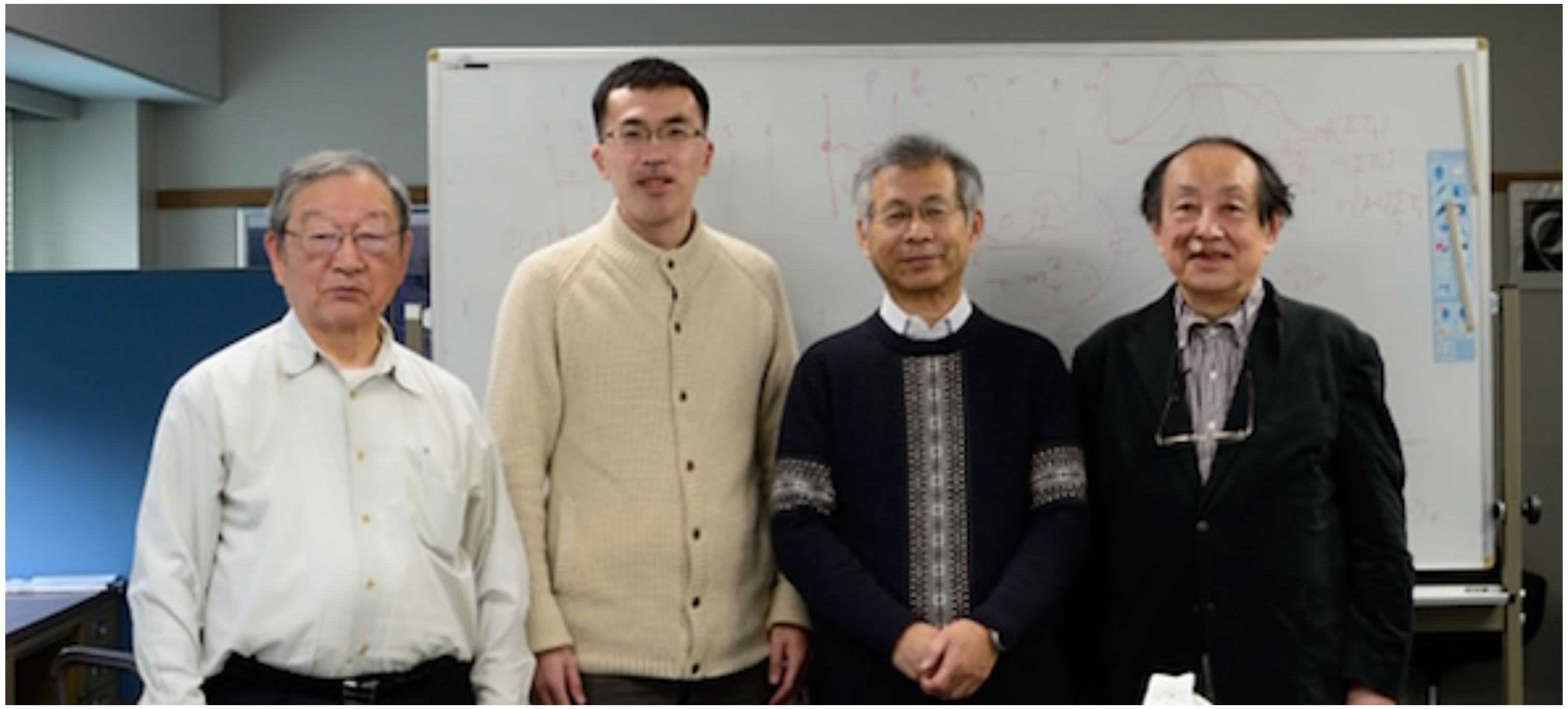
Hadron Nuclear Physics
- dominant role of pion -

Hiroshi Toki (RCNP/Osaka)

with

Takayuki Myo
Kiyomi Ikeda
Hisashi Horiuchi
+
Many Chinese friends
(young)



Three lectures (1hour X 3)

1. **First lecture**
   preliminary(QCD, NJL, Skyrmion)
   what is nucleon and what is pion?
   pion produces nucleus (homework)
   importance of tensor interaction
   Tensor optimized shell model (TOSM)
2. **Second lecture**
   Tensor saturates nuclear matter
   Tensor blocking shell model
3. **Third lecture**
   Tensor optimized antisymmetrized molecular dynamics (TOAMD)
   Delta isobar in nuclei

First lecture

1. preliminary (QCD, NJL, Skyrmion)
   what is nucleon and what is pion
2. pion produces nucleus (homework)
3. importance of tensor interaction
4. Tensor optimized shell model (TOSM) for light nuclei

1. Preliminary

Quantum chromo-dynamics (QCD)

Fundamental theory of nucleon and nucleus

$$L_{\mathrm{QCD}} = \bar{\psi}(i\gamma_\mu(\partial^\mu - igA^\mu) - m)\psi + \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

quark

gluon

Maxwell equation

Dirac equation

$$\psi \to e^{i\theta}\psi$$
$$A_\mu \to A_\mu + \frac{1}{g}\partial_\mu\theta$$

This is all (Gauge theory)!!

All particles are interacting each other.

(I encountered this Lagrangian when I am 30.)

Confinement→Dual Ginzburg-Landau theory

Chiral symmetry→NJL model

Chiral symmetry and its breaking

PHYSICAL REVIEW VOLUME 122, NUMBER 1 APRIL 1, 1961

Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. I*

Y. NAMBU AND G. JONA-LASINIO†
The Enrico Fermi Institute for Nuclear Studies and the Department of Physics, The University of Chicago, Chicago, Illinois

$$L_{\rm NJL} = \bar{\psi}(i\gamma_\mu \partial^\mu - m)\psi + \frac{g}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\psi)^2]$$

Vector transformation $\psi \to \psi' = e^{i\alpha}\psi$

Chiral transformation $\psi \to \psi' = e^{i\gamma_5\beta}\psi$

If m is zero, chiral invariant!!

However, nucleon has mass and quark should have mass!

Chiral symmetry is broken

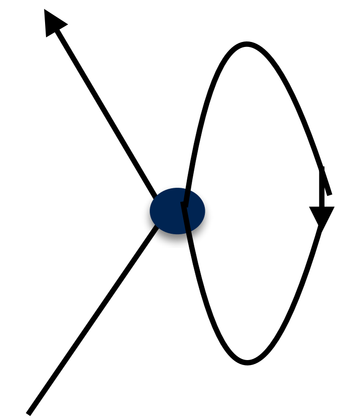
$$E = \sqrt{(e-\lambda)^2 + \Delta^2}$$

$$E = \sqrt{p^2 + M^2}$$

Mean field approximation

$$< \bar{\psi}\psi > = \sigma \neq 0$$

$$M = m - g\sigma$$

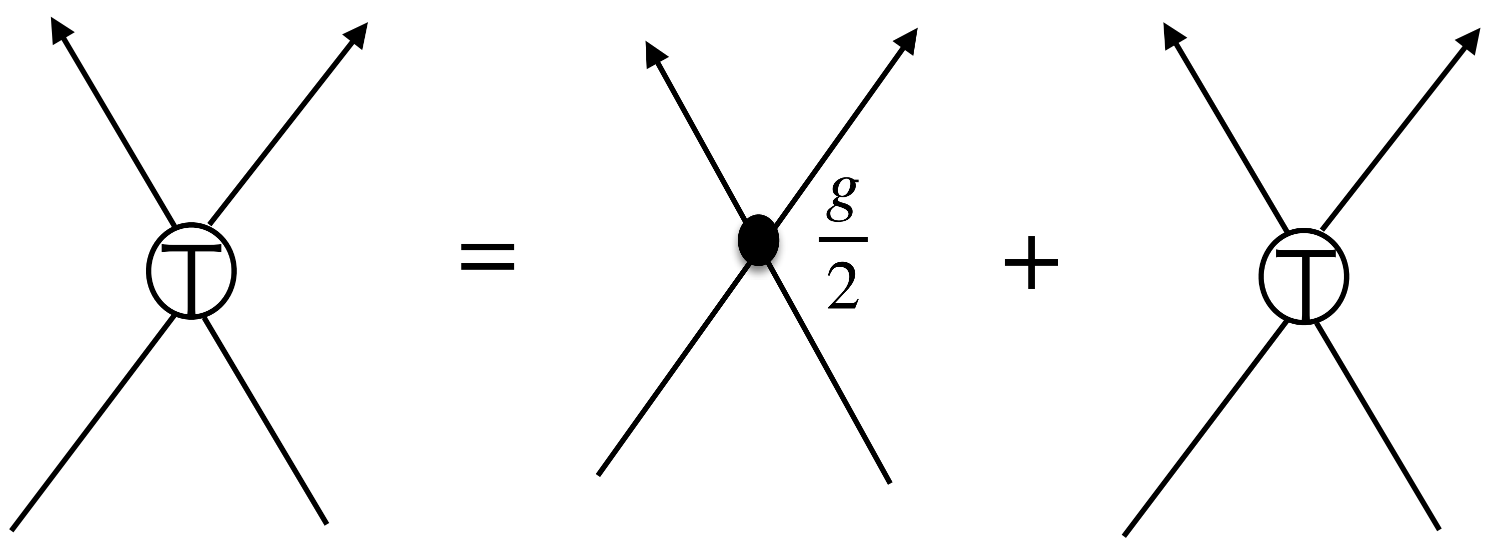


Quarks have mass and nucleon mass is $M_{\mathrm{N}} \sim 3M$

Bethe-Salpeter equation (RPA)

Amazing is pion mass becomes close to zero

Nambu-Goldstone boson



Nucleon has mass and interact with small-mass pion.

Hedgehog state: delta is a brother of nucleon

Skyrmion

Hedgehog

$\overrightarrow{\phi} \parallel \vec{r}$

$(1/2^+, 1/2)$ N^*(1440MeV)Roper

$(3/2^+, 3/2)$ Δ(1230MeV)Delta

$(1/2^+, 1/2)$ N (939MeV)

Pion connects N and Δ strongly!

Nuclear Physics

Ingredient

Nucleon(composite of quarks and pions)

Delta(partner of nucleon)

Pion(light meson)

$$H = T + V$$

⇩

$$H = T + V + U(\text{three body})$$

Hadron Nuclear Physics

$$H = (T + V)_N + (T + V)_\Delta + V_{N\Delta}$$

Future is in your hand!!

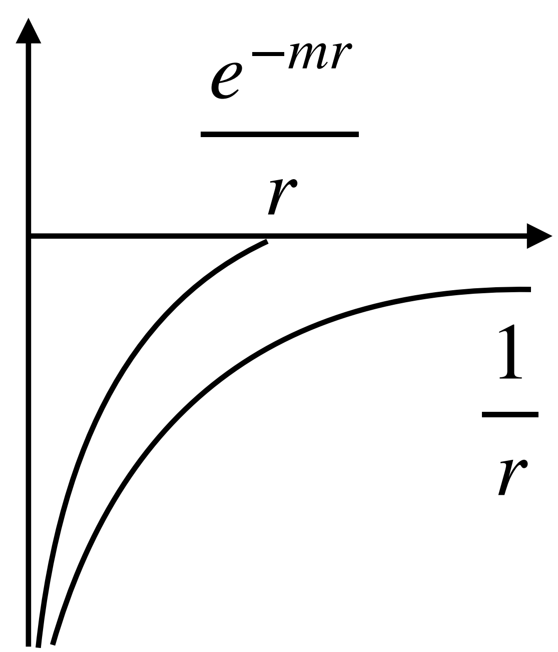
Pion produces nucleus

Yukawa theory (1934)

$$V_C(m=0:r) \rightarrow V_C(m:r)$$

$$V_C(m:r) = -\frac{g^2}{4\pi}\frac{e^{-mr}}{r}$$

$$H = -\frac{\hbar^2}{2m}\nabla_1^2 - \frac{\hbar^2}{2m}\nabla_2^2 + V_C(m:r)$$
$$= -\frac{\hbar^2}{m}\nabla_R^2 - \frac{\hbar^2}{4m}\nabla_r^2 + V_C(m:r)$$



$$H\psi = E\psi$$

$$E = -2.2 \text{ MeV}$$

$$m = 140 \text{ MeV}$$

Homework

Solve the Schroedinger equation for deuteron using the Yukawa potential.

$$H = -\frac{\hbar^2}{2\mu}\nabla^2 - V_0 e^{-ar^2}$$

$$\mu = m/2$$

$$H\psi = E\psi$$

$$\psi(r) = N e^{-\frac{1}{2}br^2}$$

$$\hbar c = 197\ MeV\ fm$$

$$mc^2 = 940\ MeV$$

$$E = -2.2\ MeV$$

Use the gauss integral

$$\int d^3 r e^{-br^2} = \left(\frac{\pi}{b}\right)^{3/2}$$

$$\int d^3 r e^{-\frac{1}{2}br^2}\nabla^2 e^{-\frac{1}{2}br^2} = -\frac{3}{2}b\left(\frac{\pi}{b}\right)^{3/2}$$

$$E = < H > = \frac{(197)^2}{940}\frac{3}{2}b - V_0\left(\frac{b}{a+b}\right)^{3/2}$$

$$\frac{dE}{db} = 62 - V_0\frac{3}{2}\frac{ab^{0.5}}{(a+b)^{2.5}} = 0$$

Yukawa form

$$\int d^3 r e^{-ar^2} \frac{e^{-mr}}{r} = 4\pi \int_0^\infty r dr e^{-a(r+m/(2a))^2 + m^2/(4a)}$$

$$r + m/(2a) = x \quad r = x - m/(2a)$$

$$= 4\pi \int_{m/(2a)}^\infty dx (x - m/(2a)) e^{-ax^2} e^{m^2/(4a)}$$

If $m = 0$, we can perform integral

$$= 4\pi [e^{-ax^2}(-1)/(2a)]_0^\infty = 2\pi/a$$

If $m \neq 0$, we get error function

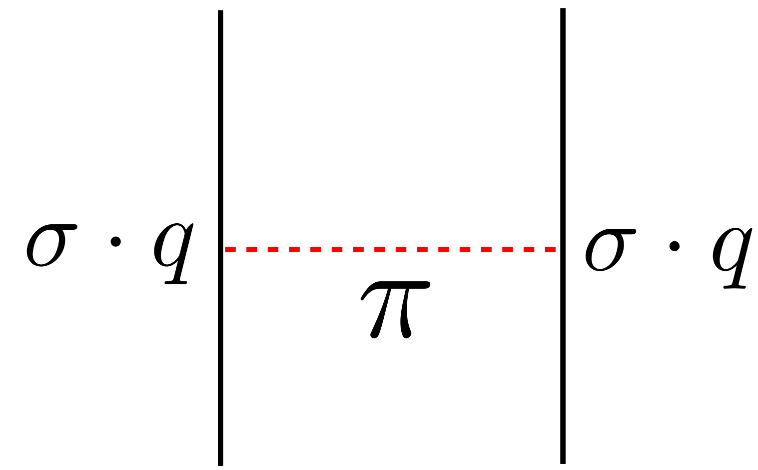
(Python) import math; math.erf(1)

Please work out numbers!!

Pion is a pseudoscalar meson $J^\pi = 0^-$

Pion interaction = tensor interaction

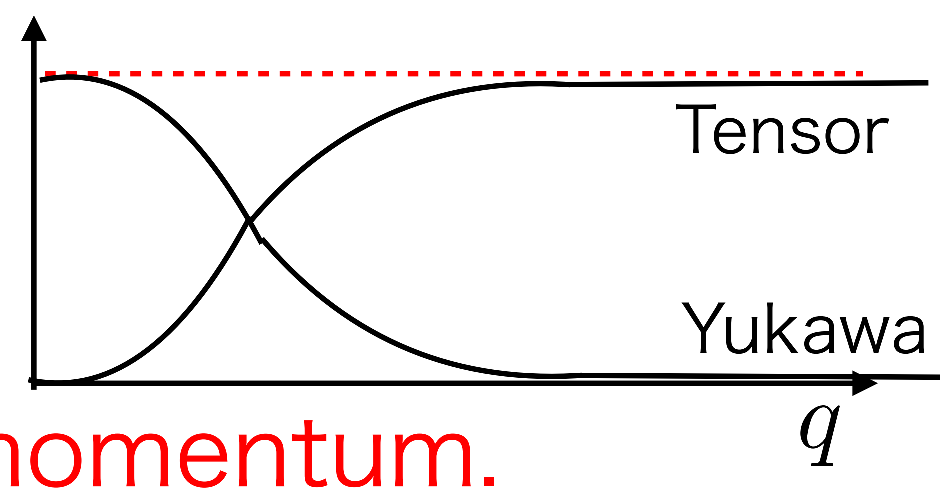
$$V_\pi = \frac{f_\pi^2}{m_\pi^2} \frac{(\sigma_1 \cdot q)(\sigma_2 \cdot q)}{m_\pi^2 + q^2} \tau_1 \tau_2$$



$$\frac{(\sigma_1 \cdot q)(\sigma_2 \cdot q)}{m_\pi^2 + q^2} = \frac{1}{3}\sigma_1\sigma_2 \left[\frac{m_\pi^2 + q^2}{m_\pi^2 + q^2} - \frac{m_\pi^2}{m_\pi^2 + q^2} \right] + \frac{1}{3} S_{12}(q) \frac{q^2}{m_\pi^2 + q^2}$$

δ function

$$S_{12}(q) = [[\sigma_1 \sigma_2]_2 \times Y_2(q)]_0$$



Tensor interaction grows with momentum.

Fourier transformation

$$\int \frac{d^3q}{(2\pi)^3} \frac{1}{m^2+q^2} e^{-i\vec{q}\vec{r}} = \frac{1}{4\pi} \frac{e^{-mr}}{r}$$

Yukawa interaction

$$\int \frac{d^3q}{(2\pi)^3} \frac{q^2}{m^2+q^2} Y_{2M}(q) e^{-i\vec{q}\vec{r}} = \frac{m^2}{4\pi} \left(1 + \frac{3}{mr} + \frac{3}{(mr)^2}\right) \frac{e^{-mr}}{r} Y_{2M}(r)$$

Tensor interaction

Argonne choice

$$Y(r) = \frac{e^{-mr}}{r}(1 - e^{-cr^2}) \qquad c = 2.1\ \mathrm{fm}^{-2}$$

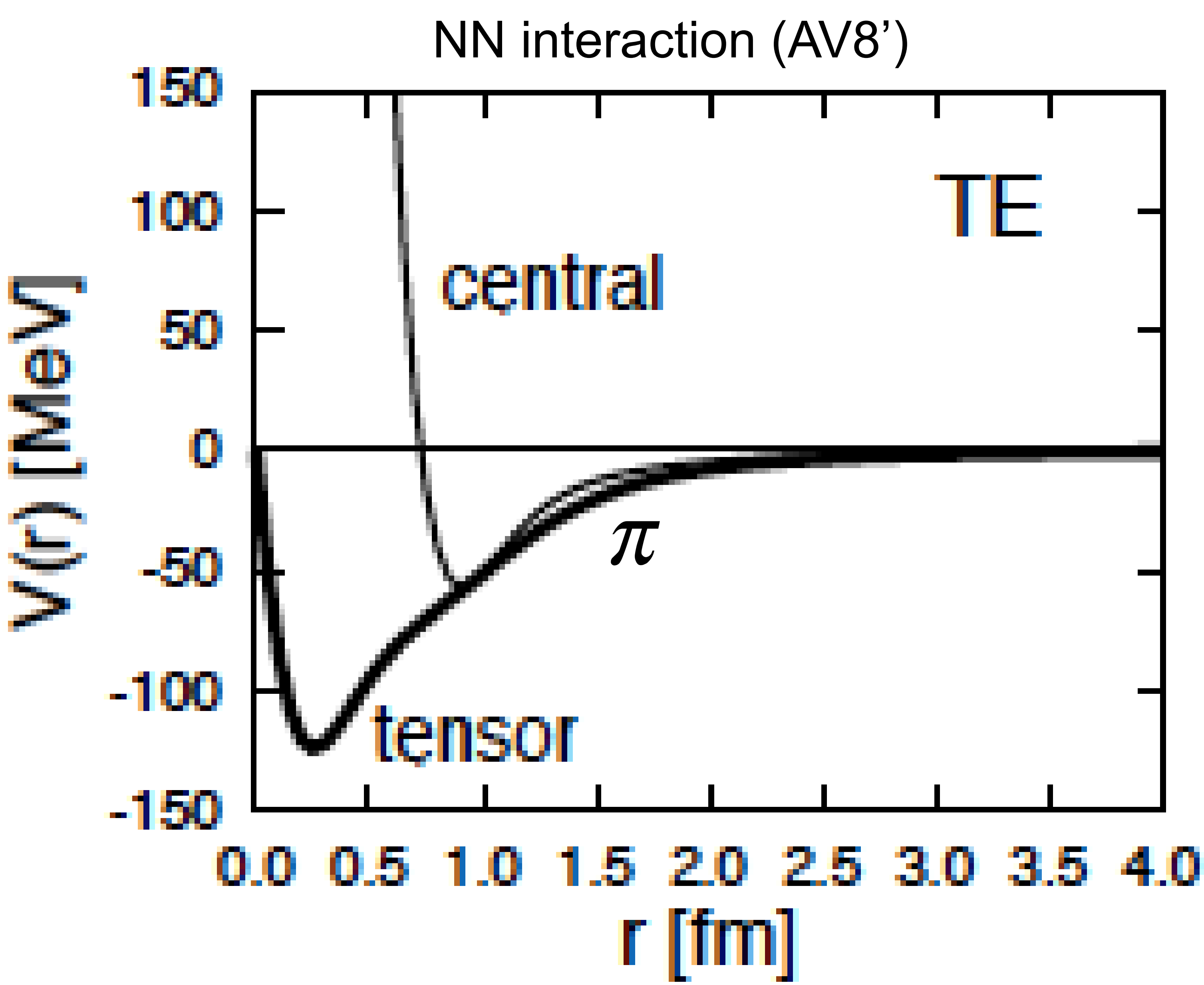
$$T(r) = \left(1 + \frac{3}{mr} + \frac{3}{(mr)^2}\right) \frac{e^{-mr}}{r}(1 - e^{-cr^2})^2$$

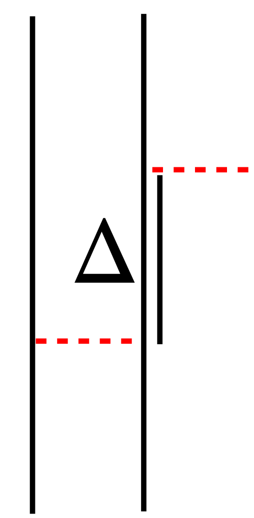
PHYSICAL REVIEW C VOLUME 51, NUMBER 1 JANUARY 1995

Accurate nucleon-nucleon potential with charge-independence breaking

Wiringa, Stoks, Schiavila

$$H = \sum_i \frac{p_i^2}{2M} + \sum_{ij} \left[V_\pi(i,j) + V_S(i,j)\right] + \sum_{ijk} V_\Delta(i,j,k)$$



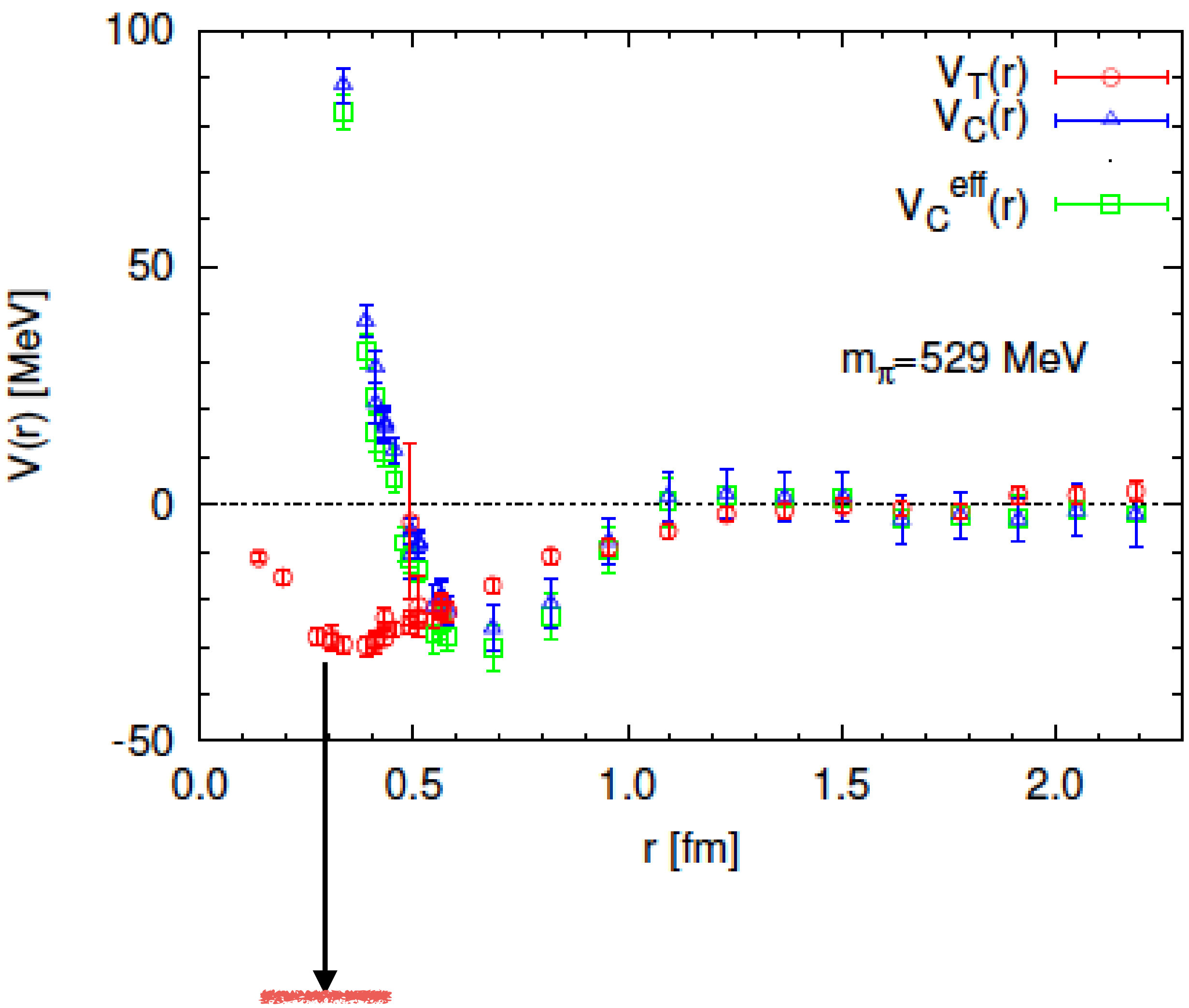


Progress of Theoretical Physics, Vol. 123, No. 1, January 2010

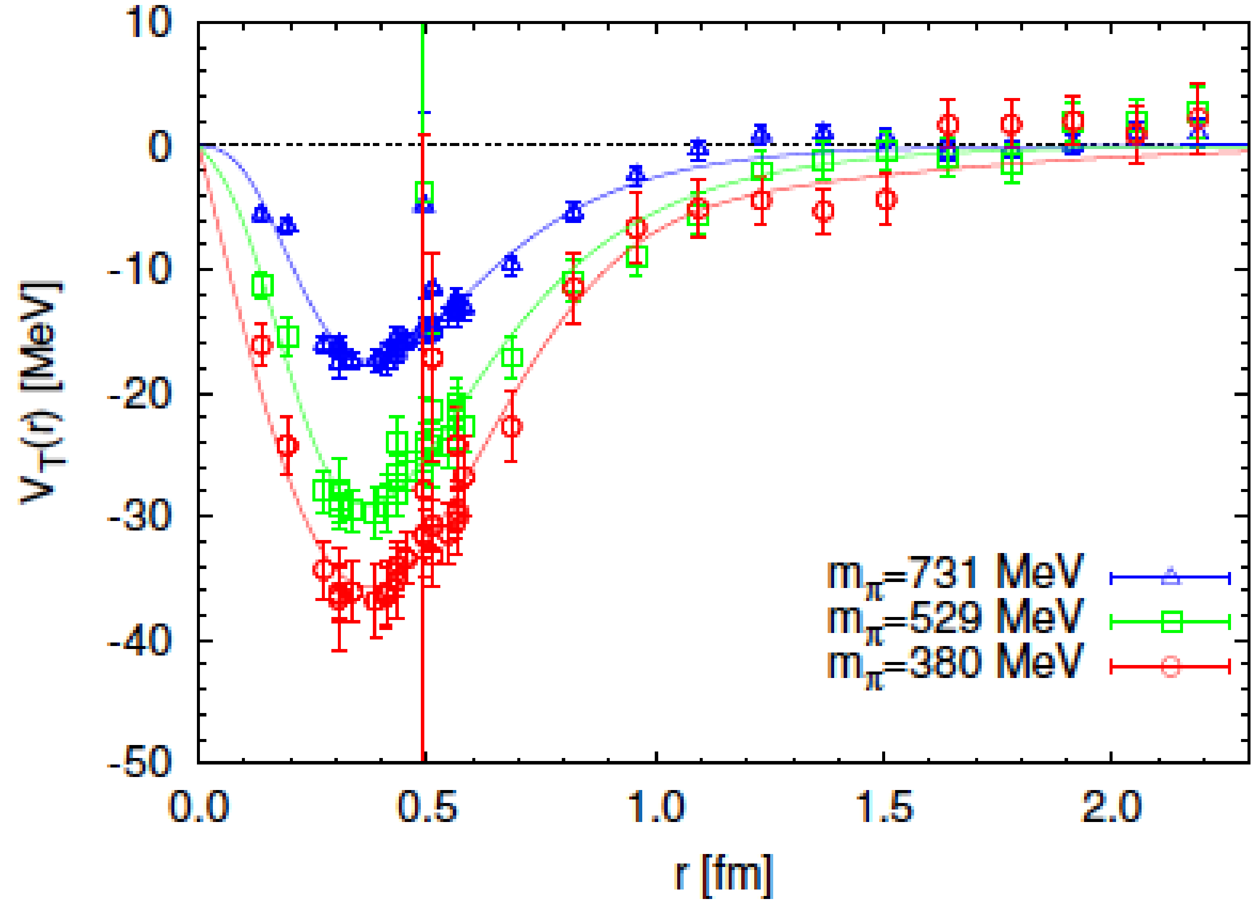
Theoretical Foundation of the Nuclear Force in QCD and Its Applications to Central and Tensor Forces in Quenched Lattice QCD Simulations

Sinya AOKI,[1] Tetsuo HATSUDA[2] and Noriyoshi ISHII[2]

Central+Tensor



Tensor interaction

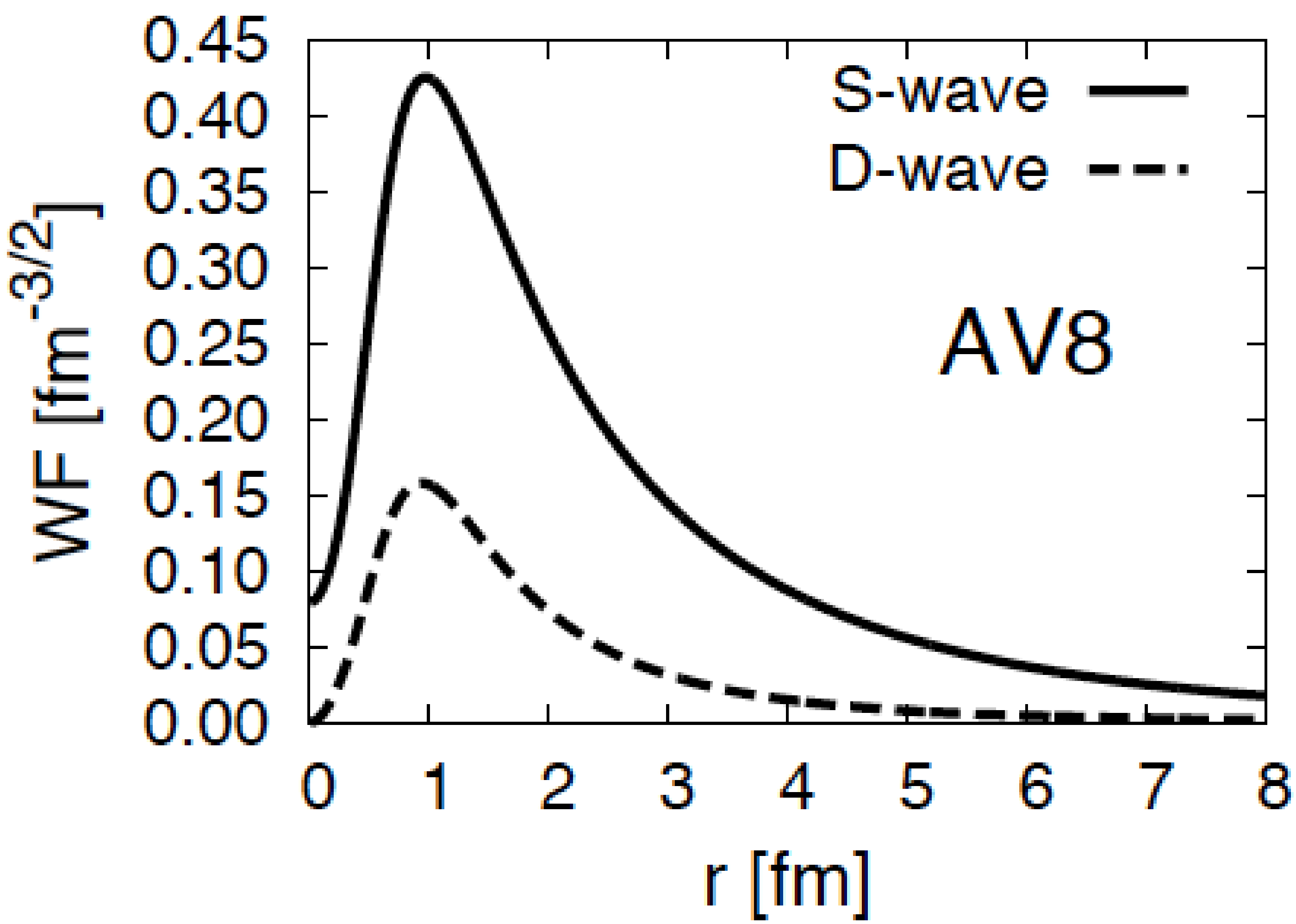


$$m_\pi = 140 MeV$$

Nuclear Physics=Tensor physics

Deuteron (1^+)

S=1 and L=0 or 2



Energy	−2.24 [MeV]
Kinetic	19.88
(SS)	11.31
(DD)	8.57
Central	−4.46
(SS)	−3.96
(DD)	−0.50
Tensorc	−16.64
(SD)	−18.93
(DD)	2.29
LS	−1.02
P(D)	5.78 [%]
Radius	1.96 [fm]
(SS)	2.00 [fm]
(DD)	1.22 [fm]

Tensor interaction is strong!!

P(D) is only 6%

1% provides 3MeV

$$\Psi = |S\rangle + |D\rangle$$

Second lecture

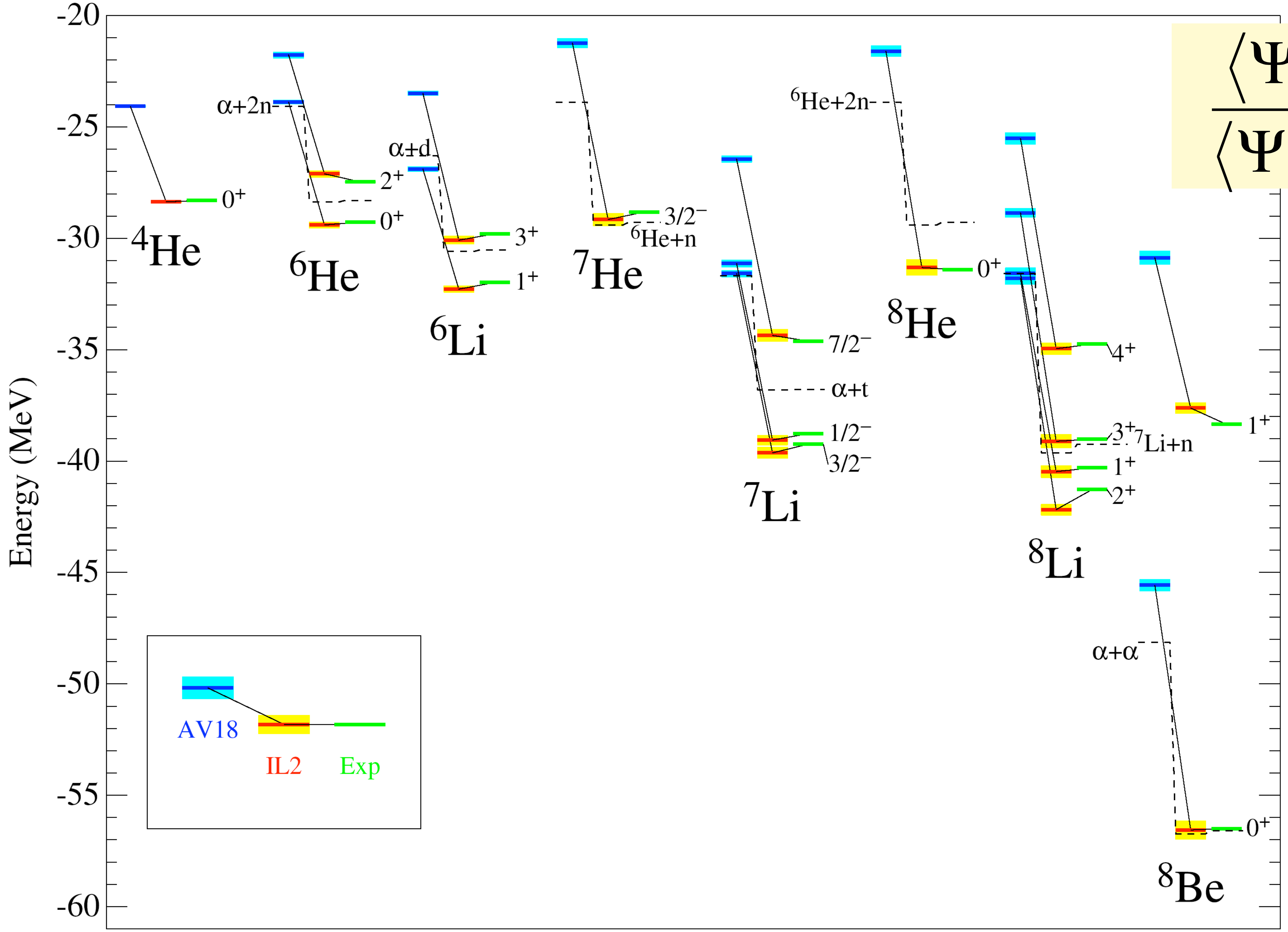
1. Argonne description of nucleus
2. TOSM description of nucleus and nuclear matter

Variational calculation of light nuclei with NN interaction

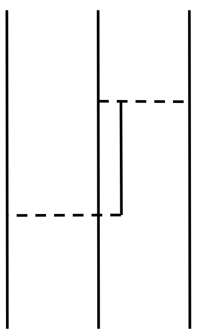
(Argonne group)

VMC+GFMC

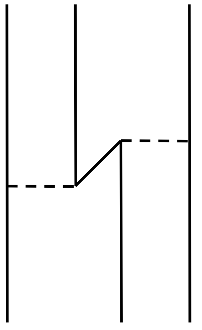
$$\frac{\langle \Psi | V_\pi | \Psi \rangle}{\langle \Psi | V_{NN} | \Psi \rangle} \sim 80\%$$



V_{NNN}



Fujita-Miyazawa



Relativistic

C. Pieper and R. B. Wiringa, Annu. Rev. Nucl. Part. Sci.51(2001)

Heavy nuclei (Super model)

Pion is key

Argonne group

(Jastrow function)

$$\Psi = \Pi_{i<j}(1 + U_{ij})\Phi(\text{shell model})$$
$$= (1 + U_{12})(1 + U_{13})(1 + U_{34})\ldots\Phi(\text{shell model})$$
$$= (1 + \Sigma_{i<j}U_{ij} + \ldots + \Pi_{i<j}U_{ij})\Phi(\text{shell model})$$

2, 3, 4,.. A body operators

difficult

TOSM

Tensor optimized shell model

$$\Psi = C_0|0\rangle + \sum_\alpha C_\alpha|2p2h:\alpha\rangle$$
$$= (1 + \Sigma_{i<j}U_{ij})\Phi(\text{shell model})$$

Two body operator

simpler

$$\Sigma_\alpha|2p-2h:\alpha\rangle\langle 2p-2h:\alpha|\Sigma U_{ij}|0\rangle$$

Tensor optimized shell model
(TOSM)

Myo, Toki, Ikeda, Kato, Sugimoto, PTP 117 (2006)

0p-0h + 2p-2h

$$\Psi = C_0 |0\rangle + \sum_\alpha C_\alpha |2p2h : \alpha\rangle$$

$$\Phi(^4\mathrm{He}) = \Sigma_i \, C_i \, \psi_i(\{b_\alpha\}) \;=\; C_1 \,(0s)^4 + C_2 \,(0s)^2 (\overline{0p}_{1/2})^2 + \cdots$$

$b_{0s} \neq b_{\overline{0p}}$ (size parameter)

Energy variation

$$H = \sum_{i=1}^{A} t_i - T_G + \sum_{i<j}^{A} v_{ij}, \qquad v_{ij} = v_{ij}^{C} + v_{ij}^{T} + v_{ij}^{LS} + v_{ij}^{Clmb},$$

$$\delta \frac{\langle \Phi | \, H \, | \Phi \rangle}{\langle \Phi | \Phi \rangle} = 0 \quad \Rightarrow \quad \frac{\partial \langle H - E \rangle}{\partial b_\alpha} = 0 \,, \quad \frac{\partial \langle H - E \rangle}{\partial C_i} = 0.$$

Unitary Correlation Operator Method

(UCOM)

$$\Psi_{\text{corr.}} = C \cdot \Phi_{\text{uncorr.}} \quad \leftarrow \text{SM, HF, FMD}$$

short-range correlator

$C^{\dagger} = C^{-}$ (Unitary trans.)

$$H\Psi = E\Psi \qquad C^{+}HC\Phi = E\Phi$$

Bare Hamiltonian

Shift operator depending on the relative distance r

$$C = \exp(-i\sum_{i<j} g_{ij}), \quad g = \tfrac{1}{2}\{p_r s(r) + s(r) p_r\} \qquad \vec{p} = \vec{p}_r + \vec{p}_\Omega$$

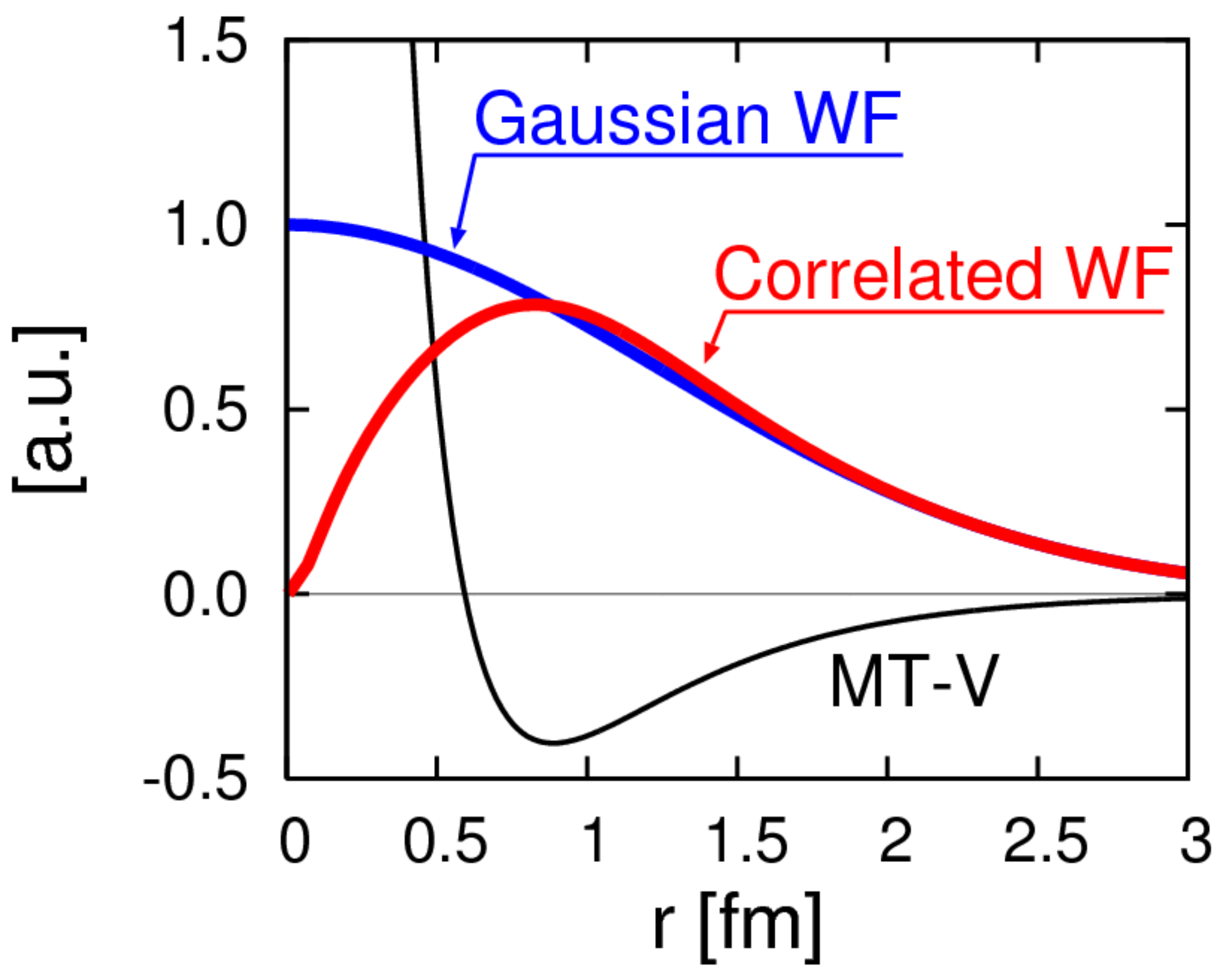
$g = g^{\dagger}$: Hermitian generator

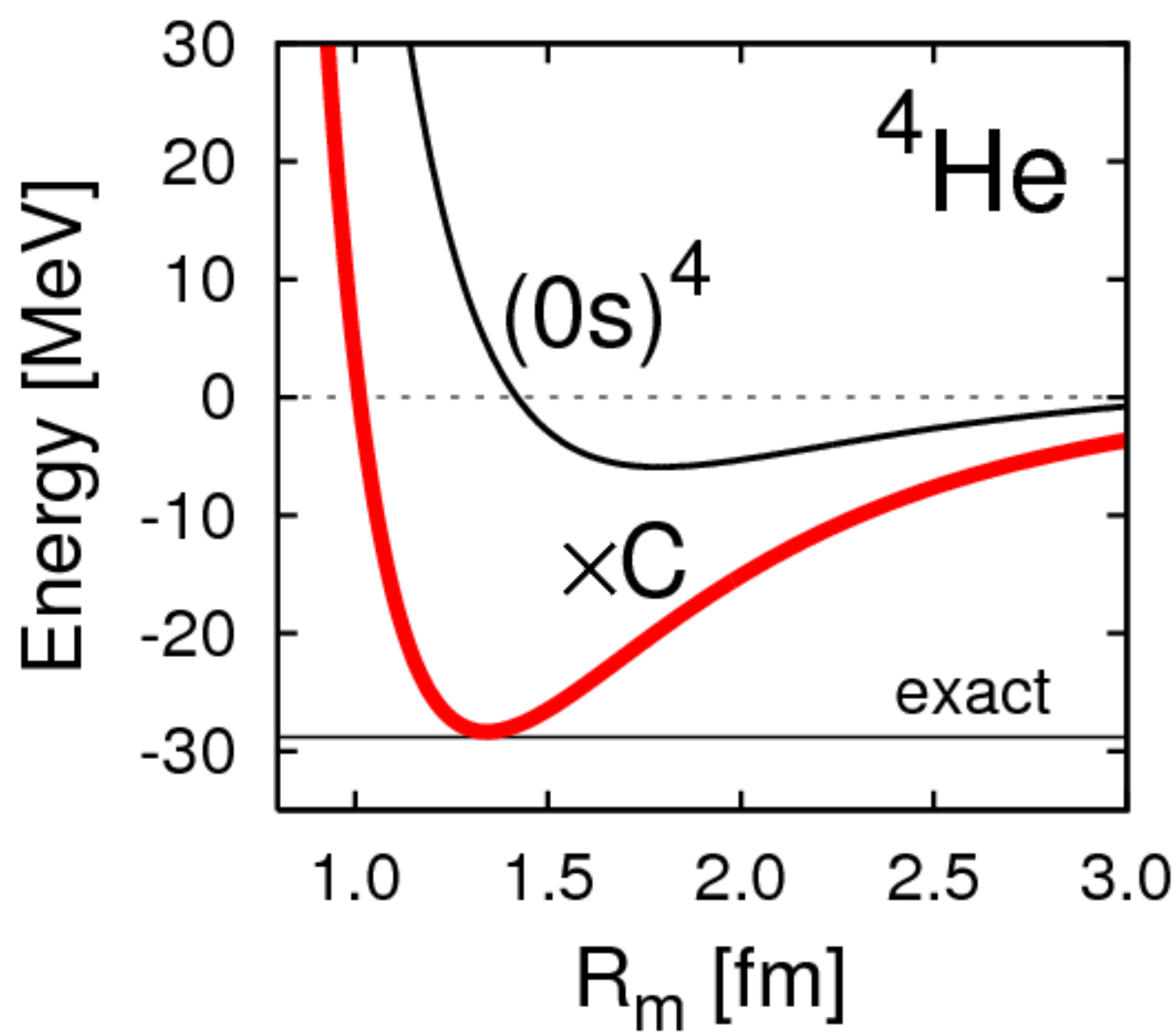
$$C^{\dagger} r\ C = R_{+}(r)$$

H. Feldmeier, T. Neff, R. Roth,
J. Schnack, NPA632(1998)61

^{4}He with UCOM

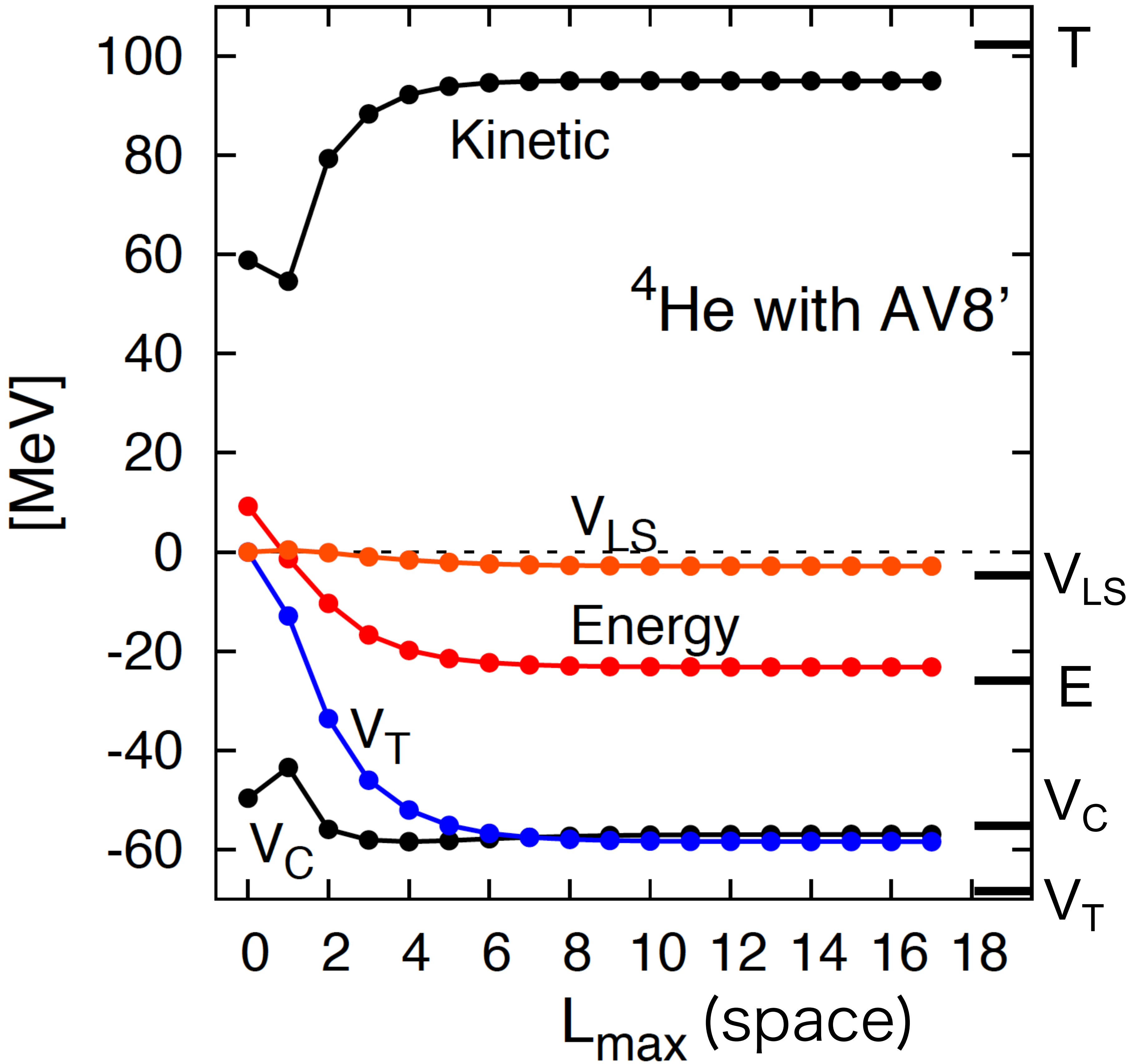
$$^{A}\text{He} \;:\; (0s)^{A} \quad \text{with } b\text{-parameter}$$





TOSM+UCOM with AV8'

$$\Psi = C_0|0\rangle + \sum_{\alpha} C_{\alpha}|2p2h:\alpha\rangle$$



T.Myo H.Toki K.Ikeda
PTP121(2009)511

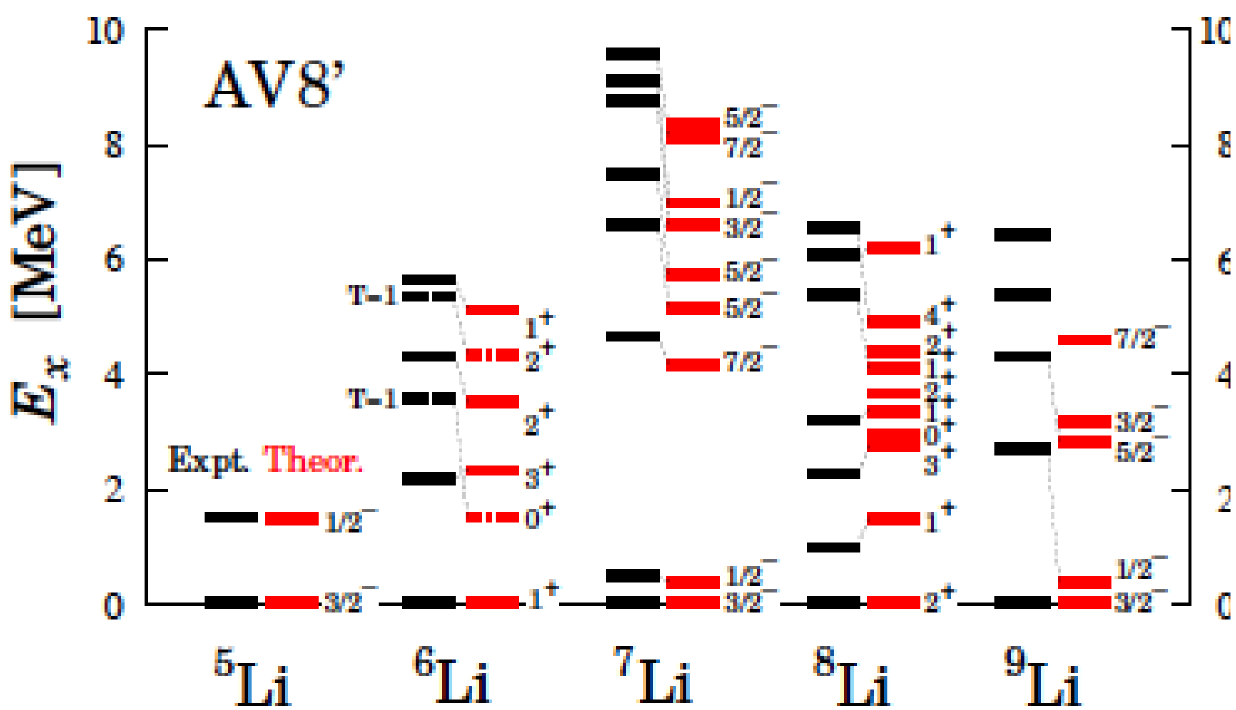
TOSM
Tensor optimized
Shell model

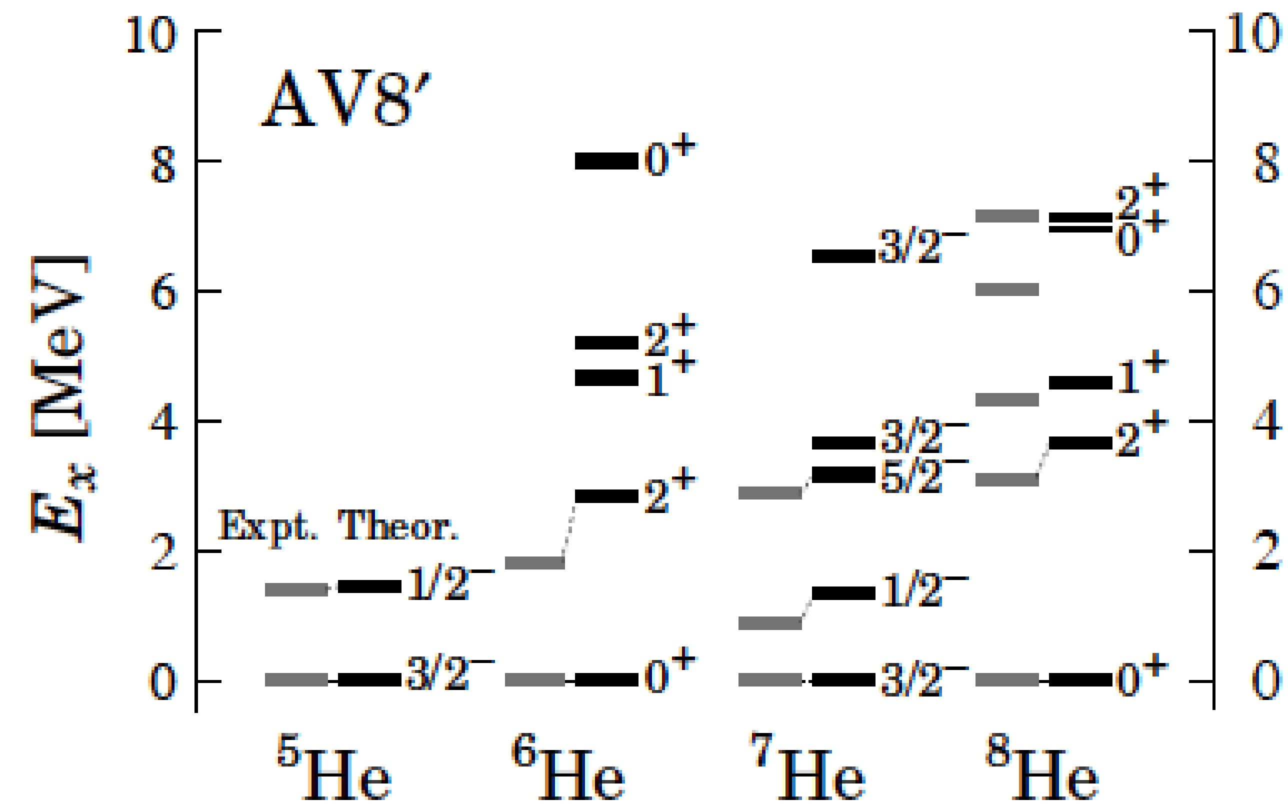
Few body
Calculation
(Kamda et al)

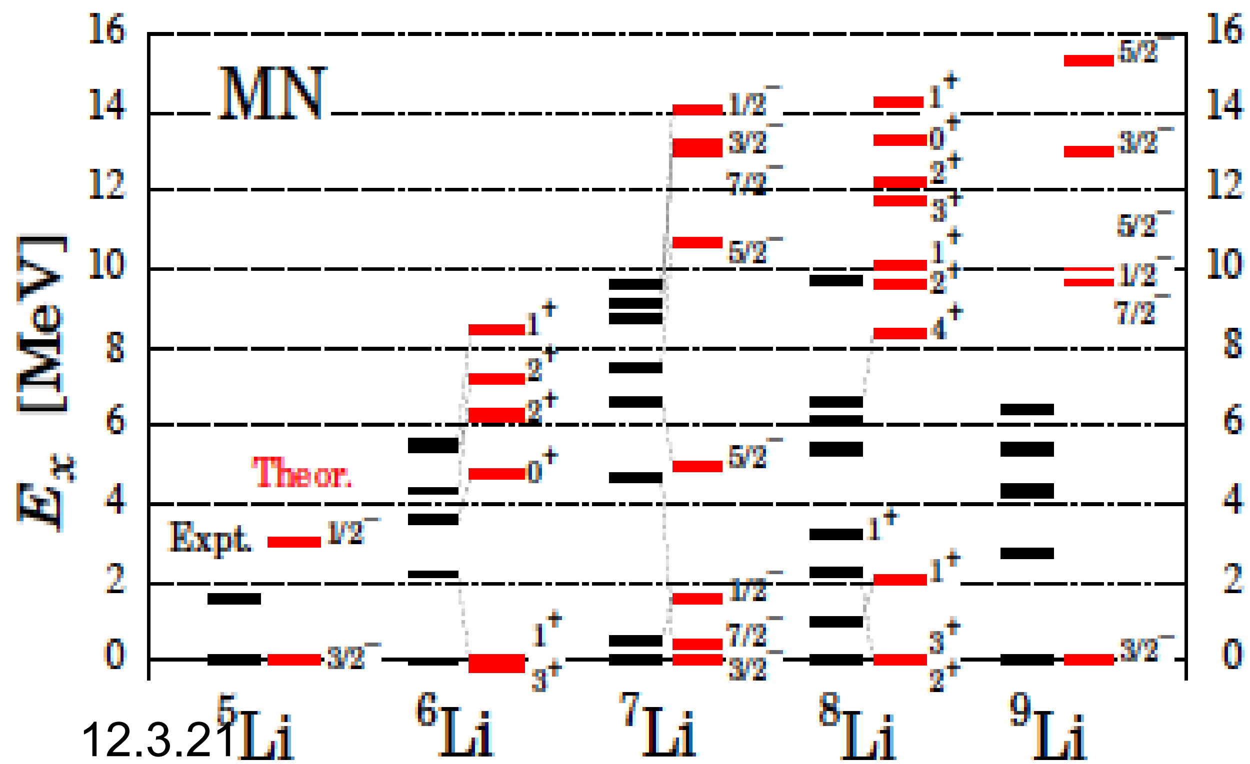
PHYSICAL REVIEW C **84**, 034315 (2011)

Role of the tensor interaction in He isotopes with a tensor-optimized shell model

Takayuki Myo,[1,2,*] Atsushi Umeya,[3,†] Hiroshi Toki,[2,‡] and Kiyomi Ikeda[4,§]







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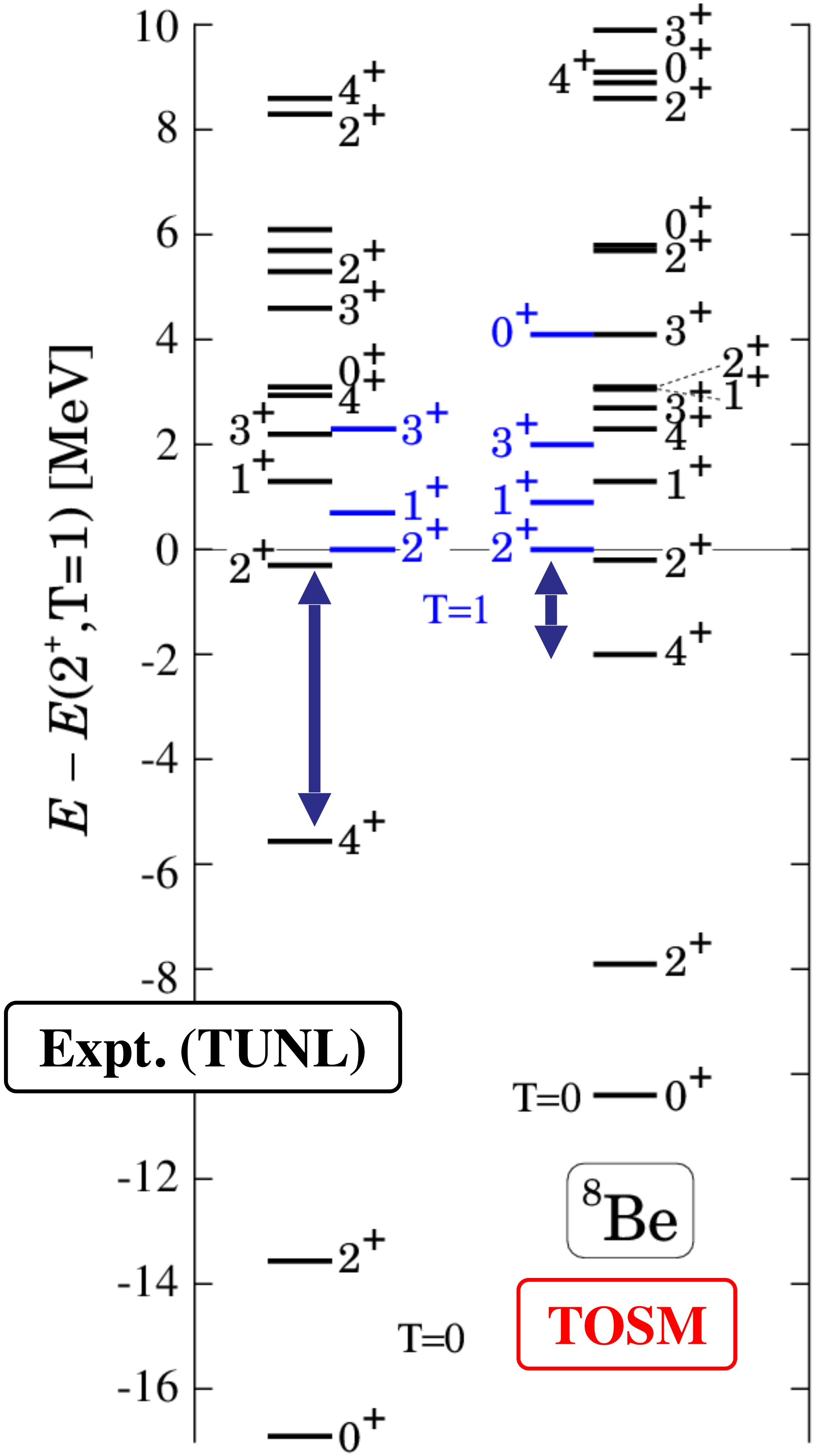
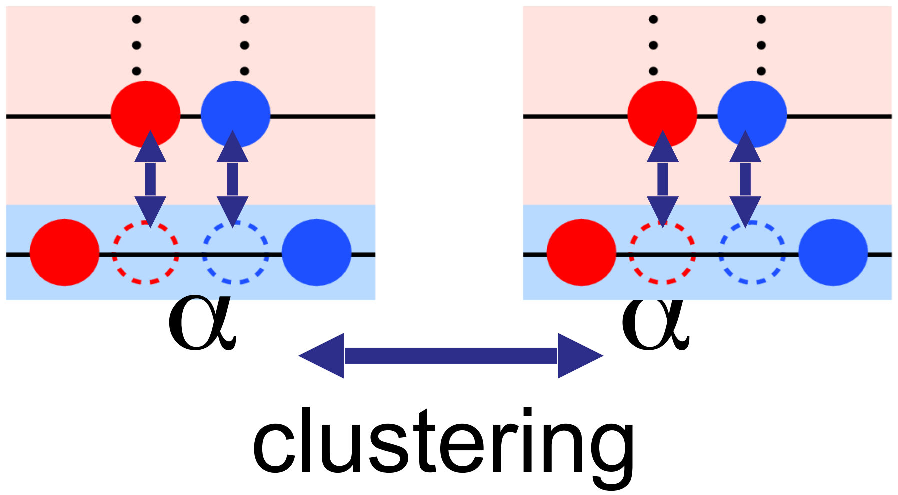


^{8}Be in TOSM – AV8' –

- correct level order (T=0,1)
- tensor contribution : T=0 > T=1
- α : 0p0h+2p2h with high-k
  - 2α needs 4p4h.
  - spatial asymptotic form of 2α



⇒ TOAMD

Extended Brueckner–Hartree–Fock theory with pionic correlation in finite nuclei

Yoko Ogawa *, Hiroshi Toki

Annals of Physics (2011)

$$\langle 0|S_{12}|0\rangle = 0, \qquad S_{12} = \sqrt{\frac{24\pi}{5}}[Y_2(\hat{r}) \times [\sigma_1 \times \sigma_2]_2]^{(0)}.$$

Hartree-Fock theory cannot handle tensor interaction.

$$|\Psi\rangle = C_0|0\rangle + \sum_\alpha C_\alpha |2p-2h:\alpha\rangle$$

$$\delta \frac{\langle\Psi|H|\Psi\rangle}{\langle\Psi|\Psi\rangle} = 0 \qquad \langle\Psi|\Psi\rangle = |C_0|^2 + \sum_\alpha |C_\alpha|^2 = 1$$

Total energy

TOSM Hartree-Fock theory

$$\langle\Psi|H|\Psi\rangle = |C_0|^2\langle 0|H|0\rangle + C_0^*\sum_\alpha C_\alpha\langle 0|H|2p-2h:\alpha\rangle$$

$$+\,C_0\sum_\alpha C_\alpha^*\langle 2p-2h:\alpha|H|0\rangle + \sum_{\alpha\beta} C_\alpha^* C_\beta\langle\alpha|H|\beta\rangle$$

$$|2p-2h:\alpha\rangle \equiv |\alpha\rangle$$

Variational principle

$$\frac{\partial}{\partial C_\alpha^*}\langle\Psi|H-E|\Psi\rangle = 0$$

$$C_0\langle\alpha|H|0\rangle + \sum_\beta C_\beta\langle\alpha|H|\beta\rangle = EC_\alpha$$

$$\frac{\partial}{\partial\psi_a^*(x)}\left[\langle\Psi|H|\Psi\rangle - \sum_b e_b\psi_b^*(x)\psi_b(x)\right] = 0$$

$$|0\rangle = \prod_a \psi_a(x)$$

$$|C_0|^2\frac{\partial}{\partial\psi_a^*}\langle 0|H|0\rangle + C_0^*\sum_\alpha C_\alpha\frac{\partial}{\partial\psi_a^*}\langle 0|H|\alpha\rangle + \sum_{\alpha\beta} C_\alpha^* C_\beta\frac{\partial}{\partial\psi_a^*}\langle\alpha|H|\beta\rangle = e_a\psi_a(x)$$

Extender BHF theory

$$C_\alpha = -C_0 \sum_\beta \frac{1}{\langle \alpha | H | \beta \rangle - E\delta_{\alpha\beta}} \langle \beta | H | 0 \rangle = -C_0 \sum_\beta \langle \alpha | \frac{1}{H-E} | \beta \rangle \langle \beta | H | 0 \rangle$$

$$|C_0|^2 \frac{\partial}{\partial \psi_a^*} \langle 0 | H | 0 \rangle - |C_0|^2 \sum_{\alpha\beta} \frac{\partial}{\partial \psi_a^*} \langle 0 | H | \alpha \rangle \langle \alpha | \frac{1}{H-E} | \beta \rangle \langle \beta | H | 0 \rangle$$

$$+ |C_0|^2 \sum_{\alpha\alpha'\beta\beta'} \langle 0 | H | \alpha' \rangle \langle \alpha' | \frac{1}{H-E} | \alpha \rangle \frac{\partial}{\partial \psi_a^*} \langle \alpha | H | \beta \rangle \langle \beta | \frac{1}{H-E} | \beta' \rangle \langle \beta' | H | 0 \rangle = e_a \psi_a(x)$$

Effective Hamiltonian for HF calculation

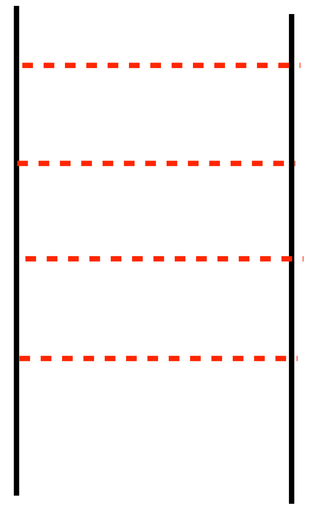
$$H_{eff} = |C_0|^2 H - |C_0|^2 \sum_{\alpha\beta} H | \alpha \rangle \langle \alpha | \frac{1}{H-E} | \beta \rangle \langle \beta | H$$

$$\frac{\partial}{\partial \psi_a^*(x)} \langle 0 | H_{eff} - \sum_b e_b \psi_b^* \psi_b | 0 \rangle = 0$$

Nuclear matter

Brueckner-Hartree-Fock theory

$$G = V + V\frac{Q}{e}G$$
$$= V + V\frac{Q}{e}V + V\frac{Q}{e}V\frac{Q}{e}V + \ldots$$



$$G = V + \Sigma_{\alpha\beta}^{(2p)}\langle 0|V|2p-2h:\alpha\rangle\frac{1}{E-\langle 2p-2h:\alpha|V|2p-2h:\beta\rangle}\langle 2p-2h:\beta|V|0\rangle$$

TOSM

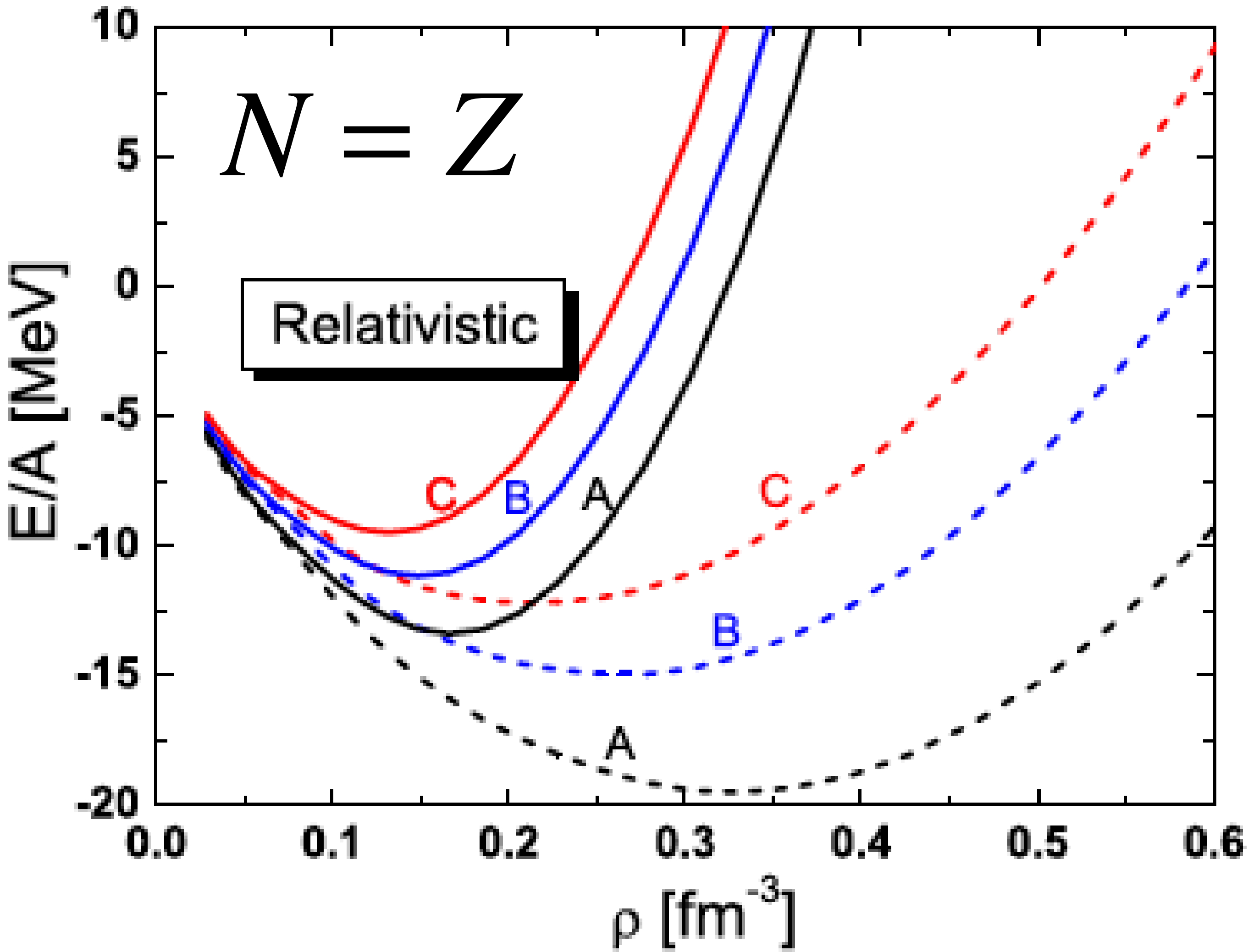
$$\Psi = C_0|0\rangle + \sum_{\alpha} C_\alpha |2p2h:\alpha\rangle$$

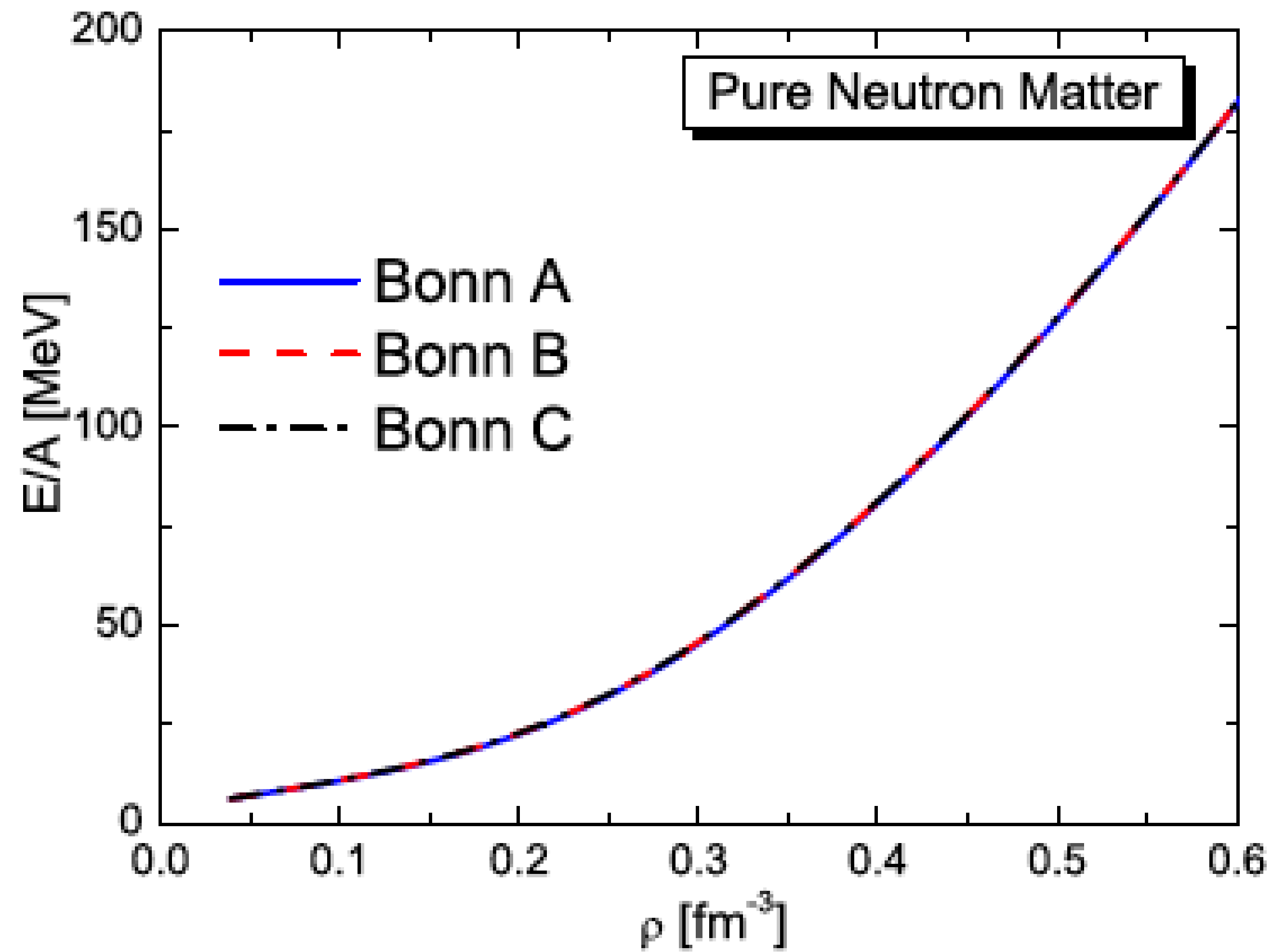
$$V_{\rm eff} = |C_0|^2 V + |C_0|^2 \Sigma_{\alpha\beta}\langle 0|V|2p-2h:\alpha\rangle\frac{1}{E-\langle 2p-2h:\alpha|V|2p-2h:\beta\rangle}\langle 2p-2h:\beta|V|0\rangle$$

Nuclear matter

Prog. Theor. Exp. Phys. **2013**, 103D02 (17 pages)

J. Hu, Toki and Y. Ogawa

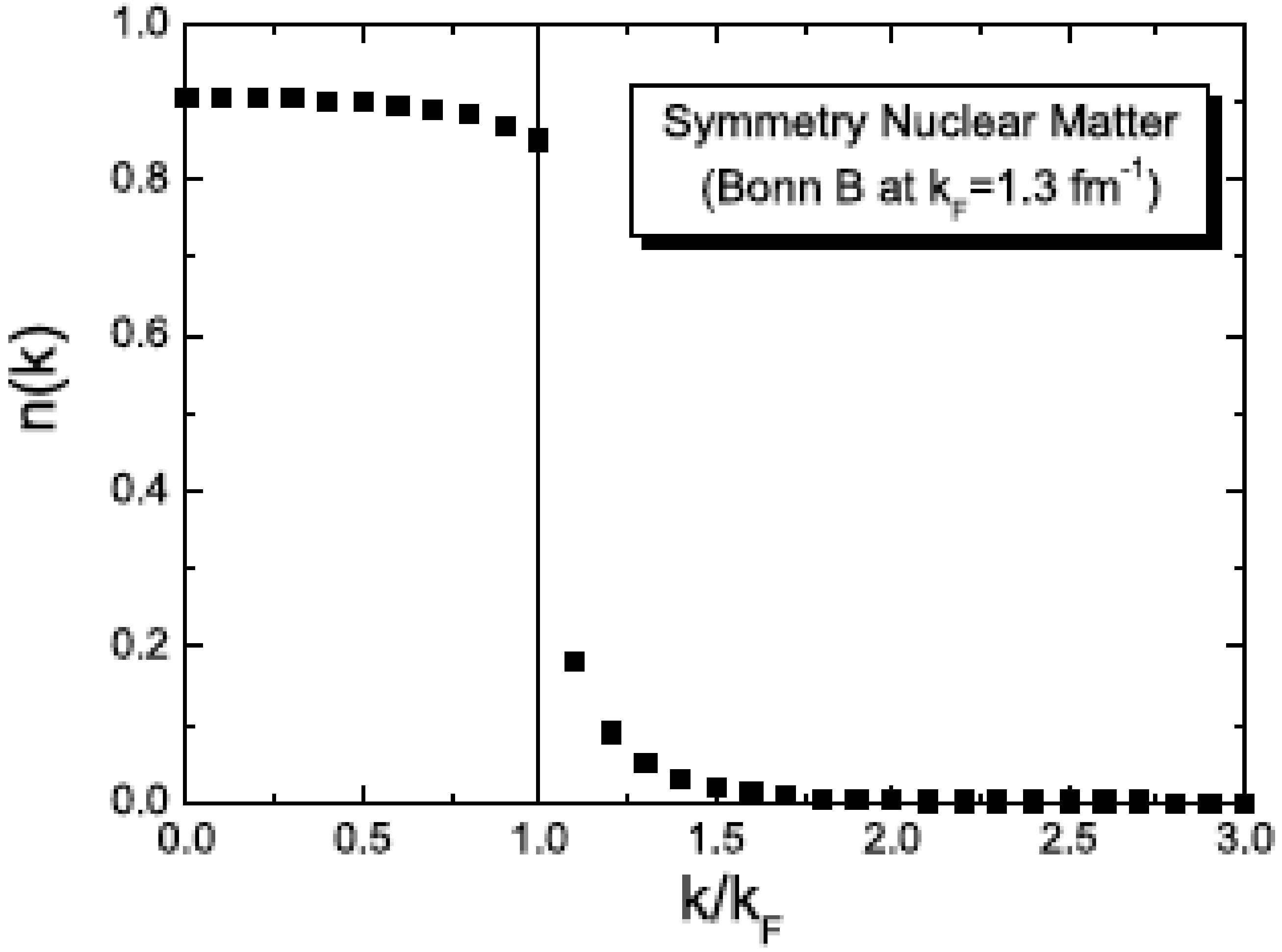


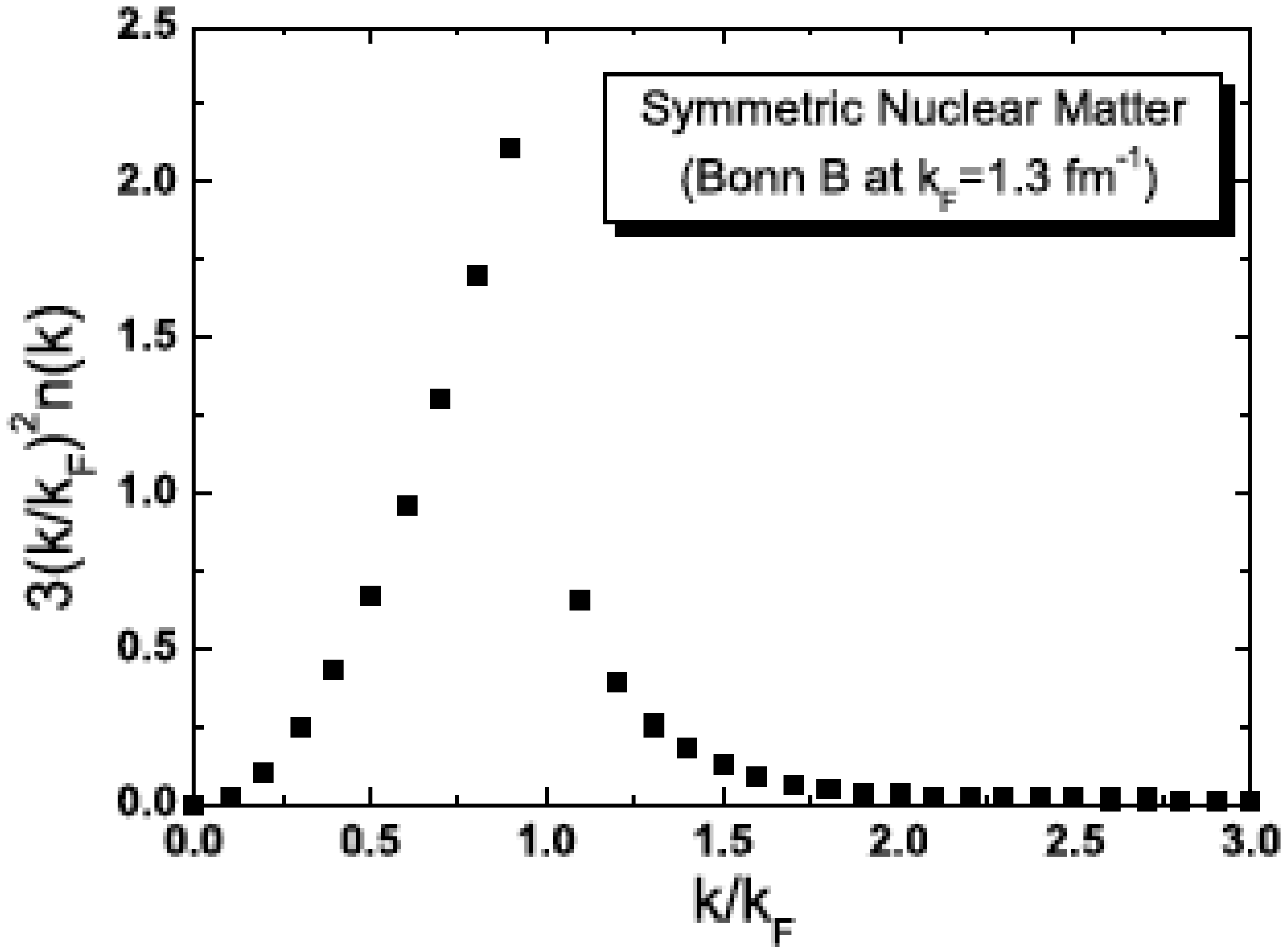


Attraction due to tensor interaction decreases with density

Relativistic framework is important

Momentum distribution in nuclear matter



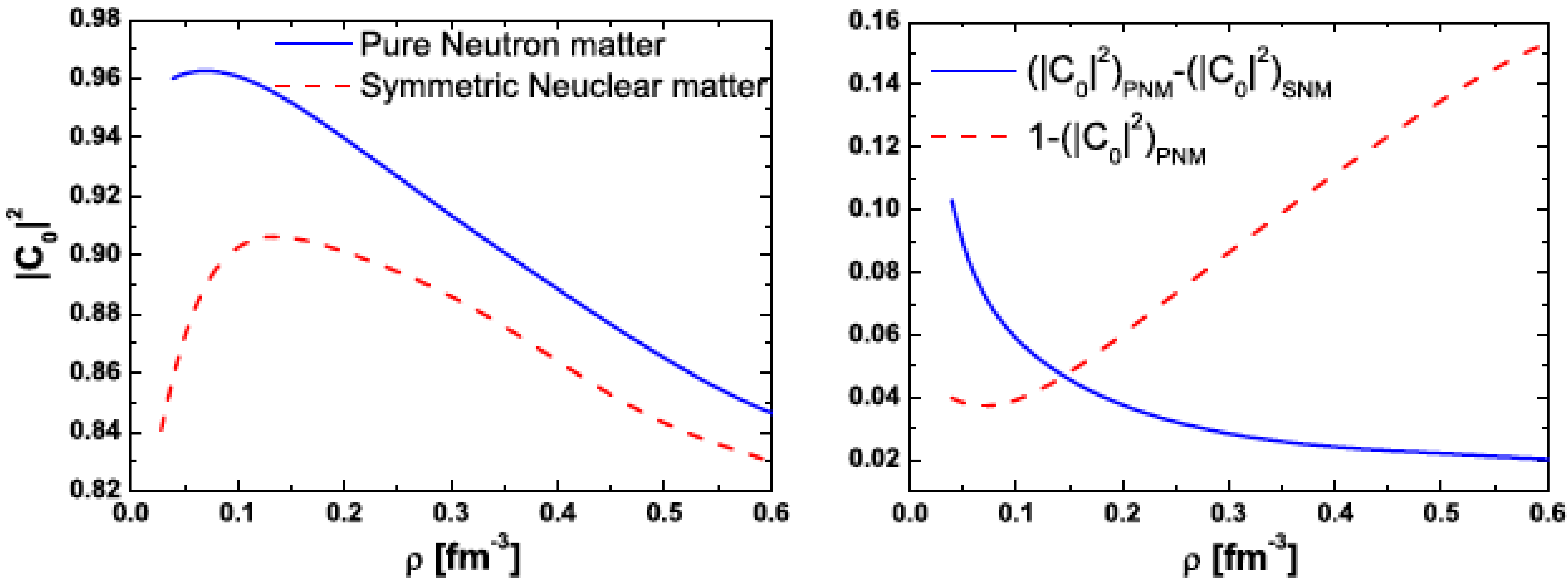


1. Low momentum component decreases
2. High momentum component appears

URCA process

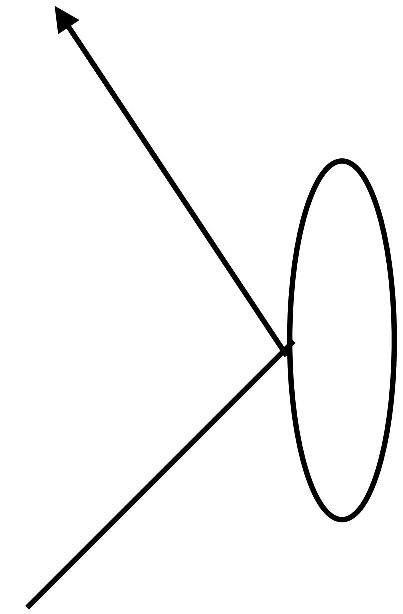
$$n \rightarrow p + e^- + \nu$$

Tensor correlation and short range correlation



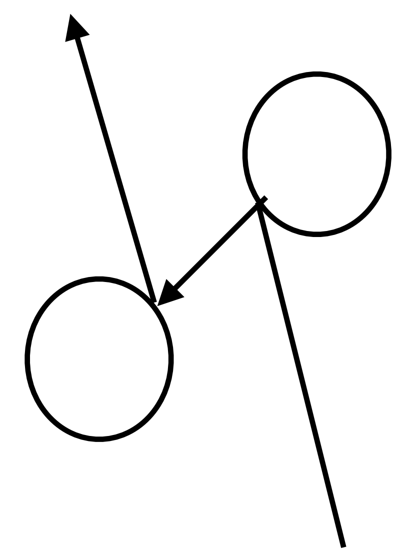
1. Short range correlation increases with density
2. Tensor correlation decreases with density

Relativistic kinematics generates
Strong repulsive effect!!



$$E = \sqrt{p^2 + (m+U)^2} = \sqrt{p^2 + m^2}\left(1 + \frac{2mU + U^2}{2\varepsilon^2}\right)$$

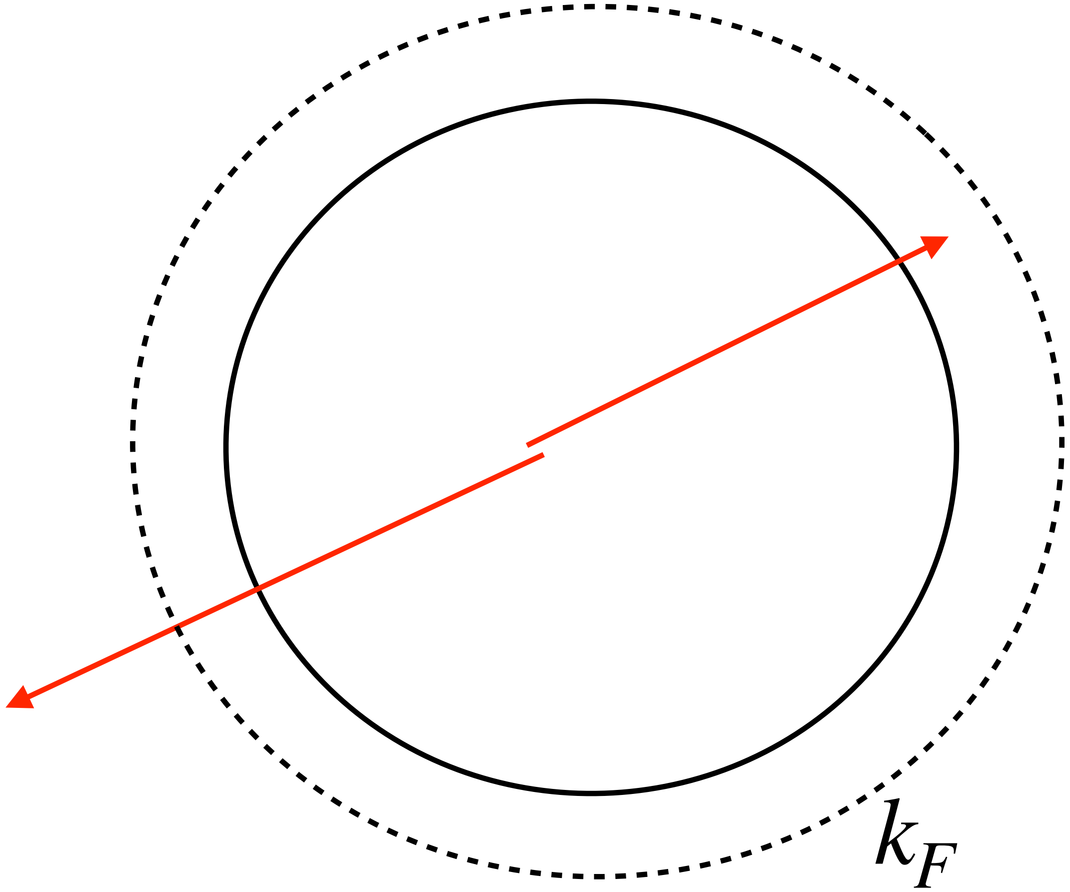
$$= \varepsilon + \frac{m}{\varepsilon}U + \frac{U^2}{2\varepsilon} - \frac{m^2U^2}{2\varepsilon^3} = \varepsilon + \frac{m}{\varepsilon}U + \frac{p^2}{2\varepsilon^3}U^2$$



$$U \sim -400\ MeV$$

$$\frac{p^2}{2\varepsilon^3}U^2 \sim 2\ MeV\ (\rho/\rho_0)^{8/3}$$

Brockmann-Machleight
1990



Tensor interaction is generating strong attraction

Attraction is decreased with density

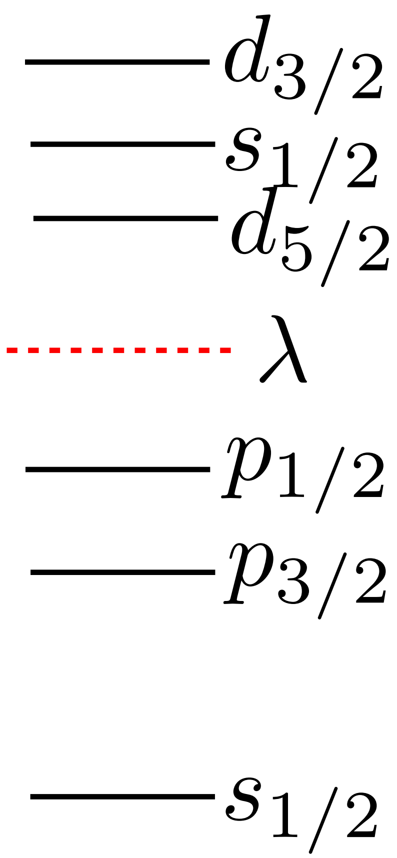
$$\rho = \frac{4}{(2\pi)^3}\frac{4\pi}{3}k_F^3 = \frac{2}{3\pi^2}k_F^3$$

Saturation:

1. 2p-2h states appear with small excitation energy
2. Some 2p-2h states can be blocked by shell model state

^{15}O Level scheme

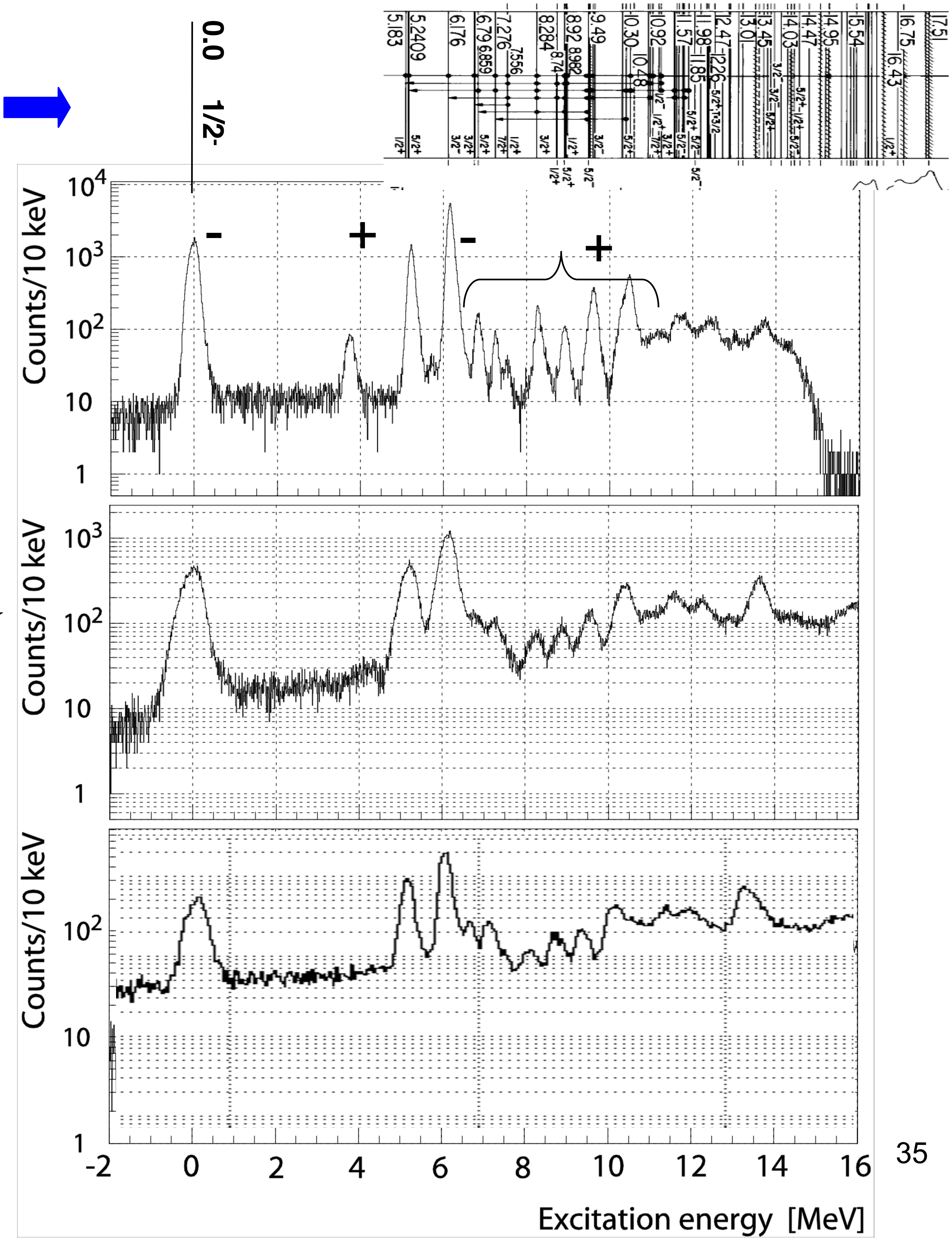
Ong, Tanihata et al



^{16}O (*p*,*d*)
$E_p = 198$ MeV
$\Theta_d = 10°$

^{16}O (*p*,*d*)
$E_p = 295$ MeV
$\Theta_d = 10°$

^{16}O (*p*,*d*)
$E_p = 392$ MeV
$\Theta_d = 10°$



12.3.21

Third lecture

1. Tensor blocking shell model
2. Tensor optimized antisymmetrized molecular dynamics (TOAMD)
3. Delta isobar effect

Tensor blocking shell model

Understanding Magic Numbers in Neutron-Rich Nuclei by Tensor Blocking Mechanism

I. Tanihata[a,b], H. Toki[b], S. Terashima[a], and H.-J. Ong[b]

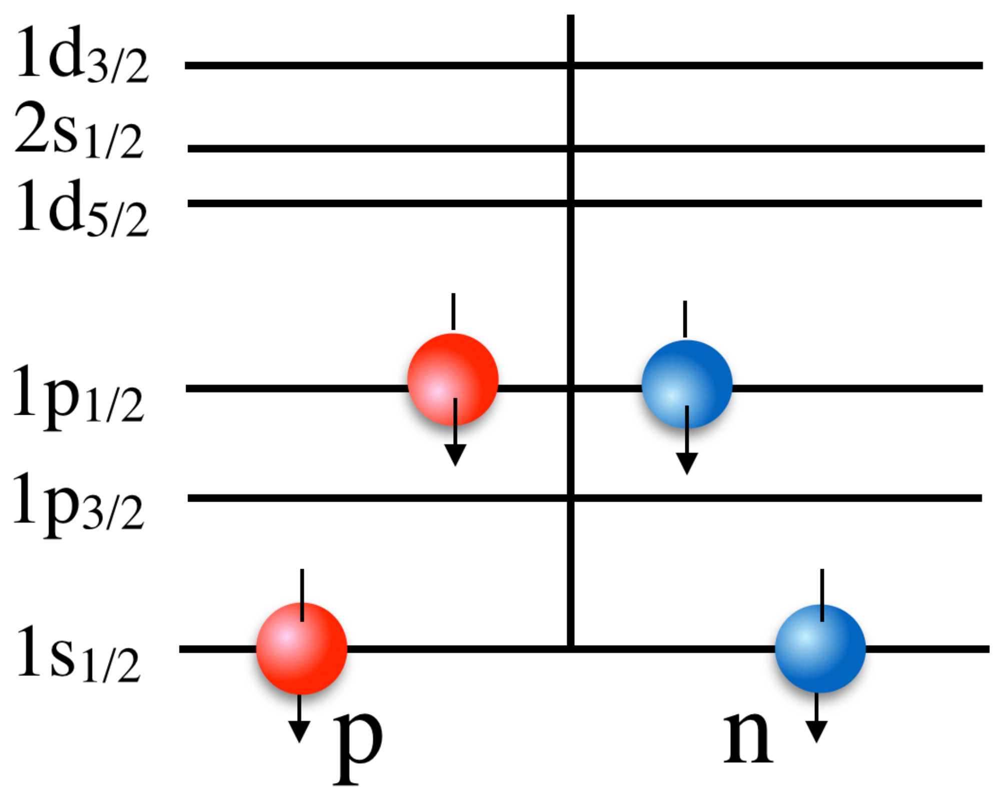


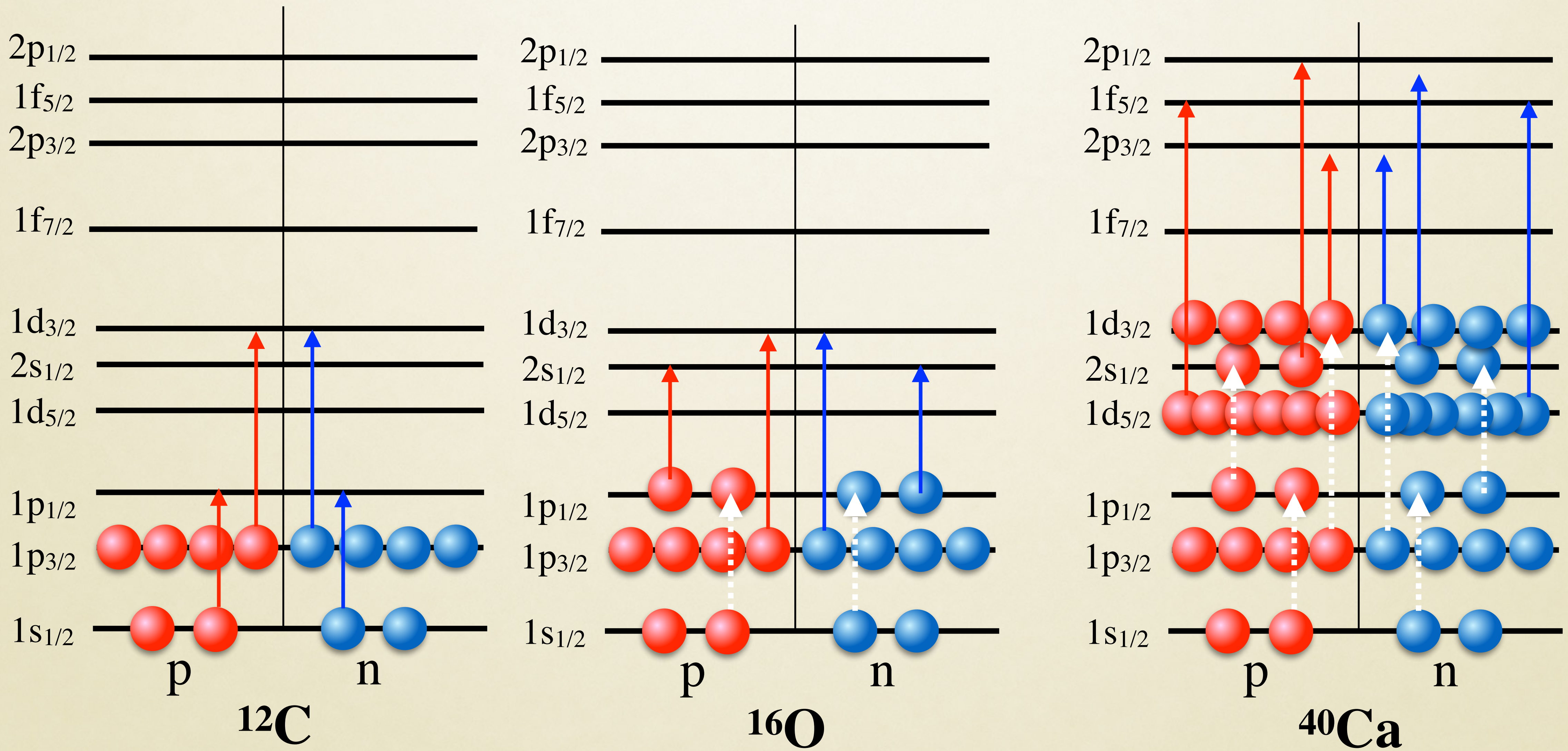
Fig. 1 The most important 2p-2h excitation by the tensor interaction in ^{4}He. This configuration alone, $(1p_{1/2})^2(1s_{1/2})^{-2}$, provides 8.4 MeV of potential energy in a ^{4}He nucleus.

Table I Outstanding questions in neutron-rich nuclei

1. *Why are ^{10}He and ^{28}O not bound while ^{48}Ca is bound and magic?*
2. *Why does the dripline suddenly extend so much in F isotopes?*
3. *How are new magic numbers N=6, 14, 16, 32 and 34 realized?*
4. *Why do the magic numbers N=8 and N=20 disappear in neutron-rich nuclei?*
5. *How can we understand the peculiar configurations of ^{11}Li, ^{11}Be and ^{12}Be consistently?*

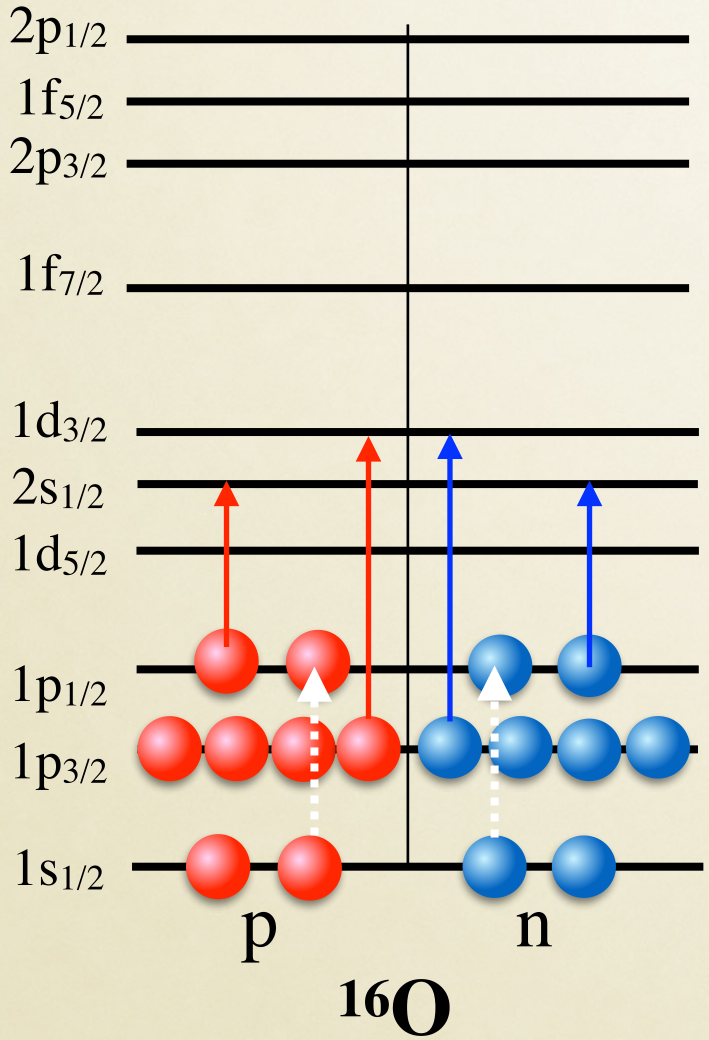
Nuclear Saturation by Tensor Blocking

Blocking and Opening occurs simultaneously and keep the binding per nucleon to be almost constant.



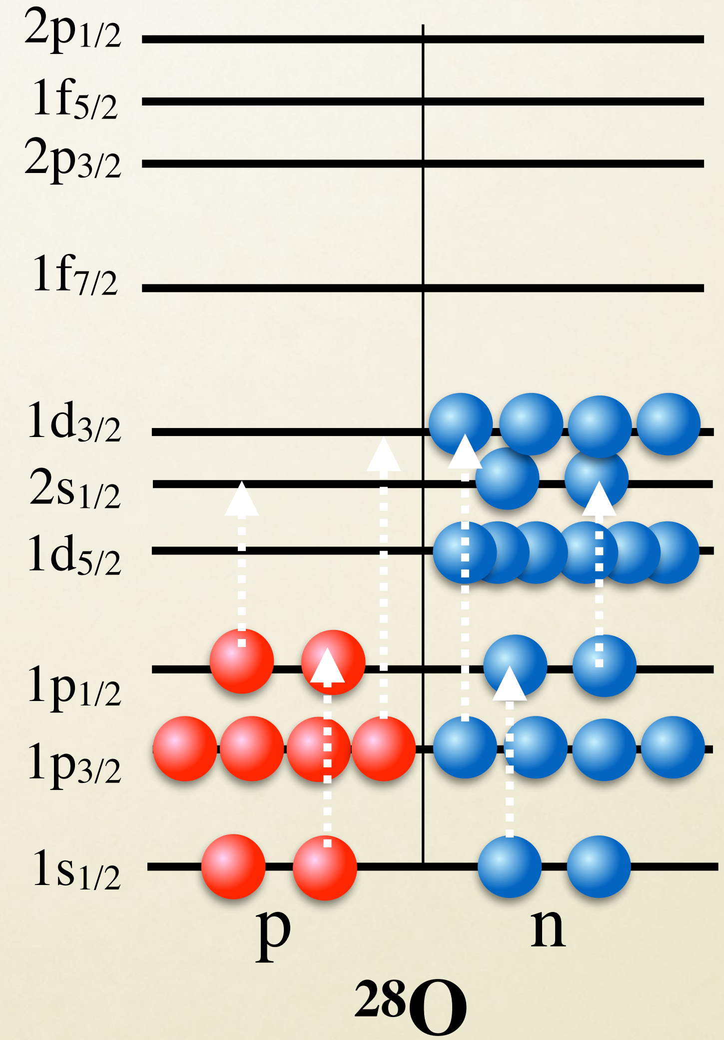
What is the difference between stable and neutron rich nuclei?

Symmetric nuclei



Blocking and Opening occur simultaneously.

Neutron rich nuclei



Only tensor blocking occurs.

Tensor Blocking Shell Model

New!

- Use spirit of TOSM (include 2p-2h configurations as base wave function so that tensor force is treated properly).
- Treat only Δl=1 orbital separately. All light nuclei so far fills only up to Δl=1 orbitals.

Tensor Blocking in Shell Model

$$H = T + V_C + V_T \qquad \Psi = \psi_{Sh} + \psi_{2p-2h}$$

ψ_{sh} **only low momentum**
$\psi_{2p\text{-}2h}$ **includes high-momentum**

Potential energy

$$\langle\Psi|V_c + V_T|\Psi\rangle = \langle\psi_{Sh}|V_C|\psi_{Sh}\rangle + \langle\psi_{Sh}|V_T|\psi_{Sh}\rangle + 2\langle\psi_{Sh}|V_T|\psi_{2p-2h}\rangle + \langle\psi_{2p-2h}|V|\psi_{2p-2h}\rangle$$

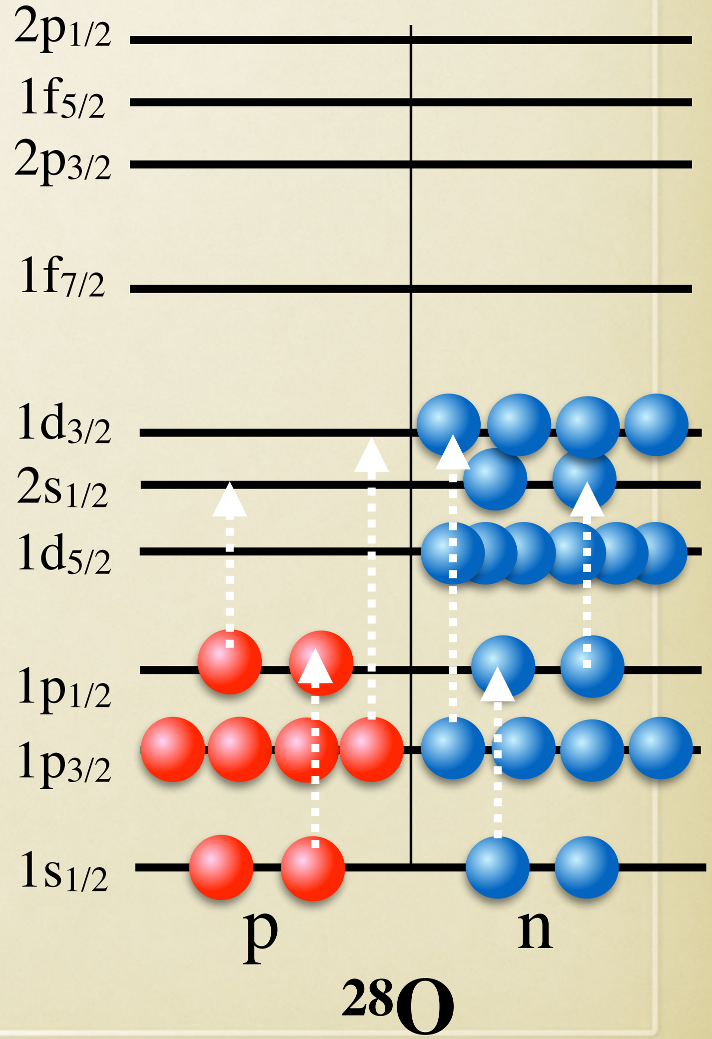
HF_tensor TOSM_tensor

Δl=1 gives 5~8 MeV additional energy in the binding.

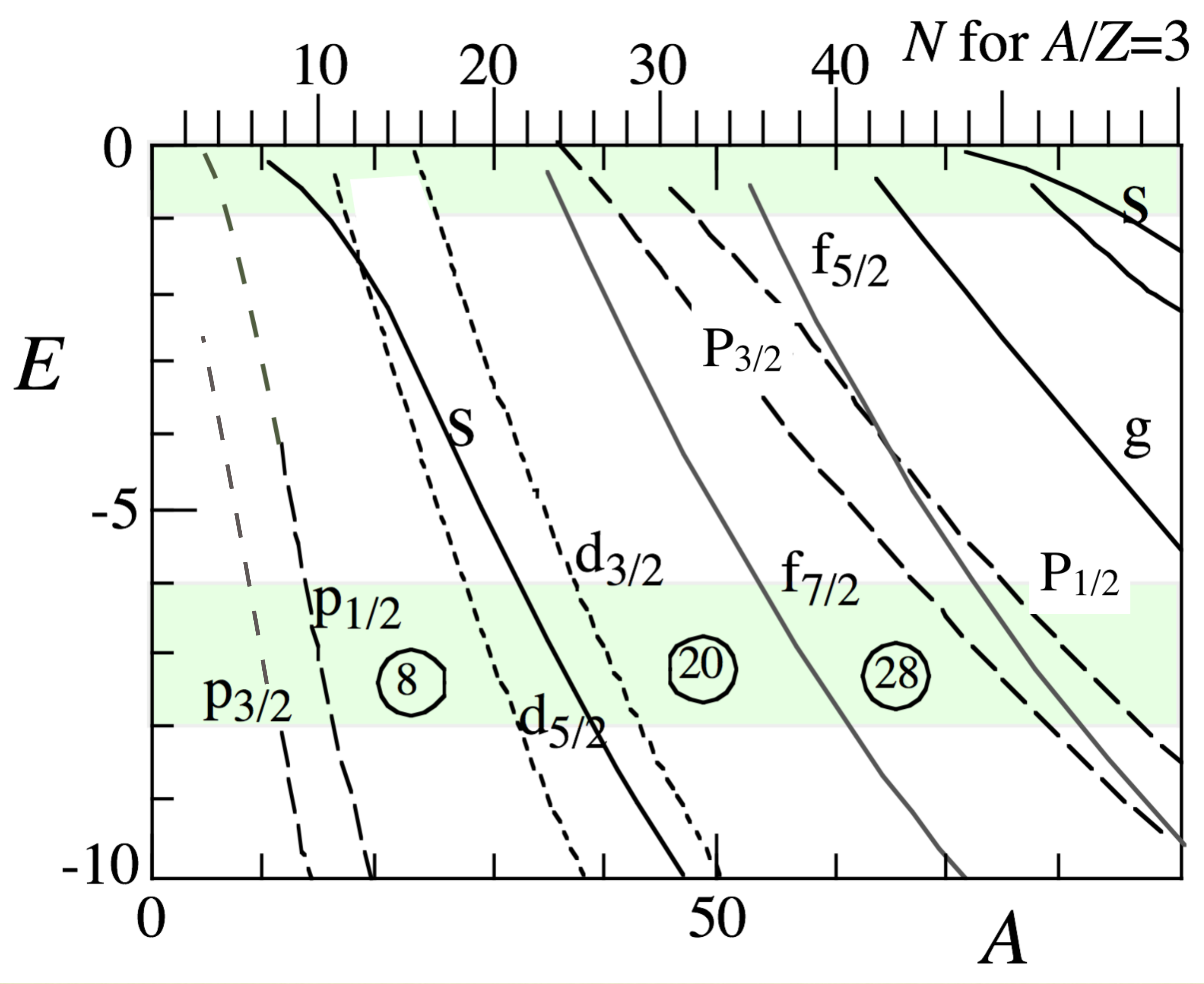
$$\psi_{2p-2h} = |\Delta l = 1\rangle + \Sigma_{higher\ l}|2p - 2h\rangle$$

So we treat all other terms as a mean field potential given by phenomenologically accepted Woods-Saxon potential.

Any mean field mode can be used.



Orbitals in W-S potential and blocking of 2p-2h excitation (>5MeV)



Woods-Saxon potential parameters are from the book of Bohr and Mottelson.
Calculations are made for A/Z=3 nuclei.

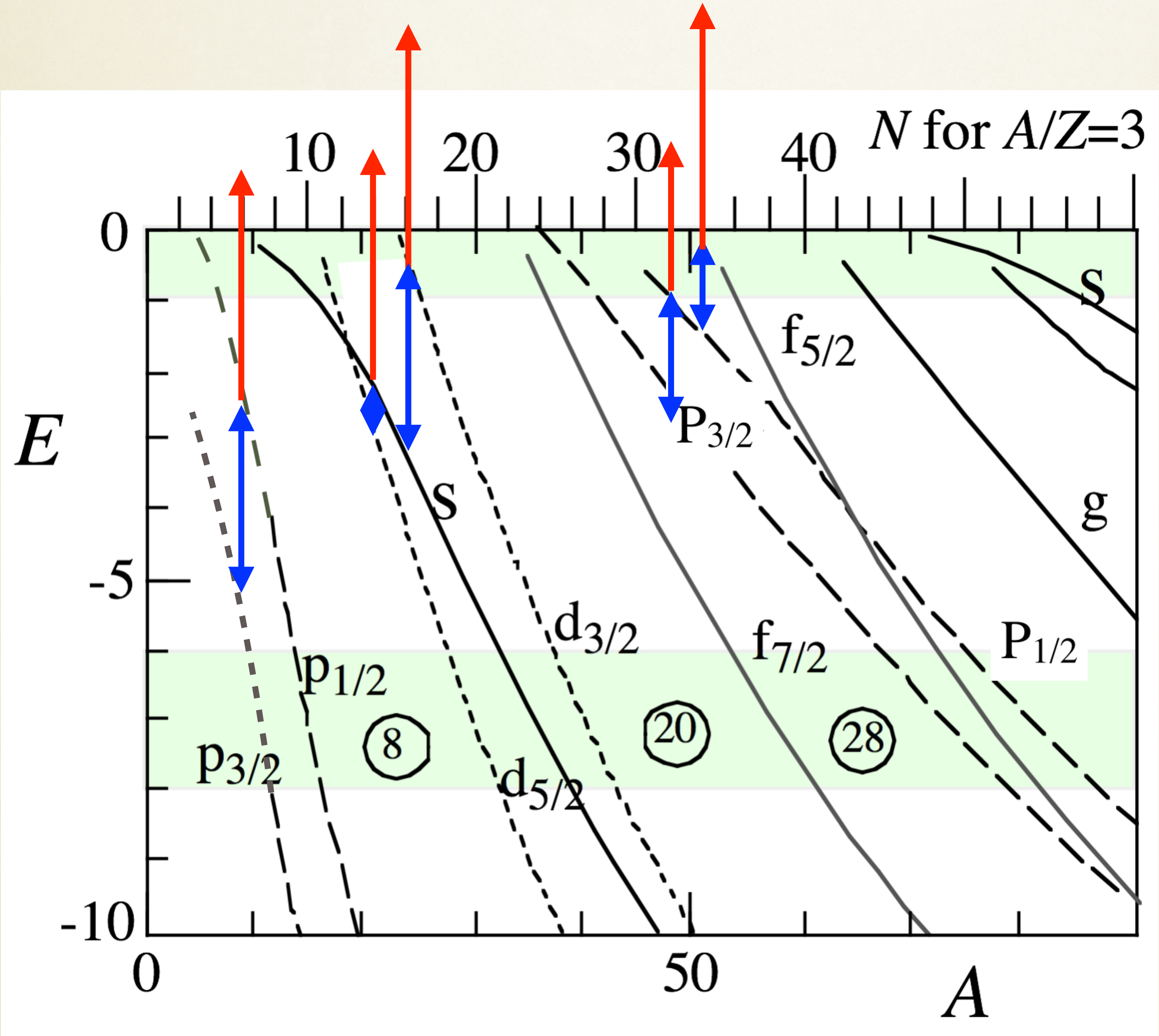
Nucleus	Orbit	Woods-Saxon energy [MeV]	Tensor blocking (TB) [MeV]
^{10}He	$p_{1/2}$	$E(p_{1/2}) > 0$ (=-0.49)	$5.0 \rightarrow$ unbound
^{22}O	$s_{1/2}$	$E(s_{1/2}) - E(d_{5/2}) = 2.8$	$1.9 \rightarrow 4.7$ MeV
^{24}O	$d_{3/2}$	$E(d_{3/2}) - E(s_{1/2}) = 3.7$	$1.9 \rightarrow 5.6$ MeV
^{52}Ca	$p_{1/2}$	$E(p_{1/2}) - E(p_{3/2}) = 2.3$	$1.4 \rightarrow 3.7$ MeV
^{54}Ca	$f_{5/2}$	$E(f_{5/2}) - E(p_{1/2}) = 0.8$	$1.4 \rightarrow 2.2$ MeV
^{10}Be	$p_{1/2}$	$E(s_{1/2}) - E(p_{1/2}) > 3.8$	$2.5 \rightarrow 1.3$ MeV
^{12}Be	$p_{1/2}$	$E(d_{5/2}) - E(p_{1/2}) > 4.2$	$2.4 \rightarrow 1.8$ MeV
^{30}Ne	$d_{3/2}$	$E(f_{7/2}) - E(d_{3/2}) = 3.9$	$1.7 \rightarrow 2.2$ MeV

$2\Delta E(TB) = 5\ MeV$ for $A = 10$

$\Delta E(TB) = 2.5\ MeV\ (A/10)^{-1/3}$

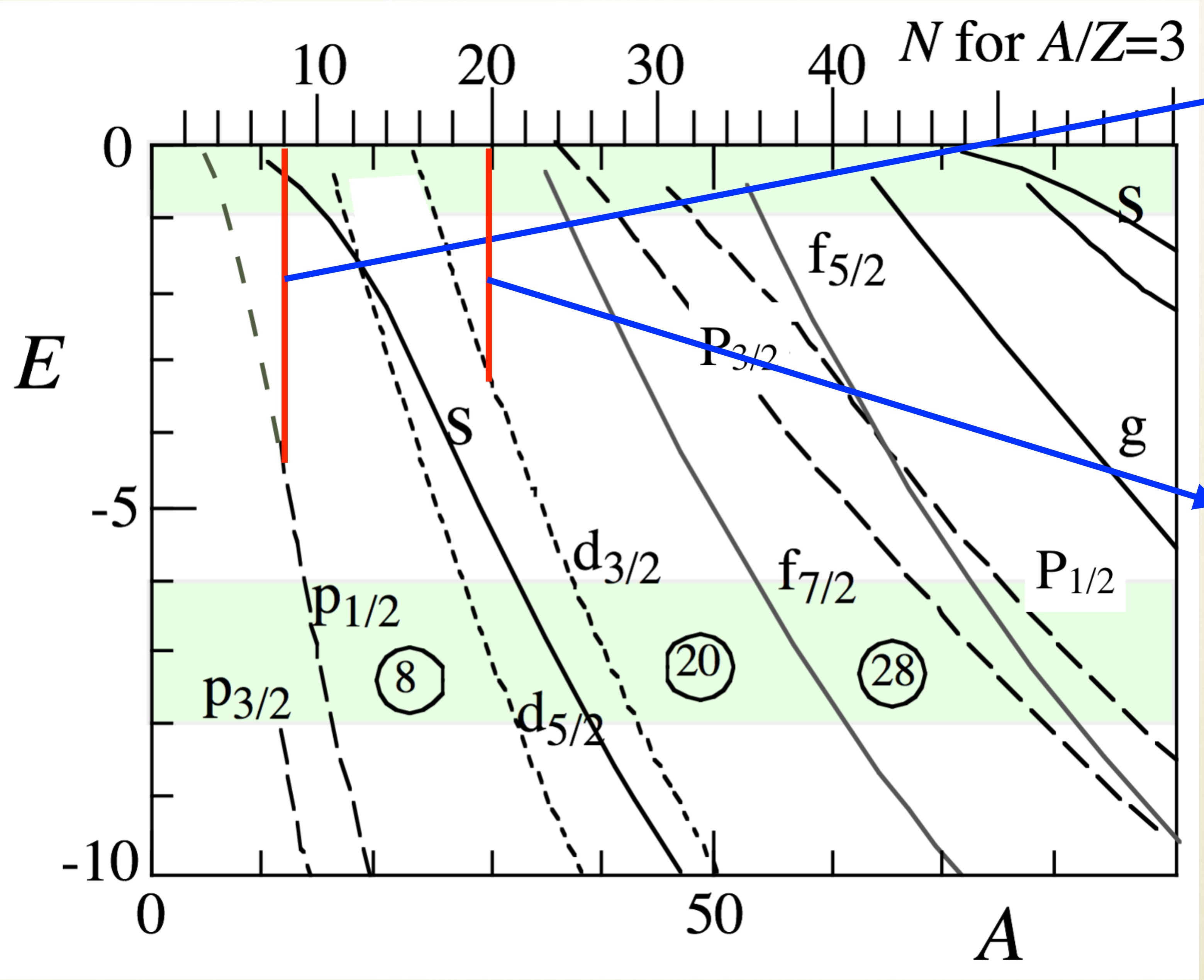
From TOSM calculation

How are new magic numbers N=6,14,16,32,34 made?



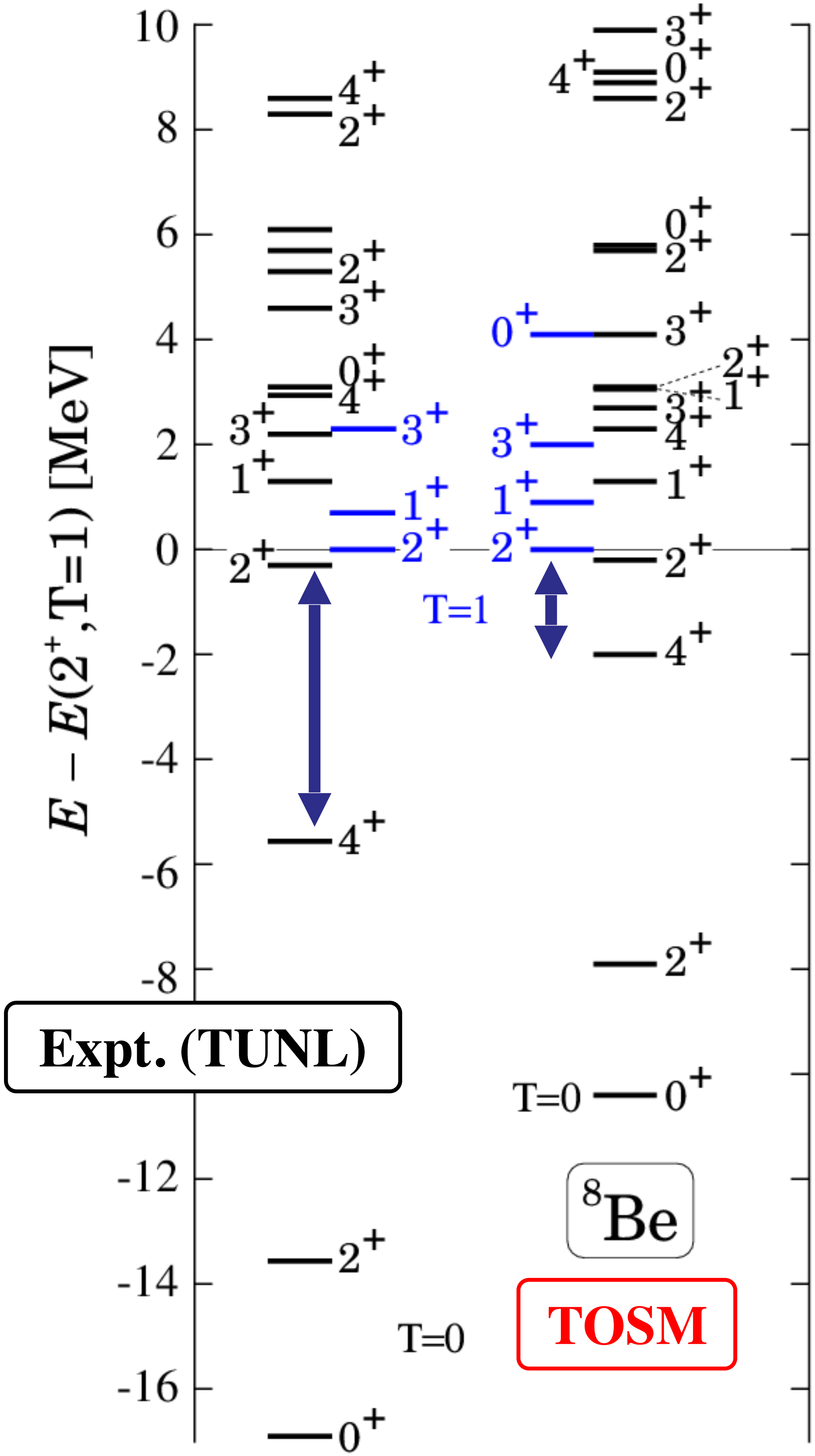
energy gaps become more than factor of two larger due to the tensor blocking.

Why magic numbers N=8 and N=20 disappear in neutron-rich nuclei?



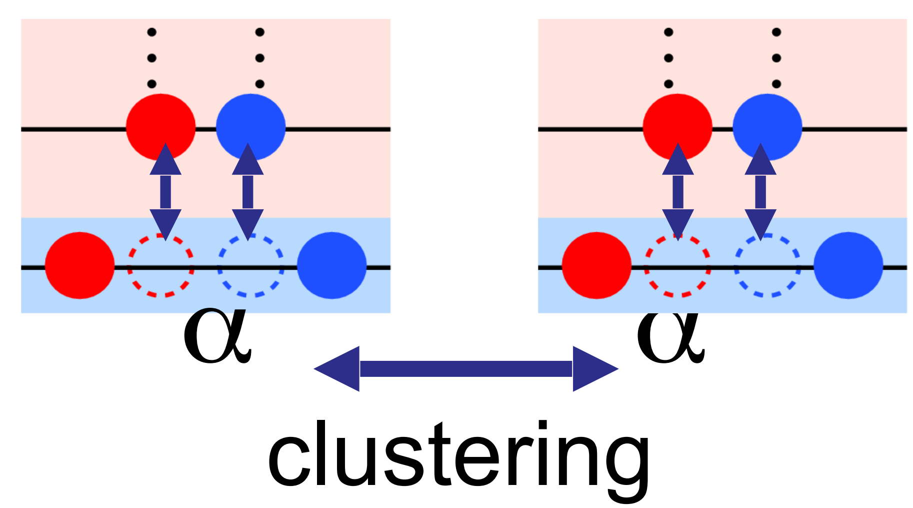
Originally a large gap but the tensor blocking effectively bring $p_{1/2}$ much loosely bound and mixes with sd-shell. Blocking occurs for $s_{1/2}$ until proton fills $p_{1/2}$.

Originally the energy gap is larger than ~4 MeV but the tensor blocking effectively bring $d_{3/2}$ much loosely bound and mixes with fp-shell. For loosely bound nuclei not only $f_{7/2}$ but also $p_{3/2}$ comes closer. $f_{7/2}$ has no blocking effect and $p_{3/2}$ does not until proton fills $d_{3/2}$.



^{8}Be in TOSM – AV8' –

- correct level order (T=0,1)
- tensor contribution : T=0 > T=1
- α : 0p0h+2p2h with high-k
  - 2α needs 4p4h.
  - spatial asymptotic form of 2α



⇒ TOAMD

Tensor Optimized Antisymmetrized Molecular Dynamics (TOAMD)

Myo Toki Ikeda

Tensor optimized shell model (TOSM)

1. We include tensor interaction most effectively to shell model
2. Difficult to treat cluster structure

+

Horiuchi Enyo Kimura..

Antisymmetrized molecular dynamics (AMD)

1. Cluster+shell structure is handled on the same footing using effective interaction
2. Difficult to treat bare nucleon-nucleon interaction

↓

Study nuclear structure based on bare NN interaction

Prog. Theor. Exp. Phys. **2015**, 073D02 (38 pages)
DOI: 10.1093/ptep/ptv087

Tensor-optimized antisymmetrized molecular dynamics in nuclear physics

Takayuki Myo[1,2,*], Hiroshi Toki[2], Kiyomi Ikeda[3], Hisashi Horiuchi[2], and Tadahiro Suhara[4]

Tensor-optimized antisymmetrized molecular dynamics as a successive variational method in nuclear many-body system

Takayuki Myo [a,b,*], Hiroshi Toki [b], Kiyomi Ikeda [c], Hisashi Horiuchi [b], Tadahiro Suhara [d]

Phys. Lett. B769 (2017) 213

PHYSICAL REVIEW C **95**, 044314 (2017)

Successive variational method of the tensor-optimized antisymmetrized molecular dynamics for central interaction in finite nuclei

Takayuki Myo,[1,2,*] Hiroshi Toki,[2,†] Kiyomi Ikeda,[3,‡] Hisashi Horiuchi,[2,§] and Tadahiro Suhara[4,‖]

Hybridization of tensor-optimized and high-momentum antisymmetrized molecular dynamics for light nuclei with bare interaction

Prog. Theor. Exp. Phys. **2018**, 011D01 (9 pages)

Mengjiao Lyu[1,*], Masahiro Isaka[1], Takayuki Myo[1,2,*], Hiroshi Toki[1], Kiyomi Ikeda[3], Hisashi Horiuchi[1], Tadahiro Suhara[4], and Taiichi Yamada[5]

TOAMD

(Tensor optimized antisymmetrized molecular dynamics)

$$\left|\Psi\right\rangle = \left|AMD\right\rangle + F_D\left|AMD\right\rangle$$

$$\Psi = C_0\left|0\right\rangle + \sum_\alpha C_\alpha \left|2p2h:\alpha\right\rangle$$

(TOSM)

$$\left|AMD\right\rangle = A\prod_{i=1}^{A}\psi_{p_i}(\vec{r}_i)\chi_{p_i}(s_i)\xi_{p_i}(t_i)$$

$$\psi_{p_i}(\vec{r}_i) = \left(\frac{2\nu}{\pi}\right)^{3/4} e^{-\nu(\vec{r}_i-\vec{D}_{p_i})^2}$$

(shifted Gaussian)

$$\chi_{p_i}(s_i) = \beta_{p_i}\left|\uparrow\right\rangle + (1-\beta_{p_i})\left|\downarrow\right\rangle$$

$$\xi_{p_i}(t_i) = \left|proton\right\rangle \quad or \quad \left|neutron\right\rangle$$

$$F_D = \frac{1}{2}\sum_{i\neq j} f_D(r_{ij})S_{12}(r_{ij})\tau_i\cdot\tau_j$$

$$S_{12}(r_{ij}) = 3(\sigma_i\cdot\hat{r}_{ij})(\sigma_j\cdot\hat{r}_{ij}) - (\sigma_i\cdot\sigma_j)$$

$$= \sum_{xyx'y'} \hat{r}_{ijx}\hat{r}_{ijy}\sigma_{ix'}\sigma_{jy'}(3\delta_{xx'}\delta_{yy'} - \delta_{xy}\delta_{x'y'})$$

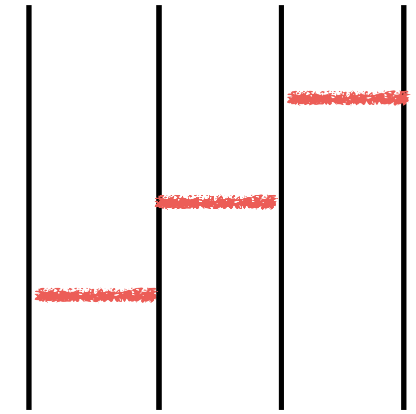
$$f_D(r_{ij}) = \sum_\mu C_\mu r_{ij}^2 e^{-a_\mu r_{ij}^2}$$

(Tensor correlation function)

Argonne wave function (Monte-Carlo method)

$$\left|\Psi\right\rangle = \prod_{i\neq j}\left(1+U_{ij}(r_{ij})\right)\left|\Psi_J(SM)\right\rangle$$

Jastrow(1955)

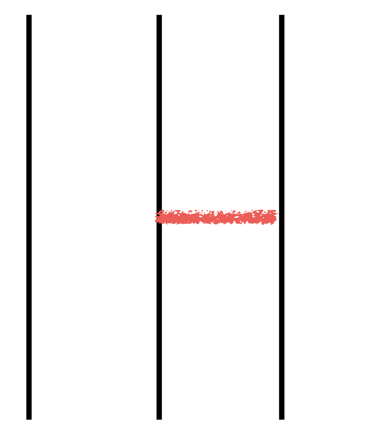


Multiple correlation functions are contained.

This is the reason why the calculations are time consuming. (very complicated) → VMC method

TOSM (Tensor optimized shell model)

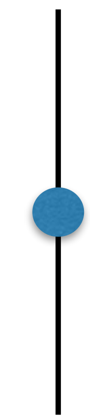
$$\left|\Psi\right\rangle = C_0\left|SM\right\rangle + \sum_{\alpha} C_{\alpha}\left|2p2h;\alpha\right\rangle$$



Relative correlation function is expressed by 2p2h states. (include 4p4h states are highly complicated)

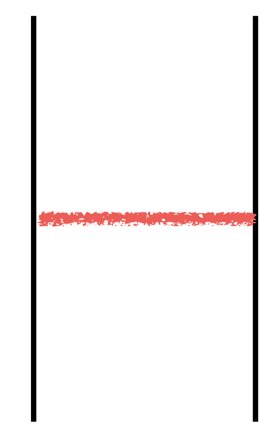
Hamiltonian (AV18)

$$H = T + V + U$$



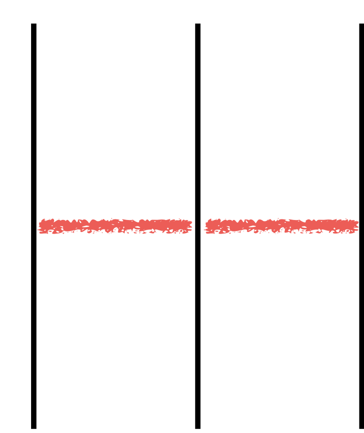
$$T = \sum_{i=1}^{A} \left(\frac{p_i^2}{2m} \right) - T_{CM}$$

$$V = \frac{1}{2} \sum_{p} \sum_{i \neq j} V^{p}(r_{ij}) O^{p}(ij)$$



$$V^{p}(r_{ij}) = \sum_{\mu} C_{\mu} e^{-a_{\mu}^{p} r_{ij}^{2}}$$

$$U = \frac{1}{2} \sum_{p} \sum_{i \neq j \neq k} U^{p}(ijk)$$



$$U^{p}(ijk) = V^{\pi}(ij) V^{\pi}(jk)$$

Energy (minimization)

$$E = \frac{\langle AMD | (1+F_D) H (1+F_D) | AMD \rangle}{\langle AMD | (1+F_D)(1+F_D) | AMD \rangle}$$

Difficulty

1. many body matrix elements (7 body)
2. antisymmetrization (exchange of particles)
3. tensor interaction (high momentum)

→ momentum space (separable)

$$e^{-ar_{ij}^2} = \left(\frac{\pi}{a}\right)^{3/2} \int_k e^{-k^2/4a} e^{ik(r_i - r_j)}$$

Advantage

1. Gaussian integral (analytical)
2. antisymmetrization (matrix technique)

Two-body matrix element

$$\left\langle AMD \middle| O_1 O_2 \middle| AMD \right\rangle = \left\langle p_1 p_2 .. p_A \middle| \sum_{i \neq j}^{A} O(i) O(j) \middle| \det \middle| q_1 q_2 .. q_A \right| \right\rangle$$

$$M(O_1 O_2) = \sum_{r_1 \neq r_2}^{A} \begin{vmatrix} \langle p_1 | q_1 \rangle & \langle p_1 | q_2 \rangle & \cdots & \langle p_1 | q_A \rangle \\ \langle p_{r_1} | O_1 | q_1 \rangle & \langle p_{r_1} | O_1 | q_2 \rangle & \cdots & \langle p_{r_1} | O_1 | q_A \rangle \\ \langle p_{r_2} | O_2 | q_1 \rangle & \langle p_{r_2} | O_2 | q_1 \rangle & \cdots & \langle p_{r_2} | O_2 | q_1 \rangle \\ \langle p_A | q_1 \rangle & \langle p_A | q_2 \rangle & \cdots & \langle p_A | q_A \rangle \end{vmatrix}$$

$$M(O_1 O_2) = \sum_{r_1 \neq r_2}^{A} \sum_{l_1 \neq l_2}^{A} \left\langle p_{r_1} \middle| O_1 \middle| q_{l_1} \right\rangle \left\langle p_{r_2} \middle| O_2 \middle| q_{l_2} \right\rangle C(r_1 r_2 : l_1 l_2)$$

$C(r_1 r_2 : l_1 l_2)$ is a co-factor matrix of B.

Central interaction

$$V^c = \frac{1}{2}\sum_{i\neq j}\sum_{\mu} C_\mu \left(\frac{\pi}{a_\mu}\right)^{3/2} \int_k e^{-k^2/4a_\mu} e^{ikr_i} e^{-ikr_j}$$

$$\langle AMD|V^c|AMD\rangle = \frac{1}{2}\sum_{p_1\neq p_2:q_1\neq q_2}\sum_{\mu} \tilde{C}_\mu^{(0)} \int_k e^{-k^{2/4a_\mu}} \langle p_1|e^{ikr_1}|q_1\rangle\langle p_2|e^{ikr_2}|q_2\rangle C(p_1p_2:q_1q_2)$$

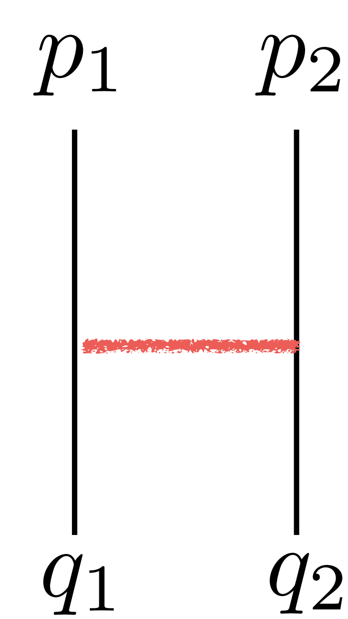
$$= \frac{1}{2}\sum_{p_1\neq p_2:q_1\neq q_2}\sum_{\mu} \tilde{C}_\mu^{(0)} I^{(12)}(A,B,C) M^{p_1q_1} M^{p_2q_2} \bar{M}^{p_1q_1} \bar{M}^{p_2q_2} C(p_1p_2:q_1q_2)$$

$$I^{(12)}(A,B,C) = \frac{1}{(2\pi)^3}\left(\frac{\pi}{A}\right)^{3/2} e^{-B^\dagger A^{-1}B/4+C}$$

$$A = 1/4\nu + 1/4a_\mu$$

$$B = \frac{1}{2}\left(\vec{D}_{p_1} + \vec{D}_{q_1}\right) - \frac{1}{2}\left(\vec{D}_{p_2} + \vec{D}_{q_2}\right)$$

$$C = -\frac{1}{2}\nu\left[\left(D_{p_1} - D_{q_1}\right)^2 + \left(D_{p_2} - D_{q_2}\right)^2\right]$$



Many correlations

$$\left|\Psi\right\rangle = \left|AMD\right\rangle + F_D\left|AMD\right\rangle \to$$

$$\to \left|AMD\right\rangle + F_D\left|AMD\right\rangle + F_D F_D\left|AMD\right\rangle ..$$

$$\to (1 + F_S)(1 + F_D + F_D F_D + ..)\left|AMD\right\rangle$$

$$F_D = \frac{1}{2}\sum_{i\neq j} C_\mu r_{ij}^2 e^{-a_\mu r_{ij}^2} S_{12}(r_{ij})$$ Tensor correlation

$$F_S = \frac{1}{2}\sum_{i\neq j} C_\mu e^{-a_\mu r_{ij}^2}$$ Short range correlation

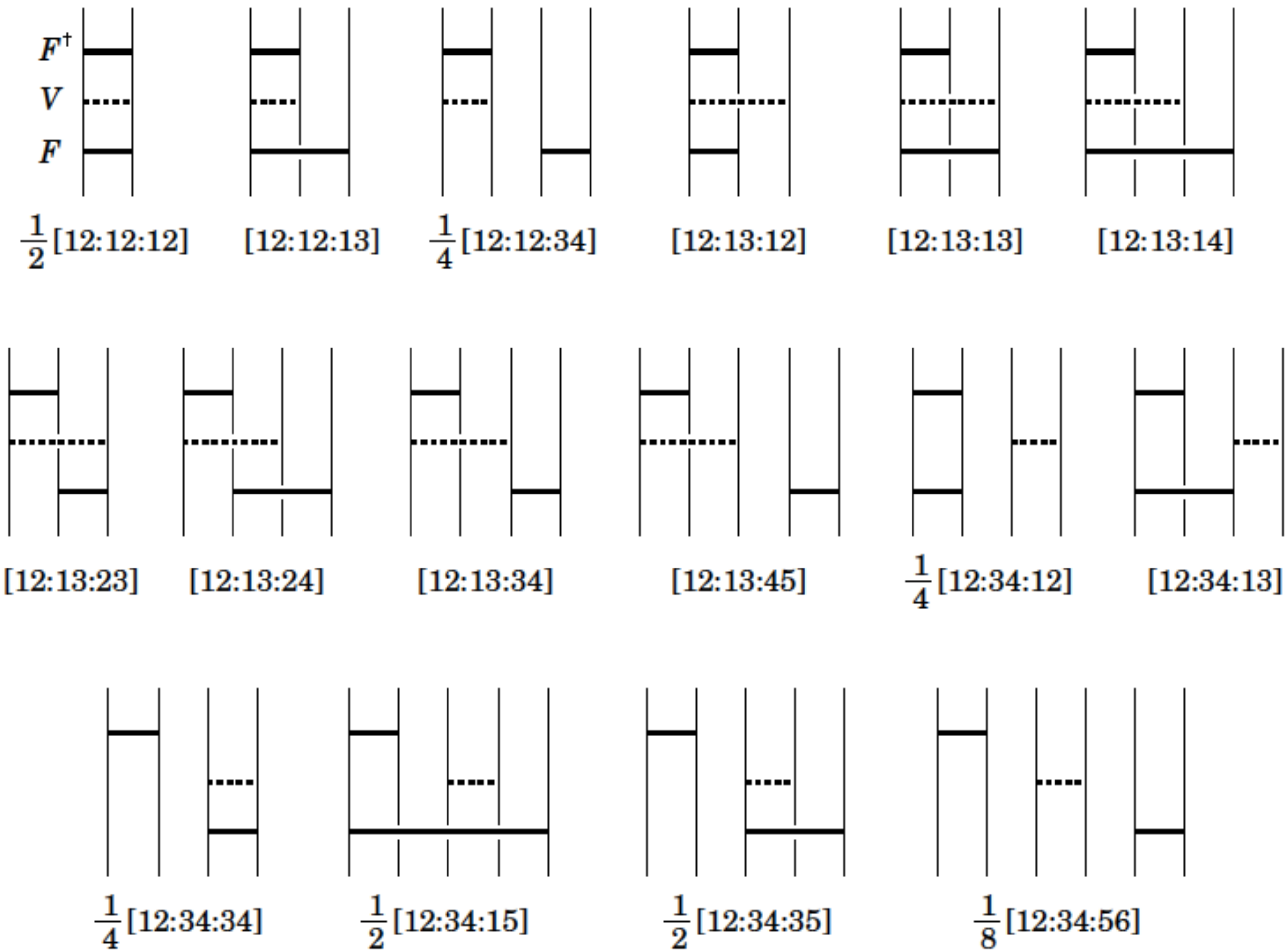


Table 1 Numbers of diagrams of many-body operators in the cluster expansion of $F^\dagger F^\dagger FF$ (norm), $F^\dagger F^\dagger TFF$ and $F^\dagger F^\dagger VFF$ appearing in the double TOAMD.

n-body	2	3	4	5	6	7	8	9	10
norm	1	13	46	47	25	6	1	–	–
T	1	40	183	259	163	55	10	1	–
V	1	40	295	587	516	235	65	10	1

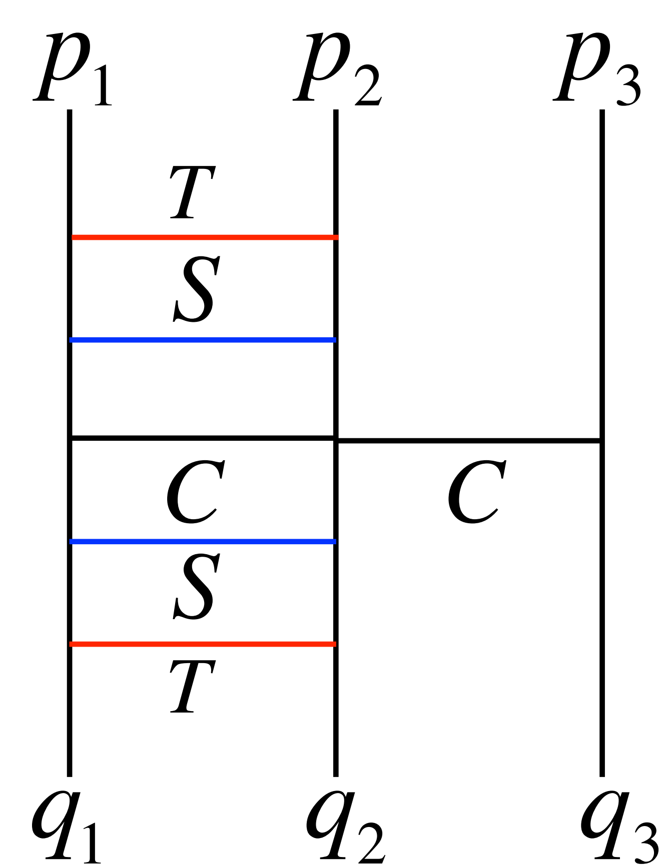
Some example of matrix element

$$\langle AMD|O|AMD\rangle = \sum_{p_1\neq p_2\neq p_3:q_1\neq q_2\neq q_3} \sum_{\mu_1\mu_2\mu_3\mu_4\mu_5\mu_6} \tilde{C}^{(2)}_{\mu_1}\tilde{C}^{(0)}_{\mu_2}\tilde{C}^{(0)}_{\mu_3}\tilde{C}^{(0)}_{\mu_4}\tilde{C}^{(0)}_{\mu_5}\tilde{C}^{(2)}_{\mu_6}$$

$$\sum_{xyzux'y'z'u'} I^{((12)^3:23:(12)^2)}_{1x1y6z6u}(A,B,C) M^{p_1q_1}_{x'z'} M^{p_2q_2}_{y'u'} M^{p_3q_3} \bar{M}^{p_1q_1} \bar{M}^{p_2q_2} \bar{M}^{p_3q_3}$$

$$(3\delta_{xx'}\delta_{yy'} - \delta_{xy}\delta_{x'y'})(3\delta_{zz'}\delta_{uu'} - \delta_{zu}\delta_{z'u'}) C(p_1p_2p_3 : q_1q_2q_3)$$

Any complicated matrix elements can be written in similar expressions

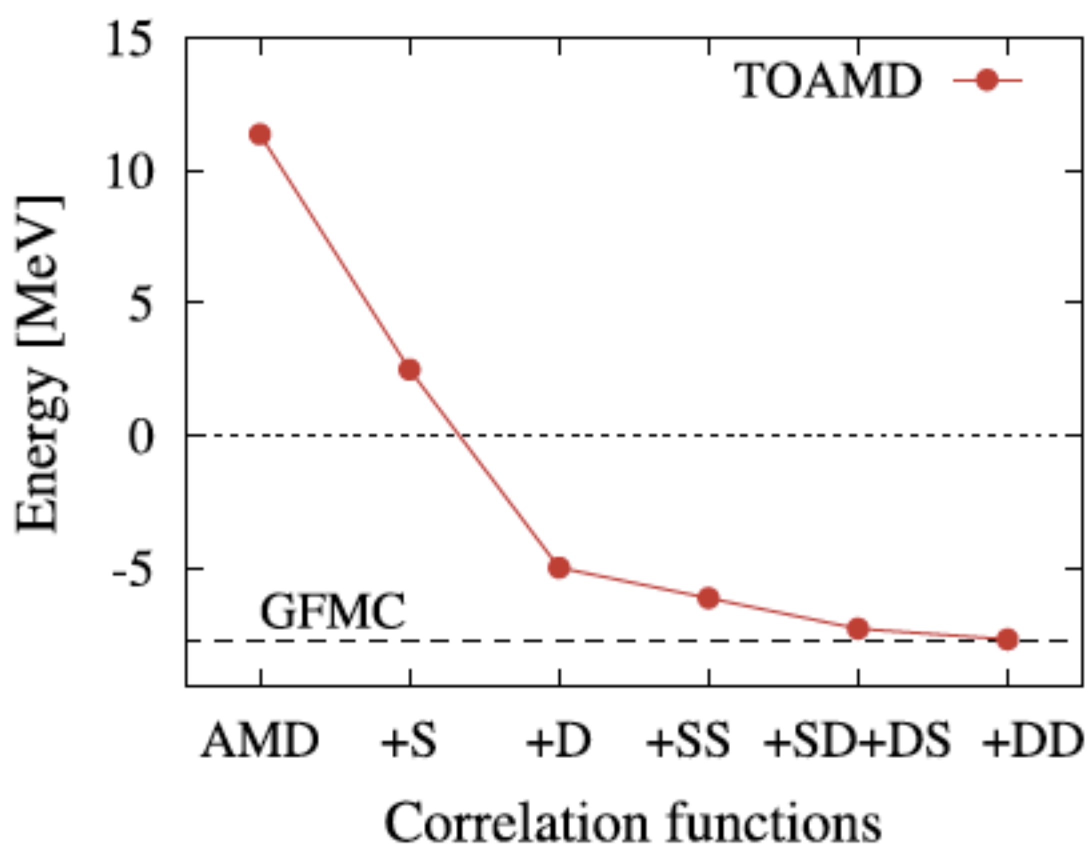


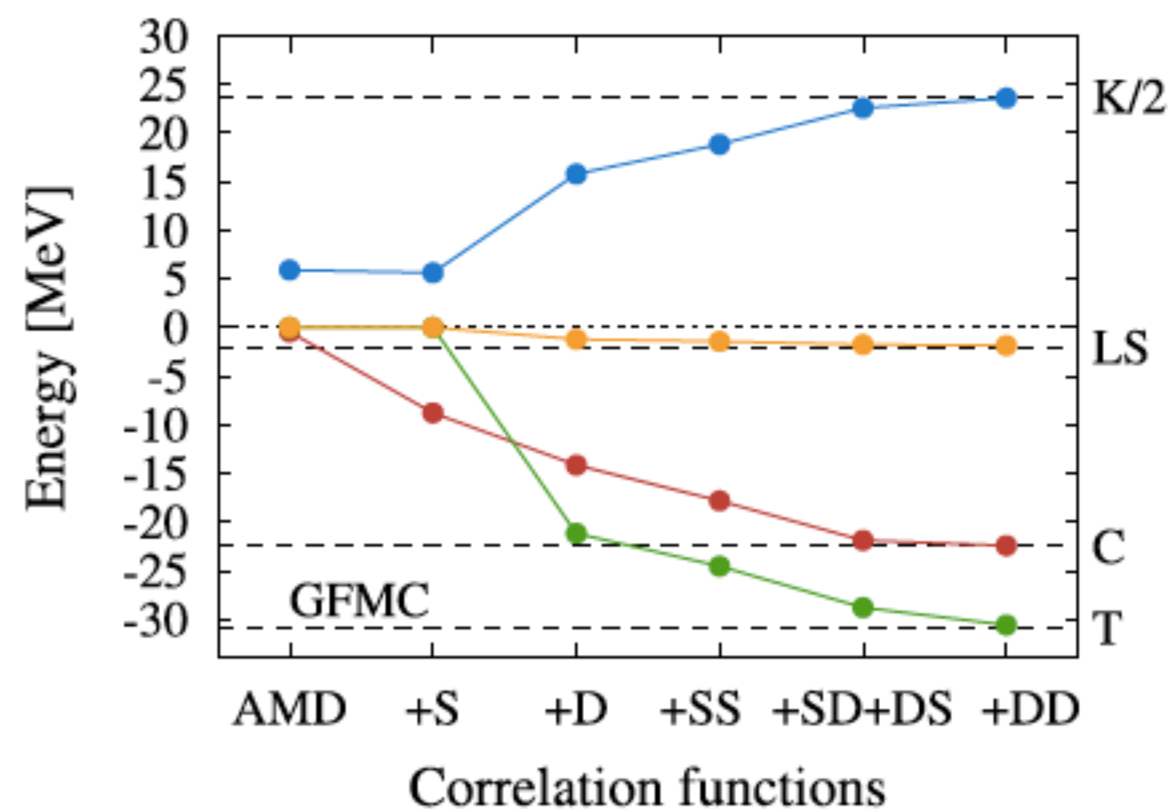
Nuclear Feynmann diagram

He(A=3)

Interaction is AV8'

TOAMD group: Phys. Lett. B769 (2017) 213





$$\Phi_{TOAMD} = (1 + F_S + F_D + F_S F_S + F_S F_D + F_D F_D)\Phi_{AMD}$$

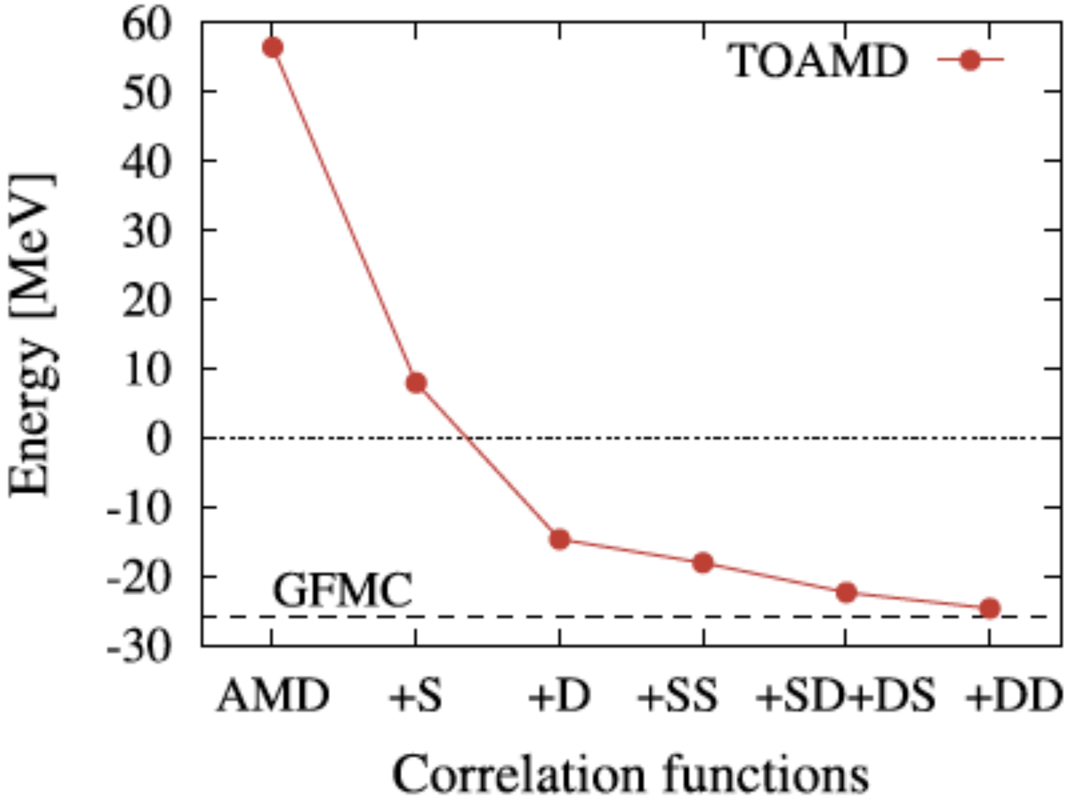
We achieve convergence successively.

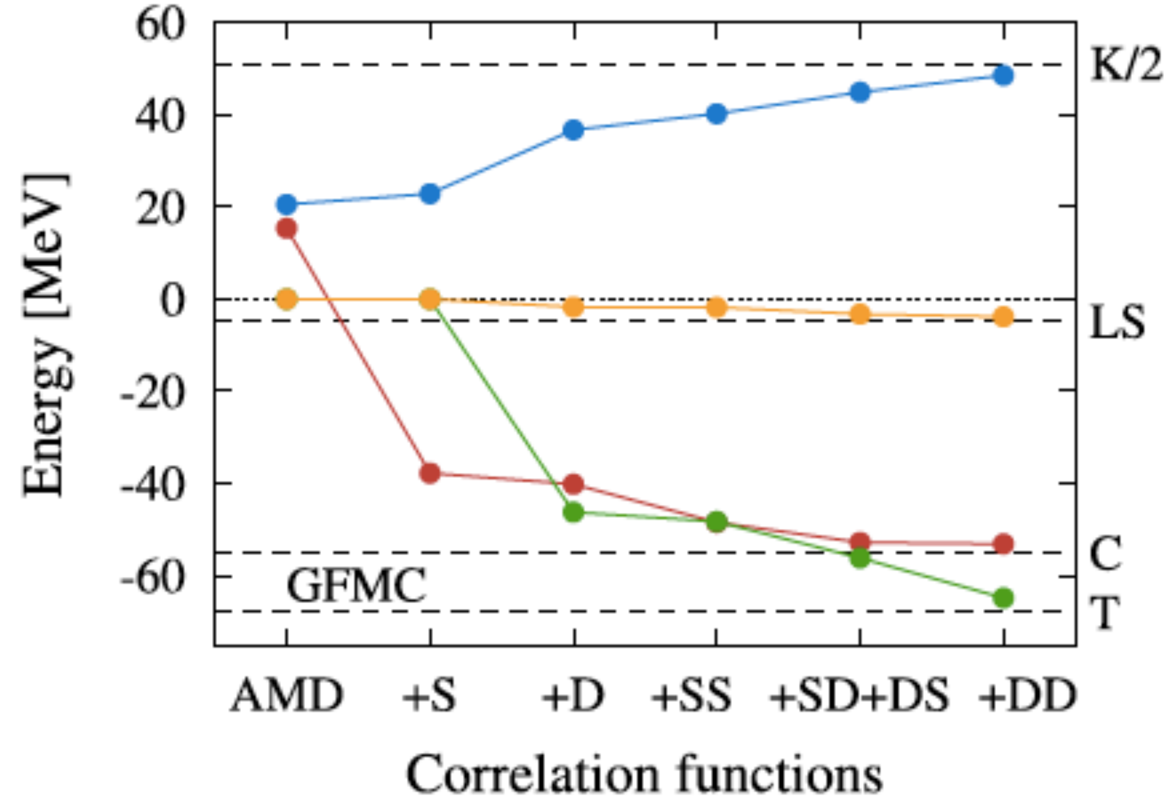
(Successive variational method)

He(A=4)

Interaction is AV8'

TOAMD group: Phys. Lett. B769 (2017) 213

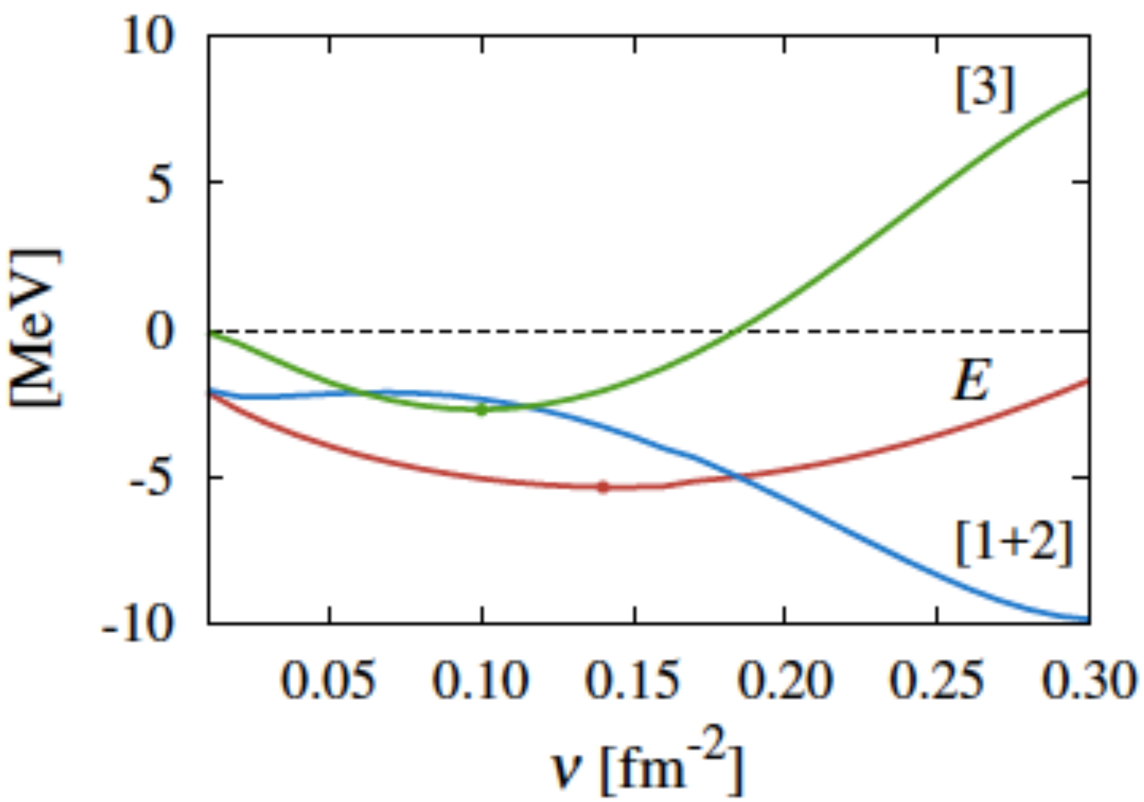




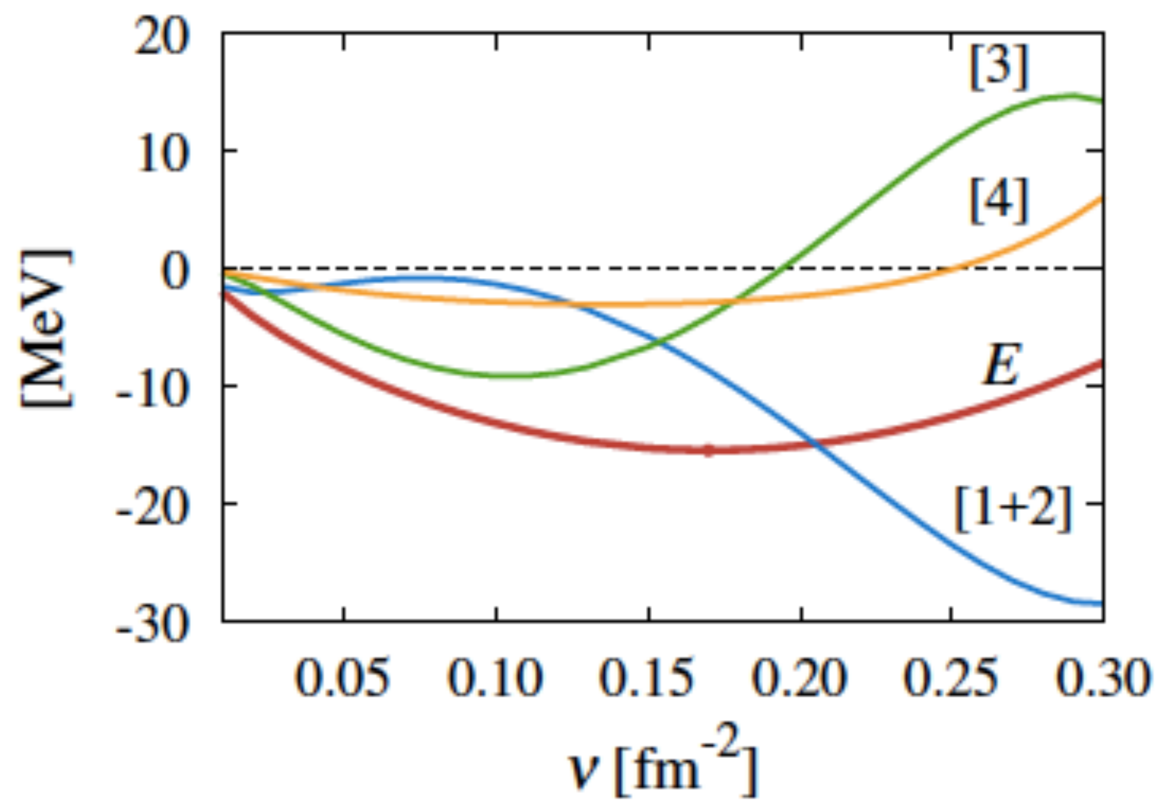
$$\Phi_{TOAMD} = (1 + F_S + F_D + F_S F_S + F_S F_D + F_D F_D)\Phi_{AMD}$$

TOAMD calculation

A=3



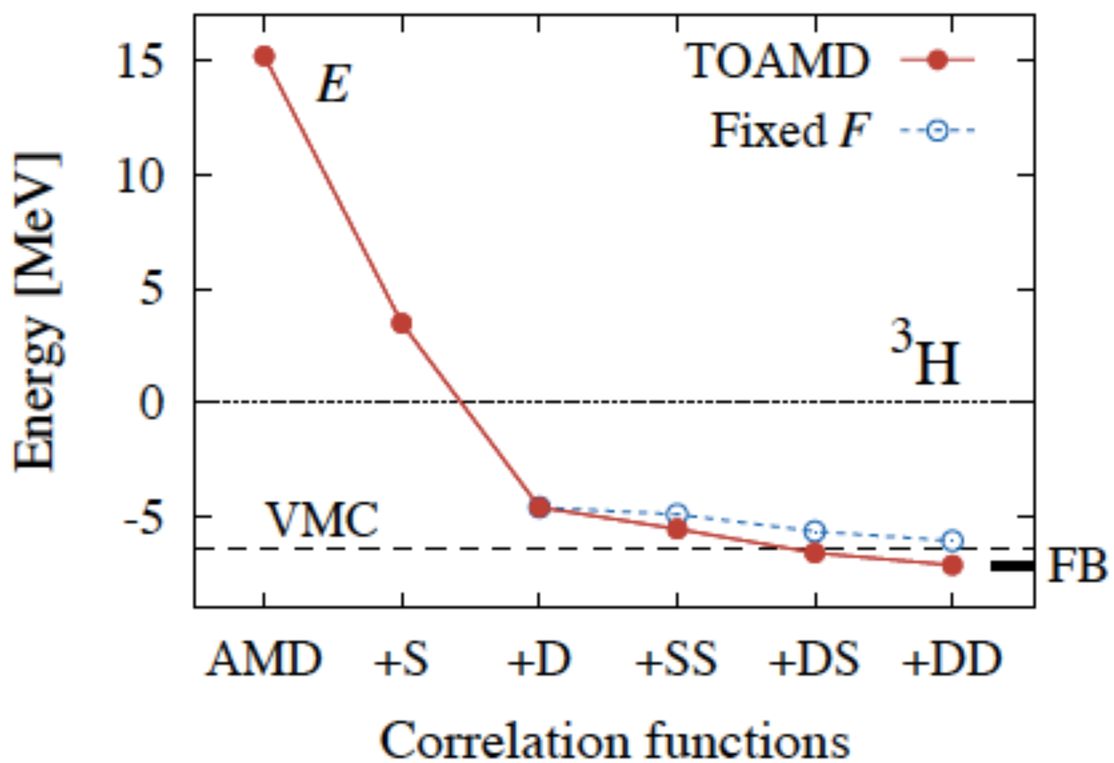
A=4

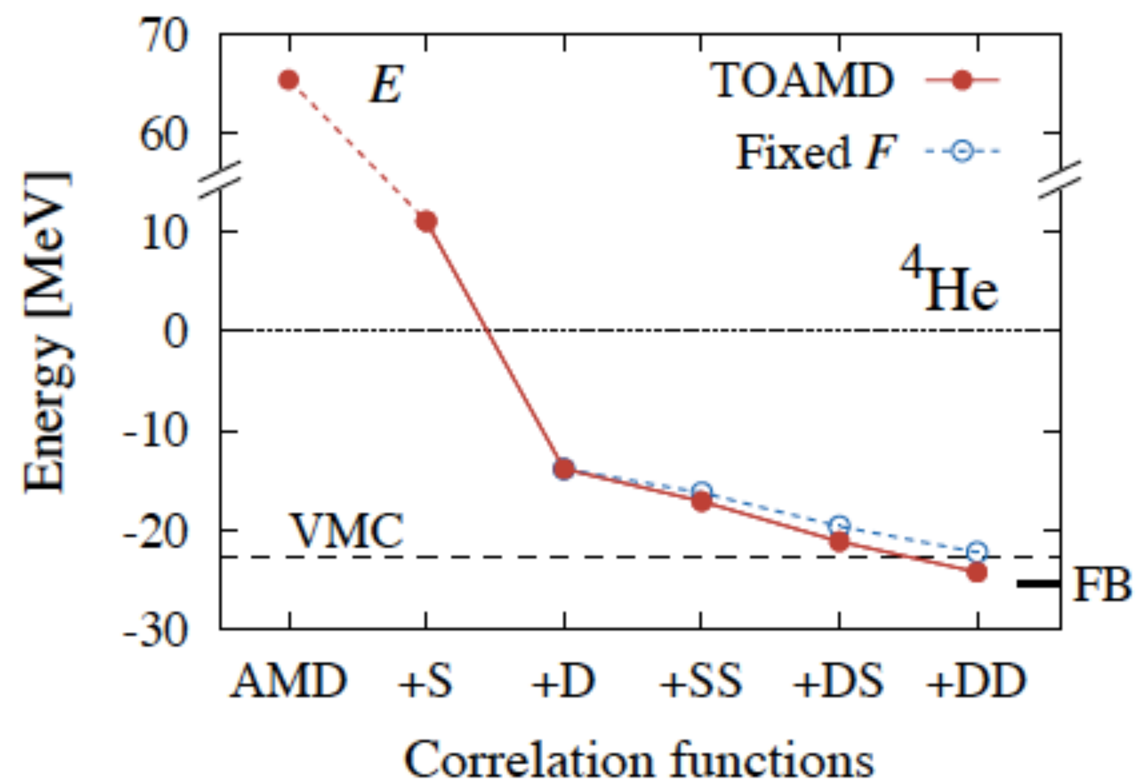


We have to calculate all the multi-body terms in order to get a variationally stable state

TOAMD vs Jastrow correlation (VMC)

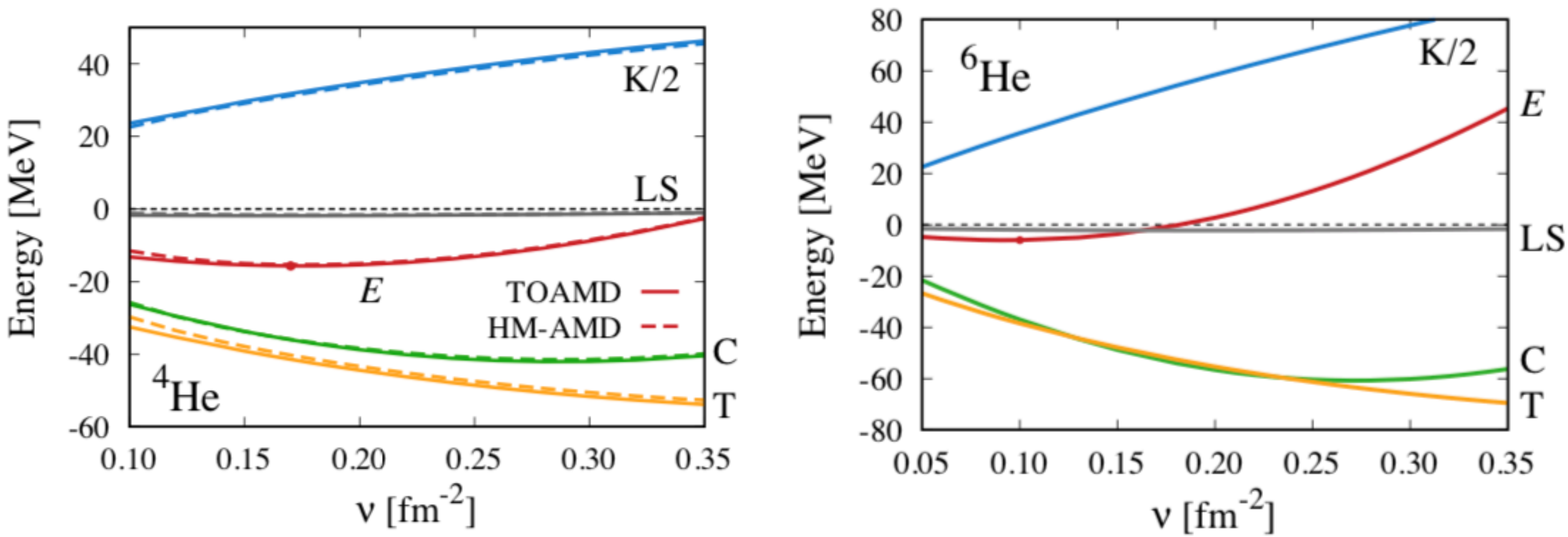
AV8 interaction





1. TOAMD is better than Jastrow correlation method
2. $F(1) \neq F(2)$ is significantly lower than $F(1) = F(2)$

P-shell nuclei
In TOAMD



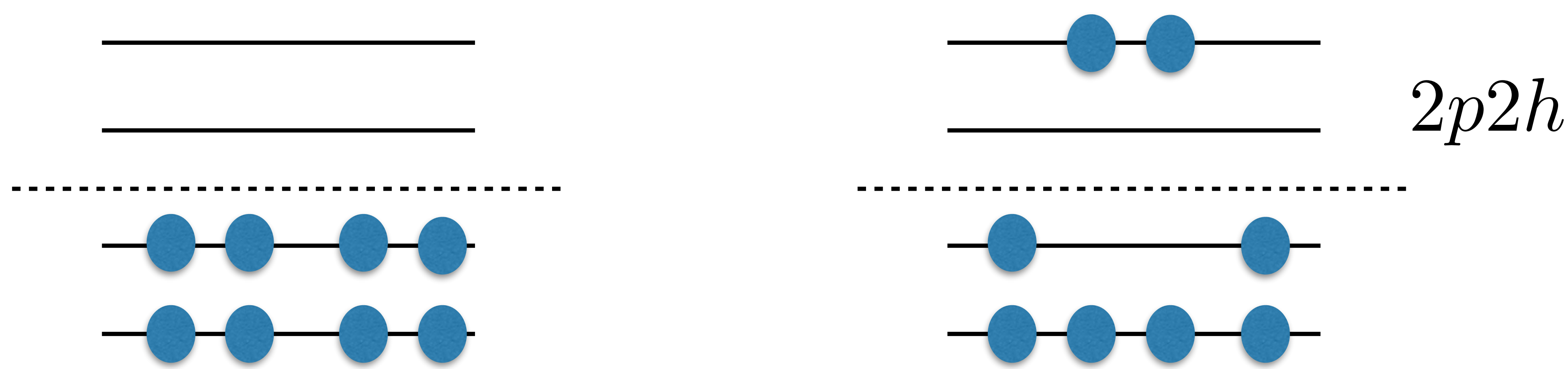
$$\Phi^{J\pm}_{MK,\mathrm{TOAMD}} = (1 + F_S + F_D)\, P^J_{MK} P^{\pm}\, \Phi_{\mathrm{AMD}}$$

Angular momentum projection

Experiments

We have wave function of ground state

$$|A\rangle = (1 + F)|AMD\rangle$$



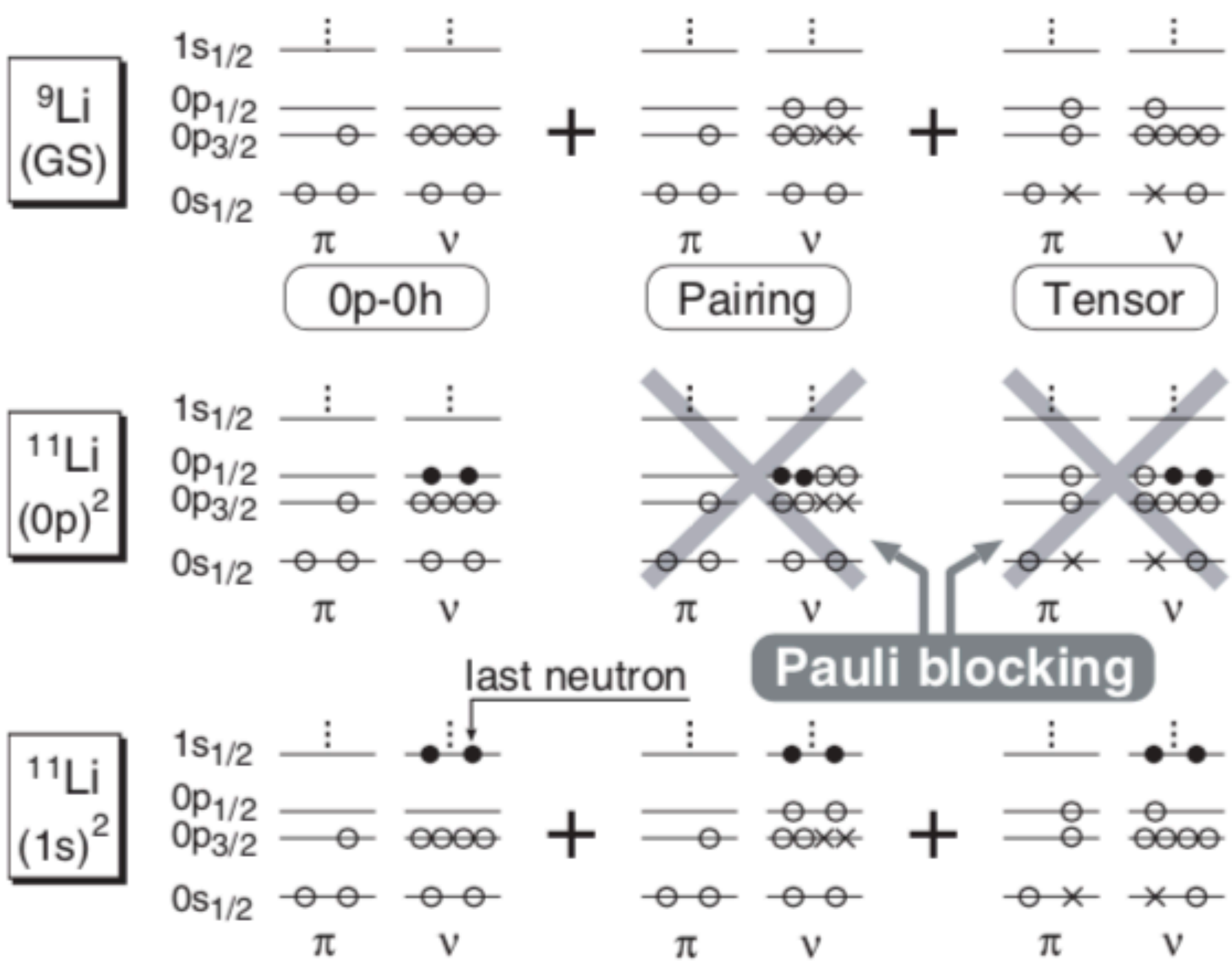
$$\langle A|O|A\rangle \approx \langle \text{model:A}|O|\text{model:A}\rangle + \langle \text{model:A}|F_D O F_D|\text{model:A}\rangle$$

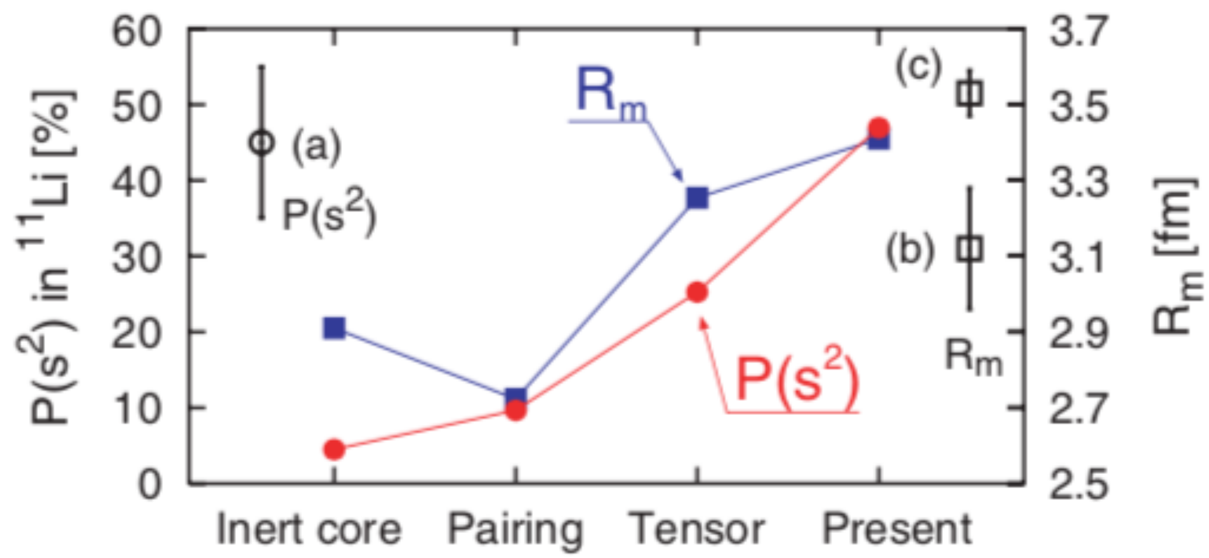
μ	Magnetic moment
$(S_p + S_n)^2$	Spin operators
e^{ikr}	Form factor

PHYSICAL REVIEW C **76**, 024305 (2007)

Roles of tensor and pairing correlations on halo formation in ^{11}Li

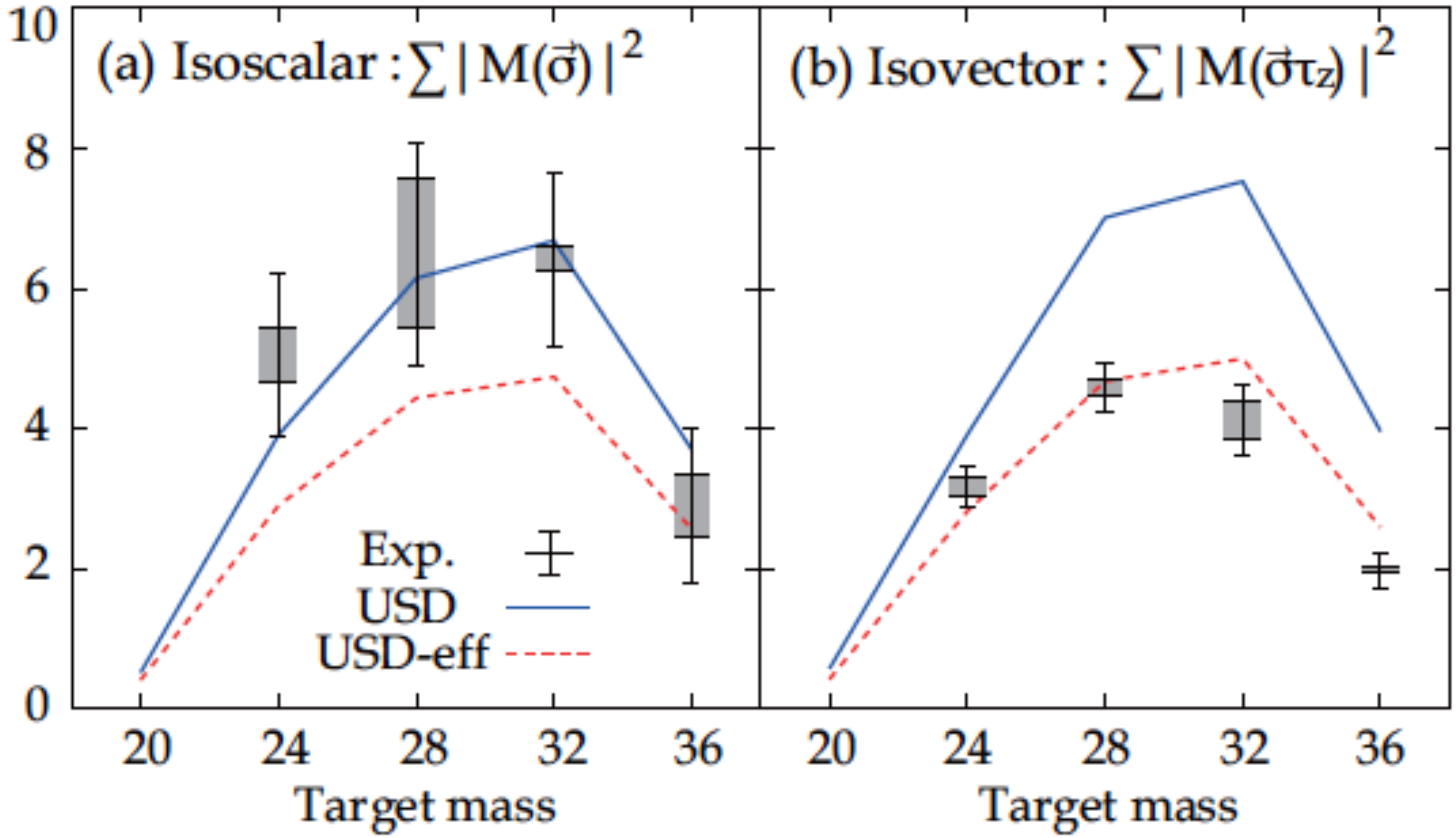
Takayuki Myo,[1,*] Kiyoshi Katō,[2,†] Hiroshi Toki,[1,‡] and Kiyomi Ikeda[3,§]





Tensor correlation reproduces the halo structure of ^{11}Li

Matsubara Tamii..PRL(2015)



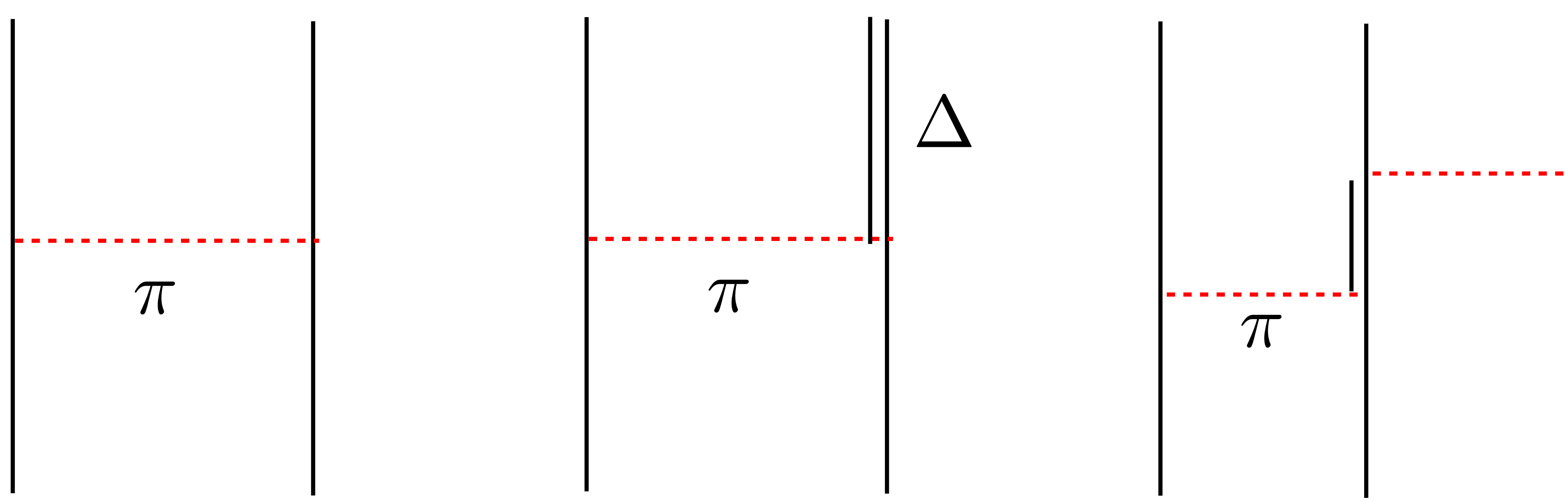
$$(S_p + S_n)^2 \qquad (S_p - S_n)^2$$

$$L_{QCD} = \bar{\psi}(i\gamma_\mu(\partial^\mu - eA^\mu) - m)\psi + \frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a}$$

Chiral symmetry $m \sim 0$

$$m \to M$$

Chiral symmetry breaking (Nambu-Jana Lasinio)



Three body Interaction

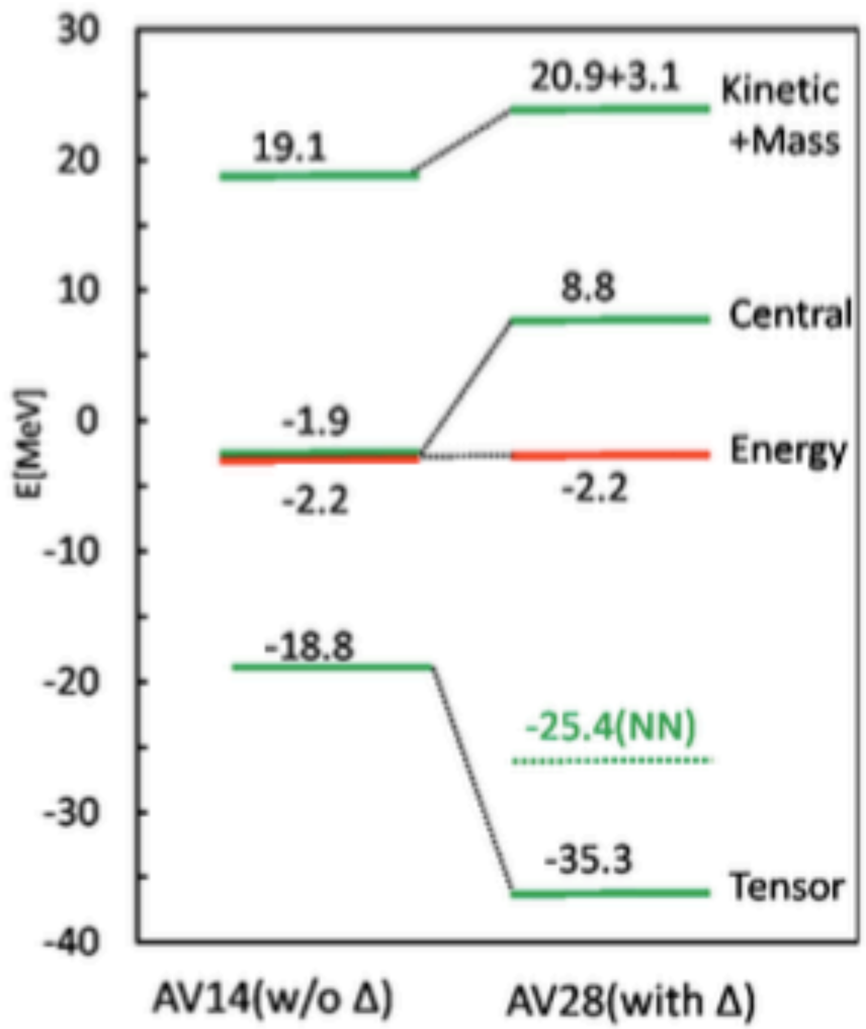
Tensor force and delta excitation for the structure of light nuclei

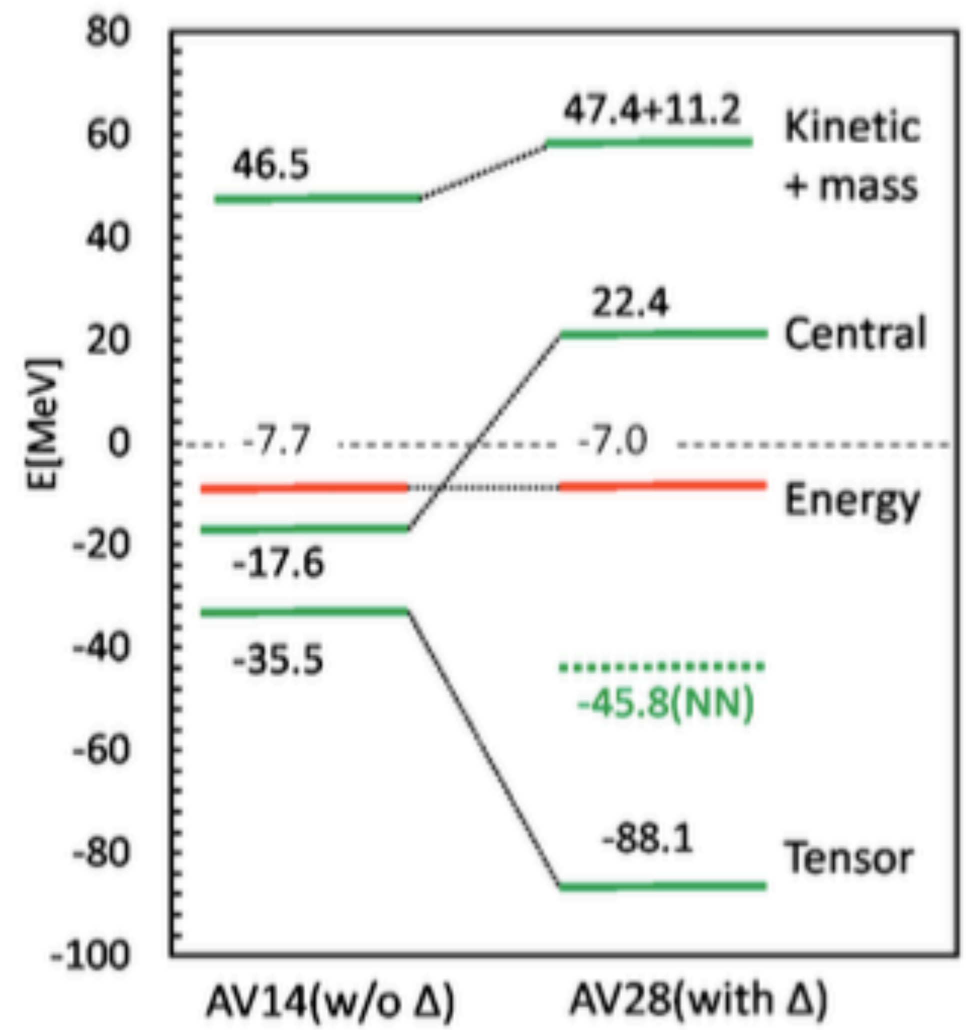
Journal of Physics: Conference Series **569** (2014) 012076

K. Horii[1], T. Myo[2,3], and H. Toki[3]

PRC (1985)
Wiringa et al
AV28

$$H = (T+V)_N + (T+V)_\Delta + V_{N\Delta}$$





All the attraction comes from the tensor interaction due to the delta excitations

Conclusion:

Nuclear Physics = Tensor physics

Pion generates strong tensor interaction

We formulate TOSM + TOAMD

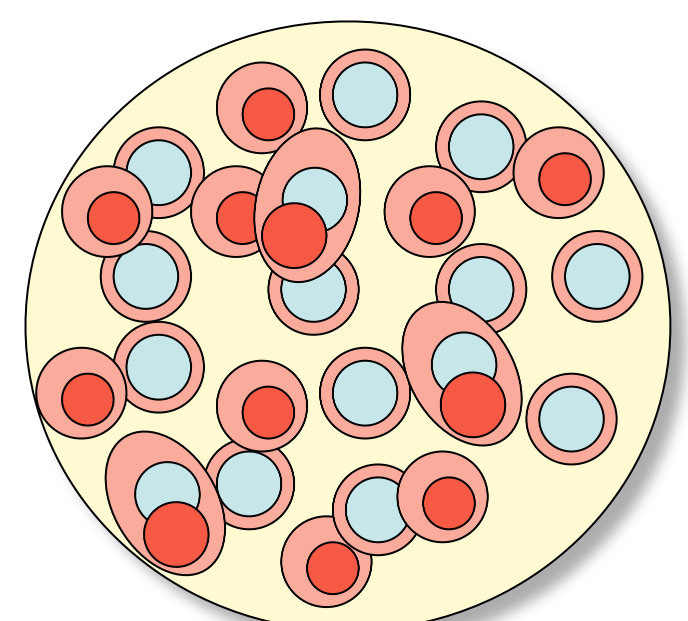
We calculated He3 and He4 using TOAMD

We treat delta excitation explicitly for He3

We develop tensor blocking shell model

Delta is treated explicitly

Hadron nuclear physics



謝謝　Thank you!!

Fruitful collaborations are welcome!