Three-Point Energy Correlator in the Coplanar Limit

Hua Xing Zhu
Zhejiang University

Work in preparation with Anjie Gao, Ming-Xing Luo, Tong-Zhi Yang

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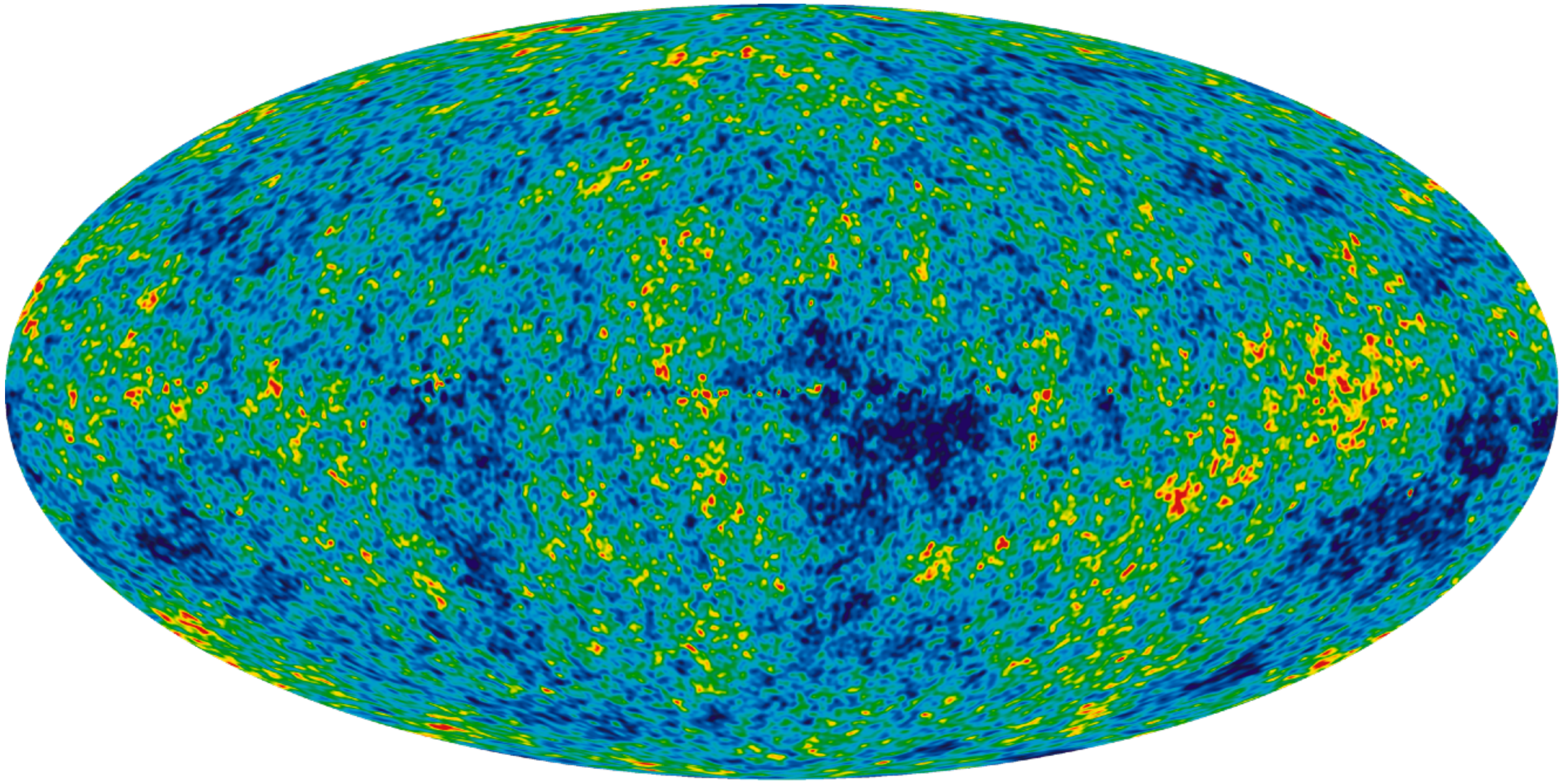
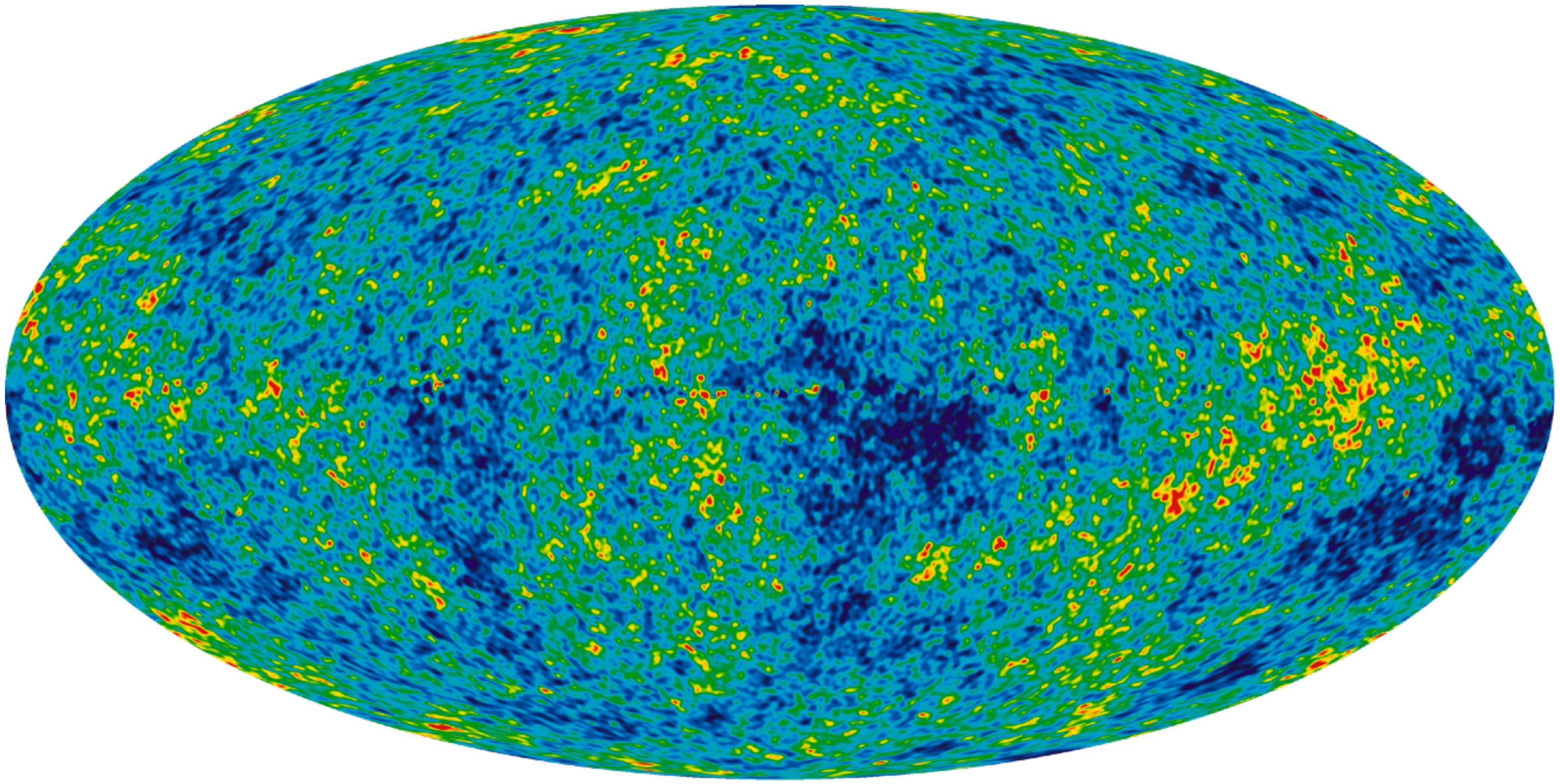
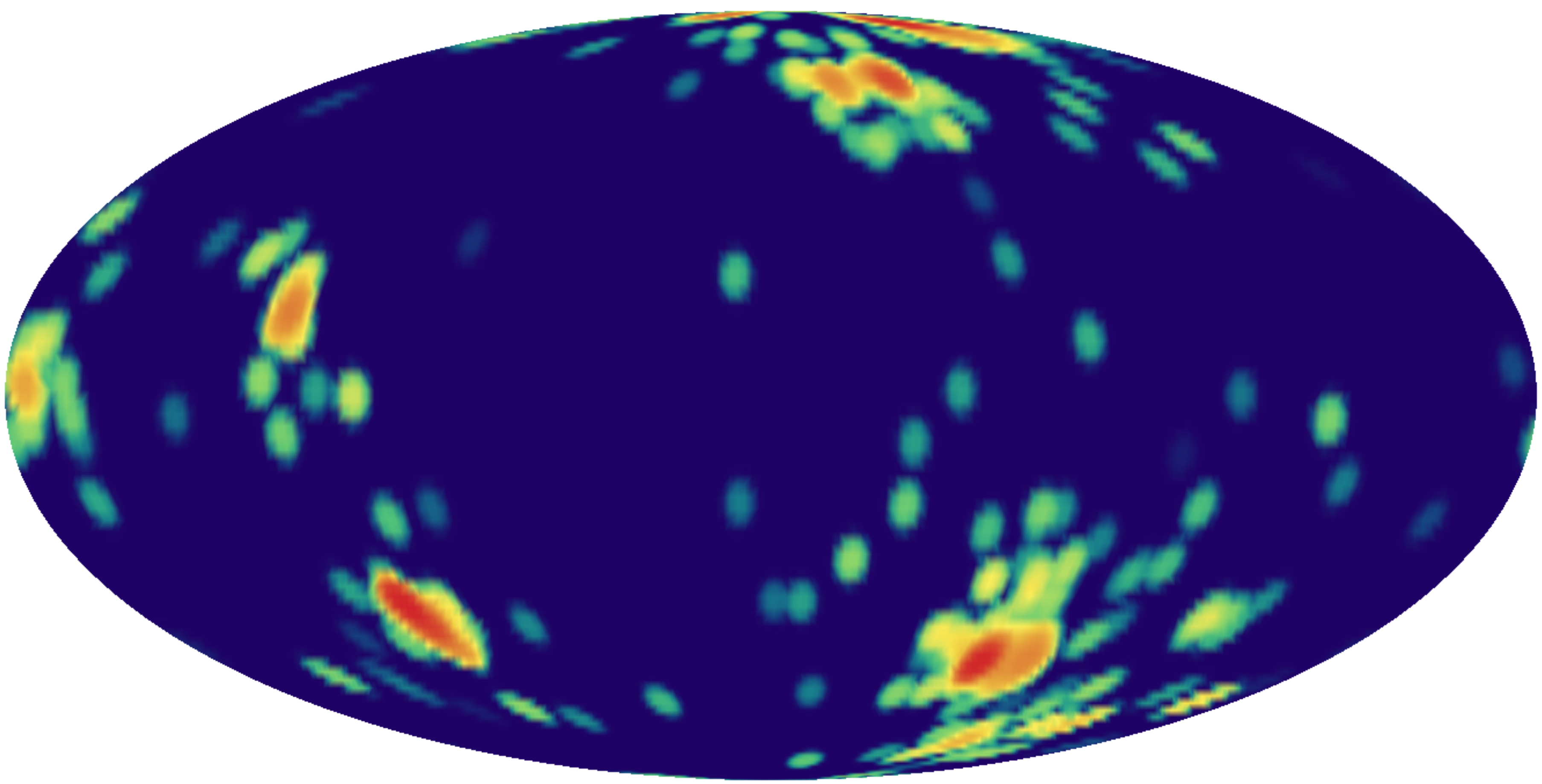
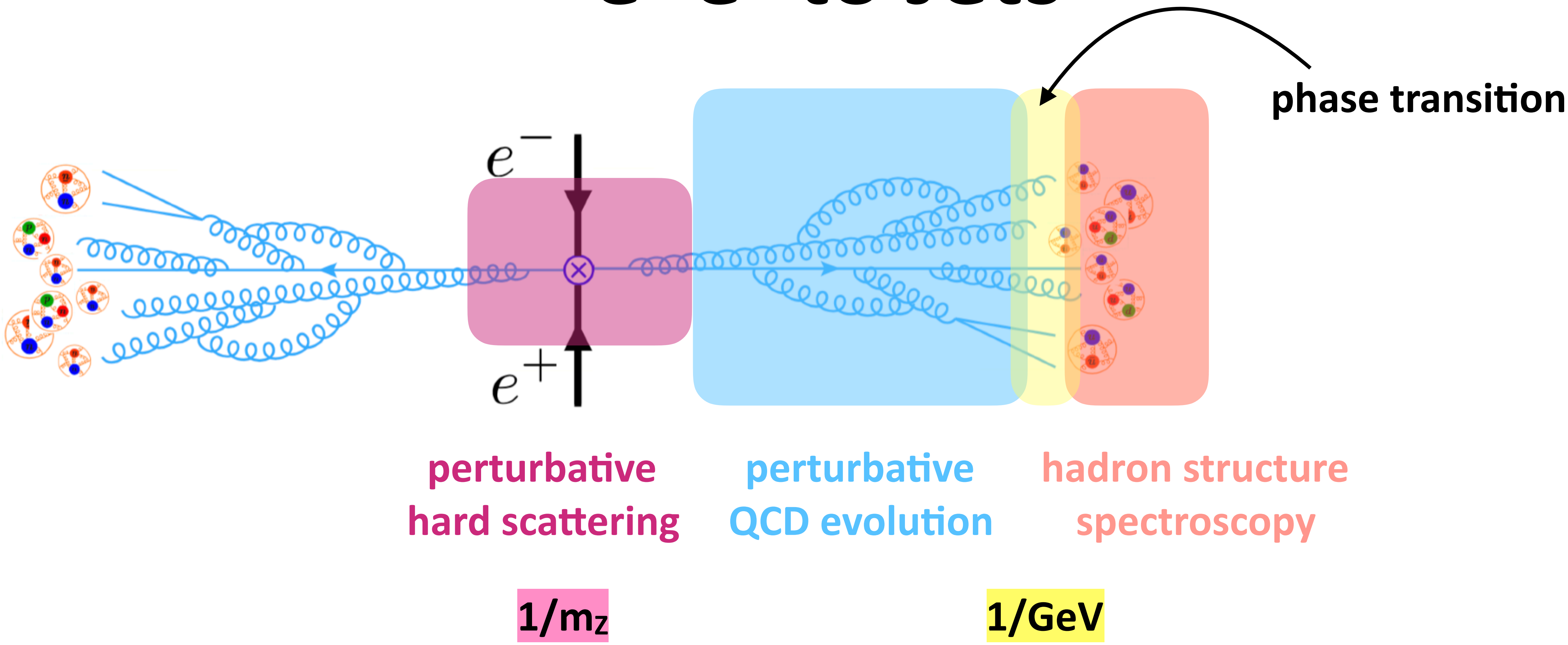


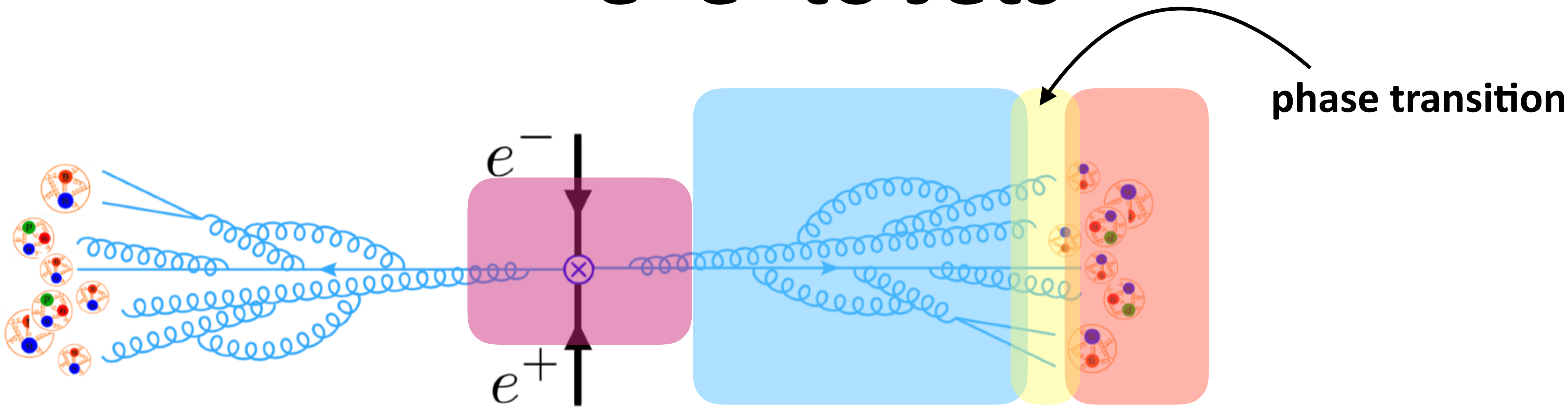




e+e- to Jets


phase transition
e^-
e^+
perturbative
hard scattering
perturbative
QCD evolution
hadron structure
spectroscopy
$1/m_Z$
1/GeV

e+e- to Jets



perturbative hard scattering

perturbative QCD evolution

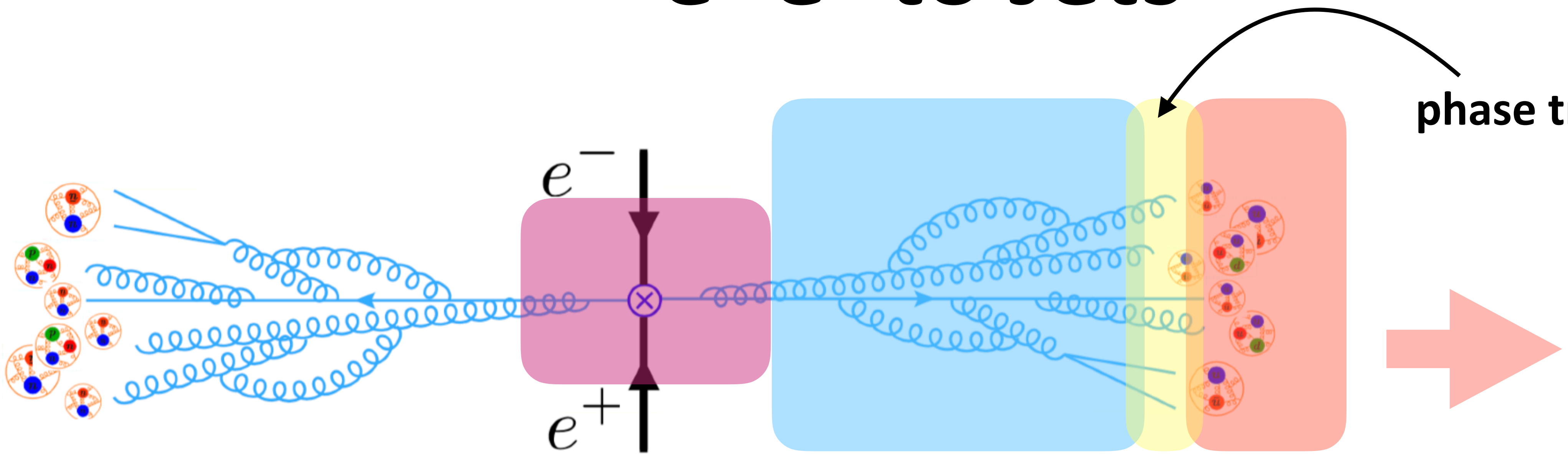
hadron structure spectroscopy

$1/m_Z$

1/GeV

But isn't all of these physics can be studied at LEP?

e+e- to Jets



$$\left(\frac{\Lambda_{\rm QCD}}{Q}\right)^{1-\gamma}$$

perturbative hard scattering

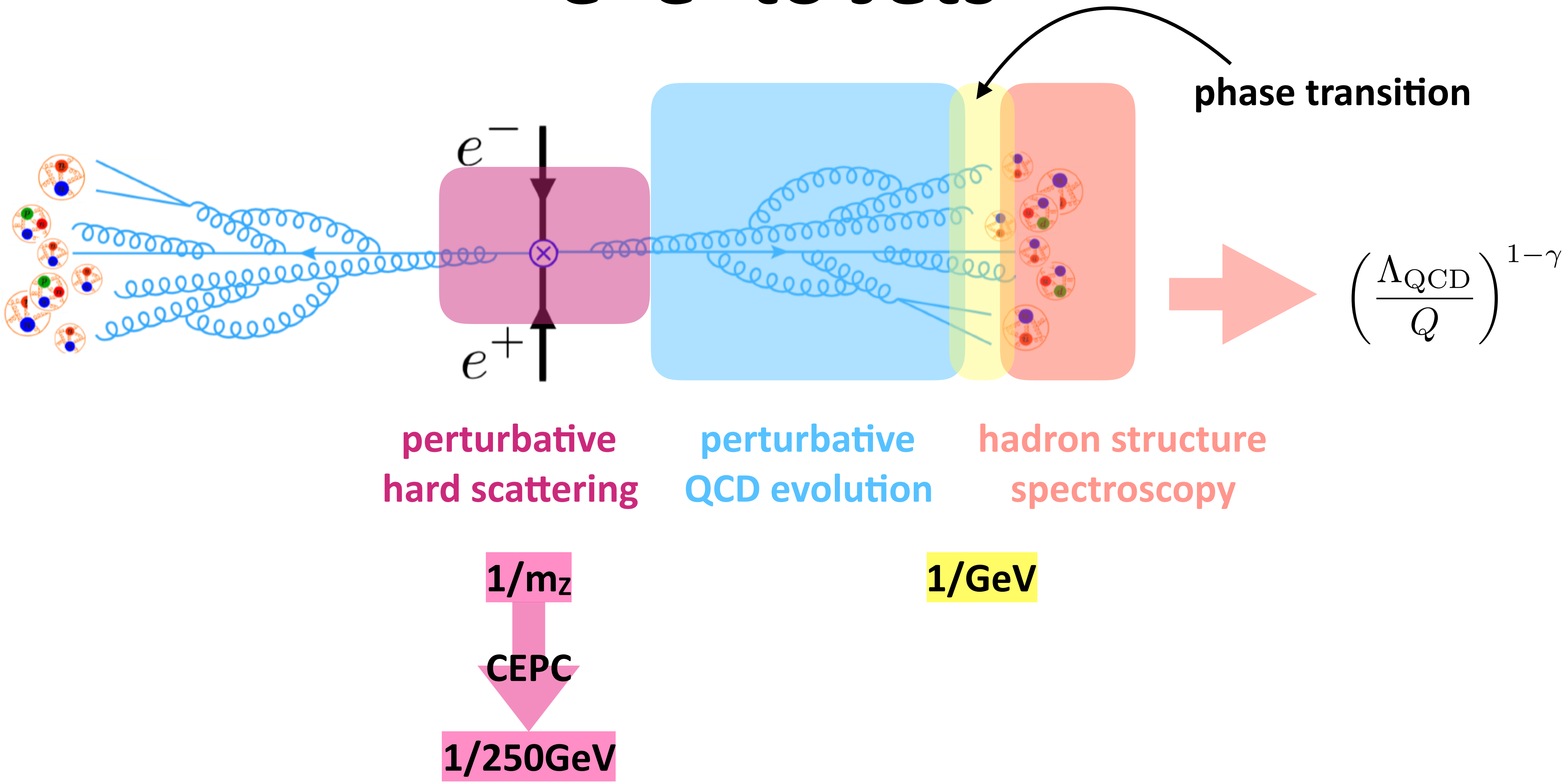
perturbative QCD evolution

hadron structure spectroscopy

$1/m_Z$

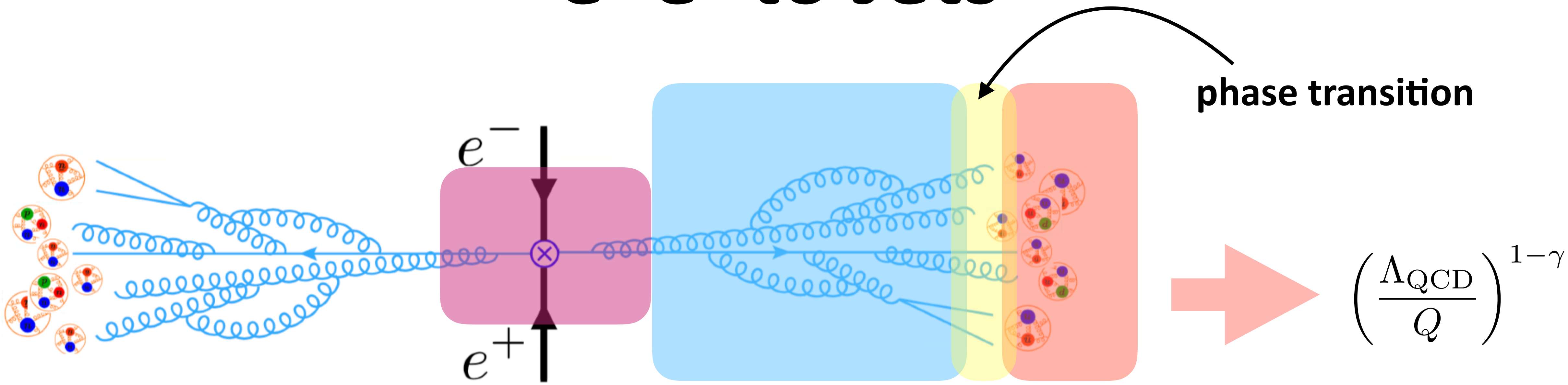
1/GeV

e+e- to Jets



phase transition
e^-
e^+
$\left(\frac{\Lambda_{QCD}}{Q}\right)^{1-\gamma}$
perturbative
hard scattering
perturbative
QCD evolution
hadron structure
spectroscopy
$1/m_Z$
CEPC
1/250GeV
1/GeV

e+e- to Jets



perturbative hard scattering

perturbative QCD evolution

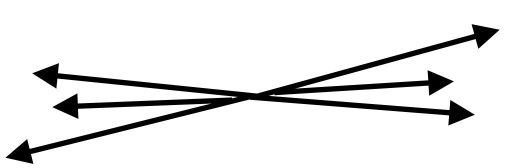
hadron structure spectroscopy

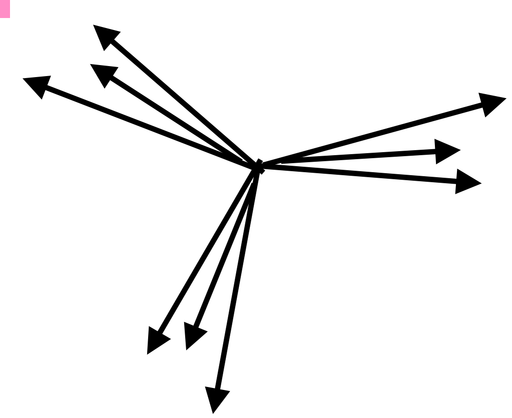
$1/m_Z$

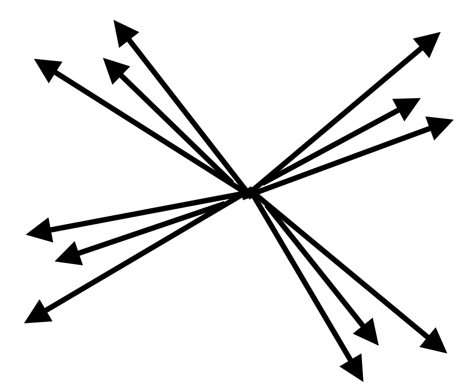
CEPC

1/250GeV

1/GeV







@250GeV

$\mathcal{O}(\alpha_s^0)$ 10^8

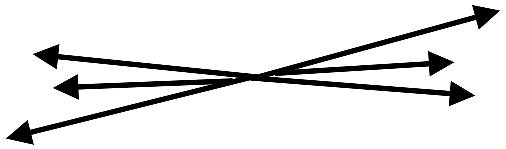
$\mathcal{O}(\alpha_s^1)$ 10^7

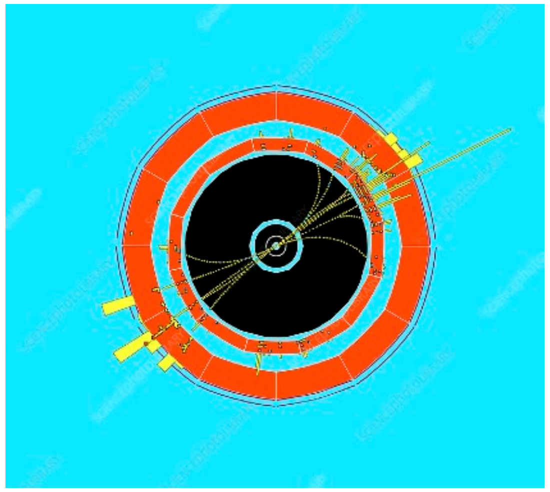
$\mathcal{O}(\alpha_s^2)$ 10^6

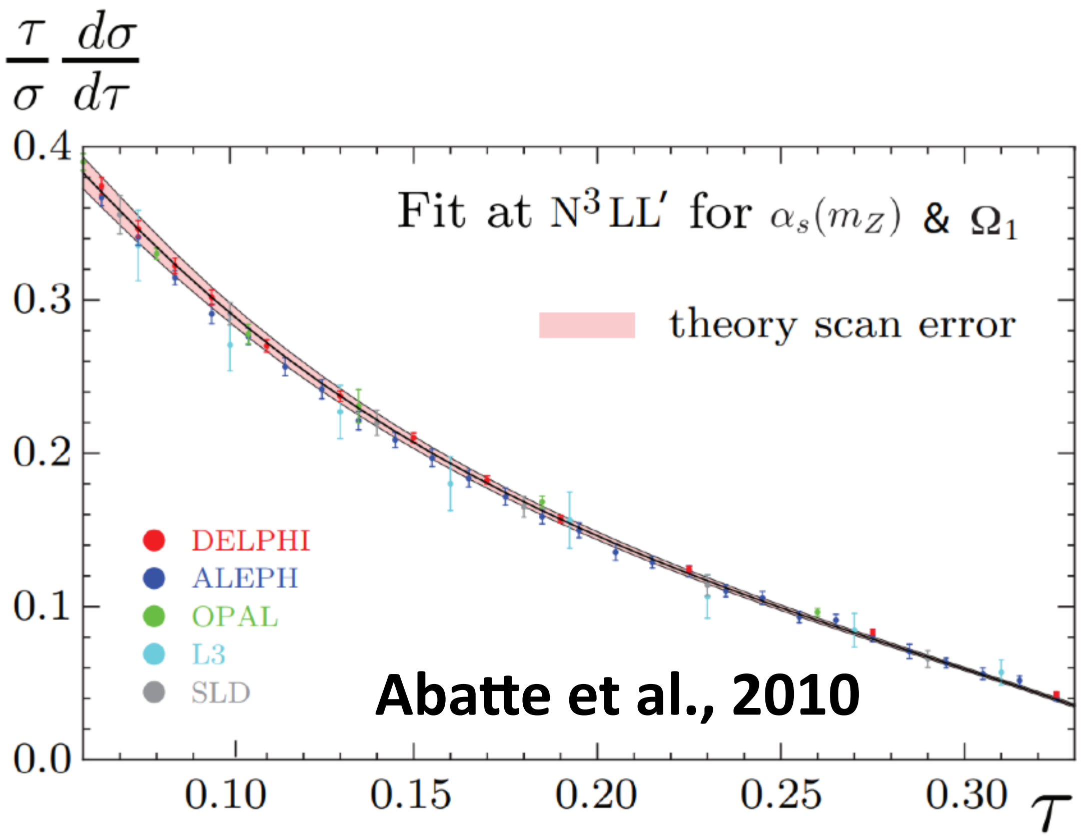
Dijet Event Shape Observables

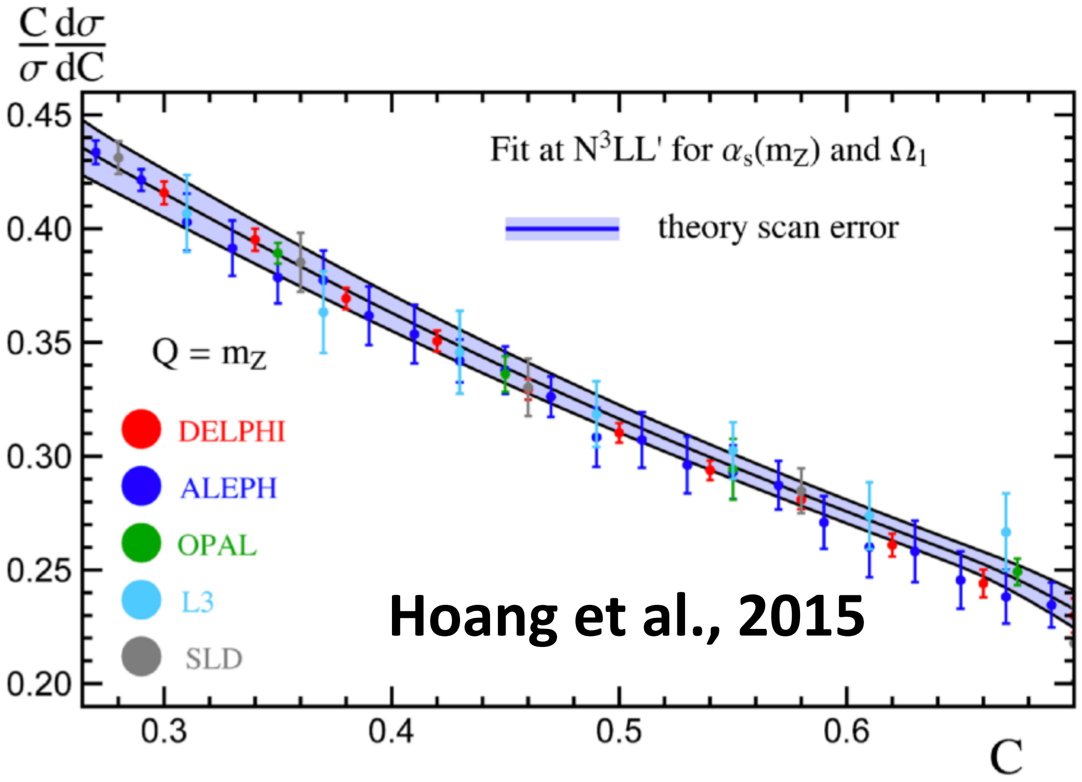
Energy flow observable which vanish in the dijet limit

Example: Thrust, C parameter, wide and total jet broadening, heavy jet mass, et al.







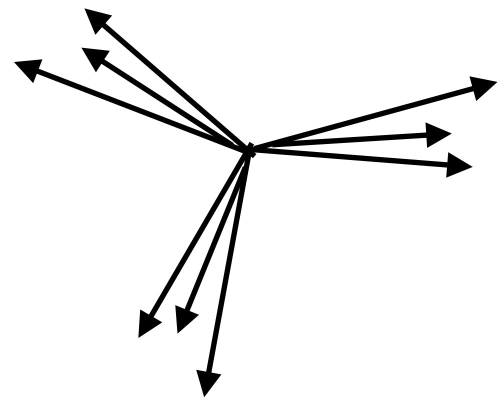


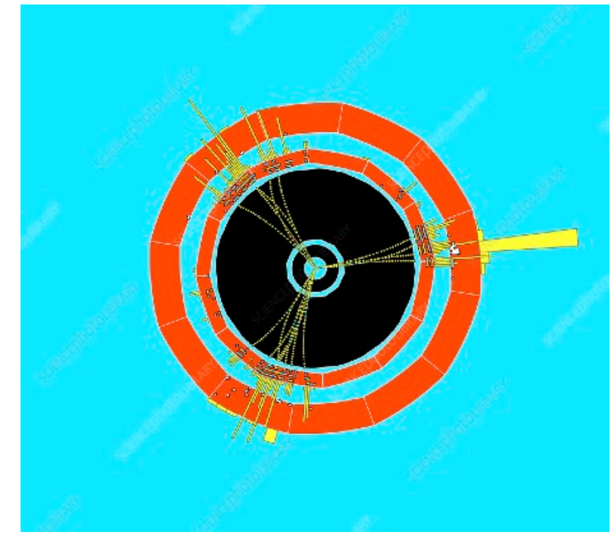
$$\alpha_s(m_Z) = 0.1135 \pm (0.0002)_{\rm exp} \pm (0.0005)_{\rm hadr} \pm (0.0009)_{\rm pert}$$

See also Andrii Verbytskyi's talk

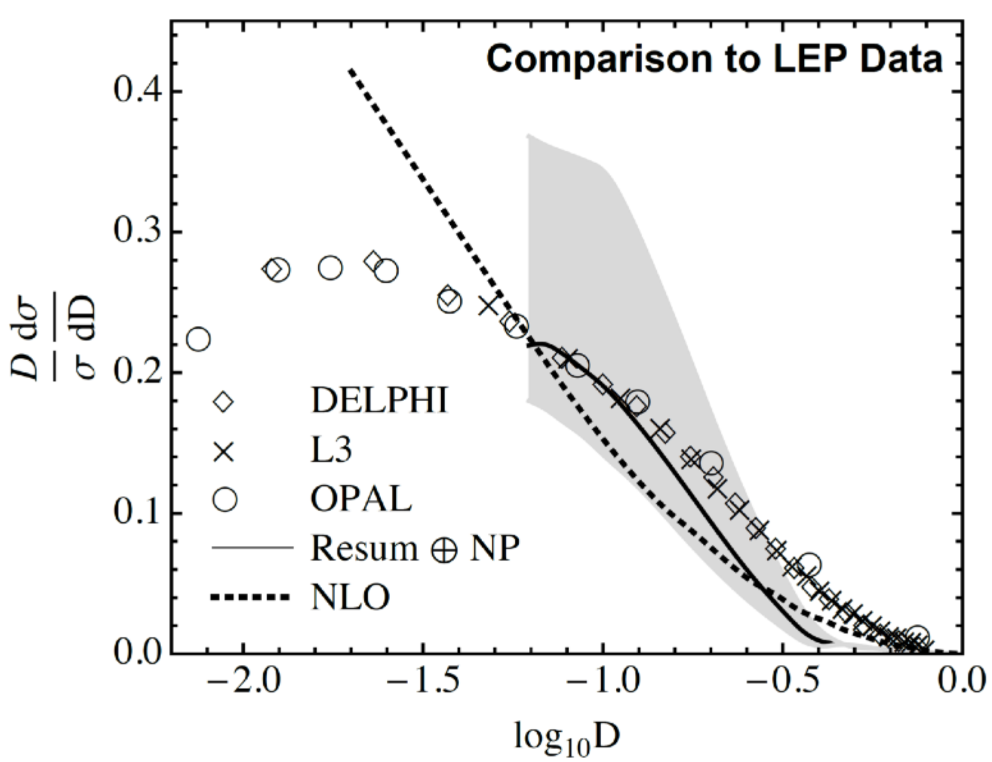
Three Jet Event Shape Observables

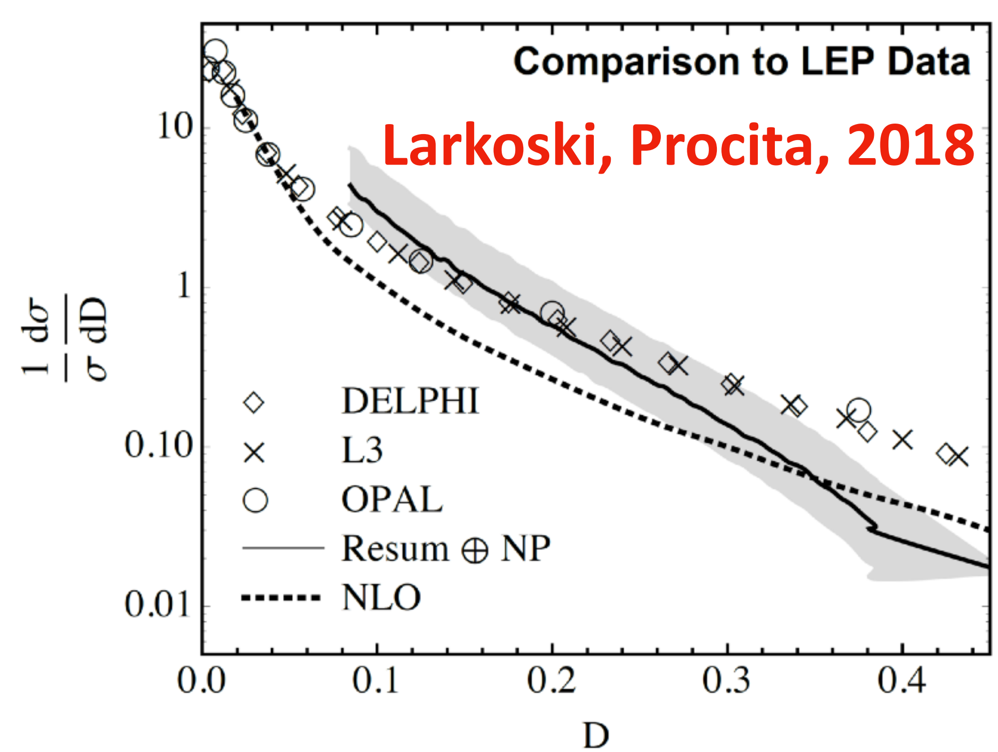
Energy flow observable which vanish in the three jet limit





- **Expect to be more sensitive to α_s**
- **Much less studied in the literature before**
  - Thrust minor [Banfi et al., 2001]; D parameter [Banfi et al., 2001; Larkoski, Procita, 2018]. **All limited to NLL study**





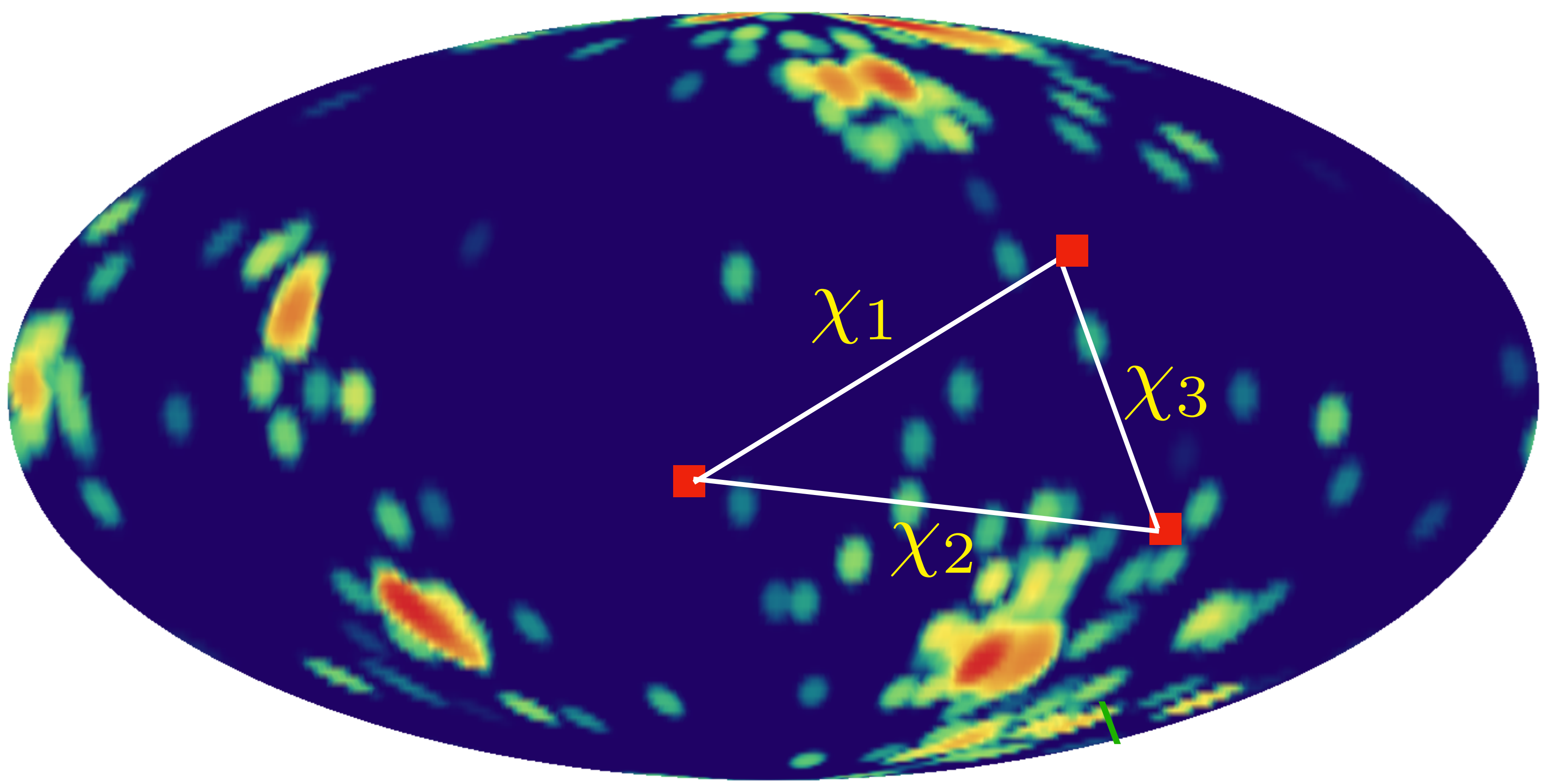
Three-Point Energy Correlator

$$\sum_{ijk} \int d\sigma \frac{E_i E_j E_k}{Q^3 \sigma_{\mathrm{tot}}} \delta(\cos\chi_1 - \cos\theta_{ij}) \delta(\cos\chi_2 - \cos\theta_{ik}) \delta(\cos\chi_3 - \cos\theta_{jk})$$

$$\sum \sum$$

Sum over all events

Sum over all inequivalent configuration, keeping χ_1, χ_2, χ_3 fixed



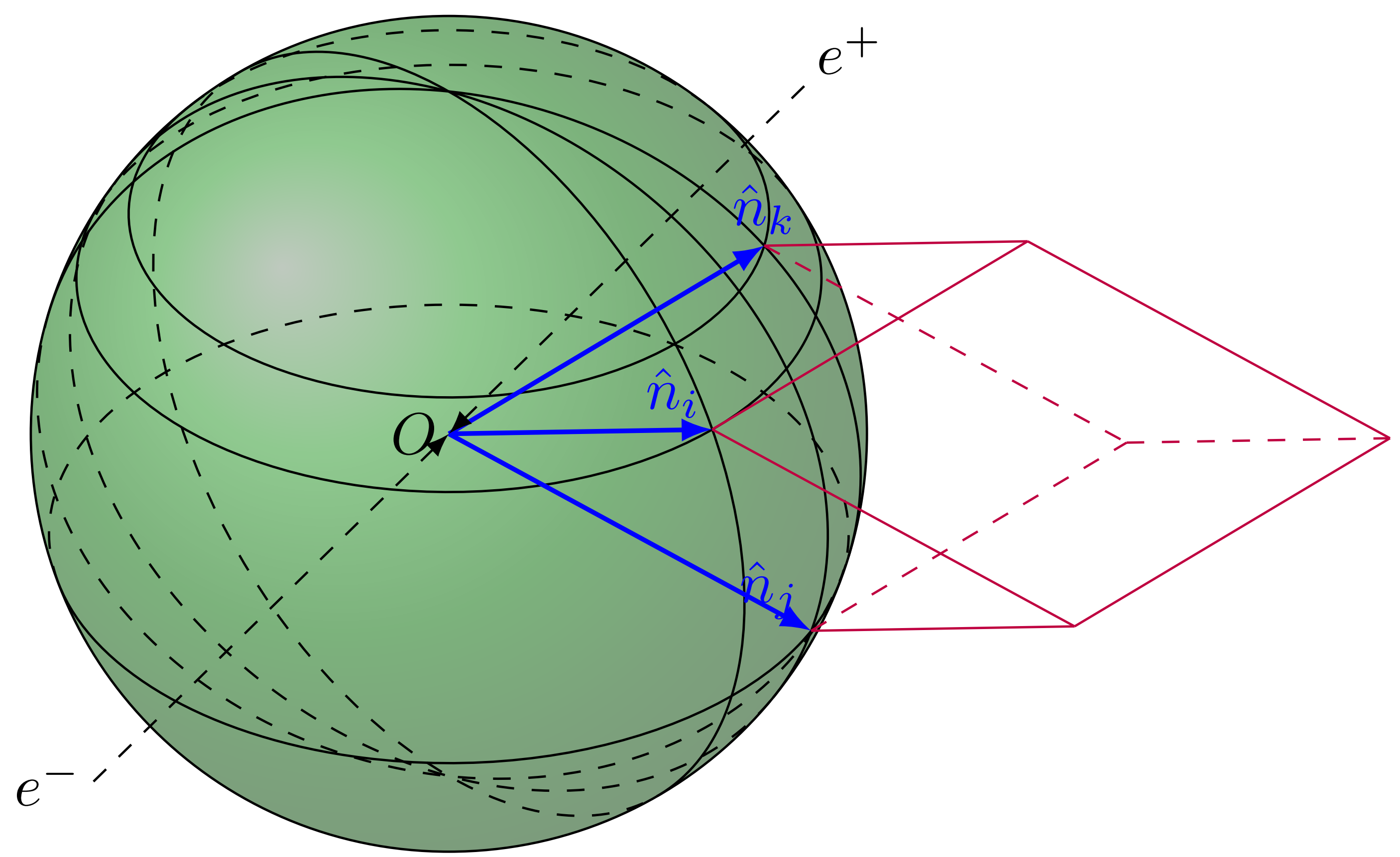
- Natural generalization of the Energy-Energy Correlator [Basham, Brown, Ellis, Love, 1978]
- Function of three angle χ_1, χ_2, χ_3

Energy Triple-Product Correlator

- The three angle χ_1, χ_2, χ_3 can also be specified by three unit vector n_1, n_2, n_3
- The Energy Triple-Production Correlator (ETPC):

$$\tau_p = \left|\vec{n}_i \cdot (\vec{n}_j \times \vec{n}_k)\right|$$

$$\tau_p = \mathrm{Vol}_{\mathrm{hexahedron}}(\vec{n}_1, \vec{n}_2, \vec{n}_3)$$



Relation to D Parameter

- The D parameter is the product of the three eigenvalues of the spherocity tensor [Parisi, 1978; Donoghue et al., 1979]

$$\Theta_{\alpha\beta} = \frac{1}{Q} \sum_i \frac{p_{i\alpha} p_{i\beta}}{E_i}$$

$$\begin{aligned} D &= 27\lambda_1\lambda_2\lambda_3 \\ &= \frac{27}{6}\left\{(\lambda_1+\lambda_2+\lambda_3)\left[(\lambda_1+\lambda_2+\lambda_3)^2 - 3\left(\lambda_1^2+\lambda_2^2+\lambda_3^2\right)\right] + 2\left(\lambda_1^3+\lambda_2^3+\lambda_3^3\right)\right\} \\ &= \frac{27}{6}\left\{Tr\Theta\left[(Tr\Theta)^2 - 3Tr\Theta^2\right] + 2Tr\Theta^3\right\} \\ &= \frac{27}{Q^3}\sum_{i<j<k}\frac{\left|(\vec{p}_i \times \vec{p}_j)\cdot\vec{p}_k\right|^2}{E_iE_jE_k} \\ &= \frac{27}{Q^3}\sum_{i<j<k} E_iE_jE_k\tau_{ijk}^2 \end{aligned}$$

Massless limit

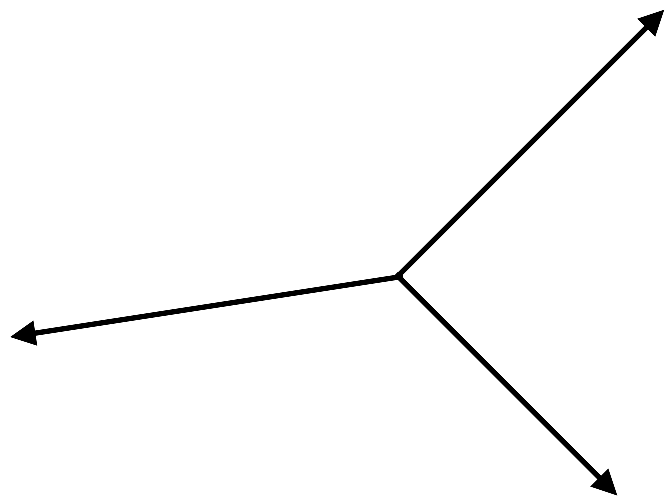
$$\langle D\rangle = \frac{9}{2}\int d\tau_p \; \tau_p^2 \; ETPC\left(\tau_p\right)$$

Average of D parameter

Third moment of ETPC

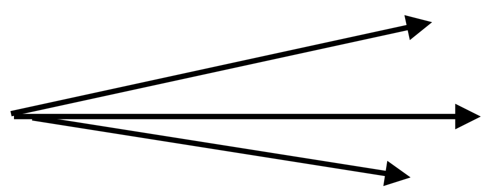
ETPC and Three Jet Configuration

3 jet coplanar limit



$$\tau_p \rightarrow 0$$

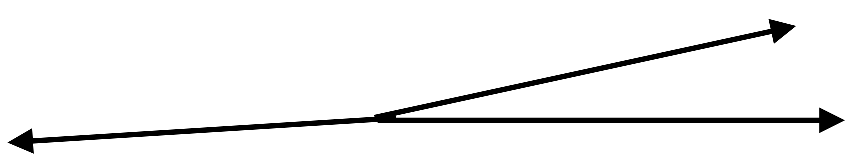
Triple collinear limit



$$\tau_p \rightarrow 0$$

See also Ian Moult's talk

Dijet double collinear limit

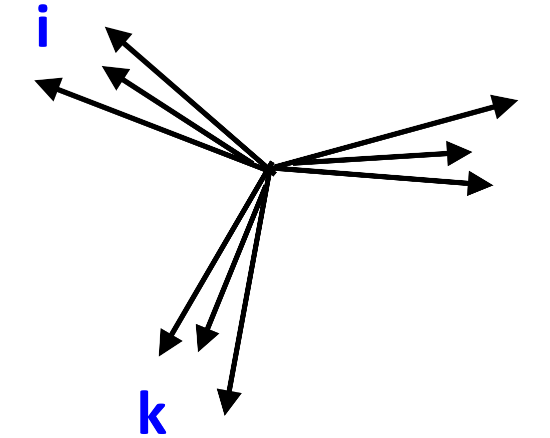


$$\tau_p \rightarrow 0$$

We are interested in the 3 jet coplanar limit only in this talk

Phase Space Cut for 3 Jet Configuration

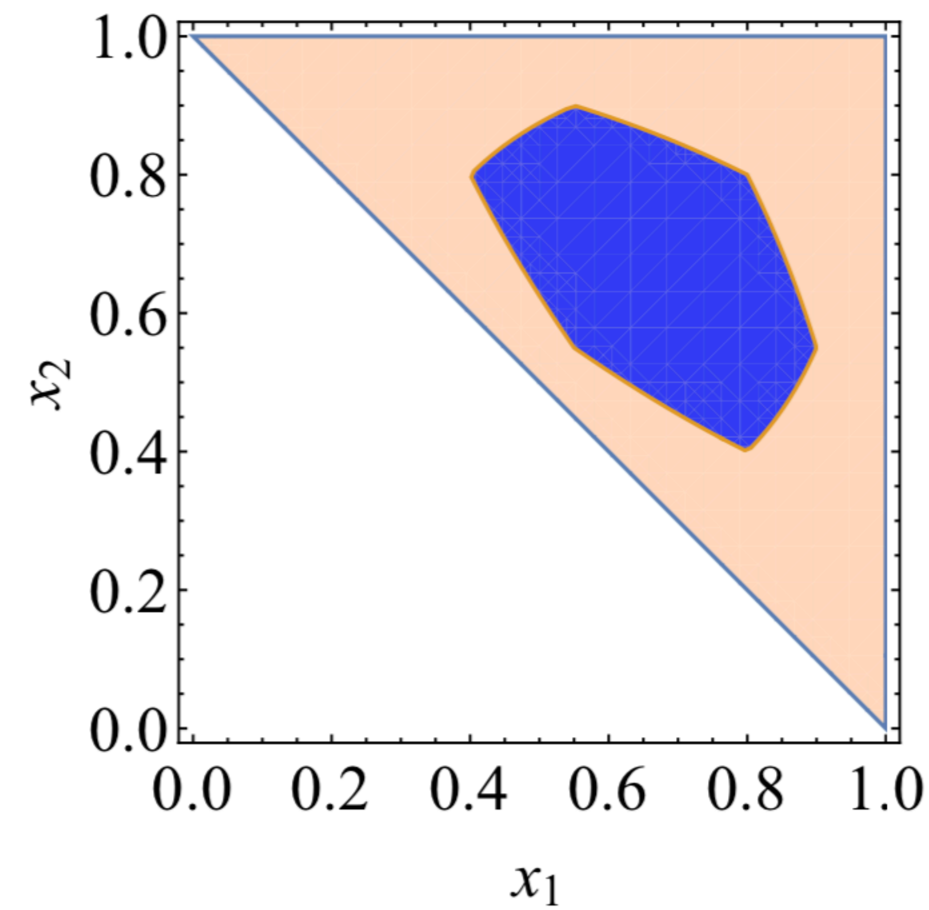
Use the k_T algorithm to find three jets, we keep the event only if $y_3 > y_{\text{cut}}$, and modify the definition of the ETPC to



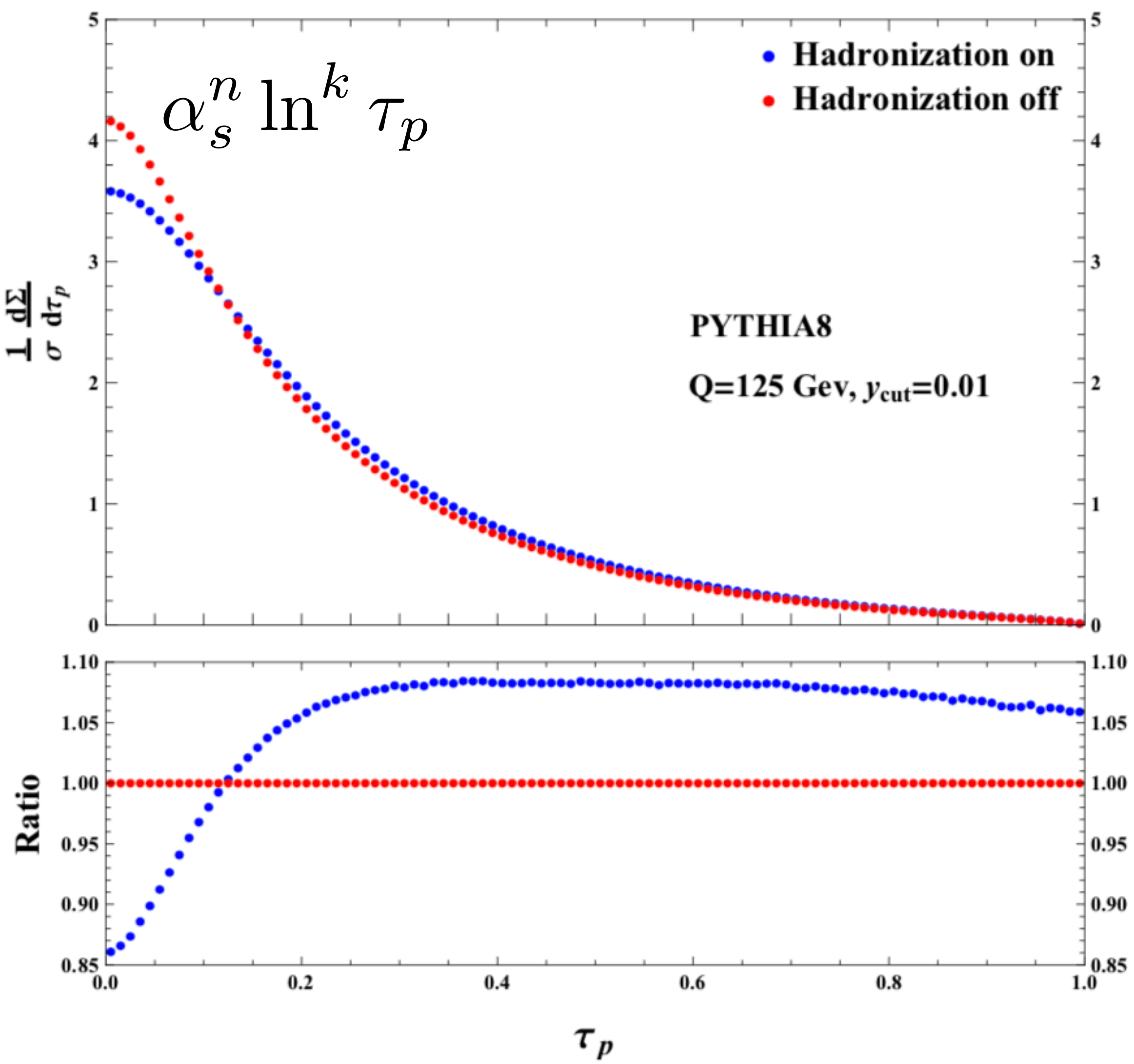
$$\sum_{\substack{i\in J_1\\ j\in J_2\\ k\in J_3}} \int d\sigma \frac{E_i E_j E_k}{E_{J_1} E_{J_2} E_{J_3} \sigma_{\text{tot}}} \delta\left(\tau_p - \tau_{ijk}\right)$$

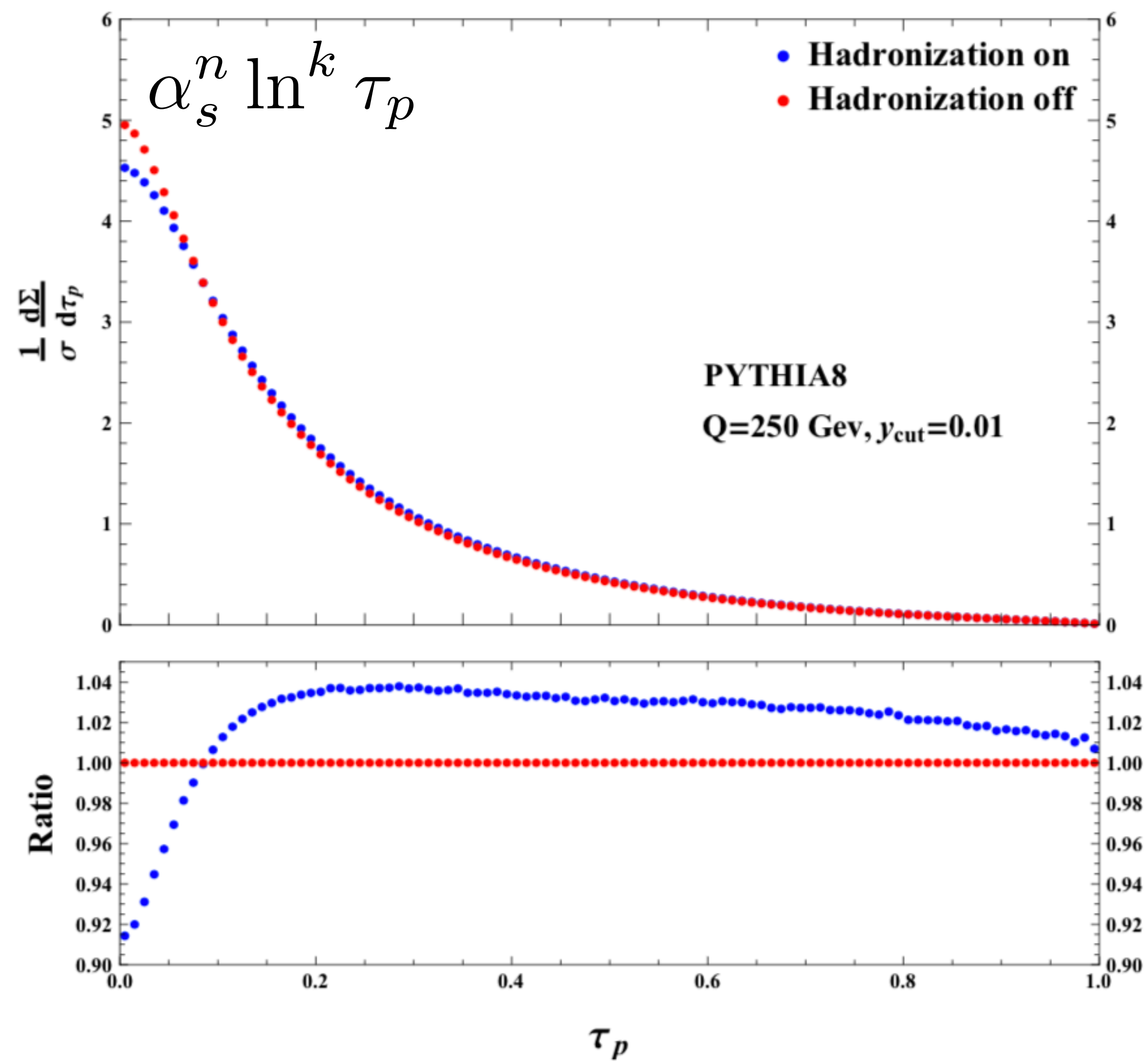
where J_1, J_2, J_3 denote three jets.

The phase space of three particles, with $y_{\text{cut}} = 0.1$
Here $x_i = 2E_i/Q$



Pythia simulation for ETPC

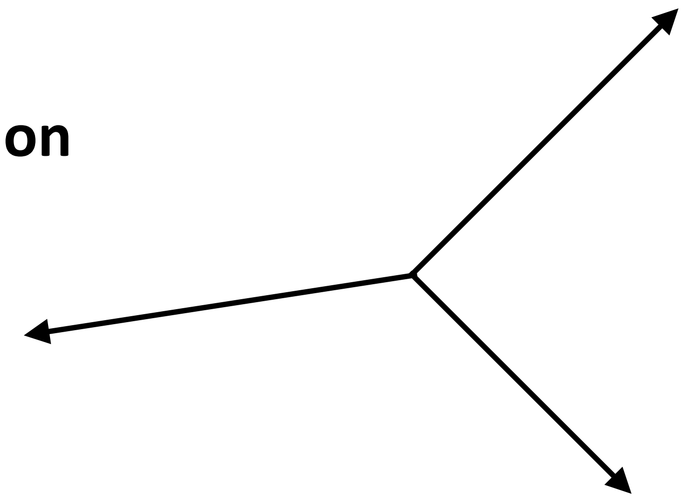




- Substantial reduction of hadronization corrections from Z pole to 250 GeV
- Pythia simulation based on ee->qq configuration. No guarantee to get the normalization and shape right
- A first principle perturbative QCD calculation is needed

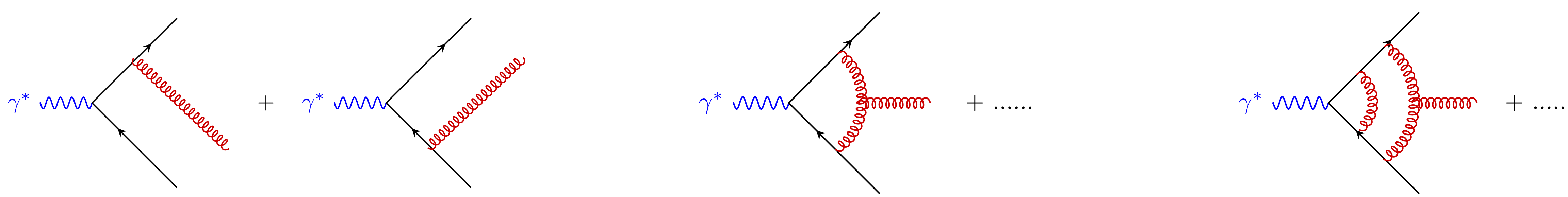
Short Distance Contribution

Born level
3 jet production



$$\tau_p \equiv 0$$

- Short distance corrections from O(Q) hard virtual gluon do not change τ_p

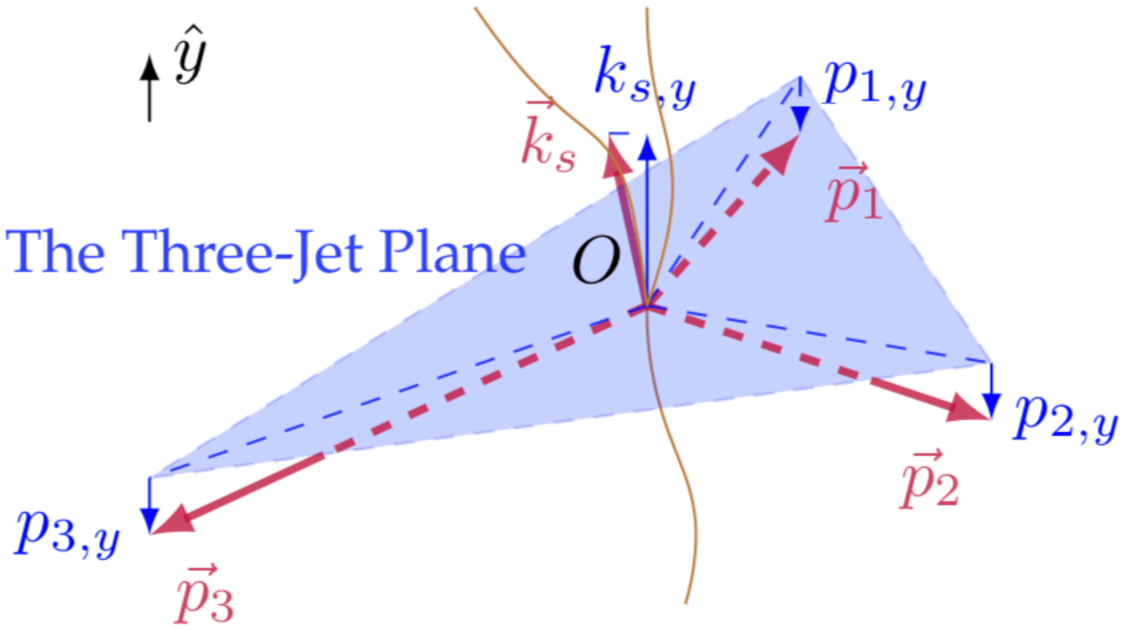


LO **NLO** **NNLO**

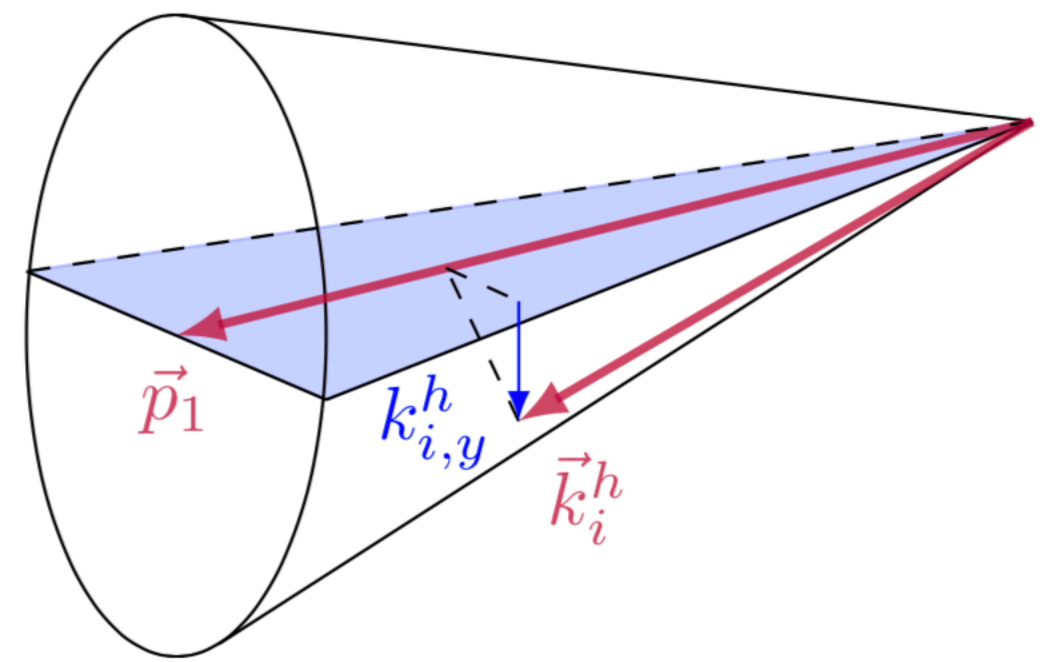
- Higher order corrections are important to get the normalization right

Soft/Collinear Contributions

- The recoil effect of soft radiations.



- Collinear fragmentation



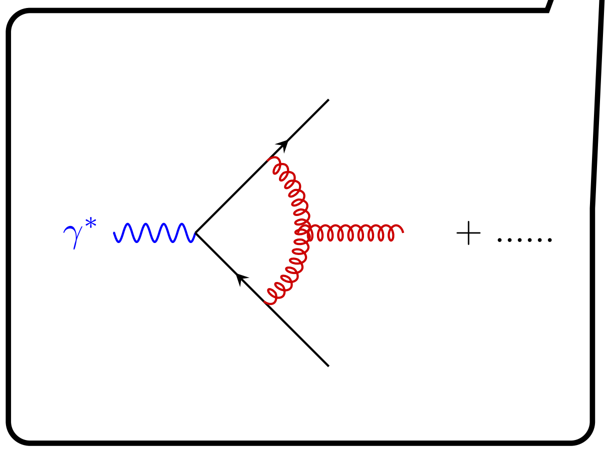
- The soft radiations *don't change the energy* of three jets, only make the three jets deviate the trijet plane slightly in the *opposite* direction.

- The final state hardron carrys longitudinal fraction momentum z_i^h from it's parent jet, i.e. $\vec{k}_i^h = z_i^h \vec{p}_i$.

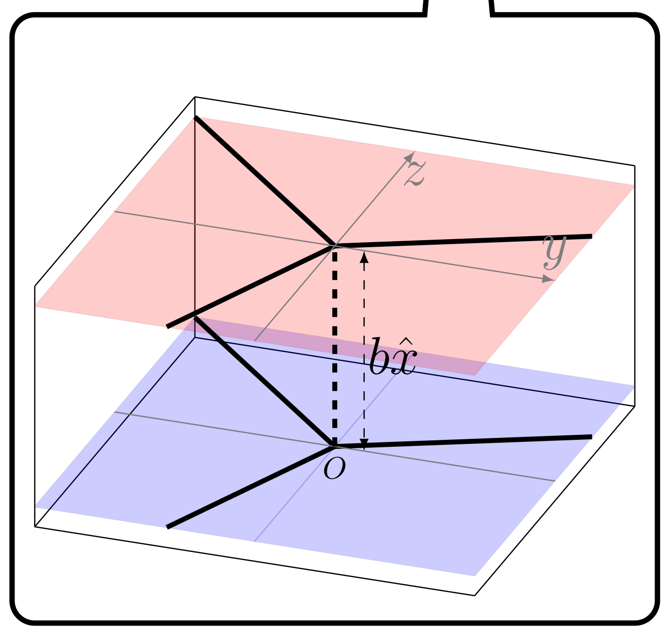
$$k_{s,y} \sim k^h_{i,y} \sim \tau_p Q$$

Factorization in the Coplanar Limit

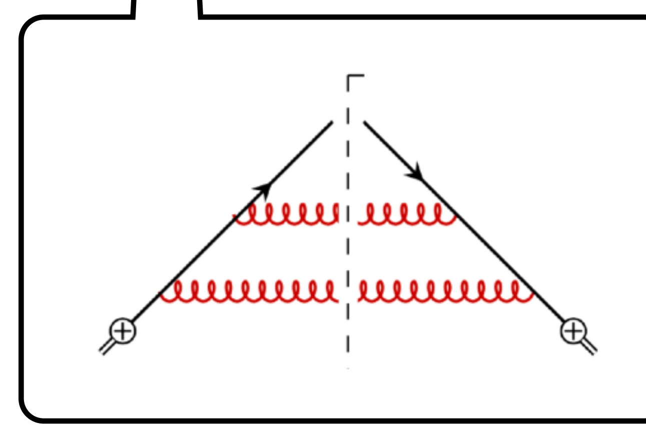
$$\frac{1}{\hat{\sigma}_0}\frac{d\sigma}{d\tau_p} = \int_D dv d\omega H(v,\omega,\mu) \int \frac{db}{2\pi\xi} 2\cos\left(b\tau_p/\xi\right) S(b,\mu,\nu) J_q\left(b,\mu,\nu\right) J_{\bar{q}}\left(b,\mu,\nu\right) J_g\left(b,\mu,\nu\right)$$



Matrix Element of 3 jet production



VEV of semi-infinity Wilson Loop



Second moment of quark/gluon TMD fragmentation function

Probe the TMD gluon fragmentation at CEPC!

Resummation by RGE

RG

$$\frac{dH}{d\ln\mu^2} = \left[\frac{C_A+2C_F}{2}\gamma_{\rm cusp}\left(\alpha_s\right)\ln\frac{Q^2}{\mu^2} + \gamma_H\left(y,z,\alpha_s\right)\right]H\,,$$

$$\frac{d\ln S}{d\ln\mu^2} = \left[\frac{2C_F+C_A}{2}\left(\gamma_{\rm cusp}\left[\alpha_s\right]\ln\frac{\mu^2}{\nu^2} - \gamma_s\left[\alpha_s\right]\right) + \frac{C_A}{2}\gamma_{\rm cusp}\left[\alpha_s\right]\ln\frac{(1-u)^2u}{\upsilon\omega} + C_F\gamma_{\rm cusp}\left[\alpha_s\right]\ln\frac{(1-\upsilon)(1-\omega)}{u}\right]\,,$$

$$\frac{d\ln S}{d\ln\nu^2} = \frac{2C_F+C_A}{2}\left(\int_{\mu^2}^{b_0^2/b^2}\frac{d\overline{\mu}^2}{\overline{\mu}^2}\gamma_{\rm cusp}\left(\alpha_s[\overline{\mu}]\right) + \gamma_r\left(\alpha_s\left[b_0/b\right]\right)\right)\,,$$

rapidity RG

$$\frac{dJ_i}{d\ln\mu^2} = \left(-\frac{1}{2}C_i\gamma_{\rm cusp}\ln\frac{\left(2p_i^0\right)^2}{\nu^2} + \gamma_{J,i}\right)J_i\,,$$

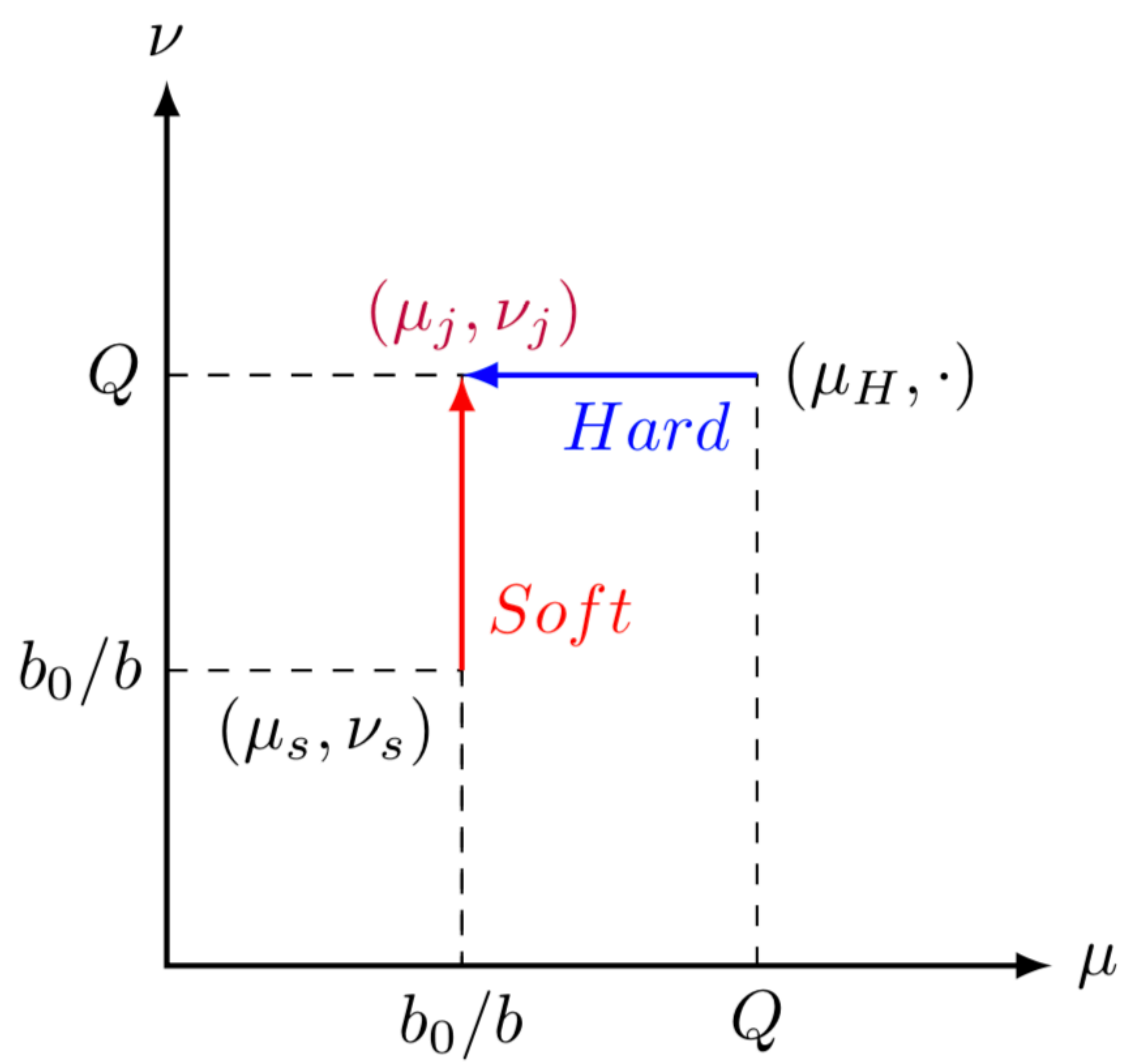
$$\frac{dJ_i}{d\ln\nu^2} = \frac{C_i}{2}\left(\int_{b_0^2/b^2}^{\mu^2}\frac{d\overline{\mu}^2}{\overline{\mu}^2}\gamma_{\rm cusp}\left[\alpha_s(\overline{\mu})\right] - \gamma_r\left[\alpha_s\left(b_0/b\right)\right]\right)J_i\,.$$

- All the anomalous dimensions are known to at least three loops
- RG invariant condition

$$\gamma_H - \frac{C_A+2C_F}{2}\gamma_s - 2\gamma_{J,q} - \gamma_{J,g} = 0$$

RG Evolution in both Virtuality and Rapidity

Setting $\mu = \mu_j = b_0/b$, $\nu = \nu_j = Q$,

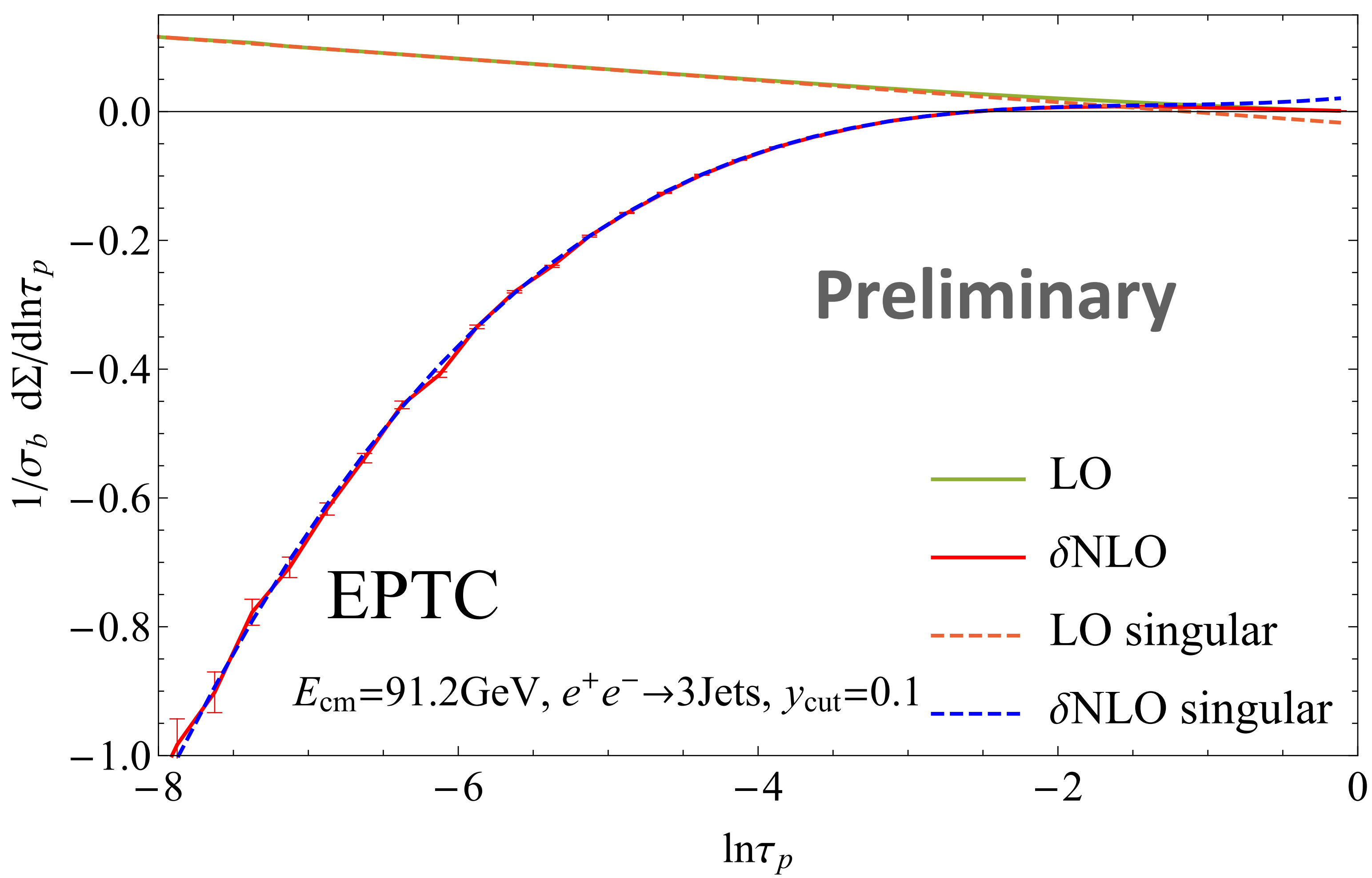


There is rapidity evolution for the soft function

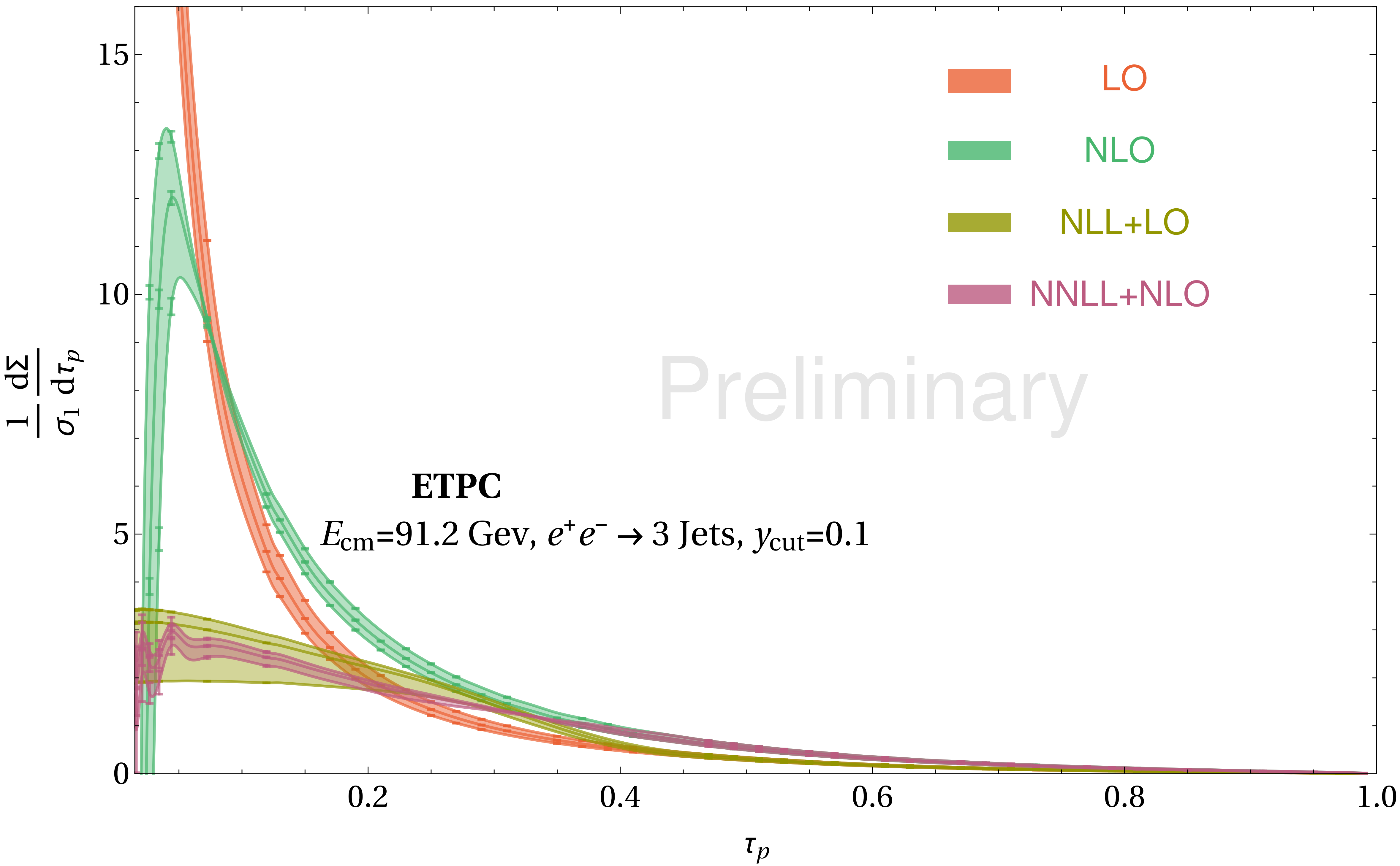
Validation of the Factorization Formula

$$\frac{d\Sigma}{d\ln\tau_p} = \int_D dv d\omega \Big\{ \left(\frac{\alpha_s}{4\pi}\right)^2 (c_1 \ln\tau_p + c_2) + \left(\frac{\alpha_s}{4\pi}\right)^3 (c_3 \ln^3\tau_p + c_4 \ln^2\tau_p + c_5 \ln\tau_p + c_6) + \mathcal{O}(\tau_p^2) \Big\}.$$

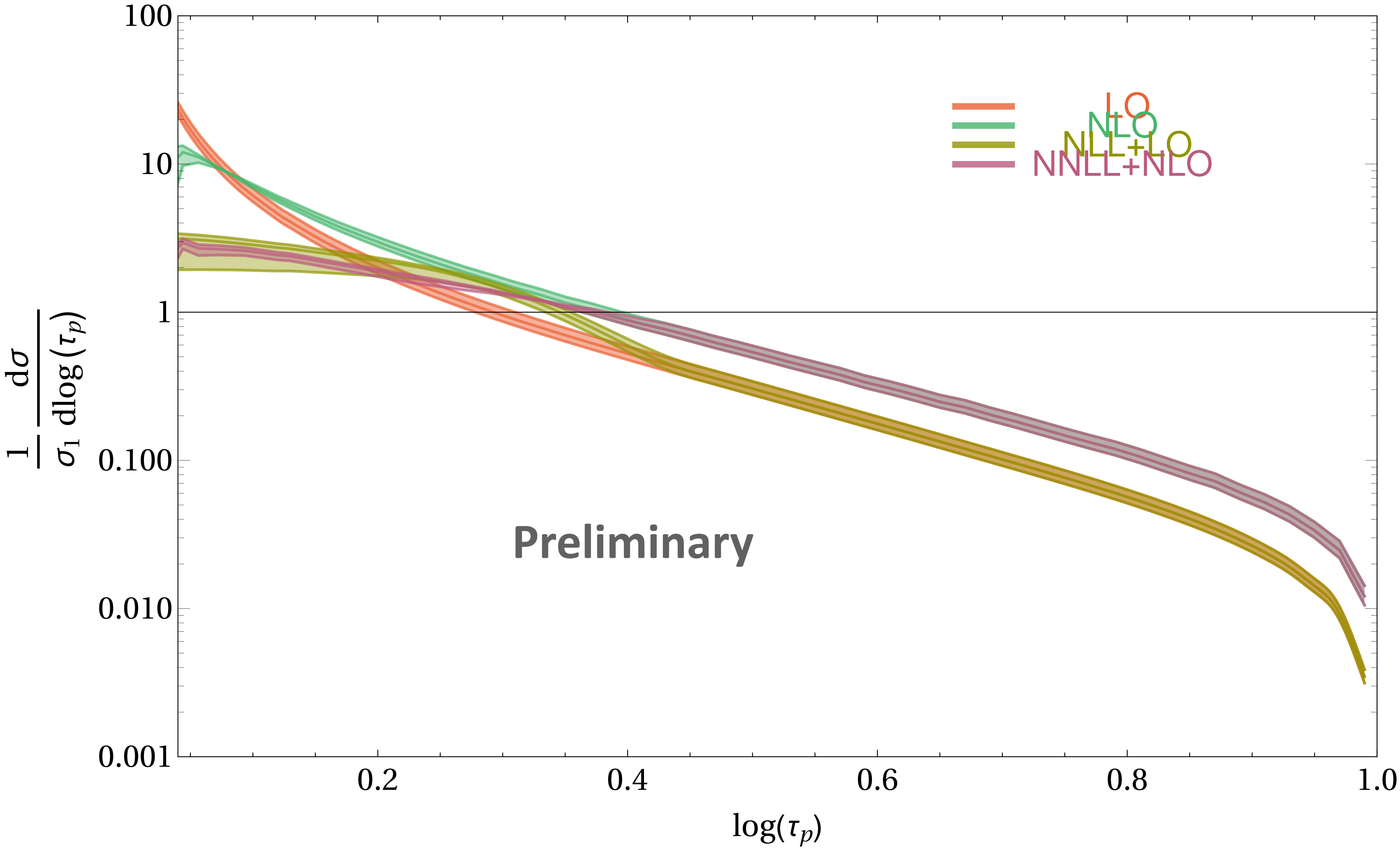
- Expand the factorization formula to NNLO, and compare with e+e- to 4 jet production at NLO (NLOJet++)



Resummation Prediction



Resummation Prediction



Summary

- A new three jet event shape: Energy Triple-Product Correlator
- Singular limit controlled by off-plane soft/collinear radiation
- All order factorization formula in terms well definite operator matrix element
- Application at the CEPC: α_s measurement, TMD fragmentation function, testing parton shower