

Nuclear Astrophysics with low-energy RI beams

Lecture by

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the University of Tokyo/

National Astronomical Observatory of Japan



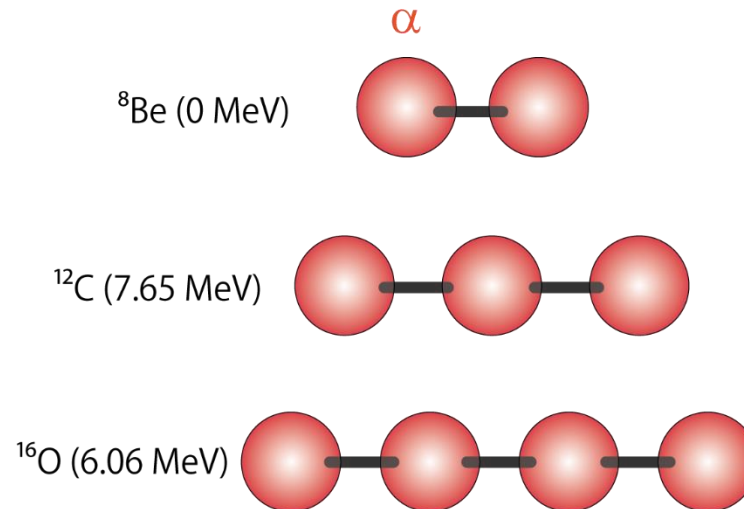
NUSYS@Lanzhou

Lecture #3

- $^{10}\text{Be} + \alpha$ and linear chain with TTIK
(continued)...main interest on nuclear cluster structure
- Trojan Horse Method (THM)
 - ◆ How it works
 - ◆ $^{18}\text{F}(p, \alpha)$ S. Cherubini et al., Phys. Rev. C (2015)...The first THM+RI beam experiment in the world
 - ◆ $^7\text{Be}(n, p)$ and (n, α) for cosmological ^7Li abundance problem
- r-process study at RIKEN RIBF

Morinaga (1956) and linear chain

- Discussed on $4n$ -nuclei based on the [alpha particle model](#)
- Predicted linear-chains in ^{12}C , ^{16}O , etc., from their high momenta of inertia.

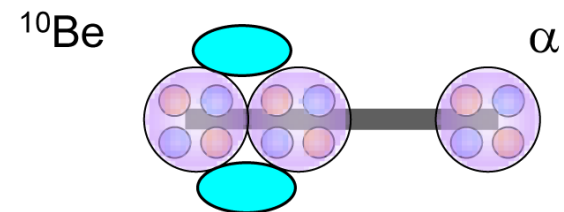
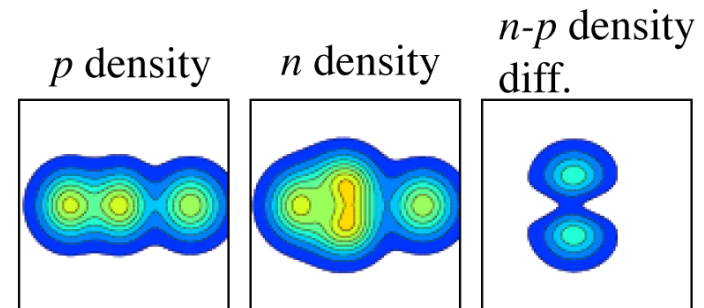


- It was shown in later studies that the Hoyle state is **NOT** a linear-chain state.

$^{10}\text{Be} + \alpha$

- Linear-chain cluster levels in ^{14}C were predicted in Suhara & En'yo papers.
- Asymmetric, $^{10}\text{Be} + \alpha$ configuration ...likely to be observed with $^{10}\text{Be} + \alpha$ alpha-resonant scattering.
- May form a band with $J^\pi = 0^+, 2^+, 4^+$ a few MeV above α -threshold.
- Scattering of two 0^+ particles...only l -dependent resonant profile.

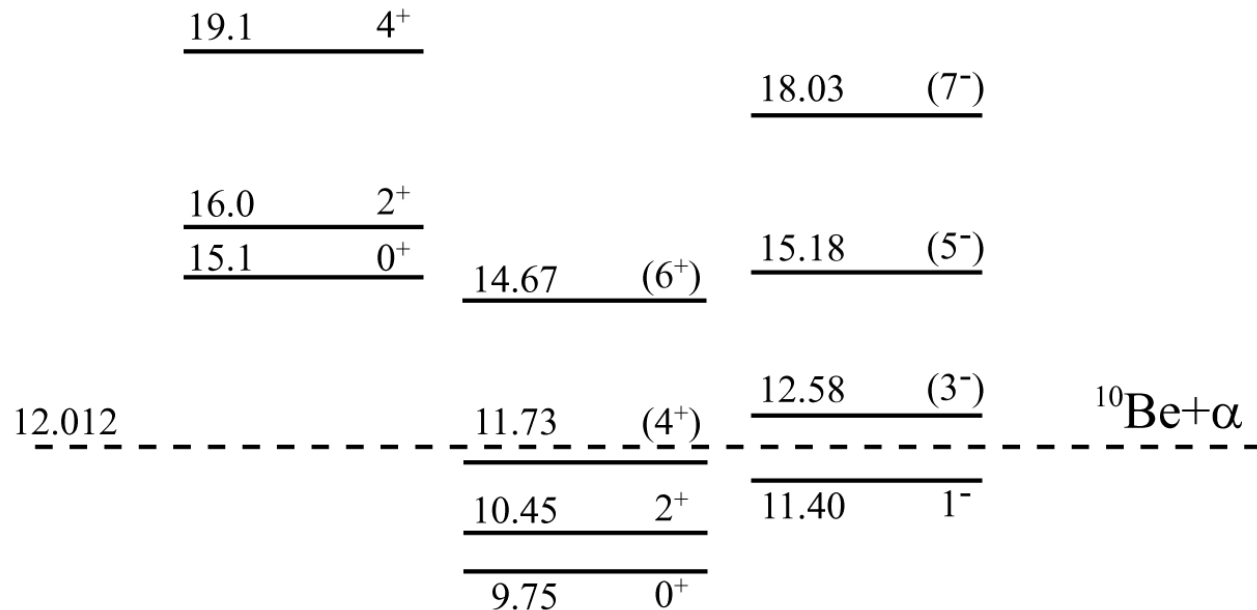
Suhara & En'yo, PRC 2010 and 2011:



Similar experiments independently conducted by Birmingham group [M. Freer et al., PRC 2014] and MSU group [A. Frisch et al., PRC 2016]

Cluster bands

- Predicted energy...few MeV above the $^{10}\text{Be}+\alpha$ threshold



Linear chain states
in the calculation by
Suhara&En'yo (2010)

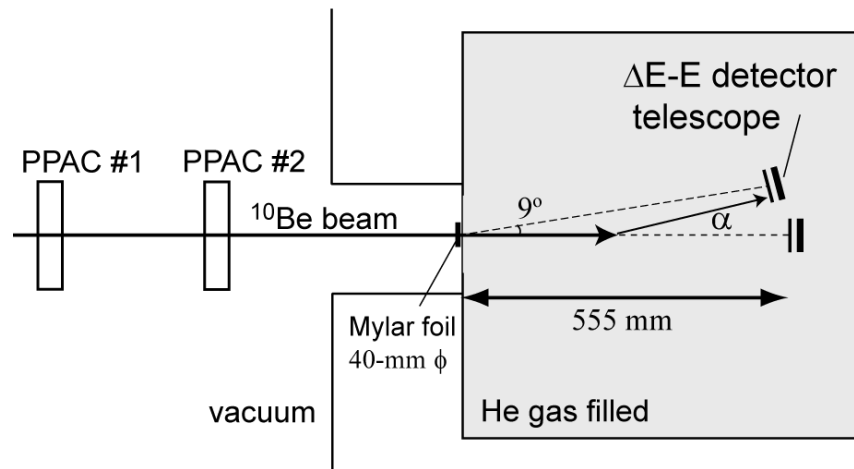
$K=0^+$

$K=0^-$

Prolate rotational bands
in Oertzen et al., (2004)

Experimental setup

Thick target method in inverse kinematics,
similar to the previous ${}^7\text{Be}+\alpha$.

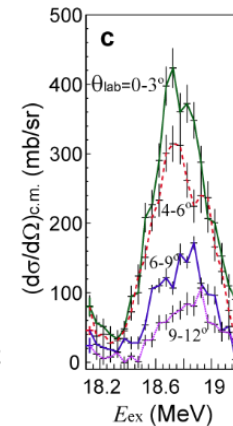
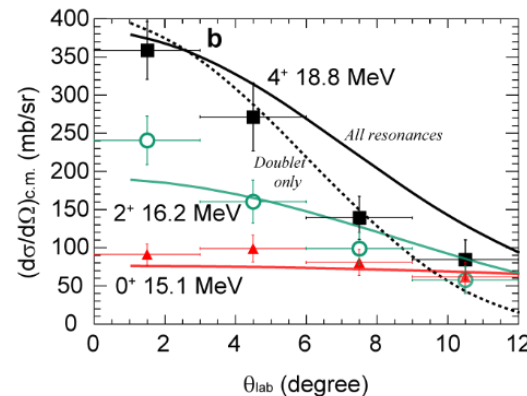
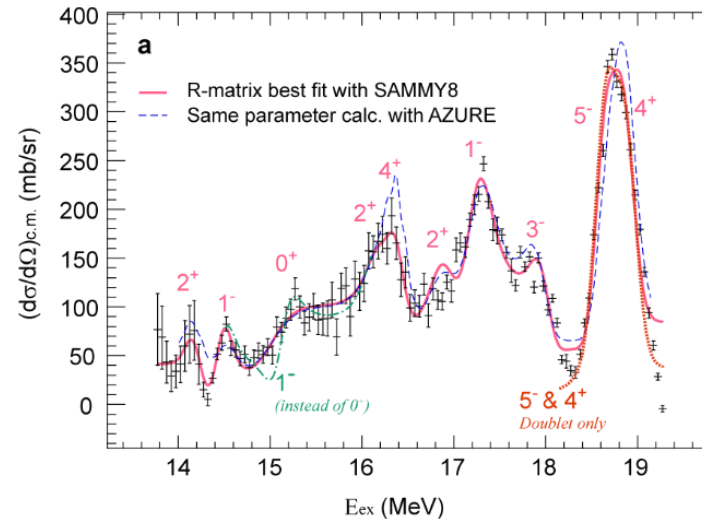


- Two PPACs for the beam PI, trajectory, number of particles.
- Two silicon detector telescopes for recoiling α particles.
- E_{cm} and θ obtained by event-by-event kinematic reconstruction.

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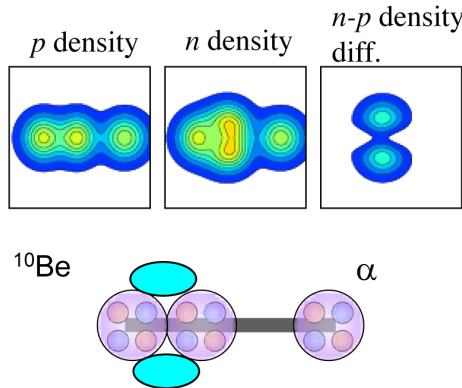
Excitation function

- The excitation function we obtained for 13.8-19.2 MeV exhibits many resonances.
- R-matrix analysis performed, and some of the resonance parameters (E , J^π , Γ_α) were determined.



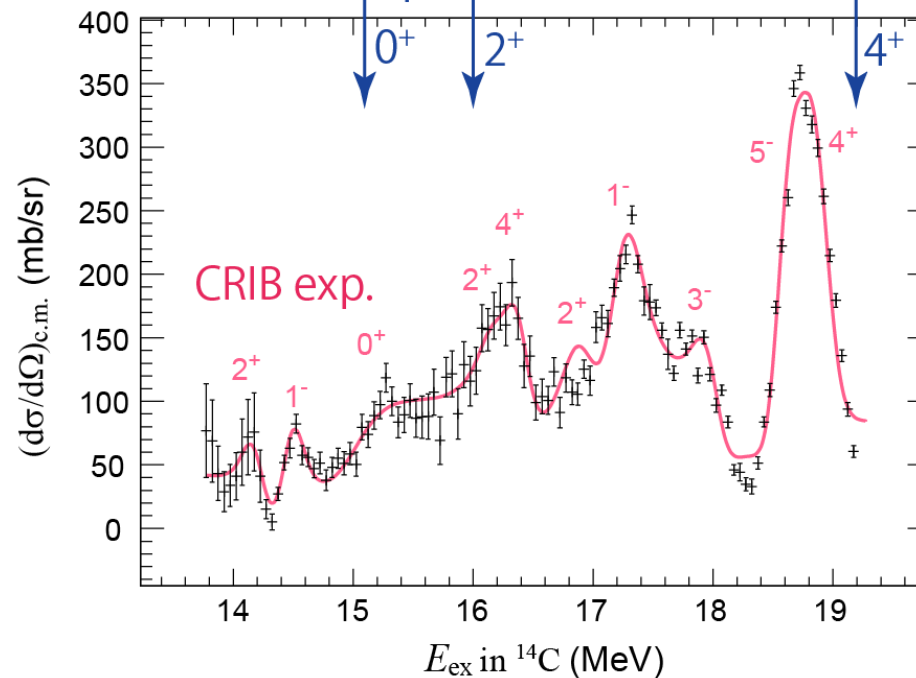
Result of the linear chain search

Suhara & En'yo, PRC 2010 and 2011:



Excellent agreement
between exp. and
theory for the (0^+ ,
 2^+ , 4^+) states.

Theoretical prediction of linear-chain states



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Experimental investigation of a linear-chain structure in the nucleus ^{14}C

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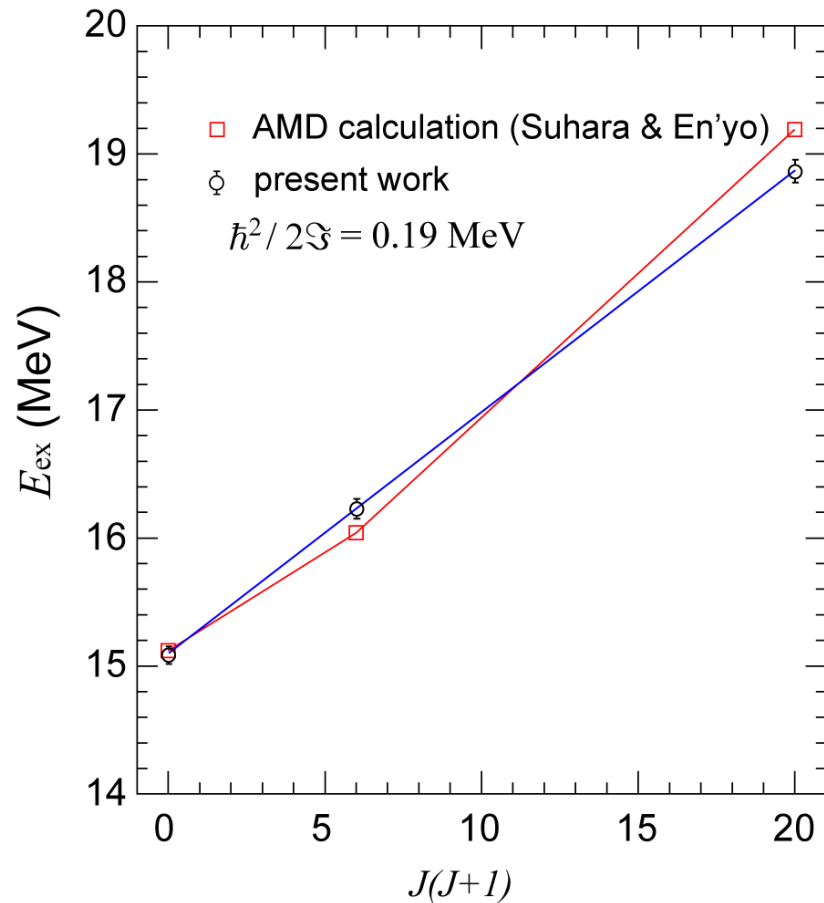
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Rotational Band

The set of resonances we observed ($0+$, $2+$, $4+$) is proportional to $J(J+1)$...
consistent with a view of rotational band.

Also perfectly consistent with the theoretical prediction.



Experiments in other facilities

Results on two other $^{10}\text{Be}+\alpha$ TTIK experiments were published before our publication was made.

- M. Freer et al., Phys. Rev. C (2014)
Birmingham group+ at ORNL
 - High-intensity ^{10}Be beam, spectrum at very forward angle, no PI

Agreement over $E_x > 16$ MeV, in spite of the difference in the absolute c.s.

- A. Fritsch et al., Phys. Rev. C (2016)
MSU group at Notre Dame
 - Low-intensity ^{10}Be beam, Active target, only side angles.

Cannot compare directly, but not good agreement?

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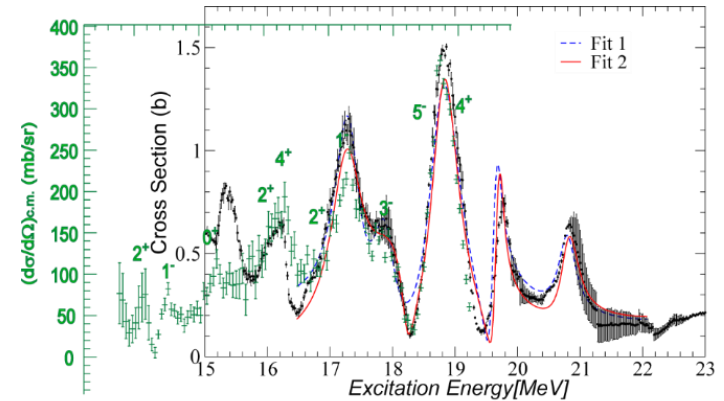
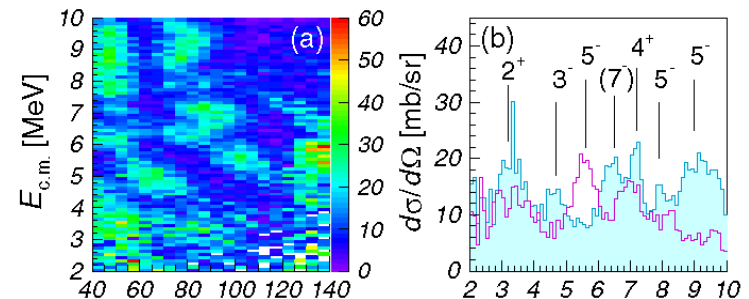


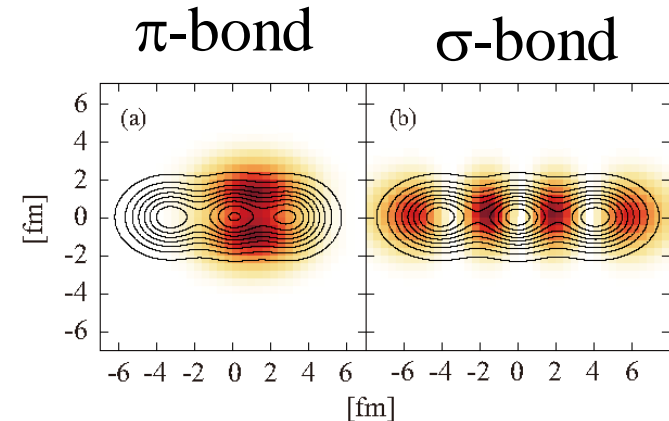
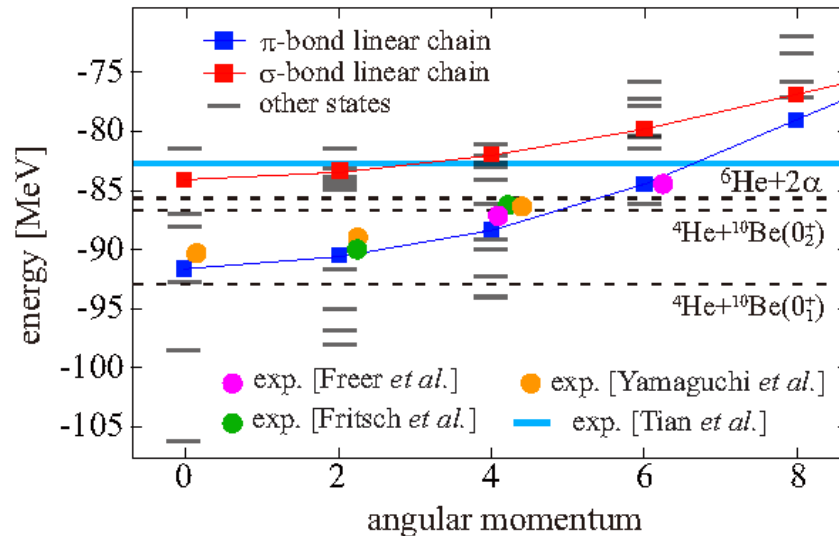
FIG. 11. (Color online) R-matrix fit to the data in the region between $E_x = 16.5$ and 22 MeV (red-solid and blue-dashed lines). The difference between the two fits is the inclusion of an additional 4^+ state in the calculation shown by the red line. See Table for the parameters of fits 1 and 2.



$$E_{\text{cm}} = E_x - 12 \text{ MeV}$$

Baba and Kimura (2016 & 2017)

PHYSICAL REVIEW C **95**, 064318 (2017)



Another AMD calculation,

“ π -bond” linear chain band, consistent with 3 experiments

“ σ -bond” linear chain band at higher energy (studied by Peking Univ. group).

How certain are the linear-chain states?

- Identification of the 0^+ state... 1^- was excluded with 3σ significance, but the error can be **systematic**.
 - ◆ Limited statistics and angular range
 - ◆ Background subtraction
 - ◆ Inelastic scattering?
- We planned the 4th experiment at INFN-LNS (Catania, Italy):
 - ◆ With offline-production ^{10}Be beam
 - ◆ Inelastic scattering separation with TOF.

⇒ Performed in Oct., 2018.

The “CHAIN” experiment at INFN-LNS (Catania, Italy)

$^{10}\text{Be} + \alpha$ with more intense beam, higher energy and angular resolution: ~2 weeks beamtime.

Investigation of α -chain structures in ^{14}C .

H. Yamaguchi¹, A. Di Pietro², R. Dressler³, J. P. Fernández-García⁴, P. Figuera², S. Hayakawa¹, S. Heinitz³, D. Lattuada⁵, M. Lattuada^{2,6}, E. Mauger³, M. Milin⁷, H. Shimizu¹, A. C. Shotton⁸, D. Shumann³, N. Soic⁹, D. Torresi², L. Yang¹, M. Zadro⁹,

¹ *Center for Nuclear Study (CNS), University of Tokyo, RIKEN, Wako, Japan*

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⁴ *Paul Scherrer Institute, Villigen, Switzerland*

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⁶ *Dipartimento di Fisica e Astronomia, Catania, Italy*

⁷ *Physics Department, Faculty of Science, University of Zagreb, Zagreb, Croatia*

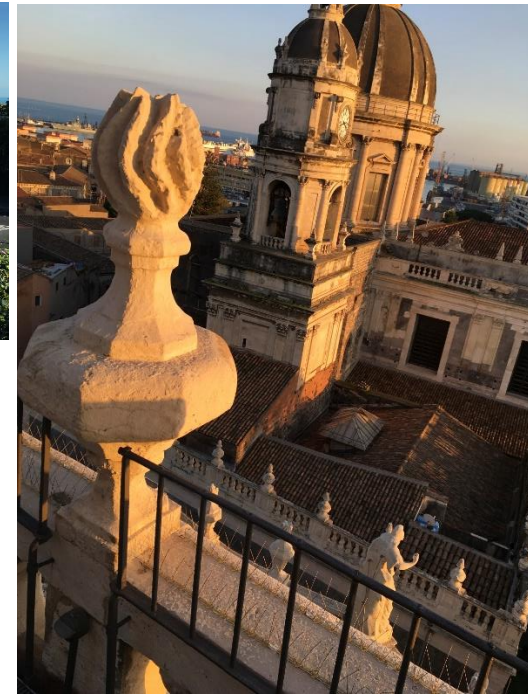
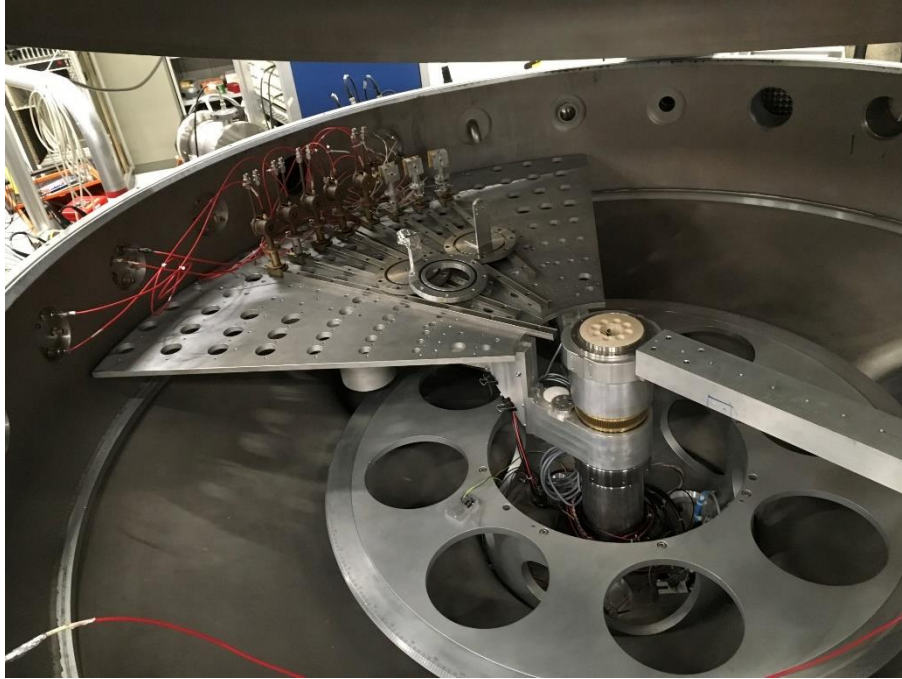
⁸ *School of Physics and Astronomy, University of Edinburgh, UK*

⁹ *Ruder Bošković Institute, Bijenička, Zagreb, Croatia*

Abstract

We propose to measure the excitation function for the elastic scattering process $^{10}\text{Be} + ^4\text{He}$, in order to shed some light upon the existence of linear-chain cluster states in the n-rich ^{14}C nucleus. These states are expected to have a configuration in which ^{10}Be and α are spatially separated and thus they can be observed by the $^{10}\text{Be} + \alpha$ resonant elastic scattering. In order

Experiment at Catania, Oct. 2018



Result (very preliminary)

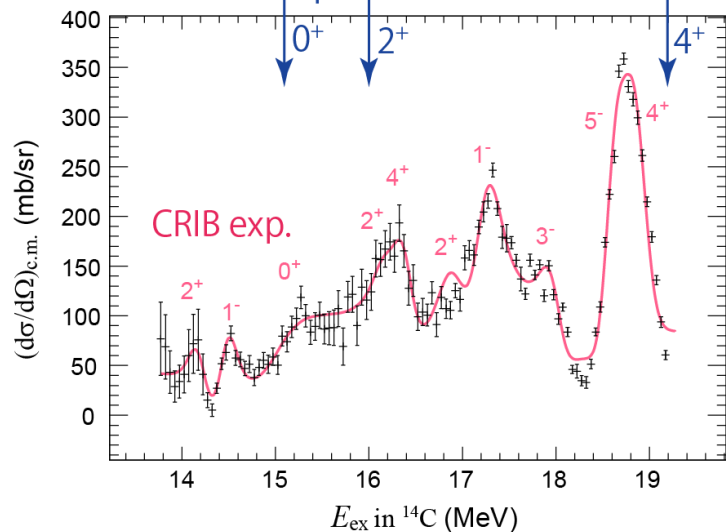
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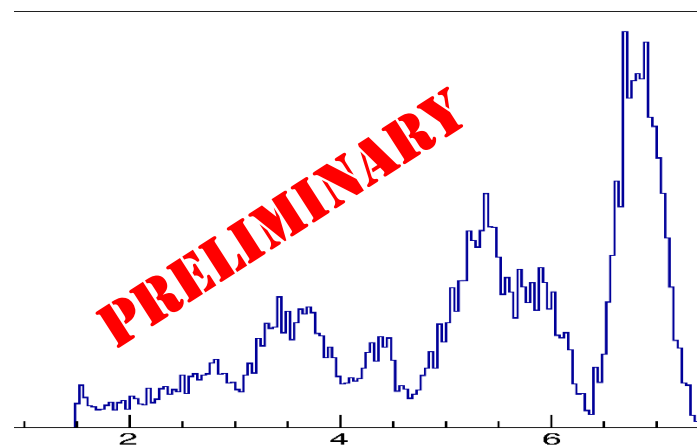
LNS(Tandem)

Including 0-8 deg events

Theoretical prediction of linear-chain states



@5 deg, No normalization for the effective target thickness/absolute cross section yet

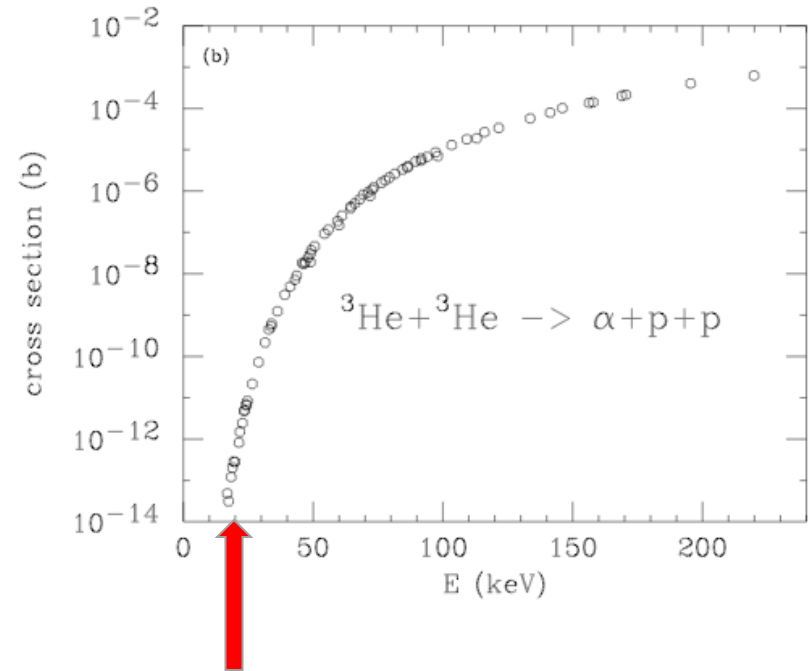


Need of indirect method

Stellar reaction cross section often has a strong dependence on energy (or temperature), changing by orders of magnitude.

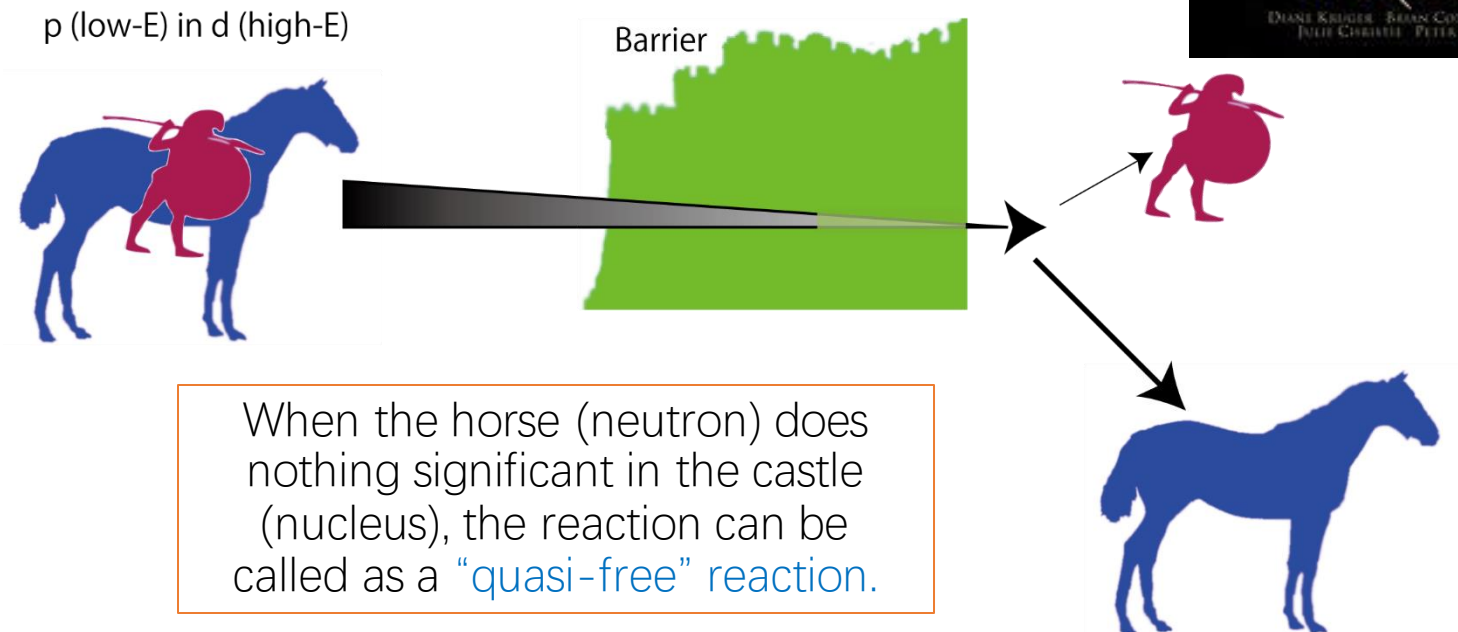
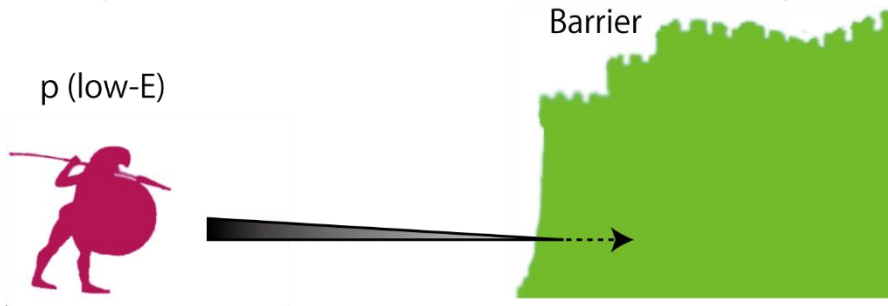
...This is because of the **tunneling probability of the Coulomb barrier**.

Experimentally, this causes much trouble. We need a clever way.



Gamow energy
of Solar T

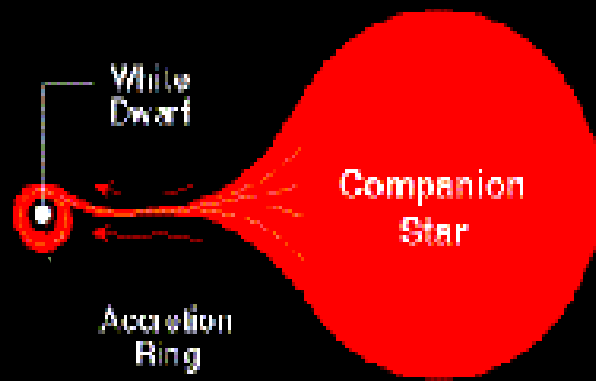
The Trojan Horse Method (THM)



The $^{18}\text{F}(\text{p},\alpha)$ project (with THM)

- $^{18}\text{F}(\text{p},\alpha)$... an astrophysical reaction important in novae, and other high-T environments.
- Measurement with the **Trojan Horse Method** performed in 2008
...**The first THM+RI beam experiment in the world.**
- The RI Beam at CRIB (after development):
Primary beam: $^{18}\text{O}^{8+}$, 4.5-5 MeVA
Production target: H_2
Production reaction: $^{18}\text{O}(\text{p},\text{n})^{18}\text{F}$
 - ◆ Purity **nearly 100%**
 - ◆ Intensity **$> 5 \times 10^5$ pps**

A NOVA MICKEY MOUSE PICTURE AND $^{18}\text{F}(p, \alpha)^{15}\text{O}$



Thin hydrogen surface layer
accumulated on white dwarf
through accretion ring

Observed γ - rays come from e^+e^-

e^+ come from ^{18}F decay mostly

At novae temperatures (100-500
keV) ^{18}F can be mainly destroyed
by



Surface
Layer

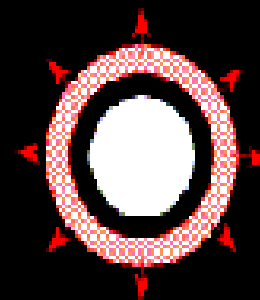


White
Dwarf

Ignition of surface layer
under degenerate
conditions



Thermonuclear
runaway until
degeneracy lifted



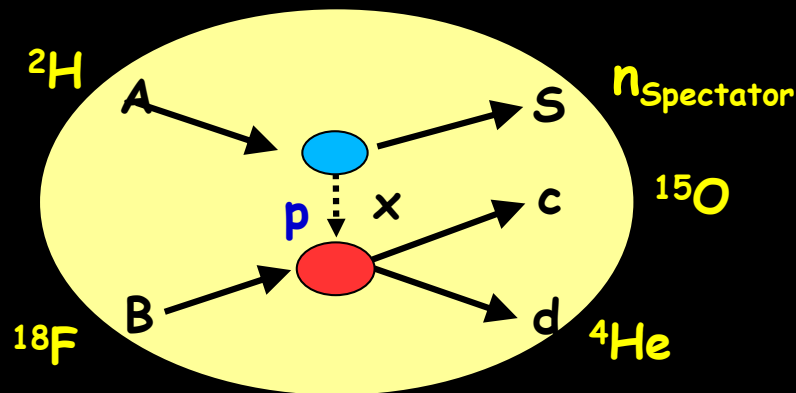
Explosive
Burning of
Hydrogen
Shell

*Slides by
S. Cherubini*

*For the star energetics
this is peanuts!*

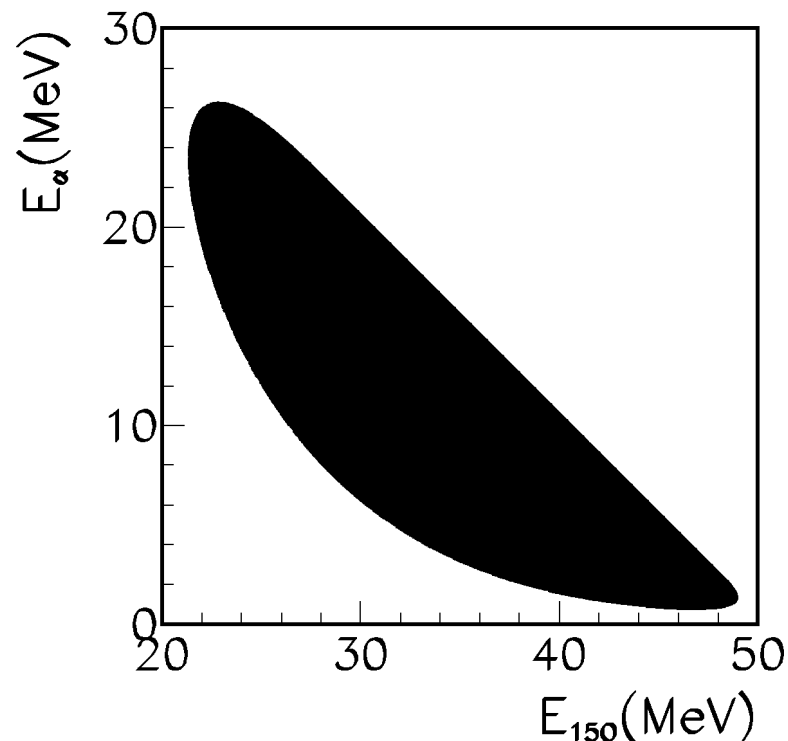
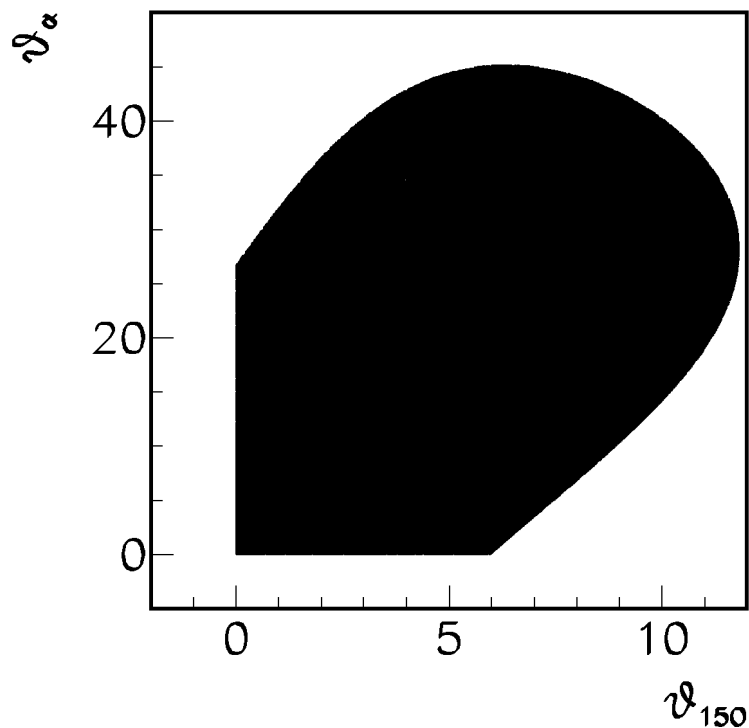
THM measurement: $^{18}\text{F}(p,\alpha)^{15}\text{O}$ via $^2\text{H}(^{18}\text{F},\alpha^{15}\text{O})n$

Kinematics

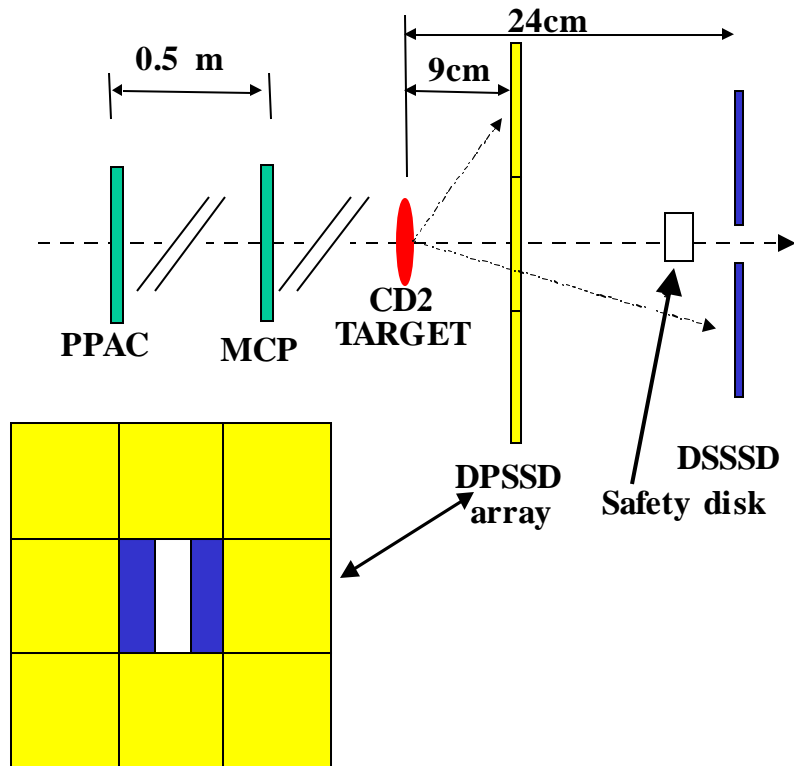


$^2\text{H}(^{18}\text{F},\alpha^{15}\text{O})n$

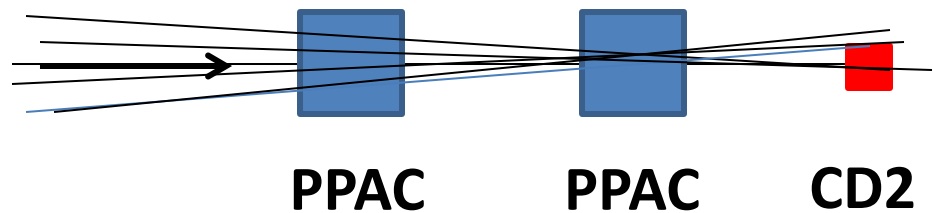
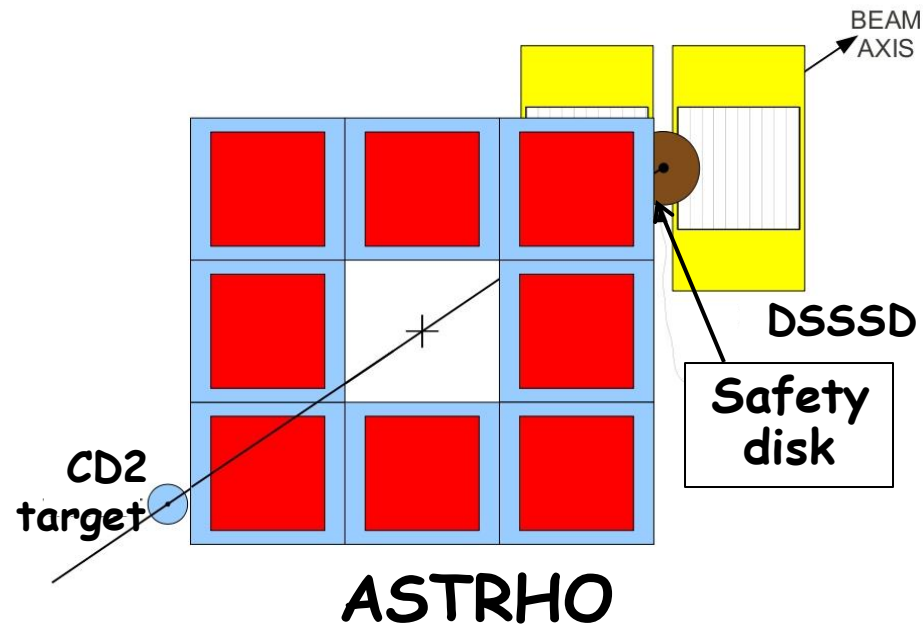
$E(^{18}\text{F}) = 50 \text{ MeV}$



EXPERIMENTAL SETUP



ASTRHO:
Array of Silicons for
TRojan HOrse



Beam track reconstruction

EVENT SELECTION

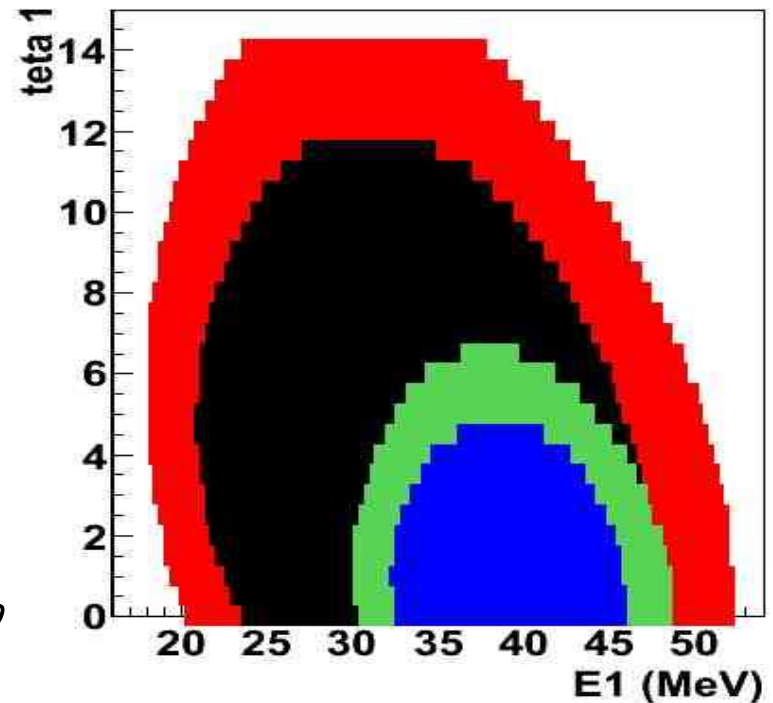
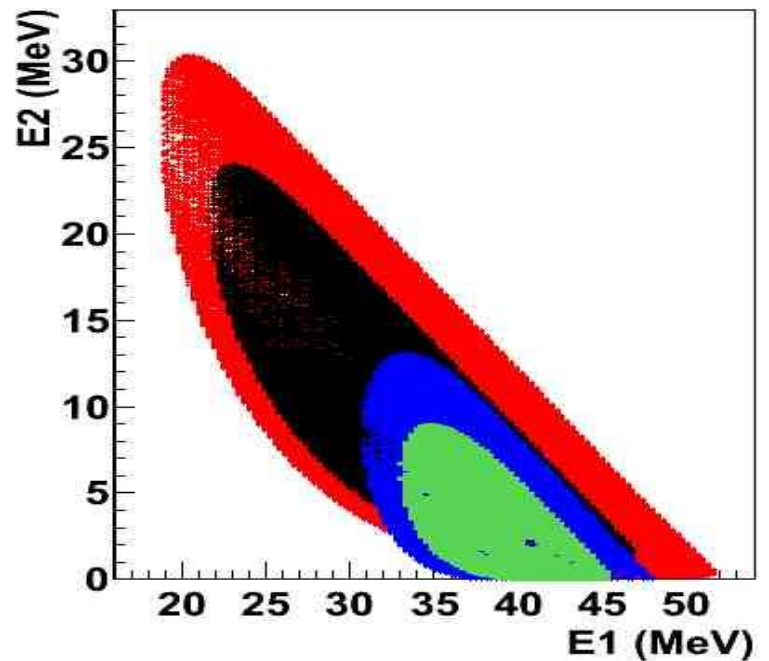
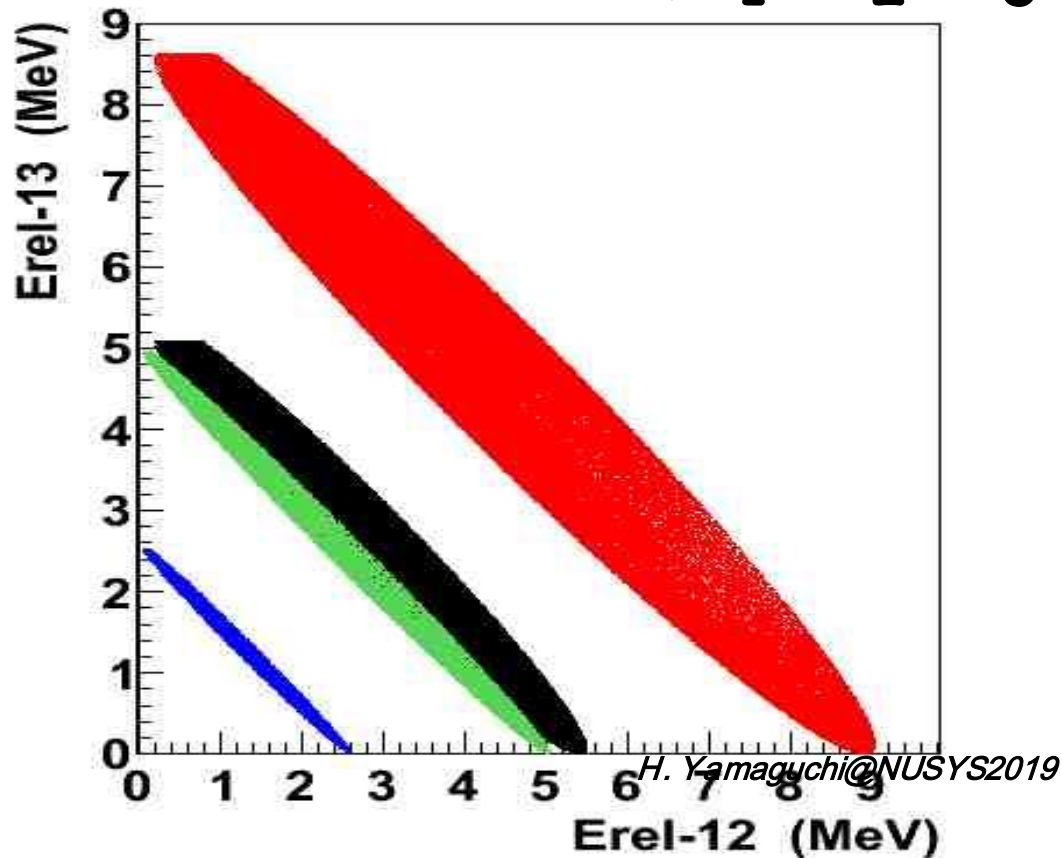
Red: $^{18}\text{F} + d \rightarrow ^{15}\text{N} + \alpha + p$

Black: $^{18}\text{F} + d \rightarrow ^{15}\text{O} + \alpha + n$

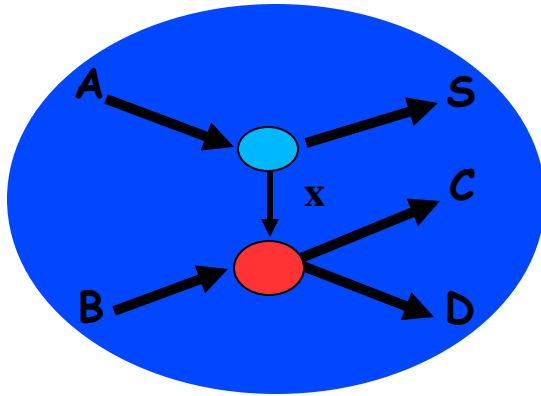
Blue: $^{18}\text{F} + d \rightarrow ^{18}\text{F} + p + n$

Green: $^{18}\text{F} + d \rightarrow ^{18}\text{O} + p + p$

» 1 + 2 + 3

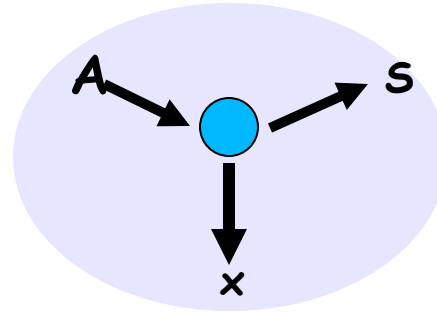


Assuming that a Quasi-free mechanism is dominant one can use the (PW)IA:



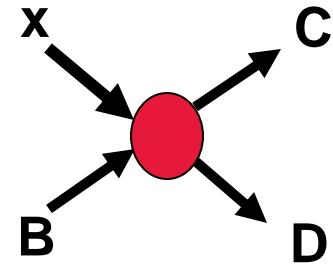
3-body Reaction

=

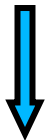


Virtual Decay

\otimes



Virtual reaction
(astrophysical process)



$$\frac{d^3 \sigma}{d\Omega_C d\Omega_D dE_{cm}}$$

Measured
at high
energy



$$KF \cdot |\Phi(P_s)|^2$$

Calculated
e.g.
Montecarlo



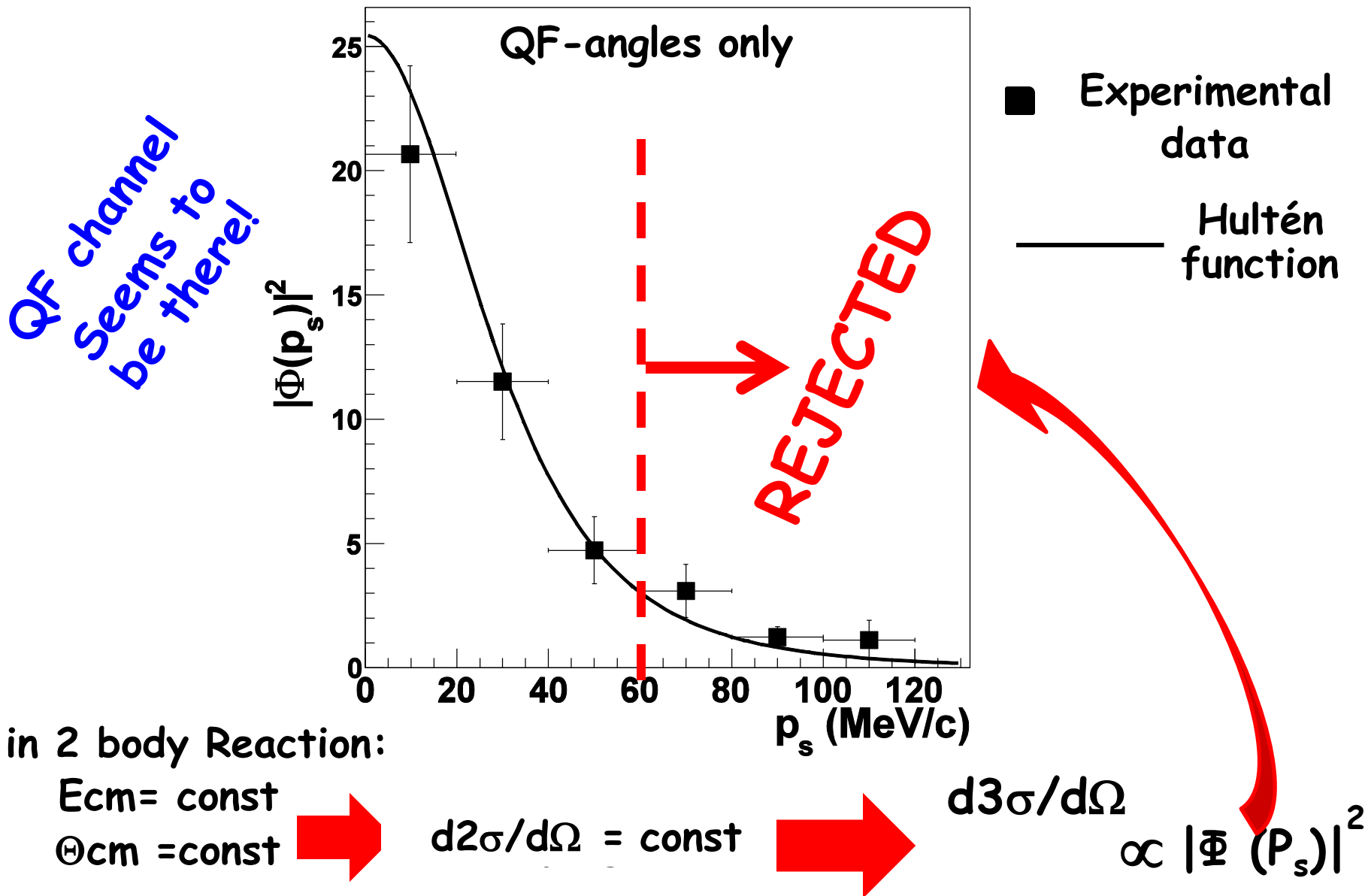
$$\frac{d\sigma^N}{d\Omega}$$

Indirectly
Measured

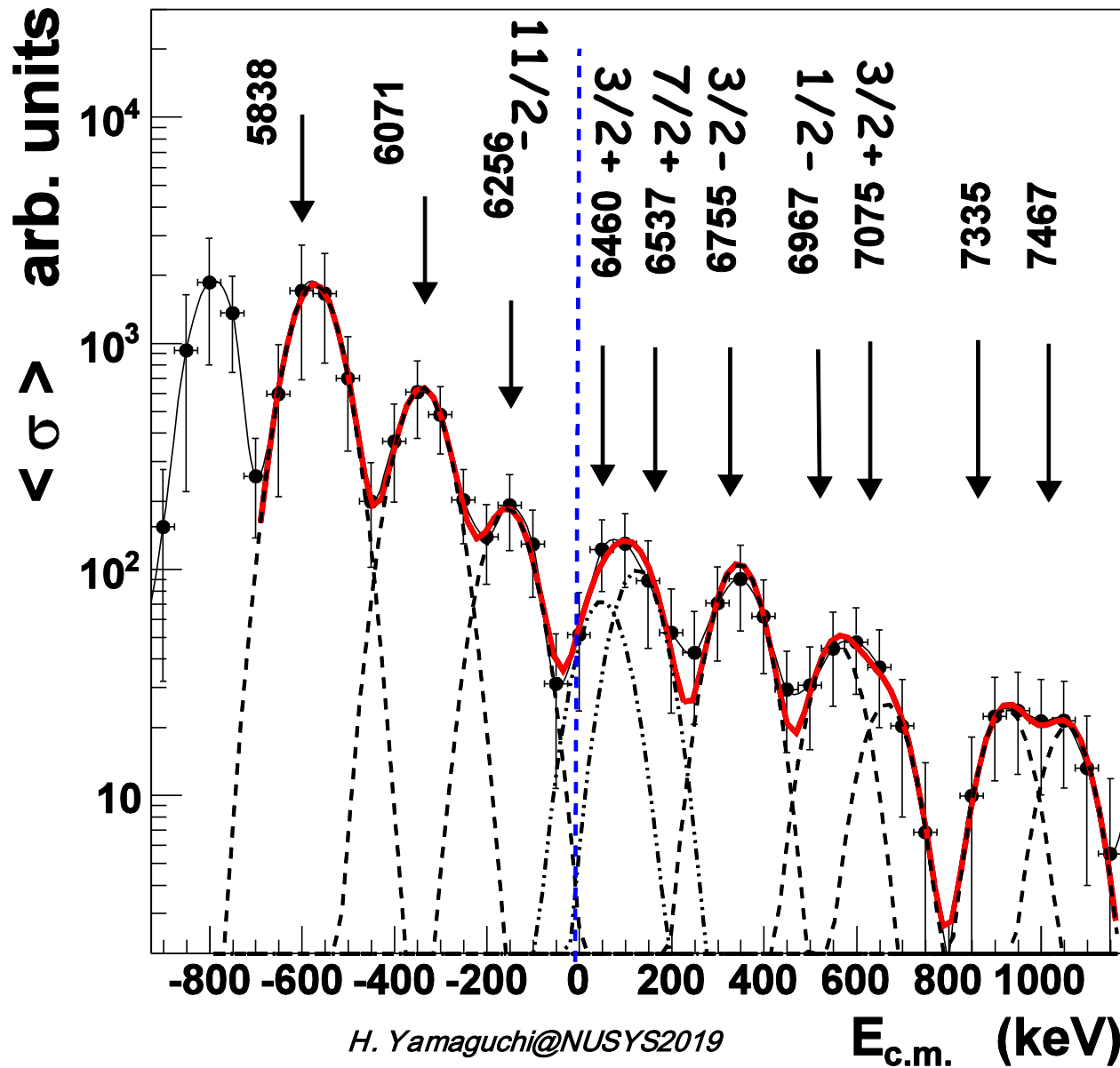
$$E_{Bx} = E_{CD} - Q_{2b}$$

\propto

EXPERIMENTAL IMPULSE DISTRIBUTION



THM(=barriers free) CROSS SECTION



$S(E)$ from THM 8 keV 3/2+

S. Cherubini et al., PRC (2015).

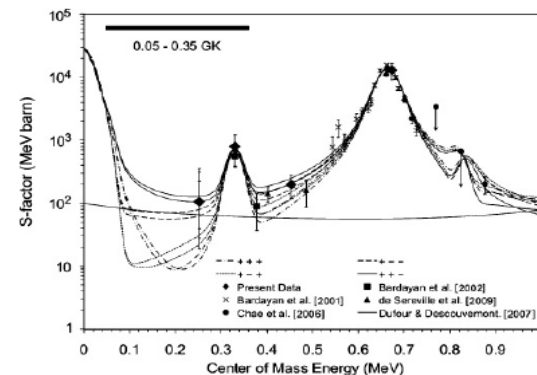
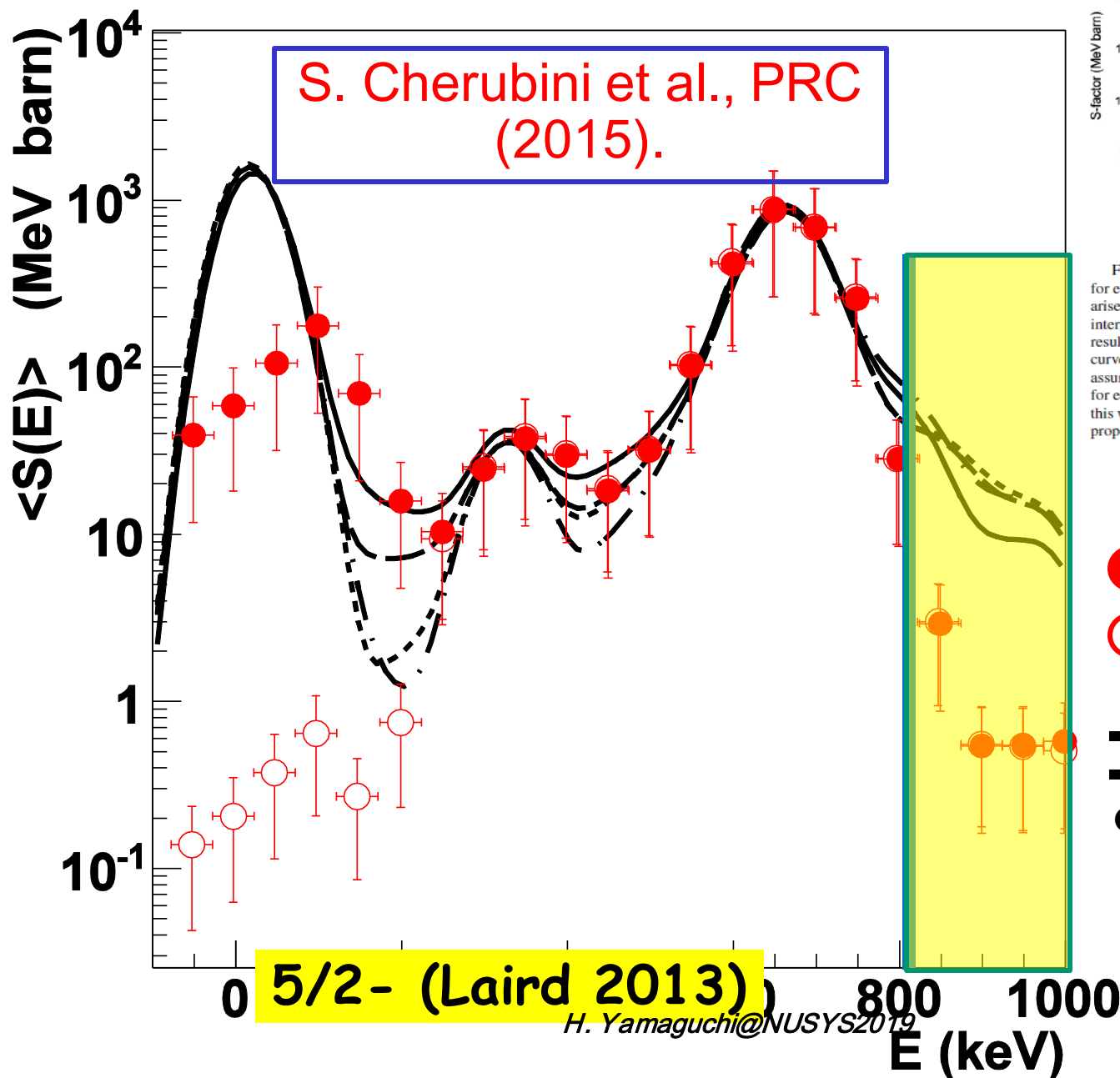


FIG. 3. The $^{18}\text{F}(p, \alpha)^{15}\text{O}$ S factors, calculated using the R matrix, for eight possible interference terms. The range in possible S factors arises from the interference between the $J^\pi = 3/2^+$ resonances. The interference between resonances dominates in the region of interest, resulting in four groups of S -factor curves. The upper and lower curves of each group are shown in the figure. The legend gives the assumed phase, for the 8-, 38-, and 665 keV resonances, respectively, for each pair of curves. Also plotted are the measured S factors from this work, those from previously published data [4,10,12,19], and the proposed contribution from $1/2^+$ states predicted in Ref. [6].

Direct data...C.E. Beer, et al.

$^7\text{Be}(n,p)^7\text{Li}$ and the $^7\text{Be}(n,\alpha)^4\text{He}$ reactions with THM for cosmological lithium problem

S. Hayakawa¹, M. La Cognata², L. Lamia², H. Shimizu¹, L. Yang¹, H. Yamaguchi¹, K. Abe¹, O. Beliuskina¹, S. M. Cha⁴, K. Y. Chae⁴, S. Cherubini^{2,3}, P. Figuera^{2,3}, Z. Ge⁵, M. Gulino^{2,6}, J. Hu⁷, A. Inoue⁸, N. Iwasa⁹, D. Kahl¹⁰,
A. Kim¹¹, D. H. Kim¹¹, G. Kiss⁵, S. Kubono^{1,5,7}, M. La Commara^{12,13}, M. Lattuada^{2,3}, E. J. Lee⁴, J. Y. Moon¹⁴,
S. Palmerini^{15,16}, C. Parascandolo¹³, S. Y. Park¹¹, D. Pierrotsakou¹³, R. G. Pizzone^{2,3}, G. G. Rapisarda²,
S. Romano^{2,3}, C. Spitaleri^{2,3}, X. D. Tang⁷, O. Trippella^{15,16}, A. Tumino^{2,6}, P. Vi⁵ and N. T. Zhang⁷

¹Center for Nuclear Study (CNS), University of Tokyo, ²INFN - Laboratori Nazionali del Sud,

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⁷Institute of Modern Physics, Chinese Academy of Sciences, ⁸Research Center for Nuclear Physics (RCNP), Osaka University, ⁹Tohoku University, ¹⁰University of Edinburgh, ¹¹Ewha Womans University,

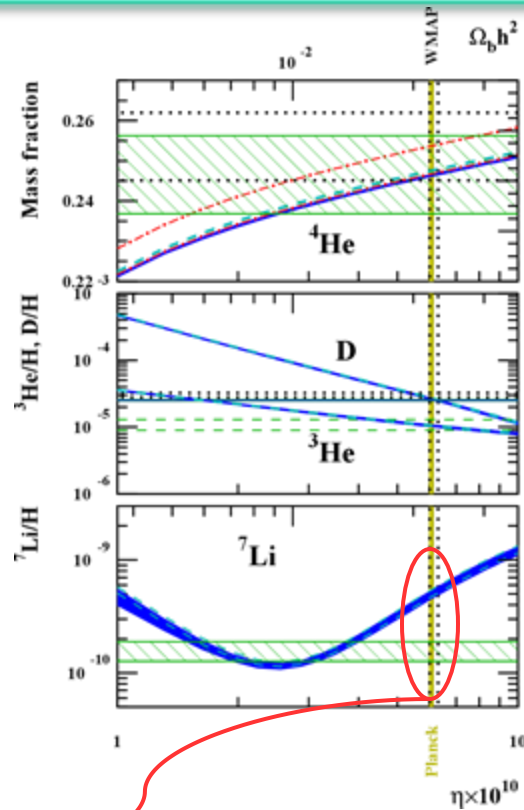
¹²University of Naples Federico II, ¹³INFN – Naples, ¹⁴Institute for Basic Science (IBS),

•¹⁵INFN - Perugia, ¹⁶University of Perugia

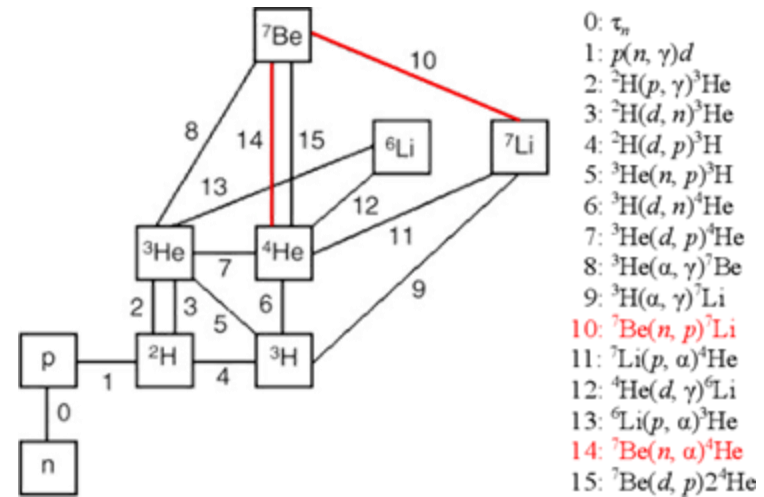


+ many others

Cosmological ^7Li problem



A. Coc et al. J. Cos. Astropart. Phys. 2014

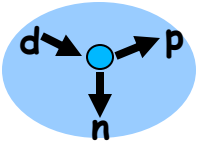


locco et al. Phys. Rep. 2009

- ^7Li problem... disagreement between theory and observation by a factor of 3–4
 - Due to CMB obs.? Low-metallicity stars obs.? Standard BBN model? **Nuclear Physics?**
 - ^7Be abundance in the end of BBN determines ^7Li predominantly
 - $p(n,\gamma)d$, $^3\text{He}(d,p)^4\text{He}$, $^7\text{Be}(n,p)^7\text{Li}$, $^7\text{Be}(n,\alpha)^4\text{He}$, $^7\text{Be}(d,p)2\alpha$, etc.
- Temperature $\sim 10^{10} - 3 \times 10^8$ K, Energy: **1 MeV – 25 keV**

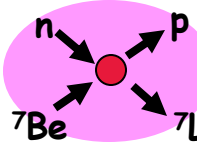
Trojan Horse Method for RI + neutron

- Trojan Horse method: (Spitaleri+ Phys. Atom. Nucl. 2011)
- ${}^7\text{Be}(n,p){}^7\text{Li}$, ${}^7\text{Be}(n,\alpha){}^4\text{He}$ via ${}^2\text{H}({}^7\text{Be}, {}^7\text{Li}p){}^1\text{H}$, ${}^2\text{H}({}^7\text{Be}, \alpha\alpha){}^1\text{H}$
- PWIA applicable when Quasi-free mechanism is dominant



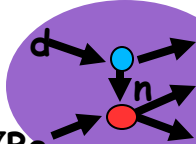
Virtual Decay

\otimes



Virtual reaction

$=$



3-body Reaction

$(\text{Kinematic Factor}) \cdot |\Phi(P_s)|^2 \times$
 Calculated by
 Monte Carlo simulation

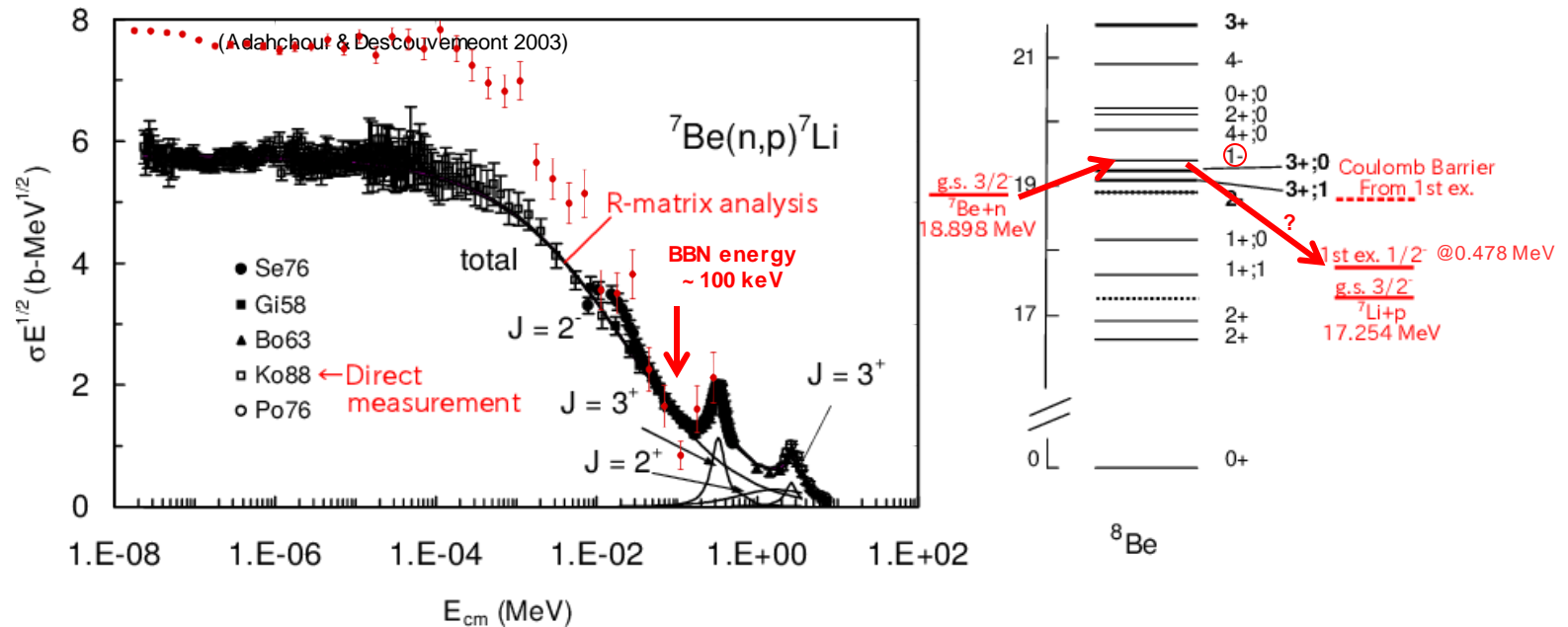
$\times \frac{d\sigma^{\text{HOES}}}{d\Omega}$
Half-off energy-shell
2-body cross section

\propto

$\frac{d^3\sigma}{d\Omega_p d\Omega_{7\text{Li}} dE_{\text{cm}}}$
 Measured at
 $E_{d-7\text{Be}} > \text{Coulomb barrier}$

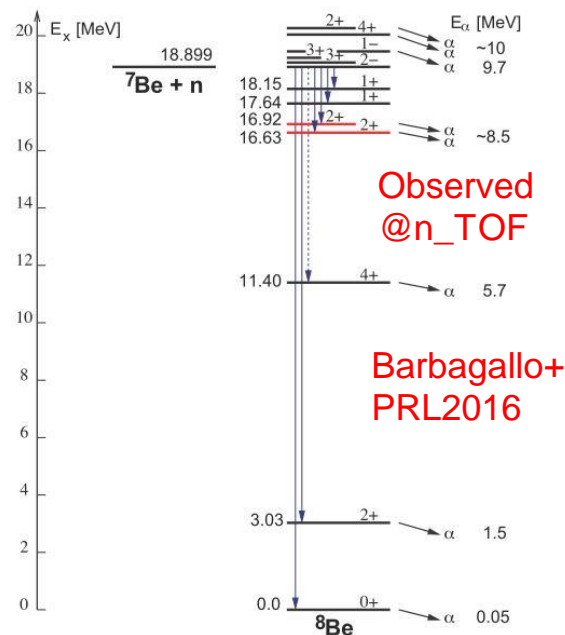
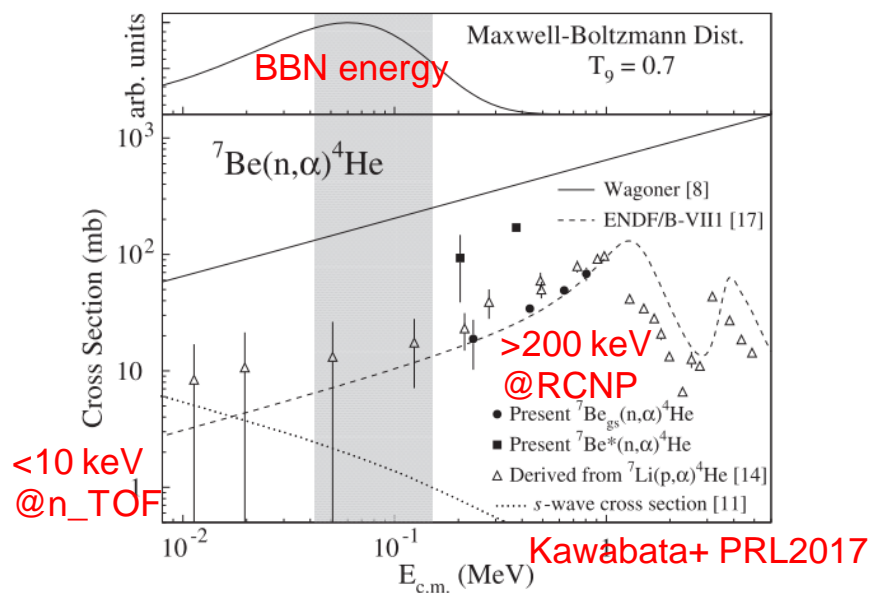
\uparrow
 $\sigma^{\text{OES}} = \text{Normalization} \times \sigma^{\text{HOES}} \times \text{Penetrability}$

${}^7\text{Be}(n, p){}^7\text{Li}$ ($Q = 1.644$ MeV)



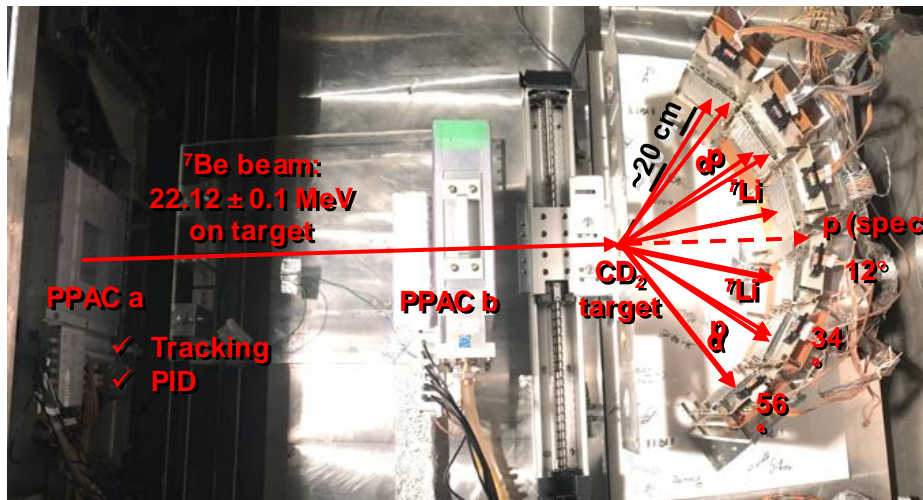
- Sensitivity: $\partial \log Y_{7\text{Li}} / \partial \log \langle \sigma v \rangle_{7\text{Be}} = -0.71$ (Coc & Vangioni 2010, Cyburt+ 2016, etc.)
If $5 \times$ higher rate \rightarrow ${}^7\text{Li}$ problem solved
- Direct measurement up to **13.5 keV**, time-reversal reactions at higher energies.
- R-matrix analysis: Adahchour & Descouvemont 2003.
- **New n_TOF measurement: enhancement below BBN energies** (Damone+ PRL 2018)

${}^7\text{Be}(n, \alpha){}^4\text{He}$ ($Q = 18.990$ MeV)



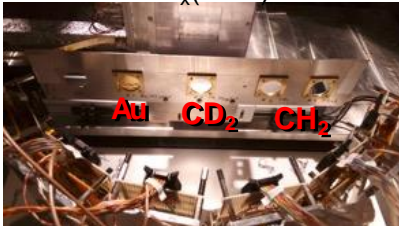
- Hou et al. PRC 2015: evaluation from ${}^4\text{He}(\alpha, p){}^7\text{Li}$
- Barbagallo et al. PRL 2016: s-wave measurement @ nTOF
- Kawabata et al. PRL 2017: p-wave measurement @RCNP
- Lamia et al. APJ 2017: evaluation of ${}^7\text{Li}(p, \alpha)$ data measured by THM.
- Recent works consistent... Yet no direct data in the BBN range.

Experimental setup



- 6 ΔE -E position sensitive silicon telescopes
- ^7Li -p and α - α coincidence measurements
... spectator not measured

- CD_2 : $64 \mu\text{g}/\text{cm}^2$
- ➡ $\Delta E_{\text{beam}} \sim 150$ keV
- ➡ To resolve $E_x(^7\text{Li}^{1\text{st}}) = 478$ keV



- Hamamatsu Charge-division PSD:
position resolution ~ 0.5 mm

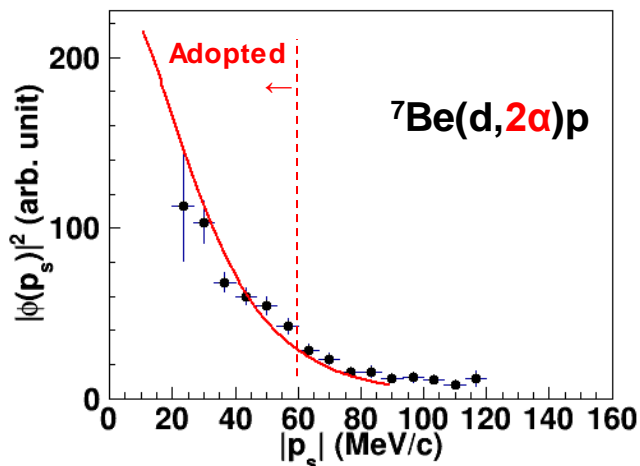
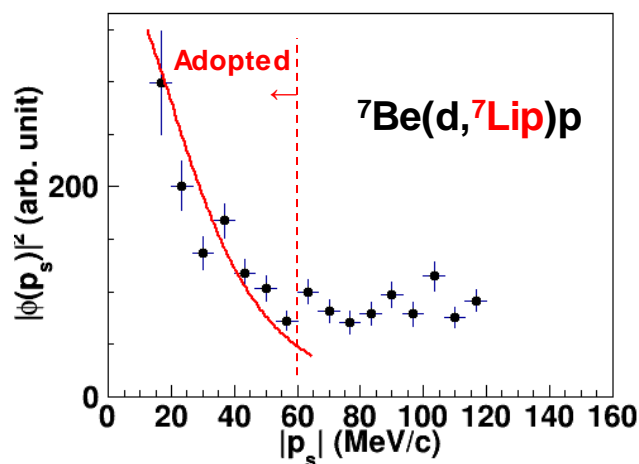


- ➡ Total angular resolution (PPACs & PSDs & alignment)
 $\sim 0.5^\circ$ ➡ $\Delta E_{\text{cm}} \sim 60$ keV

Momentum distributions of the spectator p

$$Y_{\text{exp}}/Y_{\text{sim}} \propto d^3\sigma/(d\Omega_p d\Omega_{7\text{Li}} dE_{\text{cm}}) / KF \propto |\Phi(p_s)|^2 d\sigma/d\Omega$$

$\sim |\Phi(p_s)|^2$ at a fixed $E_{\text{c.m.}}$ and $\theta_{\text{c.m.}}$ (\Leftrightarrow 2-body cross section is const.)

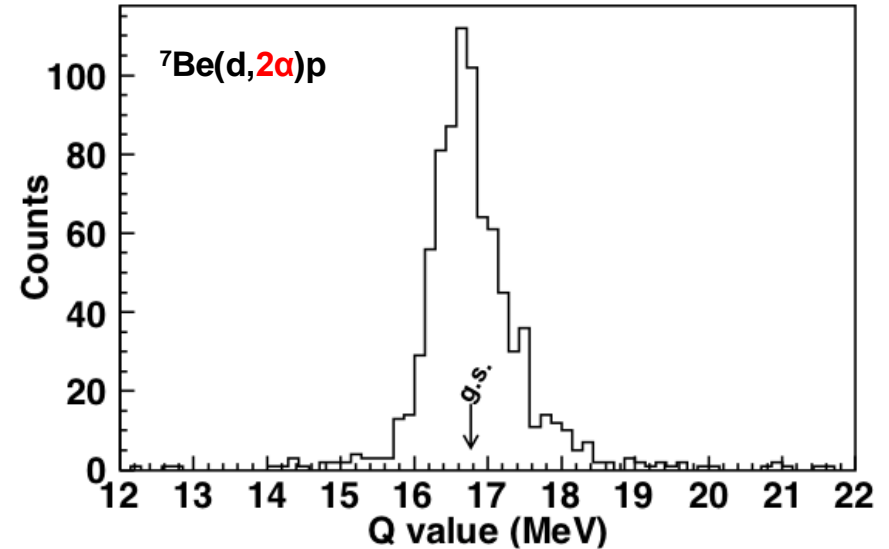
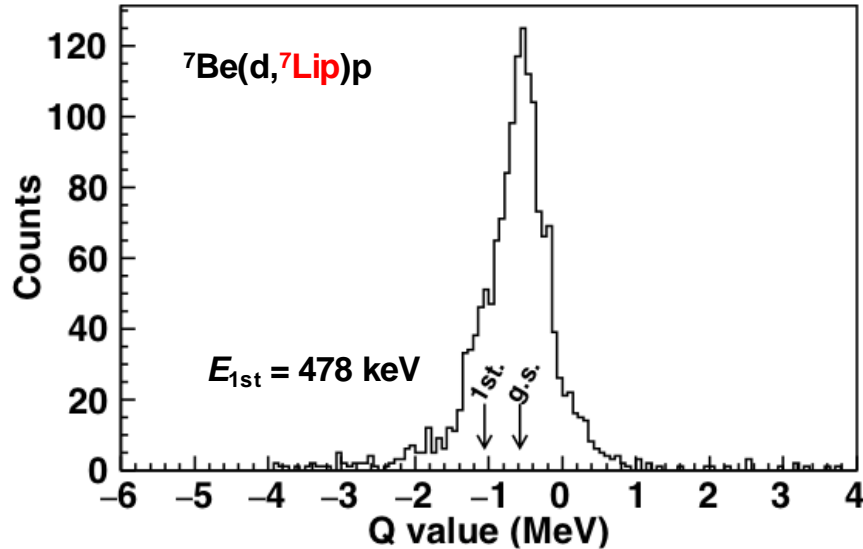


**Hulthén function in momentum space
for p-n intercluster motion (PWIA app.)**

Good agreement up to 60 MeV/c

Evidence that quasi-free contribution is dominant. \rightarrow THM is valid!

Q-value spectra of the 3-body channels



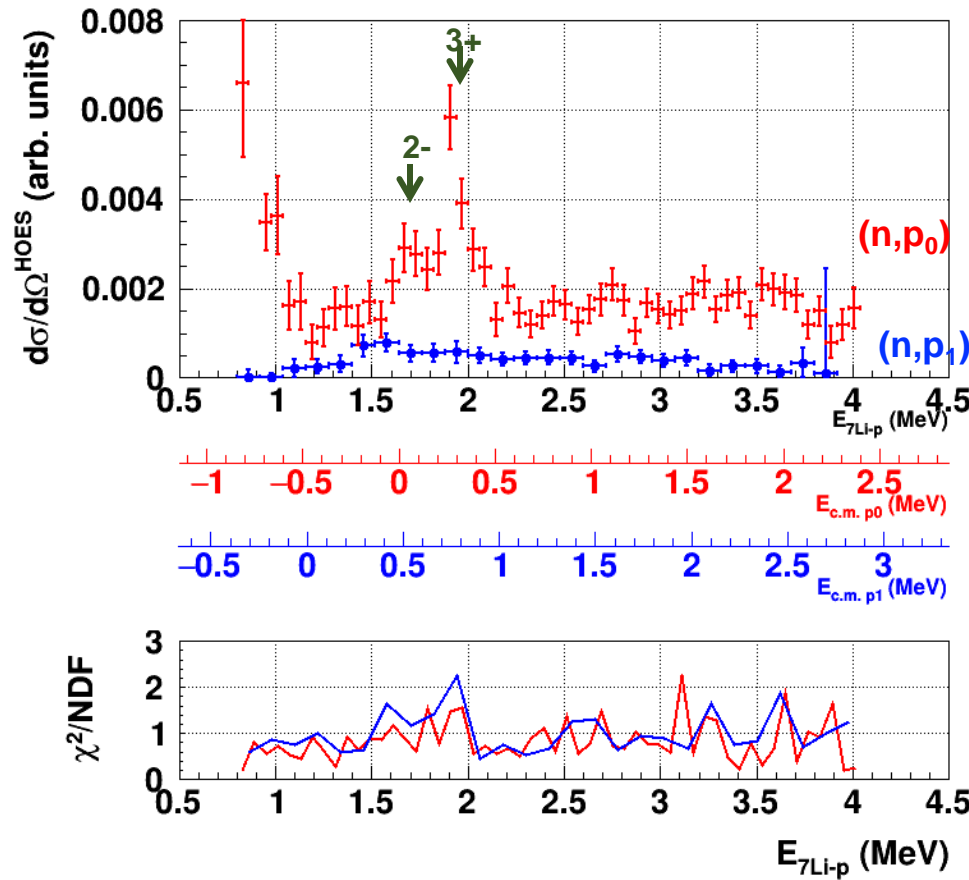
Reaction	Q-value (MeV)
$p+2\alpha$	16.766
${}^7\text{Li}+2p$	-0.589
${}^7\text{Be}+n+p$	-2.225
${}^5\text{He}+p+{}^3\text{He}$	-4.547

$$Q_{3\text{body}} = E_1 + E_2 + E_3 - E_{\text{beam}}$$

$$\Delta Q_{3\text{body}} \sim \sqrt{(\Delta E_1^2 + \Delta E_2^2 + \Delta E_3^2 + \Delta E_{\text{beam}}^2)}$$

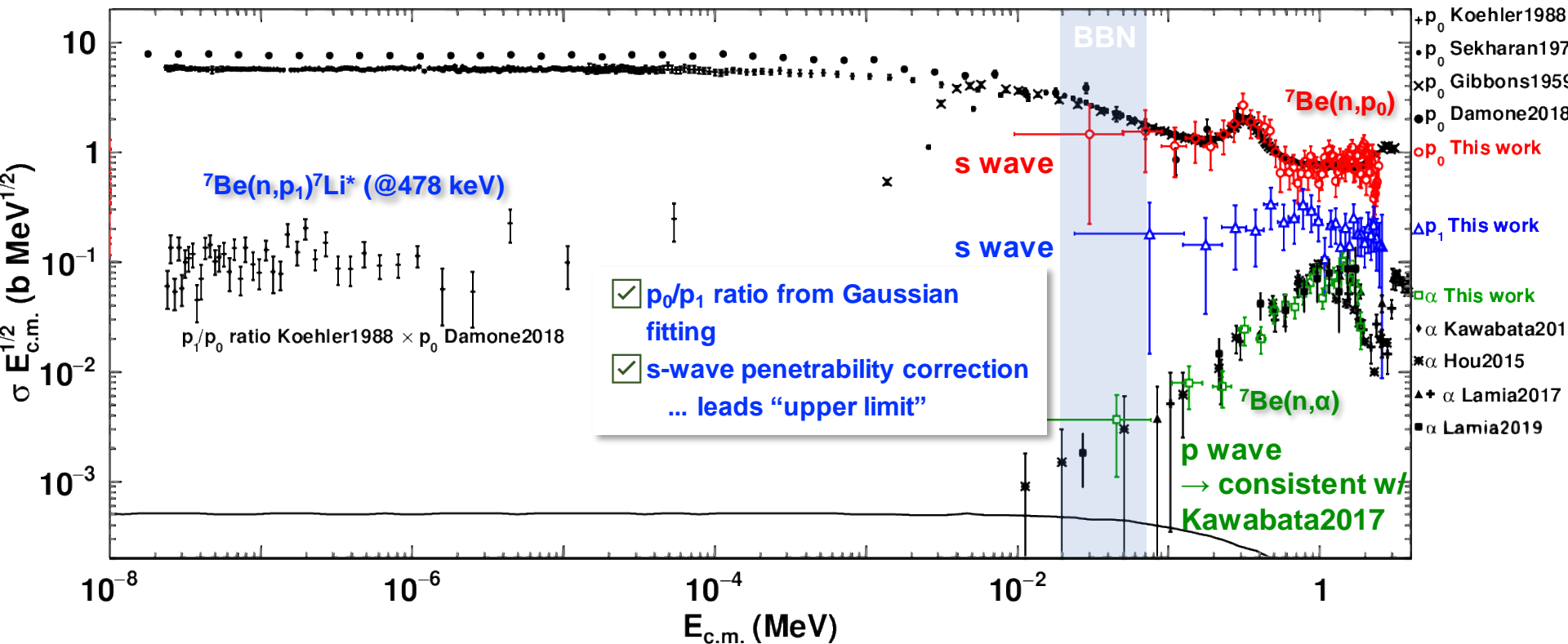
~ 200 keV expected with $64 \mu\text{g}/\text{cm}^2$ CD_2

Gaussian fitting to Q-value spectra

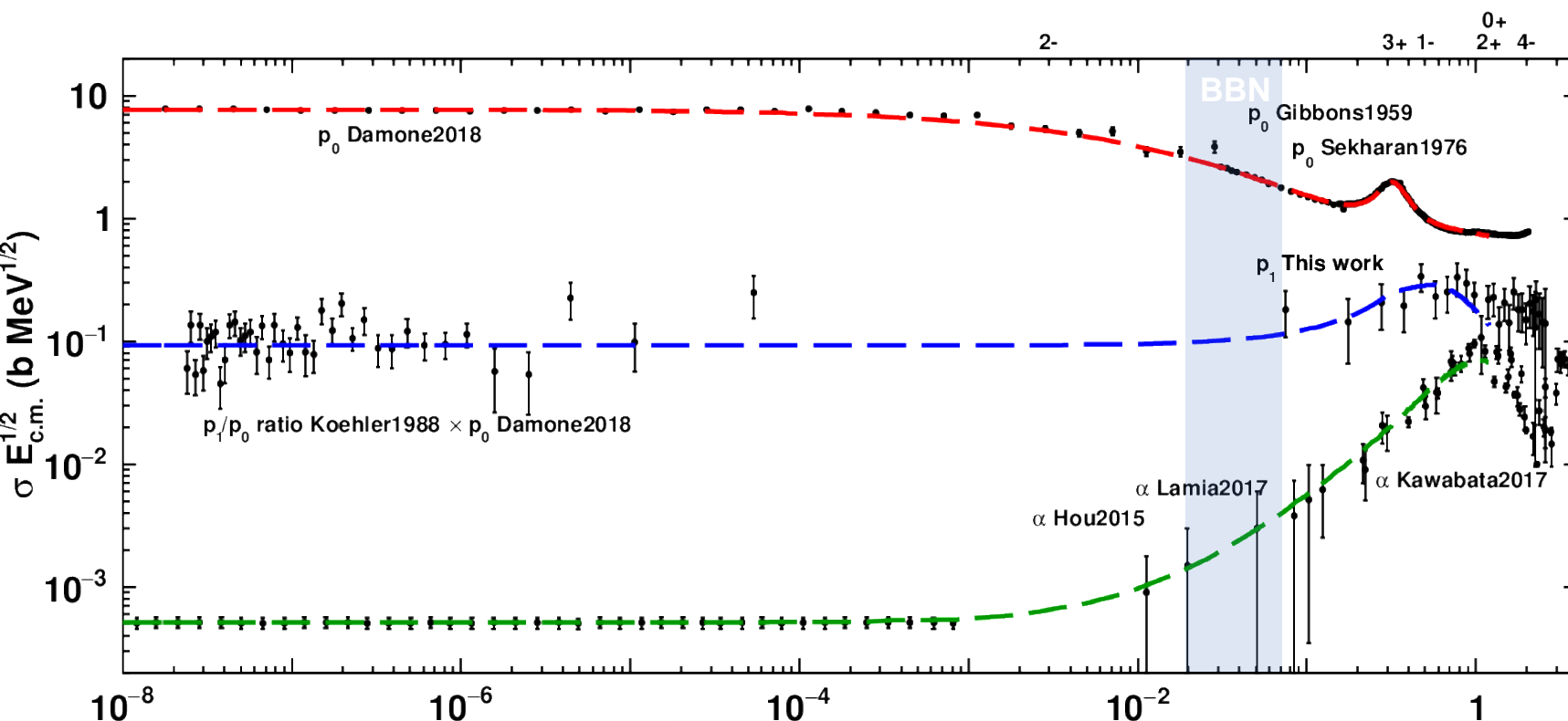


- Isotropy assumed (as no strong angular dependence seen)
- Checked systematic change of widths & peaks
➔ Reduces errors

${}^7\text{Be}(n,p_0)$, (n,p_1) & (n,α_0) cross sections by CRIB



(Preliminary) R-matrix fitting by AZURE2



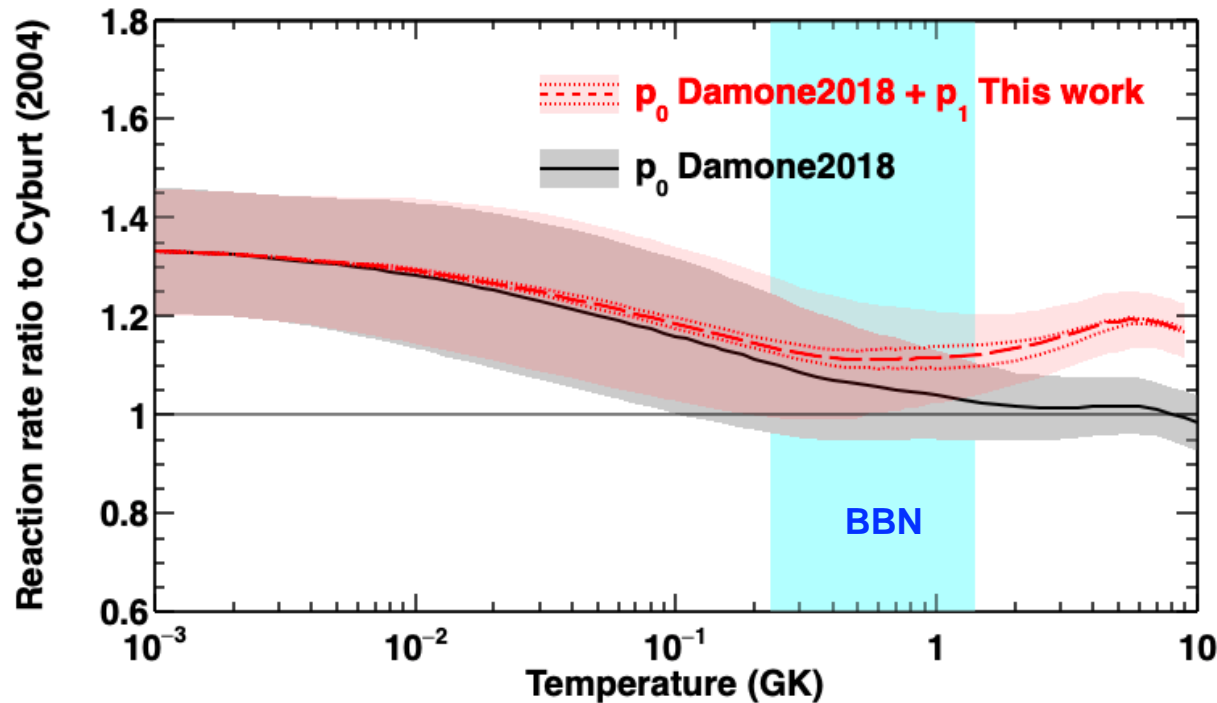
Condition (starting from simple assumptions):

- ✓ Fix known J^π and $E_{\text{resonance}}$ (except 1-)
- ✓ Adopt $l \leq 3$
- ✓ Exclude no neutron emission states
- ✓ Fit Only $E_{\text{c.m.}} < 1.2$ MeV

Procedure:

- ✓ Start from (n,p₀) channel with Ada.&Desc.2003 parameters.
- ✓ Fit only (n,p₁) and (n,α) channels.
- ✓ Fit (n,p₀) channels again.
- ✓ Fix converged parameters and iterate.
- ✓ χ^2 converged (preliminary): $\chi^2_{p_0}/\text{NDF} = 1.59$, $\chi^2_{p_1}/\text{NDF} = 1.33$, $\chi^2_{\alpha}/\text{NDF} = 0.68$

Revised ${}^7\text{Be}(n,p)$ Reaction rate



n_TOF result (Damone+ PRL2018):

~ 5% higher rate in BBN range

➔ 96% ${}^7\text{Li}$ abundance

This work:

~ $15 \pm 15\%$ higher rate (preliminary)

➔ ~ 90% ${}^7\text{Li}$ abundance (preliminary)

(with the sensitivity

$$\partial \log Y_{7\text{Li}} / \partial \log \langle \sigma v \rangle_{7\text{Be}} = -0.71)$$

Recent ${}^7\text{Be}+d$ work @ FSU

(Rijal+ PRL122, 182701 (2019)):

Resonance of (d, α) channel just in BBN Gamow window

➔ ~ 87% ${}^7\text{Li}$ abundance

Considering (n_TOF + CRIB) x FSU

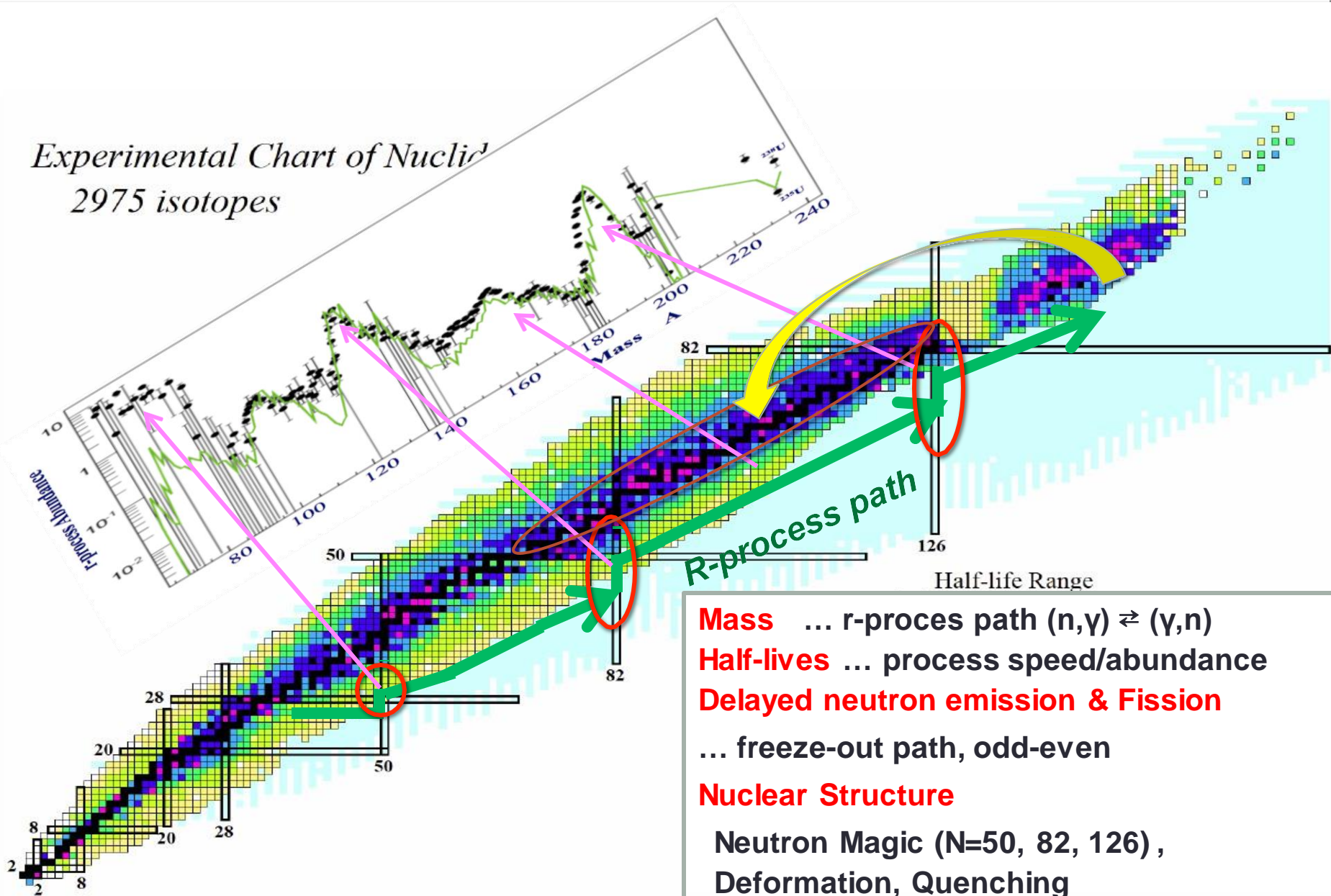
➔ ~ 80% ${}^7\text{Li}$ abundance

How to study r-process experimentally?

- r-process path nuclei...very **neutron rich**, still hard to study
- **RIKEN RIBF** can produce some of them, but not with a high intensity (i.e. too few to make a reaction study)
 - What we can do first is to study the basic properties of nuclei, such as mass and lifetime.

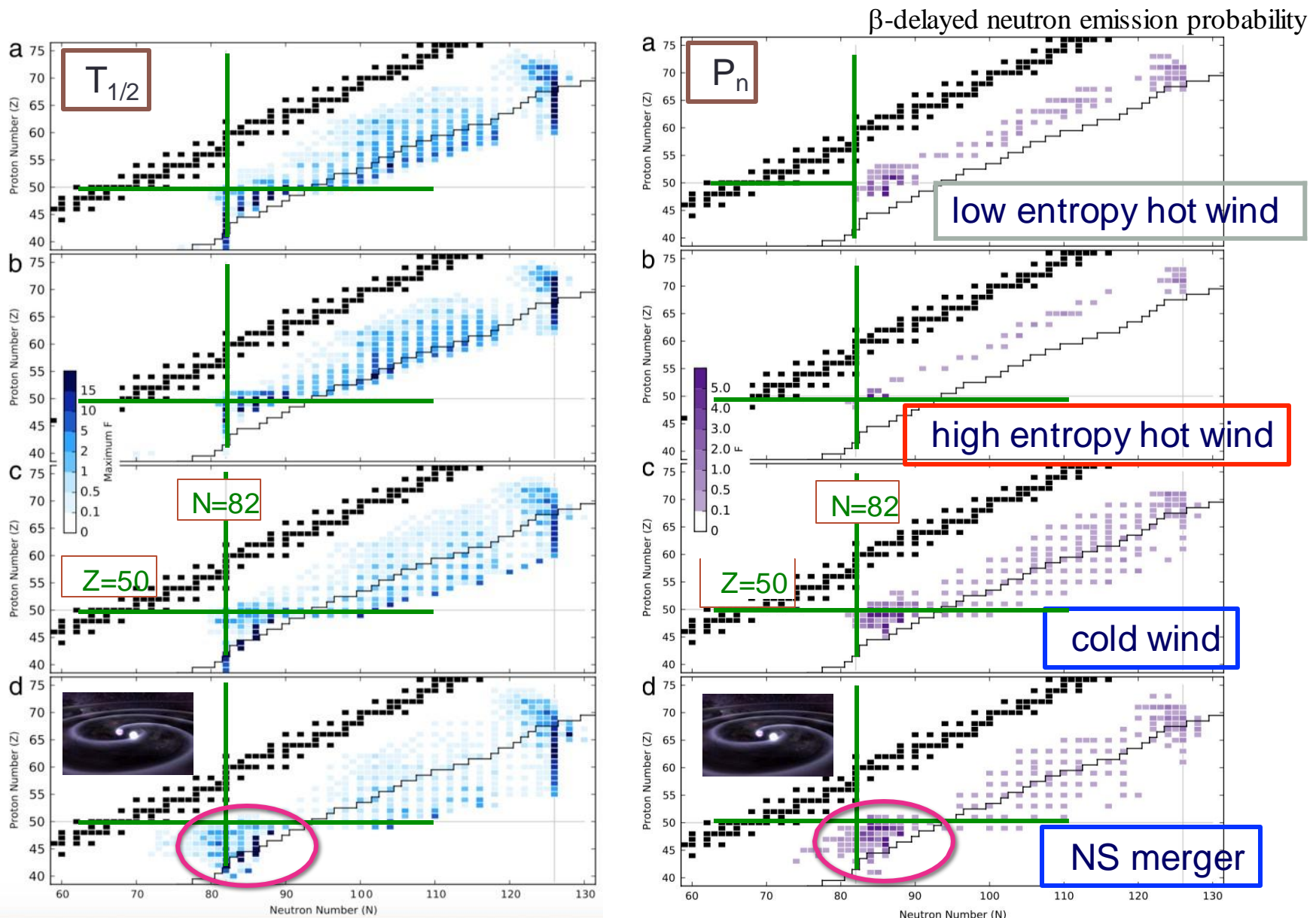
Nucleosynthesis of Heavy Elements (r-Process)

Experimental Chart of Nuclides
2975 isotopes



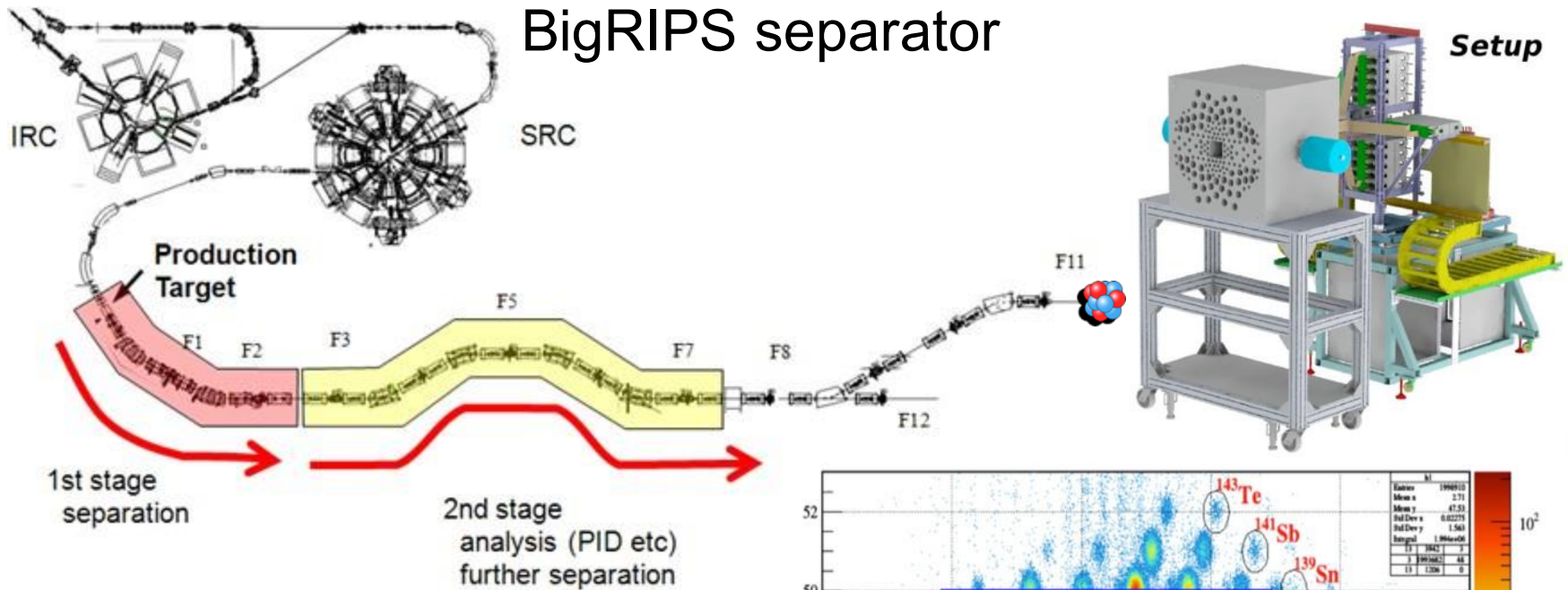
Mass ... r-proces path $(n, \gamma) \rightleftharpoons (\gamma, n)$
Half-lives ... process speed/abundance
Delayed neutron emission & Fission
 ... freeze-out path, odd-even
Nuclear Structure
 Neutron Magic (N=50, 82, 126),
 Deformation, Quenching

Sensitivity study of decay properties in r-process

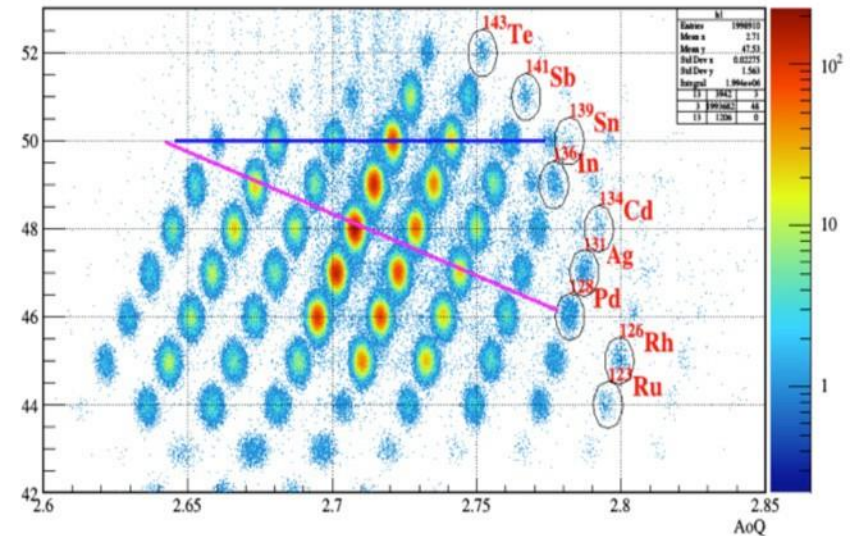


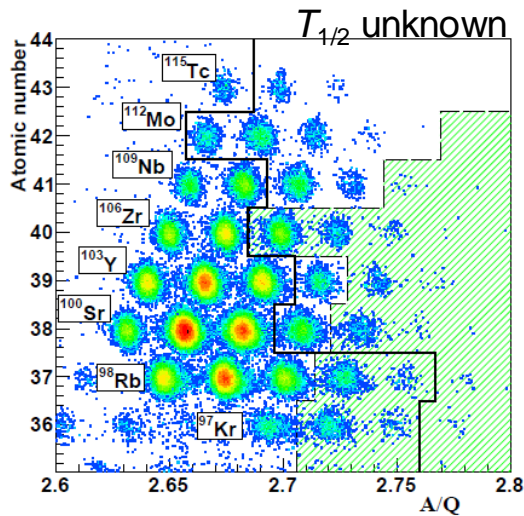
Beam Production & Decay Station

RIKEN RIBF, BigRIPS separator

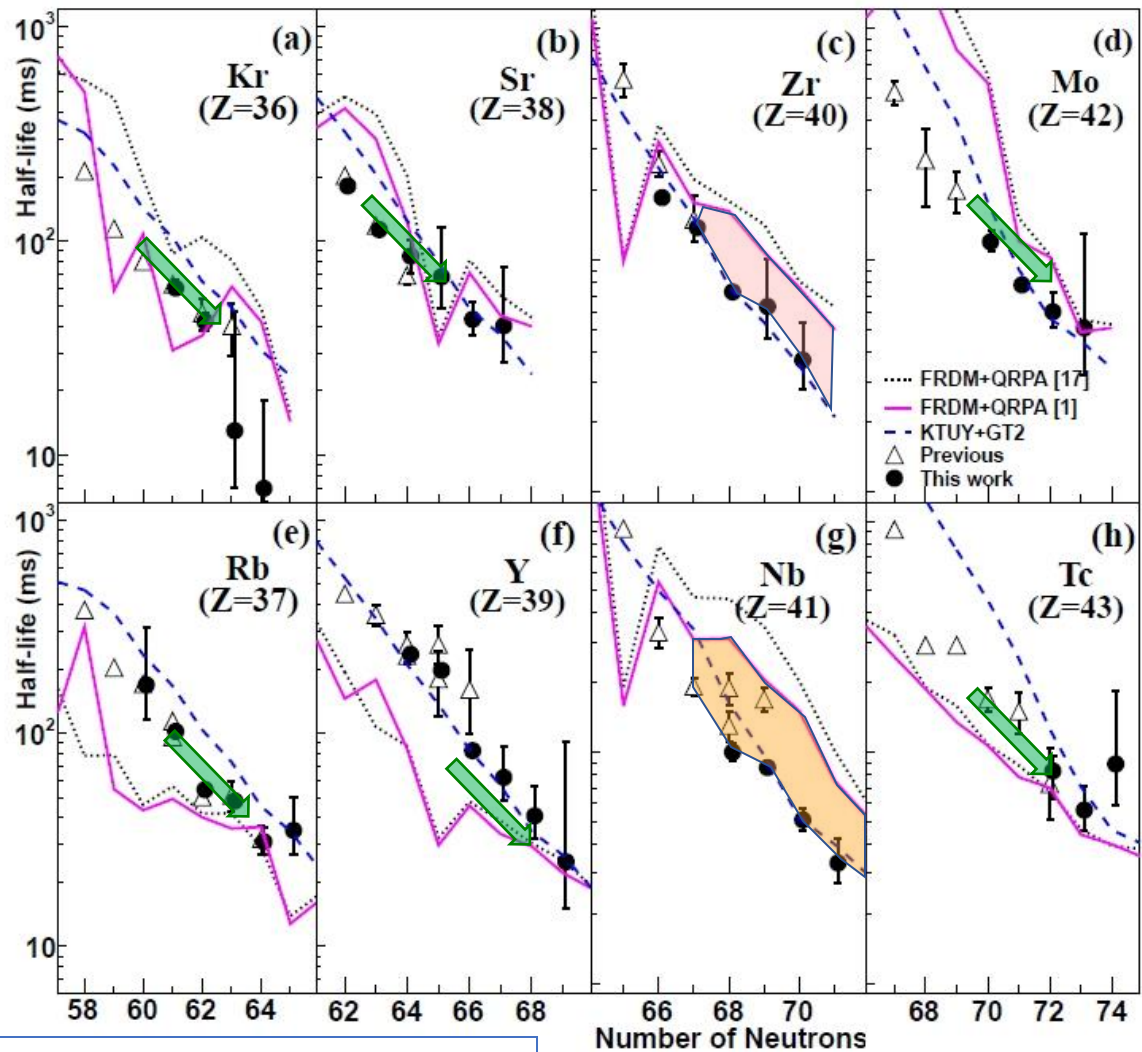


The implantation of an identified RI is associated with the following β -decay events that are detected in the same silicon pixel (DSSSD).





$T_{1/2}$ for 38 isotopes, **18 among them were first measurement**



$T_{1/2}$ is shorter than calculation (KTUY) for Zr/Nb
 \rightarrow r-process flows faster

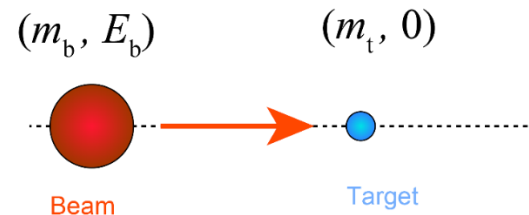
Summary

- Experimental information is essential for understanding stars and other phenomena in the universe
- Study on astrophysical reactions with (low-energy) RI beams:
Not easy, but possible for some cases. Successful cases:
 - Direct measurement (for large cross section reaction)
 - Resonant scattering to study resonances (with TTIK)
 - Indirect methods (such as THM and Coulomb dissociation)
 - Mass/lifetime measurements
- **CRIB at CNS, the University of Tokyo**, providing unique low-energy ($<10\text{MeV/u}$) RI beams...we welcome new collaborators and new ideas.

<http://www.cns.s.u-tokyo.ac.jp/crib/crib-new/>

Homework (In-flight RI beam)

[1] A ${}^7\text{Be}$ beam is created by the in-flight method, using a ${}^7\text{Li}$ beam (mass: M_b) at an energy of E_b and a hydrogen (Mass M_t) target. How much is the maximum angle deviation of the produced ${}^7\text{Be}$ particle from the original ${}^7\text{Li}$ beam trajectory?



For simplicity, you can assume

- The maximum angle deviation occurs when $\theta_{\text{c.m.}}$ is close to 90° .
- Q-value in the production reaction (p,n) is negligible. (${}^7\text{Li}/{}^7\text{Be}$ masses are the same.)
- The energy loss in the target is ignorable.

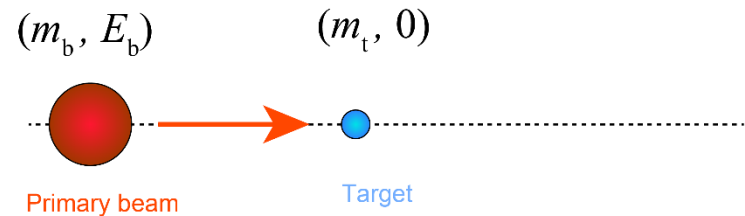
Hint) You can use the formula, $\cos\theta_{\text{lab}} = \frac{x + \cos\theta_{\text{c.m.}}}{\sqrt{1 + x^2 + 2x\cos\theta_{\text{c.m.}}}}, x = \frac{M_b}{M_t}$

Homework

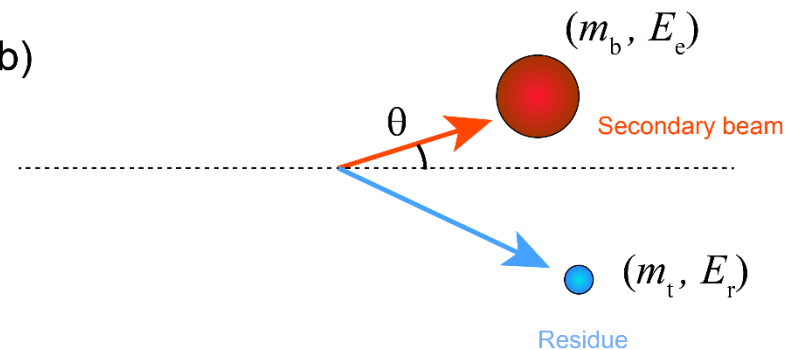
[2] When the ${}^7\text{Li}$ beam energy is $E_b = 10\text{MeV/u}$ ($\sim 70\text{ MeV}$) and ${}^7\text{Be}$ produced with the angle $\theta_{\text{lab}} < 3^\circ$ is accepted, how much is the energy spread $\Delta E_e/E_e$? Here we define ΔE_e as the energy difference of the ${}^7\text{Be}$ beam particle at 0° and 3° .

Hint) Consider energy
-momentum conservation.

a)



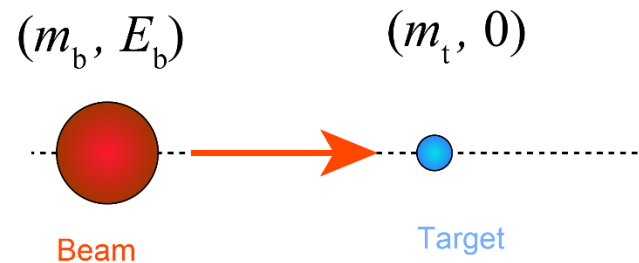
b)



Homework (TTIK)

[1] Suppose we make a scattering experiment by irradiating a beam (kinetic energy E_b , mass M_b) onto a target (Mass M_t). Show that the center-of-mass energy $E_{c.m.}$ (energy of the system in the center-of-mass frame) at the scattering is given by the following formula for non-relativistic energy:

$$E_{c.m.} = \frac{M_t}{(M_b + M_t)} E_b$$



Hint) In c.m. frame, the sum of the momentum vectors will be zero.

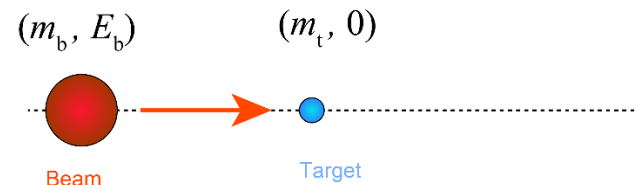
Note) This result implies that the $E_{c.m.}$ resolution can be better than the uncertainty of the beam energy in the inverse kinematics condition, $M_b > M_t$.

Homework

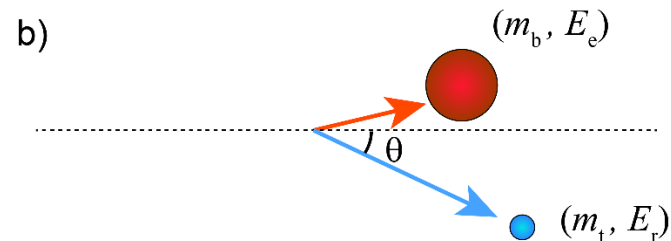
[2] In the resonant scattering experiments in inverse kinematics, we measure the energy and the angle of the **recoiling** ion, E_r and θ . First we consider a thin-target case, where the energy loss in the target is negligible.

Assuming the particle masses and the beam energy E_b are known, how do you obtain the $E_{c.m.}$ of the scattering events?
a)

a) Before scattering



b) After scattering



Homework

[3] How the formula can be modified when we use a **thick-target** in which the beam energy is significantly degraded.

(Can we still obtain $E_{c.m.}$ from the measured E_r and θ ?)

[4] What are the **advantages and disadvantages** of the TTIK (thick-target in inverse kinematics) method, as compared to the traditional, normal kinematics method?