Precise measurement of m_W and Γ_W using threshold scan method

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Outline

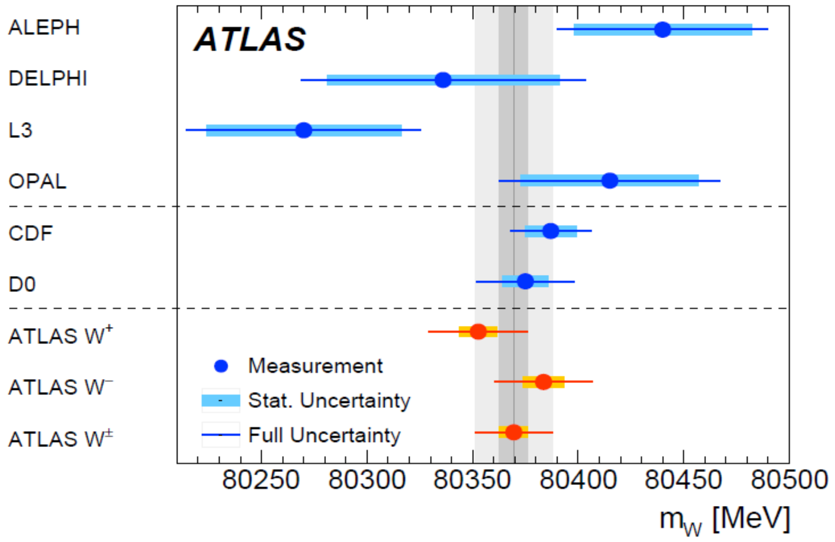
- Motivation
- Methodology
- Statistical and systematic uncertainties
- Data taking schemes
- Summary

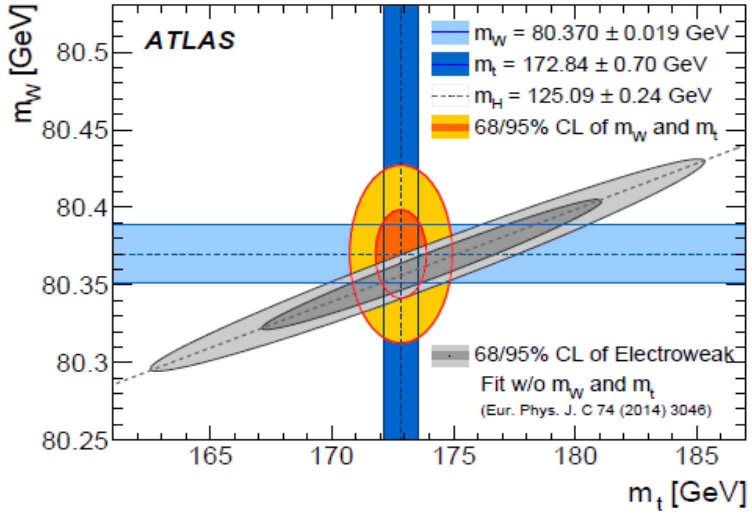
Motivation

- The m_W plays a central role in precision EW measurements and in constraint on the SM model through global fit.

$$G_F = \frac{\pi\alpha}{\sqrt{2}m_W^2 \sin^2\theta_W}\frac{1}{(1+\Delta r)}$$

Δr is the correction, whose leading-order contributions depend on the m_t and m_H





- Several ways to measure m_W:
  - The direct method, with kinematically-constrained or mass reconstructions
  - Using the lepton end-point energy
  - W^+W^- threshold scan method (this study)

Methodology

- Why?

$$\sigma_{WW}(m_W, \Gamma_W, \sqrt{s}) = \frac{N_{obs}}{L\epsilon P} \qquad (P = \frac{N_{WW}}{N_{WW}+N_{bkg}})$$

so m_W, Γ_W can be obtained by comparing the N_{obs} / LεP, with predicted σ_{WW}

- How?

$\Delta m_W, \Delta\Gamma_W$						
N_{obs}	L	ϵ	N_{bkg}	E	σ_E	……

In general, these uncertainties are dependent on $\sqrt{s}$, so it is an optimization problem when considering the data taking.

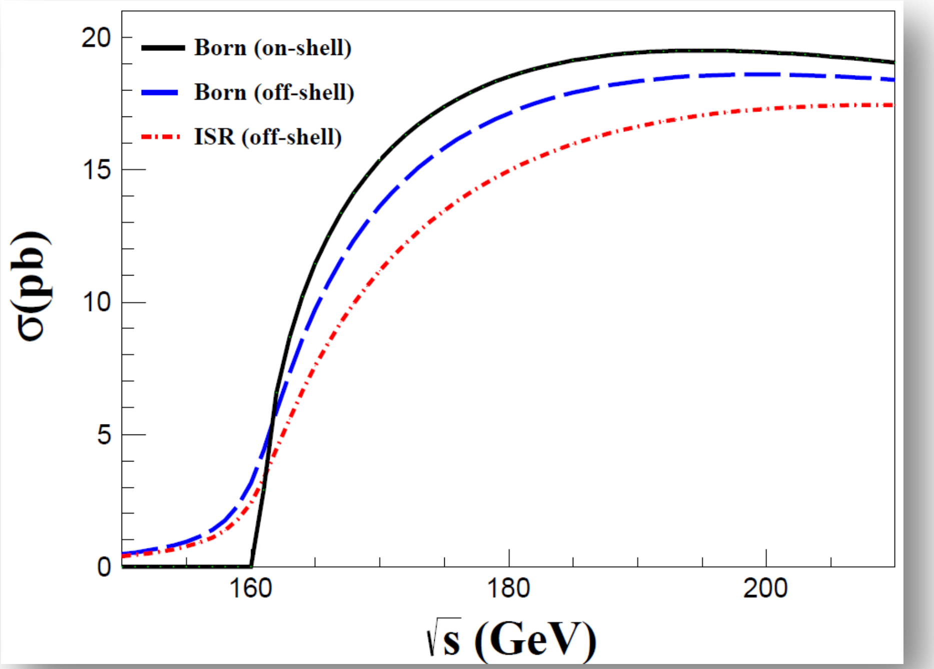
- If …, then?

With the configurations of $L, \Delta L, \Delta E$ …, we can obtain: $m_W \sim ?\ \Gamma_W \sim ?$

Theoretical Tool

- The σ_{WW} is a function of $\sqrt{s}$, m_W and Γ_W, calculated with the GENTLE package in this work (CC03)

- The ISR correction calculated by convoluting the Born cross sections with QED structure function, with the radiator up to NLO(α^2) and O(β^3)



Statistical and systematic uncertainties

Statistical uncertainty

- $\Delta\sigma_{WW} = \sigma_{WW} \times \frac{\Delta N_{WW}}{N_{WW}} = \sigma_{WW} \times \frac{\sqrt{N_{WW}+N_{bkg}}}{N_{WW}}$

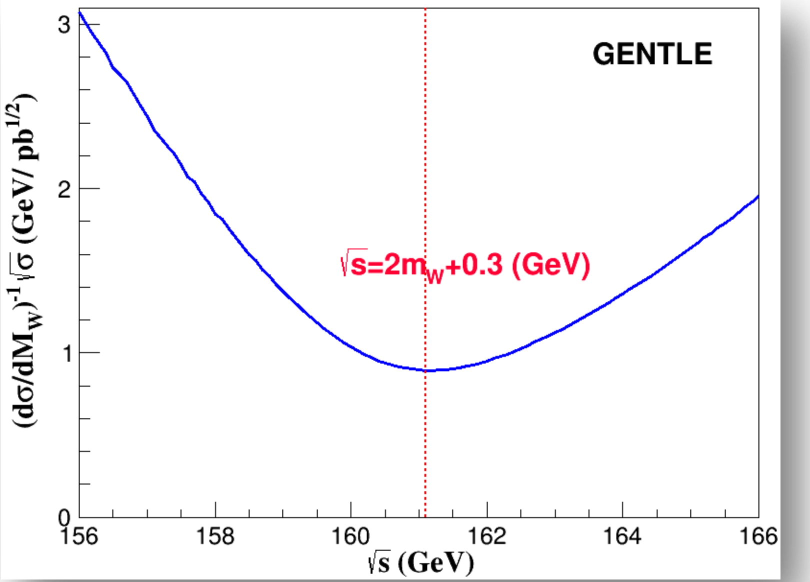
$$= \sqrt{\frac{\sigma_{WW}}{L\epsilon P}} \qquad (P = \frac{N_{WW}}{N_{WW}+N_{bkg}})$$

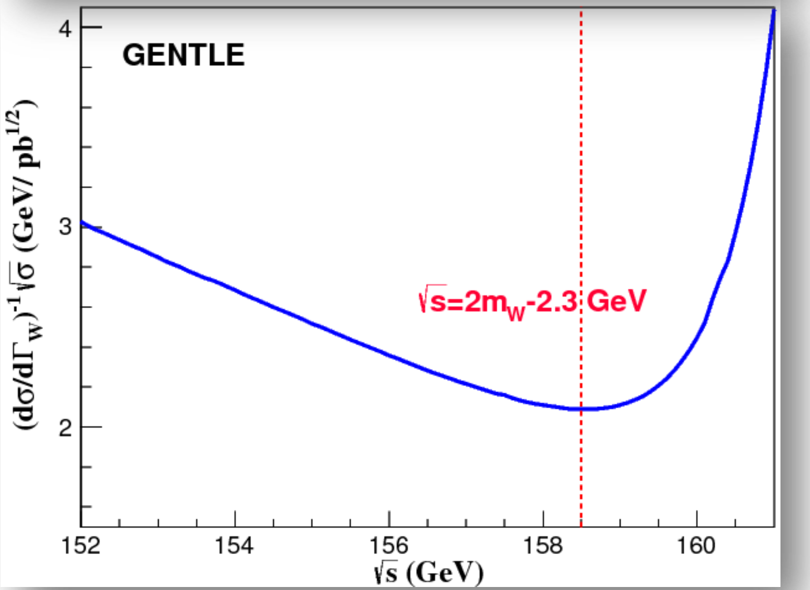
- $\Delta m_W = \left(\frac{\partial \sigma_{WW}}{\partial m_W}\right)^{-1} \times \Delta\sigma_{WW} = \left(\frac{\partial \sigma_{WW}}{\partial m_W}\right)^{-1} \times \sqrt{\frac{\sigma_{WW}}{L\epsilon P}}$

- $\Delta \Gamma_W = \left(\frac{\partial \sigma_{WW}}{\partial \Gamma_W}\right)^{-1} \times \Delta\sigma_{WW} = \left(\frac{\partial \sigma_{WW}}{\partial \Gamma_W}\right)^{-1} \times \sqrt{\frac{\sigma_{WW}}{L\epsilon P}}$

With L=3.2ab^{-1}, ϵ=0.8, P=0.9:

Δm_W=0.6 MeV, $\Delta\Gamma_W$=1.4 MeV (individually)





Statistical uncertainty

- When there are more than one data point, we can measure both m_W and Γ_W.
- With the χ^2 defined as:

$$\chi^2 = \sum_i \frac{(N_{\mathrm{fit}^i} - N^i_{\mathrm{obs}})^2}{N^i_{\mathrm{obs}}} = \frac{(\mathcal{L}\epsilon P)^i(\sigma^i_{\mathrm{fit}} - \sigma^i_{\mathrm{obs}})^2}{\sigma^i_{\mathrm{obs}}}$$

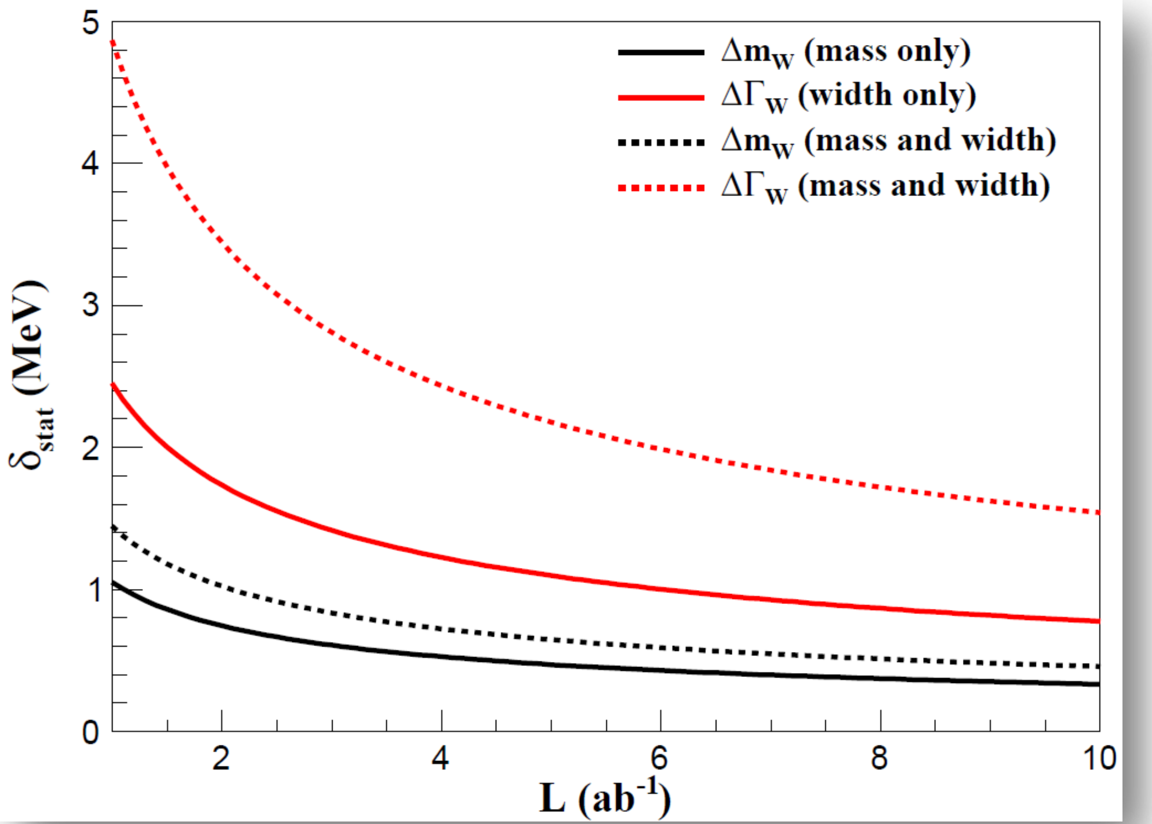
the error matrix is in the form:

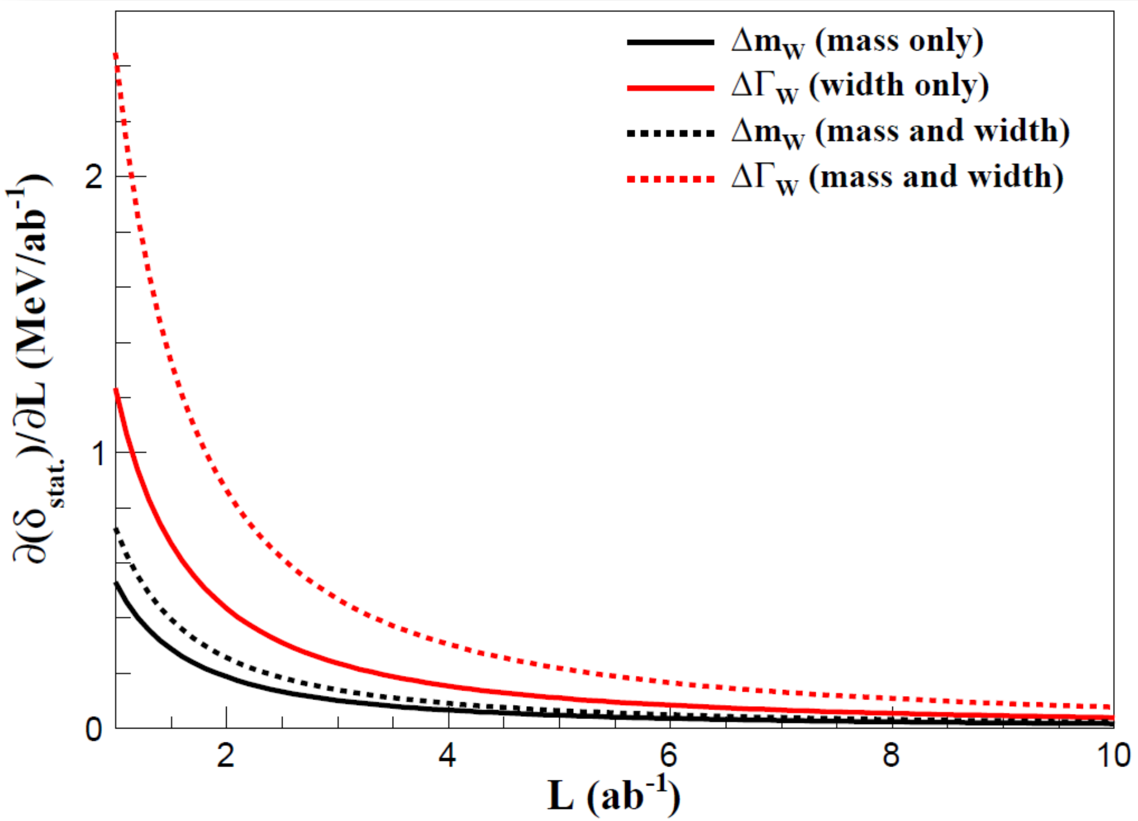
$$V = \frac{1}{2} \times \begin{pmatrix} \frac{\partial^2 \chi 2}{\partial m_W^2} & \frac{\partial^2 \chi 2}{\partial m_W \partial \Gamma_W} \\ \frac{\partial^2 \chi 2}{\partial m_W \partial \Gamma_W} & \frac{\partial^2 \chi 2}{\partial m_{\Gamma_W}^2} \end{pmatrix}^{-1} = \sum_i \begin{pmatrix} \frac{(\mathcal{L}\epsilon P)^i}{\sigma^i_{\mathrm{obs}}}(\frac{\partial \sigma}{\partial m_W})^2 & \frac{(\mathcal{L}\epsilon P)^i}{\sigma^i_{\mathrm{obs}}}\frac{\partial \sigma}{\partial m_W}\frac{\partial \sigma}{\partial \Gamma_W} \\ \frac{(\mathcal{L}\epsilon P)^i}{\sigma^i_{\mathrm{obs}}}\frac{\partial \sigma}{\partial m_W}\frac{\partial \sigma}{\partial \Gamma_W} & \frac{(\mathcal{L}\epsilon P)^i}{\sigma^i_{\mathrm{obs}}}(\frac{\partial \sigma}{\partial m_W})^2 \end{pmatrix}^{-1}$$

- When the number of fit parameter reduce to 1:

$$\Delta m_W = \left(\frac{\partial \sigma_{WW}}{\partial m_W}\right)^{-1} \times \Delta\sigma_{WW} = \left(\frac{\partial \sigma_{WW}}{\partial m_W}\right)^{-1} \times \sqrt{\frac{\sigma_{WW}}{L\epsilon P}}$$

Statistical uncertainty





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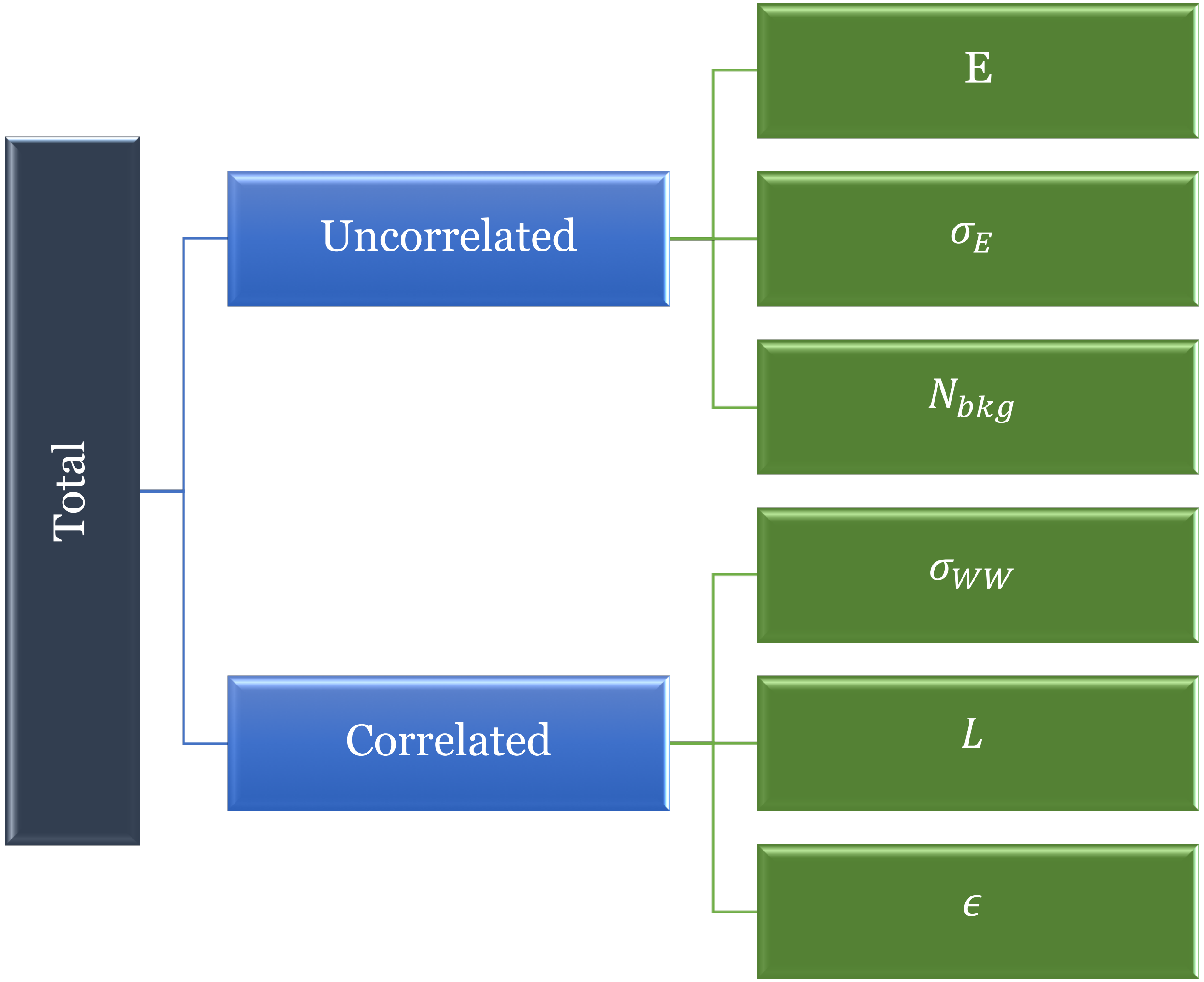
Systematic uncertainty

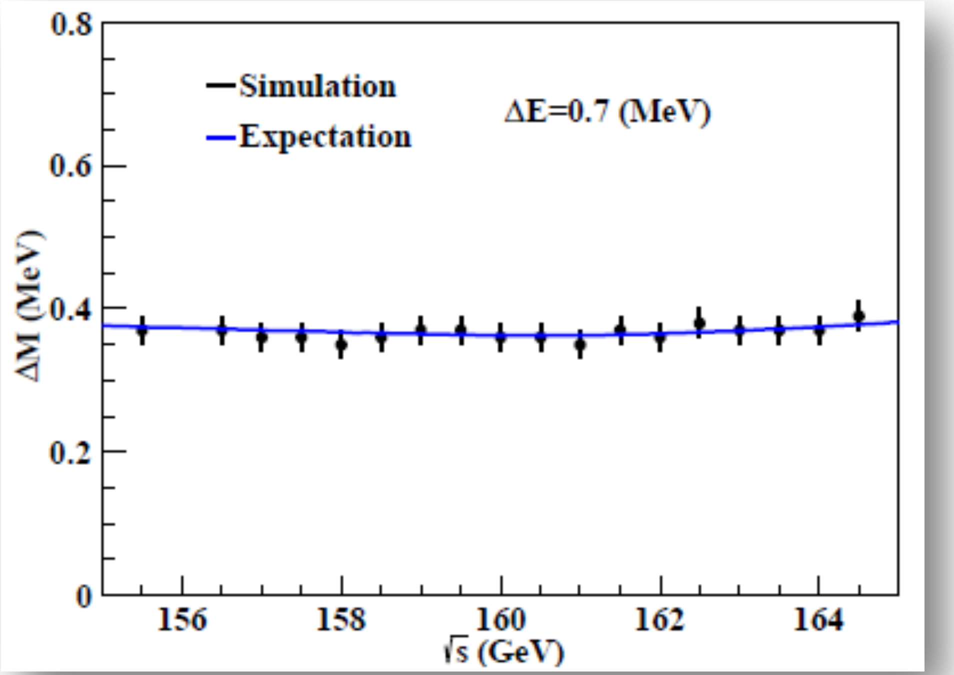


Energy calibration uncertainty

- With ΔE, the total energy becomes:

$$E = G\left(E_p, \Delta E\right) + G(E_m, \Delta E)$$

- $\Delta m_W = \frac{\partial m_W}{\partial \sigma_{WW}} \frac{\partial \sigma_{WW}}{\partial E} \Delta E$
- The $\Delta \mathrm{m_W}$ will be large when ΔE increase, but **almost independent on $\sqrt{s}$.**



Energy spread uncertainty

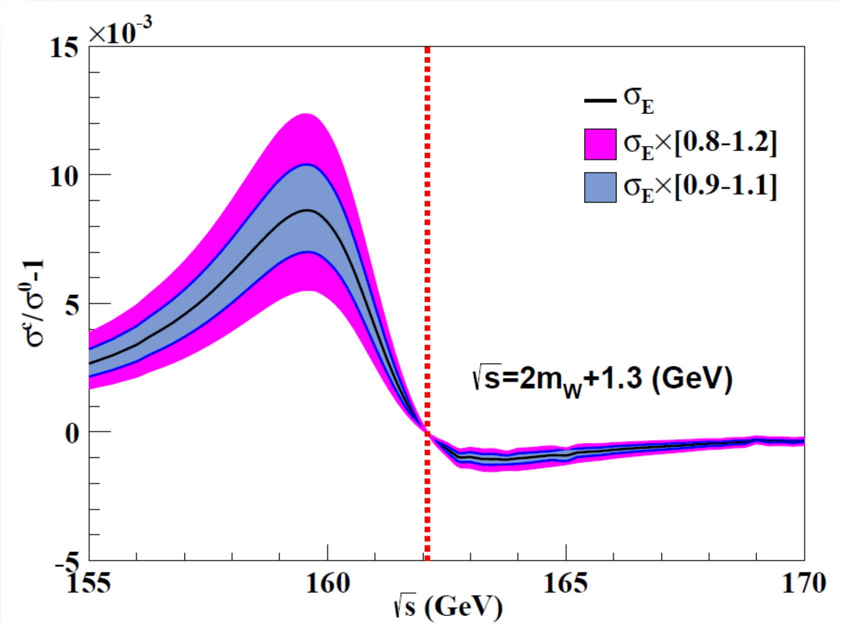
- With E_{BS}, the σ_{WW} becomes:

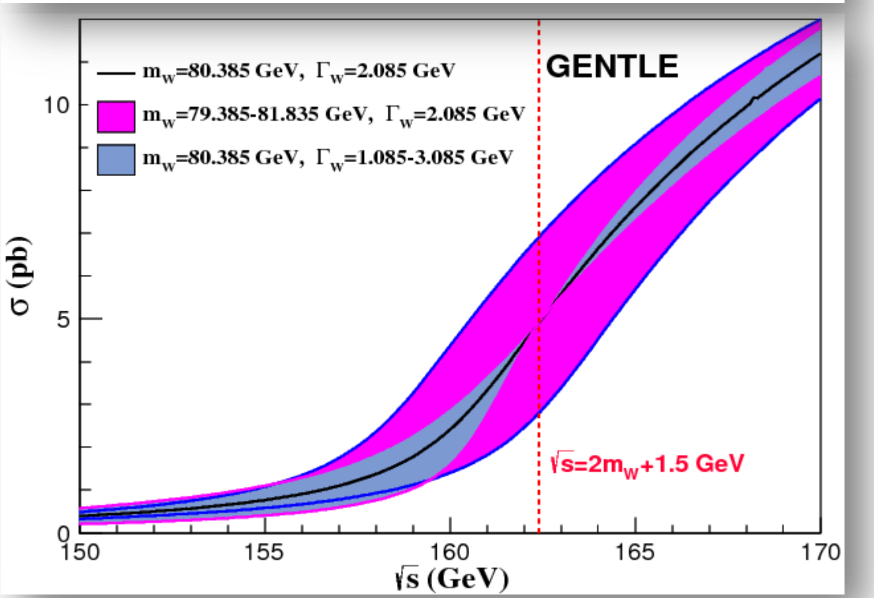
$$\sigma_{WW}(E) = \int_0^{\infty} \sigma_{WW}(E')\times G(E,E')dE'$$

$$= \int \sigma(E')\times \frac{1}{\sqrt{2\pi}\delta_E} e^{\frac{-(E-E')^2}{2\sigma_E^2}} dE'$$

- $\sigma_E + \Delta\sigma_E$ is used in the simulation, and σ_E is for the fit formula.

- **The m_W insensitive to δ_E when taking data around 162.3 GeV**





Background uncertainty

The effect of background are in two different ways

1. Stat. part: $\Delta m_W(N_B) = \frac{\partial m_W}{\partial \sigma_{WW}} \cdot \frac{\sqrt{L\epsilon_B\sigma_B}}{L\epsilon}$

2. Sys. part: $\Delta m_W(\sigma_B) = \frac{\partial m_W}{\partial \sigma_{WW}} \cdot \frac{L\epsilon_B\sigma_B}{L\epsilon} \cdot \Delta\sigma_B$

With L=3.2ab^{-1}, $\epsilon_B\sigma_B = 0.3$pb, $\Delta\sigma_B = 10^{-3}$:

$\Delta m_W(N_B)$~0.2 MeV, which has been embodied in the product of $\epsilon \cdot P$, and $\Delta m_W(\sigma_B)$ is considerable with the former.

Correlated sys. uncertainty

- The correlated sys. uncertainty includes: $\Delta L, \ \Delta\epsilon, \ \Delta\sigma_{WW}$...
- Since $N_{obs} = L \cdot \sigma \cdot \epsilon$, these uncertainties affect σ_{WW} in same way.
- We use the total correlated sys. uncertainty in data taking optimization:

$$\delta_c = \sqrt{\Delta L^2 + \Delta\epsilon^2}$$

$$\Delta m_W = \frac{\partial m_W}{\partial \sigma_{WW}} \sigma_{WW} \cdot \delta_c \, , \Delta\Gamma_W = \frac{\partial \Gamma_W}{\partial \sigma_{WW}} \sigma_{WW} \cdot \delta_c$$

Correlated sys. uncertainty

$$\Delta m_W = \frac{\partial m_W}{\partial \sigma_{WW}} \sigma_{WW} \cdot \delta_c$$

Two ways to consider to effect:

(a) Gaussian distribution

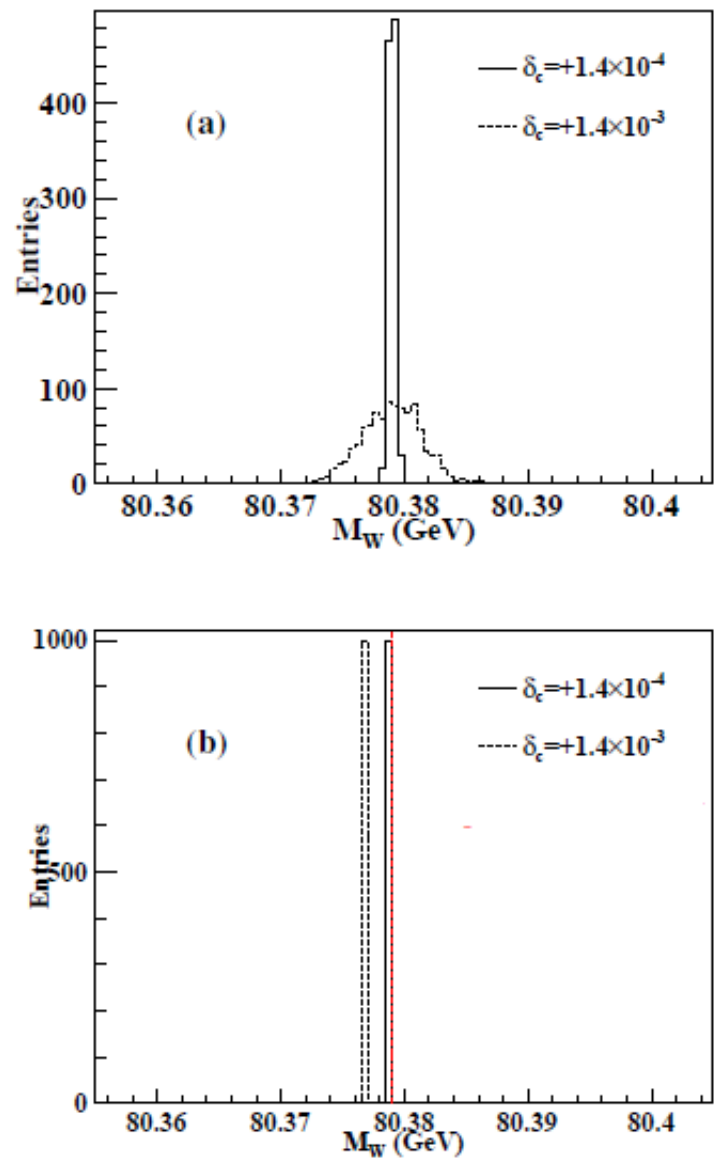
$$\sigma_{WW} = G(\sigma_{WW}^0, \delta_c \cdot \sigma_{WW}^0)$$

(b) Non-Gaussian (will cause shift)

$$\sigma_{WW} = \sigma_{WW}^0 \times (1 + \delta_c)$$

With $\delta_c = +1.4 \cdot 10^{-4} (10^{-3})$ at 161.2GeV

$\Delta m_W \sim 0.24$MeV (3MeV)



Correlated sys. uncertainty

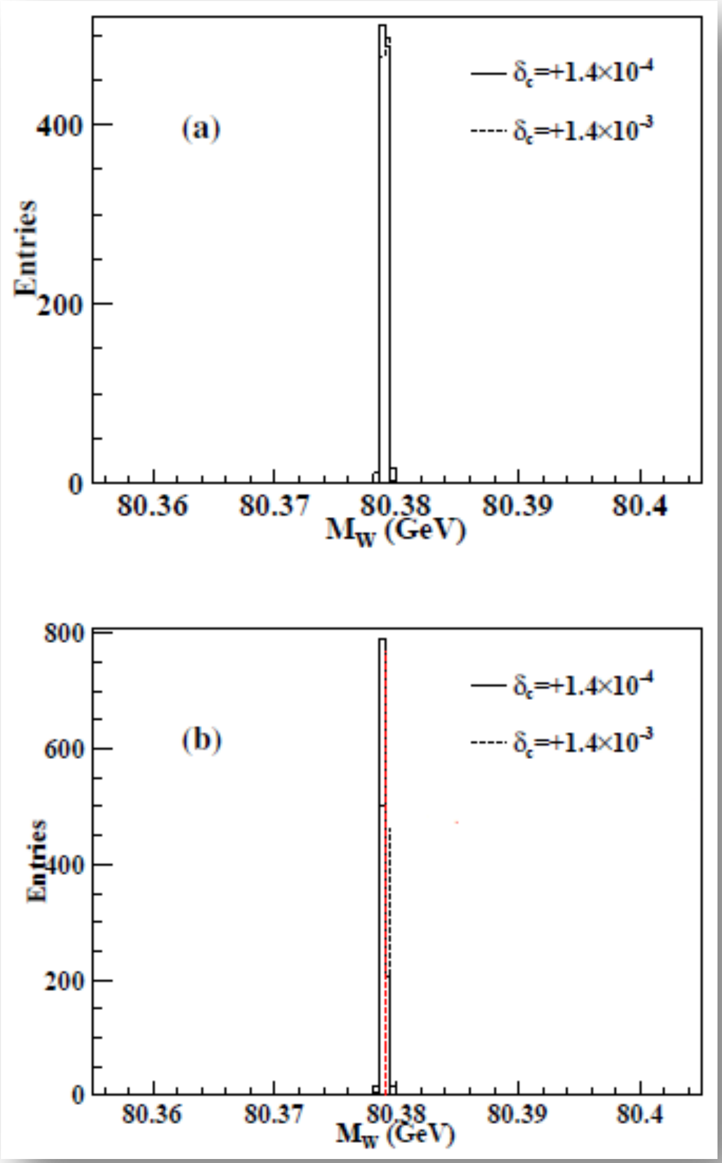
To consider the correlation, the scale factor method is used,

$$\chi^2 = \sum_i^n \frac{(y_i - h\cdot x_i)^2}{\delta_i^2} + \frac{(h-1)^2}{\delta_c^2},$$

where y_i, x_i are the true and fit results, h is a free parameter, δ_i and δ_c are the independent and correlated uncertainties.

For the Gaussian consideration, the scale factor can reduce the effect.

For the non-Gaussian case, the shift of the m_W is controlled well



Data taking scheme

Data taking scheme

One point

- Smallest Δm_W, $\Delta\Gamma_W$ (stat.)
- Large sys. uncertainties
- Only for m_W or Γ_W, without correlation

Two points

- Measure m_W and Γ_W simultanously
- Without the correlation

Three points or more

- Measure m_W and Γ_W simultaneously, with the correlation

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Taking data at one point (just for $\boldsymbol{m_W}$)

There are two special energy points :

- The one most statistical sensitivity to m_W:

  Δm_W(stat.) ~0.59 MeV at E=161.2 GeV

  (with $\Delta\Gamma_W$ and ΔE_{BS} effect)

- The one Δm_W(stat)~0.65 MeV at $E \approx 162.3$ GeV

  (with negligible $\Delta\Gamma_W, \Delta E_{BS}$ effects)

With ΔL $(\Delta\epsilon) < 10^{-4}, \Delta\sigma_B < 10^{-3}$, ΔE=0.7MeV, $\Delta\sigma_E$=0.1, $\Delta\Gamma_W$=42MeV)

$\sqrt{s}$(GeV)	161.2	162.3
E	0.36	0.37
σ_E	0.20	-
σ_B	0.17	0.17
δ_c	0.24	0.34
Γ_W	7.49	-
Stat.	0.59	0.65
Δm_W(MeV)	7.53	0.84

Taking data at two energy points

- To measure Δm_W and $\Delta\Gamma_W$, we scan the energies and the luminosity fraction of the two data points:

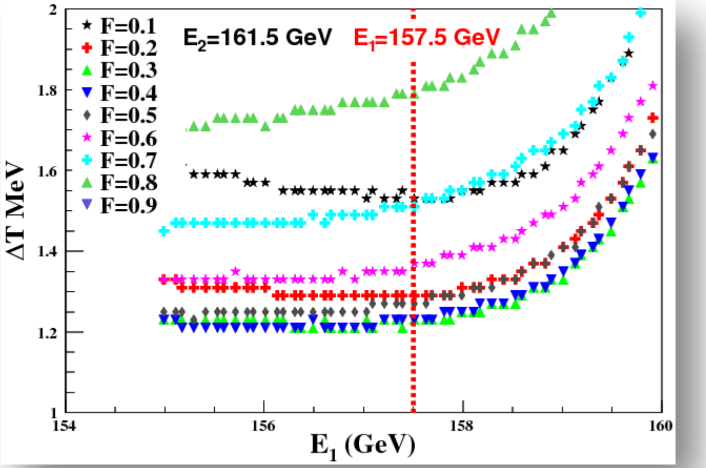
   1. $E_1, E_2 \in [155,\ 165]$ GeV, $\Delta E = 0.1$ GeV

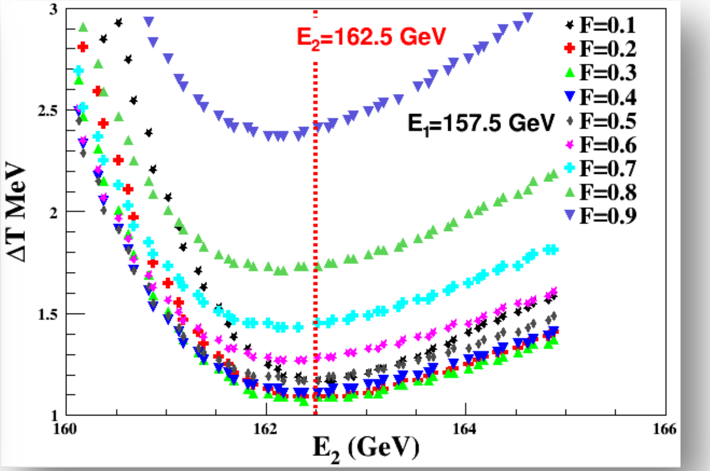
   2. $F \equiv \left(\frac{L_1}{L_2}\right) \in (0, 1)$, $\Delta F = 0.05$

- We define the object function: $T = \mathrm{m_W} + 0.1\Gamma_W$ to optimize the scan parameters (assuming m_W is more important than Γ_W).

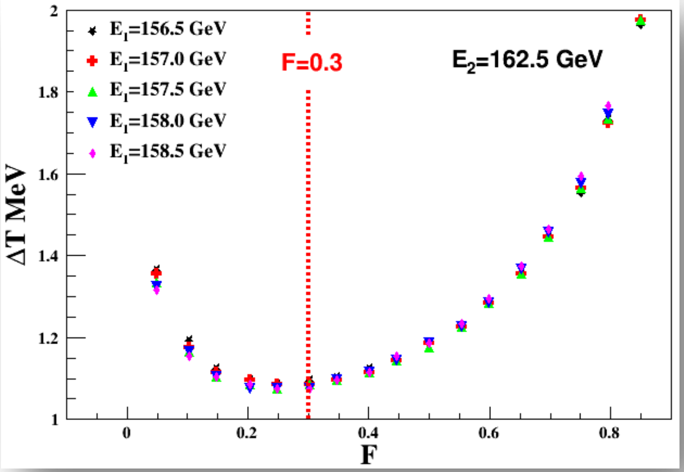
Taking data at two energy points

- The 3D scan is performed, and 2D plots are used to illustrate the optimization results;
- When draw the ΔT change with one parameter, another is fixed with scanning of the third one;
- E_1=157.5 GeV, E_2=162.5 GeV (around $\frac{\partial \sigma_{WW}}{\partial \Gamma_W}=0$, $\frac{\partial \sigma_{WW}}{\partial E_{BS}}=0$) and F=0.3 are taken as the result.





$\Delta L(\Delta\epsilon)<10^{-4}, \Delta\sigma_B<10^{-3}$
$\sigma_E=1\times10^{-3}, \Delta E=0.7\text{MeV}$
$\Delta\sigma_E=0.1\%$



(MeV)	E	σ_E	σ_B	δ_c	Stat.	Total
Δm_W	0.38	-	0.21	0.33	0.80	0.97
$\Delta\Gamma_W$	0.54	0.56	1.38	0.20	2.92	3.32

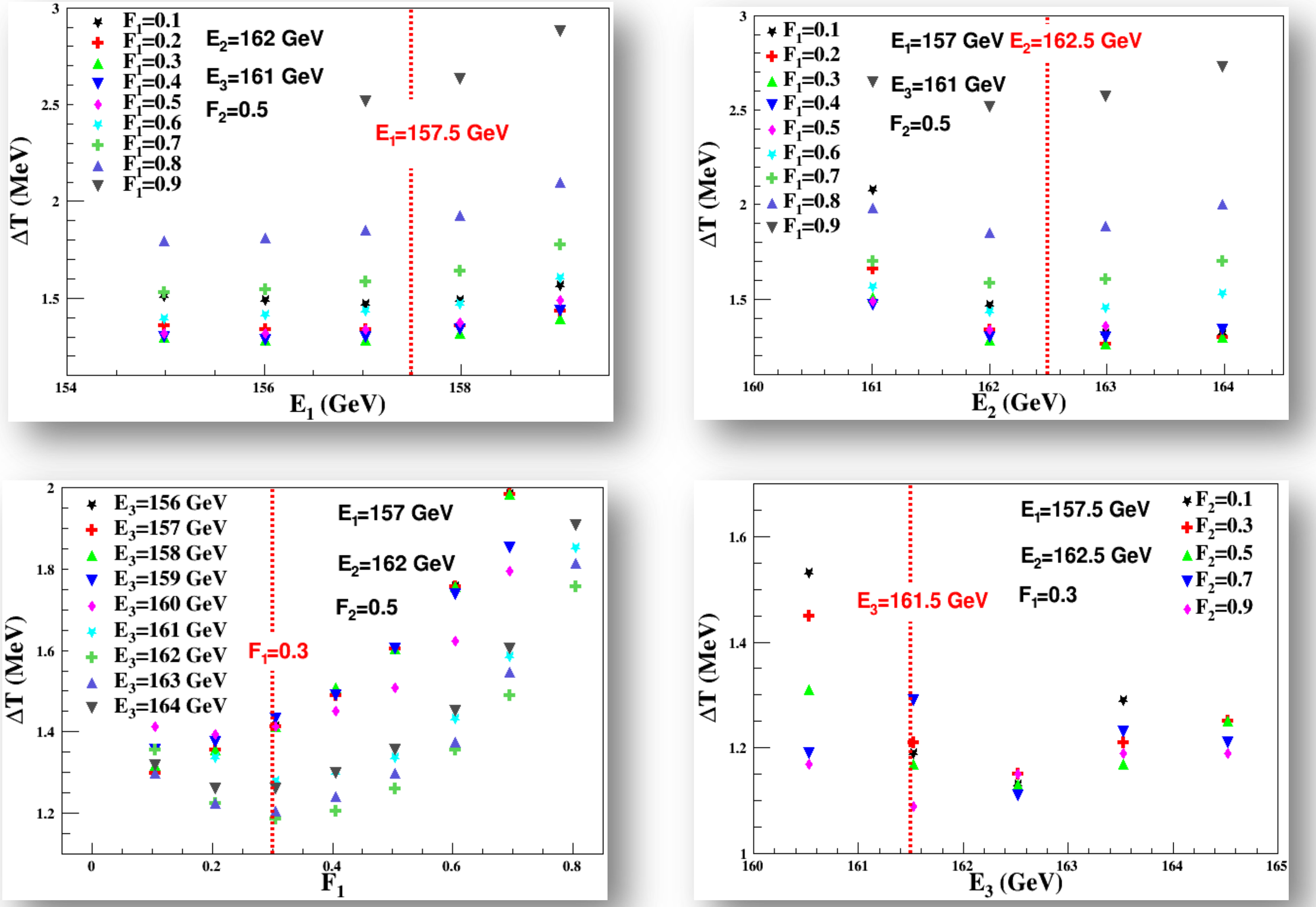
Taking data at three energy points

The procedure of three points optimization is similar to two points

E_1	157.5 GeV
E_2	162.5 GeV
E_3	161.5 GeV
F_1	0.3
F_2	0.9

$\Delta m_W \sim 0.98$ MeV
$\Delta\Gamma_W \sim 3.37$ MeV

$\Delta L(\Delta\epsilon)<10^{-4}, \Delta\sigma_B<10^{-3}$
$\sigma_E=1\times10^{-3}, \Delta E=0.7\text{MeV}$
$\Delta\sigma_E=0.1$



Summary

- Measurement of m_W (Γ_W) with threshold scan method studied
- Different data taking schemes investigated, take stat. and sys. into account.
- CEPC&FCC-ee work together, EPJC refereeing
- With assumptions

$\Delta L(\Delta\epsilon)<10^{-4}, \Delta\sigma_B<10^{-3}$
$\sigma_E=1\times10^{-3}, \Delta E=0.7\text{MeV}$
$\Delta\Gamma_W=42\text{MeV}, \Delta\sigma_E=0.1$

Data-taking scheme	mass or width	δ_{stat} (MeV)	δ_{sys} (MeV)				Total (MeV)
			ΔE	$\Delta\sigma_E$	δ_B	δ_c	
One point	Δm_W	0.65	0.37	-	0.17	0.34	0.84
Two points	Δm_W	0.80	0.38	-	0.21	0.33	0.97
	$\Delta\Gamma_W$	2.92	0.54	0.56	1.38	0.20	3.32
Three points	Δm_W	0.81	0.30	-	0.23	0.29	0.98
	$\Delta\Gamma_W$	2.93	0.52	0.55	1.38	0.20	3.37

Thank you !

Backup Slides

Covariance matrix method

- $$y_i = \frac{n_i}{\epsilon},\ v_{ii} = \sigma_i^2 + y_i^2\sigma_f^2$$

where σ_i is the stat. error of n_i, σ_f is the relative error of ϵ

H. J. Behrend et al. (CELLO Collaboration) Phys. Lett. B 183 (1987) 400
D' Agostini G. Nucl. Instrum. Meth. A346 (1994)

- The correlation between data points i, j contributes to the off-diagonal matrix element v_{ij}:

$$V = \begin{pmatrix} \sigma_1^2 + y_1^2\sigma_f^2 & y_1y_2\sigma_f^2 & \cdots & y_1y_n\sigma_f^2 \\ y_2y_1\sigma_f^2 & \sigma_2^2 + y_2^2\sigma_f^2 & \cdots & y_2y_n\sigma_f^2 \\ \vdots & \vdots & \ddots & \vdots \\ y_ny_1\sigma_f^2 & y_ny_2\sigma_f^2 & \cdots & \sigma_n^2 + y_n^2\sigma_f^2 \end{pmatrix}$$

- Then we minimize: $\chi_1^2 = \eta^T V^{-1} \eta$

$$\eta = \begin{pmatrix} y_1 - k_1 \\ y_2 - k_2 \\ \vdots \\ y_n - k_n \end{pmatrix}$$

For this method, The biasness is uncontrollable

(MO Xiao-Hu HEPNP 30 (2006) 140-146)

Scale factor method

- This method is used by introducing a free fit parameter to the χ^2:

$$\chi_2^2 = \sum_i \frac{(f y_i - k_i)^2}{\sigma_i^2} + \frac{(f-1)^2}{\sigma_f^2}$$

σ_i includes stat. and uncorrelated sys errors, σ_f are the correlated errors.
The equivalence of this form and the one from matrix method is proved in : MO Xiao-Hu HEPNP 30 (2006) 140-146 .

Brandelik R et al(TASSO Collab.). Phys. Lett., 1982, **B113**: 499—508; Brandelik R et al(TASSO Collab.). Z. Phys., 1980, **C4**: 87—93
Bartel W et al(JADE Collab.). Phys. Lett., 1983, **B129**: 145—152
D'Agostini G. Nucl. Instrum. Methods, 1994, **A346**: 306—311

- Both the matrix and the factor approach have bias, which may be considerably striking when the data points are quite many or the scale factor is rather large.

- According to ref: MO Xiao-Hu HEPNP 31 (2007) 745-749, the unbiased χ^2 is constructed as:

$$\chi_3^2 = \sum_i \frac{(y_i - g k_i)^2}{\sigma_i^2} + \frac{(g-1)^2}{\sigma_f^2} \quad \text{(used in our previous results)}$$

The central value from χ_2^2 can be re-scaled, the relative error is still larger than those from χ_3^2 estimation.