m} = \quad & \int d\Gamma^{VVV}_{H\to b\bar{b}} F_2^m(\Phi_2) d\Phi_2 \\
+ & \int d\Gamma^{RVV}_{H\to b\bar{b}} F_3^m(\Phi_3) d\Phi_3 \\
+ & \int d\Gamma^{RRV}_{H\to b\bar{b}} F_4^m(\Phi_4) d\Phi_4 \\
+ & \int d\Gamma^{RRR}_{H\to b\bar{b}} F_5^m(\Phi_5) d\Phi_5
\end{aligned}
$$

Overview of the calculation

Differential N3LO coefficient:

$$\frac{d\,\Delta\Gamma^{\rm N3LO}_{H\to b\bar{b}}}{d\,\mathcal{O}_m} = \int d\Gamma^{VVV}_{H\to b\bar{b}} F_2^m(\Phi_2)d\Phi_2$$

$$+ \int d\Gamma^{RVV}_{H\to b\bar{b}} F_3^m(\Phi_3)d\Phi_3$$

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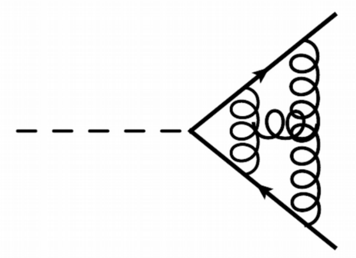

triple-virtual (3 loops, 2 partons)

Overview of the calculation

Differential N3LO coefficient:

$$\frac{d\,\Delta\Gamma^{\mathrm{N3LO}}_{H\to b\bar{b}}}{d\,\mathcal{O}_m} = \int d\Gamma^{VVV}_{H\to b\bar{b}} F_2^m(\Phi_2) d\Phi_2$$
$$+ \int d\Gamma^{RVV}_{H\to b\bar{b}} F_3^m(\Phi_3) d\Phi_3$$
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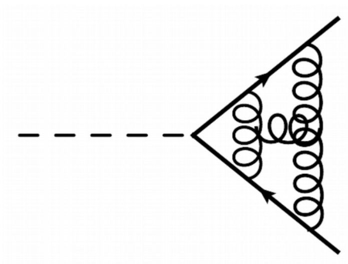
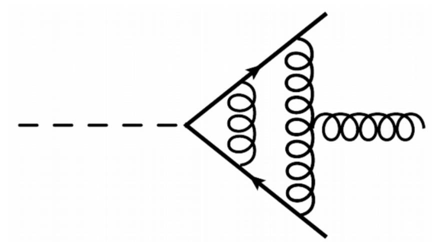


triple-virtual (3 loops, 2 partons)

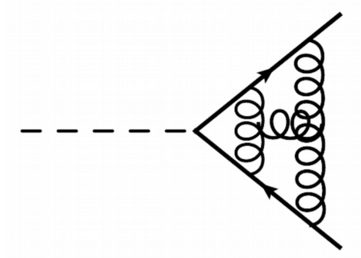


real double-virtual (2 loops, 3 partons)

Overview of the calculation

Differential N3LO coefficient:

$$\frac{d\,\Delta\Gamma^{\rm N3LO}_{H\to b\bar{b}}}{d\,\mathcal{O}_m} = \int d\Gamma^{VVV}_{H\to b\bar{b}} F_2^m(\Phi_2)d\Phi_2 + \int d\Gamma^{RVV}_{H\to b\bar{b}} F_3^m(\Phi_3)d\Phi_3 + \int d\Gamma^{RRV}_{H\to b\bar{b}} F_4^m(\Phi_4)d\Phi_4 + \int d\Gamma^{RRR}_{H\to b\bar{b}} F_5^m(\Phi_5)d\Phi_5$$



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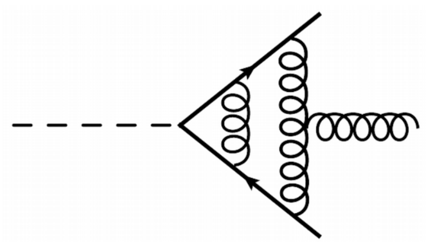

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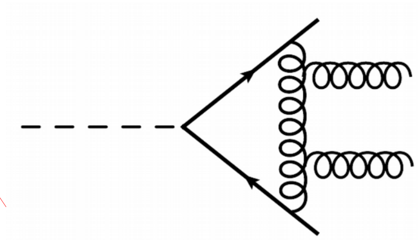
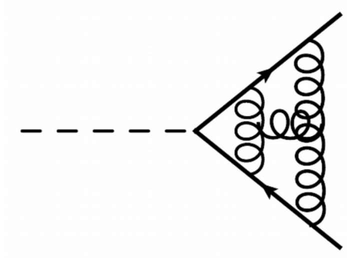


double-real virtual (1 loop, 4 partons)

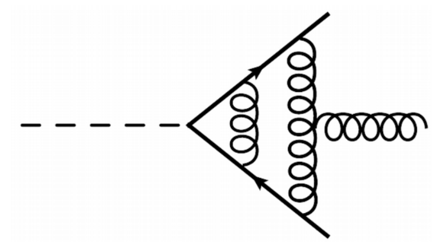
Overview of the calculation

Differential N3LO coefficient:

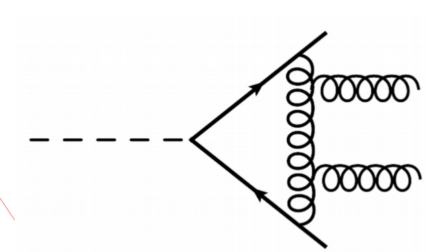
$$\frac{d\,\Delta\Gamma^{\mathrm{N3LO}}_{H\to b\bar{b}}}{d\,\mathcal{O}_m} = \int d\Gamma^{VVV}_{H\to b\bar{b}} F_2^m(\Phi_2)d\Phi_2 + \int d\Gamma^{RVV}_{H\to b\bar{b}} F_3^m(\Phi_3)d\Phi_3 + \int d\Gamma^{RRV}_{H\to b\bar{b}} F_4^m(\Phi_4)d\Phi_4 + \int d\Gamma^{RRR}_{H\to b\bar{b}} F_5^m(\Phi_5)d\Phi_5$$



triple-virtual (3 loops, 2 partons)



real double-virtual (2 loops, 3 partons)



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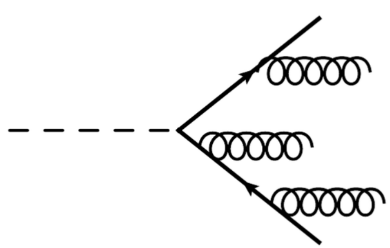

triple-real (0 loops, 5 partons)

Overview of the calculation

$$\frac{d\,\Delta\Gamma^{\mathrm{N3LO}}_{H\to b\bar{b}}}{d\,\mathcal{O}_m} = \int d\Gamma^{VVV}_{H\to b\bar{b}} F^m_2(\Phi_2)\,d\Phi_2 + \int d\Gamma^{RVV}_{H\to b\bar{b}} F^m_3(\Phi_3)\,d\Phi_3 + \int d\Gamma^{RRV}_{H\to b\bar{b}} F^m_4(\Phi_4)\,d\Phi_4 + \int d\Gamma^{RRR}_{H\to b\bar{b}} F^m_5(\Phi_5)\,d\Phi_5$$

$F^m_i(\Phi_i)$ uses a jet-clustering algorithm to define an *m*-jet observable from *i* final-state partons

Each contribution contains soft and collinear IR divergences that cancel upon combination into a suitably-inclusive observable

Projection-to-Born method

We use the Projection-to-Born (P2B) method to deal with the IR divergences
[Cacciari, Dreyer, Karlberg, Salam, Zanderighi 1506.02660]

Main idea: construct local counter-terms for the matrix elements projected onto a LO (Born) phase space.

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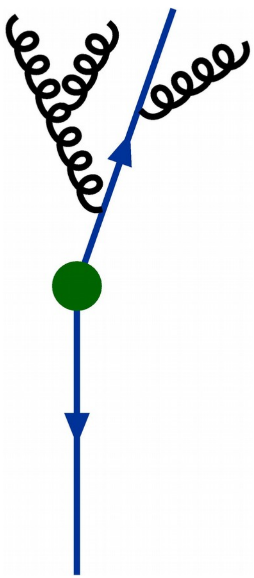
Example with $i=5$ partons clustered into $m=2$ jets:



Generated event with $|\mathcal{M}|^2$

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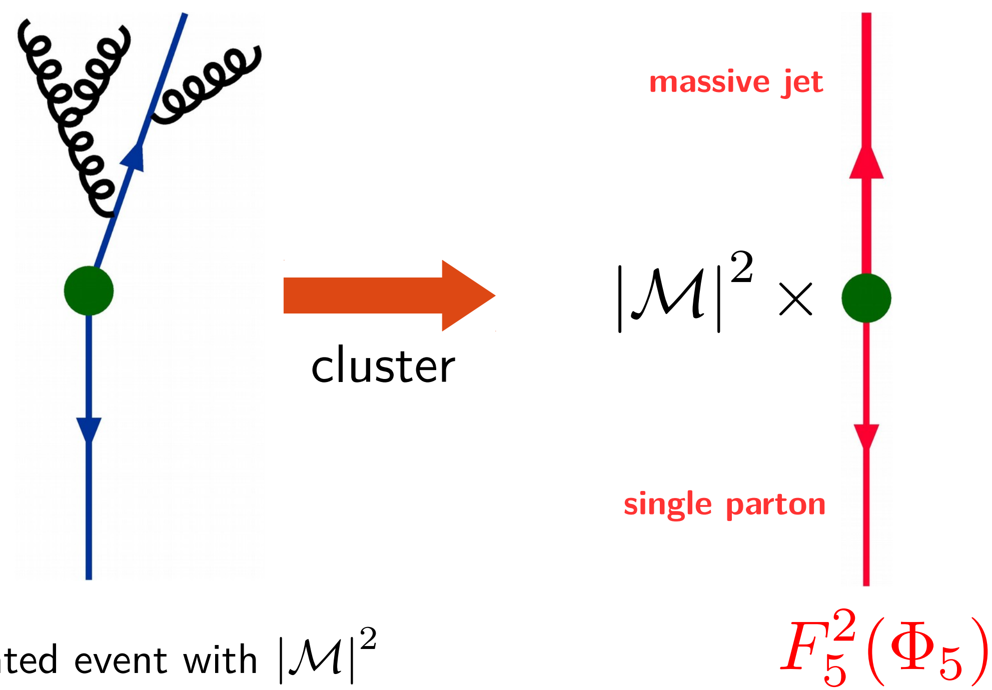
Example with *i*=*5* partons clustered into *m*=*2* jets:



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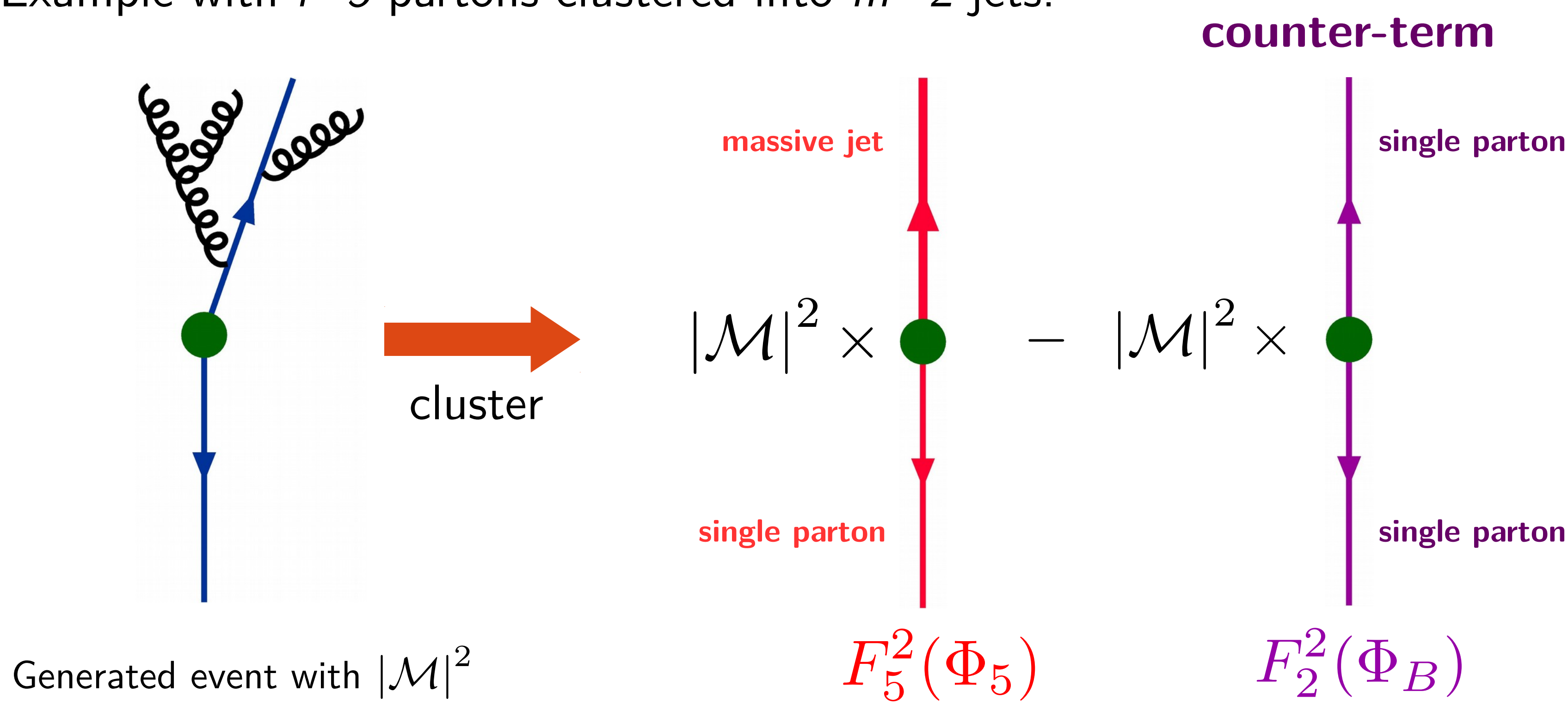
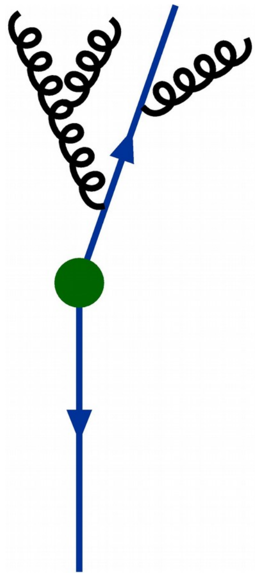


Projection-to-Born method



$$|\mathcal{M}|^2 \times \left(F_5^2(\Phi_5) - F_2^2(\Phi_B)\right)$$

The IR divergences cancel *exactly* when the full phase space matches the Born-projected phase space.

This is the triple-unresolved region.

Born phase space in the Higgs rest frame:

$$\Phi_B = \{p_1, p_2\} \qquad p_1 = \frac{m_H}{2}(1, \mathbf{n}_j) \quad p_2 = \frac{m_H}{2}(1, -\mathbf{n}_j)$$

with $\mathbf{n}_j$ the direction of the leading jet.

Projection-to-Born method

To restore the N3LO coefficient we need to add back the counter-term that we arbitrarily subtracted:

$$
\begin{aligned}
\frac{d\,\Delta\Gamma^{\rm N3LO}_{H\to b\bar{b}}}{d\,\mathcal{O}_m} = &+\int d\Gamma^{RVV}_{H\to b\bar{b}}\,[F_3^m(\Phi_3) - F_2^m(\Phi_B)]\,d\Phi_3 \\
&+\int d\Gamma^{RRV}_{H\to b\bar{b}}\,[F_4^m(\Phi_4) - F_2^m(\Phi_B)]\,d\Phi_4 \\
&+\int d\Gamma^{RRR}_{H\to b\bar{b}}\,[F_5^m(\Phi_5) - F_2^m(\Phi_B)]\,d\Phi_5 \\
&+\int d\Gamma^{VVV}_{H\to b\bar{b}} F_2^m(\Phi_B) d\Phi_2 + \int d\Gamma^{RVV}_{H\to b\bar{b}} F_2^m(\Phi_B) d\Phi_3 \\
&+\int d\Gamma^{RRV}_{H\to b\bar{b}} F_2^m(\Phi_B) d\Phi_4 + \int d\Gamma^{RRR}_{H\to b\bar{b}} F_2^m(\Phi_B) d\Phi_5
\end{aligned}
$$

Projection-to-Born method

To restore the N3LO coefficient we need to add back the counter-term that we arbitrarily subtracted:

$$\frac{d\,\Delta\Gamma^{\rm N3LO}_{H\to b\bar{b}}}{d\,\mathcal{O}_m} = +\int d\Gamma^{RVV}_{H\to b\bar{b}}\,[F_3^m(\Phi_3) - F_2^m(\Phi_B)]\,d\Phi_3$$
$$+\int d\Gamma^{RRV}_{H\to b\bar{b}}\,[F_4^m(\Phi_4) - F_2^m(\Phi_B)]\,d\Phi_4$$
$$+\int d\Gamma^{RRR}_{H\to b\bar{b}}\,[F_5^m(\Phi_5) - F_2^m(\Phi_B)]\,d\Phi_5$$
$$+\int d\Gamma^{VVV}_{H\to b\bar{b}} F_2^m(\Phi_B) d\Phi_2 + \int d\Gamma^{RVV}_{H\to b\bar{b}} F_2^m(\Phi_B) d\Phi_3$$
$$+\int d\Gamma^{RRV}_{H\to b\bar{b}} F_2^m(\Phi_B) d\Phi_4 + \int d\Gamma^{RRR}_{H\to b\bar{b}} F_2^m(\Phi_B) d\Phi_5$$

$$\frac{d\Delta\Gamma^{\rm N3LO,\,incl}_{H\to b\bar{b}}}{d\mathcal{O}_m^B} = \int \Delta\Gamma^{\rm N3LO}_{H\to b\bar{b}} F_2^m(\Phi_B) d\Phi_B$$

Ingredient 1: Inclusive N3LO H→b$\bar{\text{b}}$ width as a function of the Born kinematics

Projection-to-Born method

To restore the N3LO coefficient we need to add back the counter-term that we arbitrarily subtracted:

$$
\begin{aligned}
\frac{d\,\Delta\Gamma^{\rm N3LO}_{H\to b\bar b}}{d\,\mathcal{O}_m} = &+\int d\Gamma^{RVV}_{H\to b\bar b}\left[F^m_3(\Phi_3) - F^m_2(\Phi_B)\right] d\Phi_3 \\
&+\int d\Gamma^{RRV}_{H\to b\bar b}\left[F^m_4(\Phi_4) - F^m_2(\Phi_B)\right] d\Phi_4 \\
&+\int d\Gamma^{RRR}_{H\to b\bar b}\left[F^m_5(\Phi_5) - F^m_2(\Phi_B)\right] d\Phi_5 \\
&+\int d\Gamma^{VVV}_{H\to b\bar b} F^m_2(\Phi_B) d\Phi_2 + \int d\Gamma^{RVV}_{H\to b\bar b} F^m_2(\Phi_B) d\Phi_3 \\
&+\int d\Gamma^{RRV}_{H\to b\bar b} F^m_2(\Phi_B) d\Phi_4 + \int d\Gamma^{RRR}_{H\to b\bar b} F^m_2(\Phi_B) d\Phi_5
\end{aligned}
$$

$$
\frac{d\,\Delta\Gamma^{\rm NNLO}_{H\to b\bar b j}}{d\,\mathcal{O}_m} - \frac{d\,\Delta\Gamma^{\rm NNLO}_{H\to b\bar b j}}{d\,\mathcal{O}^B_m}
$$

Ingredient 2: Differential NNLO $H\to b\bar{b}j$ width and its Born projection

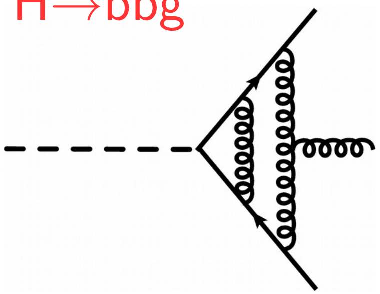
Differential NNLO H→b$\bar{\text{b}}$j width

$$\frac{d\,\Delta\Gamma^{\mathrm{NNLO}}_{H\to b\bar{b}j}}{d\,\mathcal{O}_m} = \int d\Gamma^{VV}_{H\to b\bar{b}j} F_3^m(\Phi_3) d\Phi_3 + \int d\Gamma^{RV}_{H\to b\bar{b}j} F_4^m(\Phi_4) d\Phi_4 + \int d\Gamma^{RR}_{H\to b\bar{b}j} F_5^m(\Phi_5) d\Phi_5$$

Differential NNLO H→b$\bar{\text{b}}$j width

$$\frac{d\,\Delta\Gamma^{\rm NNLO}_{H\to b\bar{b}j}}{d\,\mathcal{O}_m} = \int d\Gamma^{VV}_{H\to b\bar{b}j} F_3^m(\Phi_3) d\Phi_3 + \int d\Gamma^{RV}_{H\to b\bar{b}j} F_4^m(\Phi_4) d\Phi_4 + \int d\Gamma^{RR}_{H\to b\bar{b}j} F_5^m(\Phi_5) d\Phi_5$$

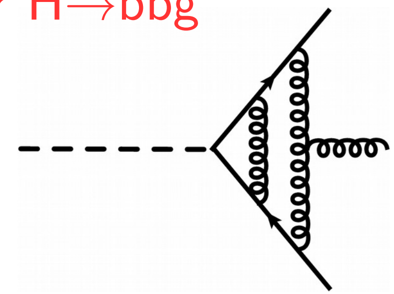
two-loop amplitudes for H→b$\bar{\text{b}}$g



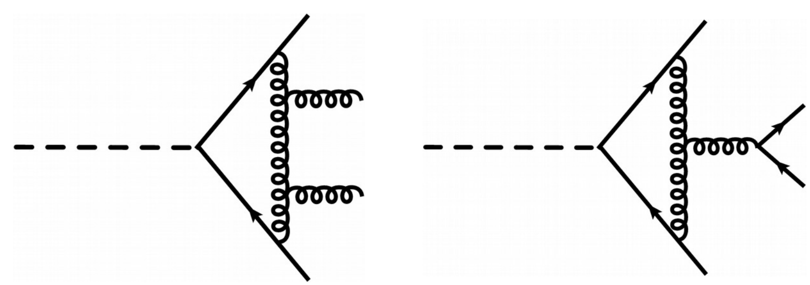
Differential NNLO H→b$\bar{\text{b}}$j width

$$\frac{d\,\Delta\Gamma^{\text{NNLO}}_{H\to b\bar{b}j}}{d\,\mathcal{O}_m} = \int d\Gamma^{VV}_{H\to b\bar{b}j} F_3^m(\Phi_3)d\Phi_3 + \int d\Gamma^{RV}_{H\to b\bar{b}j} F_4^m(\Phi_4)d\Phi_4 + \int d\Gamma^{RR}_{H\to b\bar{b}j} F_5^m(\Phi_5)d\Phi_5$$

two-loop amplitudes
for H→b$\bar{\text{b}}$g



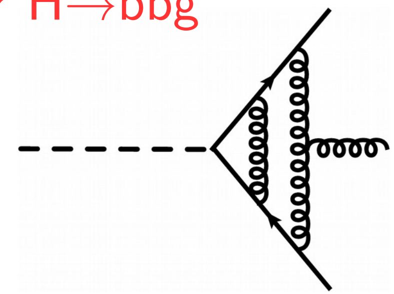
one-loop amplitudes
for H→b$\bar{\text{b}}$gg and H→b$\bar{\text{b}}$q$\bar{\text{q}}$



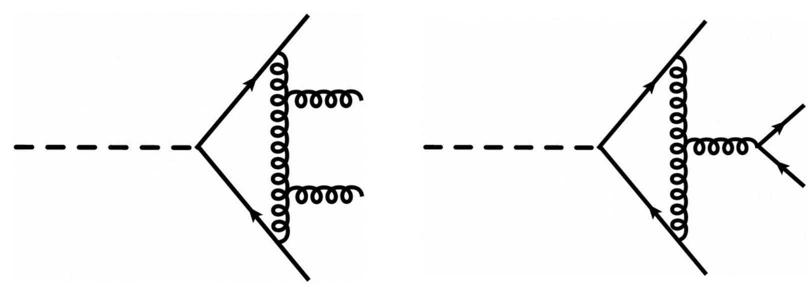
Differential NNLO H→b$\bar{\text{b}}$j width

$$\frac{d\,\Delta\Gamma^{\rm NNLO}_{H\to b\bar{b}j}}{d\,\mathcal{O}_m} = \int d\Gamma^{VV}_{H\to b\bar{b}j} F_3^m(\Phi_3)d\Phi_3 + \int d\Gamma^{RV}_{H\to b\bar{b}j} F_4^m(\Phi_4)d\Phi_4 + \int d\Gamma^{RR}_{H\to b\bar{b}j} F_5^m(\Phi_5)d\Phi_5$$

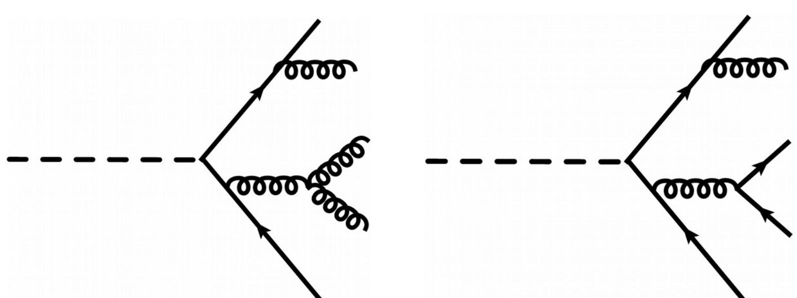
two-loop amplitudes for H→b$\bar{\text{b}}$g



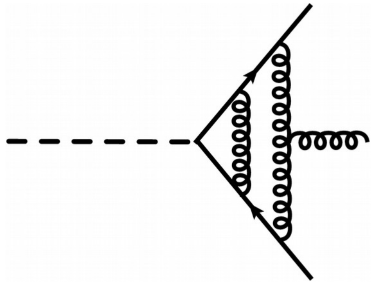
one-loop amplitudes for H→b$\bar{\text{b}}$gg and H→b$\bar{\text{b}}$q$\bar{\text{q}}$



tree-level amplitudes for H→b$\bar{\text{b}}$ggg and H→b$\bar{\text{b}}$q$\bar{\text{q}}$g



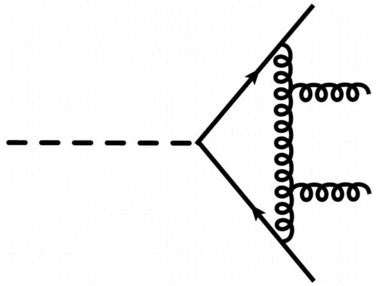
Differential NNLO $H \to b\bar{b}j$ width



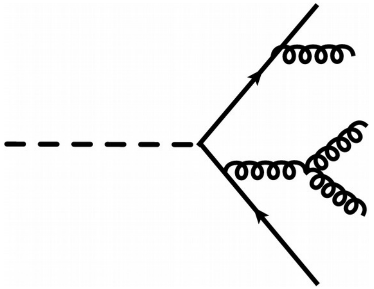
Two-loop $H \to b\bar{b}g$ amplitudes calculated using the MIs from [Gehrmann, Remiddi hep-ph/0008287 and hep-ph/0101124]

Checks:

- IR poles against the known IR structure [Catani hep-ph/9802439]
- Finite part against an independent calculation [Ahmed, Mahakhud, Mathews, Rana, Ravindran 1405.2324]
- Two-loop soft/collinear-gluon limits



One-loop $H \to 4$ partons amplitudes calculated analytically using generalized unitarity for helicity amplitudes [Bern, Dixon, Dunbar, Kosower hep-ph/9403226]



Tree-level $H \to 5$ partons amplitudes calculated using BCFW recursion relations [Britto, Cachazo, Feng, Witten hep-th/0501052]

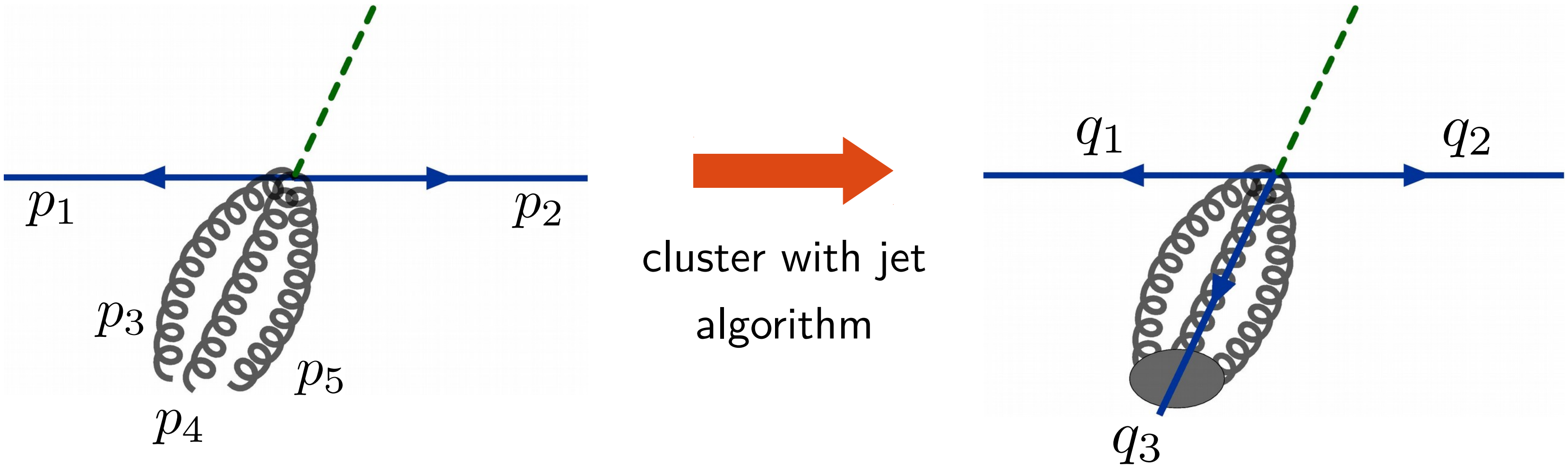
N-jettiness slicing

We regulate the IR divergences present in our NNLO H→b$\bar{\text{b}}$j calculation by using **N-jettiness slicing** [Boughezal, Focke, Liu, Petriello 1504.02131; Gaunt, Stahlhofen, Tackmann, Walsh 1505.04794]. For a parton-level event we define the 3-jettiness variable [Stewart, Tackmann, Waalewijn 1004.2489]:

$$\tau_3 = \sum_{j=1,m} \min_{i=1,2,3} \left\{ \frac{2q_i \cdot p_j}{Q_i} \right\}$$

- The index j runs over the m partons in the phase space
- The momenta q_i are the momenta of the three most energetic jets
- $Q_i = 2E_i$ with E_i the energy of the i-th jet.

$$H \to b\bar{b}j \text{ at NNLO}$$



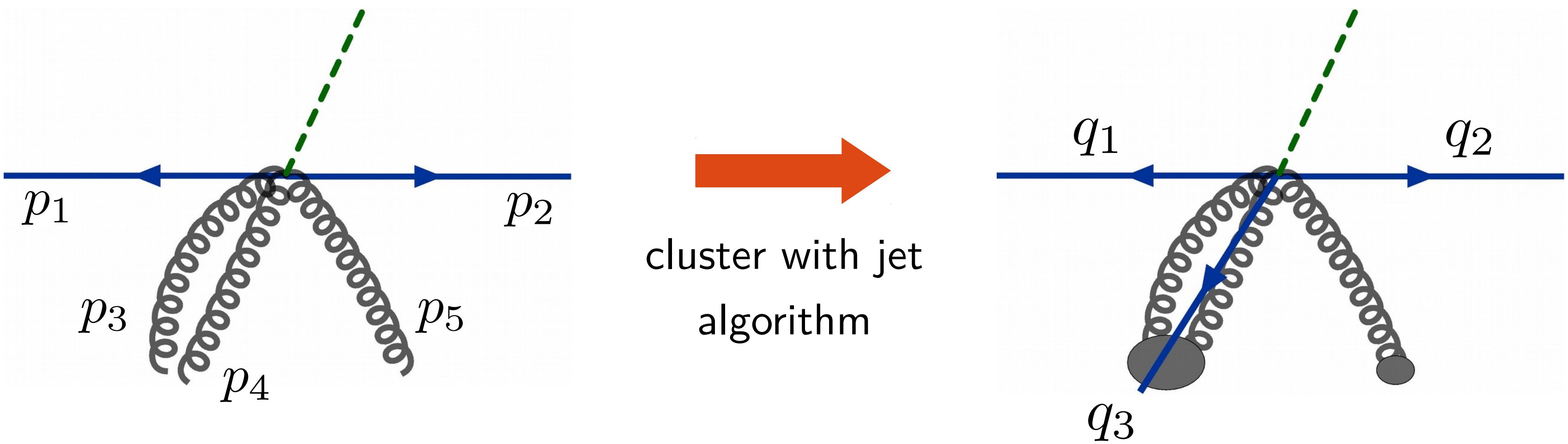
$$\tau_3 = \sum_{j=1,m} \min_{i=1,2,3} \left\{ \frac{2q_i \cdot p_j}{Q_i} \right\} \approx 0$$

Doubly-unresolved region

All radiation is either soft or collinear

N-jettiness slicing

$$H \to b\bar{b}j \text{ at NNLO}$$



$$\tau_3 = \sum_{j=1,m} \min_{i=1,2,3} \left\{ \frac{2q_i \cdot p_j}{Q_i} \right\} > 0$$

Singly-unresolved region

At least one parton is resolved

N-jettiness slicing

Introduce a variable τ_3^{cut} that separates the phase space into two regions:

N-jettiness slicing

Introduce a variable τ_3^{cut} that separates the phase space into two regions:

- The region $\tau_3 < \tau_3^{\text{cut}}$ contains all of the *doubly-unresolved* regions of phase space and here the decay width is approximated using this factorization theorem from SCET [Stewart, Tackmann, Waalewijn 0910.0467]:

$$\Gamma_{H\to b\bar{b}j}\left(\tau_3 < \tau_3^{\text{cut}}\right) \approx \int \prod_{i=1}^{3} \mathcal{J}_i \otimes \mathcal{S} \otimes \mathcal{H} + \mathcal{O}(\tau_3^{\text{cut}})$$

Jet functions
[Becher, Neubert hep-ph/0603140]

Soft function
[Boughezal, Liu, Petriello 1504.02540; Campbell, Ellis, RM, Williams 1711.09984]

Hard function
(finite part of the two-loop amplitudes)

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Hard function
(finite part of the two-loop amplitudes)

- The region $\tau_3 > \tau_3^{\mathrm{cut}}$ contains the *singly-unresolved* and *fully-resolved* regions. It is the NLO calculation of $H \to b\bar{b}jj$. In our case we regulate the IR divergences using Catani-Seymour dipoles [hep-ph/9605323].

Results

We have implemented our NNLO $H \to b\bar{b}j$ calculation into a parton-level MC code based on MCFM [Campbell, Ellis et al].

We use the **Durham jet algorithm**. Starting at the parton level, for every pair of partons (i,j):

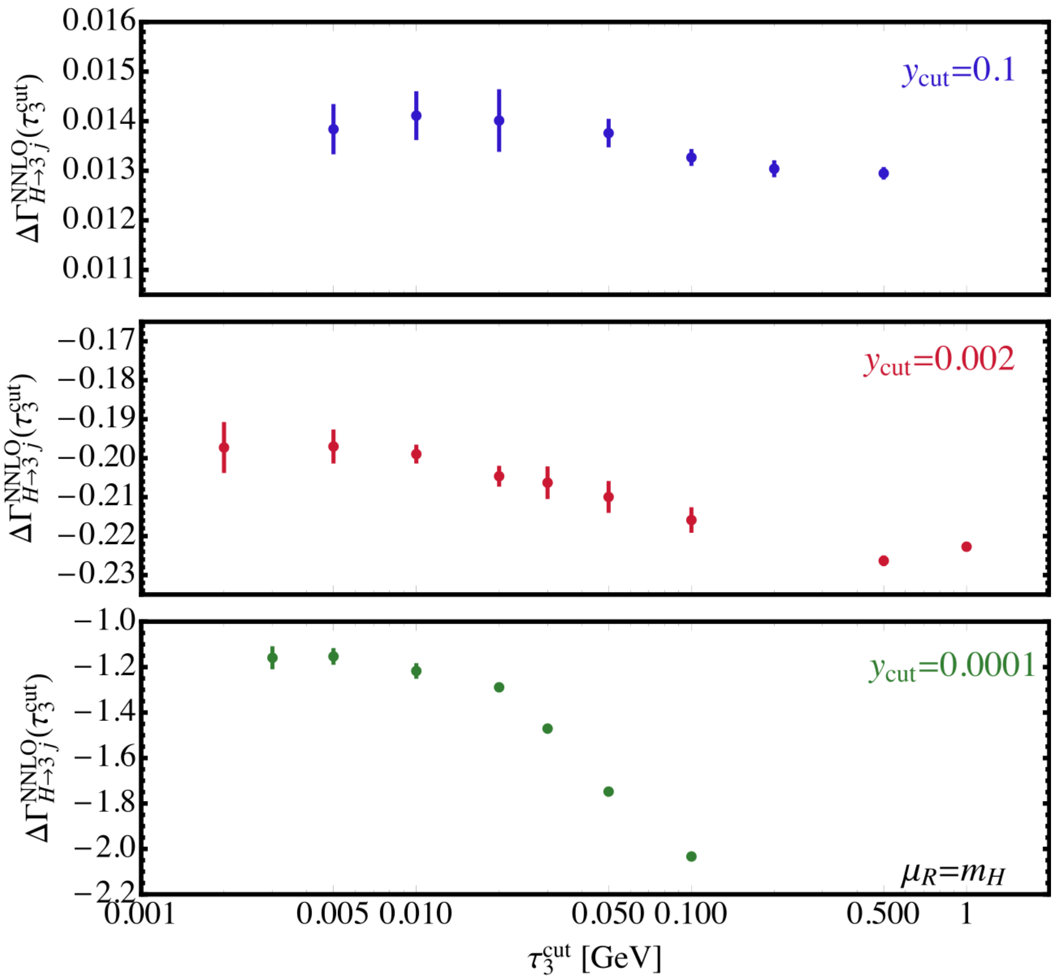
$$y_{ij} = \frac{2\min(E_i^2, E_j^2)(1-\cos\theta_{ij})}{Q^2}$$

If $y_{ij} < y_{cut}$ the pairs are combined into a new object with momentum $p_i + p_j$. The algorithm repeats until no further clusterings are possible and the remaining objects are classified as jets.

We present results in the Higgs *rest frame*.

Validation of the H→$b\bar{b}$j NNLO N-jettiness calculation

Dependence of the NNLO H→3j coefficient on the unphysical parameter τ_3^{cut} for three clustering options



Asymptotic behavior is established in each region.

$y_{\mathrm{cut}} = 0.0001$ corresponds to imposing a very weak jet cut

P2B with N-jettiness slicing

$$\frac{d\,\Delta\Gamma^{\rm N3LO}_{H\to b\bar b}}{d\,\mathcal{O}_m} = \frac{d\Delta\Gamma^{\rm N3LO,\,incl}_{H\to b\bar b}}{d\mathcal{O}^B_m}$$
$$+\int d\Gamma^{RVV}_{H\to b\bar b}\,[F^m_3(\Phi_3)-F^m_2(\Phi_B)]\,d\Phi_3$$
$$+\int d\Gamma^{RRV}_{H\to b\bar b}\,[F^m_4(\Phi_4)-F^m_2(\Phi_B)]\,d\Phi_4$$
$$+\int d\Gamma^{RRR}_{H\to b\bar b}\,[F^m_5(\Phi_5)-F^m_2(\Phi_B)]\,d\Phi_5$$

Differential NNLO $H\to b\bar{b}j$ calculation using N-jettiness slicing

Problem when *m=2*: how to define 3-jettiness for 2-jet events?

P2B with N-jettiness slicing

Focus on triple-real contribution as an example:

$$\int d\Gamma^{RRR}_{H\to b\bar{b}}\left[F^m_5(\Phi_5) - F^m_2(\Phi_B)\right] d\Phi_5$$

$F^m_5(\Phi_5)$ picks out the various jet topologies (2-, 3-, 4-, or 5-jet events):

P2B with N-jettiness slicing

Focus on triple-real contribution as an example:

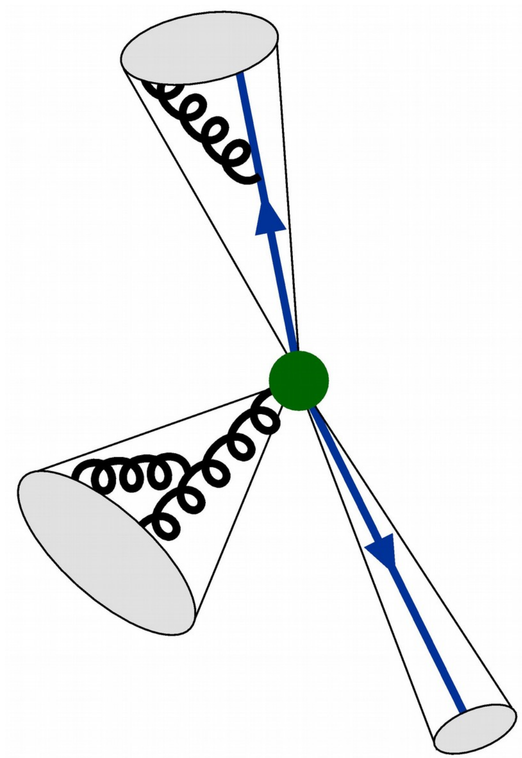
$$\int d\Gamma^{RRR}_{H\to b\bar{b}} \left[F_5^m(\Phi_5) - F_2^m(\Phi_B)\right] d\Phi_5$$

$F_5^m(\Phi_5)$ picks out the various jet topologies (2-, 3-, 4-, or 5-jet events):

a) events with 3 or more jets:

straightforward to

compute 3-jettiness



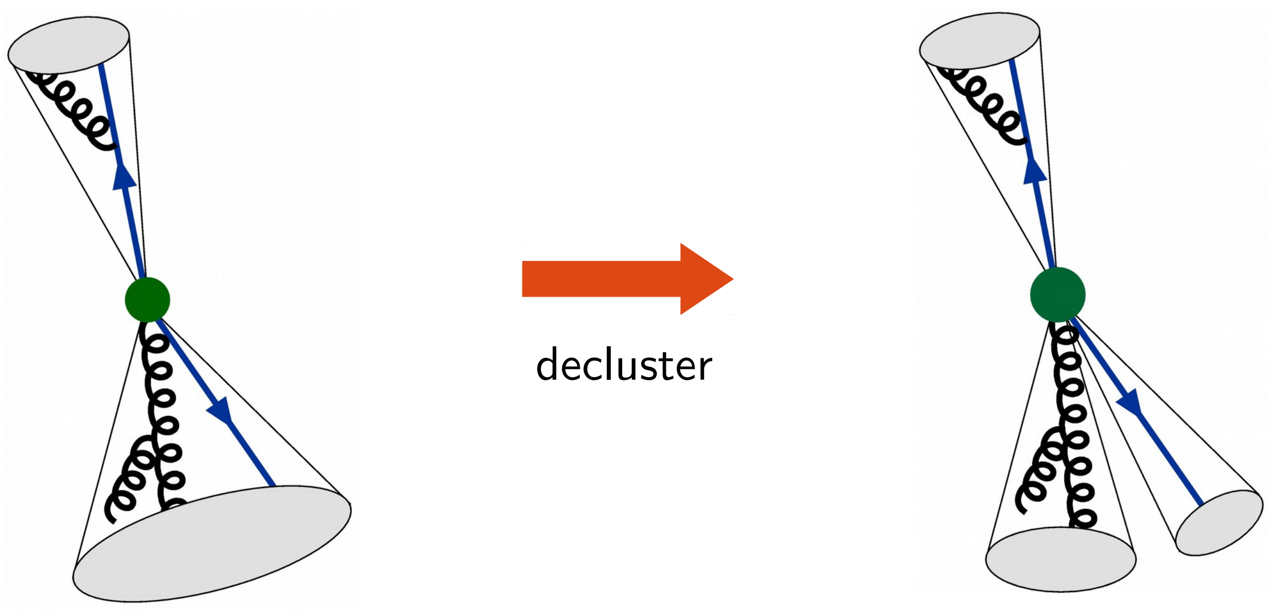
P2B with N-jettiness slicing

Focus on triple-real contribution as an example:

$$\int d\Gamma^{RRR}_{H\to b\bar{b}} \left[F_5^m(\Phi_5) - F_2^m(\Phi_B)\right] d\Phi_5$$

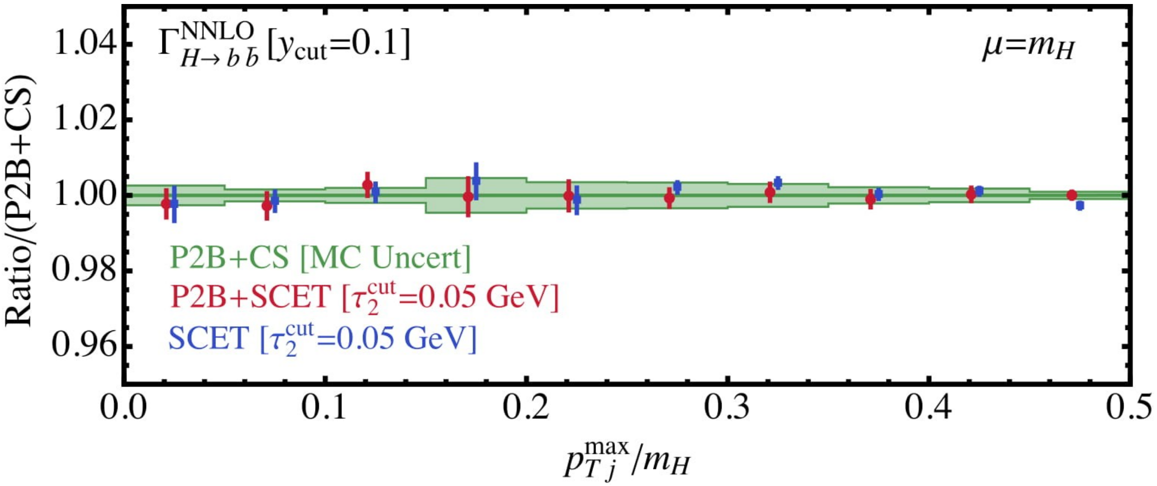
$F_5^m(\Phi_5)$ picks out the various jet topologies (2-, 3-, 4-, or 5-jet events):

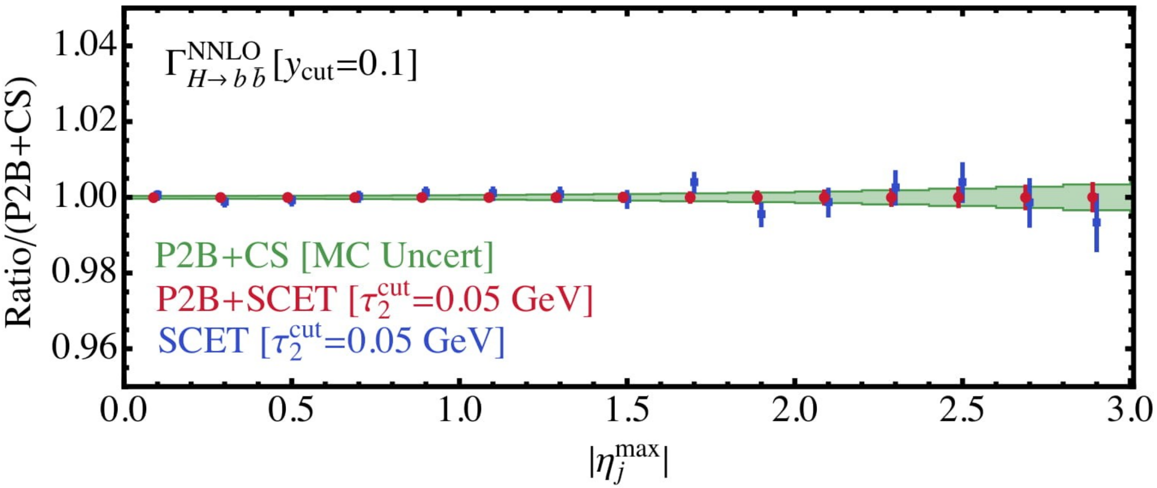
b) events with 2 jets: reverse last step of clustering to obtain exactly 3 sub-jets. Then apply 3-(sub)jettiness slicing.



Validation of the P2B+SCET method at NNLO

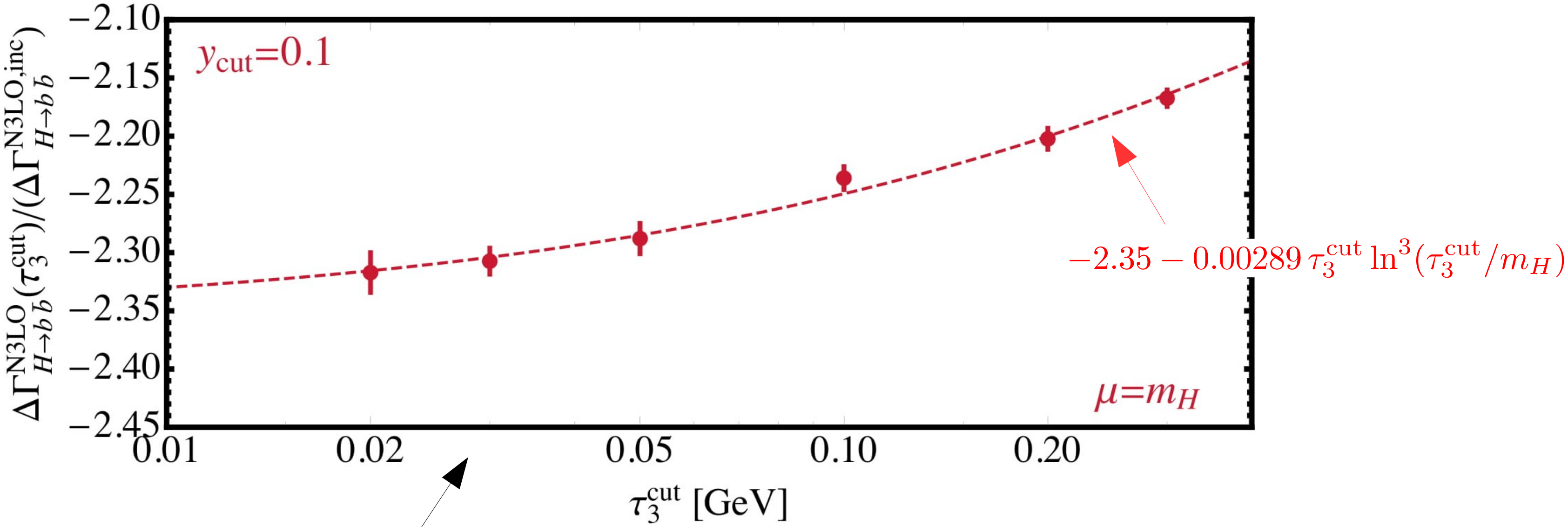
We introduce the transverse momentum and pseudo-rapidity of the leading jet with respect to a fictitious beam axis to fully test the IR cancellations





Validation at N3LO

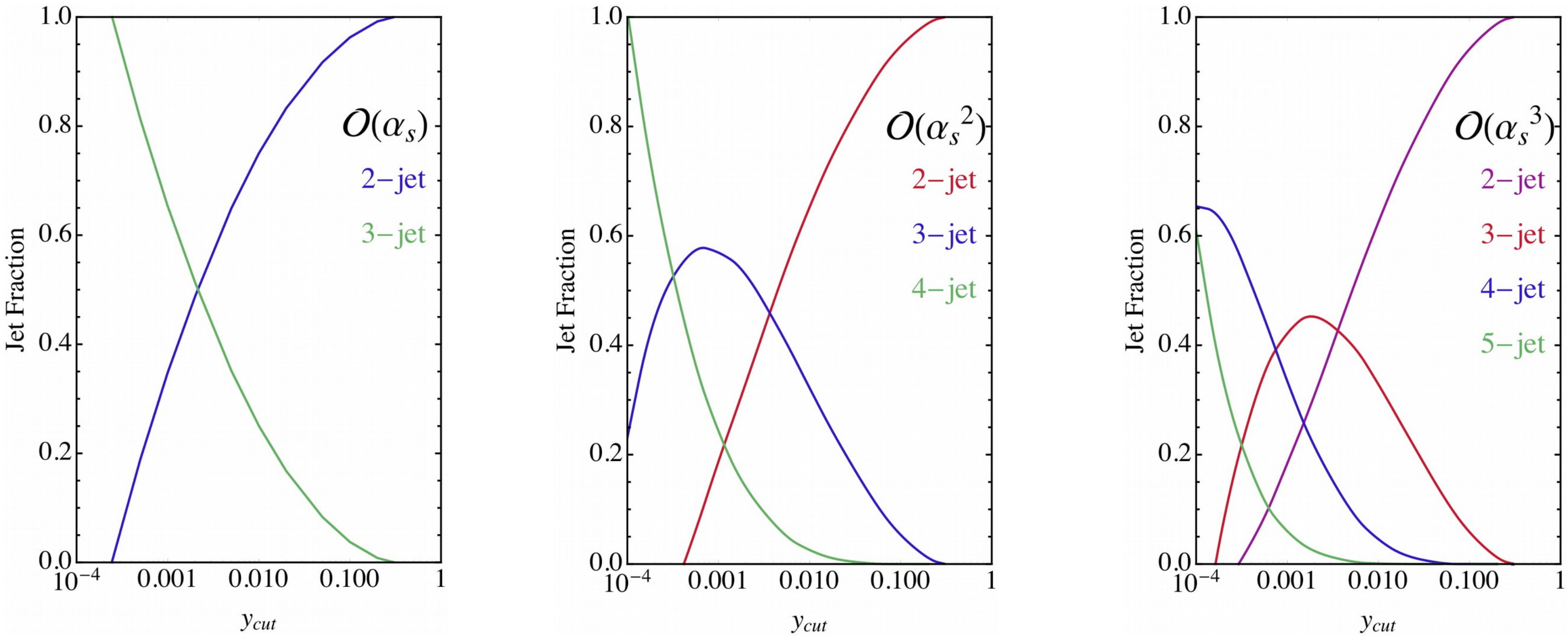
Dependence of the 2-jet N3LO coefficient on the 3-(sub)jettiness slicing parameter τ_3^{cut}



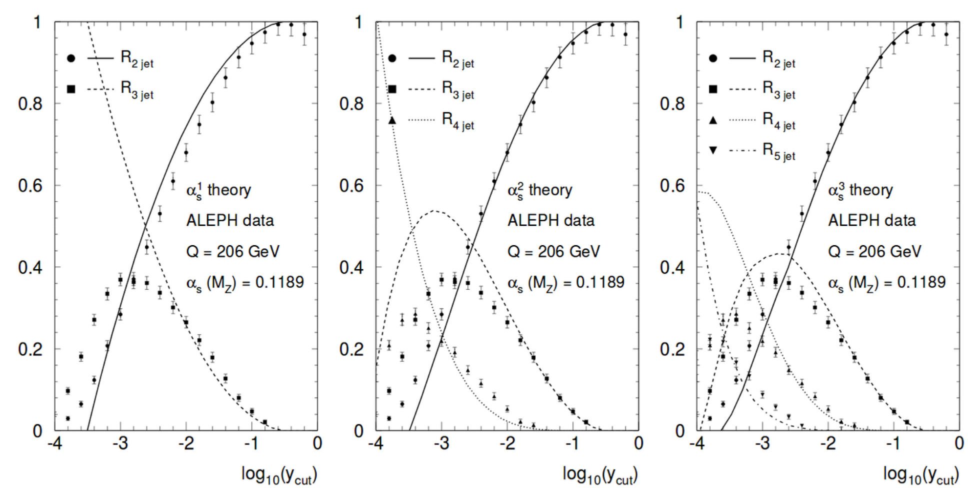
in this region change
in N3LO coefficient
is about 1%

Use
$\tau_3^{\mathrm{cut}} = 0.02\ \mathrm{GeV}$
for predictions

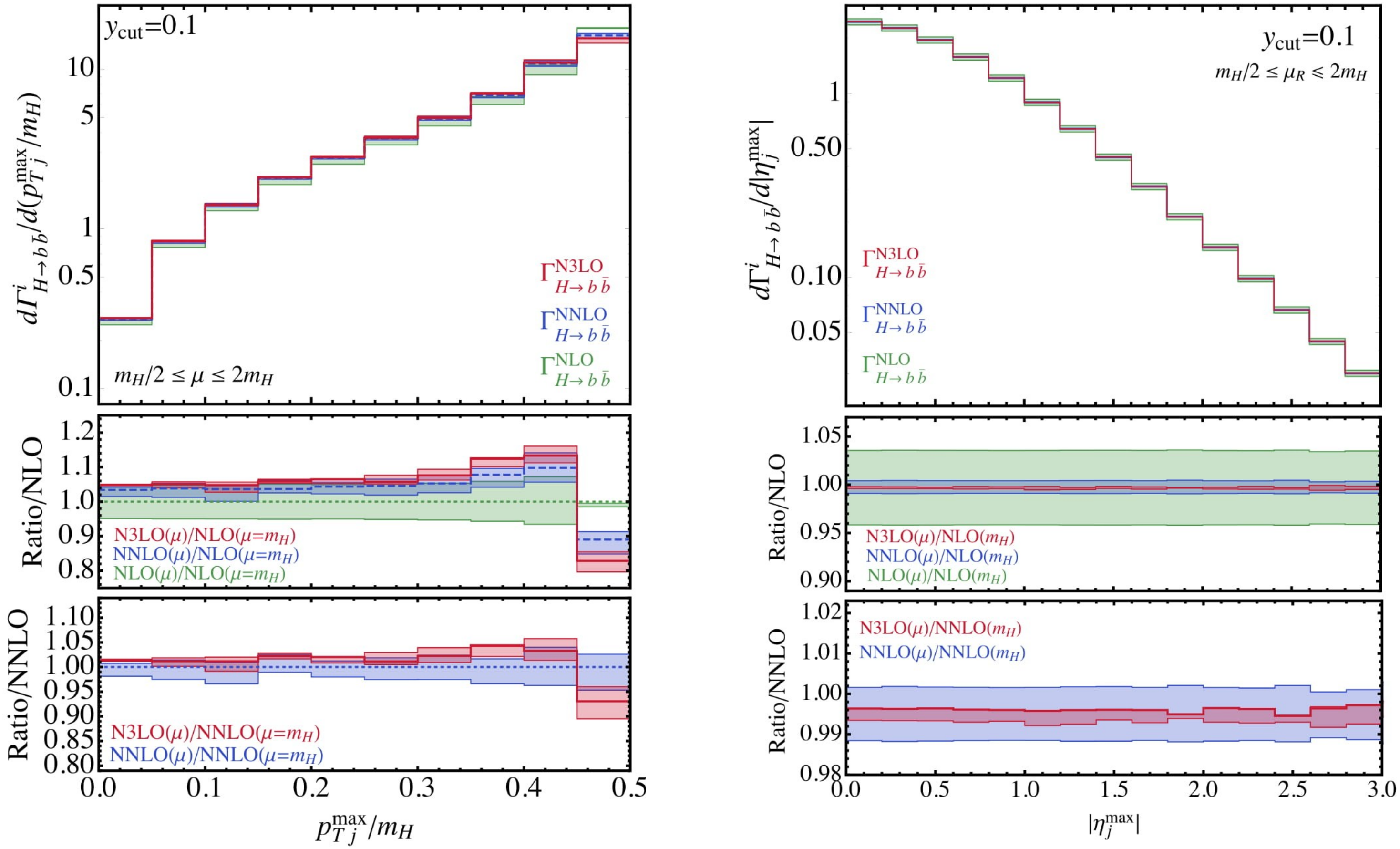
Jet fractions



The observed pattern is similar to the results obtained for $e^+e^- \to$ jets at the same order [Gehrmann-De Ridder, Gehrmann, Glover, Heinrich 0802.0813; Weinzierl 0807.3241]

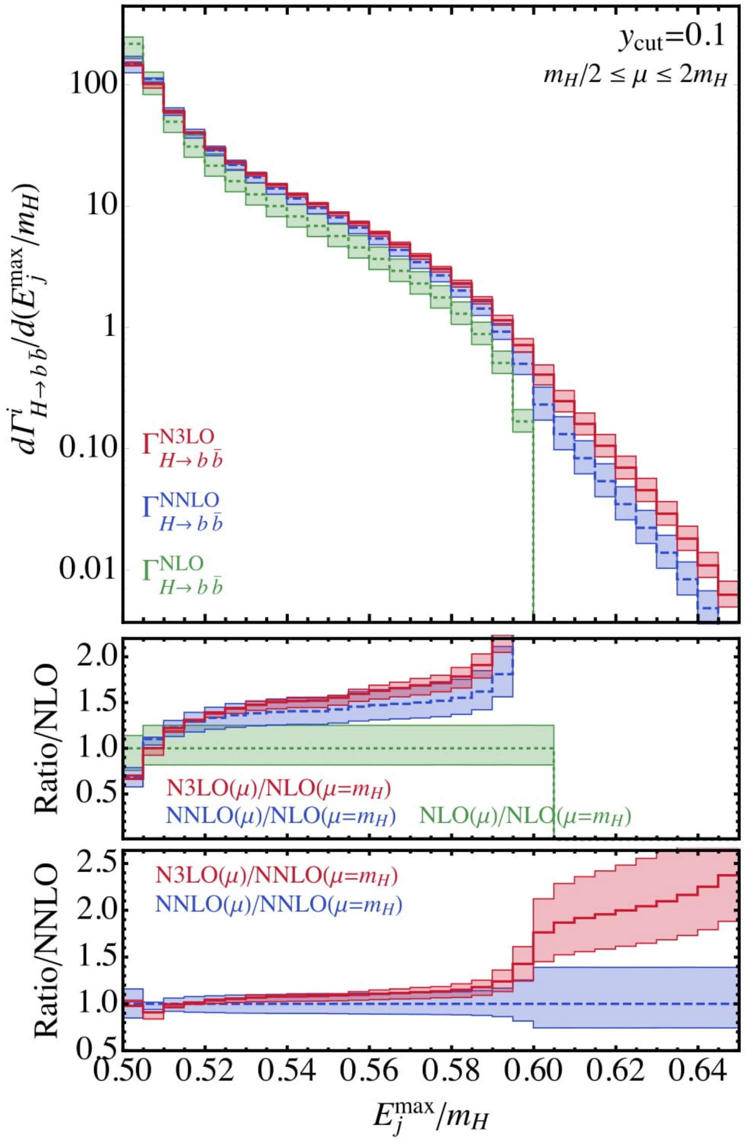


Results for $H \to b\bar{b}$ at N3LO



The size of the corrections is observable-dependent. The scale dependence is considerably reduced as higher-order terms are included.

Results for H→b$\bar{\text{b}}$ at N3LO



Can broadly observe three regions:

1) LO boundary: all phase spaces contribute, good convergence of the series and small residual scale dependence

2) "Bulk": only phase spaces with 3+ partons contribute, NNLO-like calculation

3) "Tail": only phase spaces with 4+ partons contribute, NLO-like calculation

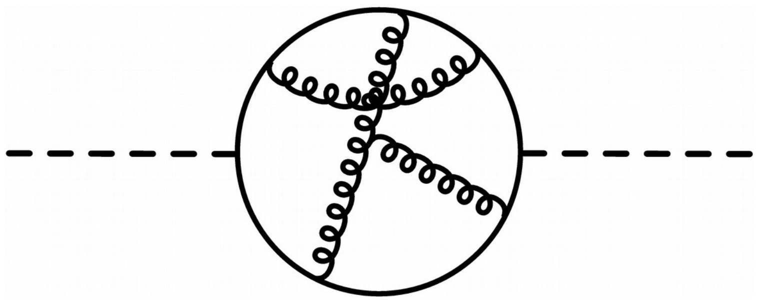
Conclusions

- At the CEPC, we will probe most Higgs couplings to the 1% level.
- Precise theoretical predictions for Higgs observables are needed to successfully compare theory and experiment.
- We computed the $H \to b\bar{b}$ decay at N3LO accuracy focusing on the contribution in which the Higgs boson couples directly to massless bottom quarks.
- Using the Projection-to-Born method + N-jettiness slicing, we produced differential distributions and jet rates in the Higgs rest frame.
- Our calculation could be used outside of the rest frame for LHC/CEPC applications.

Extra slides

Inclusive N3LO H→b$\bar{\text{b}}$ width

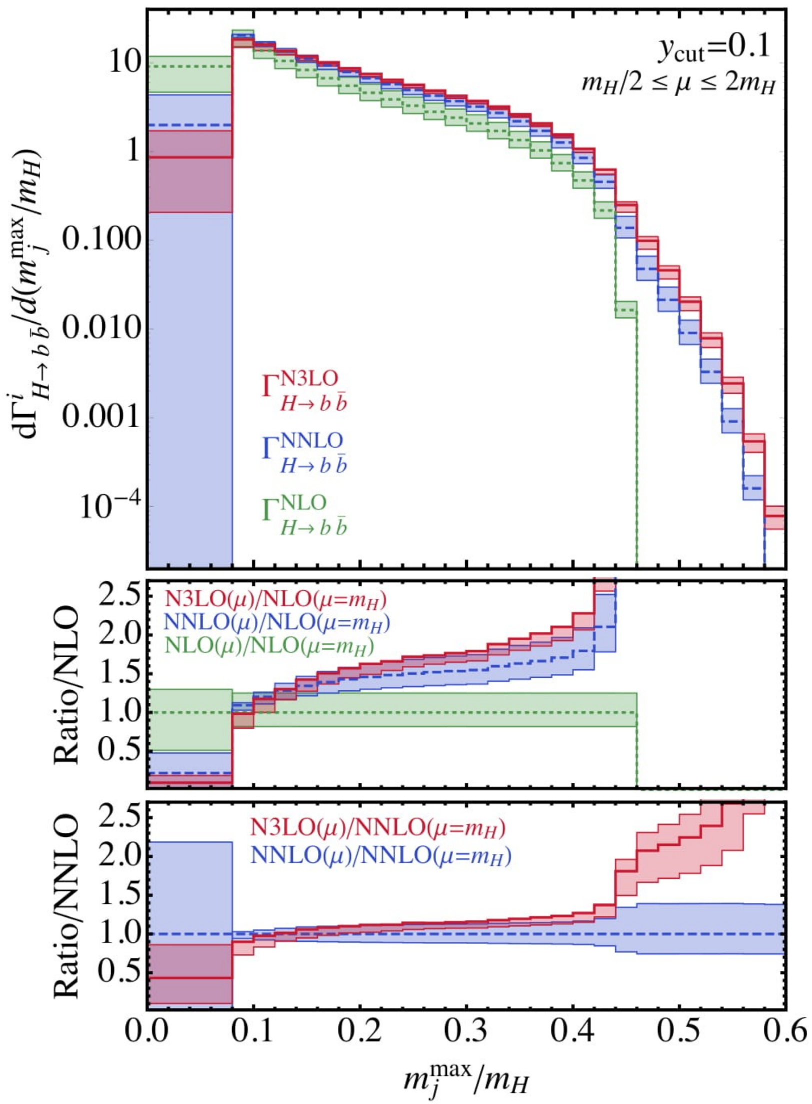
Can be obtained through the *optical theorem* by computing the massless $\mathcal{O}(\alpha_s^3)$ four-loop correlator of the quark-scalar current
[Chetyrkin hep-ph/9608318]



$$\begin{aligned}\Delta\Gamma^{\mathrm{N3LO}}_{H\to b\bar{b}} \quad = \quad & \Gamma^{\mathrm{LO}}_{H\to b\bar{b}}\left(\frac{\alpha_s}{\pi}\right)^3 \Big[s_3 + L\left(2s_2\beta_0 + s_1\beta_1 + 2s_2\gamma_m^0 + 2s_1\gamma_m^1 + 2\gamma_m^2\right) \\ & +L^2\left(s_1\beta_0^2 + 3s_1\beta_0\gamma_m^0 + \beta_1\gamma_m^0 + 2s_1(\gamma_m^0)^2 + 2\beta_0\gamma_m^1 + 4\gamma_m^0\gamma_m^1\right) \\ & +L^3\left(\frac{2}{3}\beta_0^2\gamma_m^0 + 2\beta_0(\gamma_m^0)^2 + \frac{4}{3}(\gamma_m^0)^3\right)\Big]\end{aligned}$$

$$L = \log\left(\mu^2/m_H^2\right)$$

Results for H→bb̄ at N3LO



Can broadly observe three regions:

1) At LO m_j=0. Must ensure that first bin be inclusive enough for IR cancellations. Large corrections

2) "Bulk": phase spaces with 3+ partons contribute, NNLO-like calculation

3) "Tail": phase spaces with 4+ partons contribute, NLO-like calculation

Two-loop amplitudes for H→b$\bar{b}$g

Soft-gluon limit: $p_3 \to 0$ which means y,z→0 simultaneously

$$2\,\mathrm{Re}\left(\mathcal{M}^{(2)}_{H\to b\bar{b}g}\mathcal{M}^{(0)*}_{H\to b\bar{b}g}\right) \to 2\,\mathrm{Re}\Big(S^{(0)}(y,z)\mathcal{M}^{(2)}_{H\to b\bar{b}}\mathcal{M}^{(0)*}_{H\to b\bar{b}} + S^{(1)}(y,z)\mathcal{M}^{(1)}_{H\to b\bar{b}}\mathcal{M}^{(0)*}_{H\to b\bar{b}} + S^{(2)}(y,z)\mathcal{M}^{(0)}_{H\to b\bar{b}}\mathcal{M}^{(0)*}_{H\to b\bar{b}}\Big)$$

$y = z = 10^{-10}$

Coefficient	Known limit	Our result
ϵ^{-4}	81.7702729678	81.7702729678
ϵ^{-3}	3818.49680411	3818.49680413
ϵ^{-2}	130763.8079162	130763.8079168
ϵ^{-1}	$3.26338843478 \cdot 10^6$	$3.26338843480 \cdot 10^6$
ϵ^{0}	$6.52342650778 \cdot 10^7$	$6.52342650793 \cdot 10^7$

Two-loop amplitudes for H→b$\bar{\text{b}}$g

Collinear limit: t→0 which means y→0 while z is fixed

$$2\,\mathrm{Re}\left(\mathcal{M}^{(2)}_{H\to b\bar{b}g}\mathcal{M}^{(0)*}_{H\to b\bar{b}g}\right) \to 2\,\mathrm{Re}\Big(C^{(0)}(y,z)\mathcal{M}^{(2)}_{H\to b\bar{b}}\mathcal{M}^{(0)*}_{H\to b\bar{b}} + C^{(1)}(y,z)\mathcal{M}^{(1)}_{H\to b\bar{b}}\mathcal{M}^{(0)*}_{H\to b\bar{b}} + C^{(2)}(y,z)\mathcal{M}^{(0)}_{H\to b\bar{b}}\mathcal{M}^{(0)*}_{H\to b\bar{b}}\Big)$$

$y = 10^{-12}$

$z = 0.23$

Coefficient	Known limit	Our result
ϵ^{-4}	283.156234427	283.156234427
ϵ^{-3}	8122.55721506	8122.55721505
ϵ^{-2}	170379.942318	170379.942317
ϵ^{-1}	$2.584146 \cdot 10^6$	$2.584189 \cdot 10^6$
ϵ^{0}	$3.09852 \cdot 10^7$	$3.09870 \cdot 10^7$