Nuclear Astrophysics with low-energy RI beams

Lecture by Hidetoshi Yamaguchi 山口英斉 from Center for Nuclear Study (CNS), the University of Tokyo/ National Astronomical Observatory of Japan





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What will be covered in this lecture

- Experimental nuclear astrophysics
- Astrophysical reaction study with unstable nuclei (radioactive isotope; RI) ... What does it mean?
- Low energy RI beam production (ISOL/in-flight).
 In particular, CRIB of CNS, the Univ. of Tokyo
- Physics cases, using methods to study astrophysical reactions with RI beams
 - Direct measurement
 - Resonant scattering with thick-target method in inverse kinematics (TTIK)
 - The active target, as an advanced form of TTIK
 - Indirect method (e.g. Trojan Horse Method)

How to simulate "stars"

Stars...interesting objects to study, may evolve, explode, and create elements

Our sun...important energy source of our life

However,

- Interior of the sun/stars...we cannot see directly.
- We need a theoretical model, observational and experimental evidences (especially nuclear reaction rates) to understand them completely.

The sun



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Nuclear physics experiment to astrophysical simulation

What we can measure?

reaction cross section vs energy.

Make the stellar reaction at laboratory, using a beam and a target.

 Cross section...target thickness, number of beam particle, number of detected particle, scattering angle, solid angle of the detector.

What we need for the simulation: reaction rate vs temperature



detector

-Beam energy is changed for each data point to measure excitation function

T-dependence

 To study thermonuclear reaction cross sections (or rates) → To understand stars/nucleosynthesis

["thermo"nuclear reaction... nuclear reaction thermally induced by the environmental heat (such as in stars)]

- $^{7}Be(p,\gamma)...a$ capture reaction to make ^{8}B (just an example)
- Reaction rates...Much dependent on T
- Compound nucleus...
 ⁸B structure (resonance)
 is also important.

We need experiments!



Quantum tunneling

- The large E-dependence of the cross section...because of the tunneling probability to penetrate the Coulomb barrier of the nucleus.
- Tunneling probability of square potential well:

$$P \propto \exp(-2\kappa\Delta r)$$

$$\kappa = \frac{\sqrt{2m|V-E|}}{\hbar}, \Delta r; \text{ width of the potential}$$

• Coulomb potential:

$$P = \exp(-2\pi\eta)$$
$$\eta = \frac{Z_1 Z_2 e^2}{\hbar v} \propto Z_1 Z_2 \left(\frac{\mu}{E}\right)^{1/2}$$

η: Sommerfeld parameterμ: reduced mass



Astrophysical S-factor



Reaction rate

Reaction rate per particle pair <σv>

 a particle and the other particle collide with a relative velocity v, and the cross section σ(v).

How much is the averaged rate of reactions?

- Maxwell-Boltzmann distribution
- Coulomb barrier (use S-factor)
- ♦ Barrier penetrability b

$$<\sigma v>= \left(\frac{2}{\mu}\right)^{1/2} \frac{\Delta}{(kT)^{3/2}} \int_0^\infty S(E) \exp(-\frac{E}{kT} - \frac{b}{E^{1/2}}) dE,$$

$$b = (2\mu)^{1/2} \pi e^2 Z_1 Z_2 / \hbar$$

• Total reaction rate... $r=N_xN_y < \sigma v >$

Exp. cross section ⇒ we can evaluate production rate of newly synthesized nuclides H. Yamaguchi@NUSYS2019

T and E, more in detail

- What's the relation between T and E?
 - Boltzmann distribution $kT \Leftrightarrow E$
 - However, the cross section in much dependent on E, due to the tunneling effect.

tunneling probability $\propto \exp(-\sqrt{1/E})$

 Gamow peak (at Gamow energy) is the realistic energy at which the nuclear reactions take place.



Resonant reaction

- Sometimes the reaction is dominated by resonant reactions.
- We only need to know resonance parameters (*E*, *Γ*, *J^π*) and apply the resonant reaction formula:



$$\sigma(E) = \pi \lambda^2 \frac{2J+1}{(2J_1+1)(2J_2+1)} (1+\delta_{12}) \frac{\Gamma_a \Gamma_b}{(E-E_0)^2 + (\Gamma/2)^2}$$

$$\sigma(E=E_0) \propto \frac{\Gamma_a \Gamma_b}{\Gamma^2} (\text{max.c.s.}), \int_0^\infty \sigma(E) dE \propto \frac{\Gamma_a \Gamma_b}{\Gamma} (\text{integrated c.s.}).$$

If $\Gamma_a << \Gamma_b$, the integrated c.s. $\propto \Gamma_a \Gamma_b / (\Gamma_a + \Gamma_b) \sim \Gamma_a$ ($\Gamma_\gamma << \Gamma_p$ for low-energy (p, γ) reactions.)

"Traditional" measurement



-Beam energy is changed for each data point to measure excitation function

Example: ${}^{12}C(\alpha,\gamma){}^{16}O$



Kettner et al. (1986) @Bochum

- ¹²C beam (50µA, down to 0.5 MeV)
- Windowless ⁴He target, differential pumping, 10 Torr
- Si detectors to measure elastic scattering (beam intensity normalization)
- Nal detectors to measure γ-rays

Measuring ¹⁶O is also possible, in principle (e.g. Kyushu Univ.), but very few and low-energy ¹⁶O must be separated from the intense ¹²C beam.

¹²C(α,γ)¹⁶O



•Measurements have been performed at the energy close to $E_{cm}=1MeV (\sigma \sim nb).$ \Leftrightarrow

AGB star Gamow energy~300 keV

•Background from cosmic γ-rays... requires us a low-background environment.



Underground (p,γ) reaction measurement

¹⁷O(p,γ) by LUNA collaboration



FIG. 2. Sketch of the experimental setup used for the prompt γ -ray detection, consisting of a HPGe detector placed in close geometry to the target, tilted at an angle of 55° with respect to the beam axis.

Beam: 200uA proton Target: isotope-enriched Ta_2O_5 Underground background reduction: 1/2500



Astrophysical reactions involving RI

• Example: ¹³N(p, γ)...Cold CNO cycle to Hot CNO cycle. $\tau_{1/2}$ (¹³N)=10 min, $\tau_{1/2}$ (¹⁴O)=1 min.



Difficulty of RI-beam experiments

Beam intensity

Astrophysical reactions often have small cross section. **Typical RI beam intensity**...10⁵ pps Can be much more, or much less, depending on the RI. **Light ion beam intensity**...>10¹⁴ pps 1-hour beamtime for light ion...100,000 years for RI. (Stars do not mind waiting for a long time, but we do.)

Limitation on available nuclides/energy

High quality RI beams are not available for all nuclides. **Projectile fragmentation**...covers a great variety of nuclides, but basically at high energy (not astrophysical energy).

A challenge for our intelligence.

But don't worry too much

- RI in stars... often important in high temperature (explosive) phenomena. The reaction cross section is not too small in that case.
- What if the cross section is very small and it involves very rare isotopes?

The measurement would be extremely difficult, but that reaction should not be relevant for the universe.

Methods and typical energy for astrophysical reaction study

- Direct method...measure the reaction as it is [Gamow energy (keV-MeV)]
- Indirect methods
 - ◆ANC [peripheral reaction, <10 MeV/u]
 - Coulomb Breakup [can be high, 10~100 MeV/u]
 - ◆Trojan Horse Method [motion of nucleon, ~5 MeV/u]
 - ◆Surrogate [10-50 MeV]
- Studying property of resonances
 - elastic scattering [excitation energy (~few MeV/u)]
 - transfer reaction [10-100 MeV/u]

Low energy: 1~10 MeV/u is suitable for the methods discussed in this lecture. CRIB(U-Tokyo) is a facility to provide such low energy RIB.

Methods of RI-beam production

- Offline separation (lifetime> days)
 Only available for long-lived RI (⁷Be etc).
- Online methods
 - ISOL(isotope separator on-line) (lifetime >10ms)...low energy, good quality beam, suitable for astrophysics. (In spite of the limitation by the chemical property of ions and reacceleration.)
 - in-flight production/separation (lifetime $>\mu$ s)
 - Fragmentation...high energy (>50 MeV/u)
 - direct reaction...main topic of this lecture
 - fusion/fission

Kinematics in Projectile fragmentation

- Fragmentation...the velocity of fragments is nearly equal to the projectile velocity.
 "persistence of velocity".
- Orientation peaked to the forward angle.
- Small momentum transfer between fragments.



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Low-energy RIB with in-flight method

- The fragmentation is not possible at low energy machines (<<50 MeV/u).
- Direct nuclear reaction such as (p, n) reaction can be used as the RI-beam production mechanism.
 - Texas A&M MARS, U-Tokyo CRIB, IMP RIBLL1 INFN-LNL(Italy, Padova) EXOTIC, Florida RESOLUT

(p,n) reactions...reaching to 100mb/sr. (³He,n) reactions...order of 10mb/sr.

High energy fragmentation... μ b~mb/sr (depending on how far from the stability line.)

Why low-energy RI beam?

- Stellar astrophysical reactions :
 - T ~ 10⁶-10⁹ K (typically keV to a few MeV).
 - ⇒ Low energy is not bad energy! (Good for astrophysics and structure study.)



The Sun



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•Nucleosynthesis proceeds through unstable nuclei in some processes(pp chain, CNO cycle, r-, rp-, processes etc.)

CRIB

- CNS Radio-Isotope Beam separator, constructed and operated by CNS, Univ. of Tokyo, located at RIBF (RIKEN Nishina Center).
 - Low-energy(<10MeV/u) RI beams by in-flight method.</p>
 - Primary beam from K=70 AVF cyclotron.

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- Momentum (Magnetic rigidity) separation by "double achromatic" system, and velocity separation by a Wien filter.
- Orbit radius: 90 cm, solid angle: 5.6 msr, momentum resolution: 1/850.



In-flight low-energy RI beam Production

2-body reactions such as (p,n), (d,p) and (³He,n) in inverse kinematics are mainly used for the production...large cross section (>10mb/sr)





Low-Energy RI beam Productions at CRIB

Many RI beams have been produced at CRIB: typically 10⁴-10⁶ pps

Higher intensity for ⁷Be beam with cryogenic H_2 target: 3 x 10⁸ pps.



Historical experiment: $^{13}N(p,\gamma)$

VOLUME 67, NUMBER 7

PHYSICAL REVIEW LETTERS

12 AUGUST 1991

Determination of the ${}^{13}N(p, \gamma)$ ${}^{14}O$ Reaction Cross Section Using a ${}^{13}N$ Radioactive Ion Beam

P. Decrock, ⁽²⁾ Th. Delbar, ⁽¹⁾ P. Duhamel, ⁽³⁾ W. Galster, ⁽¹⁾ M. Huyse, ⁽²⁾ P. Leleux, ⁽¹⁾ I. Licot, ⁽¹⁾ E. Liénard, ⁽¹⁾ P. Lipnik, ⁽¹⁾ M. Loiselet, ⁽¹⁾ C. Michotte, ⁽¹⁾ G. Ryckewaert, ⁽¹⁾ P. Van Duppen, ⁽²⁾ J. Vanhorenbeeck, ⁽³⁾ and J. Vervier⁽¹⁾

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⁽²⁾Instituut voor Kern- en Stralingsfysika, Katholieke Universiteit Leuven, B-3001 Leuven, Belgium ⁽³⁾Institut d'Astronomie et d'Astrophysique, Université Libre de Bruxelles, B-1050 Bruxelles, Belgium (Received 2 May 1991)

The cross section for the astrophysically important ${}^{13}N(p,\gamma){}^{14}O$ reaction has been measured directly with an intense (3×10⁸ particles/s) and pure (>99%) 8.2-MeV ${}^{13}N$ radioactive ion beam. The average

value, for the 5.8-8.2-MeV ¹³N energy range, is 106(30) μ b. The partial γ width of the resonance which occurs in this reaction at a center-of-mass energy of 0.545 MeV has been deduced to be 3.8(1.2) eV. It is compared with theoretical predictions and indirect determinations.

- The first astrophysical reaction measurement with RI beam
- Direct measurement of ¹³N(p,γ) around 0.545 MeV resonance in ¹⁴O.

Setup [Galster et al., PRC 1991]







Coulomb dissociation v.s. direct capture			
¹³ N(p,γ) ¹⁴ O (1 ⁻)			
	C.D. RIKEN('91)	Direct Louvan ('91)	ratio
beam(s ⁻¹)	3×10 ⁴	3×10 ⁸	10-4
σ	10 mb	100 µb	10 ²
target	350 mg/cm ²	200 µg/cm ²	60
efficiency	0.5	2×10 ⁻³	350
data taking	36 h	30 h	1.2
total counts	1.5×10 ⁴	85	180

Results

Two new experiments were performed at the same period, but with different methods.



• Successful! ... The two experiments (direct/indirect) yielded consistent Γ_{γ} values.

Summary of Lecture #1

- Nuclear physics experiment is essential for understanding stars and nucleosynthesis
- Difficulty...



- Unstable nuclei: short lifetime of the nuclei
- RI beam production technique ...enabled us to study stellar reactions in hot environment.

Lecture #2

- How to make RI beam experiments for astrophysical reactions?
- Thick target method in inverse kinematics
 Principle
 - Application

Direct measurement of astrophysical (capture) reactions with unstable nuclei...

- Works for at least relatively intense RI beams, such as (¹³N, ⁷Be).
- But still not easy for others, such as ${}^{15}O(\alpha, \gamma)$, because of the low RI beam intensity/reaction cross section.

Then, what can we do?

- 1. Use "indirect" methods (Coulomb dissociation, ANC, Trojan Horse Method, ...)
- 2. Use TTIK (Thick target in inverse kinematics, I will discuss on this)
 - Direct measurement with a thicker target ⇒More efficient measurement.
 - Resonant scattering⇒High cross section (~100 mb/sr), to study resonances.

Inverse kinematics



Features of (ideal) inverse kinematics: Heavy ion as the beam...keep going forward. Light ion as the target...tend to be scattered to forward angle (compared to the normal kinematics).

1. Inverse kinematics at RI-beam production...The produced RI is already like a beam (cf. ISOL).

2. Inverse kinematics at scattering/reaction measurement...discussed later.

The method...TTIK

- W.W. Daenick and R. Sherr (1963) "thick target method" ¹²C(p,p).
- K.P. Artemov et al., (1990)

Thick-Target with Inverse Kinematics

¹²C beam into thick helium (α) target

Effective method of study of α -cluster states

K.P. Artemov, O.P. Belyanin, A.L. Vetoshkin, R. Wolskj, M.S. Golovkov, V.Z. Gol'dberg, M. Madeja, V.V. Pankratov, I.N. Serikov, V.A. Timofeev, V.N. Shadrin, and J. Szmider

I. V. Kurchatov Institute of Atomic Energy (Submitted 15 February 1990) Yad. Fiz. **52**, 634–639 (September 1990)

For study of states with a large reduced α width the method of measurement of the excitation function of elastic scattering of α particles is proposed, but in a geometry which is the reverse of the traditional experimental arrangement. The targets are helium gas which is simultaneously a moderator for the primary beam of heavy ions and an absorber which shields the detector from the direct beam. The advantages of the method are obvious in those cases in which in the usual experimental arrangement the need arises of using gas targets or targets of rare isotopes or of measurements at an angle 180°. To check the method we have carried out a comparison with the known $\alpha + {}^{12}C$ interaction. New results are obtained in the interaction ${}^{15}N + \alpha$.



FIG. 1. Spectrum of α particles obtained in interaction of ¹²C ions with initial energy 28 MeV with helium. The detection angle is 0°. In the insert we have given the excitation function for elastic scattering of α particles by carbon from Ref. 4. The detection angle is 158.8°.
The thick-target method in inverse kinematics

Measurement of resonance scattering



Measurement is possible for short-lived RI which cannot be used as the target.

 $\label{eq:Ecm} \begin{array}{l} \blacklozenge E_{cm} = E_{beam} \ast A_t / (A_p + A_t) \ll E_{beam} \\ \\ \mbox{Measurement can be at low} \\ \mbox{energy with high resolution.} \end{array}$

- Simultaneous measurement for a certain energy range.(No need to change beam energy.)
- The beam can be stopped in the target...measurement at θ_{cm}=180° is possible.

Resonant reaction

- Sometimes the reaction is dominated by resonant reactions.
- We only need to know resonance parameters (E, Γ, J^π) and apply the resonant reaction formula:



$$\sigma(E) = \pi \lambda^2 \frac{2J+1}{(2J_1+1)(2J_2+1)} (1+\delta_{12}) \frac{\Gamma_a \Gamma_b}{(E-E_0)^2 + (\Gamma/2)^2}$$

$$\sigma(E=E_0) \propto \frac{\Gamma_a \Gamma_b}{\Gamma^2} (\text{max.c.s.}), \ \int_0^\infty \sigma(E) dE \propto \frac{\Gamma_a \Gamma_b}{\Gamma} (\text{integrated c.s.}).$$

If $\Gamma_a <<\!\!<\!\!\Gamma_b$, the integrated c.s. $\simeq \Gamma_a \Gamma_b / (\Gamma_a + \Gamma_b) \sim \Gamma_a$ ($\Gamma_\gamma <<\!\!<\!\!\Gamma_p$ for low-energy (p, γ) reactions.)

Resonant elastic scattering

- Elastic scattering
 - At energies far below Coulomb barrier...Simply Rutherford scattering. Cross section is higher at low energies and angles.
 - At higher energies... interference of Coulomb and nuclear potential ... "resonances" can be observed in the excitation function. Ep (MeV)



T. Teranishi et al. / Physics Letters B 556 (2003) 27-32

Breit-Wigner formula



• Breit-Wigner for nuclear resonant reaction:

$$\sigma(E) = \pi \lambda^2 \frac{2J+1}{(2J_1+1)(2J_2+1)} (1+\delta_{12}) \frac{\Gamma_a \Gamma_b}{(E-E_0)^2 + (\Gamma/2)^2}$$

 λ : de Broglie wavelength

 J_1, J_2, J : spins of projectile, target, excited state in the compound nucleus δ_{12} : 1 for identical particles, 0 otherwise

 Γ_a, Γ_b : Widths of entrance and exit channels

⁷Li+ α /⁷Be+ α study

- ⁷Li(α,γ)¹¹B ...important at high-T, as a production reaction of ¹¹B (the v-process in core-collapse supernovae).
- ⁷Be(α,γ)¹¹B ... one of the reaction in hot *p-p* chain, relevant at high-T.
- α -cluster structure in ¹¹B/¹¹C :
 - 2α+t/2α+³He cluster states are known to exist (similar to the dilute cluster structure in ¹²C.)
 - Several "bands" which have α -cluster structure could be formed. We can study the band and cluster structure more in detail.



⁷Be(α,γ) in supernovae

vp-process calculation (T₉>1) shows considerable contribution by ${}^{10}B(\alpha,p){}^{13}C$ and ${}^{7}Be(\alpha,\gamma){}^{11}C$ as much as the triple-alpha process.



Setup for ⁷Li/⁷Be+ α

- Thick target method with inverse kinematics ... An efficient method to measure excitation function.
 - ⁷Be beam is monitored by a PPAC (or an MCP detector).
 - ⁷Be beam stops in a thick helium gas target (200 mmlong, 1.6 atm).
 - Recoiled α particles are detected by ΔE-E counter (10 μm and 500 μm Si detectors) at forward angle.

 Nal array for γ-ray measurement (to identify inelastic events).



⁷Be+ α Excitation functions

• 4 excitation functions... new information on resonant widths, spin, and parity. *H. Yamaguchi et al., PRC (2013).*



Resonant contribution to ⁷Be(α,γ)

 Small but not negligible contribution compared to lower-lying states (~10%).



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Direct measurement of (α , p) reactions

¹¹C(α,p)¹⁴N@CRIB [S. Hayakawa et al., PRC 93, 065802, (2016)]

An important alpha-induced reaction as a bypass of the 3α process in explosive hydrogen-burning processes.

Reactions to excited levels identified by TOF information





Limitation of the TTIK for astrophysical reaction studies

- Resonant scattering: Very striking experimental method available even with kHz-order RI beams (thanks to the large cross section) and suitable for resonance search, however,...
 - We cannot access the low energy close to the threshold where the Coulomb scattering dominates.
 - Γ_{α} or Γ_{p} can be determined (if they are large), but we need another partial width (such as Γ_{γ}) to determine reaction cross section.
- Direct reaction:
 - Still the yield may not sufficient for capture reactions at low-T, but we can study (α, p) reactions at explosive stellar environments, for example.
- As a common problem, the reaction/scattering channel we observe must be the dominant one. (i.e. we may have backgrounds by reactions producing the same particle, such as inelastic scattering, break up reaction, and fusion evapolation)...this problem can be solved with an active target.

Measurement of ²⁵Al+p elastic scattering relevant to the ²²Mg(α, p)²⁵Al reaction

Jun Hu, X.D. Tang, S.W. Xu, L.Y. Zhang, S.B Ma, N.T. Zhang, J.J. He, H. Yamaguchi. K. Abe, S. Hayakawa, L. Yang, H. Shimizu, D. Kahl, T. Teranishi, J. Su. H.W. Wang, B. Guo et al.,

Institute of Modern Physics, Chinese Academy of Sciences, CNS. The University of Tokyo, National Astronomical Observatories, The University of Edinburgh.









1.1 αp-process in Type I X-ray bursts



1.2 Sensitivity study to the light curve of X-ray burst

(α, p) reactions that impact the burst light curve in the multi-zone x-ray burst model.

Rank	ap-process reaction	Source of reaction rates adopted by
		multi-zone model
1.	22 Mg(α ,p) 25 Al	Non-SMOKER
2.	${}^{14}O(\alpha,p){}^{17}F$	Hu et al. PRC 90 (2014) 025803
3.	¹⁸ Ne(α ,p) ²¹ Na	He et al. PRC 88 (2013) 012801
4.	${}^{26}Si(\alpha,p){}^{29}P$	Non-SMOKER
5.	$^{30}S(\alpha,p)^{33}Cl$	D. Kahl <i>et al.</i> PRC 97 (2018)
б.	34 Ar(α ,p) 37 K	Non-SMOKER
7.	38 Ca(α ,p) 41 Sc	Non-SMOKER

Ref: Cyburt et al., ApJ, 830 (2016) 55

 $^{22}Mg(\alpha,p)^{25}Al$ could be the most sensitive reaction in the αp -process and may have a prominent impact on the burst light curve.

1.3 The effect of ${}^{22}Mg(\alpha,p){}^{25}Al$ on the X-ray burst light curve



Change in multi-zone model X-ray burst light curves induced by variation of the ${}^{22}Mg(\alpha,p){}^{25}Al$ reaction up (Up rate $\times 100$) and down (Dn rate $\div 100$)

2.2 Status of level properties in ²⁶Si



2.3 Status of ${}^{22}Mg(\alpha,p){}^{25}Al$ astrophysical reaction rate



The ${}^{22}Mg(\alpha,p){}^{25}Al$ reaction rate as a function of the temperature for the Hauser-Feshbach predictions TALYS and non-SMOKER

Experimental Setup at F3 focal plane

²⁵Al beam:
 2 x 10⁵ pps, 80%
 purity





Particle Identification for the Recoiling Particles

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Preliminary R-Matrix Fit Result

1. We observed 13 resonant states in ²⁶Si.

2. The spin parities of 5 states above the α threshold were determined for the first time (in the present tentative analysis).



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The αp -process

- αp-process: (α,p),(p,γ) reactions occur faster than β-decay at high temperature. Accelerates the rp-process (Wallace and Woosley, 1981).
- Suitable objective for CRIB
 - Not many direct measurements of (α,p) reactions have been performed in other facilities.



³⁰S(α,p)

- ${}^{30}S(\alpha,p) \dots$ one of the key reaction in X-ray bursts.
- Scarce ³⁴Ar resonance information, reaction rate evaluation was by statistical model.
- ³⁰S+α resonant scattering with active target (D. Kahl et al., submitted to Phys. Rev. C).
- 3 higher-lying resonance observed:





- Acts as a He target and a detector (TPC) simultaneously
 - GEM with "backgammon" type readout pad.
- 3-dimentional trajectory and energy loss can be measured ⇒ Accurate event identification.

How to obtain the 3D-trajectory?



X: L⇔R ratio of the "Backgammon" pad.Y: Electron drift time in the TPC.Z: pad #.

γd ^Β

x₀ x

0

particle



Bragg curve

 The energy loss profile in the target (the Bragg curve) can be measured with the active target, which should be known precisely for the TTIK experiments.



Kinematic reconstruction analysis



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Astrophysical implications

Reaction rate evaluation with RCNP(Osaka) ³⁶Ar(p,t)³⁴Ar transfer reaction data + CRIB(Tokyo) resonant scattering data

 \Rightarrow Higher than the stat. model rate calculation





⇐Energy generation higher than the statistical model 25% enhancement [with a single reaction].
-Max. 30% of abundance change for A=20-80 nuclei.

Morinaga (1956) and linear chain

- Discussed on 4n-nuclei based on the alpha particle model
- Predicted linear-chains in ¹²C, ¹⁶O, etc., from their high momenta of inertia.



 It was shown in later studies that the Hoyle state is NOT a linear-chain state.

Linear-chain levels



Suhara & En'yo, PRC 2010 and 2011:

¹⁰Be+ α

- Linear-chain cluster levels in ¹⁴C were predicted in Suhara & En'yo papers.
- Asymmetric, ${}^{10}Be+\alpha$ configuration ...likely to be observed with ${}^{10}Be+\alpha$ alpha-resonant scattering.
- May form a band with $J^{\pi}=0^+,2^+,4^+$ a few MeV above α -threshold.
- Scattering of two 0⁺ particles...only *l*-dependent resonant profile.

Similar experiments independently conducted by Birmingham group and MSU group, published already.



Experimental setup

Thick target method in inverse kinematics, similar to the previous ⁷Be+ α .



- •Two PPACs for the beam PI, trajectory, number of particles.
- •Two silicon detector telescopes for recoiling α partciles.
- • E_{cm} and θ obtained by event-by-event kinematic reconstruction.

Excitation function

- The excitation function we obtained for 13.8-19.2 MeV exhibits many resonances.
- R-matrix analysis performed, and some of the resonance parameters (E, J^π, Γ_α) were determined.



Rotational Band

The set of resonances we observed (0+, 2+, 4+) is proportional to J(J+1) ... consistent with a view of rotational band.

Also perfectly consistent with the theoretical prediction.



Baba and Kimura (2016 & 2017)



Another AMD calculation,

" σ -bond" linear chain band, consistent with 3 experiments " π -bond" linear chain band at higher energy (studied by Peking Univ. group).

How certain are the linear-chain states?

- Identification of the 0⁺ state...1⁻ was excluded with 3σ significance, but the error can be systematic.
 - Limited statistics and angular range
 - Background subtraction
 - Inelastic scattering?
- We planned the 4th experiment at INFN-LNS (Catania, Italy):
 - ♦ With offline-production ¹⁰Be beam
 - Inelastic scattering separation with TOF.

⇒Performed in Oct., 2018.
The "CHAIN" experiment at INFN-LNS (Catania, Italy)

¹⁰Be+ α with more intense beam, higher energy and angular resolution: ~2 weeks beamtime.

Investigation of α -chain structures in ¹⁴C.

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Abstract

We propose to measure the excitation function for the elastic scattering process ${}^{10}\text{Be}{+}^{4}\text{He}$, in order to shed some light upon the existence of linear-chain cluster states in the n-rich ${}^{14}\text{C}$ nucleus. These states are expected to have a configuration in which ${}^{10}\text{Be}$ and α are spatially separated, and thus they can be observed by the ${}^{10}\text{Be}{+}\alpha$ resonant elastic scattering. In order

Result (very preliminary)

CRIB

VS

LNS(Tandem)

Including 0-8 deg events



@5 deg, No normalization for the effective target thickness/absolute cross section yet



The origin of galactic ²⁶Al gamma rays

²⁶Al γ -ray : The first observed cosmic γ -ray from specific nuclide (1.809 MeV)

Evidence of on-going nucleosynthesis.

Key for understanding the evolution of the galaxy ($^{26}\text{Algs}$, $t_{1/2}$ = 0.7 million years)

Production source: still uncertain. Massive stars? Supernovae? Novae?



Amount of ²⁶Al in galaxy

"RADIOACTIVE ²⁶AI IN THE GALAXY: OBSERVATION VERSUS THEORY" – Prantzos & Diehl (1996) Summarizing the theory and observation at that time, 1.5 ~ 3 M_☉ Estimation by a recent observation 26AI $2.0 \pm 0.4 M_{\odot}$ – Diehl (2016) Stellar production by calculation Nova: ~ 0.8 M_{\odot} – Bennet et al. (2013) ccSN&WR(11-120 Ms): 1.8~2 M_o – Limongi & Chieffi

(2006)

AGB: ~0.4 M_{\odot} – Mowlavi & Meynet (2000)

SAGB: ~0.3 M_{\odot} – Siess & Arnould (2008)

Total production of ²⁶Al exceeds the amount estimated by observation! (Needs destruction process?)

²⁶AI



Low-T (<<0.4 *GK*)

- Simple scheme: ²⁶Al is on the Mg-Al cycle, decay into ²⁶Mg.
- Both ^{26g}Al (ground state) and ^{26m}Al (isomer, τ=6.3s) are produced. Only ^{26g}Al decay into the excited state of ²⁶Mg and emit 1.809-MeV γ-rays.

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26AI



High-T (>> 0.4GK)

Isomeric ²⁶AI does not produce γ -rays, however,

- ^{26m}Al production by
 ²⁵Mg(p,γ) and also
 from ²⁵Al⇒²⁶Si decay.
 - Thermal equilibrium between ^{26g}AI and ^{26m}AI.
- ²⁶Al(p,γ)²⁷Si reaction destroys ²⁶Al.

²⁶Al isomer beam

- ²⁶Mg(p,n)²⁶Al reaction: At the energy of CRIB, the maximum angular momentum brought by the beam is limited, and the production of ²⁶Al ground state(5⁺) is highly suppressed. ⇒High purity ²⁶Al isomer beam production is possible.
- This seemed to be a unique idea in 2014, but...

²⁶Al^m beam @Argonne:

S. Almaraz-Calderon et al., Phys. Rev. Lett 119, 072701 (2017), B.W. Asher et al., NIM A (2018).

At CRIB:

2016 First ^{26m}Al beam production 2017 ^{26m}Al+p resonant scattering measured



FIG. 4. Excitation functions for (a) $^{26}Mg(p,n_0)^{26}Al$, (b) $^{26}Mg(p,n_1)^{26}Al$, (c) $^{26}Mg(p,n_2)^{26}Al$, and (d) the total neutron yield.

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Cocktail beam at the RI optimization focal plane



Figure: Flight time vs. residual energy. ${}^{26}Al^{13+}$ is clearly separated.Main contaminant ${}^{23}Na$. This is illustrative (not optimized).Intensity $\sim 2 \ge 10^5$ pps at F3.

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Experimental setup for ^{26m}Al(p, p) measurement



Figure: Beam is tracked by PPACs before impinging on and stopping in one of the targets. Scattered protons were detected by $\Delta E \cdot E$ Si telescopes, the first layer is 75 μ m with 16×16 strips and the other detectors 1.5 mm. An array of 10 NaI detectors was placed above the target to measure γ -rays (not depicted).

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Proof we made ^{26m}Al

• Pulsed the beam in regular tests, 12 s on - 12 s off

- Measured the β^+ 's with the Si telescope
 - (Also measured 511-keV Y's with Nal)



 β^+ decay measurements: (a) Energy spectrum and (b) Decay timing. Both are consistent with ${}^{26m}Al$.

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²⁶Al proton spectra — the method worked!



Rough normalization (factor 2 error). Clear evidence of structure arising from ^{26m}Al and not ^{26g}Al. H. Yamaguchi@NUSYS2019

Need of indirect method

Stellar reaction cross section often has a strong dependence on energy (or temperature), changing by orders of magnitude.

...This is because of the tunneling probability of the Coulomb barrier.

Experimentally, this causes much trouble. We need a clever way.



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The ¹⁸F(p, α) project (with THM)

- ¹⁸F(p,α)... an astrophysical reaction important in novae, and other high-T environments.
- Measurement with the Trojan Horse Method performed in 2008 ... The first THM+RI beam experiment in the world.
- The RI Beam at CRIB (after development): Primary beam: ¹⁸O ⁸⁺, 4.5-5 MeVA
 Production target: H₂
 Production reaction: ¹⁸O(p,n)¹⁸F
 Purity nearly 100%
 - Intensity $> 5 \times 10^5 \text{ pps}$

A NOVA MICKEY MOUSE PICTURE AND ${}^{18}F(p,\alpha){}^{15}O$



Thin hydrogen surface layer accumulated on white dwarf through accretion ring Observed γ - rays come from e tet et come from ¹⁸F decay mostly At novae temperatures (100-500 keV) ¹⁸F can be mainly destroyed by $18F(p,\alpha)^{15}O$



THM measurement: ${}^{18}F(p,\alpha) {}^{15}O$ via ${}^{2}H({}^{18}F,\alpha {}^{15}O)n$ **Kinematics** ^{2}H **N**_{Spectator} ²H(¹⁸F,α ¹⁵O)n S 15**0** С X p ÷ $E(^{18}F) = 50 MeV$ B d ⁴He 18**F** 30 $E_{a}(MeV)$ స్థ 40 20 20 10 0 5 H. Yamaguchi 30 0 40 50 20 *YS2019* ϑ_{150} $E_{150}(MeV)$





Assuming that a Quasi-free mechanism is dominant one can use the (PW)IA:



EXPERIMENTAL IMPULSE DISTRIBUTION



THM(=barriers free) CROSS SECTION



S(E) from THM 8 keV 3/2+





FIG. 3. The ¹⁸F(p, α)¹⁵O S factors, calculated using the R matrix, for eight possible interference terms. The range in possible S factors arises from the interference between the $J^{\pi} = 3/2^+$ resonances. The interference between resonances dominates in the region of interest, resulting in four groups of S-factor curves. The upper and lower curves of each group are shown in the figure. The legend gives the assumed phase, for the 8-, 38-, and 665 keV resonances, respectively, for each pair of curves. Also plotted are the measured S factors from this work, those from previously published data [4,10,12,19], and the proposed contribution from 1/2+ states predicted in Ref. [6]. Direct data...C.E. Beer, et al.



⁷Be(n,p)⁷Li and the ⁷Be(n,α)⁴He reactions with THM for cosmological lithium problem

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+ many others

Cosmological ⁷Li problem



- ⁷Li problem... disagreement between theory and observation by a factor of 3-4
 - Due to CMB obs.? Low-metallicity stars obs.? Standard BBN model? Nuclear Physics?
 - ⁷Be abundance in the end of BBN determines ⁷Li predominantly
 - $p(n,\gamma)d$, ³He(d,p)⁴He, ⁷Be(n,p)⁷Li, ⁷Be(n,α)⁴He, ⁷Be(d,p)2 α , etc.
- Temperature ~ $10^{10} 3 \times 10^8$ K, Energy: 1 MeV 25 keV

Trojan Horse Method for RI + neutron

Trojan Horse method: (Spitaleri+ Phys. Atom. Nucl. 2011)
 ⁷Be(n,p)⁷Li, ⁷Be(n,α)⁴He via ²H(⁷Be,⁷Lip)¹H, ²H(⁷Be,αα)¹H
 PWIA applicable when Quasi-free mechanism is dominant



⁷Be(n, p)⁷Li (Q = 1.644 MeV)



- Sensitivity: $\partial \log Y_{7\text{Li}} / \partial \log \langle \sigma v \rangle_{7\text{Be}} = -0.71$ (Coc & Vangioni 2010, Cyburt+ 2016, etc.) If 5 × higher rate \Rightarrow ⁷Li problem solved
- Direct measurement up to 13.5 keV, time-reversal reactions at higher energies.
- R-matrix analysis: Adahchour & Descouvemont 2003.
- New n_TOF measurement: enhancement below BBN energies (Damone+ PRL 2018)

⁷Be(n, α)⁴He (Q = 18.990 MeV)



- Hou et al. PRC 2015: evaluation from ⁴He(α,p)⁷Li
- Barbagallo et al. PRL 2016: s-wave measurement @ nTOF
- Kawabata et al. PRL 2017: p-wave measurement @RCNP
- Lamia et al. APJ 2017: evaluation of ⁷Li(p,α) data measured by THM.
- Recent works consistent... Yet no direct data in the BBN range.

Experimental setup

- YBe beam:

 22.12 ± 0.1 MeV

 on target

 PPAC a

 Tracking

 PiD
 - 6 ΔE-E position sensitive silicon telescopes
 - ⁷Li-p and α-α coincidence measurements
 ... spectator not measured

- CD₂: 64 µg/cm²
- $\Rightarrow \Delta E_{\text{beam}} \sim 150 \text{ keV}$
- To resolve $E_x(^7\text{Li}^{1\text{st}}) = 478 \text{ keV}$



• Hamamatsu Charge-division PSD: position resolution ~ 0.5 mm



→ Total angular resolution (PPACs & PSDs & alignment) $\sim 0.5^{\circ} \Rightarrow \Delta E_{cm} \sim 60 \text{ keV}$

Momentum distributions of the spectator p

 $Y_{exp}/Y_{sim} \propto d^3\sigma/(d\Omega_p d\Omega_{7Li} dE_{cm}) / KF \propto |\Phi(p_s)|^2 d\sigma/d\Omega$

~ $|\Phi(p_s)|^2$ at a fixed $E_{c.m.}$ and $\theta_{c.m.}$ (\Leftrightarrow 2-body cross section is const.)



Good agreement up to 60 MeV/c Evidence that quasi-free contribution is dominant. \rightarrow THM is valid!

Q-value spectra of the 3-body channels



Gaussian fitting to Q-value spectra



- Isotropy assumed (as no strong angular dependence seen)
- Checked systematic change of widths & peaks

Reduces errors

⁷Be(n,p₀), (n,p₁) & (n, α_0) cross sections by CRIB



(Preliminary) R-matrix fitting by AZURE2



 \square Fit Only $E_{c.m.} < 1.2 \text{ MeV}$

- Fix converged parameters and iterate.
- ✓ χ² converged (preliminary): χ²_{p0}/NDF = 1.59, χ²_{p1}/NDF = 1.33, χ²_α/NDF = 0.68

Revised ⁷Be(n,p) Reaction rate



Summary

- Study on astrophysical reactions with (low-energy) RI beams:
 - Not easy, but possible for some cases
 - 1. Direct measurement with Intense RI beam, or efficient measurement by TTIK.
 - 2. Resonant scattering to study resonances
 - 3. Indirect methods
 - Active target...new experimental technique to make a complete thick-target experiment/low-energy reaction study
 - CRIB at CNS, the University of Tokyo, providing unique lowenergy (<10MeV/u) RI beams...we welcome new collaborators and new ideas.

http://www.cns.s.u-tokyo.ac.jp/crib/crib-new/

Homework (In-flight RI beam)

[1] A ⁷Be beam is created by the in-flight method, using a ⁷Li beam (mass: M_b) at an energy of E_b and a hydrogen (Mass M_t) target. How much is the maximum angle deviation of the produced ⁷Be particle from the original ⁷Li beam trajectory?



For simplicity, you can assume

-The maximum angle deviation occurs when $\theta_{c.m.}$ is close to 90°.

-Q-value in the production reaction (p,n) is negligible. (⁷Li/⁷Be masses are the same.)

-The energy loss in the target is ignorable.

Hint) You can use the formula, $\cos\theta_{\text{lab}} = \frac{x + \cos\theta_{\text{c.m.}}}{\sqrt{1 + x^2 + 2x\cos\theta_{\text{c.m}^2}}}$, $x = \frac{M_b}{M_t}$
Homework

[2] When the ⁷Li beam energy is $E_{\rm b}$ = 10MeV/u (~70 MeV) and ⁷Be produced with the angle $\theta_{\rm lab}$ < 3° is accepted, how much is the energy spread $\Delta E_{\rm e}/E_{\rm e}$? Here we define $\Delta E_{\rm e}$ as the energy difference of the ⁷Be beam particle at 0° and 3°.

Hint) Consider energy -momentum conservation.



[1] Use the formula $\cos\theta_{\text{lab}} = \frac{x + \cos\theta_{\text{c.m.}}}{\sqrt{1 + x^2 + 2x \cos\theta_{\text{c.m}^2}}}, x = \frac{M_{\text{b}}}{M_{\text{t}}}$ At $\theta_{\text{c.m.}} \sim 90^\circ$, $\cos\theta_{\text{max}} = \frac{x}{\sqrt{1 + x^2}} = \frac{1}{\sqrt{1 + 1/x^2}}$ As $1/x^2 \ll 1$, $\cos\theta_{\text{max}} \simeq 1 - \frac{1}{2x^2}$

On the other hand, for small angles,

$$\cos\theta \simeq 1 - \frac{1}{2}\theta^2$$

Therefore,

$$\theta_{\max} \simeq \frac{1}{x} = \frac{1}{7}(\operatorname{rad}) = 8.2^{\circ}$$

[2] Energy conservation: $E_{\rm b} = E_{\rm e} + E_{\rm r}$... (1) Momentum conservation:

$$p_{\rm b} = p_{\rm e} \cos \theta + p_{\rm r} \cos \varphi \quad \dots (2)$$
$$0 = p_{\rm e} \sin \theta - p_{\rm r} \sin \varphi \quad \dots (3)$$

 φ is the (laboratory) scattering angle of the residual particle (neutron). Using (2) and (3),

$$p_{\rm r}^2 (\cos^2 \varphi + \sin^2 \varphi) = p_{\rm e}^2 + p_{\rm b}^2 - 2p_{\rm b}p_{\rm e} \cos \theta$$

From (1),

$$p_{\rm r}^2 = 2M_{\rm t}E_{\rm r} = 2M_{\rm t}(E_{\rm b} - E_{\rm e}) = (1/x)(p_{\rm b}^2 - p_{\rm e}^2) \qquad (x \equiv \frac{M_{\rm b}}{M_{\rm t}})$$

Combining these, we obtain a quadratic equation as,

$$(1 + \frac{1}{x})p_{e}^{2} - 2p_{b}p_{r}\cos\theta + (1 - \frac{1}{x})p_{b}^{2} = 0$$

The solution is,

$$p_{\rm e} = \frac{\cos\theta \pm \sqrt{\cos^2\theta - (1 - \frac{1}{x^2})}}{1 + \frac{1}{x}} p_{\rm b}$$

Take the positive sign solution of $p_{\rm e} = \frac{\cos\theta \pm \sqrt{\cos^2\theta - (1 - \frac{1}{x^2})}}{1 + \frac{1}{x}} p_{\rm b}$

• For $\theta = 0^{\circ}$

$$p_{\rm e} = \frac{1 + \sqrt{1 - (1 - \frac{1}{x^2})}}{1 + \frac{1}{x}} p_{\rm b} = p_{\rm b}$$

This means the momentum (or energy) of the secondary beam is the same as the primary beam (under present approximation).

• For
$$\theta = 3^{\circ}$$
, and $x = 7$

$$p_{\rm e} = \frac{\cos\theta + \sqrt{\cos^2\theta - (1 - \frac{1}{x^2})}}{1 + \frac{1}{x}} p_{\rm b} = \frac{0.99863 + \sqrt{0.99863^2 - (1 - \frac{1}{7^2})}}{1 + \frac{1}{7}} p_{\rm b} = 0.9901 \ p_{\rm b}$$

$$\Delta p_{\rm e}/p_{\rm e} = 1 - 0.9901 \sim 1\%$$

 $\Delta E_{\rm e}/E_{\rm e} \sim 2\%$

Implication

- The result shows the produced ⁷Be particles mostly go to the forward angle (< 8°). Taking the most forward angles, the ⁷Be particles have similar energies (within 2%). This is why it can be regarded as a secondary beam.
- For higher mass particle beam, the secondary beam is even more focused to the forward angle (while the stopping power is huge).
- The energy (10MeV/u) was not used...the solution is independent of the energy (but remember it's an approximation).
- The negative solution in [2] also makes a beam with another energy. The positive (negative) solution corresponds to events in which the residual is scattered to very backward (forward) angle.

Homework (TTIK)

[1] Suppose we make a scattering experiment by irradiating a beam (kinetic energy $E_{\rm b}$, mass $M_{\rm b}$) onto a target (Mass $M_{\rm t}$). Show that the center-of-mass energy $E_{\rm c.m.}$ (energy of the system in the center-of-mass frame) at the scattering is given by the following formula for non-relativistic energy:



Hint) In c.m. frame, the sum of the momentum vectors will be zero.

Note) This result implies that the $E_{c.m.}$ resolution can be better than the uncertainty of the beam energy in the inverse kinematics condition, $M_b > M_t$.

Homework

[2] In the resonant scattering experiments in inverse kinematics, we measure the energy and the angle of the recoiling ion, E_r and θ . First we consider a thin-target case, where the energy loss in the target is negligible.

Assuming the particle masses and the beam energy $E_{\rm b}$ are known, how do you obtain the $E_{\rm c.m.}$ of the scattering events from the measured quantities? _{a)}



Homework

[3] How the formula can be modified when we use a thicktarget in which the beam energy is significantly degraded. (Can we still obtain $E_{c.m.}$ from the measured E_r and θ ?)

[4] What are the advantages and disadvantages of the TTIK (thick-target in inverse kinematics) method, as compared to the traditional, normal kinematics method?

[1] There are several ways to calculate it.

One way is to calculate the invariant mass, $M_{INV}^2 = E^2 - \vec{p}^2$ for each frame. (Here we use a unit with c = 1, and $E^2 = m^2 + p^2$ is the squared relativistic energy.)

Momentum of b: $p_{\rm b} = \sqrt{2M_{\rm b}E_{\rm b}}$

Total energy of b: $E = \sqrt{M_b^2 + p_b^2} \sim M_b + E_b$ (non-relativistic case)

Lab frame:
$$M_{\rm INV}^2 = ((M_{\rm b} + E_{\rm b}) + M_{\rm t})^2 - p_{\rm b}^2$$

$$= ((M_{b}+M_{t}) + E_{b})^{2} - 2M_{b}E_{b}$$

$$\sim (M_{b}+M_{t})^{2} + 2E_{b}(M_{b}+M_{t}) - 2M_{b}E_{b}$$

$$= (M_{\rm b} + M_{\rm t})^2 + 2E_{\rm b}M_{\rm t}$$

CM frame: consider a combined system with a mass (M_b+M_t) , and momentum vectors are cancelled out.

$$M_{\rm INV}^2 = (M_{\rm b} + M_{\rm t})^2 + 2(M_{\rm b} + M_{\rm t})E_{\rm c.m}$$

These must be equal,

 $2(M_{b}+M_{t})E_{c.m.}=2E_{b}M_{t}$ $E_{c.m.}=E_{b}M_{t}/(M_{b}+M_{t})$

[2] Energy conservation: $E_{\rm b} = E_{\rm e} + E_{\rm r}$... (1) Momentum conservation:

 $p_{\rm b} = p_{\rm e} \cos \varphi + p_{\rm r} \cos \theta \quad \dots (2)$

 $0 = p_{\rm e} \sin \varphi - p_{\rm r} \sin \theta \dots (3)$

 φ is the (laboratory) scattering angle of the ejectile.

From (2) and (3),

$$p_{\rm e}^2 \left(\cos^2 \varphi + \sin^2 \varphi\right) = p_{\rm r}^2 + p_{\rm b}^2 - 2p_{\rm b}p_{\rm r} \cos \theta$$

Rewritten with energies:

 $2M_{\rm b}E_{\rm e} = 2M_{\rm t}E_{\rm r} + 2M_{\rm b}E_{\rm b} - 2p_{\rm b}p_{\rm r}\cos\theta$ Using (1),

 $(M_{\rm b} + M_{\rm t})E_{\rm r} = p_{\rm b}p_{\rm r}\cos\theta$

Square both sides,

$$(M_{\rm b} + M_{\rm t})^2 E_{\rm r}^2 = p_{\rm b}^2 p_{\rm r}^2 \cos^2 \theta = 4M_{\rm t} E_{\rm r} M_{\rm b} E_{\rm b} \cos^2 \theta$$
$$E_{\rm r} = \frac{4M_{\rm t} M_{\rm b} \cos^2 \theta}{(M_{\rm b} + M_{\rm t})^2} E_{\rm b} = \frac{4M_{\rm b} \cos^2 \theta}{(M_{\rm b} + M_{\rm t})} E_{\rm c.m.}$$

[3] In the thick-target method condition, the formula we obtained in [2],

$$E_{\rm r} = \frac{4M_{\rm t}M_{\rm b}\cos^2\theta}{(M_{\rm b}+M_{\rm t})^2} E_{\rm b} = \frac{4M_{\rm b}\cos^2\theta}{(M_{\rm b}+M_{\rm t})} E_{\rm c.m.}$$

is only valid at the scattering point.

 $E_{\rm b}$ is obtained from the original beam energy $E_{\rm b0}$, if the energy loss of the beam particle is known (we assume that is known by experiment or theory):

 $E_{\rm b}=E_{\rm b0}-E_{\rm loss,b}~(L),$

where *L* is the distance from the target entrance to the reaction position. The recoiling particle energy we measure (E_{meas}) is also modified by

 $E_{\rm r} = E_{\rm detec} - E_{\rm loss,b} (L_2)$

 L_2 is the length that the recoiling particle runs in the target, known from L and the scattering angle θ .

Therefore, if only *L* is known, we can get $E_{c.m.}$ from the above relationship.

[3] (continued) *L* cannot be known immediately, but can be obtained, such as, in the following way:

- (I) Assume a certain value of *L*, such that the reaction position will be inside the target.
- (II) Calculate $E_{\rm b}$, $E_{\rm r}$, $E_{\rm detec}$, using this L and measured θ .
- (III) If calculated E_{detec} is larger than the real measurement, it means the reaction actually occurred more downstream (and vice versa).
- (IV) Shift *L* to the correct direction and repeat (II)-(III), until true E_{detec} is found.

By performing this iterative calculation, we can obtain $E_{c.m.}$ event by event.

[4] (The following does not cover everything)

Advantages:

-Simultaneous measurement of a certain range of energy \rightarrow Good statistics with no systematic error for the beam intensity at each energy.

-No need to change the energy step by step (Nice feature for RI beams/cyclotron beams).

-When the beam is stopped in the target, measurement at $\theta_{c.m.}$ =180° is possible. (Coulomb scattering is minimal.)

Disadvantages:

-No identification between two or more processes emitting the same kind of ion (if we do not use an "active target"), e.g., elastic and inelastic proton scatterings.

-Resolution limited by the energy straggling of the beam.

-Precise energy loss function is needed (otherwise we easily get shift, skew, and wrong normalization of the spectrum.)