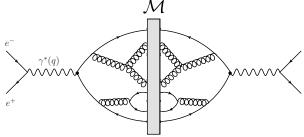


Event Shape Observables

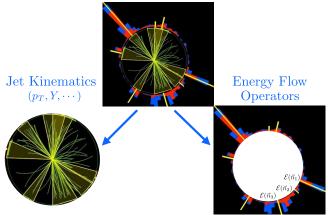
• Despite much progress in understanding of amplitudes, much less progress in cross section level infrared safe "event shapes":



- Event Shapes play a crucial role in precision tests of QCD at e^+e^- colliders, and in jet substructure at the LHC.
- Complicated nature of $\mathcal M$ for "old" event shapes often prevents higher order calculations and use of modern techniques.
- CEPC motivates reconsidering QCD event shapes from a modern perspective.

From Jets to Correlation Functions

Describing energy flow of QCD radiation requires a new language.

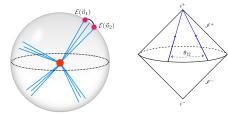


- Energy Flow Operators are the language of event shapes.
- Allows many new calculations that could provide remarkable probe of QCD at CEPC.

Energy Flow Operators

Energy Flow Operators in Quantum Field Theory:

$$\mathcal{E}(\vec{n}) = \int_{0}^{\infty} dt \lim_{r \to \infty} r^{2} n^{i} T_{0i}(t, r\vec{n})$$



$$\langle \hat{O}\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\hat{O}^{\dagger} \rangle$$

 Object of significant interest in their own right: Average Null Energy (ANEC) Operator [Sveshnikov, Tkachov]

 $[{\sf Korchemsky},\ {\sf Oderda},\ {\sf Sterman}]$

[Hofman, Maldacena]

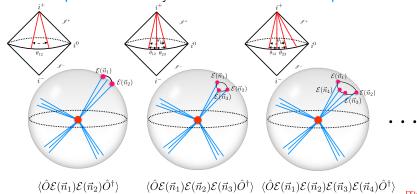
Average Null Energy Condition

$$\langle \psi | \mathcal{E} | \psi \rangle \ge 0$$



Energy Flow Operators

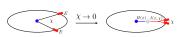
Event Shapes are the study of correlations of ANEC operators.



- All energy flow observables can be constructed from these correlations.
- Much recent progress in understanding the small angle limit of ANECs
 in (Non-)Conformal Field Theories. [Dixon, IM, Zhu]
 [Hofman, Maldacena; Belitsky, Hohenegger, Korchemsky, Sokatchev, Zhiboedov; Kravchuk, Simmons Duffin;
 Henn, Sokatchev, Yan, Zhiboedov; Belin, Hofman, Mathys; Kologlu, Kravchuk, Simmons Duffin, Zhiboedov; Korchemsky]

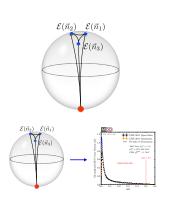
Outline

• Two Point Correlators at NNLL

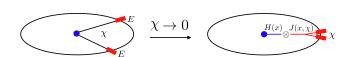


• The Three Point Correlator

• Energy Correlators in CMS Open Data



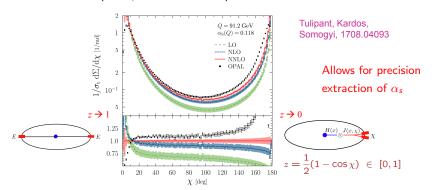
Two Point Correlators at NNLL



[Dixon, IM, Zhu:1905.01310]

EEC at e^+e^- Colliders

• The EEC (two point correlator) is a classic e^+e^- observable.



- Exhibits large perturbative corrections in $z \to 0$ (collinear) and $z \to 1$ (back-to-back) limits:
 - $z \rightarrow 1$: Known to NNNLL [IM, Zhu:1801.02627]
 - $z \rightarrow 0$: This talk.

 We can derive a timelike factorization formula for the 2-point correlator in QCD:

$$\Sigma(z, \ln \frac{Q^2}{\mu^2}, \mu) = \int_0^1 dx \, x^2 \vec{J} (\ln \frac{zx^2 Q^2}{\mu^2}, \mu) \cdot \vec{H}(x, \frac{Q^2}{\mu^2}, \mu) \qquad J$$

 The jet function satisfies the renormalization group equation:

$$\frac{d\vec{J}(\ln\frac{zQ^2}{\mu^2},\mu)}{d\ln\mu^2} = \int_0^1 dy \, y^2 \vec{J}(\ln\frac{zy^2Q^2}{\mu^2},\mu) \cdot \hat{P}_T(y,\mu)$$

• Boundary conditions for RGE analytically computed to $\mathcal{O}(\alpha_s^2)$

$$\begin{split} j_2^g &= n_f^2 \left(-\frac{8}{15} \zeta_2 + \frac{2344}{1125} \right) + C_F n_f \left(4\zeta_3 + \frac{14}{5} \zeta_2 - \frac{1528667}{108000} \right) \\ &+ C_A n_f \left(\frac{44}{5} \zeta_3 - \frac{127}{25} \zeta_2 + \frac{68111303}{1620000} \right) + C_A^2 \left(76\zeta_4 - \frac{1054}{5} \zeta_3 - \frac{2159}{75} \zeta_2 + \frac{133639871}{810000} \right) \end{split}$$

To Three Loops in One Slide

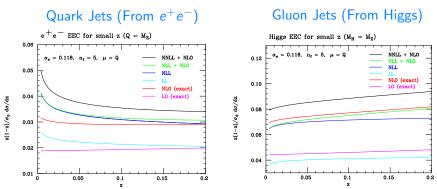
 To achieve NNLL (single log counting), one needs all anomalous dimensions at 3 loops, constants at 2 loops.

- Control of $\frac{1}{z}$, $\frac{\log z}{z}$, $\frac{\log^2 z}{z}$ at three loops, plus tower of logarithms.
- First collinear observable known at this order.

$$\begin{split} z\Sigma(z) &= \frac{\alpha_s}{4\pi} \frac{3C_F}{2} \\ &+ \left(\frac{\alpha_s}{4\pi}\right)^2 \left[C_A C_F \left(-\frac{50\zeta_2}{3} + 4\zeta_3 - \frac{107\log(z)}{15} + \frac{35366}{675} \right) + C_F n_f \left(\frac{53\log(z)}{60} - \frac{4913}{900} \right) \right. \\ &+ C_F^2 \left(\frac{86\zeta_2}{3} - 8\zeta_3 + \frac{25\log(z)}{5400} + \frac{8263}{30} - \frac{118\zeta_4}{3} + \left(-\frac{108\zeta_2}{5} + \frac{16\zeta_3}{3} + \frac{6644267}{54000} \right) \log(z) \\ &+ \left(\frac{\alpha_s}{4\pi}\right)^3 \left[C_A C_F n_f \left(\frac{379579\zeta_2}{5400} + \frac{3679\zeta_3}{30} - \frac{118\zeta_4}{3} + \left(-\frac{108\zeta_2}{5} + \frac{16\zeta_3}{3} + \frac{6644267}{540000} \right) \log(z) \right. \\ &- \frac{16259\log^2(z)}{18000} - \frac{1025118113}{21600000} \\ &+ C_A C_F^2 \left(-\frac{400\zeta_2^2}{3} + \frac{137305\zeta_2}{216} - 72\zeta_2\zeta_3 + \frac{10604\zeta_3}{15} + \frac{4541\zeta_4}{6} - 216\zeta_5 \right. \\ &+ \left(-\frac{1100\zeta_2}{3} + \frac{262\zeta_3}{3} + \frac{105425}{1444} \right) \log(z) - \frac{340}{9} \log^2(z) - \frac{105395741}{51840} \right. \\ &+ C_A^2 C_F \left(-\frac{906257\zeta_2}{2700} + 24\zeta_2\zeta_3 - \frac{4793\zeta_3}{4930} - \frac{481\zeta_4}{64} + 56\zeta_5 + \left(\frac{503\zeta_2}{5} - \frac{74\zeta_3}{3} - \frac{2916859}{6750} \right) \log(z) \right. \\ &+ \frac{8059\log^2(z)}{300} + \frac{964892415}{540000} \right. \\ &+ C_F^2 n_f \left(-\frac{15161\zeta_2}{120} - \frac{7994\zeta_3}{45} + \frac{236\zeta_4}{3} + \left(\frac{416\zeta_2}{9} - \frac{32\zeta_3}{3} - \frac{6760183}{67800} \right) \log(z) + \frac{4619\log^2(z)}{720} \right. \\ &+ \frac{164829499}{486000} + C_F n_f^2 \left(\frac{5\zeta_2}{5} + 23\log^2(z) - \frac{88673(\zeta_2)}{1350} + \frac{8803}{4500} \right) \right. \\ &+ C_F^2 \left(\frac{688\zeta_2^2}{3} - \frac{18805\zeta_2}{216} + 48\zeta_2\zeta_3 + 52\zeta_3 - 1130\zeta_4 + 208\zeta_5 + \left(\frac{1849\zeta_2}{9} - \frac{172\zeta_3}{3} - \frac{723533}{2592} \right) \log(z) \right. \\ &+ \frac{625\log^2(z)}{48} + \frac{742433}{1944} \right) \right] \\ &+ \mathcal{O}(\alpha_s^4) \end{aligned}$$

NNLL+NLO Results

Resummed results at NNLL+NLO:



- Distribution depends very sensitively on quark vs gluon!
- Large perturbative corrections observed \implies Should extend to N³LL.

Extension to NNNLL

- Extension to NNNLL in progress.
- Four loop (!!) twist 2 anomalous dimensions computed recently for several *N*.

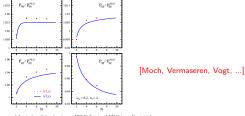
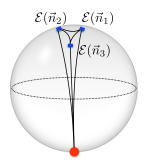


Figure 1: Moments of the singlet splitting functions at NNLO (lines) and N³LO (even-N points) for $\alpha_4 = 0.2$ and $n_f = 4$, normalized to the respective NLO approximations.

- Three loop constant for the jet function can be extracted using a trick to relate the collinear limit to the back-to-back limit.
- Provides a link between event shapes and some of the most precisely known QCD quantities.
 - \implies how can it best be used for precision QCD studies $(\alpha_s, ...)$

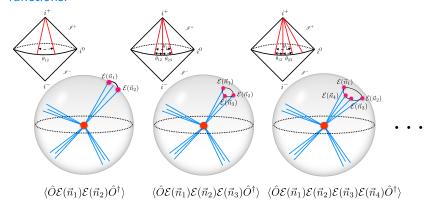
The Three Point Correlator



[Chen, Dixon, Luo, IM, Yang, Zhang, Zhu]

Multi-Point Correlators

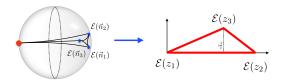
- Multiparticle correlations encode more interesting information about the internal structure of jets.
- Most basic objects are correlation functions of energy flow operators, kept fully differential in angle
 interesting multi-variable functions.



Symmetries of the Celestial Sphere

• It is convenient to exchange null reference vectors n_i^{μ} with complex coordinates z_i on the celestial sphere:

$$n_i^{\mu} = (1 + |z_i|^2, z_i + \bar{z}_i, -i(z_i - \bar{z}_i), 1 - |z_i|^2)$$



 Simple but powerful statement: Lorentz symmetries act on celestial coordinates as SL(2, C) [Penrose]

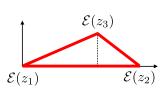
$$z_i \rightarrow z_i' = \frac{az_i + b}{cz_i + d}$$

Other recent work: [Pasterski, Shao, Strominger] [Stieberger, Taylor] [Schreiber, Volovich, Zlotnikov]

[Hofman, Maldacena]
[Dixon, IM, Zhu]
[Korchemsky]
[Kologlu, Kravchuk, Simmons Duffin, Zhiboedov]

• Correlators at the LHC/CEPC take a simple form in the small angle

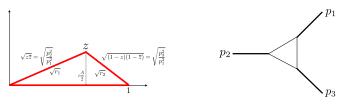
limit.



$$= f_4 \left(\frac{|z_3 - z_1|^2}{|z_2 - z_1|^2}, \frac{|z_3 - z_2|^2}{|z_2 - z_1|^2} \right) \cdot \left(\frac{1}{|z_2 - z_1|^2} \right)^{1 - \gamma_4(\alpha_s)}$$
Shape
Scaling

- Shape behaves as a conformal N+1 point function.
- Scaling controlled by twist-2 spin N+1 anomalous dimension, γ_{N+1} .

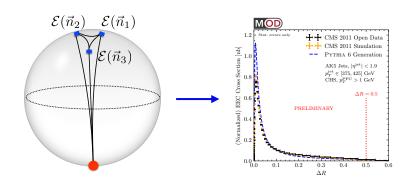
Energy Correlators



$$\frac{d^3\Sigma}{dx_L\,dz\,d\bar{z}} = \frac{g^4N_c^2}{64\pi^5} \left(\mathcal{J}^{(d=8)}(1,2,2) - \mathcal{J}^{(d=10)}(2,2,2,\widetilde{1}) + \mathcal{R}(z) \right)$$

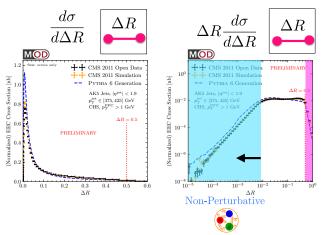
- First analytic calculation of multipoint correlators!
- Approach should also allow calculation of four point correlator.
- Extension to generic angles (away from collinear limits) in progress: Would be fascinating to measure!

Energy Correlators in CMS Open Data



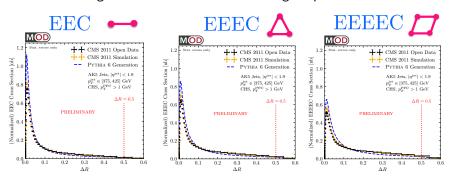
[Dixon, Komiske, IM, Thaler, Zhu]

• Analyzing N-point Energy Correlators with CMS Open Data.



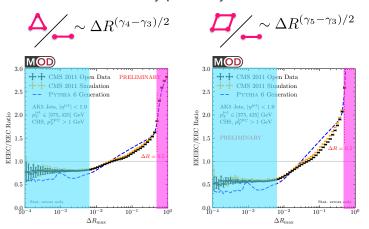
Beautiful perturbative scaling over wide range.

• Clear scaling behavior also observed for higher point correlators



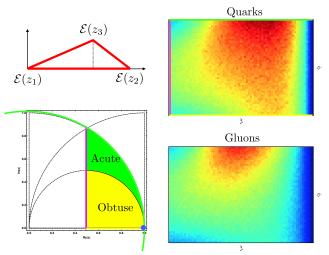
- First theoretically understood probes of higher point correlations.
- Directly probes quantum scaling on the lightcone!

Anomalous dimensions directly probed by ratios:



• Can this be used to measure α_s ? (slope is proportional to α_s)

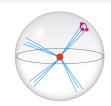
Full shape dependence can be studied/calculated.



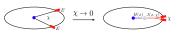
• Understanding QCD at an unprecedented level of differentiality!

Conclusions

 New Calculational Techniques allow qualitatively new probes of QCD using Energy Correlators ⇒ CEPC would provide remarkable probe of QCD



• Two Point Correlators at NNLL



Multi Point Correlators can be analytically computed!

