Nuclear Astrophysics with low-energy RI beams

Lecture by

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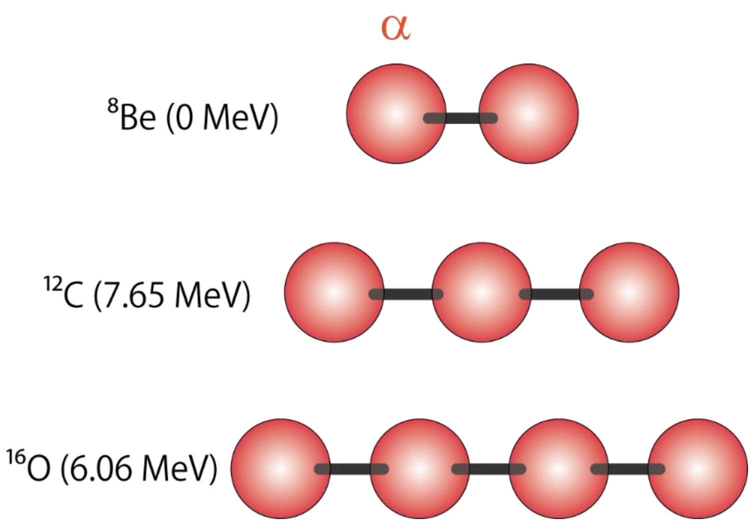
Lecture #3

- ^{10}Be+α and linear chain with TTIK (continued)…main interest on nuclear cluster structure
- Trojan Horse Method (THM)
  - ◆How it works
  - ◆^{18}F(p,α) S. Cherubini et al., Phys. Rev. C (2015)…The first THM+RI beam experiment in the world
  - ◆^{7}Be(n,p) and (n,α) for cosmological ^{7}Li abundance problem
- r-process study at RIKEN RIBF

Morinaga (1956) and linear chain

- Discussed on 4n-nuclei based on the alpha particle model
- Predicted linear-chains in ^{12}C, ^{16}O, etc., from their high momenta of inertia.



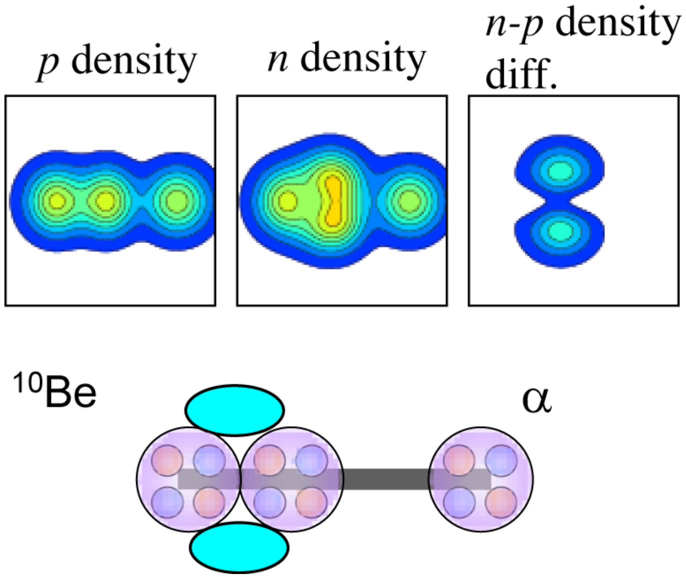
- It was shown in later studies that the Hoyle state is NOT a linear-chain state.

^{10}Be+α

- Linear-chain cluster levels in ^{14}C were predicted in Suhara & En'yo papers.
- Asymmetric, ^{10}Be+α configuration …likely to be observed with ^{10}Be+α alpha-resonant scattering.
- May form a band with $J^{\pi}=0^+,2^+,4^+$ a few MeV above α-threshold.
- Scattering of two 0^+ particles…only l-dependent resonant profile.

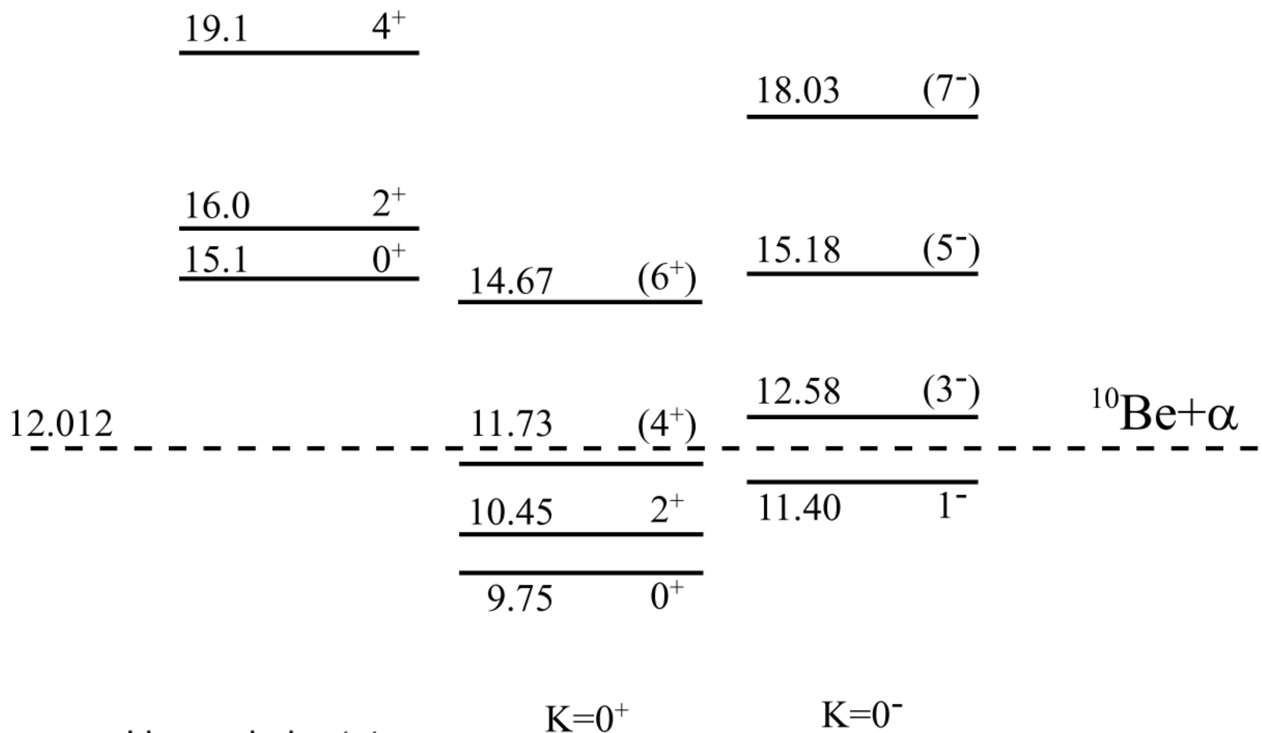
Similar experiments independently conducted by Birmingham group [M. Freer et al., PRC 2014] and MSU group [A. Frisch et al., PRC 2016]

Suhara & En'yo, PRC 2010 and 2011:



Cluster bands

- Predicted energy…few MeV above the $^{10}Be+\alpha$ threshold



Linear chain states in the calculation by Suhara&En'yo (2010)

Prolate rotational bands in Oertzen et al., (2004)

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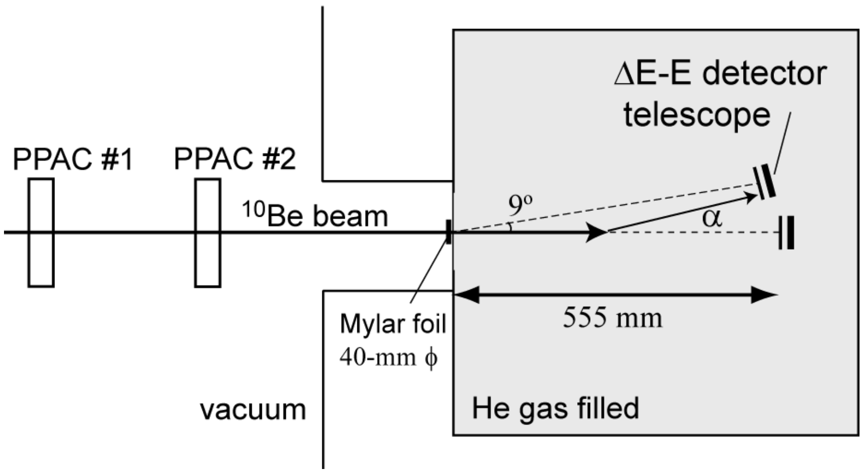
Experimental setup

Thick target method in inverse kinematics,
similar to the previous $^{7}Be+\alpha$.



- Two PPACs for the beam PI, trajectory, number of particles.
- Two silicon detector telescopes for recoiling α particles.
- E_{cm} and θ obtained by event-by-event kinematic reconstruction.

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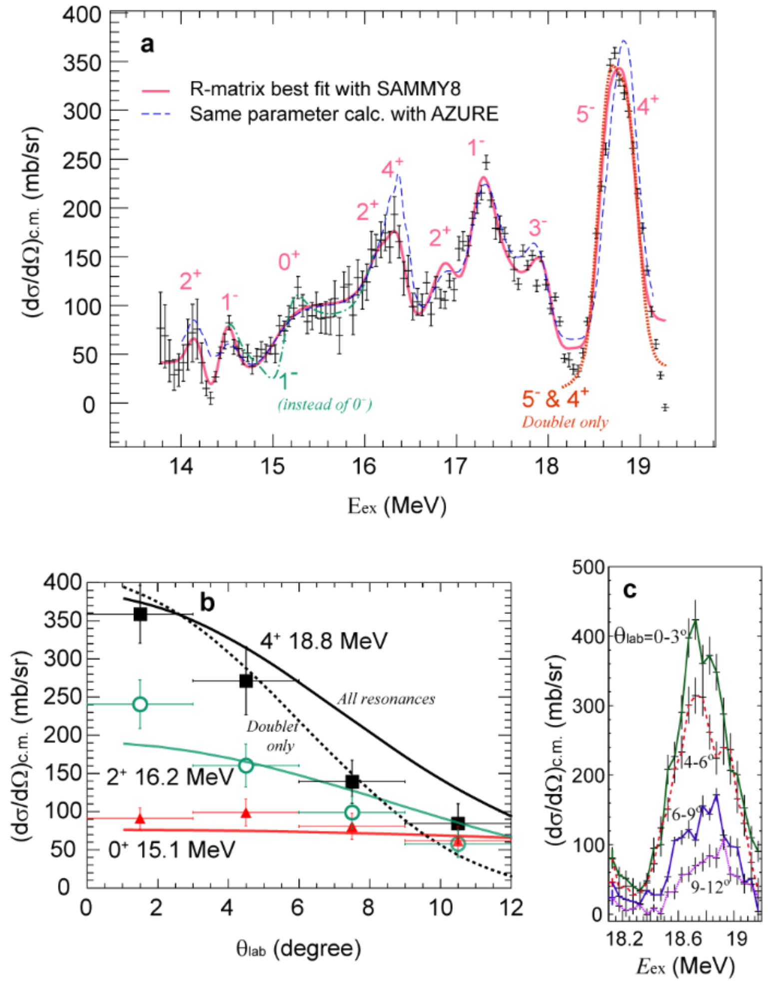
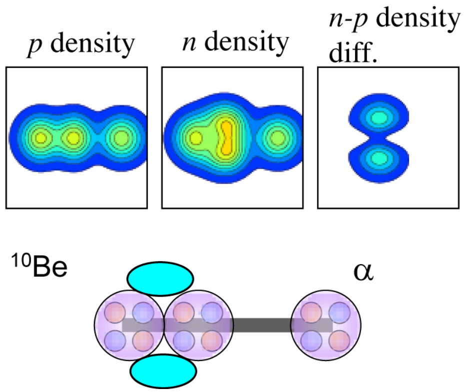
Excitation function

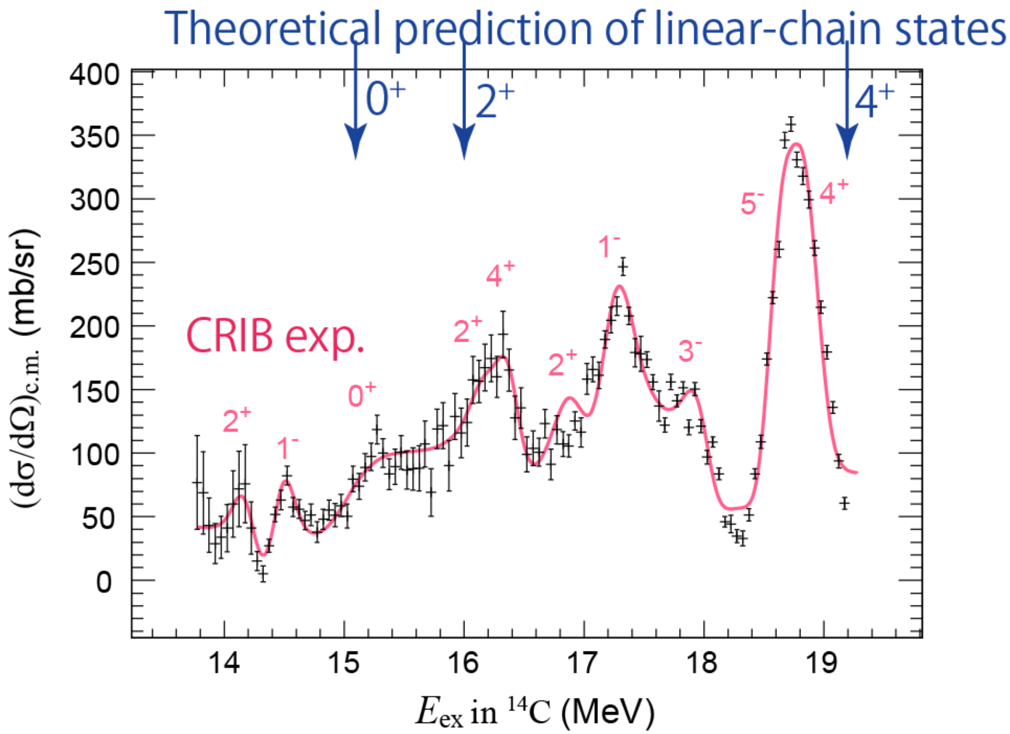
- The excitation function we obtained for 13.8-19.2 MeV exhibits many resonances.
- R-matrix analysis performed, and some of the resonance parameters (E, J^π, Γ_α) were determined.



Result of the linear chain search

Suhara & En'yo, PRC 2010 and 2011:





Excellent agreement between exp. and theory for the (0^+, 2^+, 4^+) states.

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Contents lists available at ScienceDirect
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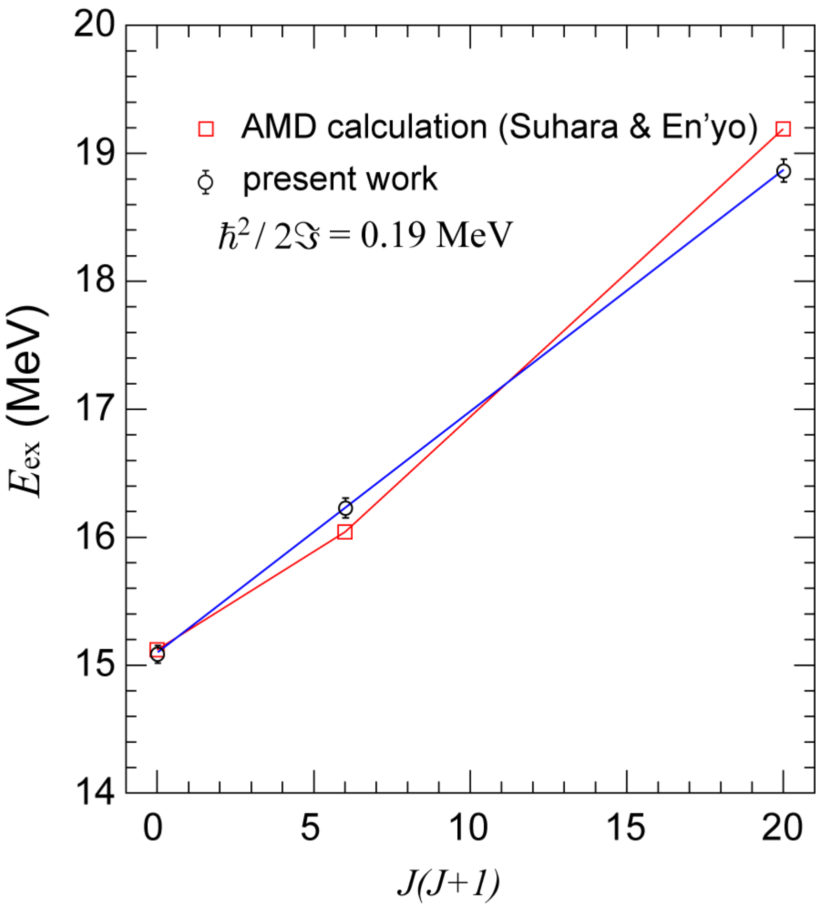
Experimental investigation of a linear-chain structure in the nucleus ^{14}C

H. Yamaguchi[a,*], D. Kahl[a,b], S. Hayakawa[a], Y. Sakaguchi[a], K. Abe[a], T. Nakao[a,c], T. Suhara[d], N. Iwasa[e], A. Kim[f,g], D.H. Kim[g], S.M. Cha[f], M.S. Kwag[f], J.H. Lee[f], E.J. Lee[f], K.Y. Chae[f], Y. Wakabayashi[h], N. Imai[a], N. Kitamura[a], P. Lee[i], J.Y. Moon[j,k], K.B. Lee[j], C. Akers[j], H.S. Jung[k], N.N. Duy[l,m], L.H. Khiem[l], C.S. Lee[i]

Rotational Band

The set of resonances we observed (0+, 2+, 4+) is proportional to *J(J+1)* … consistent with a view of rotational band.

Also perfectly consistent with the theoretical prediction.



Experiments in other facilities

Results on two other ^{10}Be+α TTIK experiments were published before our publication was made.

- M. Freer et al., Phys. Rev. C (2014)
  Birmingham group+ at ORNL
  - High-intensity ^{10}Be beam, spectrum at very forward angle, no PI

  Agreement over Ex>16 MeV, in spite of the difference in the absolute c.s.

- A. Fritsch et al., Phys. Rev. C (2016)
  MSU group at Notre Dame
  - Low-intensity ^{10}Be beam, Active target, only side angles.

  Cannot compare directly, but not good agreement?

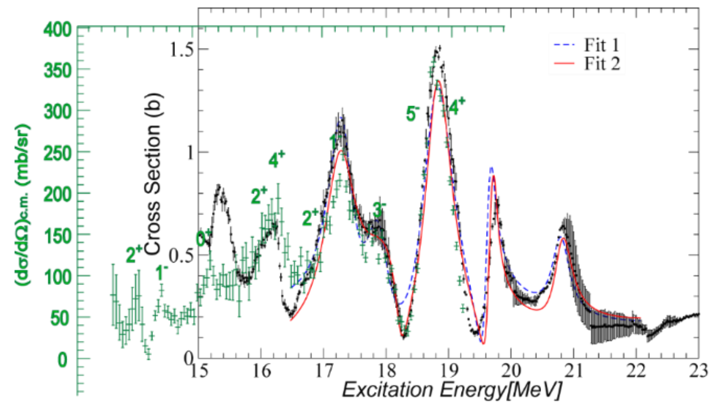
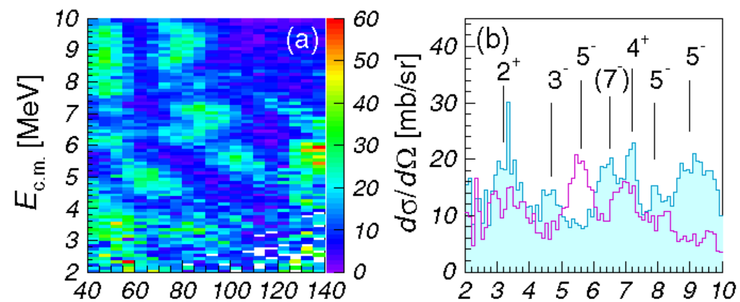


FIG. 11. (Color online) R-matrix fit to the data in the region between E_x = 16.5 and 22 MeV (red-solid and blue-dashed lines). The difference between the two fits is the inclusion of an additional 4^+ state in the calculation shown by the red line. See Table I for the parameters of fits 1 and 2.



E_{cm}=E_x-12 MeV

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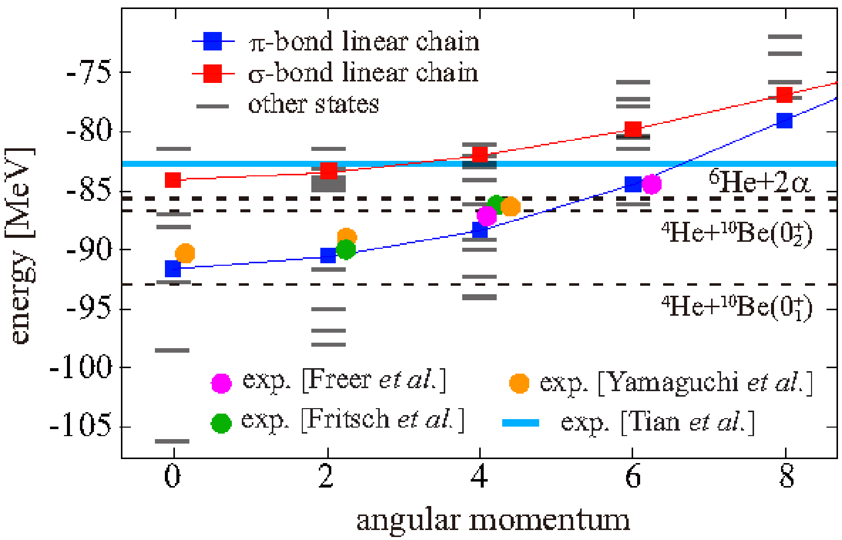
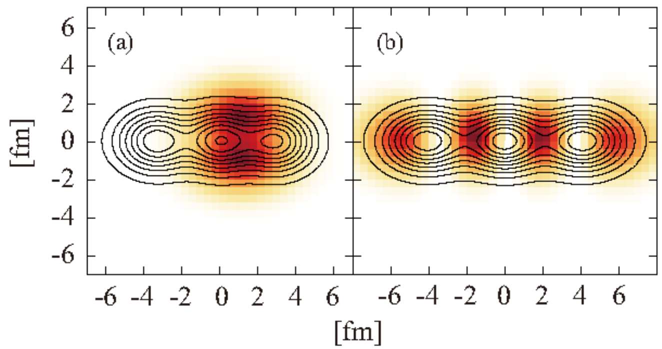
Baba and Kimura (2016 & 2017)

PHYSICAL REVIEW C **95**, 064318 (2017)



π-bond σ-bond



Another AMD calculation,

"π-bond" linear chain band, consistent with 3 experiments

"σ-bond" linear chain band at higher energy (studied by Peking Univ. group).

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How certain are the linear-chain states?

- Identification of the 0^+ state…1^- was excluded with 3σ significance, but the error can be systematic.
  - ◆Limited statistics and angular range
  - ◆Background subtraction
  - ◆Inelastic scattering?

- We planned the 4th experiment at INFN-LNS (Catania, Italy):
  - ◆With offline-production ^{10}Be beam
  - ◆Inelastic scattering separation with TOF.

⇒Performed in Oct., 2018.

The "CHAIN" experiment at INFN-LNS (Catania, Italy)

^{10}Be+α with more intense beam, higher energy and angular resolution: ~2 weeks beamtime.

Investigation of α-chain structures in ^{14}C.

H. Yamaguchi [1], A. Di Pietro[2], R. Dressler[3], J. P. Fernández-García[4], P. Figuera[2], S. Hayakawa[1], S. Heinitz[3], D. Lattuada[5], M. Lattuada[2,6], E. Maugeri[3], M. Milin[7], H. Shimizu[1], A. C. Shotter[8], D. Shumann[3], N. Soic[9], D. Torresi[2], L. Yang[1] M. Zadro[9],

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[2] *INFN, Laboratori Nazionali del Sud, Catania, Italy*
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[3] *Departamento FAMN, Universidad de Sevilla, Sevilla, Spain.*
[5] *Extreme Light Infrastructure - Nuclear Physics (ELI-NP),IFIN-HH, Magurele, Romania*
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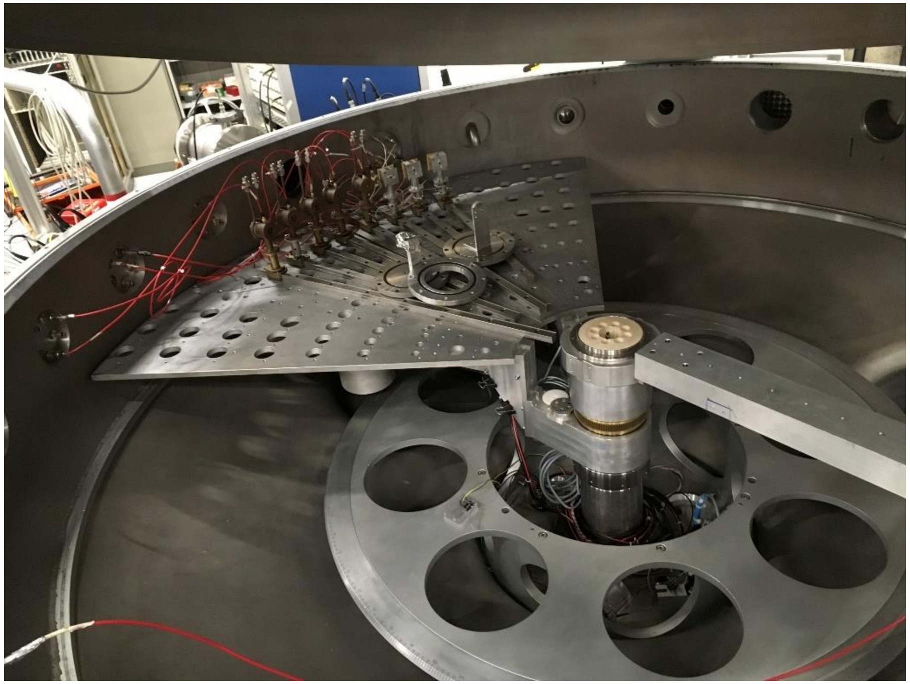
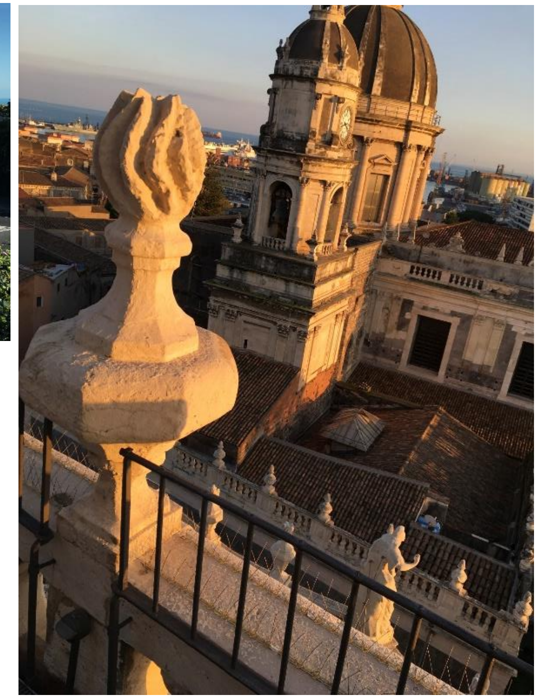
Abstract

We propose to measure the excitation function for the elastic scattering process ^{10}Be+^{4}He, in order to shed some light upon the existence of linear-chain cluster states in the n-rich ^{14}C nucleus. These states are expected to have a configuration in which ^{10}Be and α are spatially separated, and thus they can be observed by the ^{10}Be+α resonant elastic scattering. In order

Experiment at Catania, Oct. 2018



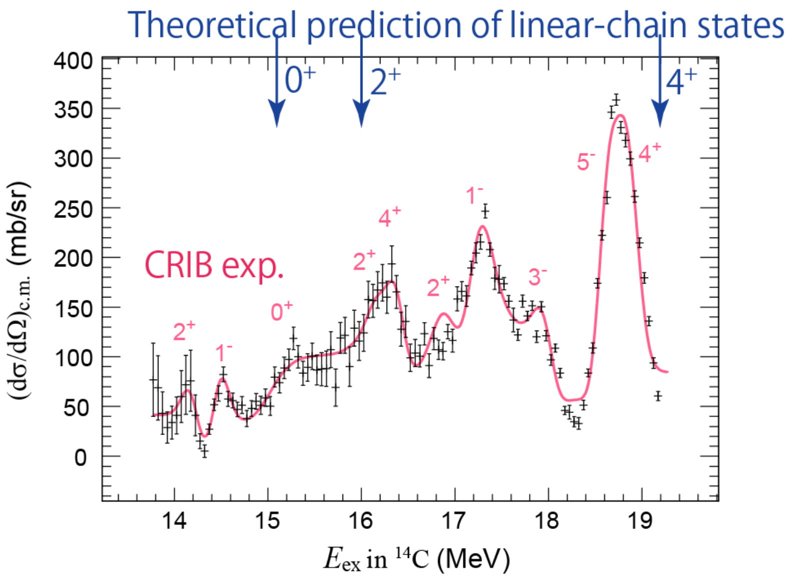




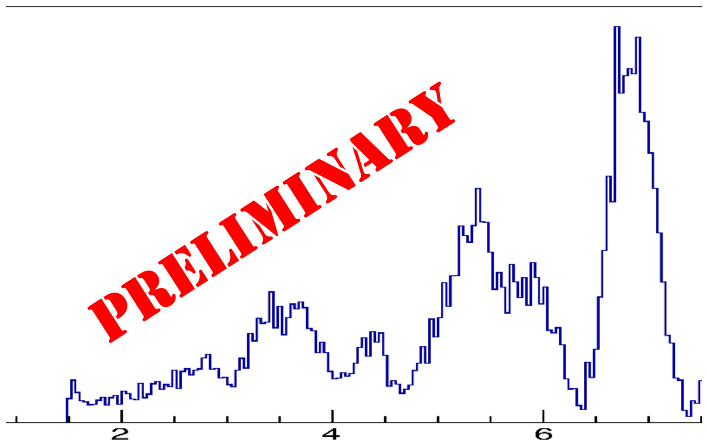
Result (very preliminary)

CRIB vs LNS(Tandem)

Including 0-8 deg events



@5 deg, No normalization for the effective target thickness/absolute cross section yet



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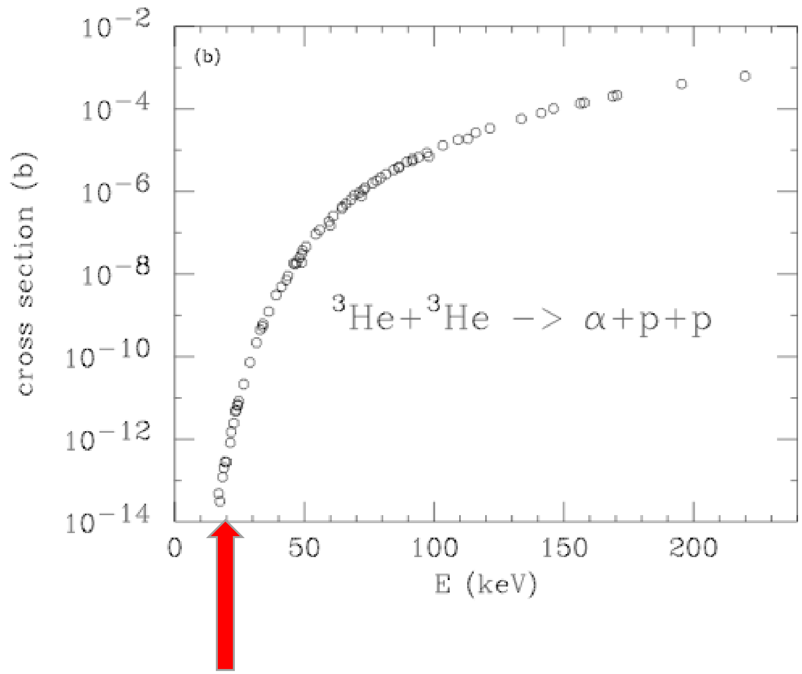
Need of indirect method

Stellar reaction cross section often has a strong dependence on energy (or temperature), changing by orders of magnitude.

…This is because of the tunneling probability of the Coulomb barrier.

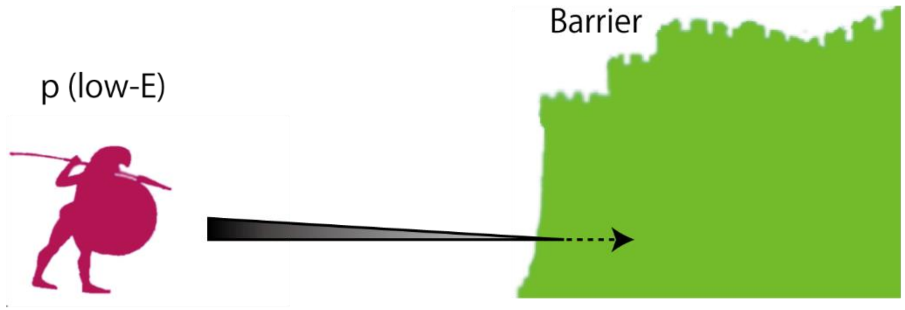
Experimentally, this causes much trouble. We need a clever way.

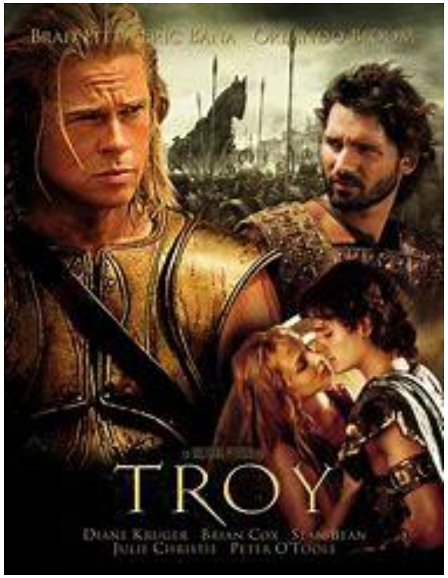


Gamow energy of Solar T

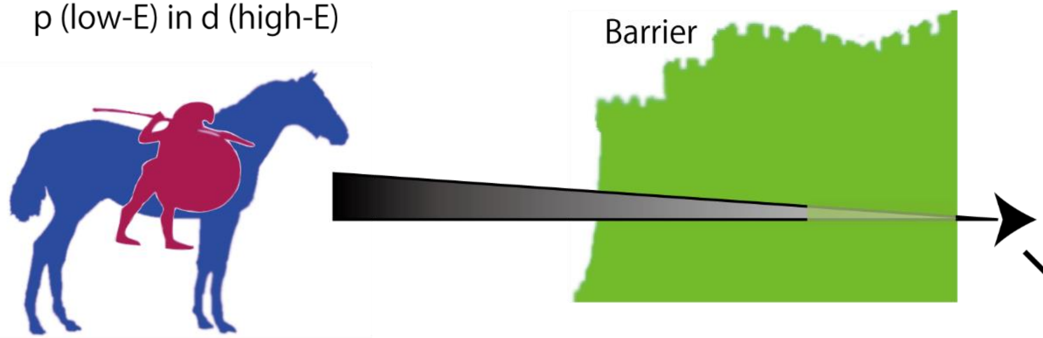
The Trojan Horse Method(THM)

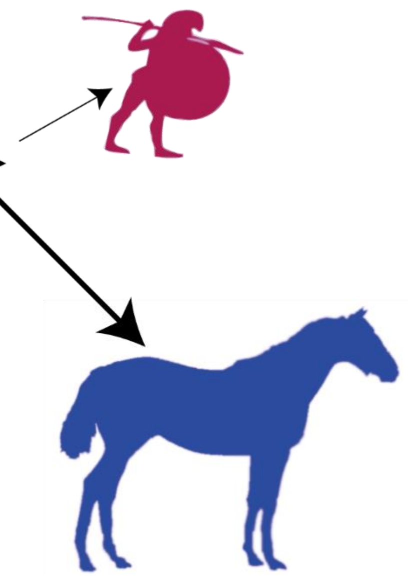
p (low-E)





p (low-E) in d (high-E)





When the horse (neutron) does nothing significant in the castle (nucleus), the reaction can be called as a "quasi-free" reaction.

The $^{18}F(p,\alpha)$ project (with THM)

- $^{18}F(p,\alpha)$… an astrophysical reaction important in novae, and other high-T environments.

- Measurement with the Trojan Horse Method performed in 2008

…**The first THM+RI beam experiment in the world.**

- The RI Beam at CRIB (after development):

Primary beam: $^{18}O^{8+}$, 4.5-5 MeVA

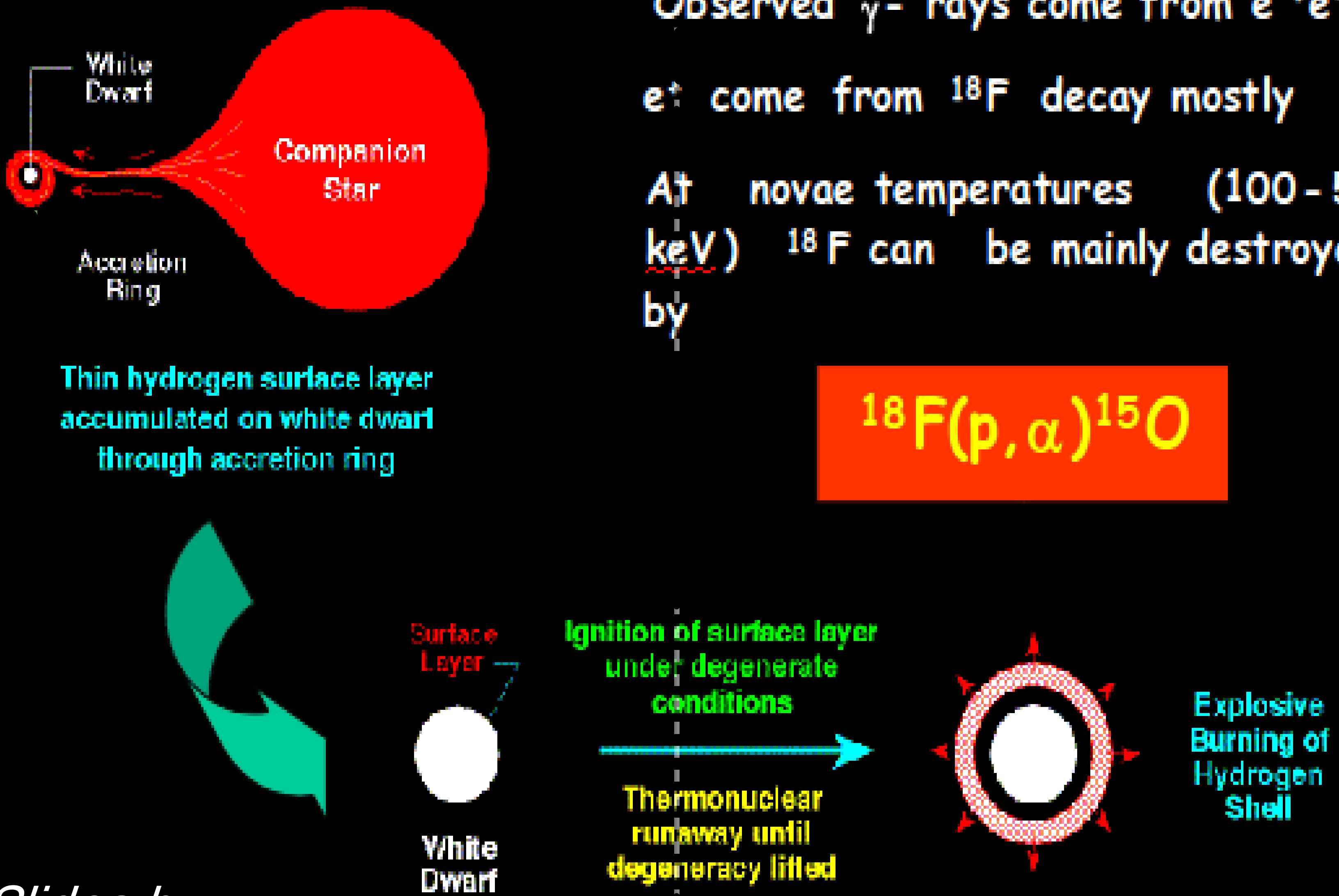
Production target: H_2

Production reaction: $^{18}O(p,n)^{18}F$

◆ Purity nearly 100%

◆ Intensity > 5 x 10^5 pps

A NOVA MICKEY MOUSE PICTURE AND $^{18}F(p,\alpha)^{15}O$



Observed γ- rays come from e^+e^-

e^+ come from ^{18}F decay mostly

At novae temperatures (100 - 500 keV) ^{18}F can be mainly destroyed by

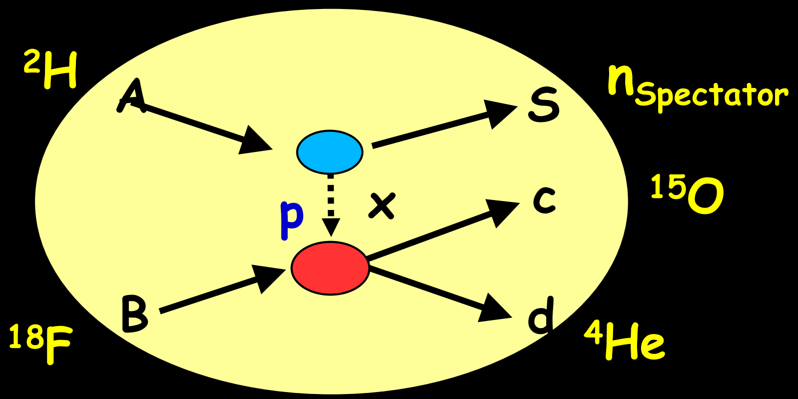
$^{18}F(p,\alpha)^{15}O$

For the star energetics this is peanuts!

Slides by S. Cherubini

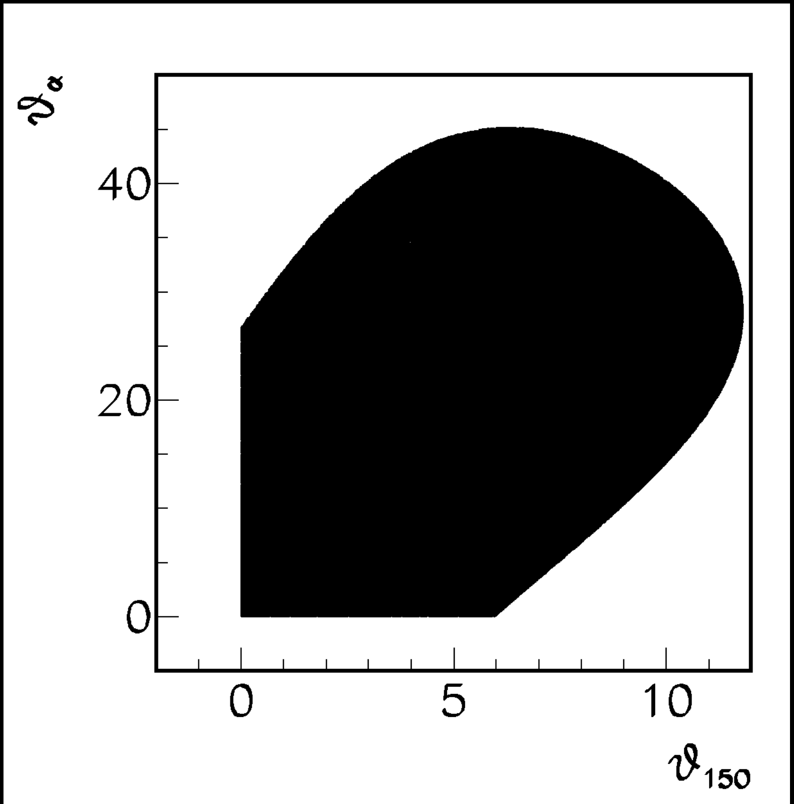
THM measurement: $^{18}F(p,\alpha)^{15}O$ via $^{2}H(^{18}F,\alpha\,^{15}O)n$

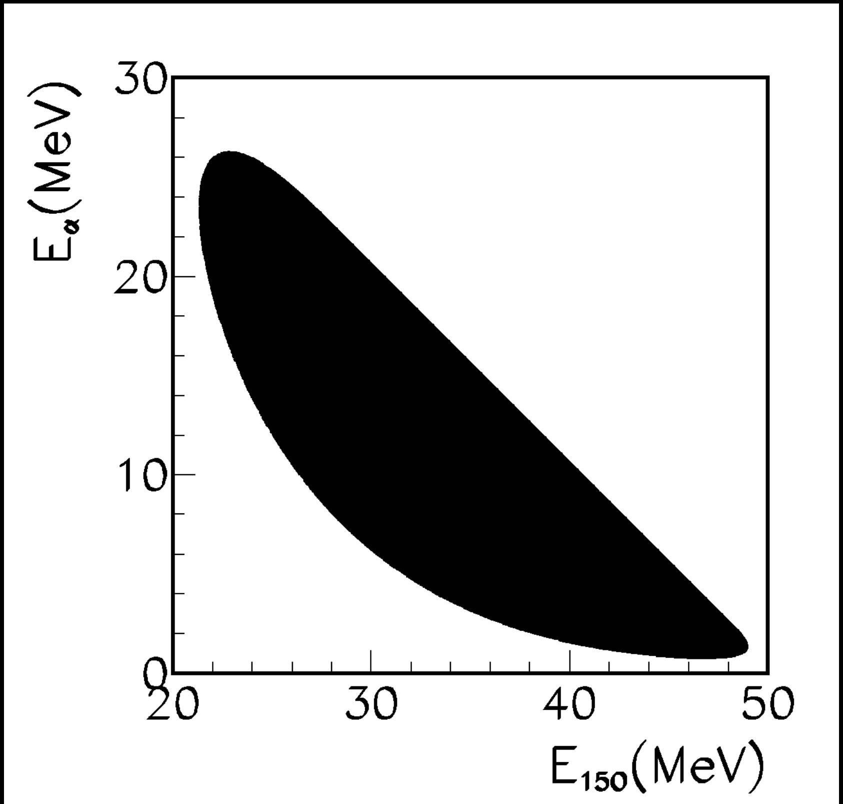
Kinematics



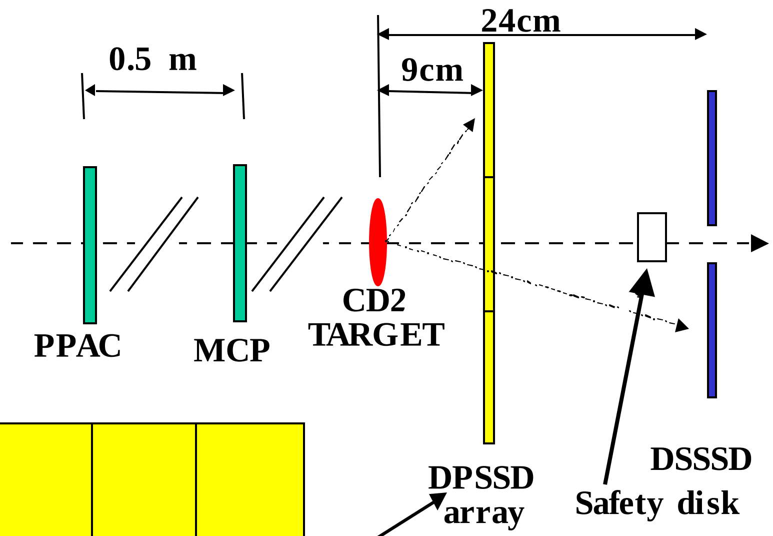
$^{2}H(^{18}F,\alpha\,^{15}O)n$

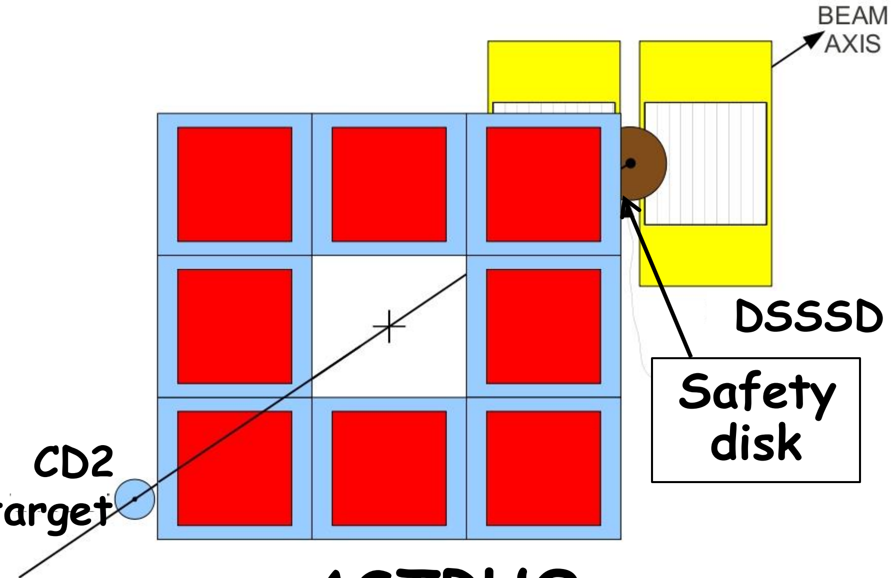
$E(^{18}F)$ = 50 MeV



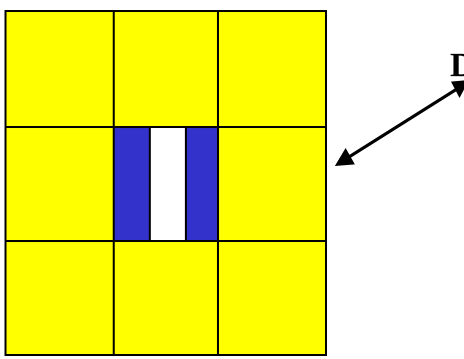


EXPERIMENTAL SETUP

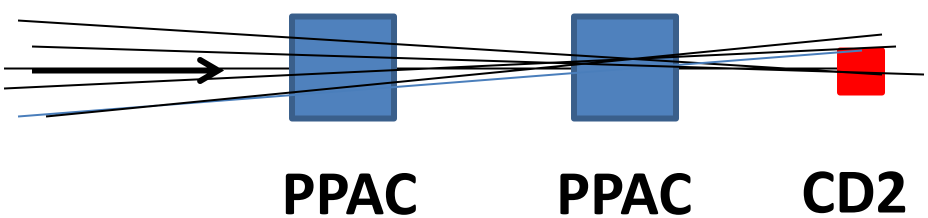




ASTRHO



ASTRHO:
Array of Silicons for
TRojan HOrse



Beam track reconstruction

EVENT SELECTION

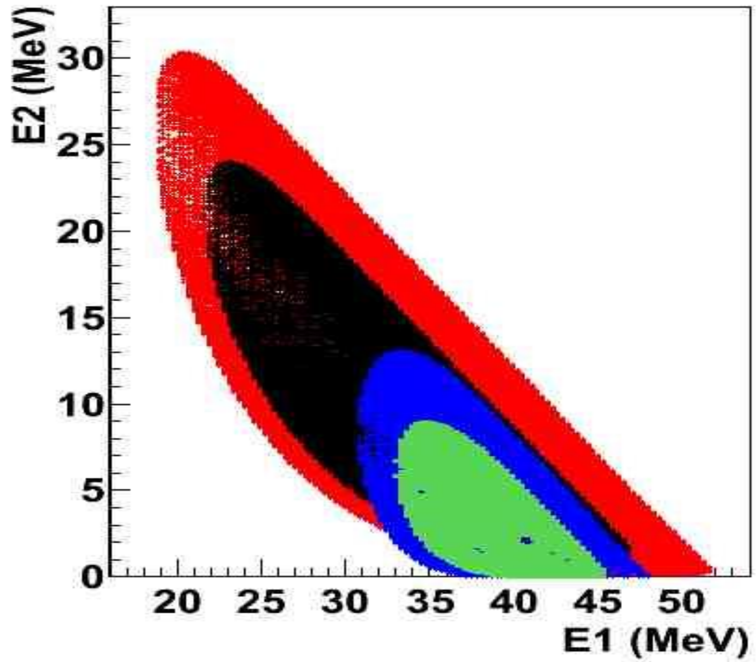
Red: $^{18}F + d \rightarrow {}^{15}N + \alpha + p$

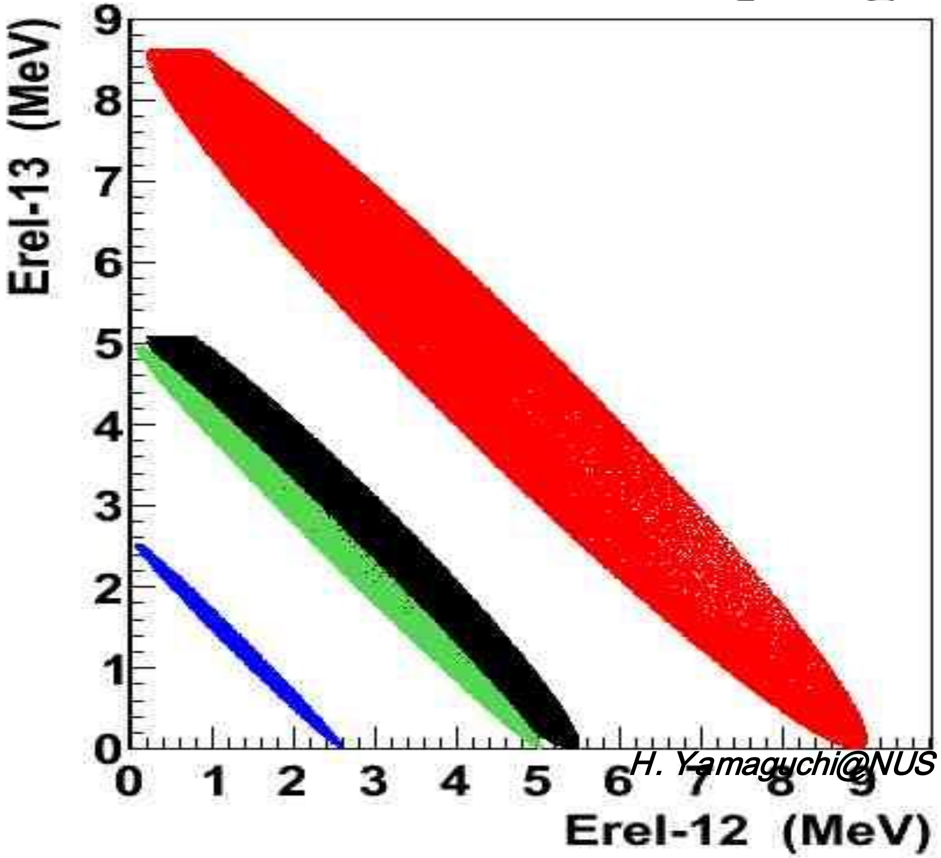
Black: $^{18}F + d \rightarrow {}^{15}O + \alpha + n$

Blue: $^{18}F + d \rightarrow {}^{18}F + p + n$

Green: $^{18}F + d \rightarrow {}^{18}O + p + p$

» 1 + 2 + 3





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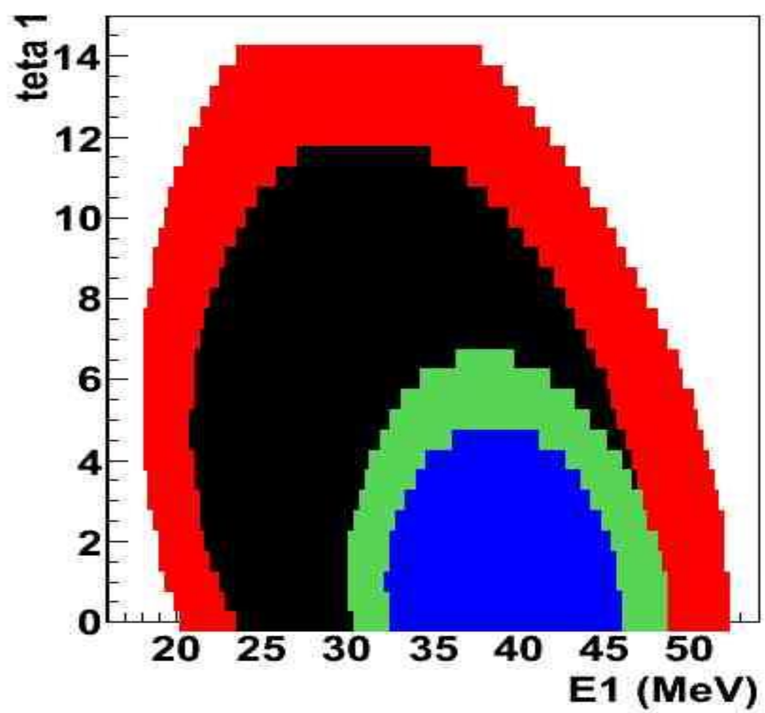
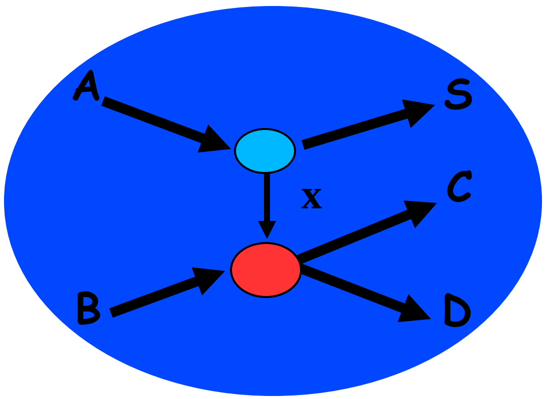


Assuming that a Quasi-free mechanism is dominant one can use the (PW)IA:



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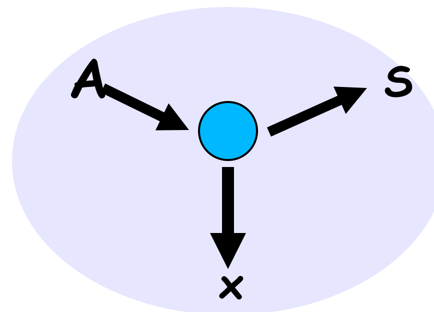

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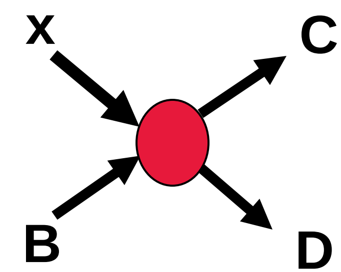


3-body Reaction

Virtual Decay

Virtual reaction
(astrophysical process)

$$\frac{d^3\sigma}{d\Omega_C \, d\Omega_D \, dE_{cm}} \Big/ \; KF \cdot |\Phi(P_s)|^2 \; \propto \; \frac{d\sigma^N}{d\Omega}$$

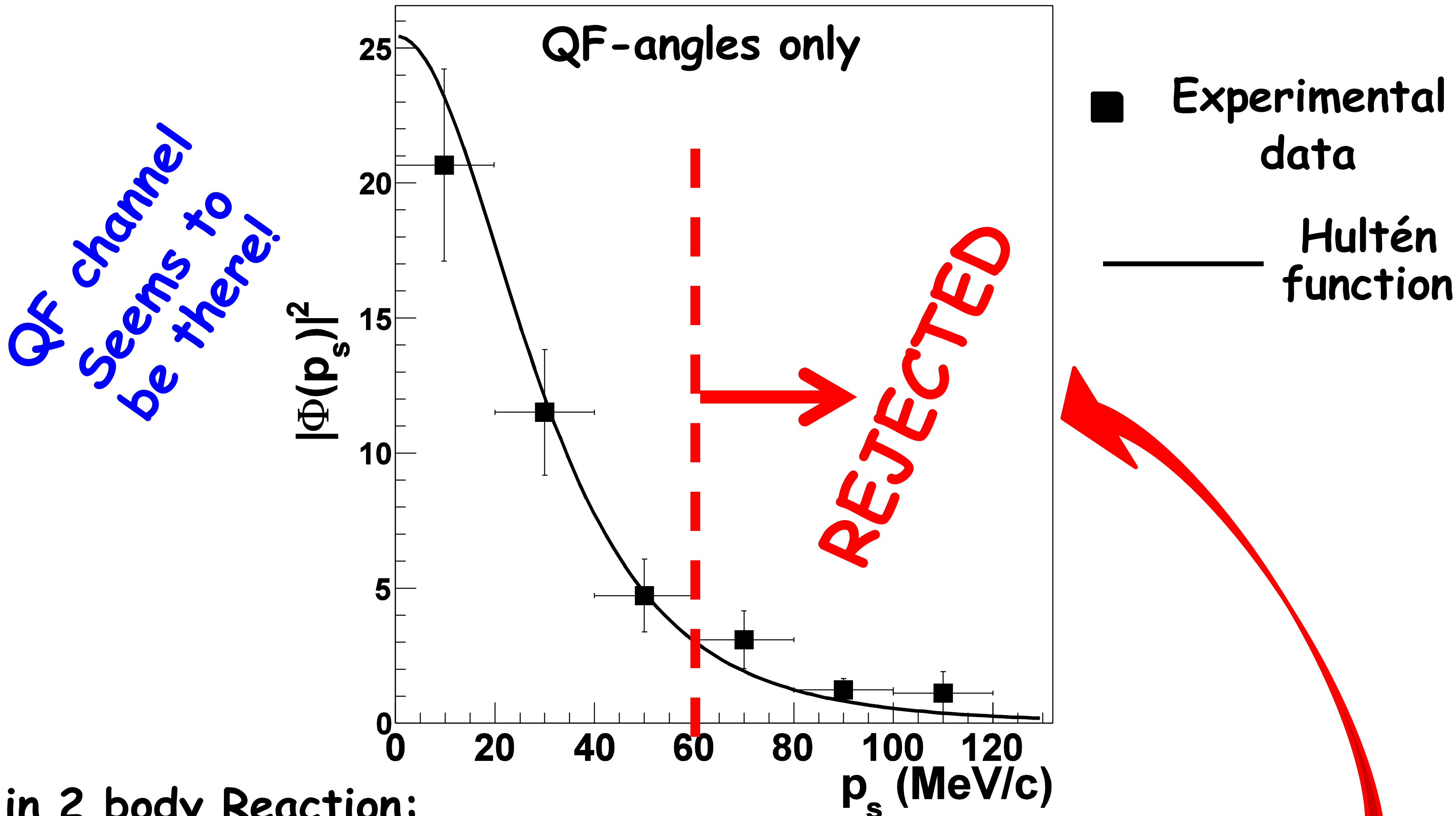
Measured at high energy

Calculated e.g. Montecarlo

Indirectly Measured

$E_{Bx} = E_{CD} - Q_{2b}$

EXPERIMENTAL IMPULSE DISTRIBUTION



If in 2 body Reaction:

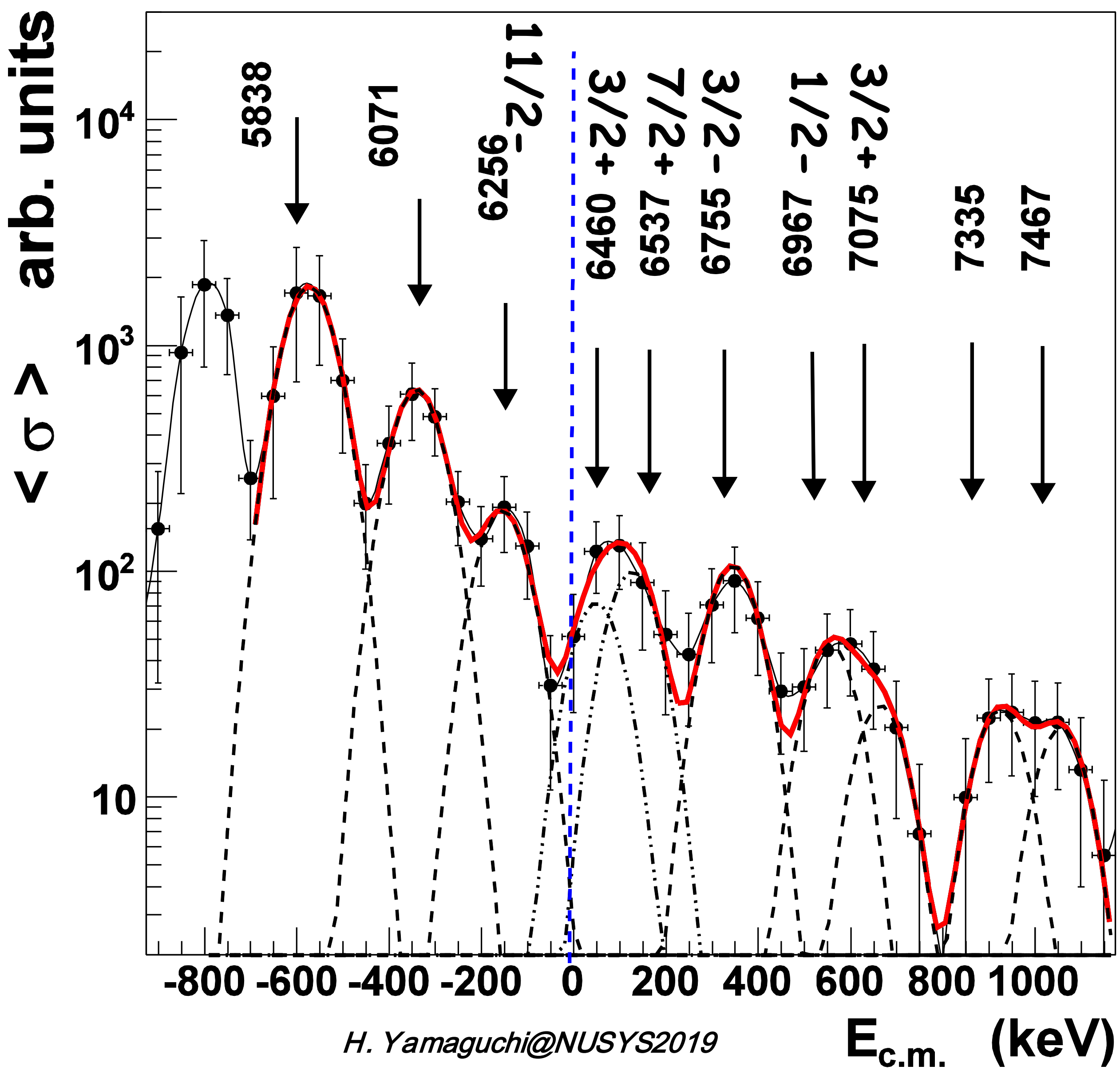
Ecm= const

Θcm =const

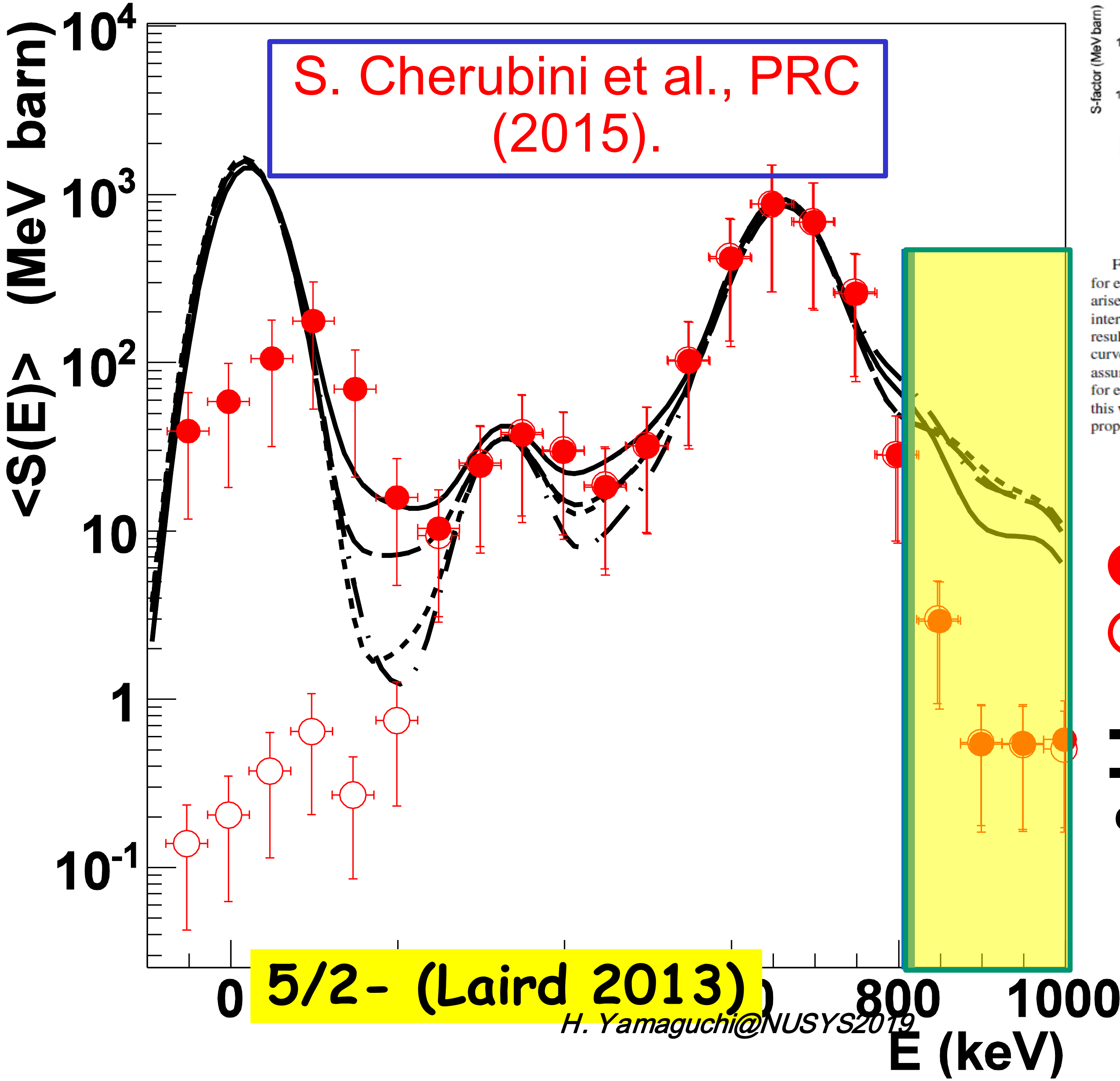
d2σ/dΩ = const

d3σ/dΩ $\propto |\Phi(P_s)|^2$

THM(=barriers free) CROSS SECTION



S(E) from THM 8 keV 3/2+



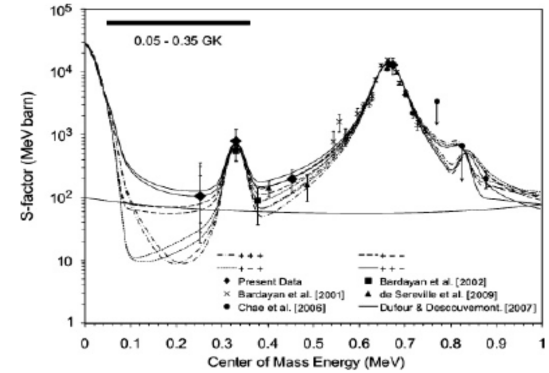


FIG. 3. The $^{18}F(p,\alpha)^{15}O$ S factors, calculated using the R matrix, for eight possible interference terms. The range in possible S factors arises from the interference between the $J^\pi = 3/2^+$ resonances. The interference between resonances dominates in the region of interest, resulting in four groups of S-factor curves. The upper and lower curves of each group are shown in the figure. The legend gives the assumed phase, for the 8-, 38-, and 665 keV resonances, respectively, for each pair of curves. Also plotted are the measured S factors from this work, those from previously published data [4,10,12,19], and the proposed contribution from $1/2^+$ states predicted in Ref. [6].

Direct data...C.E. Beer, et al.

THM data

C.E. Beer, Phys. Rev. C 83, 042801(R) (2011)

Smeared to THM resolution

^{7}Be(n,p)^{7}Li and the ^{7}Be(n,α)^{4}He reactions with THM for cosmological lithium problem

S. Hayakawa[1], M. La Cognata[2], L. Lamia[2], H. Shimizu[1], L. Yang[1], H. Yamaguchi[1], K. Abe[1], O. Beliuskina[1], S. M. Cha[4], K. Y. Chae[4], S. Cherubini[2,3], P. Figuera[2,3], Z. Ge[5], M. Gulino[2,6], J. Hu[7], A. Inoue[8], N. Iwasa[9], D. Kahl[10],
A. Kim[11], D. H. Kim[11], G. Kiss[5], S. Kubono[1,5,7], M. La Commara[12,13], M. Lattuada[2,3], E. J. Lee[4], J. Y. Moon[14],
S. Palmerini[15,16], C. Parascandolo[13], S. Y. Park[11], D. Pierroutsakou[13], R. G. Pizzone[2,3], G. G. Rapisarda[2],
S. Romano[2,3], C. Spitaleri[2,3], X. D. Tang[7], O. Trippella[15,16], A. Tumino[2,6], P. Vi[5] and N. T. Zhang[7]

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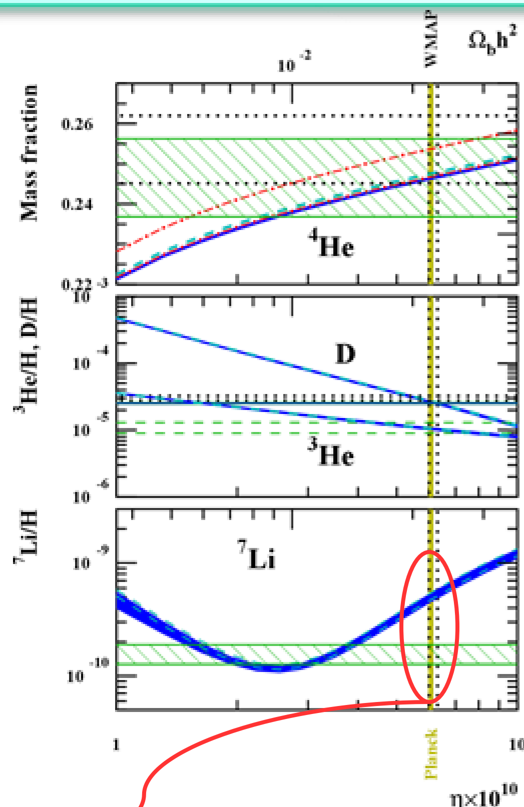
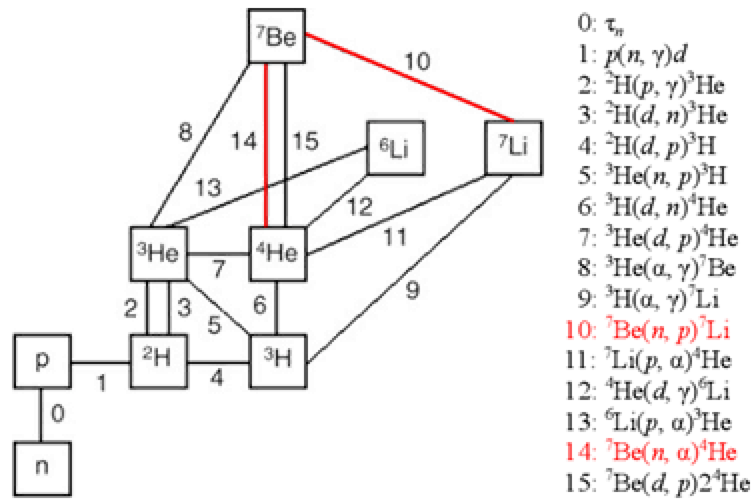
[7]Institute of Modern Physics, Chinese Academy of Sciences, [8]Research Center for Nuclear Physics (RCNP), Osaka University, [9]Tohoku University, [10]University of Edinburgh, [11]Ewha Womans University,
[12]University of Naples Federico II, [13]INFN – Naples, [14]Institute for Basic Science (IBS),
• [15]INFN - Perugia, [16]University of Perugia

+ + many others

Cosmological ^{7}Li problem



A. Coc et al. J. Cos. Astropart. Phys. 2014

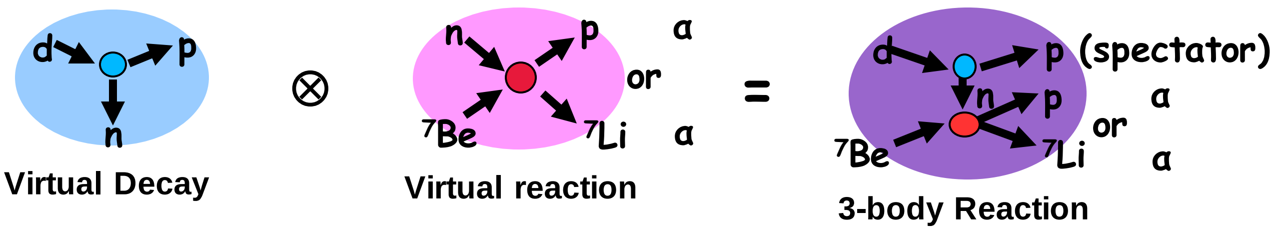


Iocco et al. Phys. Rep. 2009

- ^{7}Li problem… disagreement between theory and observation by a factor of 3−4
  - Due to CMB obs.? Low-metallicity stars obs.? Standard BBN model? Nuclear Physics?
  - ^{7}Be abundance in the end of BBN determines ^{7}Li predominantly
  - $p(n,\gamma)d$, ^{3}He$(d,p)^4$He, ^{7}Be$(n,p)^7$Li, ^{7}Be$(n,\alpha)^4$He, ^{7}Be$(d,p)2\alpha$, etc.
- Temperature ~ $10^{10} - 3\times10^8$ K, Energy: 1 MeV − 25 keV

Trojan Horse Method for RI + neutron

- Trojan Horse method: (Spitaleri+ Phys. Atom. Nucl. 2011)
- $^{7}Be(n,p)^{7}Li$, $^{7}Be(n,\alpha)^{4}He$ via $^{2}H(^{7}Be,^{7}Li\,p)^{1}H$, $^{2}H(^{7}Be,\alpha\alpha)^{1}H$
- PWIA applicable when Quasi-free mechanism is dominant



$$(\text{Kinematic Factor})\cdot|\Phi(P_s)|^2 \times \frac{d\sigma^{HOES}}{d\Omega} \propto \frac{d^3\sigma}{d\Omega_p\, d\Omega_{7Li}\, dE_{cm}}$$

Calculated by Monte Carlo simulation

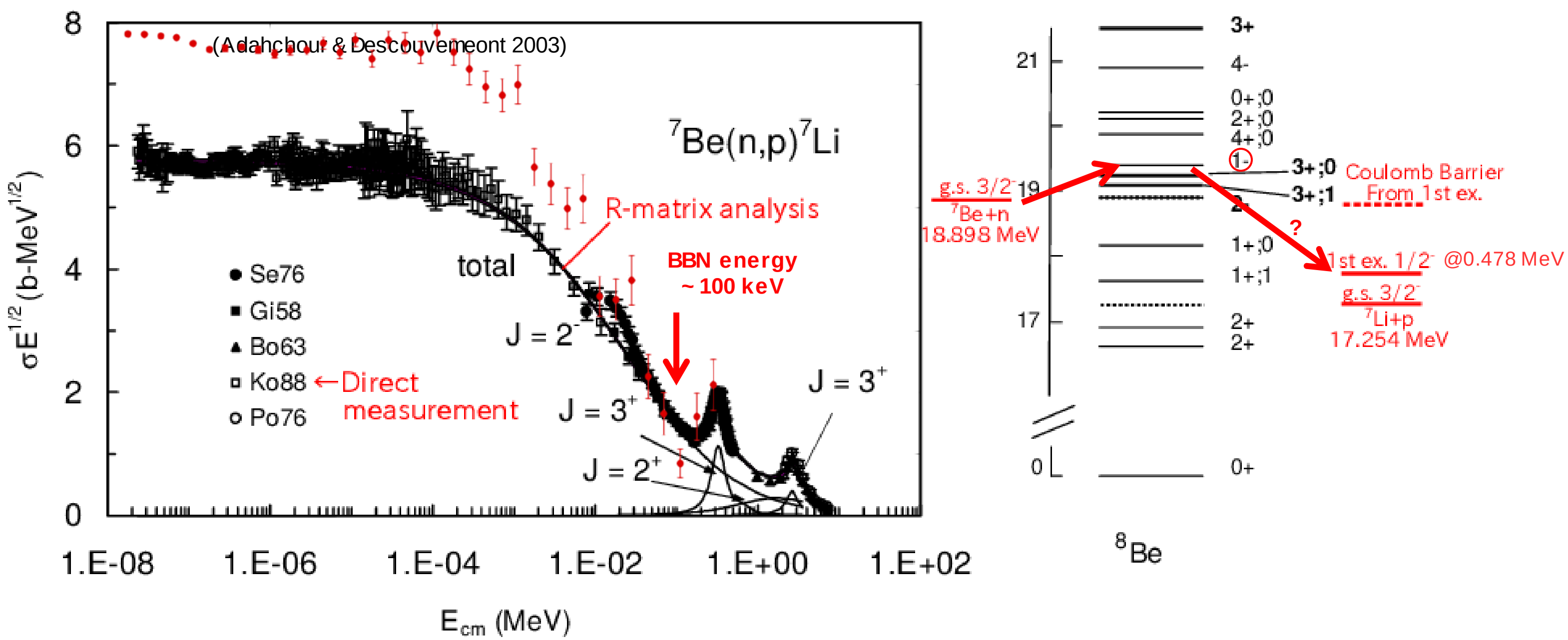
Half-off energy-shell 2-body cross section

Measured at $E_{d\text{-}7Be}$ > Coulomb barrier

↑

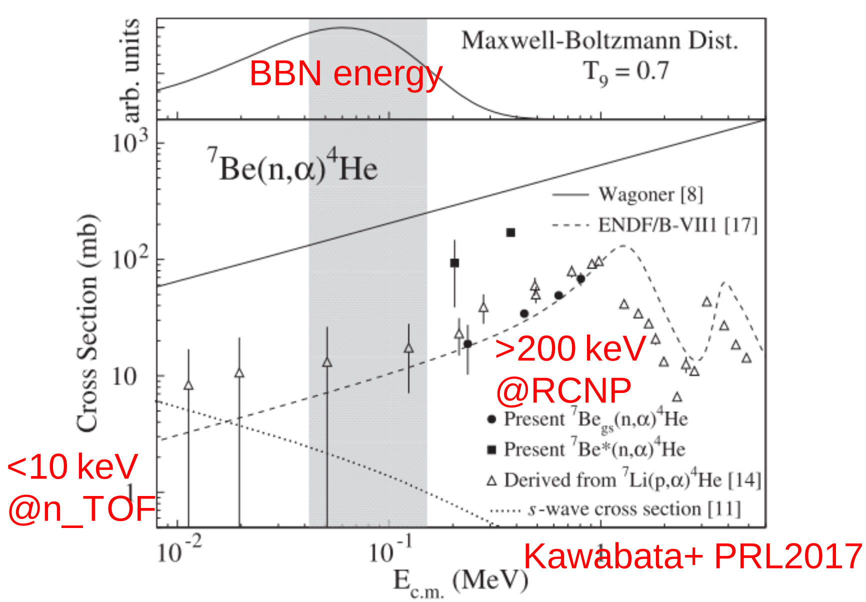
σ^{OES} = Normalization × σ^{HOES} × Penetrability

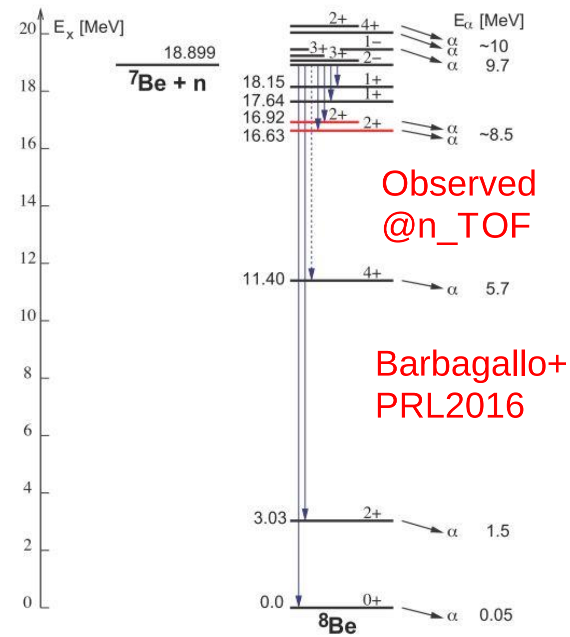
$^7Be(n, p)^7Li$ (Q = 1.644 MeV)



- Sensitivity: $\partial \log Y_{7Li} / \partial \log \langle \sigma v \rangle_{7Be} = -0.71$ (Coc & Vangioni 2010, Cyburt+ 2016, etc.)
  If 5 × higher rate ➡ ^{7}Li problem solved
- Direct measurement up to 13.5 keV, time-reversal reactions at higher energies.
- R-matrix analysis: Adahchour & Descouvemont 2003.
- New n_TOF measurement: enhancement below BBN energies (Damone+ PRL 2018)

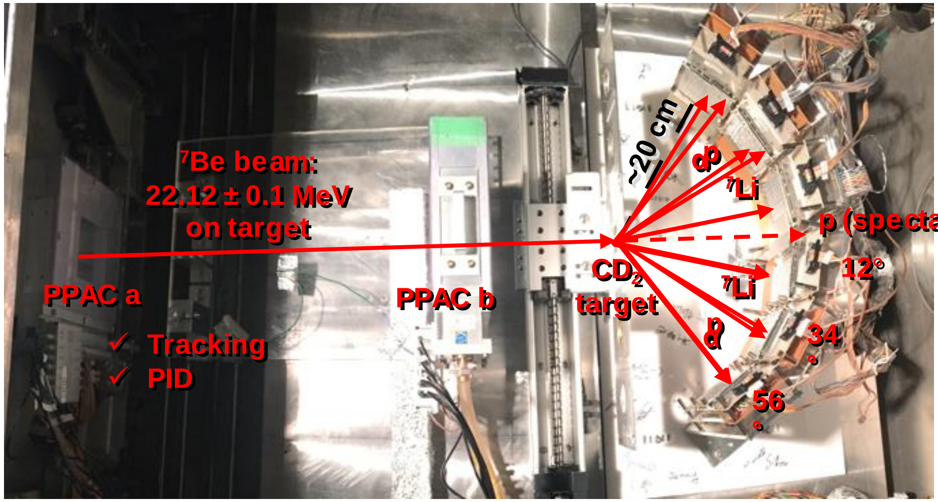
$^{7}Be(n,\alpha)^{4}He$ (Q = 18.990 MeV)





- Hou et al. PRC 2015: evaluation from $^{4}He(\alpha,p)^{7}Li$
- Barbagallo et al. PRL 2016: s-wave measurement @ nTOF
- Kawabata et al. PRL 2017: p-wave measurement @RCNP
- Lamia et al. APJ 2017: evaluation of $^{7}Li(p,\alpha)$ data measured by THM.
- Recent works consistent… Yet no direct data in the BBN range.

Experimental setup



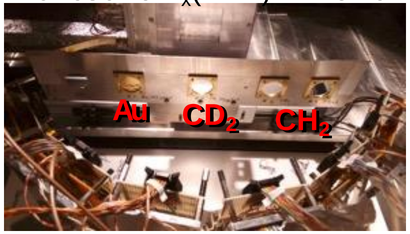
- 6 ΔE-E position sensitive silicon telescopes
- ^{7}Li-p and α-α coincidence measurements

… spectator not measured

- CD_2: 64 μg/cm^2

➡ ΔE_{beam} ~ 150 keV

➡ To resolve $E_x(^7\mathrm{Li}^{1st})$ = 478 keV



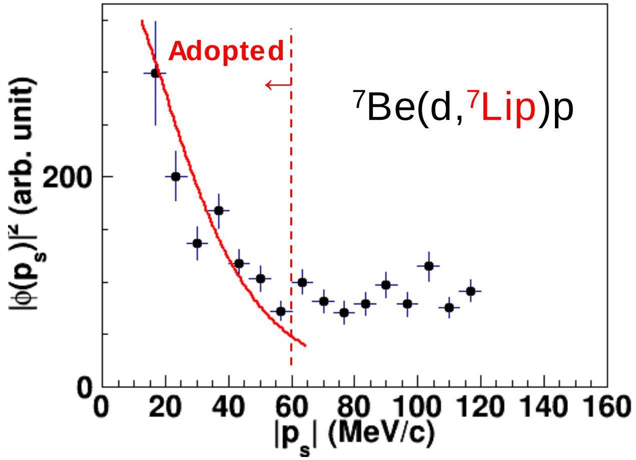
- Hamamatsu Charge-division PSD: position resolution ~ 0.5 mm

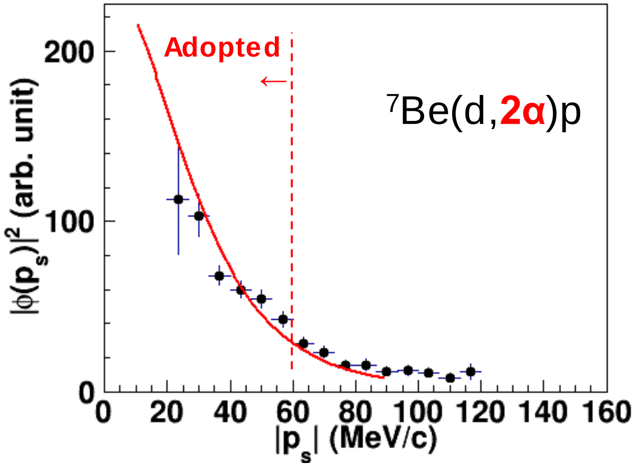
➡ Total angular resolution (PPACs & PSDs & alignment) ∽ 0.5° ➡ ΔE_{cm} ~ 60 keV

Momentum distributions of the spectator *p*

$Y_{exp}/Y_{sim} \propto d^3\sigma/(d\Omega_p d\Omega_{7Li} dE_{cm})$ / KF $\propto |\Phi(p_s)|^2 \, d\sigma/d\Omega$

$\sim |\Phi(p_s)|^2$ at a fixed $E_{c.m.}$ and $\theta_{c.m.}$ (⇔ 2-body cross section is const.)

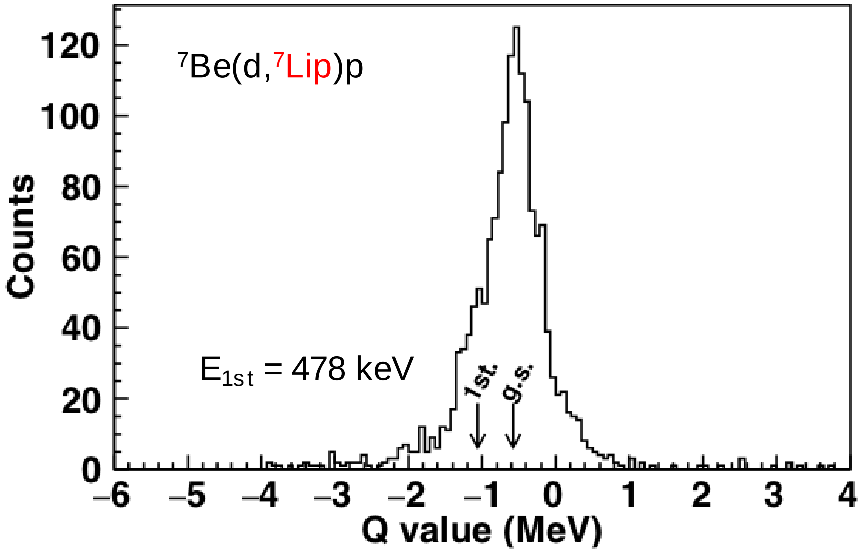


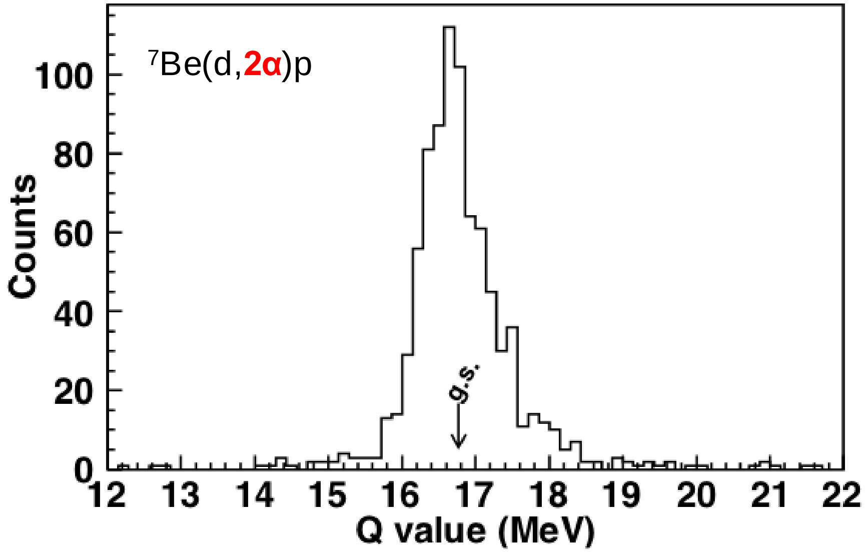


Hulthén function in momentum space
for p-n intercluster motion (PWIA app.)

Good agreement up to 60 MeV/c
Evidence that quasi-free contribution is dominant.→ THM is valid!

Q-value spectra of the 3-body channels





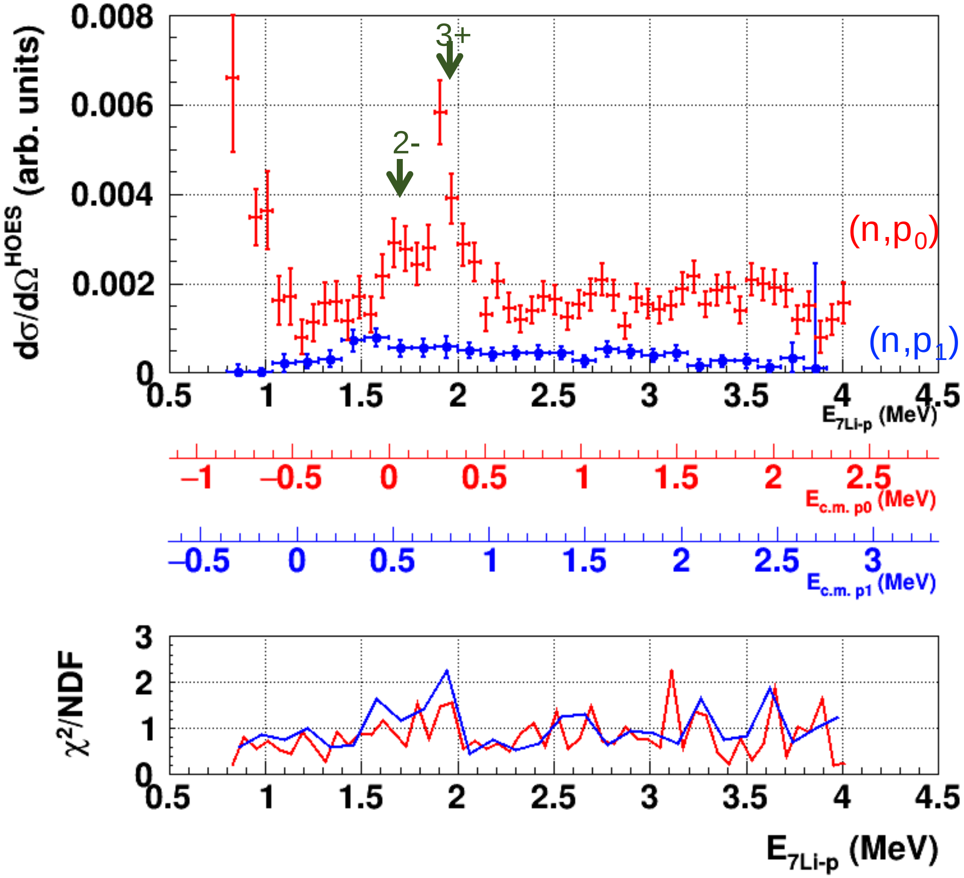
Reaction	*Q*-value (MeV)
$p+2\alpha$	16.766
$^7Li+2p$	-0.589
$^7Be+n+p$	-2.225
$^5He+p+^3He$	-4.547

$Q_{3body} = E_1 + E_2 + E_3 - E_{beam}$

$\Delta Q_{3body} \sim \sqrt{(\Delta E_1^2 + \Delta E_2^2 + \Delta E_3^2 + \Delta E_{beam}^2)}$

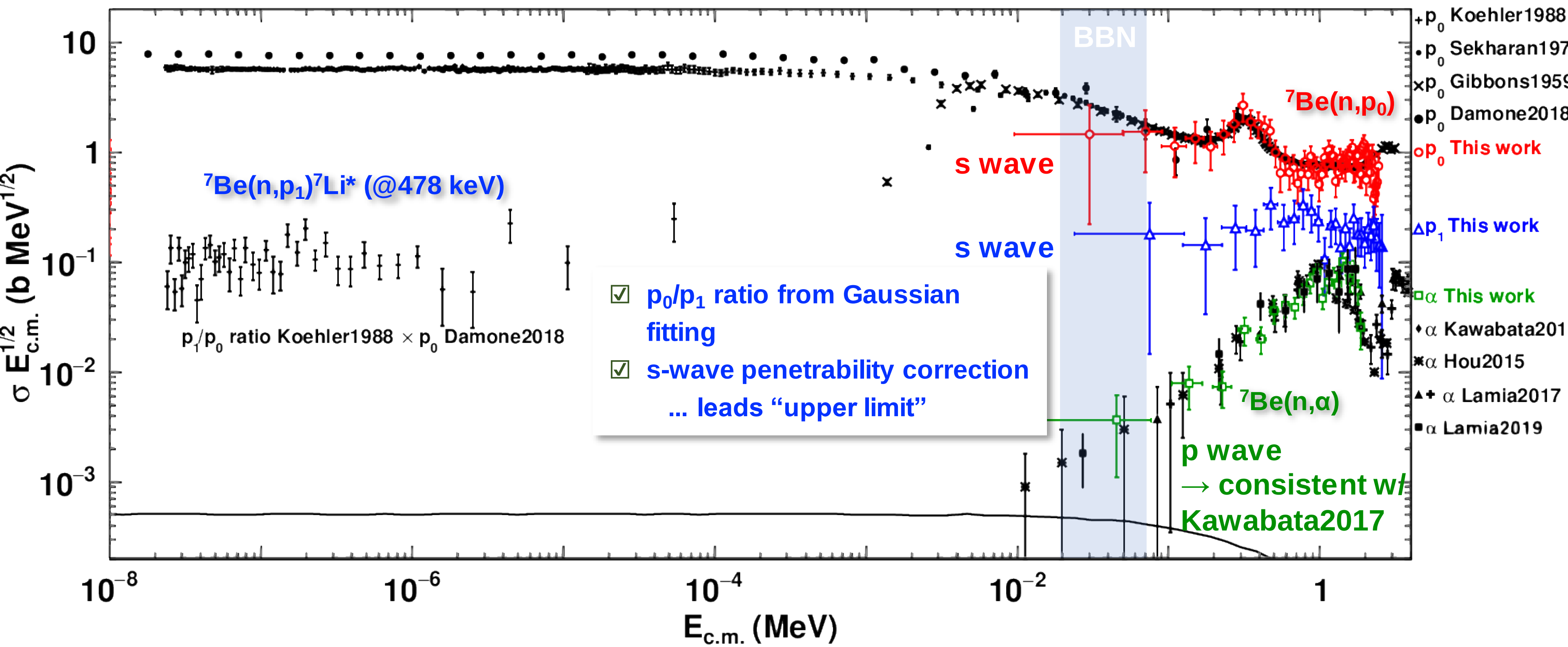
~ 200 keV expected with 64 μg/cm^2 CD_2

Gaussian fitting to Q-value spectra

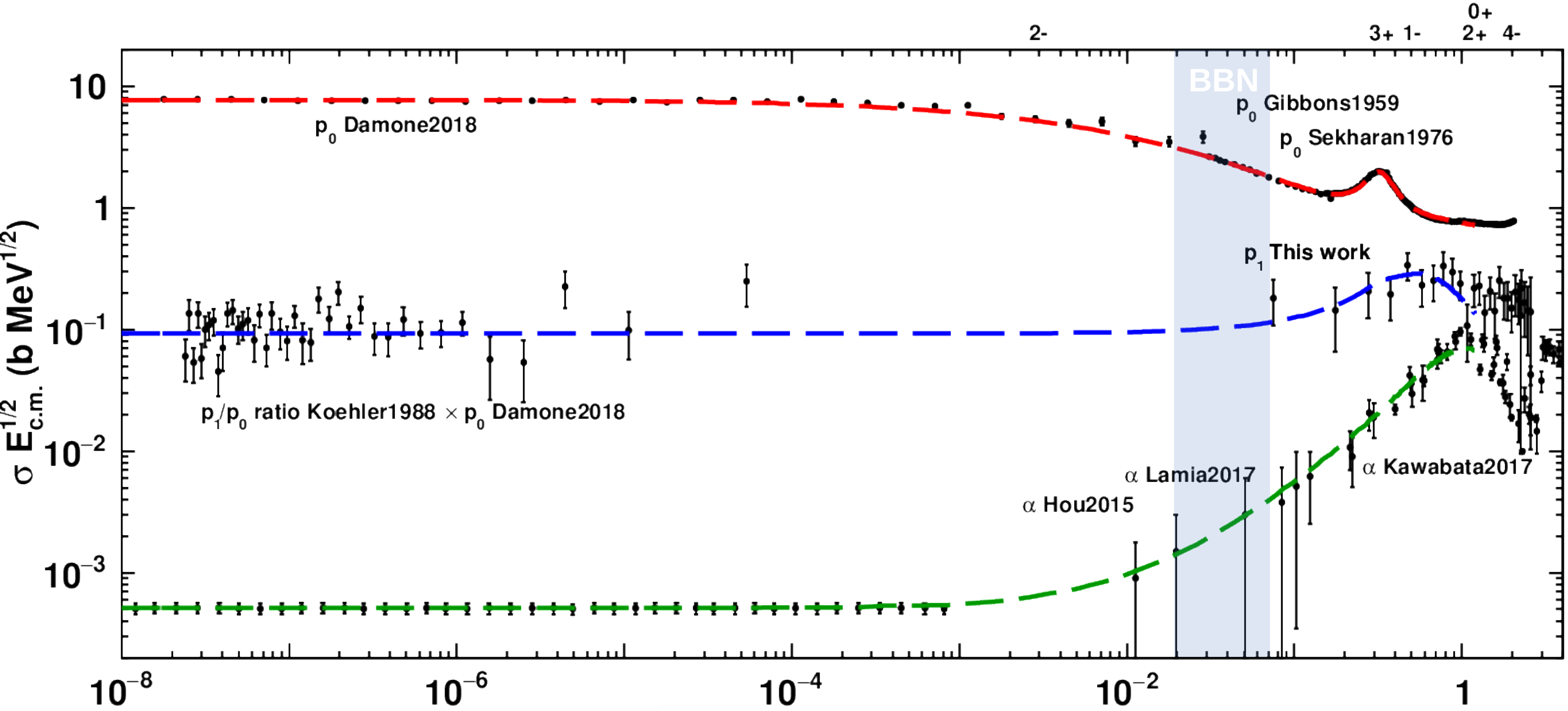


- Isotropy assumed (as no strong angular dependence seen)
- Checked systematic change of widths & peaks
  ➡ Reduces errors

$^7Be(n,p_0)$, (n,p_1) & (n,α_0) cross sections by CRIB



(Preliminary) R-matrix fitting by AZURE2



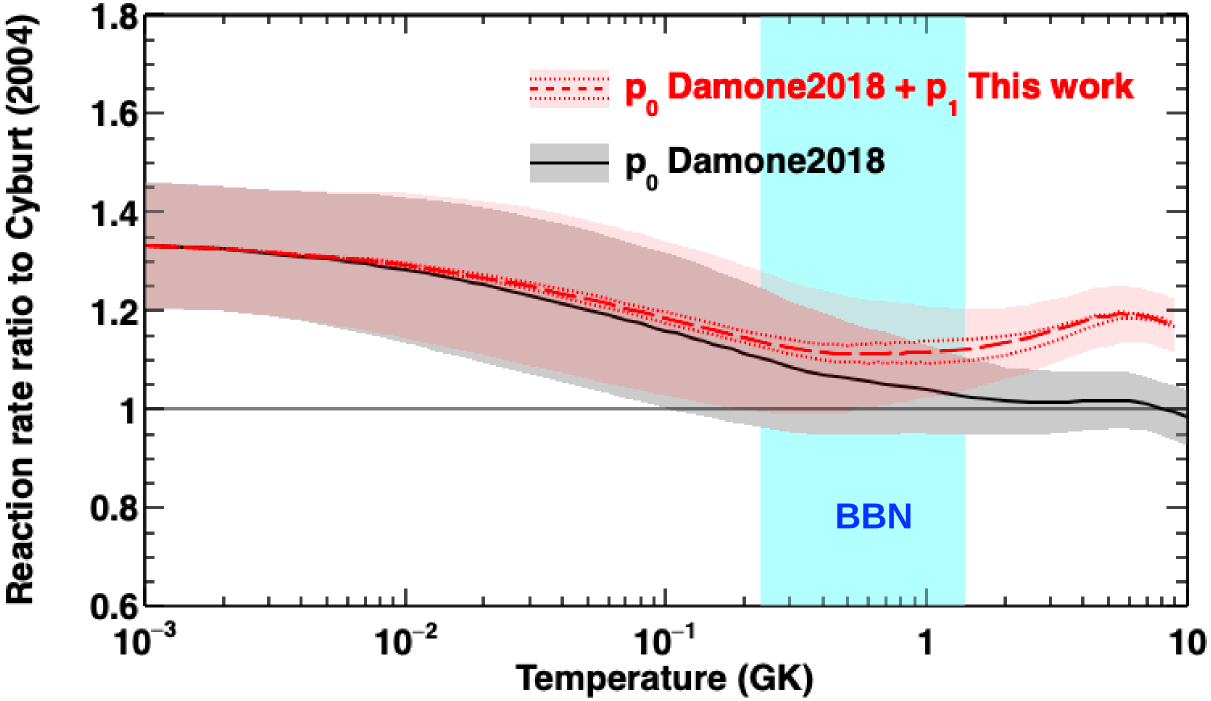
Condition (starting from simple assumptions):

- ☑ Fix known J^{π} and $E_{resonance}$ (except 1-)
- ☑ Adopt $l \leqq 3$
- ☑ Exclude no neutron emission states
- ☑ Fit Only $E_{c.m.} < 1.2$ MeV

Procedure:

- ☑ Start from (n,p_0) channel with Ada.&Desc.2003 parameters.
- ☑ Fit only (n,p_1) and (n,α) channels.
- ☑ Fit (n,p_0) channels again.
- ☑ Fix converged parameters and iterate.
- ☑ χ^2 converged (preliminary): χ^2_{p0}/NDF = 1.59, χ^2_{p1}/NDF = 1.33, χ^2_{α}/NDF = 0.68

Revised ^{7}Be(n,p) Reaction rate



<u>n_TOF result (Damone+ PRL2018):</u>
~ 5% higher rate in BBN range
➡ 96% ^{7}Li abundance

<u>This work:</u>
~ 15±15% higher rate (preliminary)
➡ ~ 90% ^{7}Li abundance (preliminary)
(with the sensitivity
$\partial \log Y_{7Li} / \partial \log \langle \sigma v \rangle_{7Be} = -0.71$)

<u>Recent ^{7}Be+d work @ FSU</u>
<u>(Rijal+ PRL122, 182701 (2019)):</u>
Resonance of (d,α) channel just in BBN Gamow window
➡ ~ 87% ^{7}Li abundance

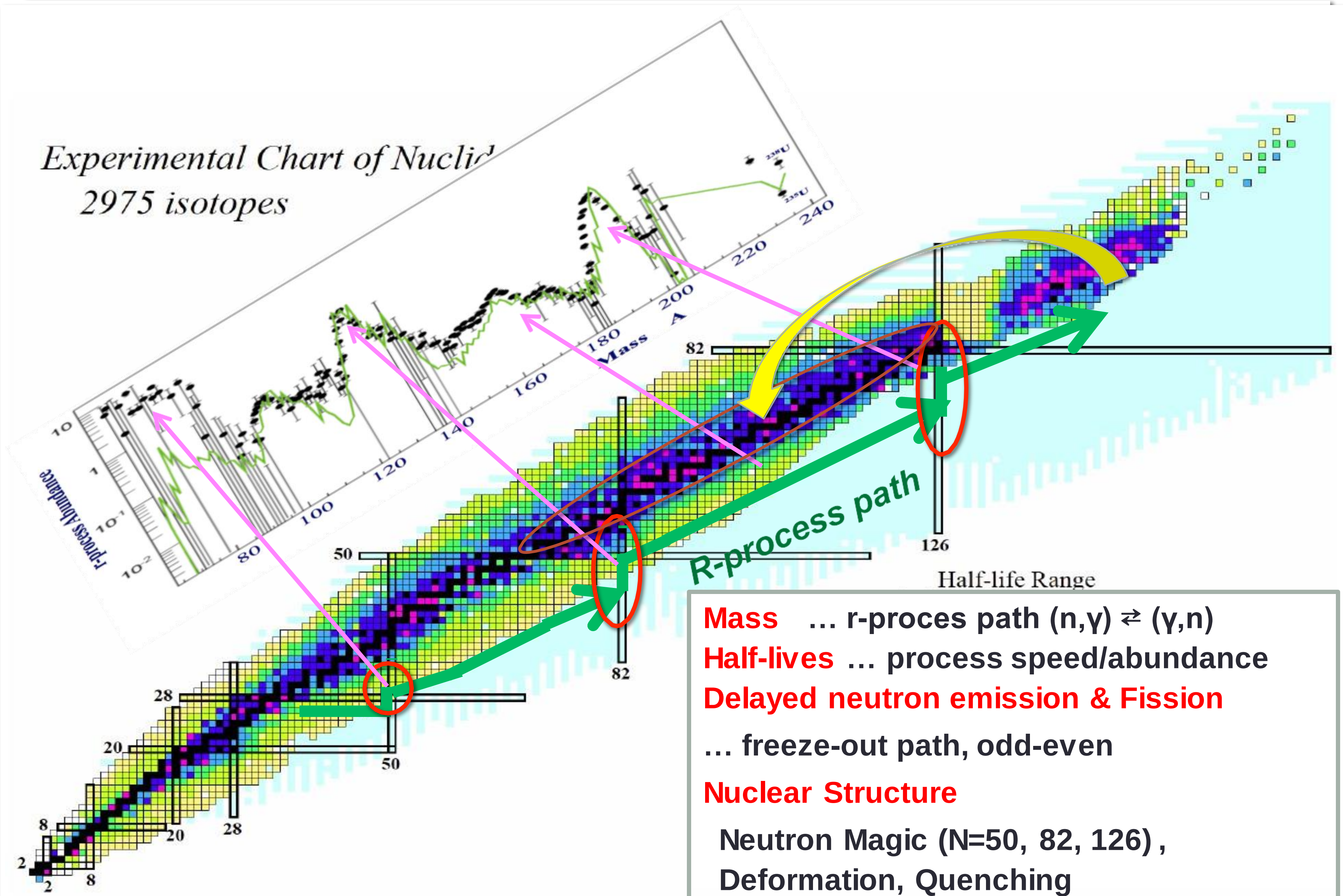
Considering (n_TOF + CRIB) x FSU
➡ ~ 80% ^{7}Li abundance

How to study r-process experimentally?

- r-process path nuclei…very neutron rich, still hard to study
- RIKEN RIBF can produce some of them, but not with a high intensity (i.e. too few to make a reaction study)

  → What we can do first is to study the basic properties of nuclei, such as mass and lifetime.

Nucleosynthesis of Heavy Elements (r-Process)



Mass … r-proces path (n,γ) ⇄ (γ,n)

Half-lives … process speed/abundance

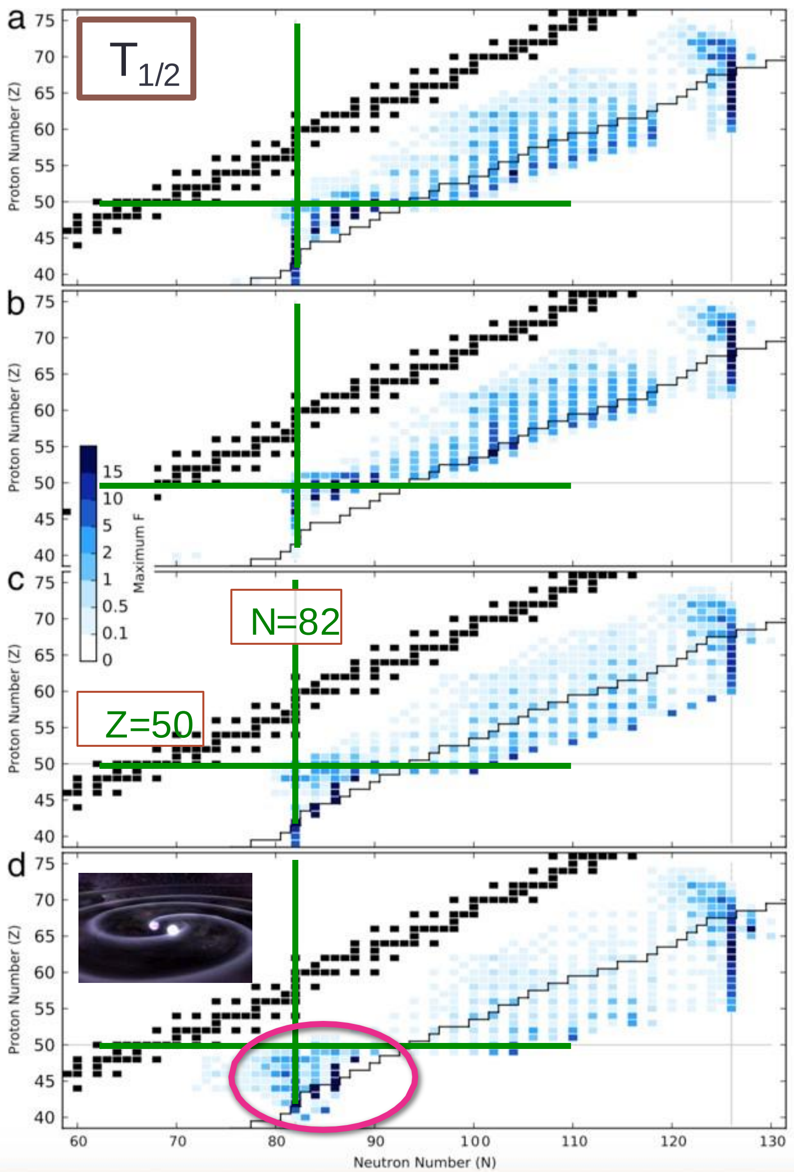
Delayed neutron emission & Fission

… freeze-out path, odd-even

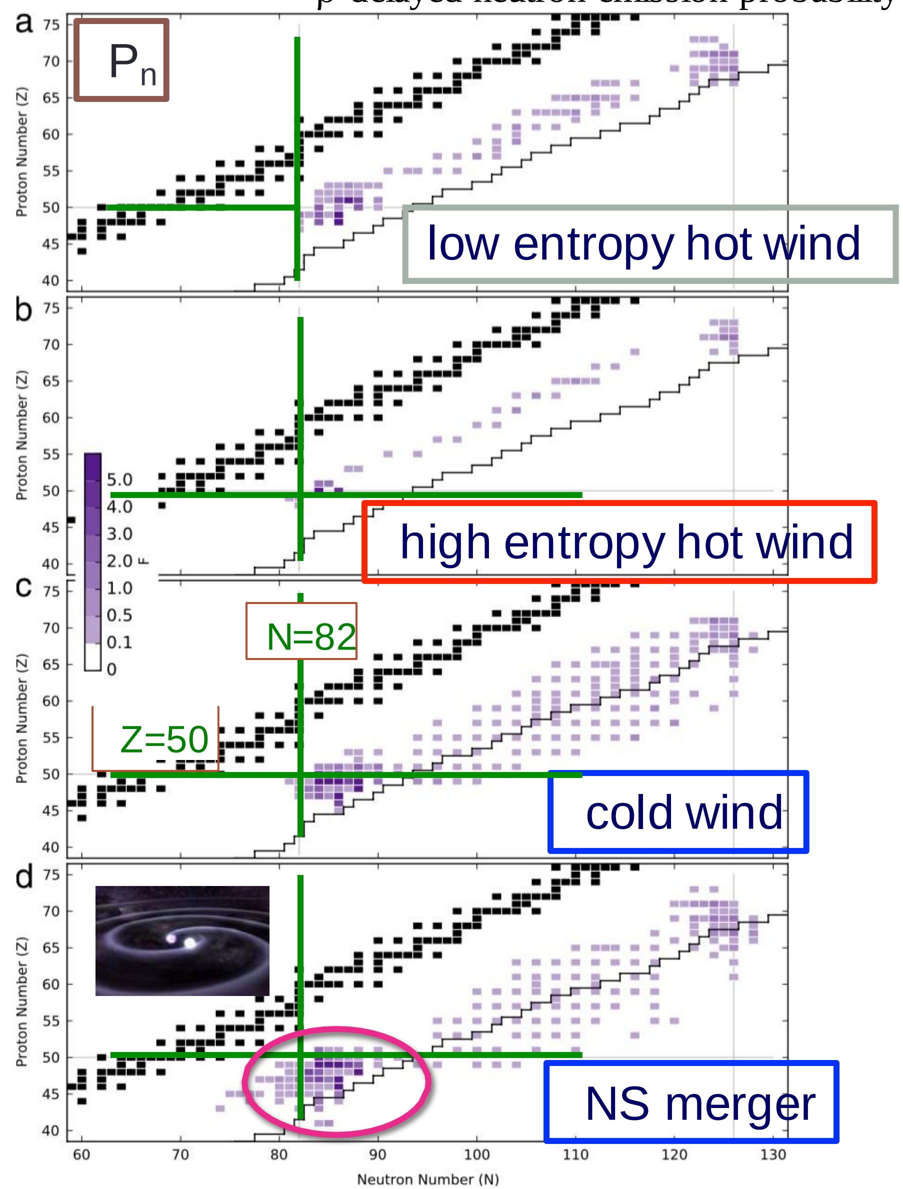
Nuclear Structure

Neutron Magic (N=50, 82, 126) ,
Deformation, Quenching

Sensitivity study of decay properties in r-process



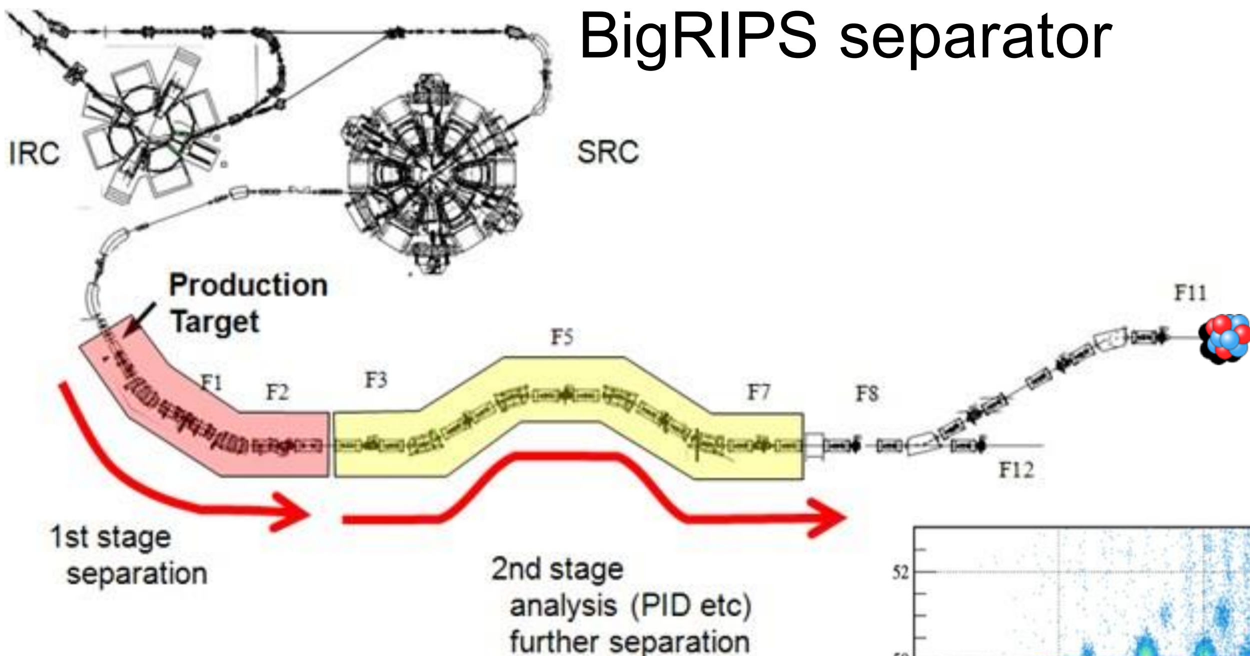
β-delayed neutron emission probability

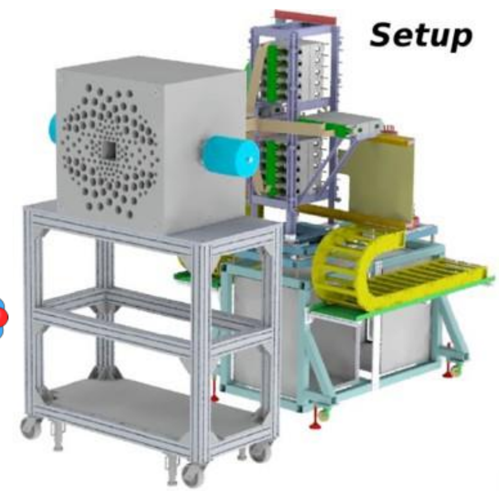


M.R. Mumpower et al.(2016)

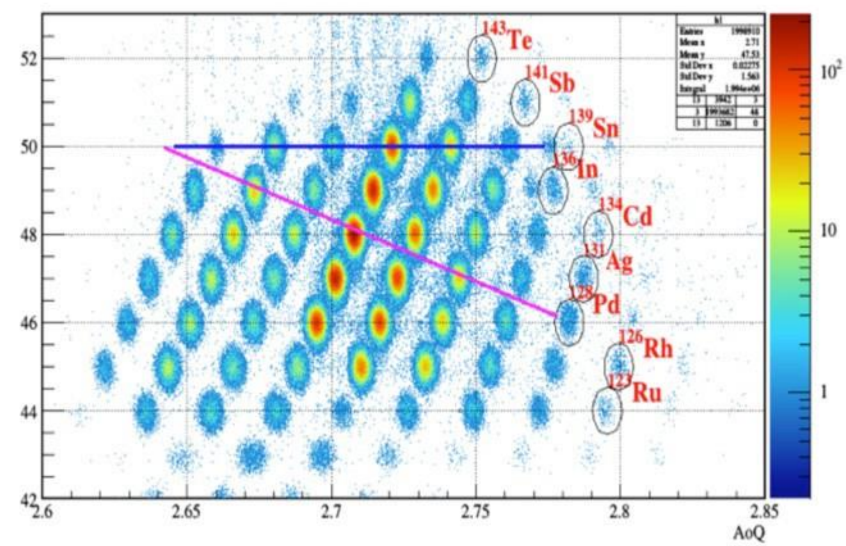
Beam Production & DecayStation

RIKEN RIBF, BigRIPS separator

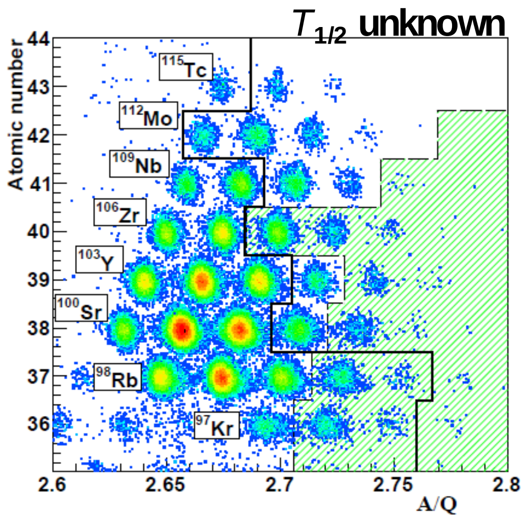


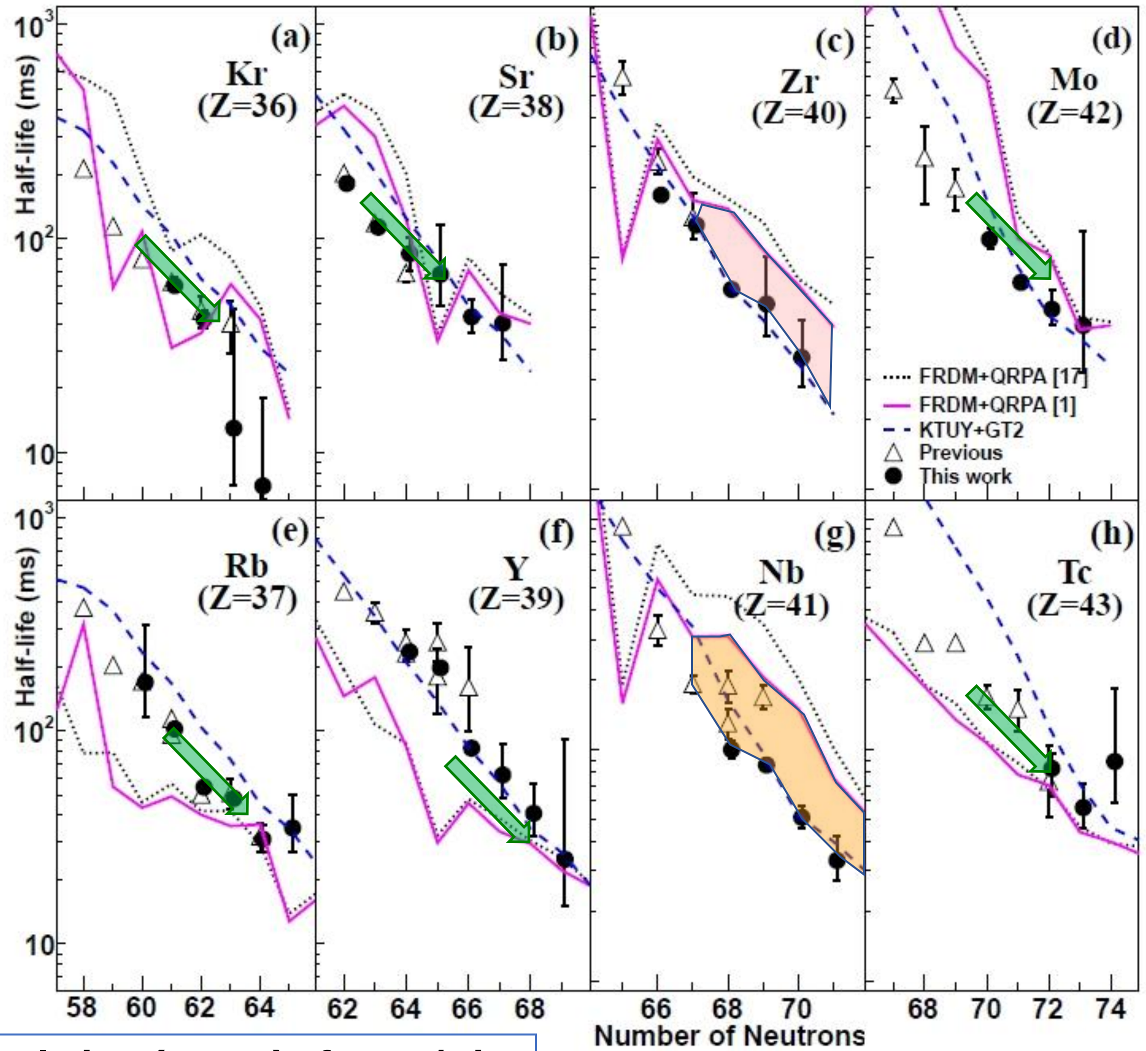


The implantation of an identified RI is associated with the following β-decay events that are detected in the same silicon pixel (DSSSD).



Nishimura *et al.* PRL106, 052502 (2011)





$T_{1/2}$ **for 38 isotopes,** 18 among them were first measurement

$T_{1/2}$ **is shorter than calculation (KTUY) for Zr/Nb**
→ r-process flows faster

Summary

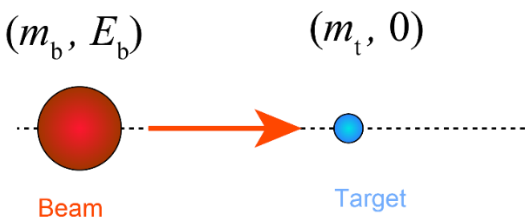
- Experimental information is essential for understanding stars and other phenomena in the universe
- Study on astrophysical reactions with (low-energy) RI beams:

  Not easy, but possible for some cases. Successful cases:
    - Direct measurement (for large cross section reaction)
    - Resonant scattering to study resonances (with TTIK)
    - Indirect methods (such as THM and Coulomb dissociation)
    - Mass/lifetime measurements
- CRIB at CNS, the University of Tokyo, providing unique low-energy (<10MeV/u) RI beams…we welcome new collaborators and new ideas.

http://www.cns.s.u-tokyo.ac.jp/crib/crib-new/

Homework (In-flight RI beam)

[1] A ^{7}Be beam is created by the in-flight method, using a ^{7}Li beam (mass: M_b) at an energy of E_b and a hydrogen (Mass M_t) target. How much is the maximum angle deviation of the produced ^{7}Be particle from the original ^{7}Li beam trajectory?



For simplicity, you can assume

-The maximum angle deviation occurs when $\theta_{\text{c.m.}}$ is close to $90°$.

-Q-value in the production reaction (p,n) is negligible. (^{7}Li/^{7}Be masses are the same.)

-The energy loss in the target is ignorable.

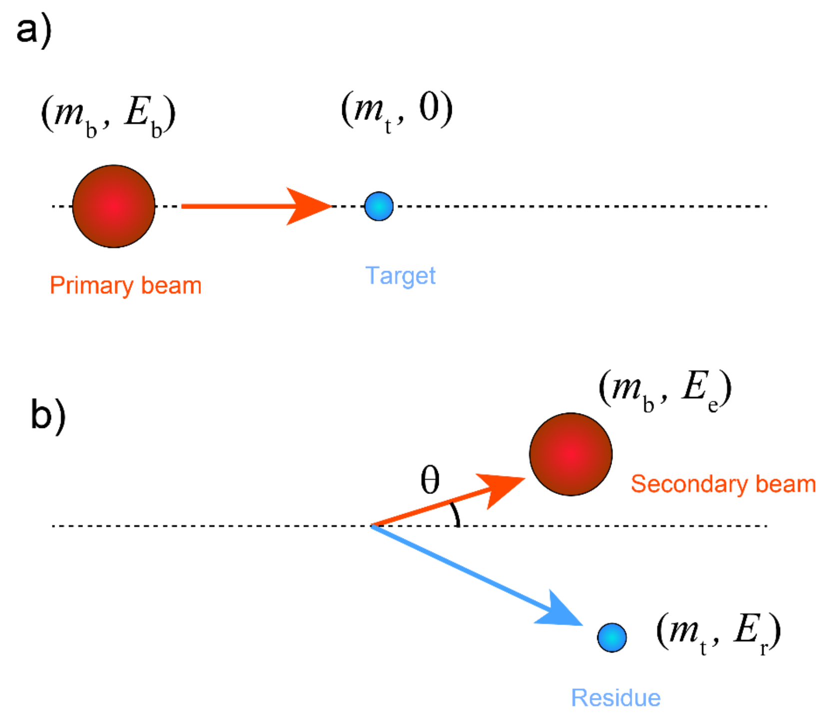
Hint) You can use the formula, $\cos\theta_{\text{lab}} = \dfrac{x+\cos\theta_{\text{c.m.}}}{\sqrt{1+x^2+2x\cos\theta_{\text{c.m}}}^2}, \; x = \dfrac{M_b}{M_t}$

H. Yamaguchi@NUSYS2019

Homework

[2] When the ^{7}Li beam energy is E_b = 10MeV/u (~70 MeV) and ^{7}Be produced with the angle $\theta_{\text{lab}} < 3°$ is accepted, how much is the energy spread $\Delta E_e / E_e$? Here we define ΔE_e as the energy difference of the ^{7}Be beam particle at $0°$ and $3°$.

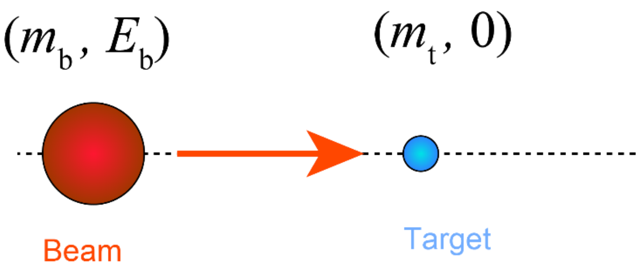
Hint) Consider energy -momentum conservation.



Homework (TTIK)

[1] Suppose we make a scattering experiment by irradiating a beam (kinetic energy E_b, mass M_b) onto a target (Mass M_t). Show that the center-of-mass energy $E_{c.m.}$ (energy of the system in the center-of-mass frame) at the scattering is given by the following formula for non-relativistic energy:

$$E_{c.m.} = \frac{M_t}{(M_b + M_t)} E_b$$



Hint) In c.m. frame, the sum of the momentum vectors will be zero.

Note) This result implies that the $E_{c.m.}$ resolution can be better than the uncertainty of the beam energy in the inverse kinematics condition, $M_b > M_t$.

Homework

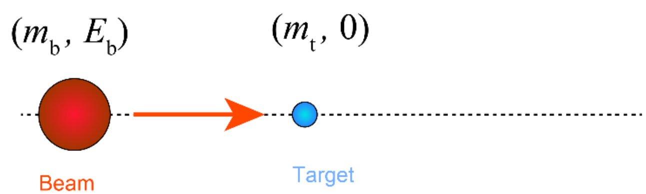
[2] In the resonant scattering experiments in inverse kinematics, we measure the energy and the angle of the recoiling ion, E_r and θ. First we consider a thin-target case, where the energy loss in the target is negligible.

Assuming the particle masses and the beam energy E_b are known, how do you obtain the $E_{c.m.}$ of the scattering events from the measured quantities?

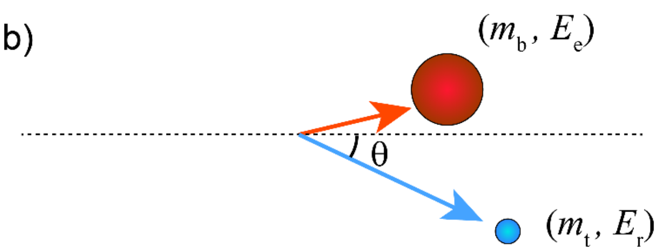
a) Before scattering

b) After scattering

a)



b)



Homework

[3] How the formula can be modified when we use a thick-target in which the beam energy is significantly degraded. (Can we still obtain $E_{\mathrm{c.m.}}$ from the measured E_{r} and θ?)

[4] What are the advantages and disadvantages of the TTIK (thick-target in inverse kinematics) method, as compared to the traditional, normal kinematics method?