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ION OPTICS: LECTURE 1

CONTENTS

- Lecture 1
  - Part 1: basics and definitions
  - Part 2: methods and instruments based on Ion Optics
- Lecture 2
  - Part 1: tools and live demos
  - Part 2: real life Ion Optics

PART1: BASICS AND DEFINITIONS

- What is the purpose of Ion Optic?
- Manipulate charged particles (not just ions!)
  - Bunching and acceleration
  - Transport and separation
  - Measure properties to realize experiments

MATRIX NOTATION

- Propagation of a straight ray
  - $x_2 = x_1 + (z_2 - z_1) \times \tan \alpha_1$
  - $y_2 = y_1 + (z_2 - z_1) \times \tan \beta_1$
- In transfer matrix notation
  - $\begin{bmatrix} x_2 \\ \tan \alpha_2 \end{bmatrix} = \begin{bmatrix} 1 & \ell \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ \tan \alpha_1 \end{bmatrix}$
  - $\begin{bmatrix} y_2 \\ \tan \beta_2 \end{bmatrix} = \begin{bmatrix} 1 & \ell \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ \tan \beta_1 \end{bmatrix}$

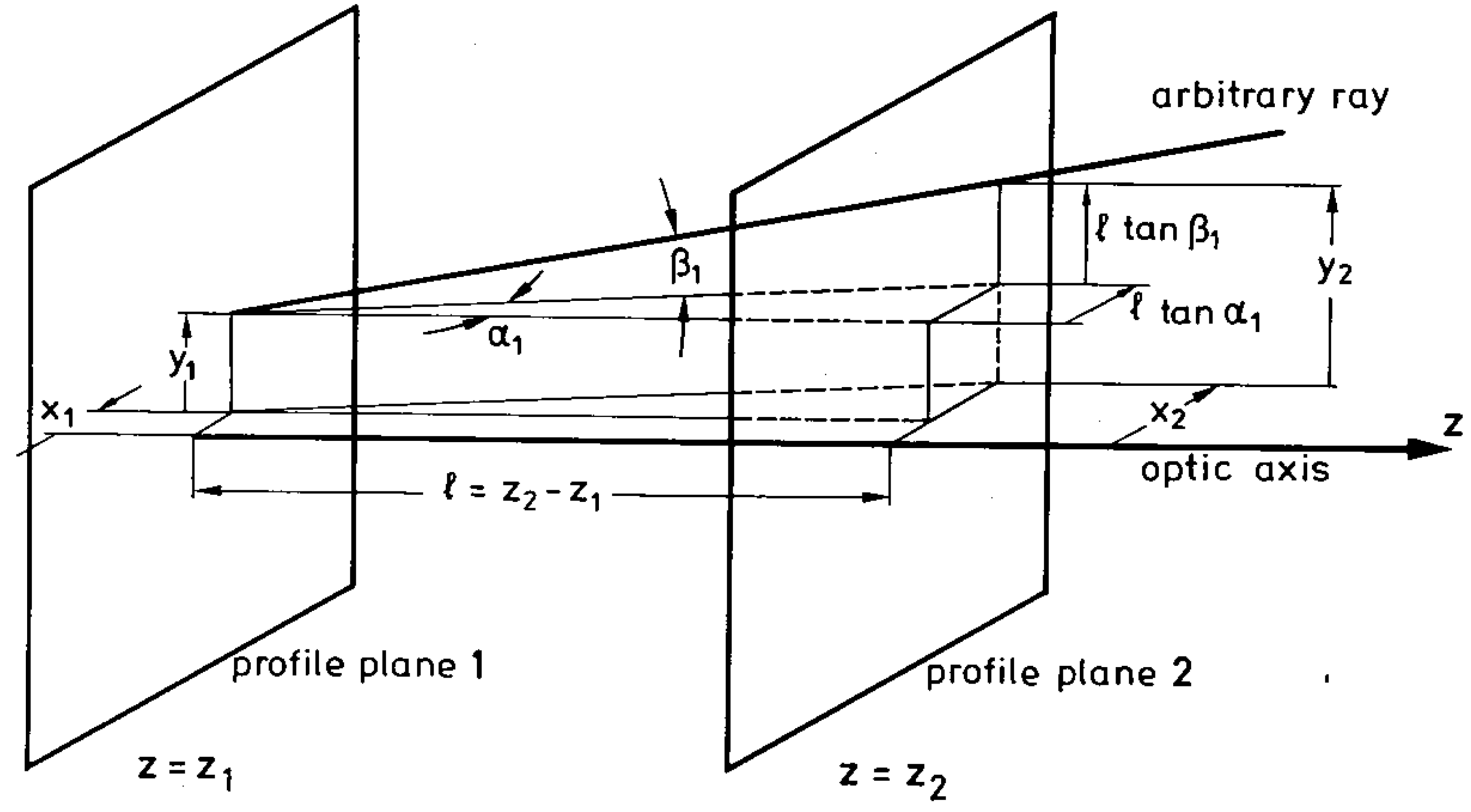


Fig. 1.1. Deviations of a light ray from the optic axis in a drift length $l = z_2 - z_1$.

H. Wollnik, Optics of charged particles, Academic Press, INC (1987)

COORDINATE SYSTEM & TRANSFER MATRIX

- Six-dimensional coordinate system
  - Necessary to fully describe the motion of particles in 3D space
  - Geometrical dimensions used on perpendicular axises (x and y)
  - Momentum (energy) dimensions used on longitudinal axis (z)
- Full matrix of a drift space of length ℓ
- Also called Transfer matrix

- $$\begin{bmatrix} x \\ a \\ y \\ b \\ l \\ \delta \end{bmatrix}_2 = \begin{bmatrix} 1 & \ell & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \ell & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ a \\ y \\ b \\ l \\ \delta \end{bmatrix}_1$$
- $$a = \frac{\sin\alpha}{\sqrt{1 + \tan^2\beta + \cos^2\alpha}}$$
- $$b = \frac{\sin\beta}{\sqrt{1 + \tan^2\alpha + \cos^2\beta}}$$
- $l =$ path length difference
- $\delta = \Delta p/p$ momentum difference

INTERPRETATION OF MATRIX ELEMENTS

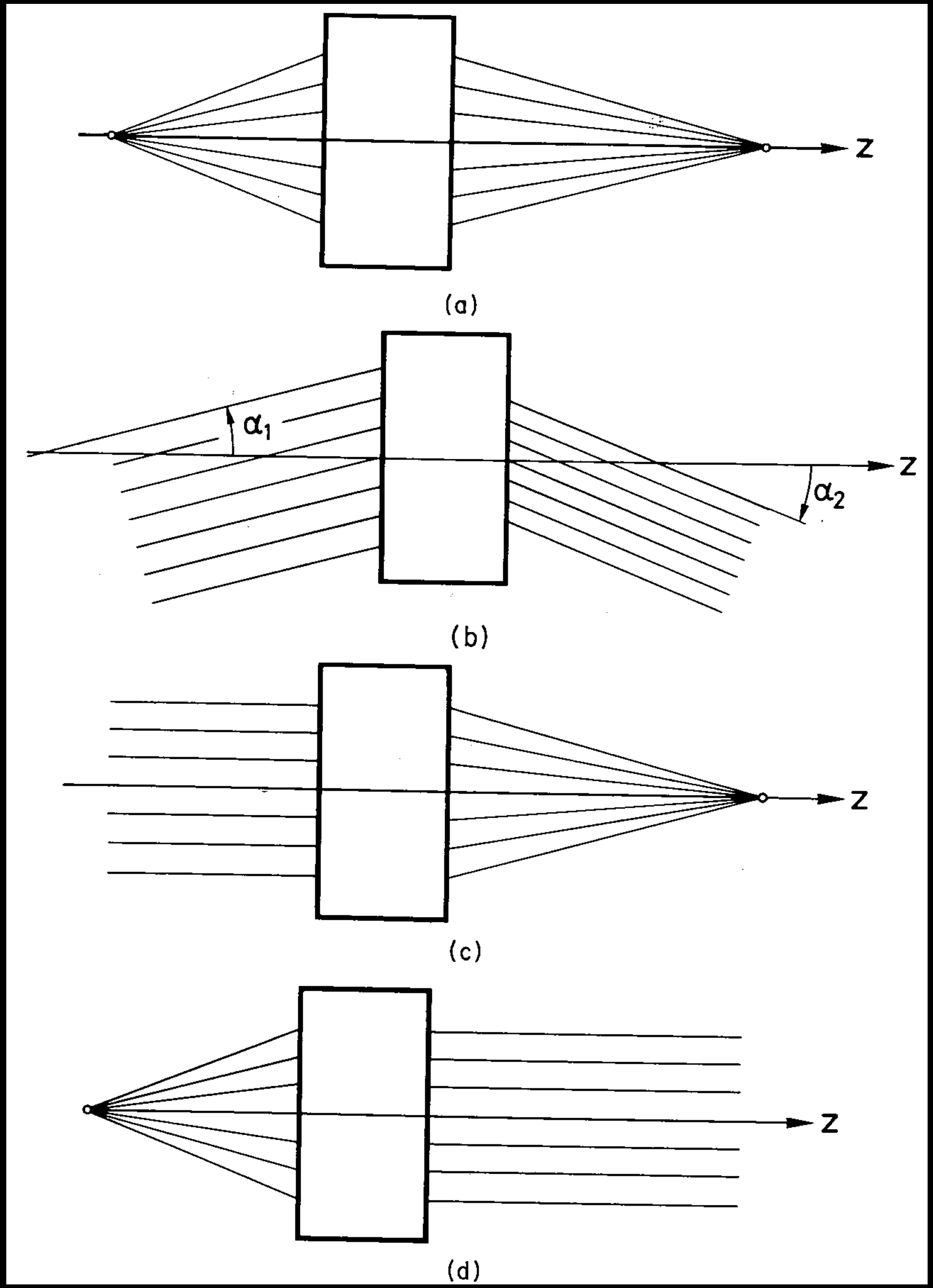
- Restricted general matrix
  - Momentum conserving
  - No cross horizontal/vertical elements
- Magnifications: diagonal elements
- Dispersions: last column
- Focussing: off-diagonal elements

$$\begin{bmatrix} x & a & y & b & l & \delta \end{bmatrix}$$

$$\begin{bmatrix} x \\ a \\ y \\ b \\ l \\ \delta \end{bmatrix}\begin{bmatrix} (x/x) & (x/a) & 0 & 0 & 0 & (x/\delta) \\ (a/x) & (a/a) & 0 & 0 & 0 & (a/\delta) \\ 0 & 0 & (y/y) & (y/b) & 0 & (y/\delta) \\ 0 & 0 & (b/y) & (b/b) & 0 & (b/\delta) \\ (l/x) & (l/a) & (l/y) & (l/b) & 1 & (l/\delta) \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

BASIC TYPES OF OPTICS SYSTEMS

- Point to point
- Parallel to parallel
- Parallel to point
- Point to parallel



H. Wollnik, Optics of charged particles, Academic Press, INC (1987)

$$(x/a) = 0$$

$$(a/x) = 0$$

$$(x/x) = 0$$

$$(a/a) = 0$$

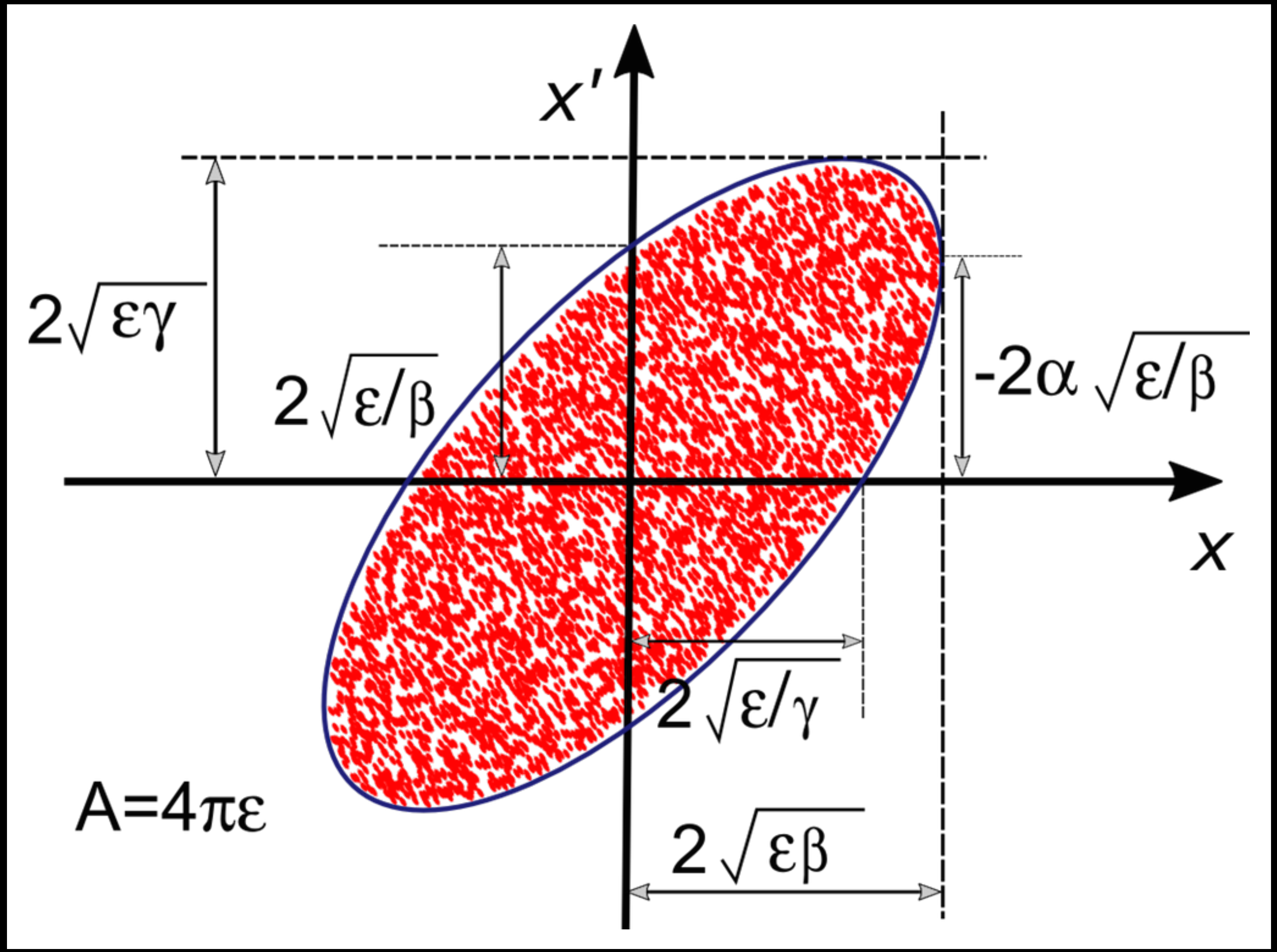
$$\begin{bmatrix} (x/x) & (x/a) \\ (a/x) & (a/a) \end{bmatrix}$$

EQUATIONS OF MOTION

- Coulomb and Lorentz forces: $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B} = \frac{d(m\vec{v})}{dt}$
  - $\vec{E}$: electric field, $\vec{B}$: magnetic field, $\vec{v}$: velocity, q: charge, m: mass
  - **Exercise 1**: can you tell why magnetic fields cannot change the kinetic energy?
- Magnetic rigidity: measure of how difficult it is to bend trajectory
  - $B\rho = 3.107\frac{m}{q}\beta\gamma, \beta = \frac{v}{c}, \gamma = \frac{1}{\sqrt{1-\beta^2}}$ in Tesla.meter (T.m.)
  - **Exercise 2**: calculate rigidity of $^{12}C^{5+}$ at 100 MeV/u

EMITTANCE AND PHASE SPACE

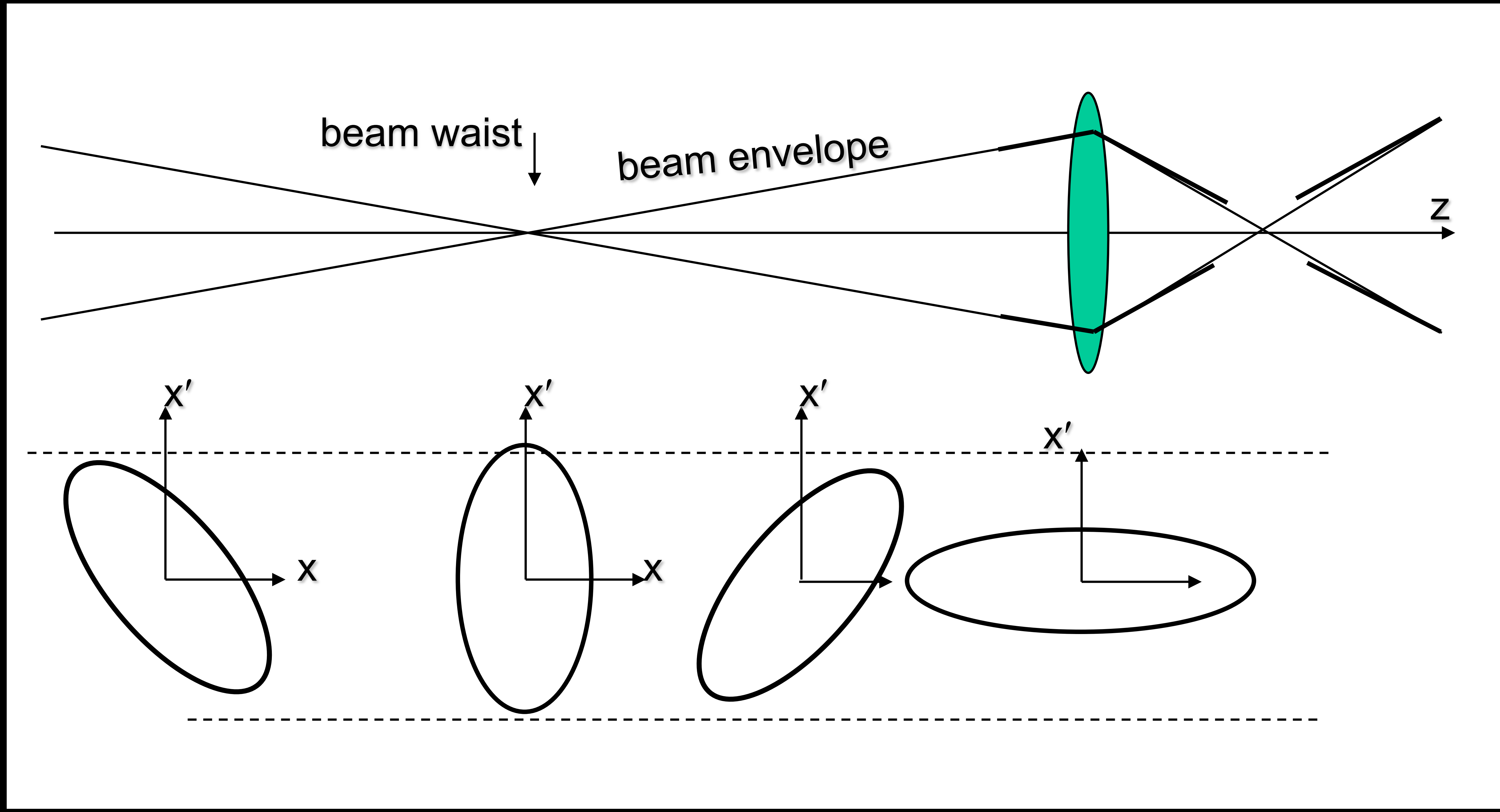
- Representation of ensemble of particles forming a beam
  - Six-dimensional ellipsoid phase space volume containing all trajectories
  - Emittance (ϵ) represents the volume of the ellipsoid
  - Orientations of the ellipsoid indicate correlations between coordinates
  - Transverse emittance (x, a) and (y, b)
  - Longitudinal emittance (ℓ, δ)



From: https://www.comsol.com/blogs/phase-space-distributions-and-emittance-in-2d-charged-particle-beams/

EVOLUTION OF BEAM PHASE SPACE

- Liouville's theorem
  - Phase space distribution function evolves along the beam axis
  - Volume of six-dimensional ellipsoid is constant in Hamiltonian (energy conserving) system
- Non energy-conserving system make emittance grow or shrink



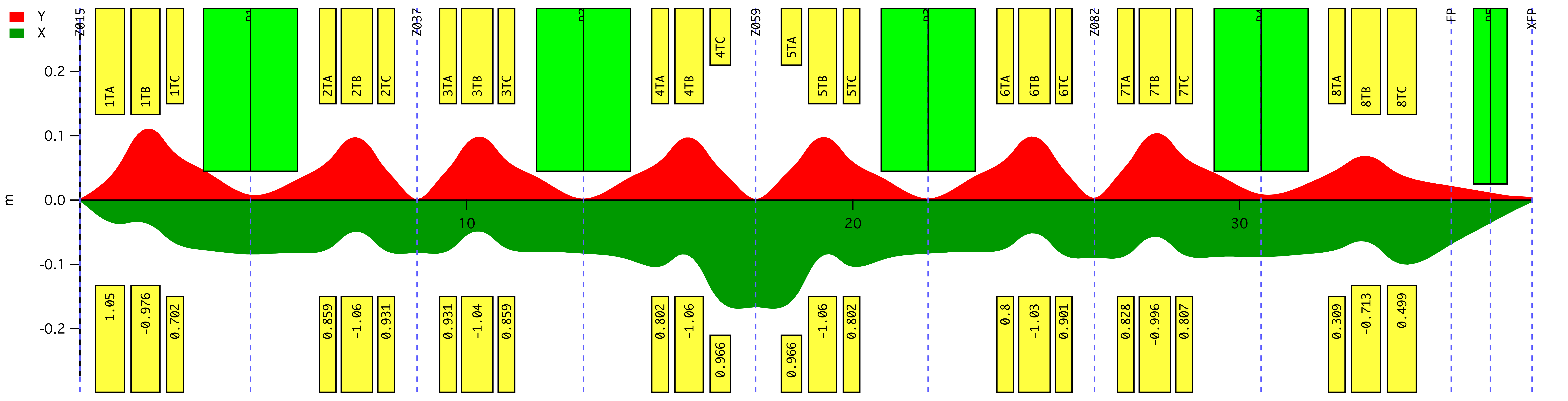
SIGMA MATRIX

- Representation of phase space ellipsoid
  - 6x6 symmetric matrix
  - Diagonal terms represent size of ellipsoid in each dimension
  - Off diagonal terms represent correlations between dimensions
- Evolution
  - Propagation of the sigma matrix follows the equation: $\sigma_2 = R\sigma_1 R^T$
  - R and R^T are the Transfer matrix and its transposed matrix

	√(sig)	x	a	y	b	l
x(mm)	2.71					
a(mr)	20.5	0.28				
y(mm)	8.43	-0.00102	0.00493			
b(mr)	15.6	-0.000725	0.00349	0.31		
l(cm)	1.37	0.135	0.687	0.0112	-0.000512	
d(%)	1	0.123	-0.591	-0.00834	-0.00592	-0.966

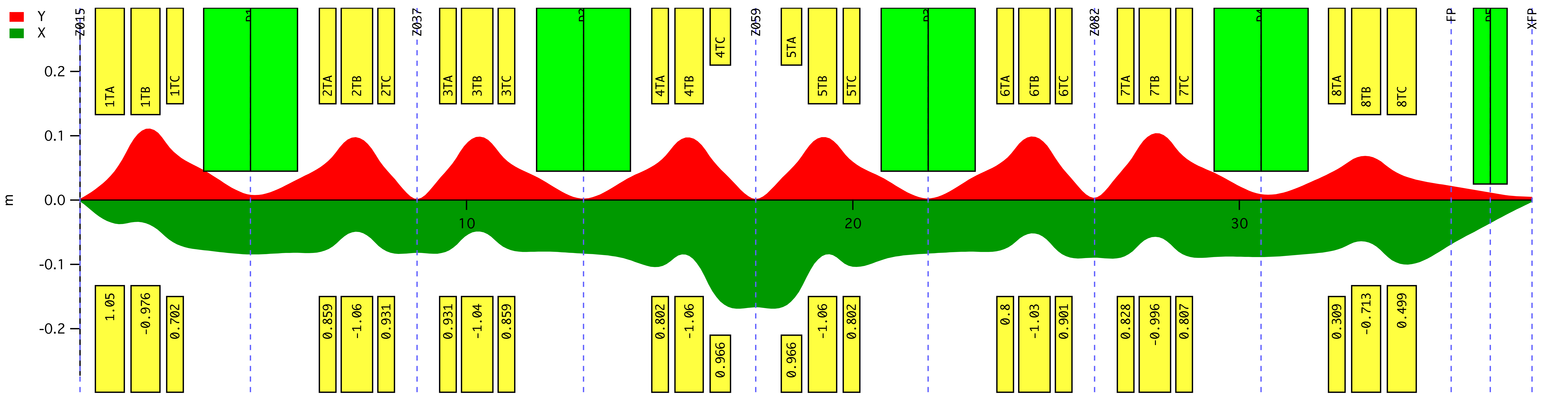
BEAM ENVELOPE

- Space taken by the beam phase space along the optics system
- Can be calculated by propagating the sigma matrix along the optics system
- Density function inside beam envelope can be assumed uniform or gaussian



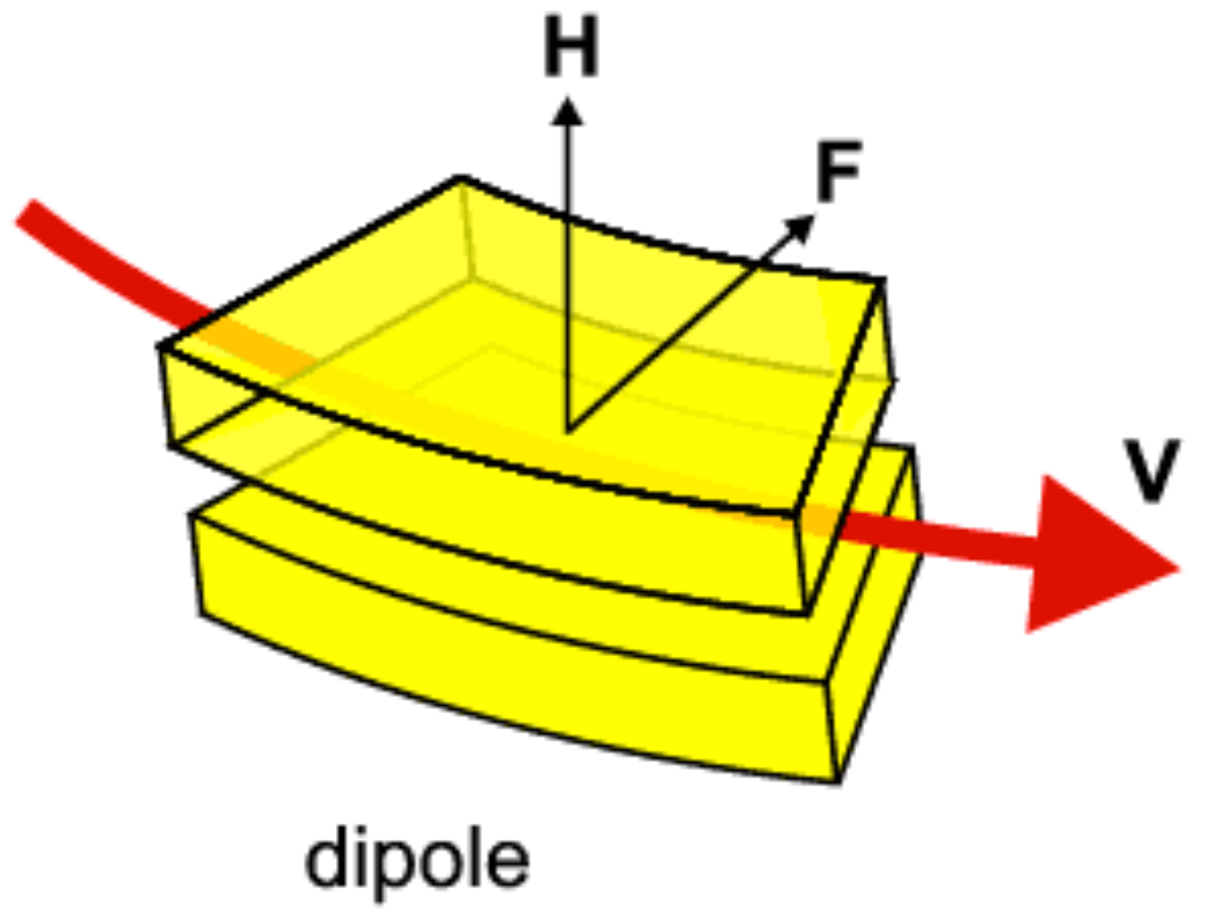
ACCEPTANCES AND TRANSMISSIONS

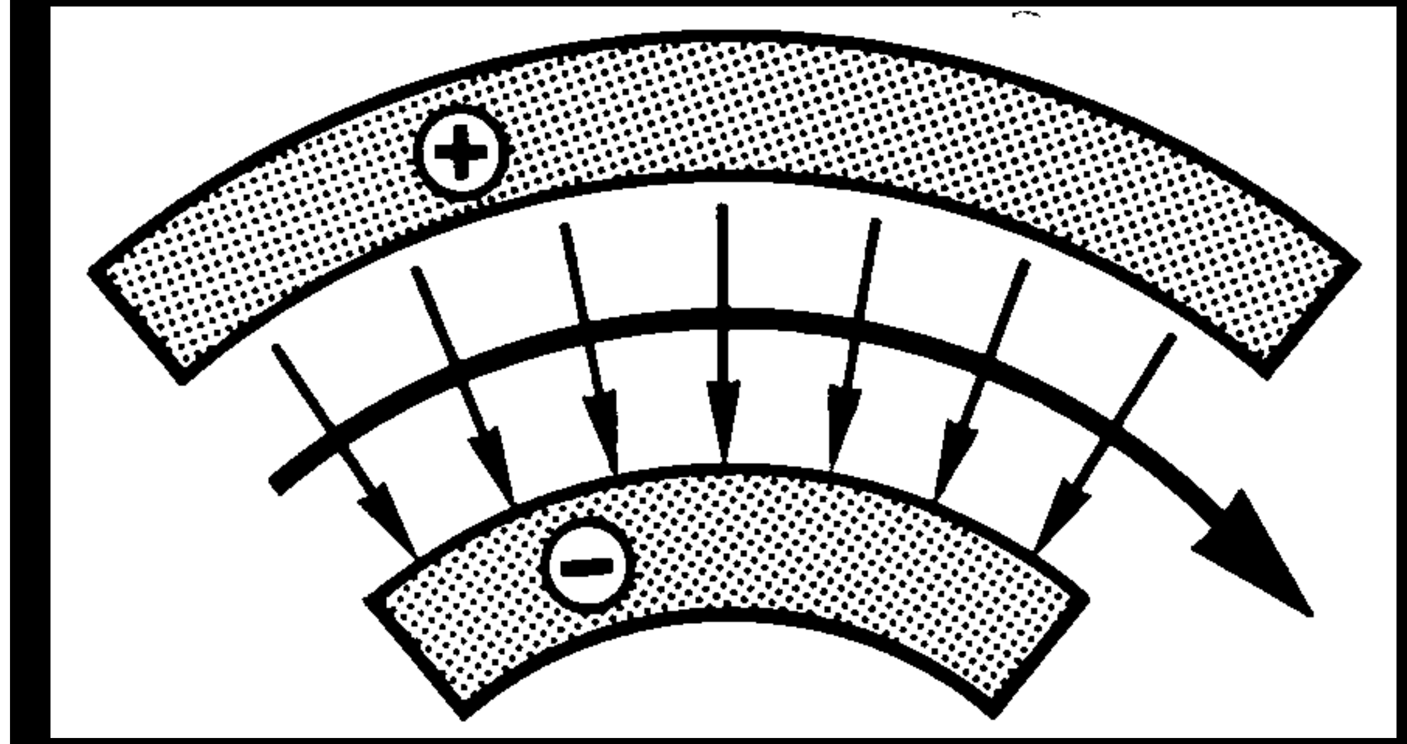
- Acceptance: physical limits of elements that compose the optics system
- Transmission: percentage of beam phase space that fits inside acceptance
- Also depends on density distributions in phase space



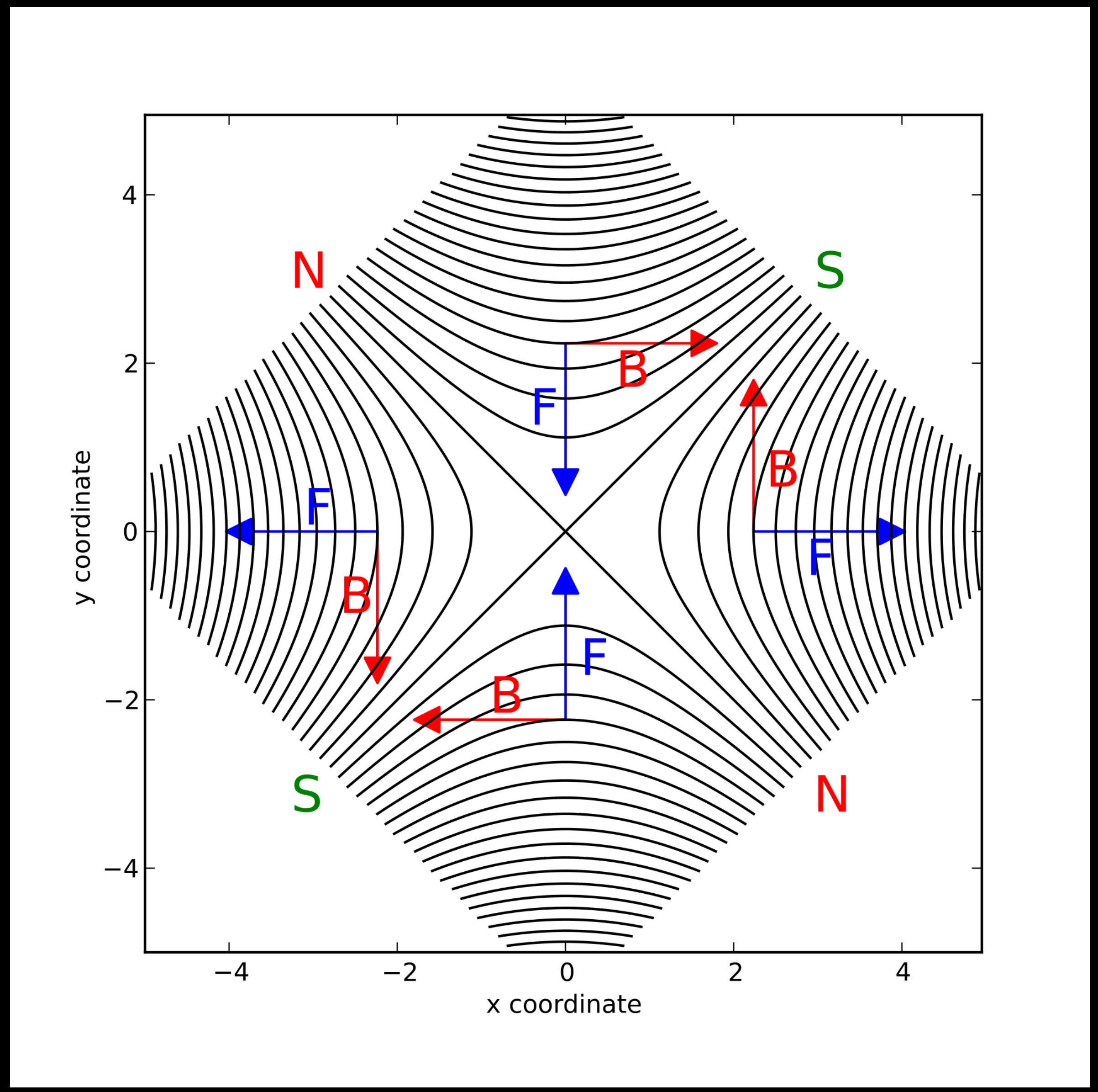
SOME TYPES OF BEAM OPTICS ELEMENTS

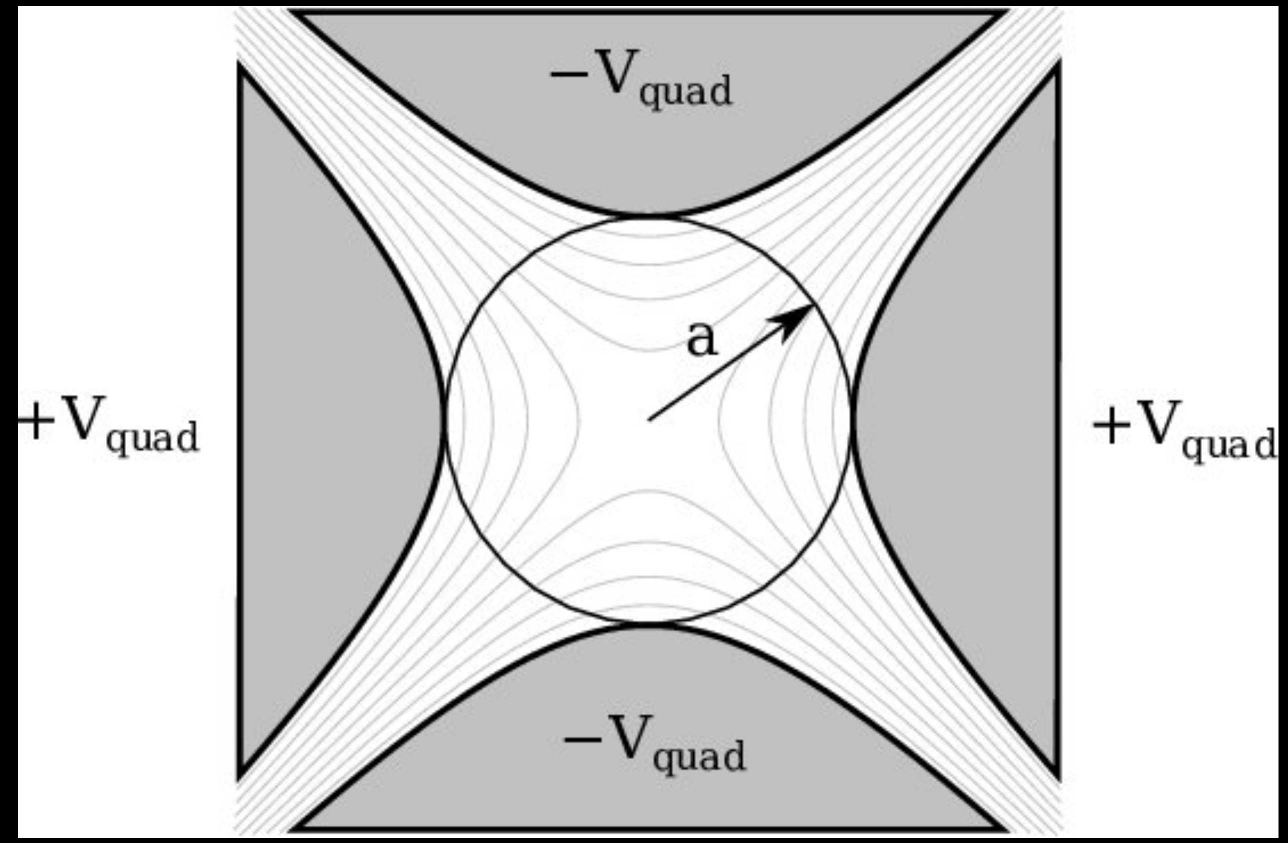
- Dipoles: deflect ions
  - Magnetic: perpendicular field
  - Electrostatic: radial field
- Quadrupoles: focus ions
  - Magnetic
- Higher order N-poles: corrections
- Cavities: accelerate





H. Wollnik, Optics of charged particles, Academic Press, INC (1987)

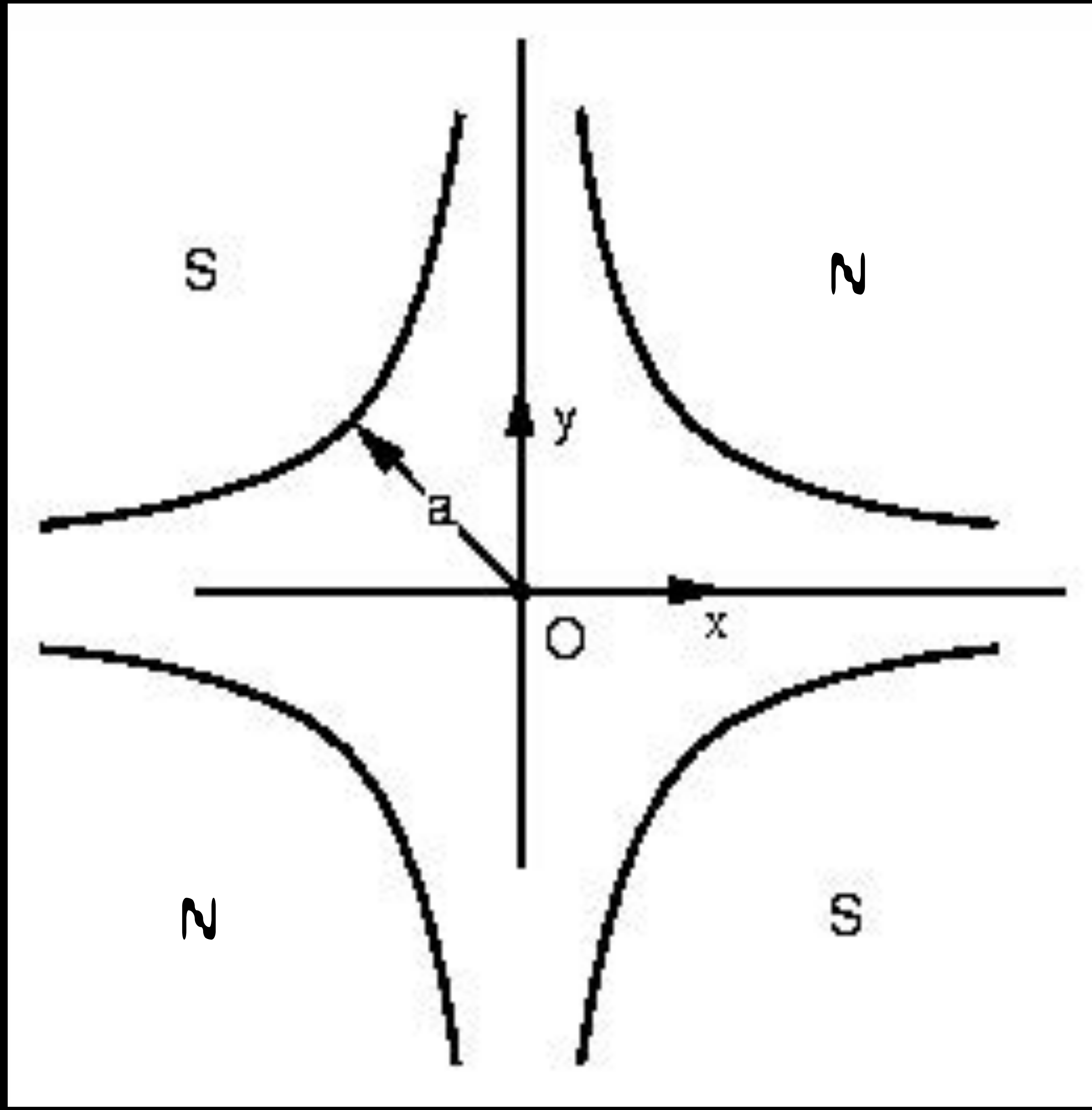




Andre Holzner

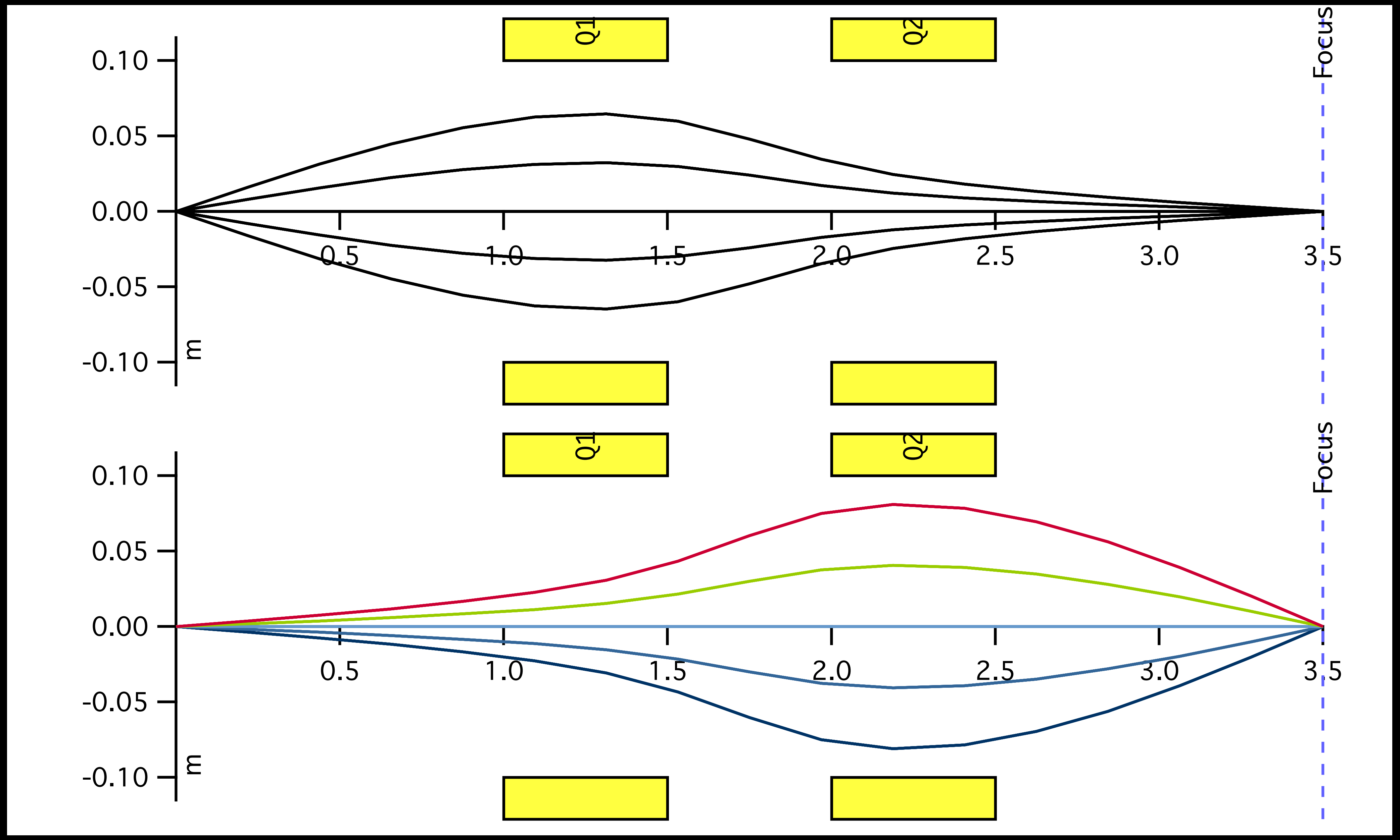
QUADRUPOLE MATRIX

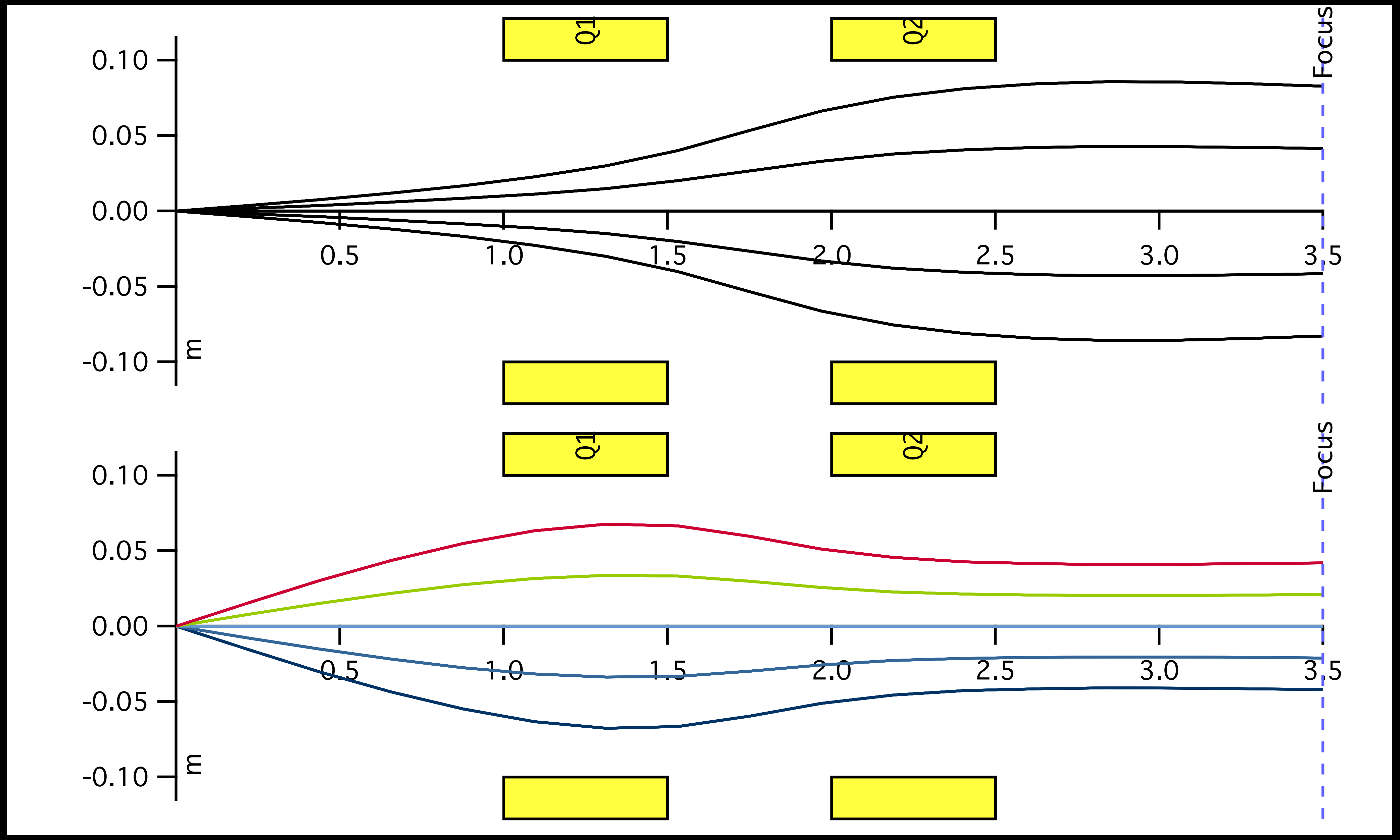
- Analytical solution for ideal quadrupole
  - $$T_Q = \begin{bmatrix} \cos kL & k^{-1}\sin kL & 0 & 0 & 0 & 0 \\ -k\sin kL & \cos kL & 0 & 0 & 0 & 0 \\ 0 & 0 & \cosh kL & k^{-1}\sinh kL & 0 & 0 \\ 0 & 0 & k\sinh kL & \cosh kL & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$
  - L: Effective length
  - $k = \sqrt{\dfrac{B_T}{aB\rho}}$ with B_T field at pole tip, a radius and $B\rho$ particle magnetic rigidity



QUADRUPOLE DOUBLET

- Necessary to achieve focussing in both horizontal and vertical planes
    - Quadrupoles are focussing in one direction and defocussing in the other
- Point-to-point optics
- Point-to-parallel optics
- **Exercise 3**: calculate the Transfer matrix of a single quadrupole with drifts D on either side (only in x direction)
    - Can you find a analytical solution for a point-to-parallel optics?
    - What is the length of the quadrupole if k=1.304 and D=1 m?

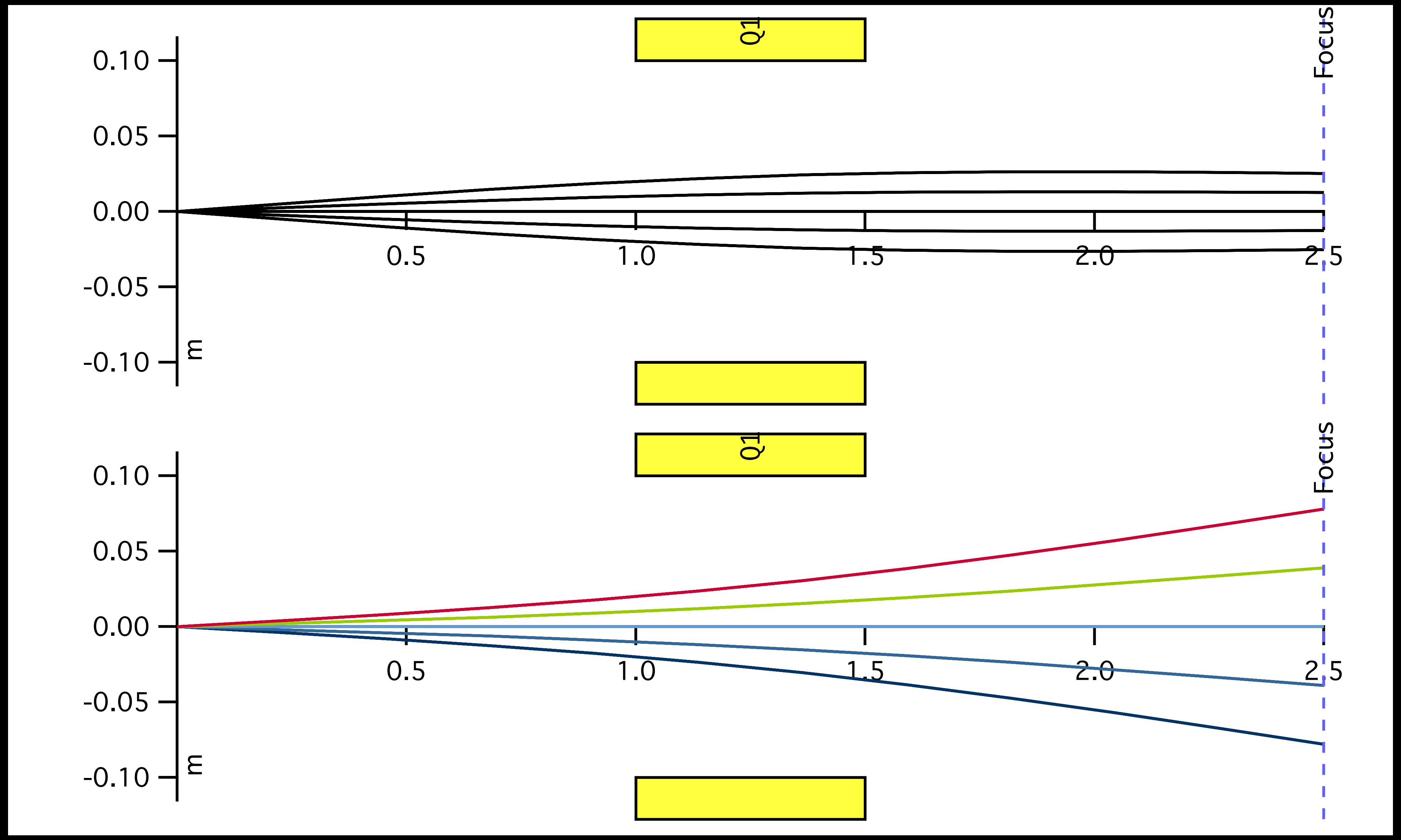




QUADRUPOLE DOUBLET: SOLUTIONS

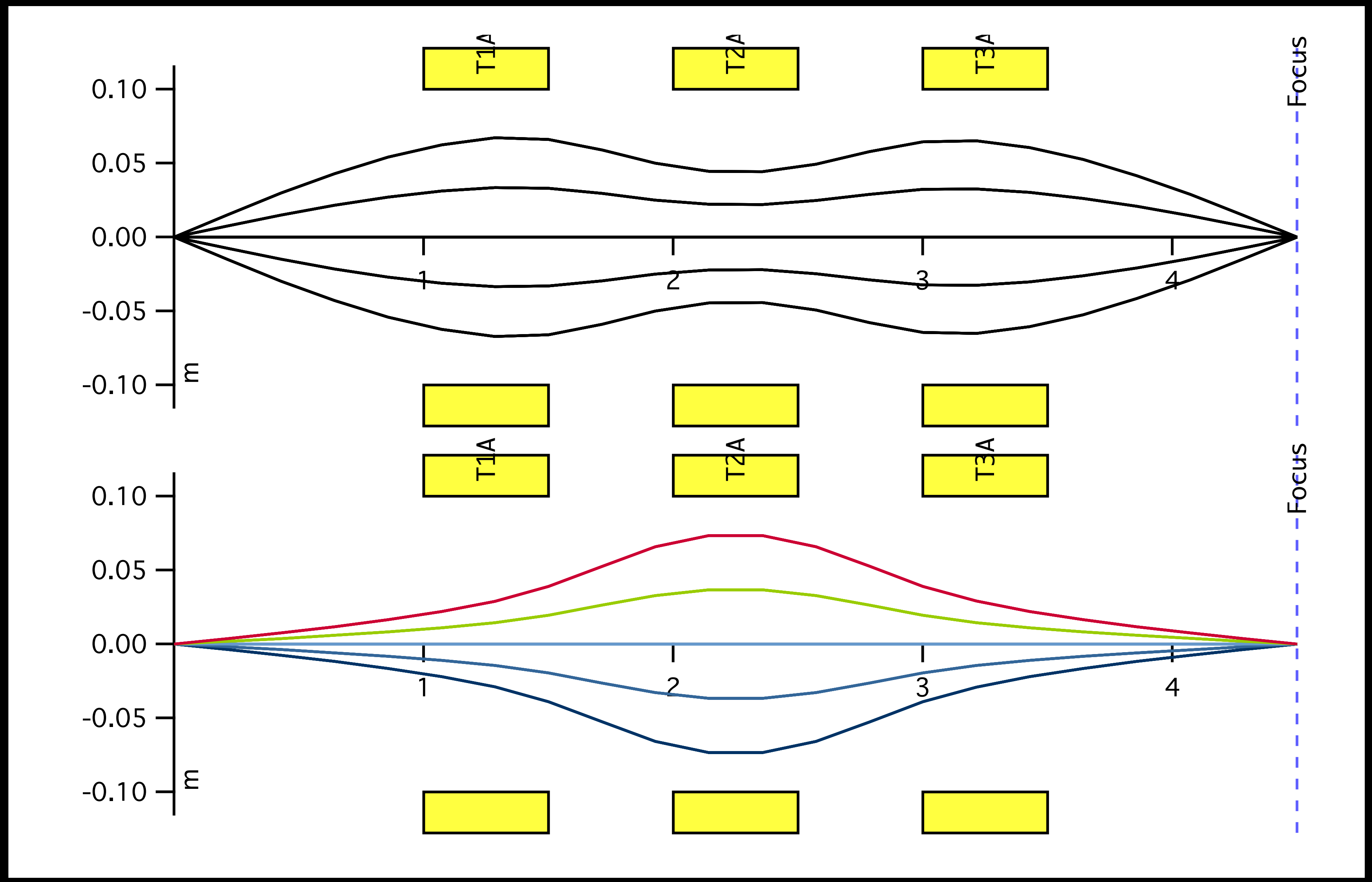
$$\begin{bmatrix}1 & D\\0 & 1\end{bmatrix}\times\begin{bmatrix}\cos kL & k^{-1}\sin kL\\-k\sin kL & \cos kL\end{bmatrix}\times\begin{bmatrix}1 & D\\0 & 1\end{bmatrix}=\begin{bmatrix}\cos kL - Dk\sin kL & 2D\cos kL + k^{-1}(1-D^2k^2)\sin kL\\-k\sin kL & \cos kL - Dk\sin kL\end{bmatrix}$$

- Point-to-parallel in x
- (a/a) = 0
- Solve: $\cos kL - Dk\sin kL = 0$
- $L = \frac{1}{k}\arctan(\frac{1}{Dk})$
- For quadrupole with k=1.304, D=1 m, the length is 0.5 m



FOCUSSING TRIPLET

- Advantages of triplet vs doublet
  - More flexibility to achieve desired optics
  - Diagonal elements can also be chosen
  - Example: system with all magnifications set to ||1||
  - Imaging optical system for a point-to-point configuration



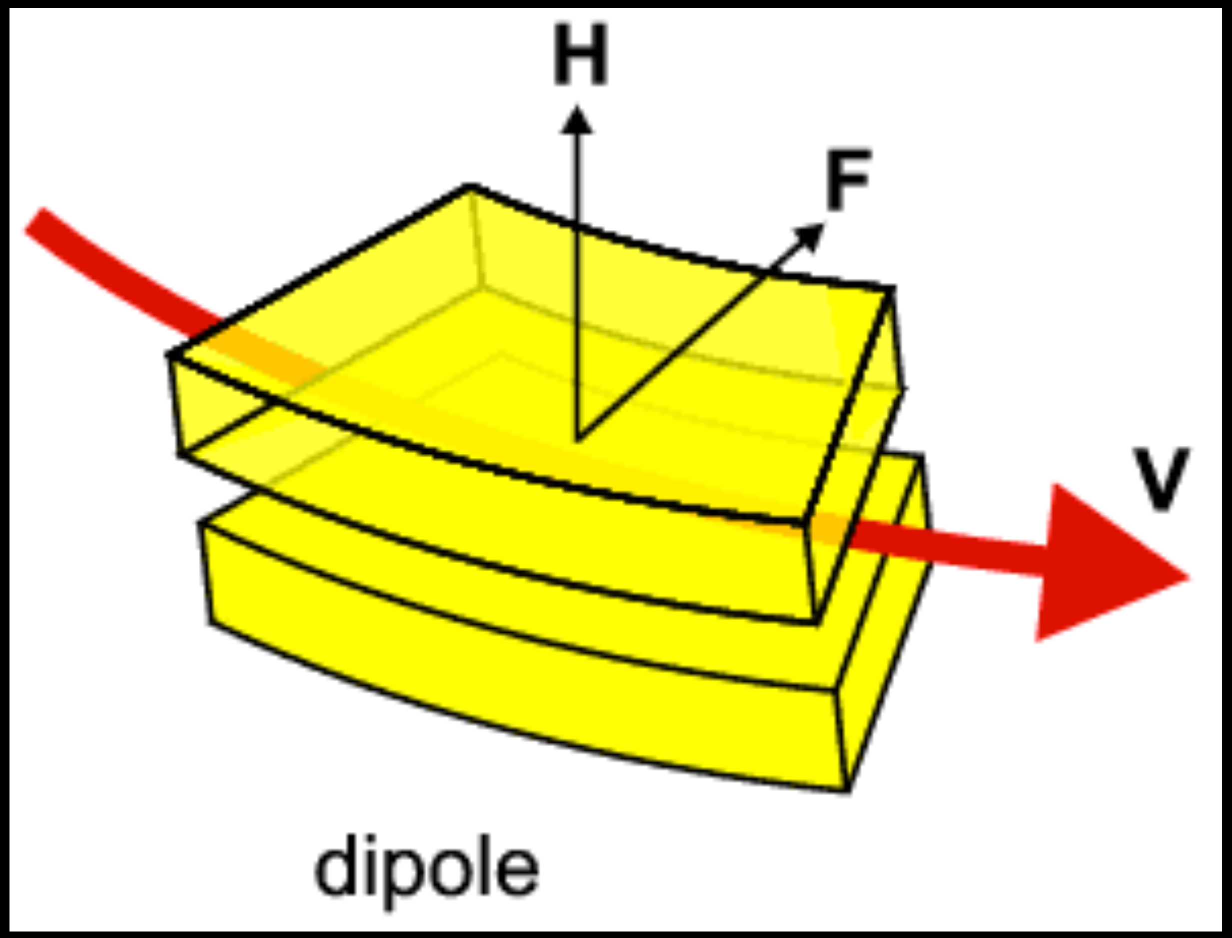
Transfer | Sigma | Inverse | Emittances

	x(m)	a(rad)	y(m)	b(rad)	l(m)	d(1)
xf	-1.03	6.46e-05	0	0	0	0
af	0.823	-0.97	0	0	0	0
yf	0	0	-0.993	-9.97e-05	0	0
bf	0	0	-1.24	-1.01	0	0
lf	0	0	0	0	1	0
df	0	0	0	0	0	1

Dismiss

DIPOLE MAGNET

- Transfer matrix includes dispersive terms
  - Only in one direction: (x/δ) lateral dispersion and (a/δ) angular dispersion
  - No field gradient (radial variation)
  - α: bending angle, ρ: bending radius
  - Resolving power for a given beam spot size x_0 is $R = -\dfrac{(x/\delta)}{x_0 \times (x/x)}$ (ability to separate particles with different momenta)
- More complex geometries are possible
  - Field gradient and/or edge angles to produce focussing in x/y directions



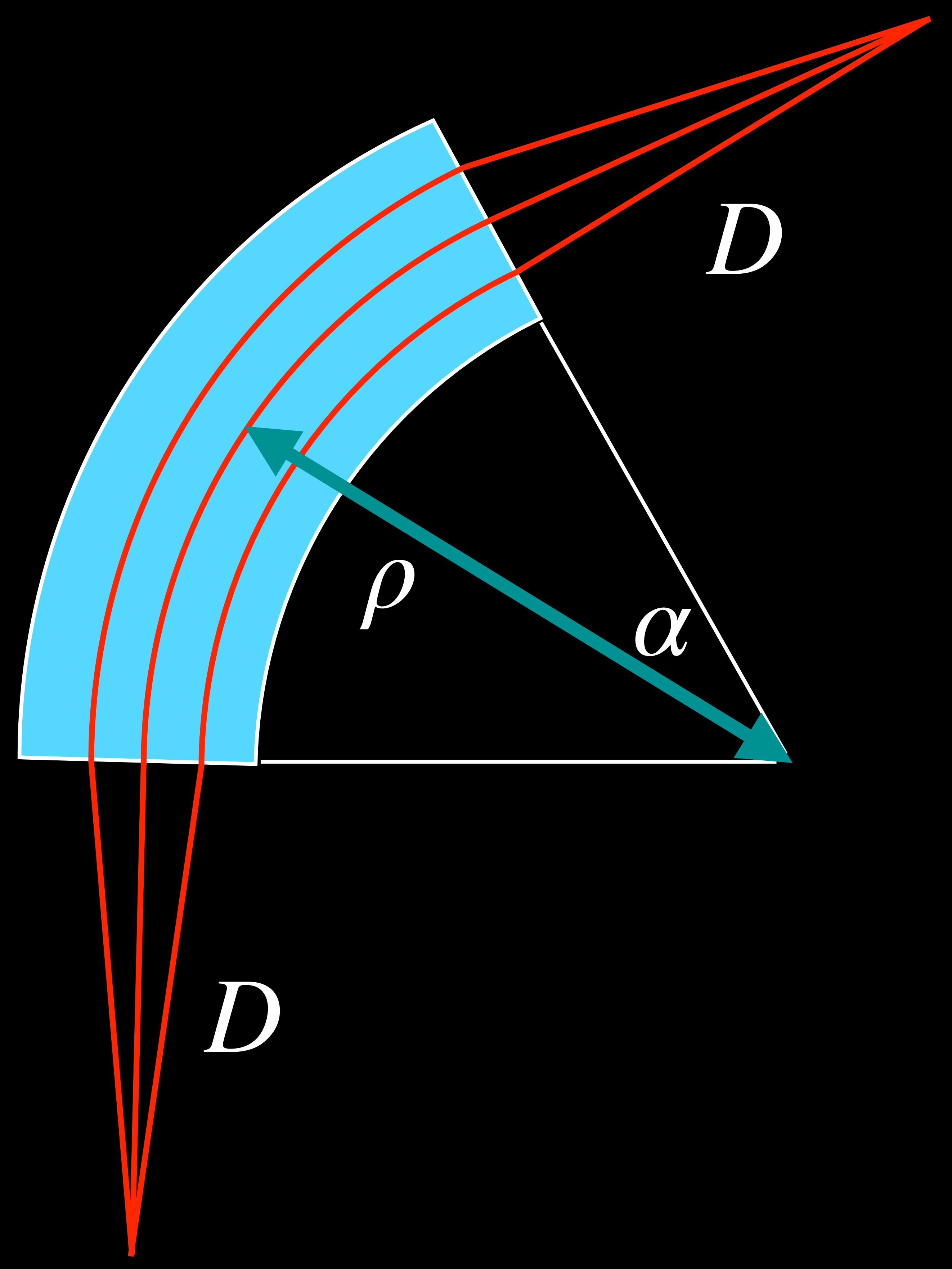
$$T_D = \begin{bmatrix} \cos\alpha & \rho\sin\alpha & 0 & 0 & 0 & \boxed{\rho(1-\cos\alpha)} \\ -\rho^{-1}\sin\alpha & \cos\alpha & 0 & 0 & 0 & \sin\alpha \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \sin\alpha & \rho(1-\cos\alpha) & 0 & 0 & 1 & -L+\rho\sin\alpha \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Dispersion (x/δ) scales with ρ

Maximum dispersion for $\alpha = 180°$

No dispersion for $\alpha = 360°$ (cyclotron)

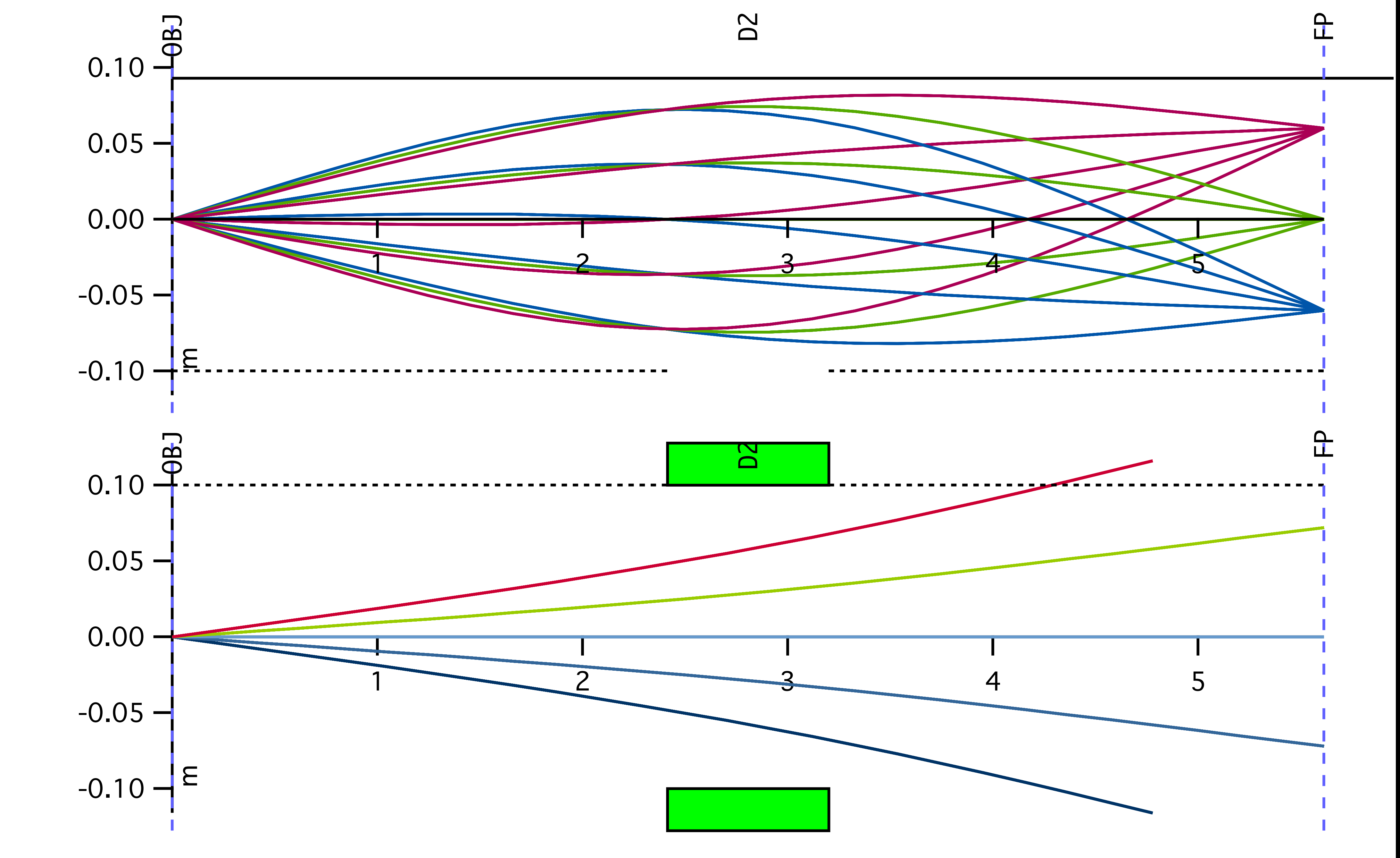
- **Exercise 4**: consider sector dipole with equal drift space D on either side
  - Calculate the transfer matrix as a function of ρ, α, D
  - Find the drift length necessary to make the system focussing in x
  - What is the lateral dispersion of the system if $\rho = 1m, \alpha = 45°$?
  - What is the resolving power assuming a beam spot of 1mm?



DIPOLE SECTOR: SOLUTIONS

$$\begin{bmatrix}1 & D & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}\times\begin{bmatrix}\cos\alpha & \rho\sin\alpha & \rho(1-\cos\alpha)\\-\rho^{-1}\sin\alpha & \cos\alpha & \sin\alpha\\0 & 0 & 1\end{bmatrix}\times\begin{bmatrix}1 & D & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}=\begin{bmatrix}\cos\alpha-\frac{D}{\rho}\sin\alpha & 2D\cos\alpha+(\rho-\frac{D^2}{\rho})\sin\alpha & \rho-\rho\cos\alpha+D\sin\alpha\\\frac{-1}{\rho}\sin\alpha & \cos\alpha-\frac{D}{\rho}\sin\alpha & \sin\alpha\\0 & 0 & 1\end{bmatrix}$$

- Focussing condition: $(x/a)=0$
- Solve $2D\cos\alpha+(\rho-\frac{D^2}{\rho})\sin\alpha=0$
- $D=\rho(\frac{1+\cos\alpha}{\sin\alpha})$
- For $\rho=1m,\alpha=45^\circ : D=2.414m$
- Lateral dispersion: $(x/\delta)=2cm/\%$
- Resolving power: $R=-\frac{2}{10^{-3}\times(-1)}=1000$



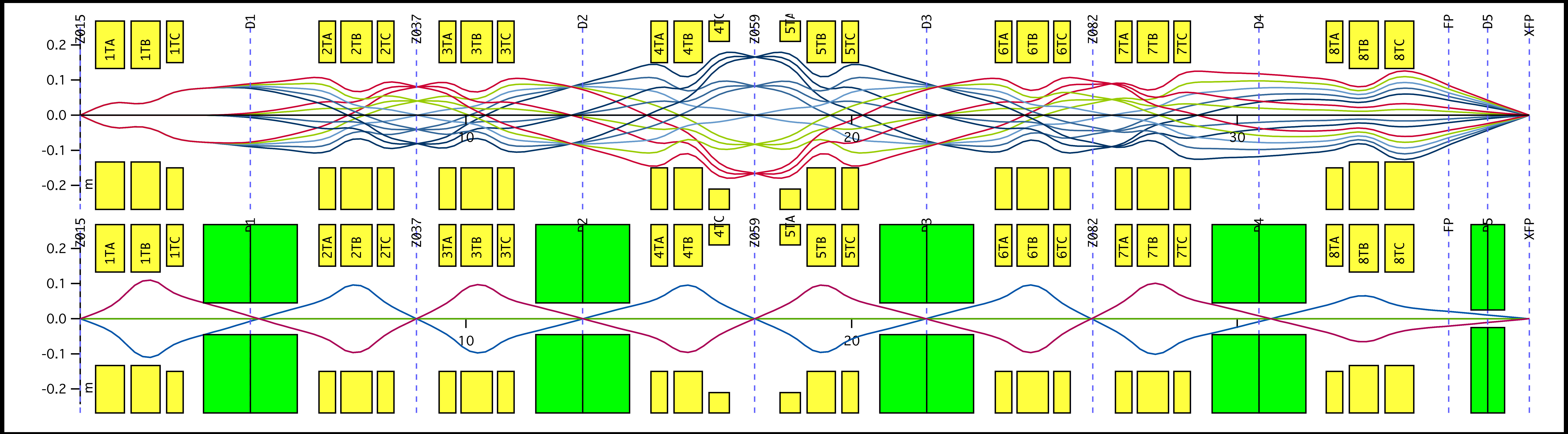
COMPLEX OPTICAL SYSTEMS

- Primarily composed of bending and focussing elements

- Several focussing, waist points and geometrical constraints

- Analytical calculations of optics in general not practical

- Numerical solutions become a multi-dimensional minimization problem (Lecture 2)



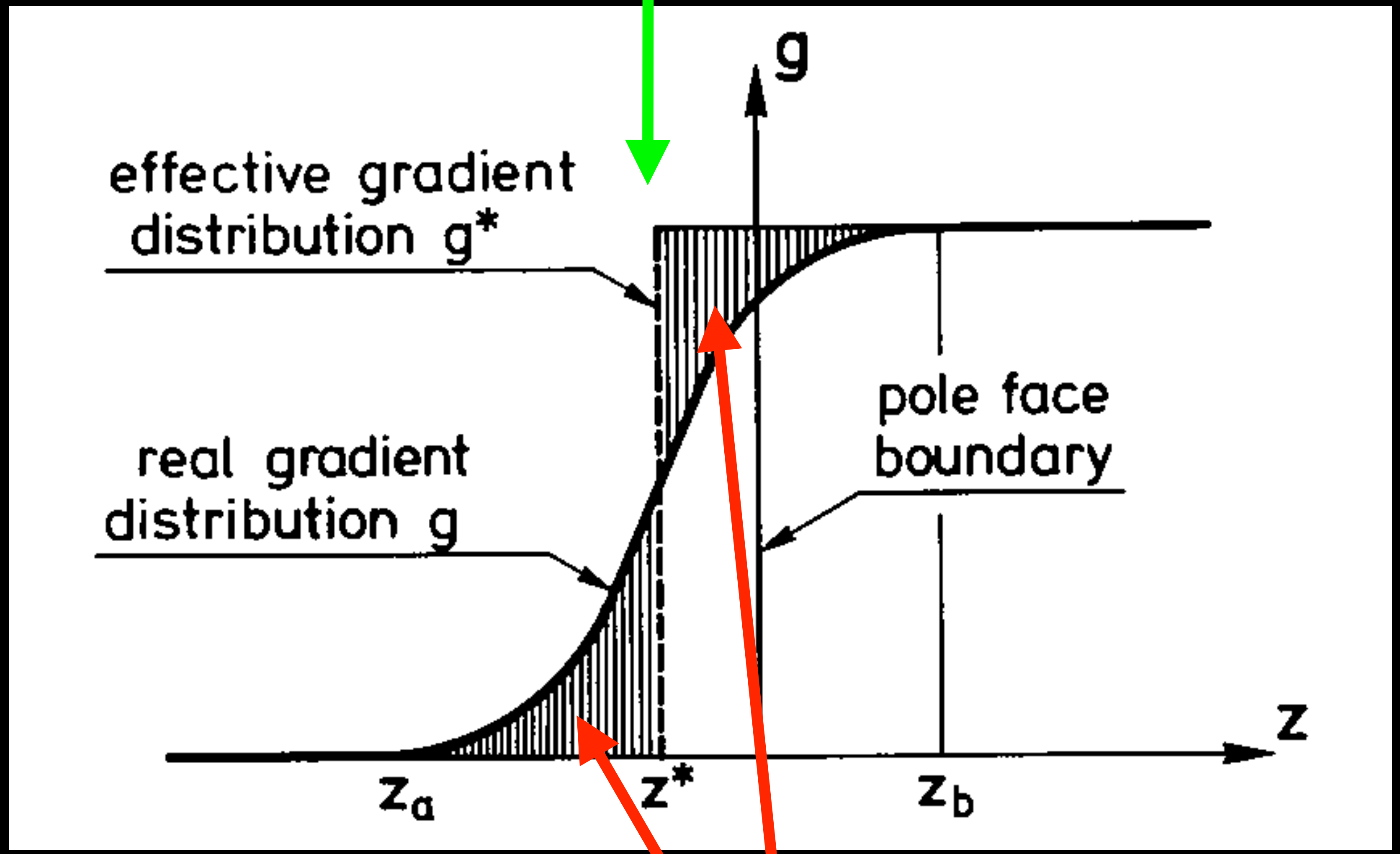
PART 2: METHODS AND INSTRUMENTS

- Ideal magnets versus real magnets
  - Effective length
  - Fringe fields
  - High order aberrations
- Instruments based on ion optics
  - Beam lines
  - Spectrometers and separators
  - Isochronous systems

REAL MAGNETS

- Ideal magnets have straight edges
- Real magnets have smooth edges
- The region where the field decays from maximum to 0 is called fringe field region
- The straight edge location corresponding to the same amount of field is called effective field boundary

EFB

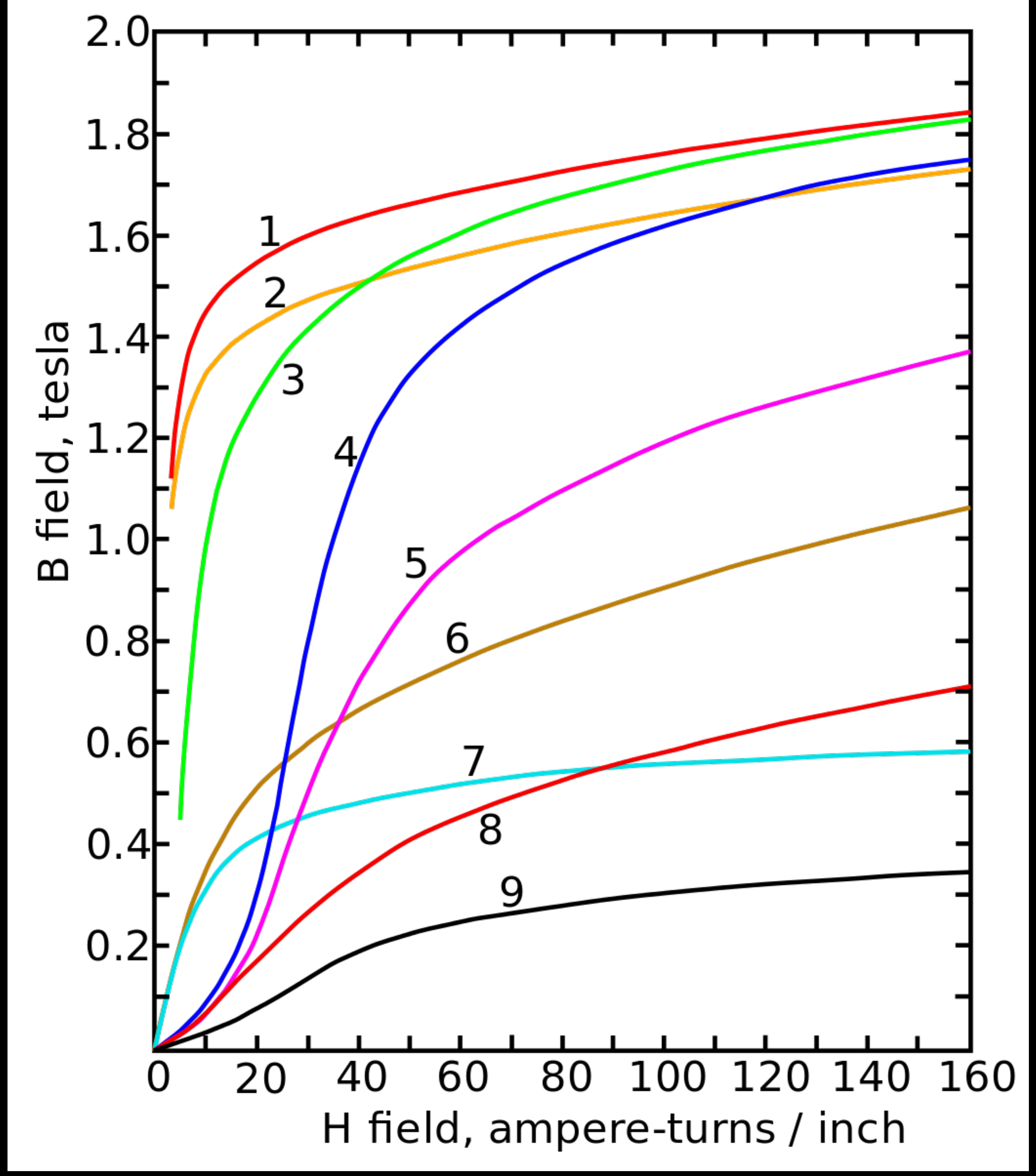


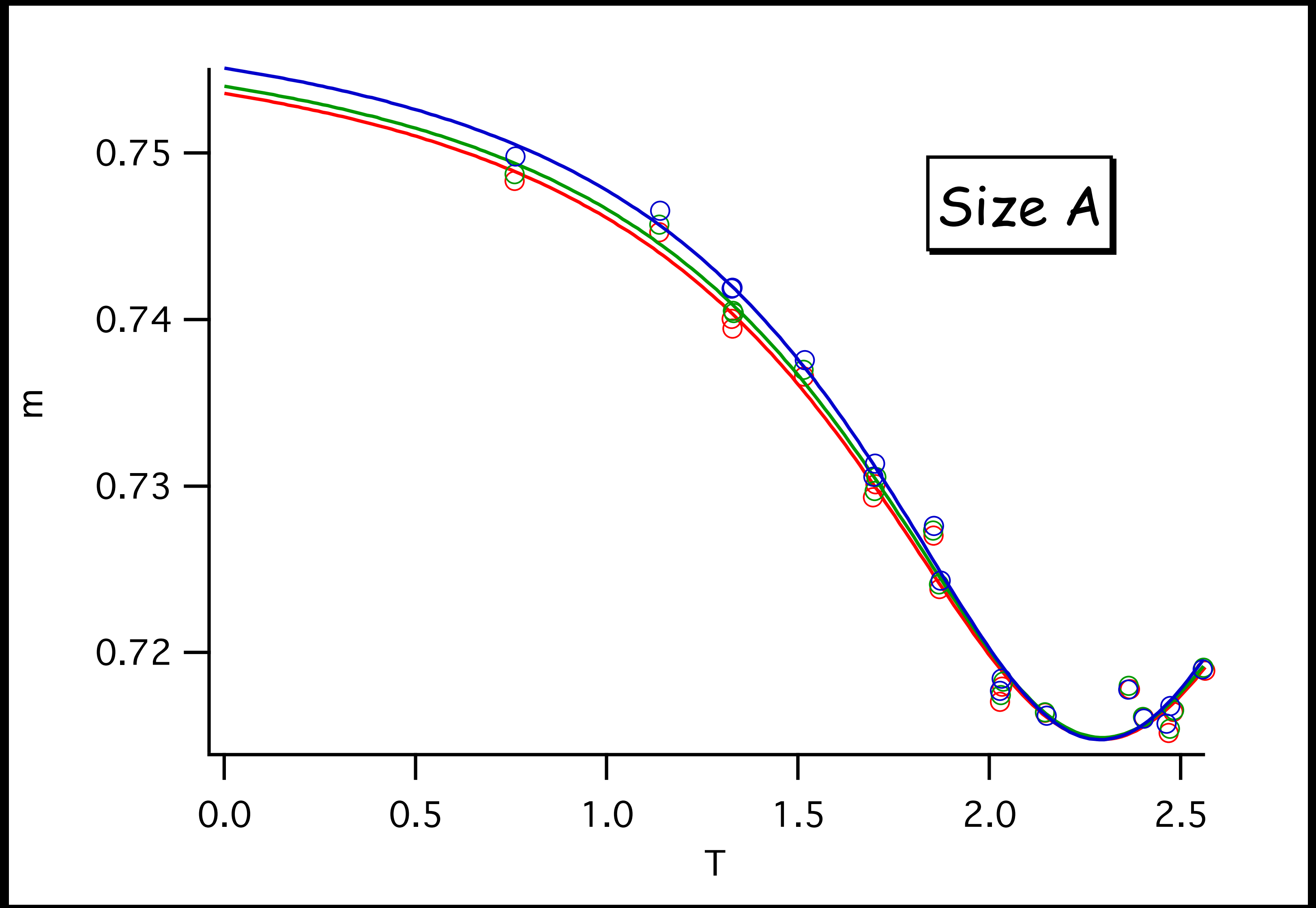
H. Wollnik, Optics of charged particles, Academic Press, INC (1987)

Same integral inside and outside EFB

VARIATIONS WITH FIELD

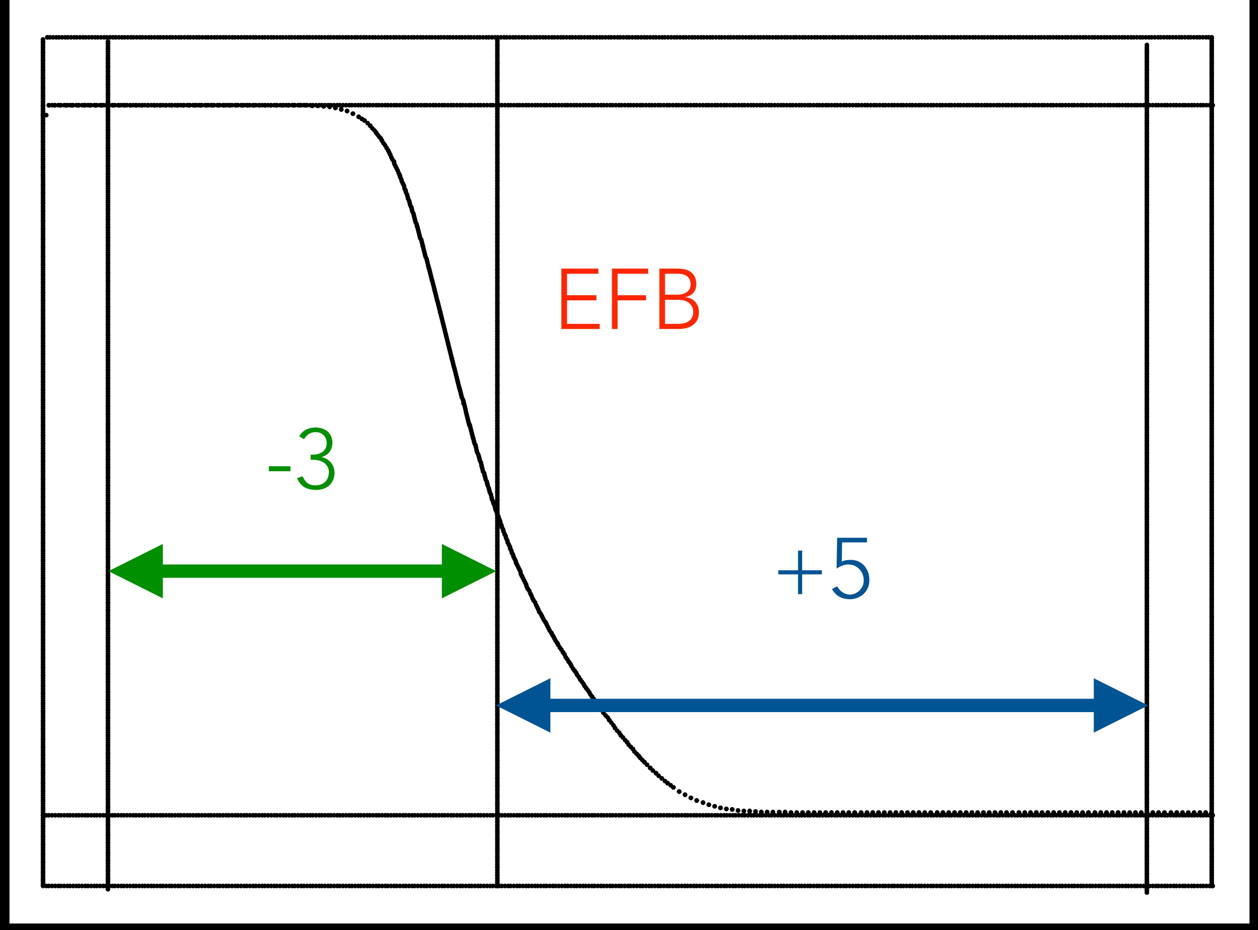
- Iron used for magnet poles saturates at about 1.2 Tesla (curve n°5)
- As the field changes in magnet, the location of the EFB moves
- Example shows effective length evolution for type A quadrupoles of the A1900
- Total variation is around 5%
- Optics calculations need to take this variation into account

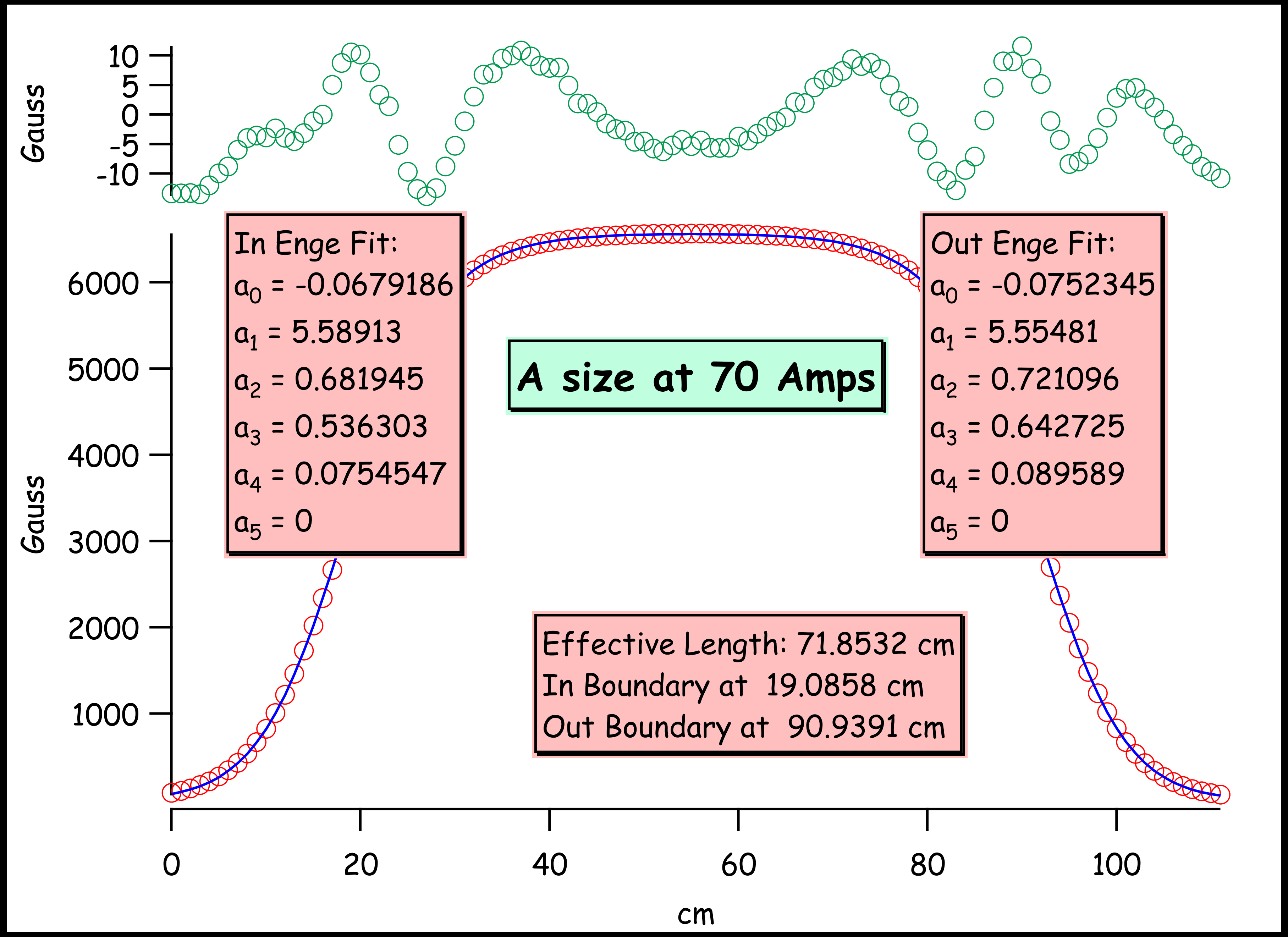




MODELING OF FRINGE FIELDS

- Shape of fringe fields modeled using Enge function
- $$F(z) = \frac{1}{1 + \exp[\sum_{i=0}^{n} a_i (\frac{z}{D})^i]}$$
- z: distance perpendicular to EFB, D: aperture of magnet, a_i: Enge coefficients
- Enge coefficients are fitted to field map data
- Stability of Enge function required from -3 to +5 of $\frac{z}{D}$ (COSY infinity)





HIGH ORDER ABERRATIONS

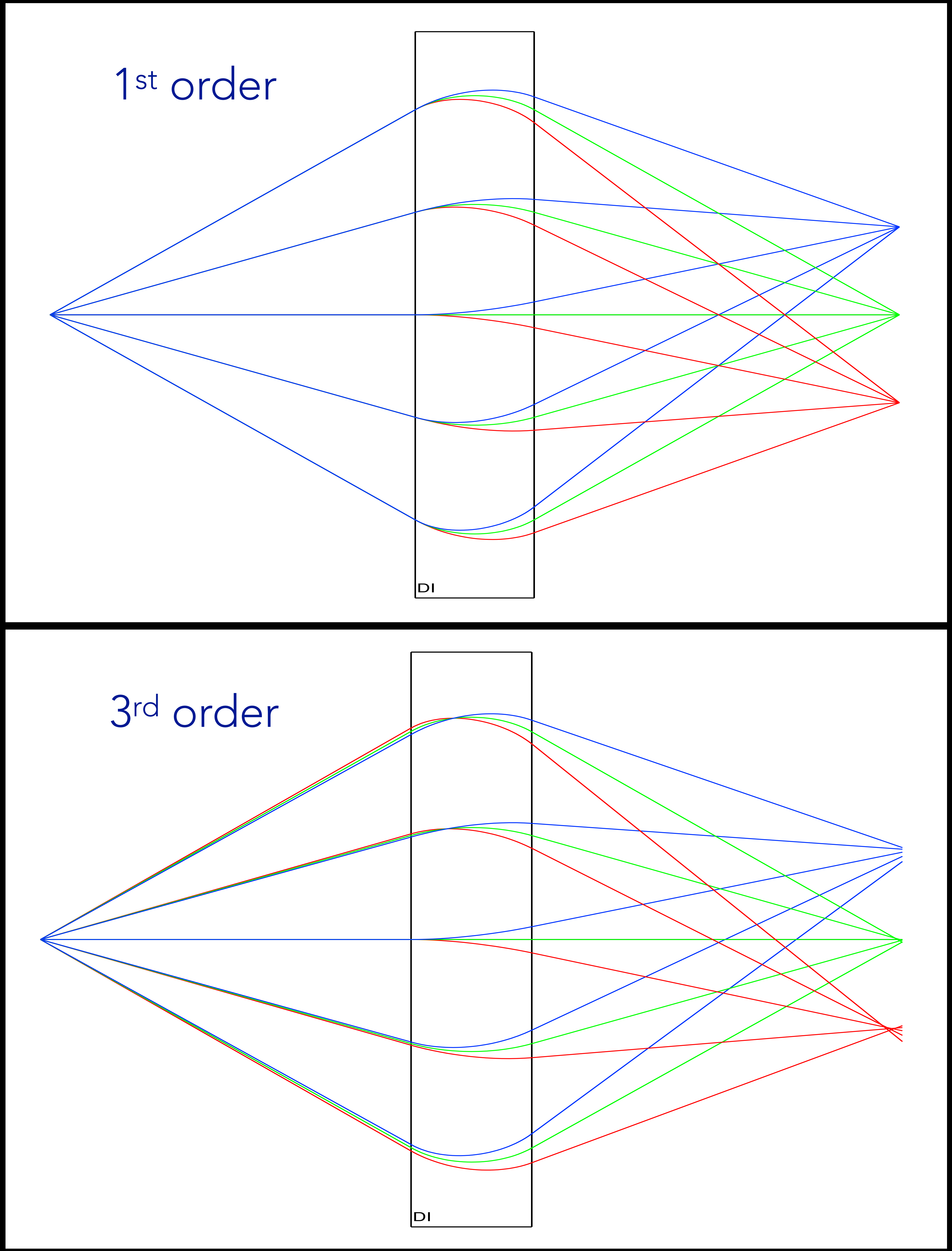
- Due to non-linear effects (such as fringe fields for instance)
- Model of optics as linear combinations between final and initial coordinates only an approximation
- Terms of order greater than 1 are called aberrations
- Terms that depend on geometrical coordinates are called geometrical aberrations
- Terms that depend on momentum/energy coordinates are called chromatic aberrations

$$x_f = \boxed{(x/x)x_i + (x/a)a_i + (x/\delta)\delta_i}$$
$$\boxed{\begin{aligned}&+(x/xx)x_i^2 + (x/xa)x_i a_i + (x/aa)a_i^2\\ &+(x/x\delta)x_i\delta_i + (x/\delta\delta)\delta_i^2\end{aligned}}$$
$$\boxed{+(x/xaa)x_i a_i^2 + (x/xa\delta)x_i a_i\delta_i + \ldots}$$

Polynomial expansion similar to Taylor series

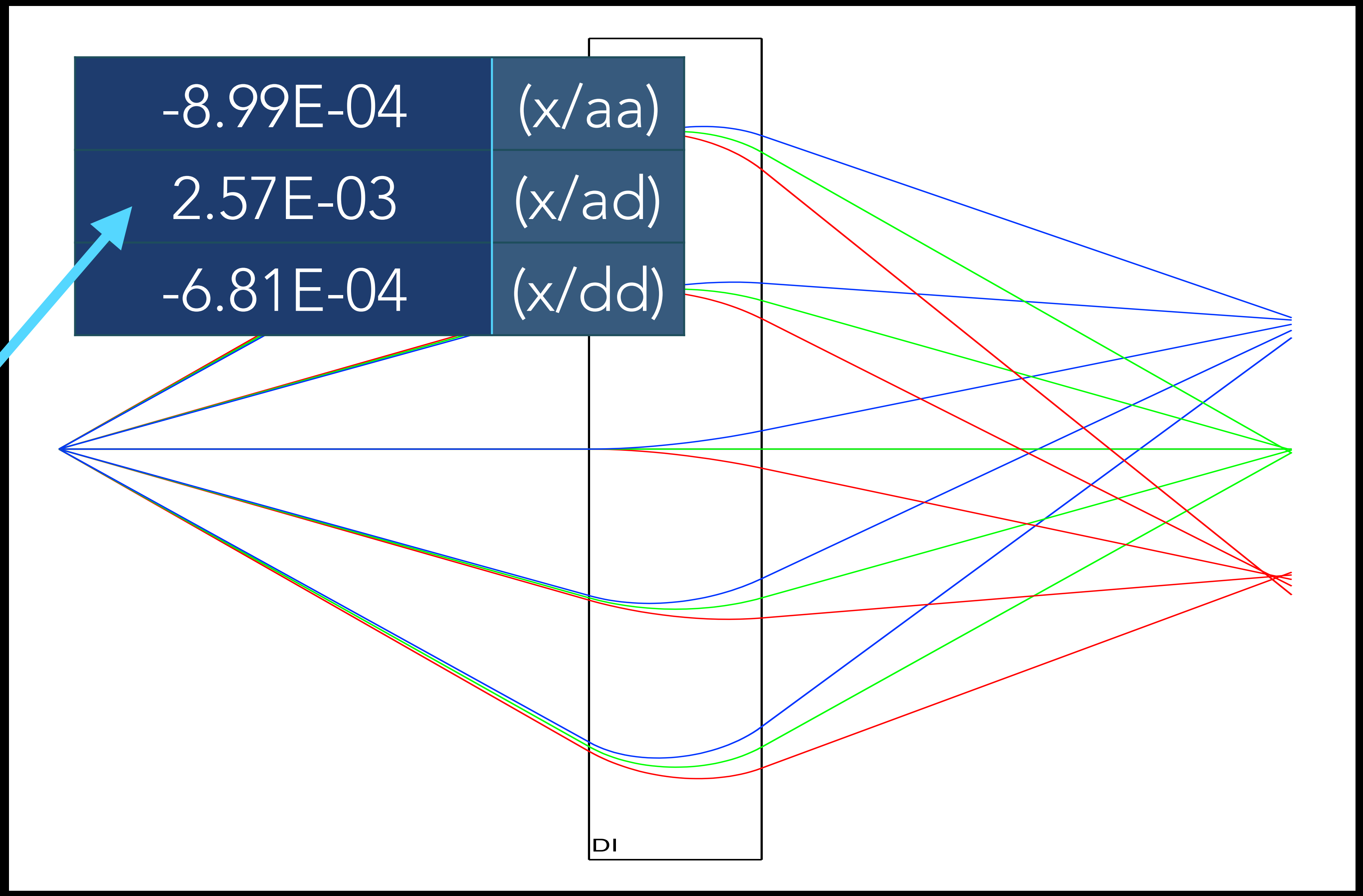
ILLUSTRATION ON SIMPLE DIPOLE

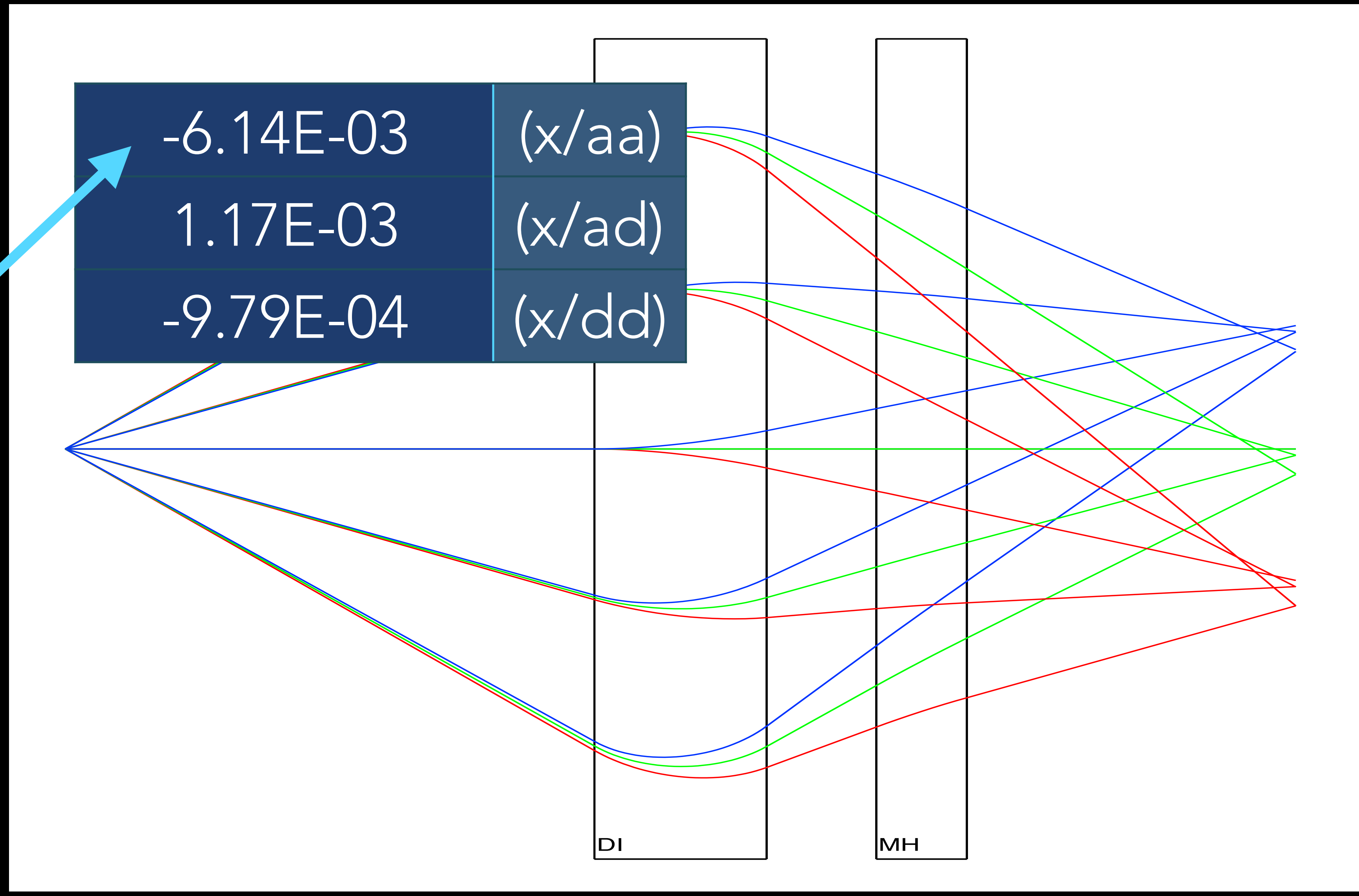
- Dipole magnet x focussing
- First order calculation shows perfect focussing of all 5 angles for the 3 different momenta
- Third order calculation shows blurring of images due to aberrations
- Blurring larger for momenta different from central ray indicate presence of chromatic aberrations



ABERRATION CORRECTIONS

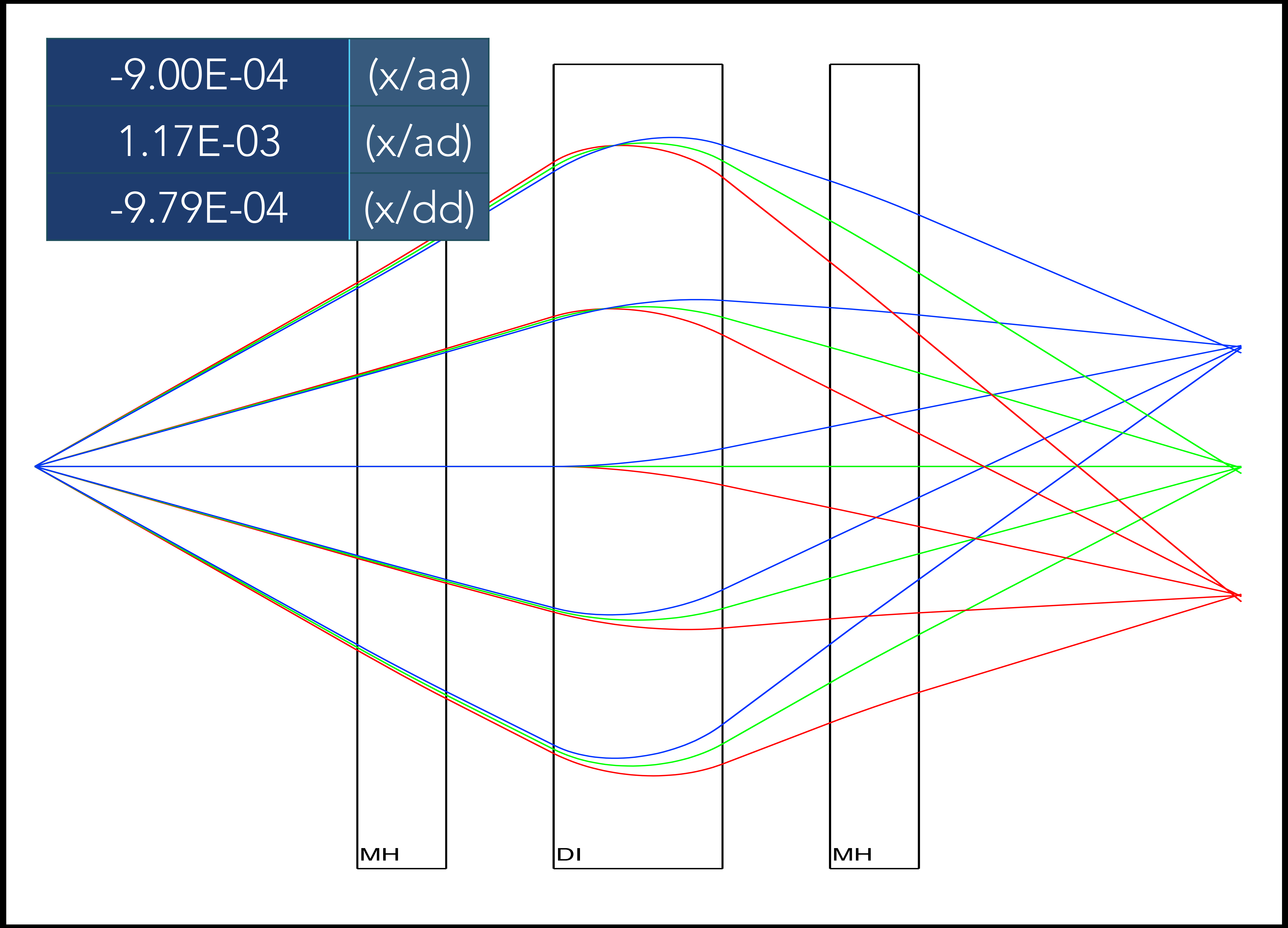
- High order multipoles can be used to correct aberrations
  - Sextupoles can correct 2^{nd} order terms
  - Octupoles can correct 3^{rd} order terms
  - And so on …
- Example on simple dipole
  - Largest aberration is $(x/a\delta)$
  - Adding a sextupole reduces it by ~ 2
  - But it increases (x/aa) by almost 7!





SYMMETRICAL SYSTEMS

- Aberrations can be better controlled when using symmetrical systems
- Many aberrations with even powers cancel out in symmetric system
- Example from simple dipole
  - Solution: add another sextupole symmetric to first with opposite field
  - (x/aa) goes back to original value while $(x/a\delta)$ is still reduced



INSTRUMENTS BASED ON ION OPTICS

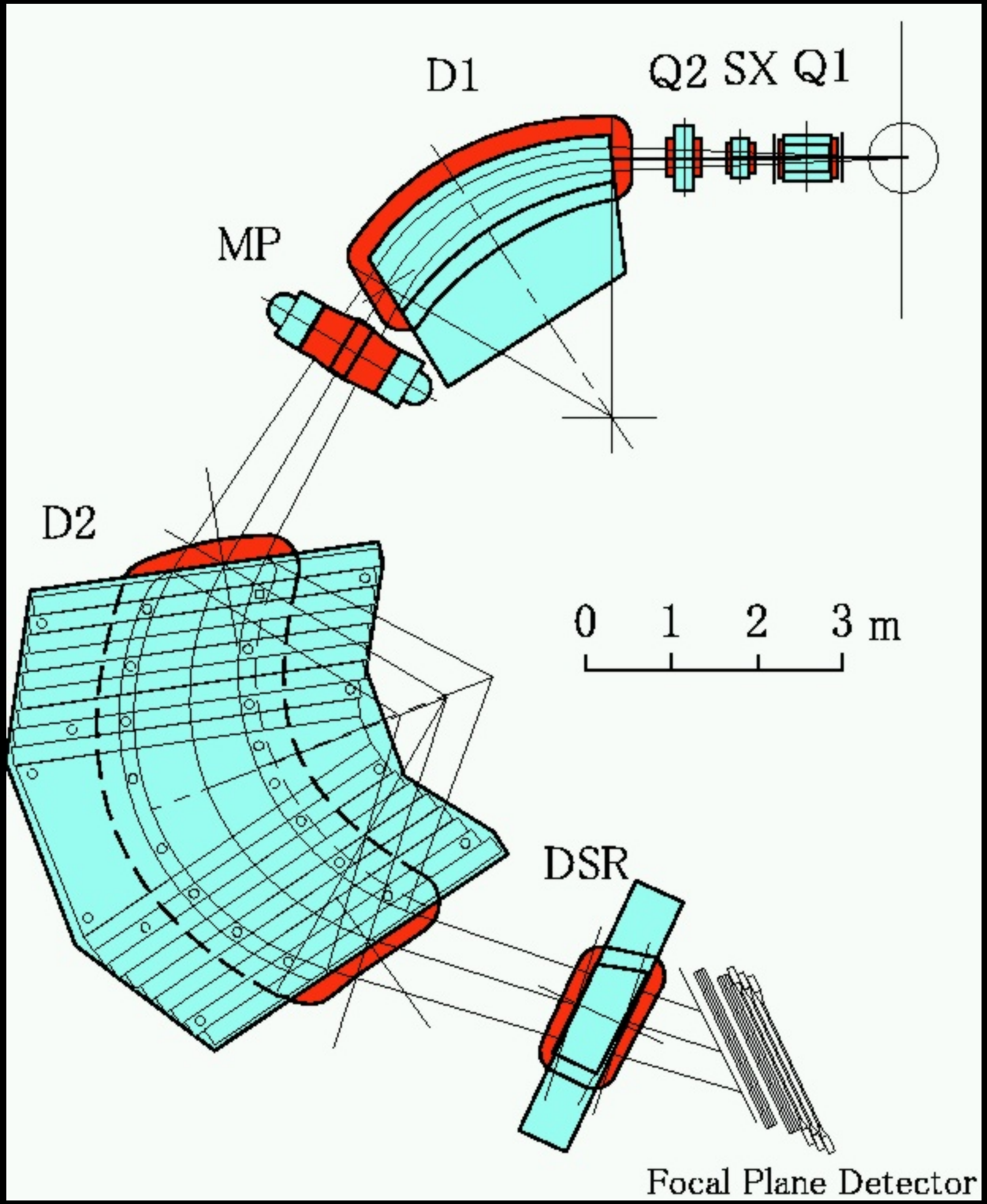
- Simplest type: beam lines
- Purpose: transmit ions from one location to another with minimal losses
- Design principally depends on type of beam to be transmitted
  - Beam from accelerator (usually called primary beam): small emittance
  - Beam from separator (also called secondary beam): large emittance

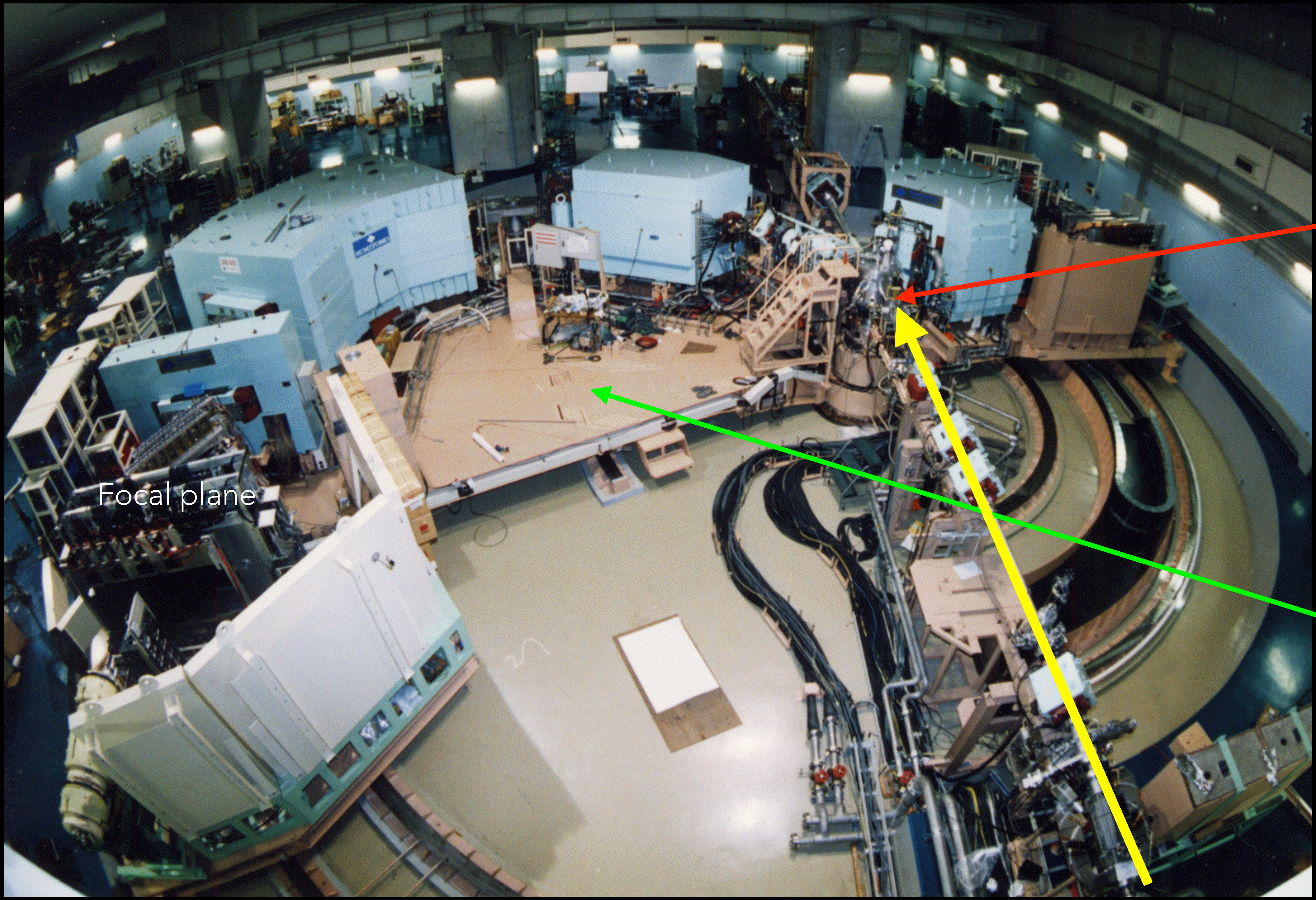
SPECTROMETERS

- Chromatic systems: final position depends on momentum
- Optics system designed to measure characteristics of charged particles
  - Usually located after a reaction target
  - Has large acceptances to collect as many reaction products as possible
  - Needs detection system to actually perform the measurements
  - Can be moved to different angles relative to the incoming beam direction
  - Resolving power designed to obtain a certain momentum resolution

EXAMPLE1: GRAND RAIDEN @ RCNP

- Optics configuration: QSQMDDD
- Characteristics
  - Maximum rigidity: 5.5 T.m.
  - Deflection angle: 162°
  - Momentum acceptance: 5%
  - Solid angle: 5 msr
  - Resolving power: 37,000 in momentum
- Used with primary beams from cyclotron

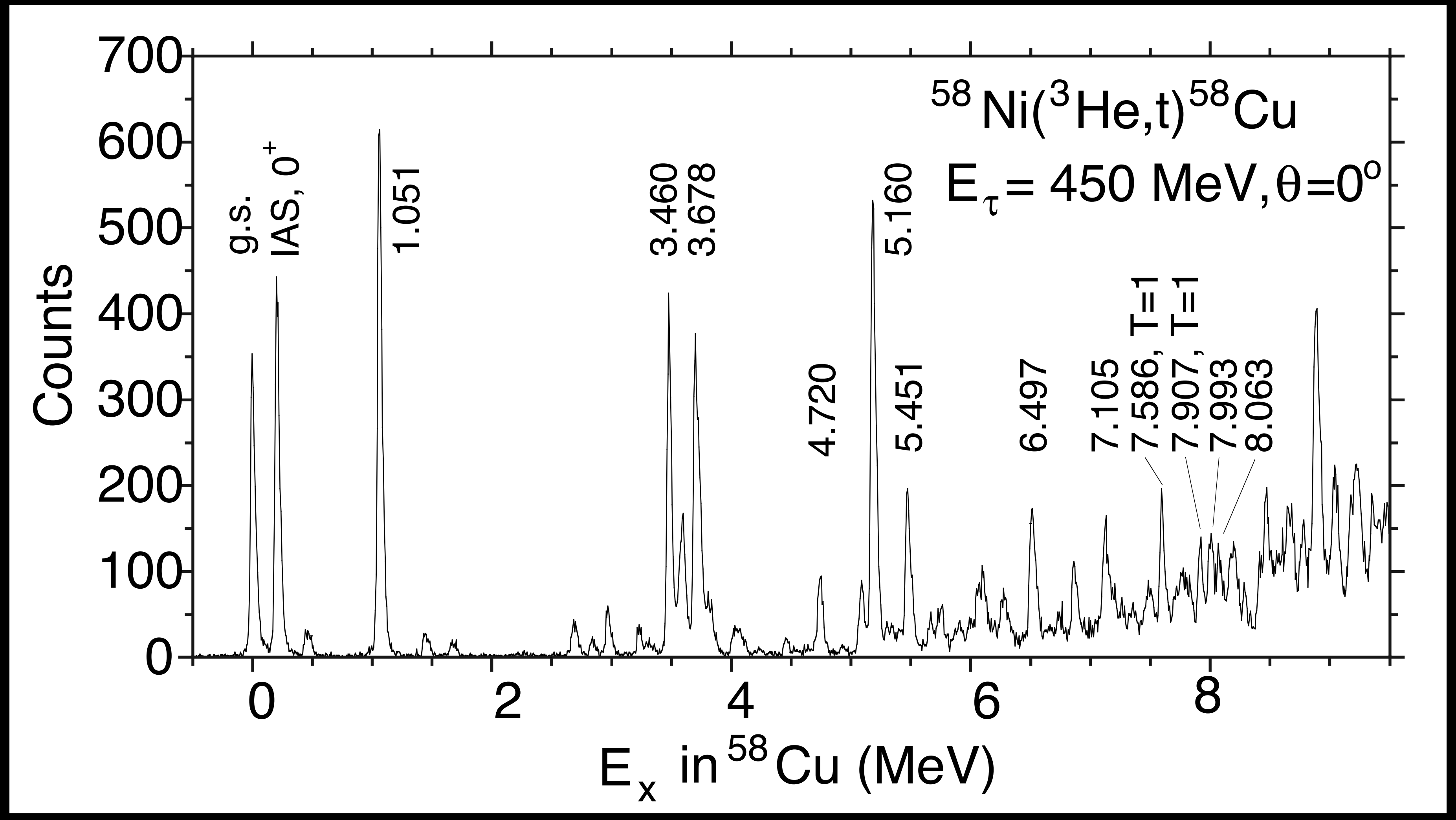


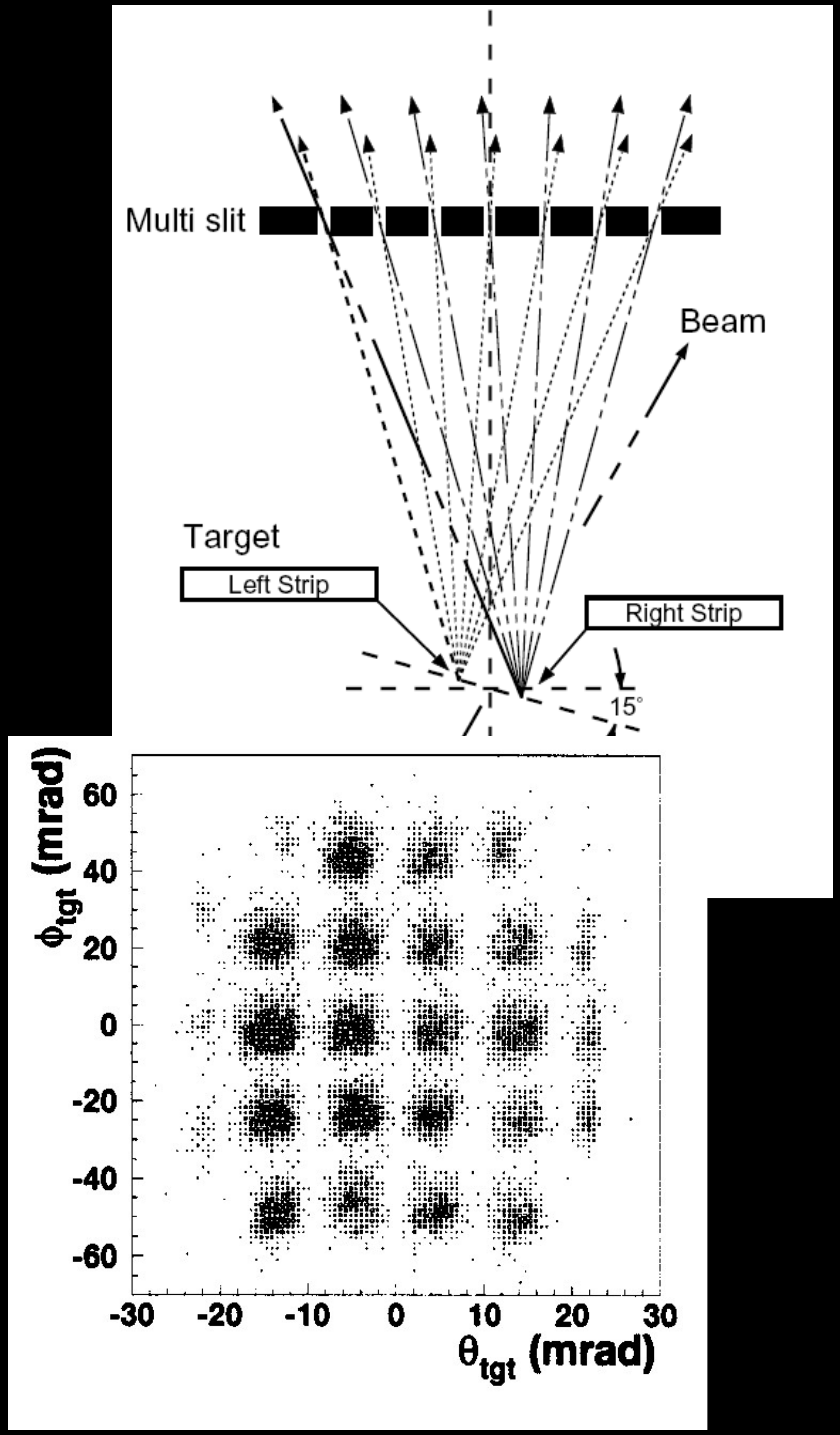

Target
Focal plane
Platform
$5^\circ < \theta < 90^\circ$
Incoming beam

HARDWARE SPECTROMETER

- Multipoles used to correct some aberrations
- Other aberrations mapped from data
  - Use of "angle sieve" to calibrate angles
- Use of dispersion matching technique (see later)

Y. Fujita et al., Eur. Phys. J. A 13, 411 (2002)
Energy resolution: 50 keV
Resolving power: 9,000 in energy

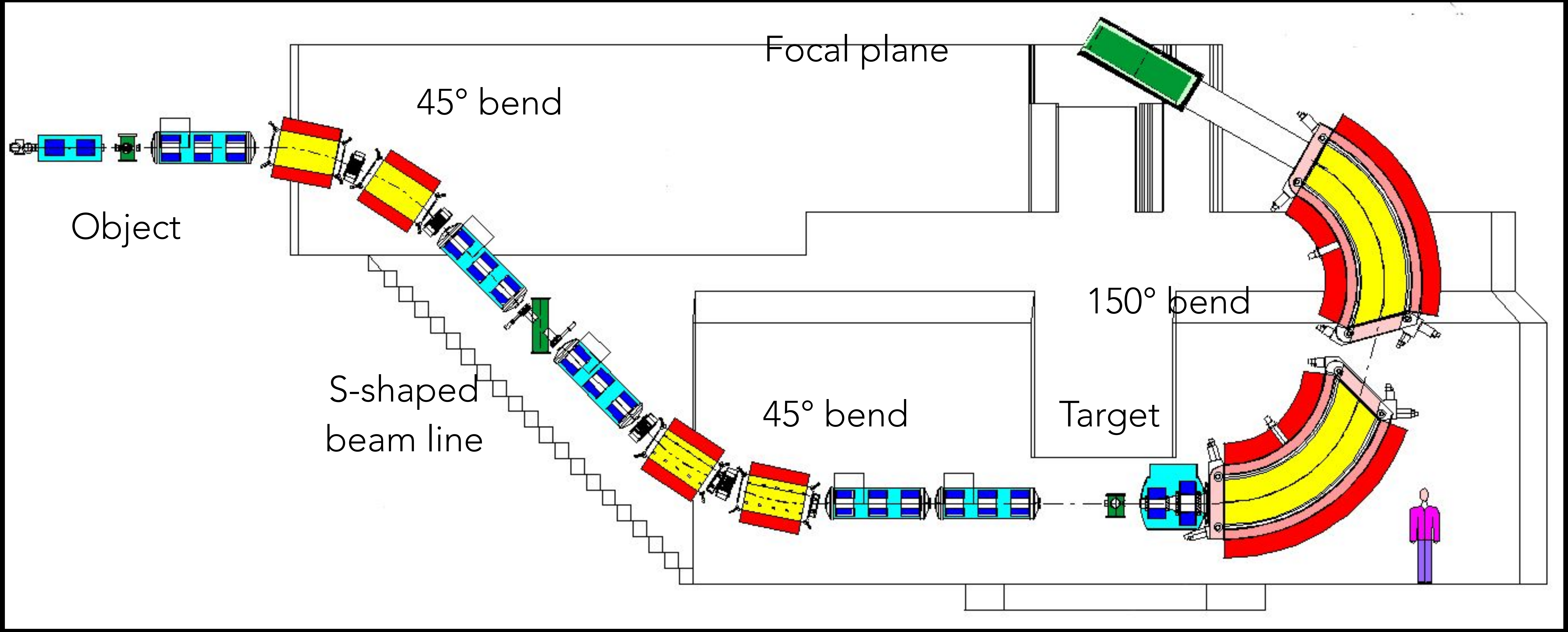


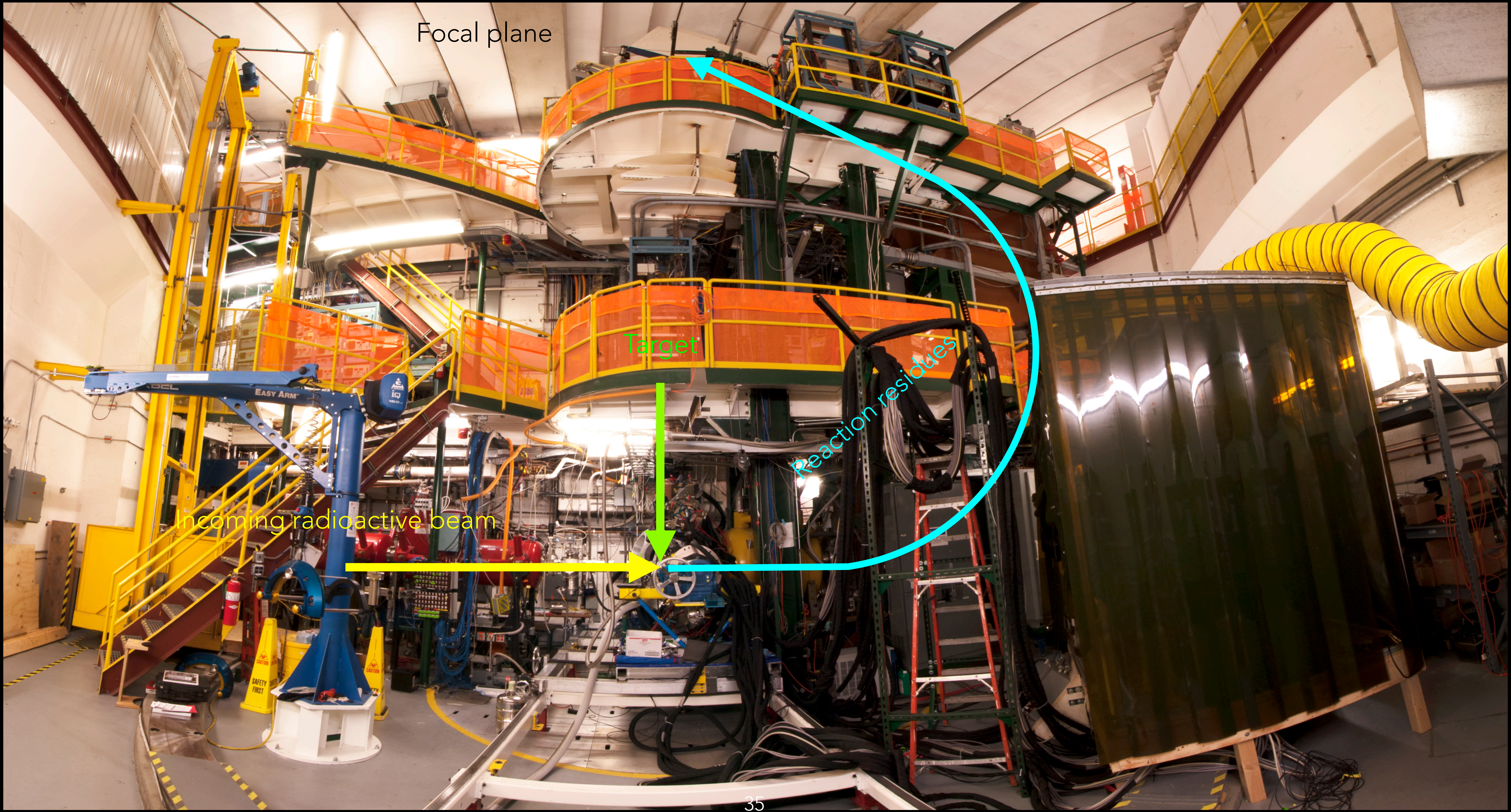


From G. Berg

EXAMPLE2: S800 @ NSCL

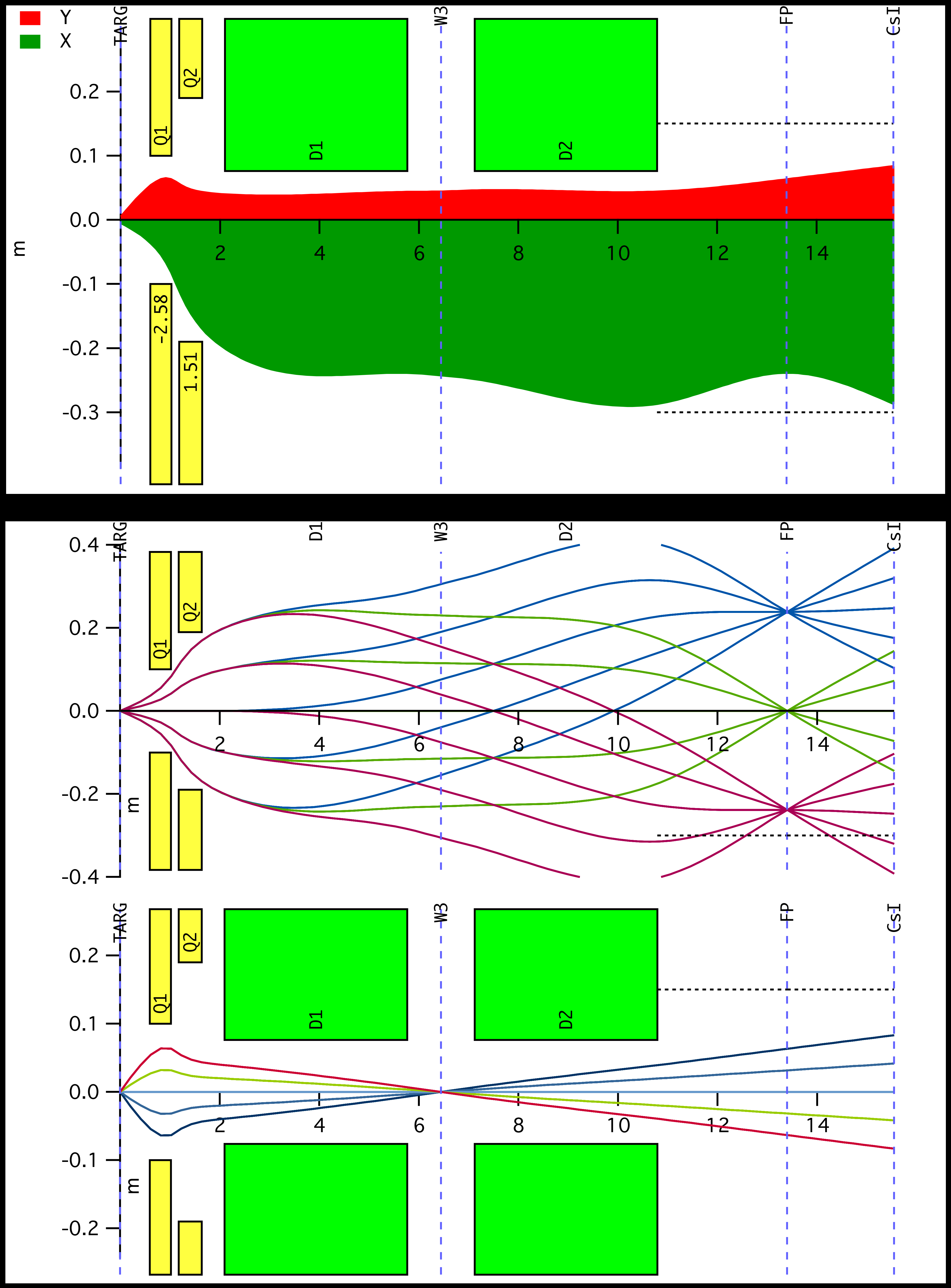
- Designed for reactions using secondary beams (large emittance)
- Large acceptances (5% momentum, 20 msr solid angle), resolving power: 10,000
- Has preceding beam line that can be used for dispersion matching (see later)




Focal plane
Target
Reaction residues
Incoming radioactive beam
EASY ARM

S800 OPTICS

- Optics design: QQDD
  - Maximize Y angular acceptance (waist between the two dipoles)
  - X focus at the focal plane
  - Dispersion: 96 mm/% (momentum)
  - Magnification (x/x): 0.9
- Focal plane measurements
  - Two tracking detectors to measure (x,y) positions
  - Followed by energy loss, timing, Energy



DISPERSION MATCHING

- Incoming secondary beam has large momentum emittance (up to 1%)
  - Momentum measurement after reaction is blurred by this large emittance
  - Use preceding beam line to compensate for spectrometer dispersion
  - Position at focal plane only depends on momentum change in target
- Lateral and angular dispersion matching calculations
  - **Exercise 5**: find b_{16} and b_{26} so that $t_{16} = 0$ and $t_{26} = 0$, assuming focussing $s_{12} = 0$

$$\begin{bmatrix} s_{11} & s_{12} & s_{16} \\ s_{21} & s_{22} & s_{26} \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & b_{16} \\ b_{21} & b_{22} & b_{26} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} & t_{16} \\ t_{21} & t_{22} & t_{26} \\ 0 & 0 & 1 \end{bmatrix}$$

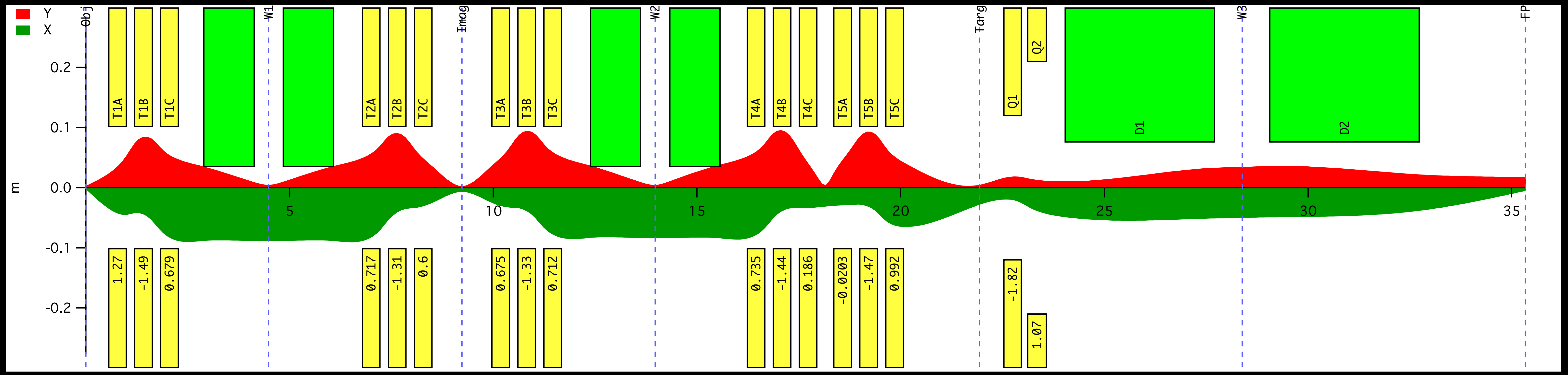
DISPERSION MATCHING: SOLUTION

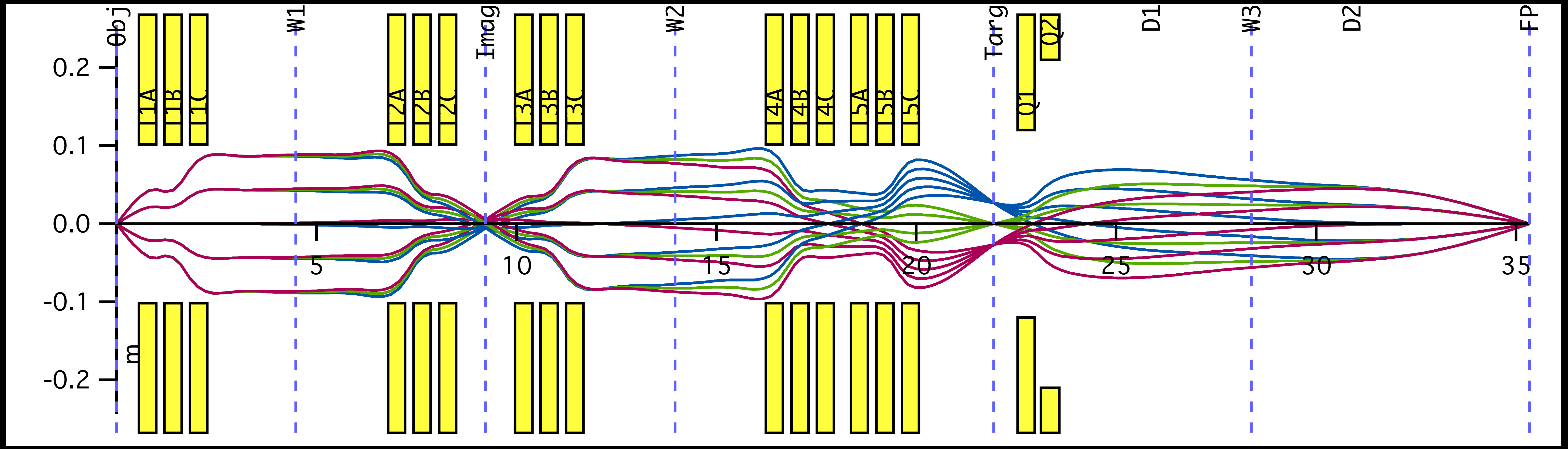
- Solve the system of equations:
  - $s_{11} \times b_{16} + s_{16} = 0$
  - $s_{21} \times b_{16} + s_{22} \times b_{26} + s_{26} = 0$
- Lateral dispersion of beam line: $b_{16} = -\dfrac{s_{16}}{s_{11}}$
- Angular dispersion of beam line: $b_{26} = s_{21} \times s_{16} - s_{11} \times s_{26}$
- Solutions more complicated if spectrometer not at 0°

DISPERSION MATCHING IN S800

Lateral dispersion at target: 107 mm/%

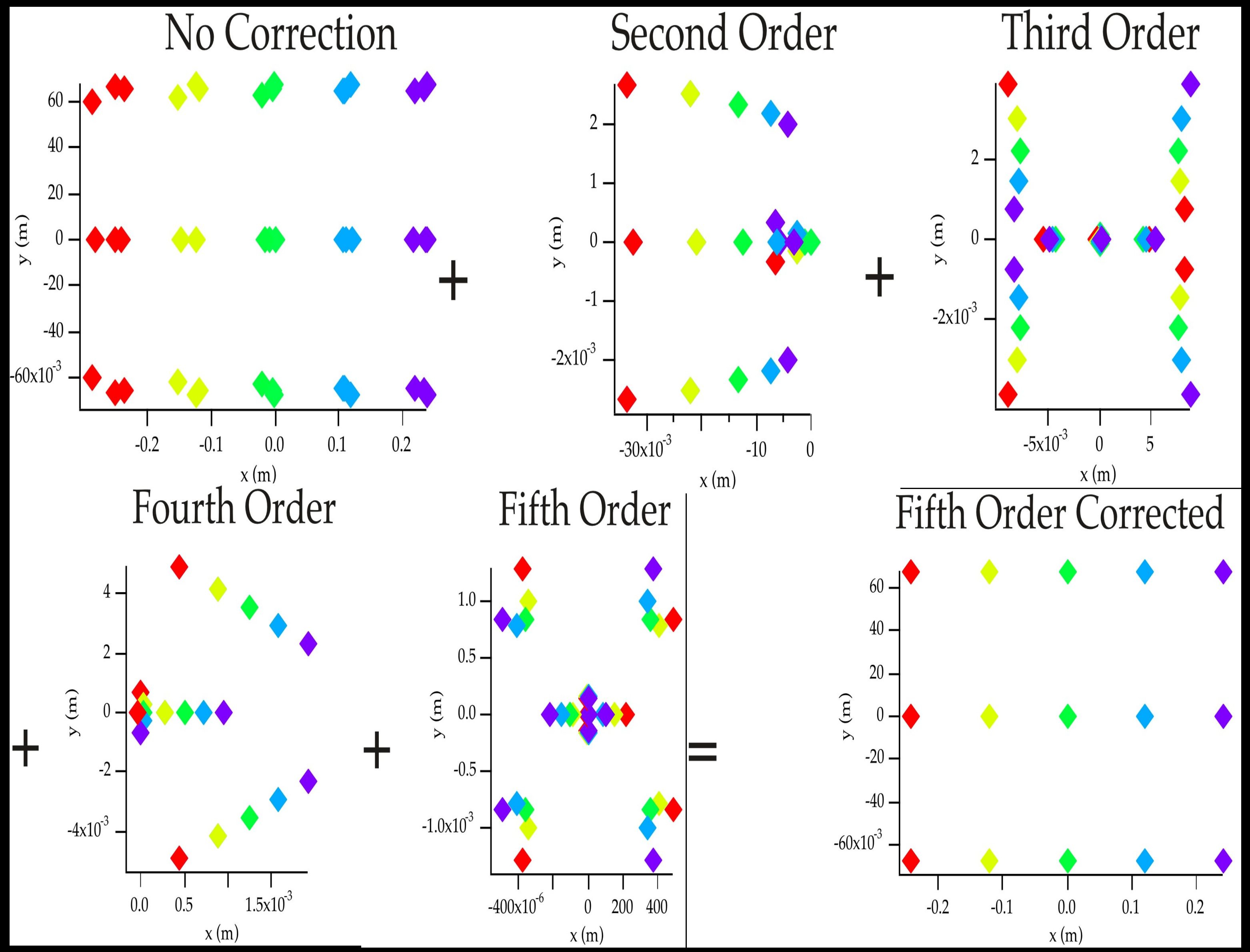
- Drawbacks
  - Reduced momentum acceptance: 0.5%
  - Large beam spot on target



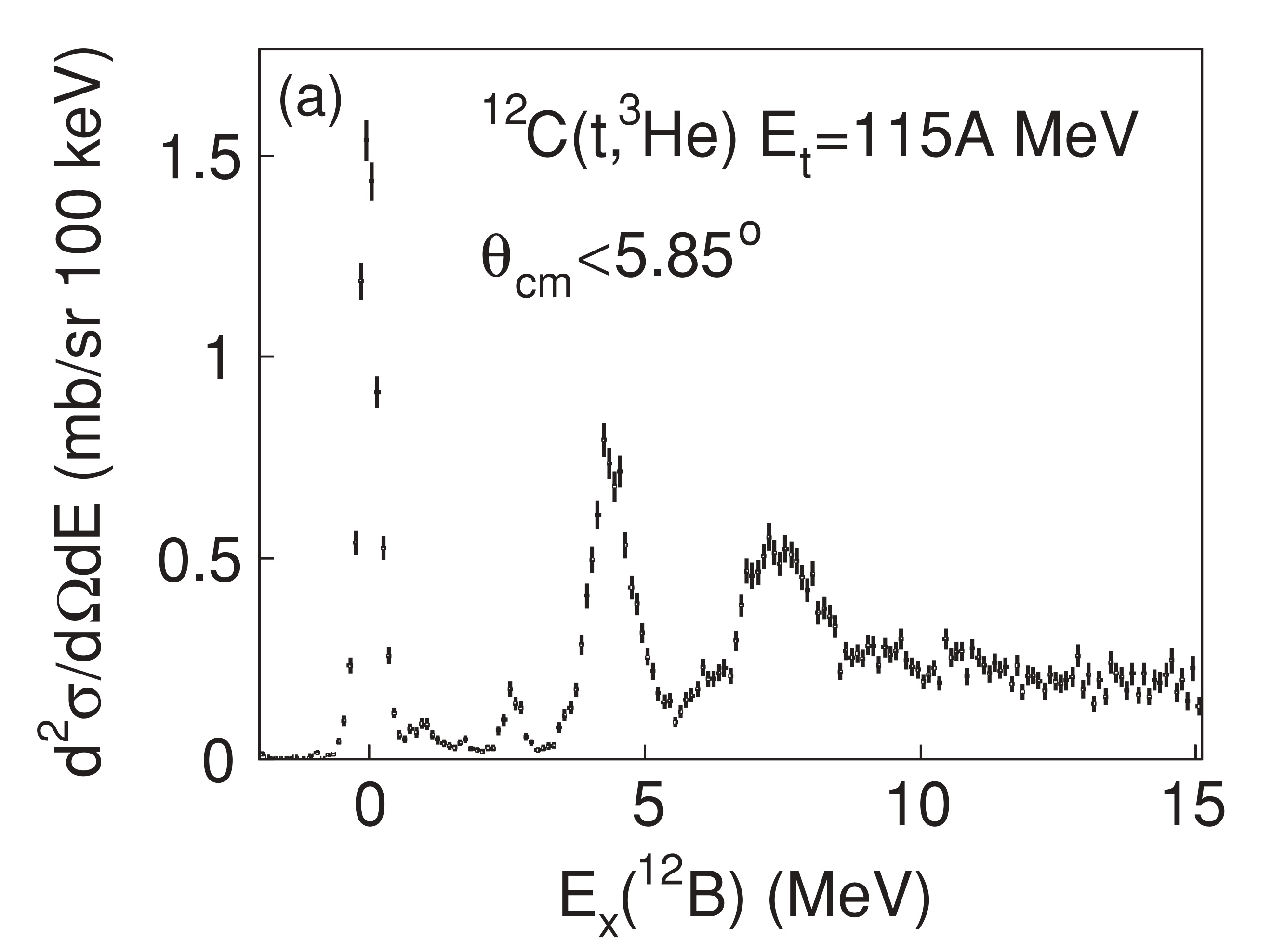


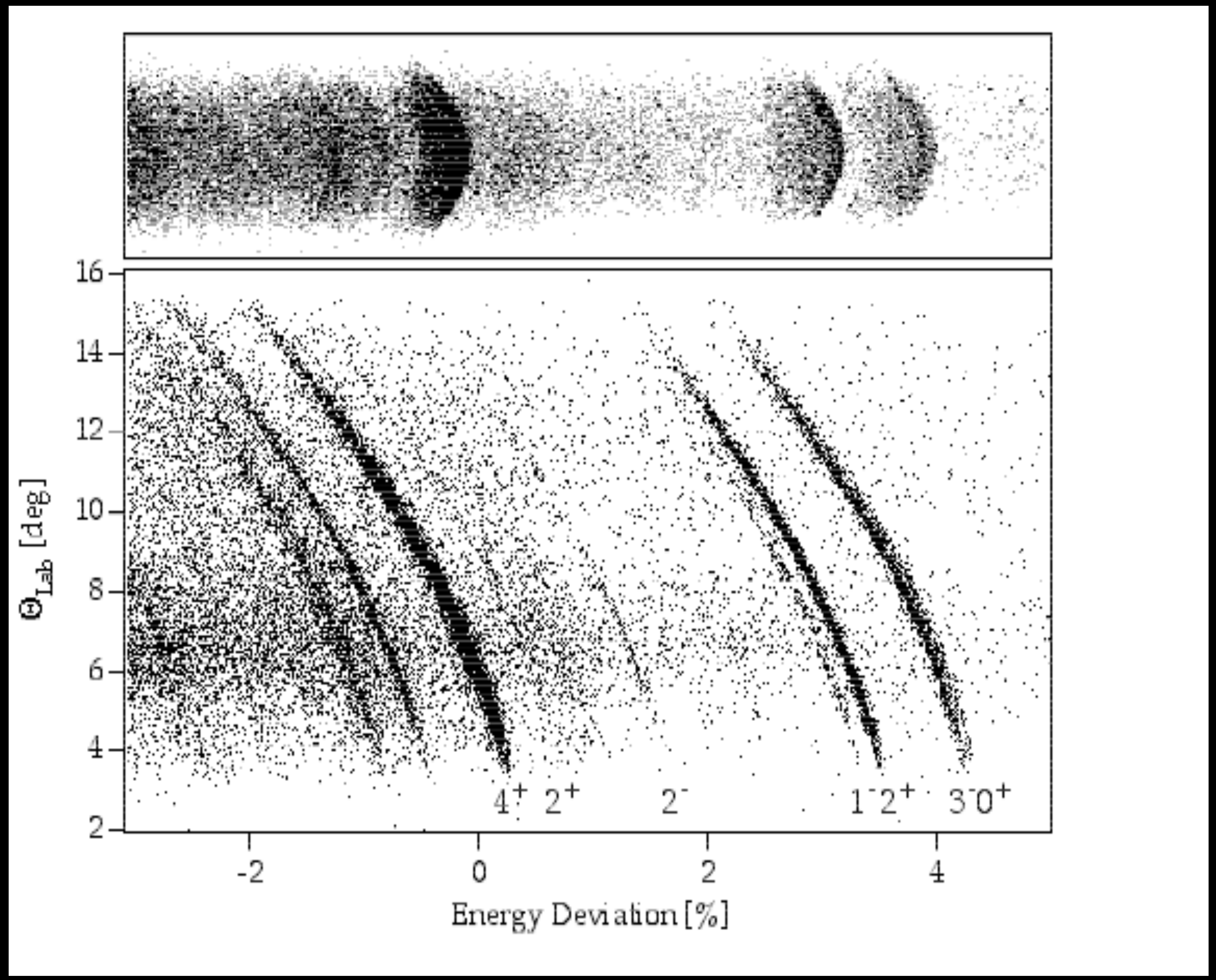
SOFTWARE SPECTROMETER

- No multipole to correct aberrations
- Aberrations are corrected "in software" from model of the spectrometer
- Model calculated from field maps of magnets and COSY calculations



Perdikakis et al., Phys. Rev. C 83, 054614 (2011)
Tritium radioactive beam from ^{16}O





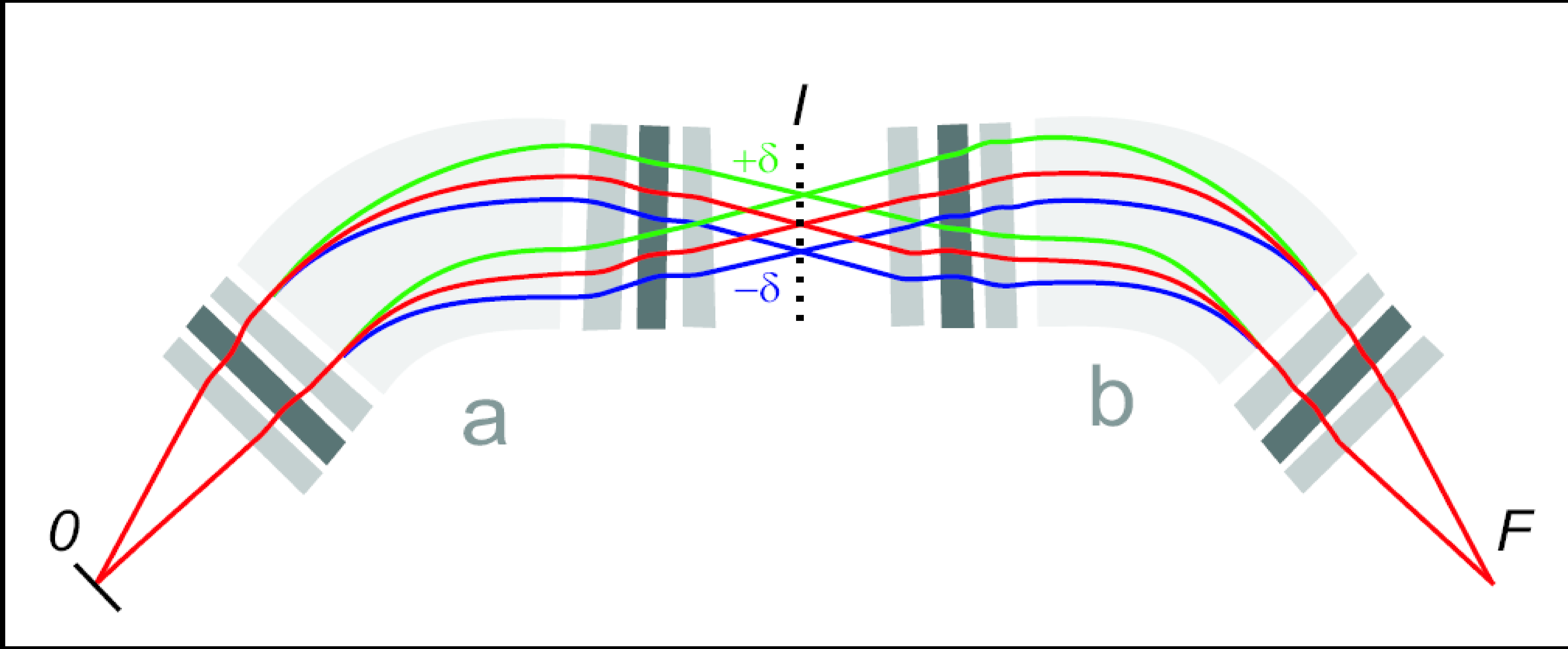
Reaction: $^{12}C(^{7}Li,^{3}H)^{16}O$ at 19 MeV/u

SEPARATORS

- Achromatic systems: final position does not depend on momentum
- This needed from separators that produce secondary beams
- Previous dispersion matching systems are in effect achromatic
- Spectrometers can also be separators: just need to add some aperture or slits at the focal plane!
- Separators are designed to transmit particles into a beam for further use
- Care is needed to avoid beam spot size growth (aberration corrections)

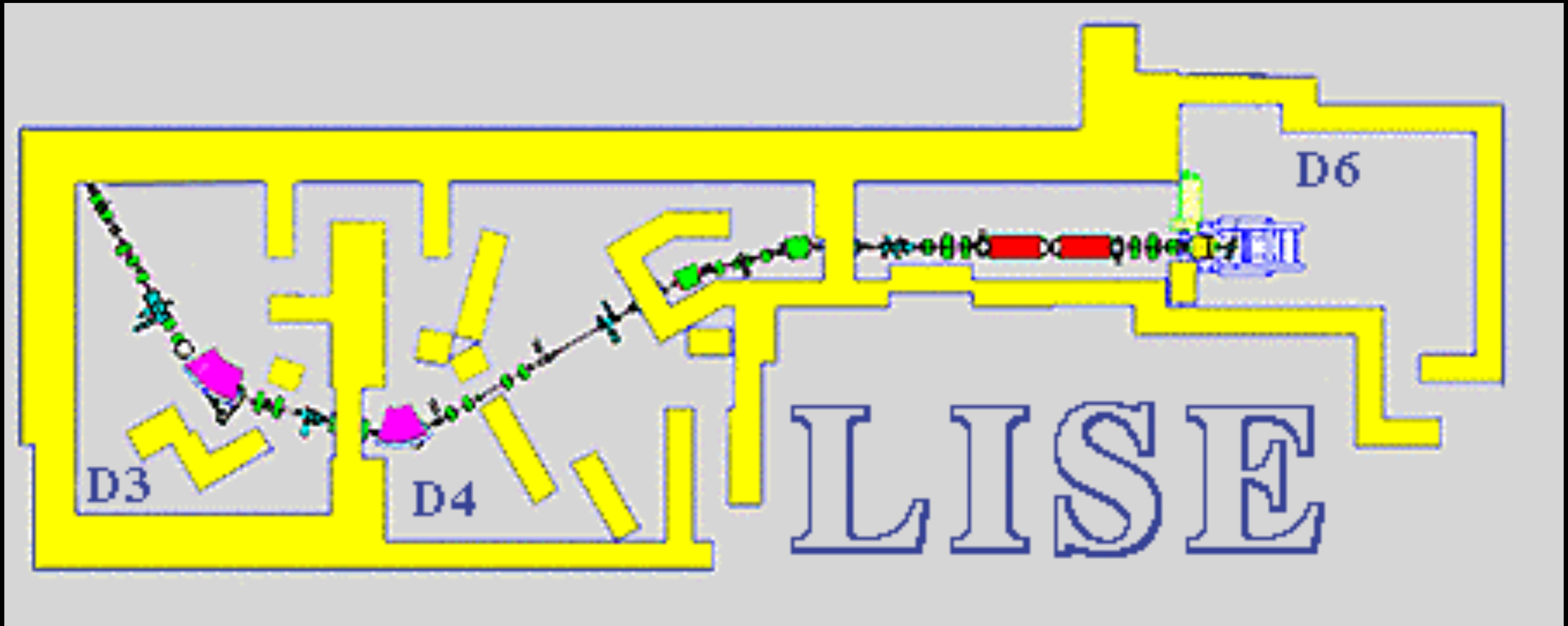
BASIC CONCEPT

- Optics configuration of a typical separator
  - Two dispersive sections mirror to each other
  - Dispersions cancel to produce an achromatic image at the focal plane
  - Slits placed at the dispersive mid point select particles with the same $B\rho$

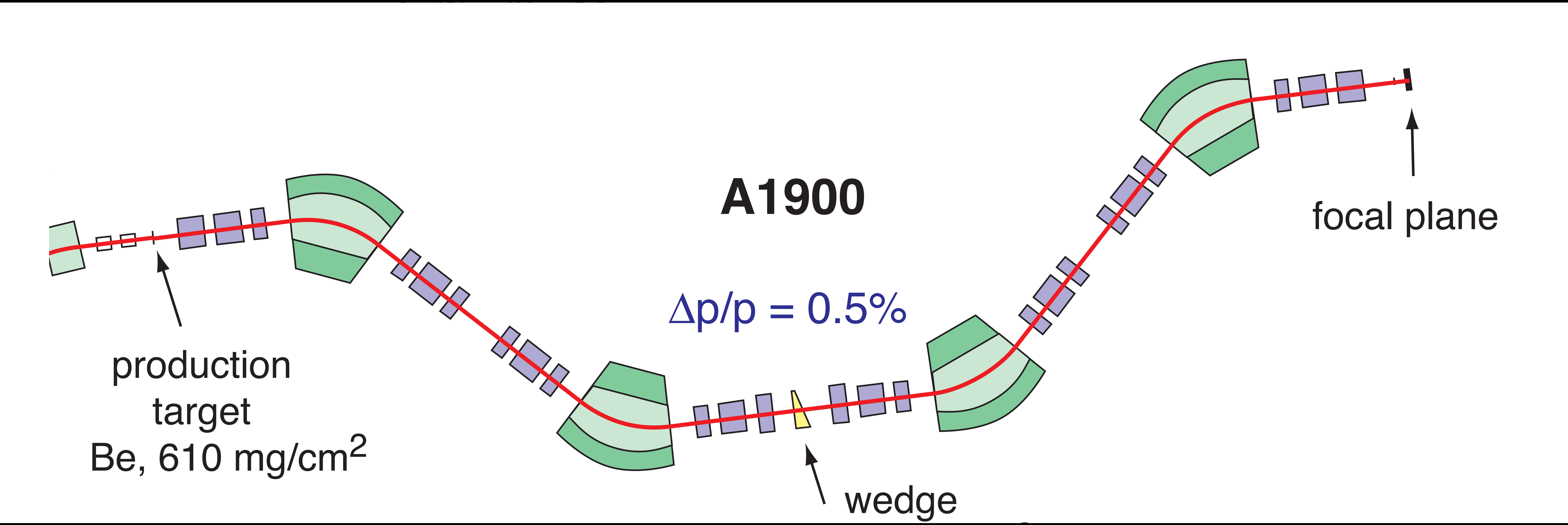


EXAMPLE 1: LISE @ GANIL

- One of the first fragment separator
- Simplest optics configuration: one dispersive focal plane in the middle
- Later expanded with Wien filter (velocity filter) to improve purity of beams

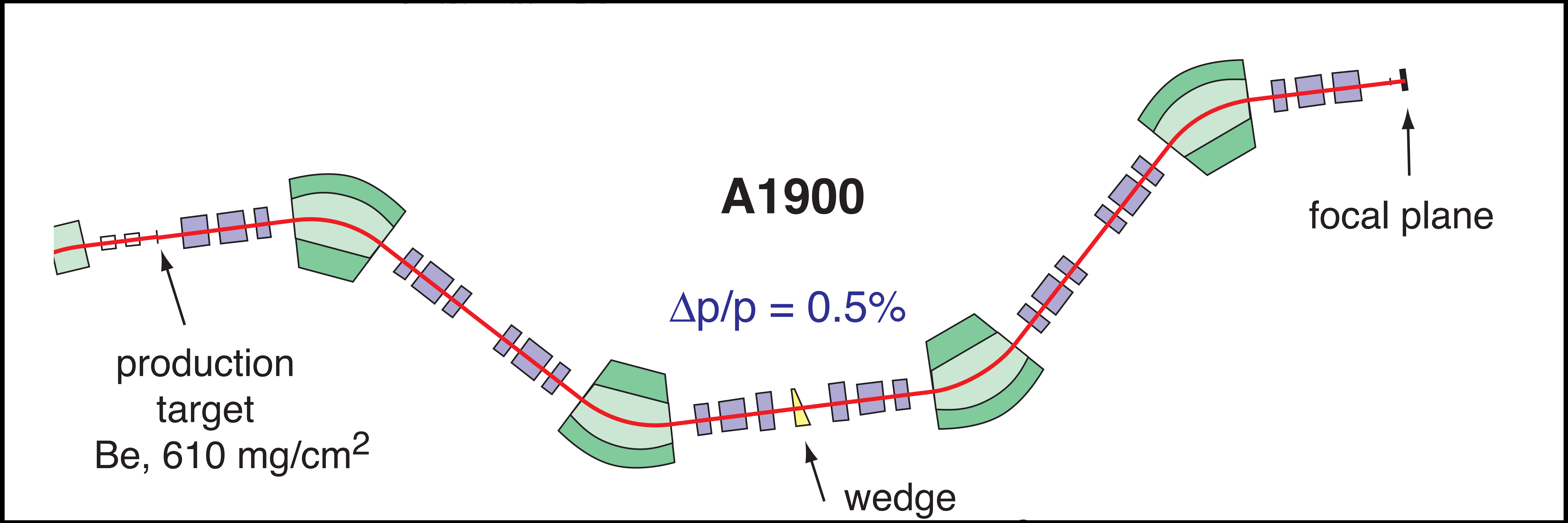


EXAMPLE 2: A1900 @ NSCL



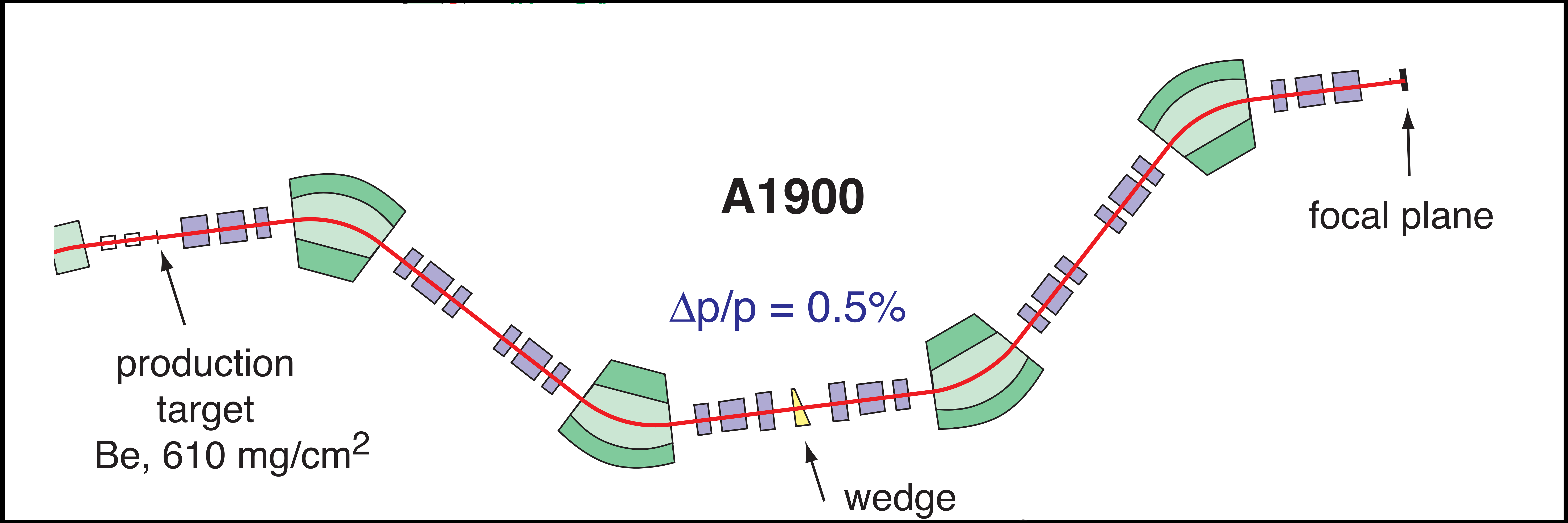
EXAMPLE 2: A1900 @ NSCL

- Second generation of fragment separator designed for radioactive beam production



EXAMPLE 2: A1900 @ NSCL

- Second generation of fragment separator designed for radioactive beam production
- 3 dispersive focal planes with dispersions 29 mm/% (sides) and 59 mm/%



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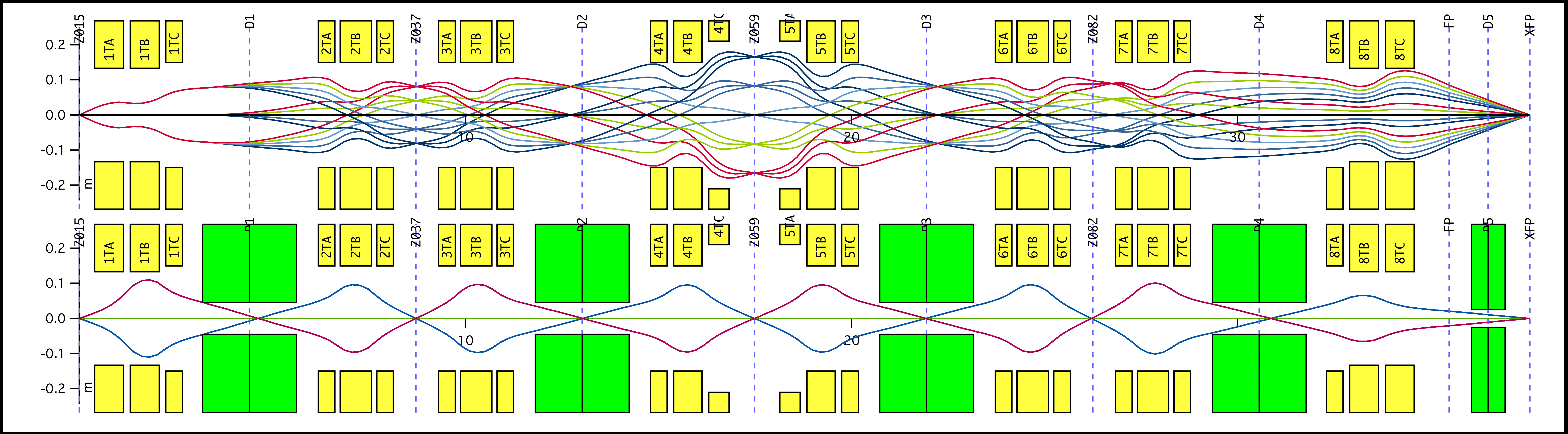
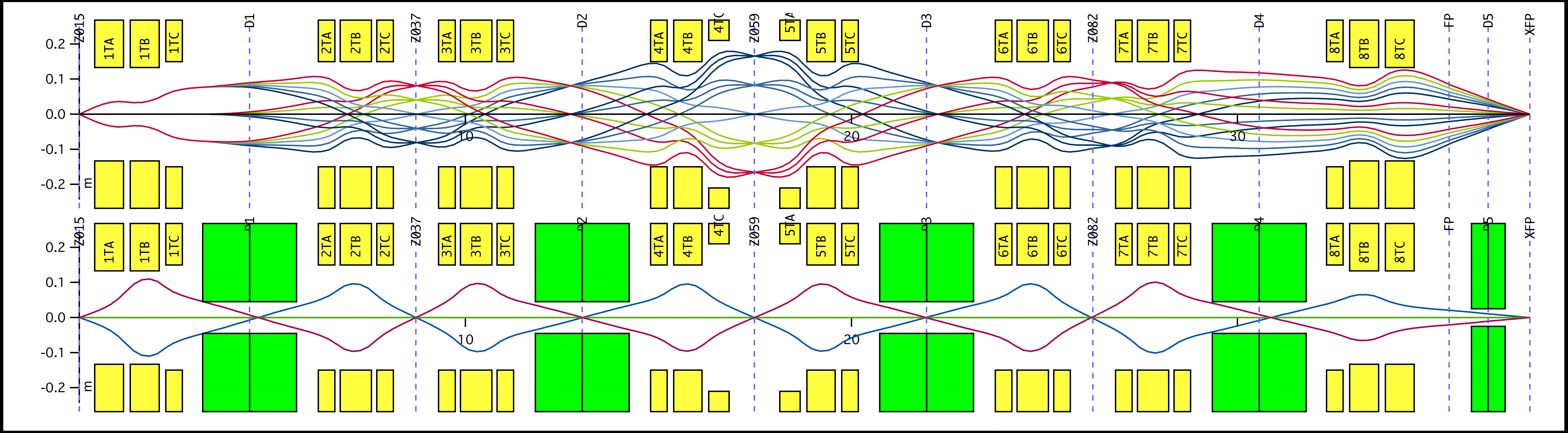


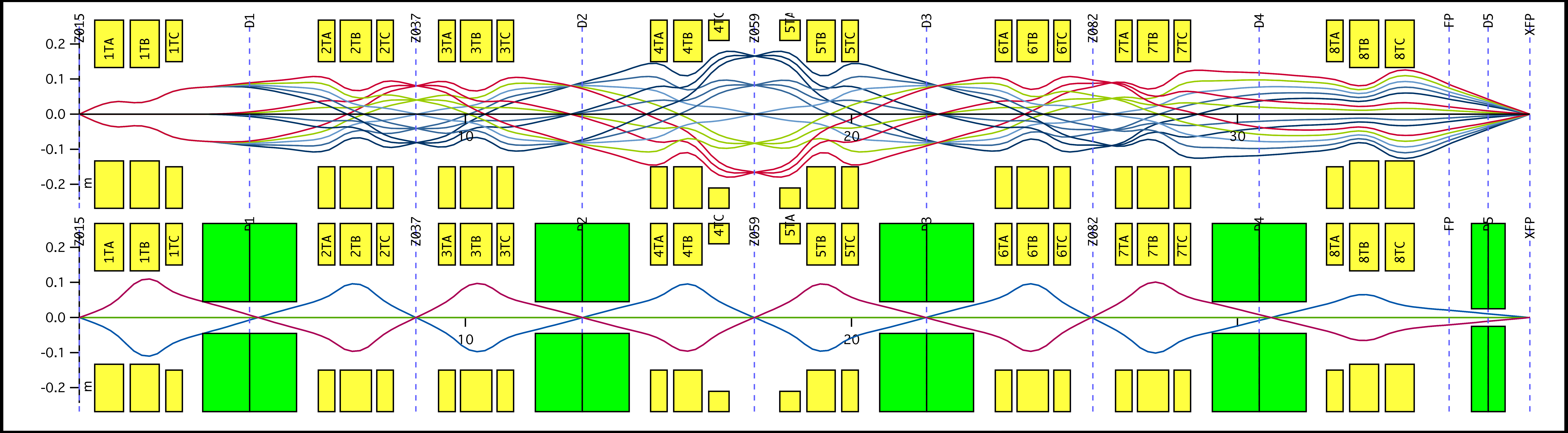
EXAMPLE 2: A1900 @ NSCL

- Second generation of fragment separator designed for radioactive beam production
- 3 dispersive focal planes with dispersions 29 mm/% (sides) and 59 mm/%
- Optics configuration maximize vertical acceptance with waists in the middle of dipole magnets



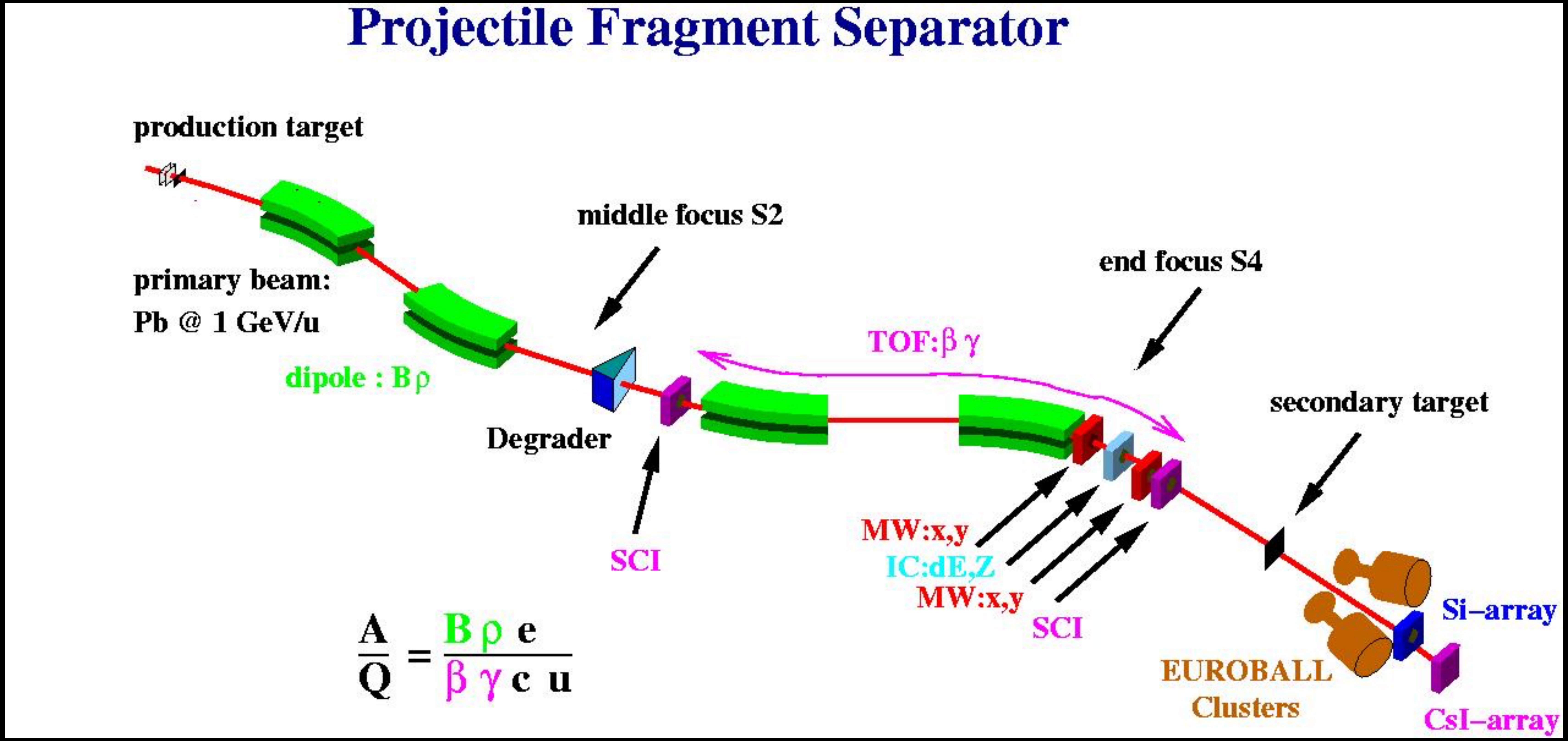
EXAMPLE 2: A1900 @ NSCL

- Second generation of fragment separator designed for radioactive beam production
- 3 dispersive focal planes with dispersions 29 mm/% (sides) and 59 mm/%
- Optics configuration maximize vertical acceptance with waists in the middle of dipole magnets
- Middle focal plane used for wedge selection (see later)



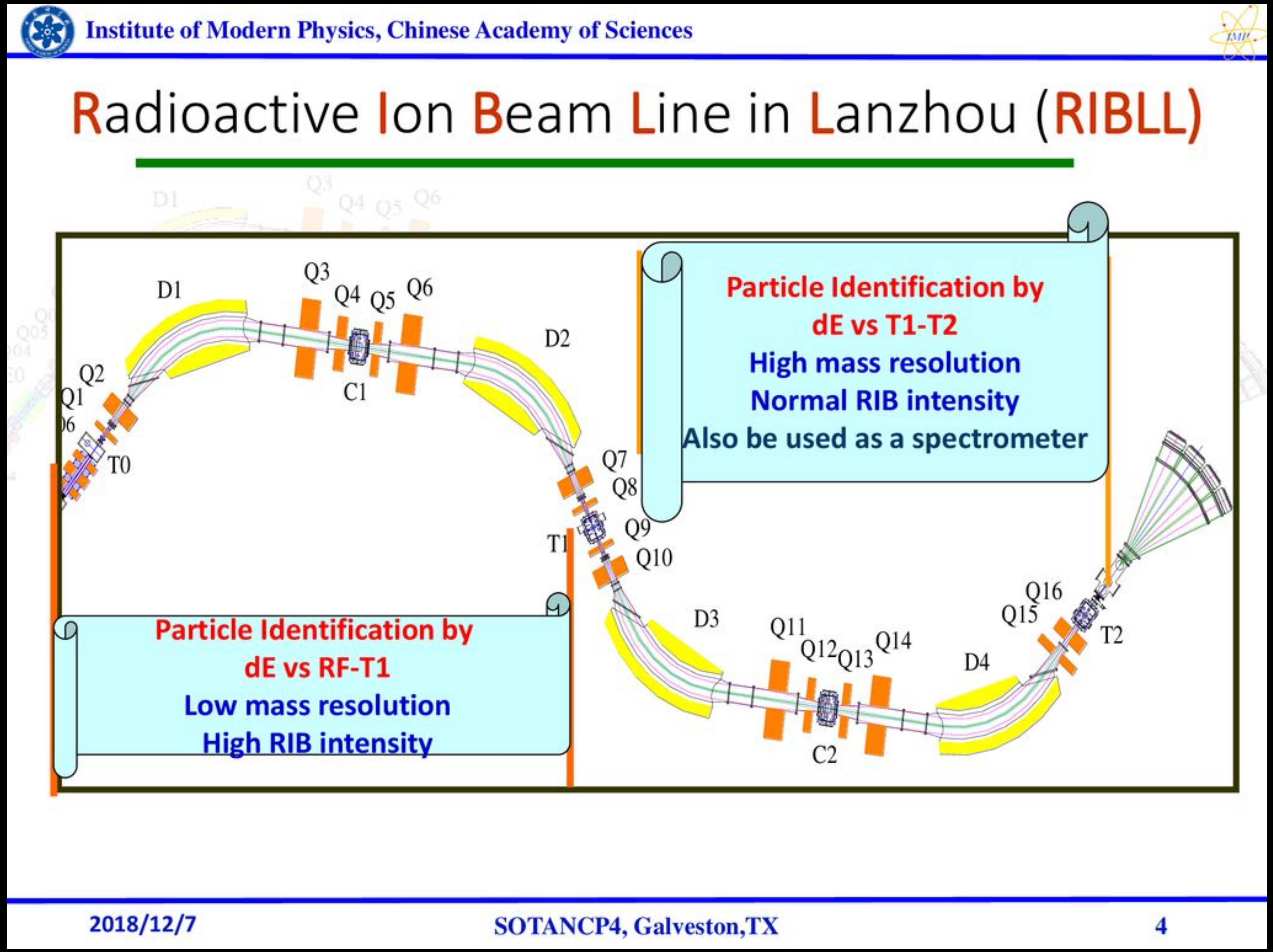
EXAMPLE 3: FRS @ GSI

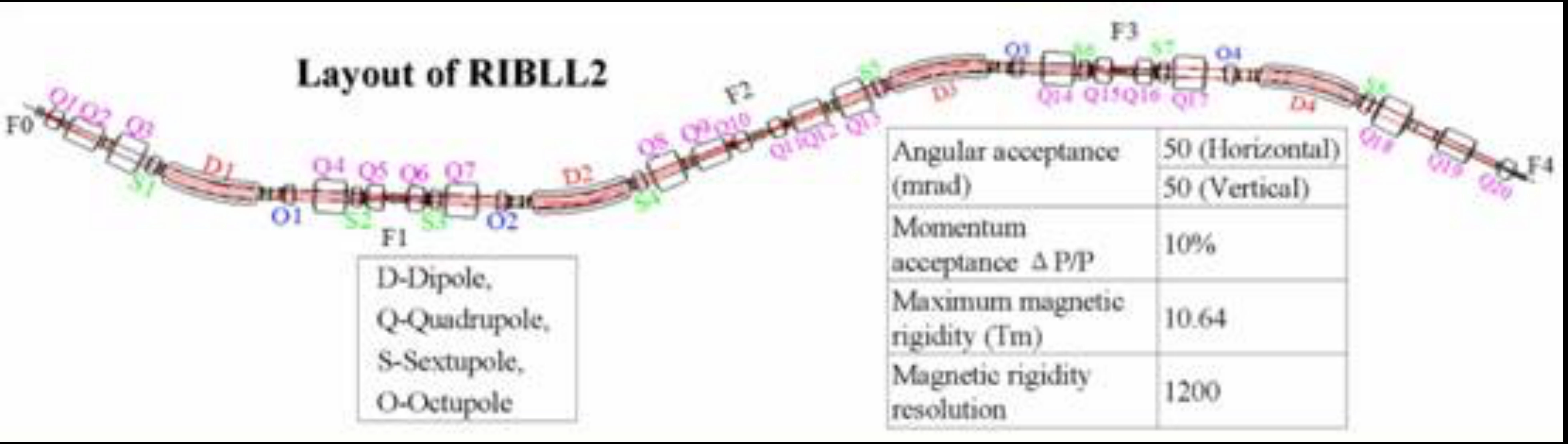
- Very similar design to A1900
- Adapted to higher energies (~1 GeV/u)



EXAMPLE 4: RIBLLS @ IMP

- Lanzhou has two fragment separators!
- RIBLLs are s-shaped: no lateral dispersion at mid-point when in achromatic mode
- Second half used for tagging or as spectrometer

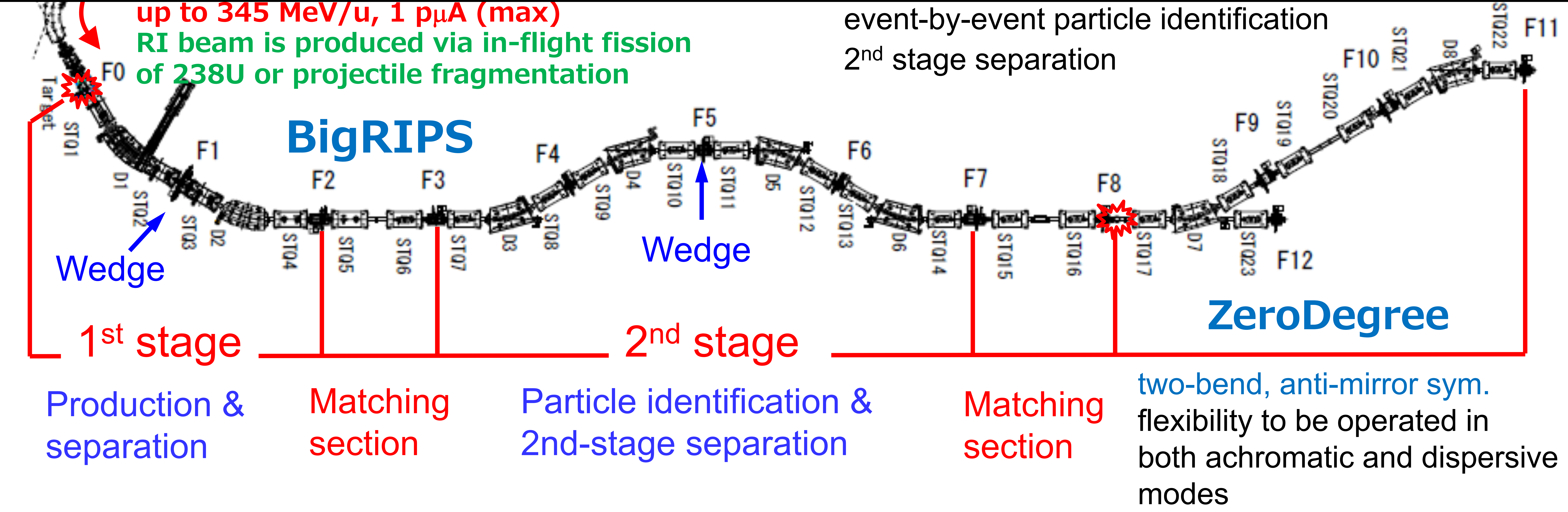




Angular acceptance (mrad)	50 (Horizontal)
	50 (Vertical)
Momentum acceptance Δ P/P	10%
Maximum magnetic rigidity (Tm)	10.64
Magnetic rigidity resolution	1200

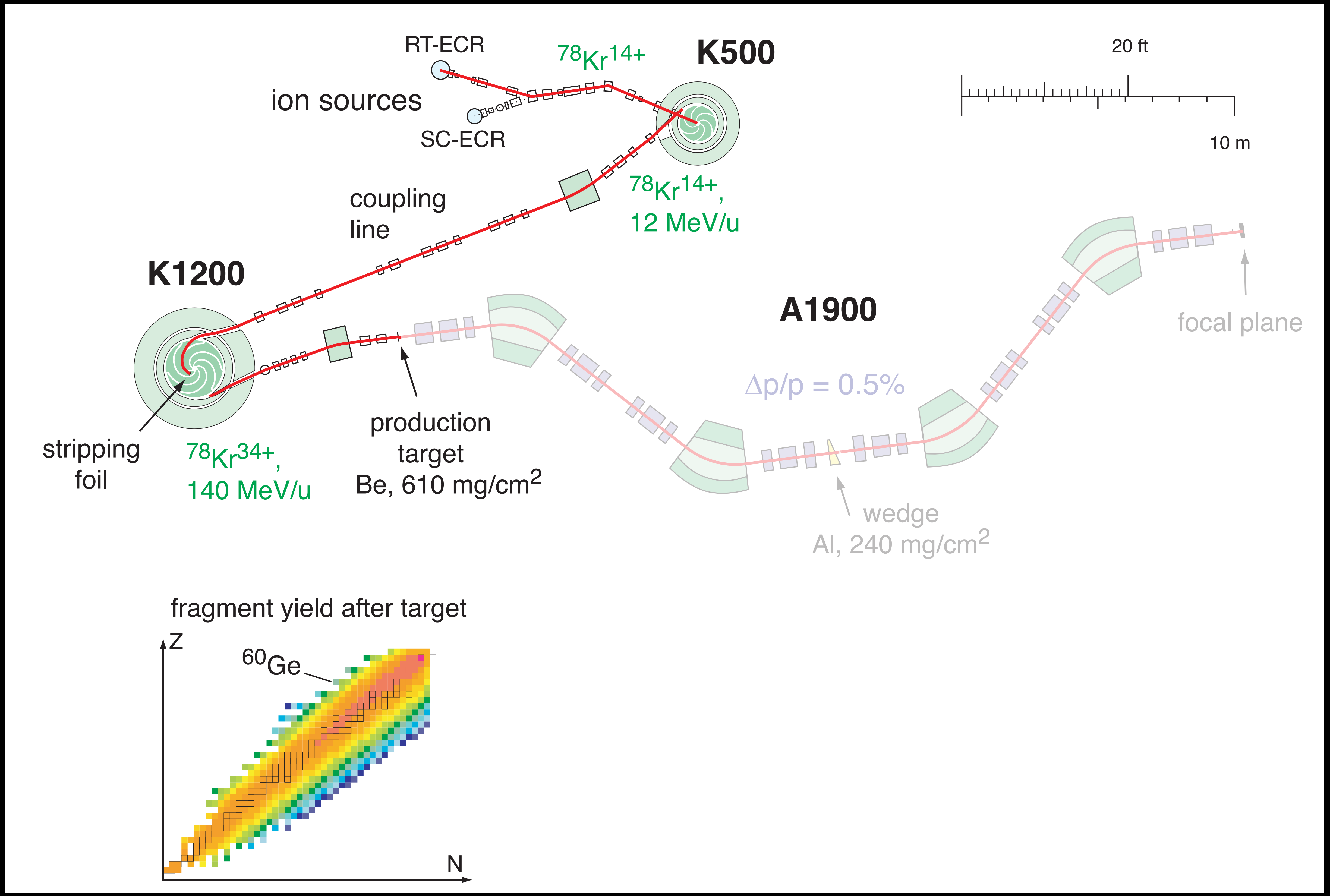
EXAMPLE 5: BIGRIPS+ZDS @ RIBF

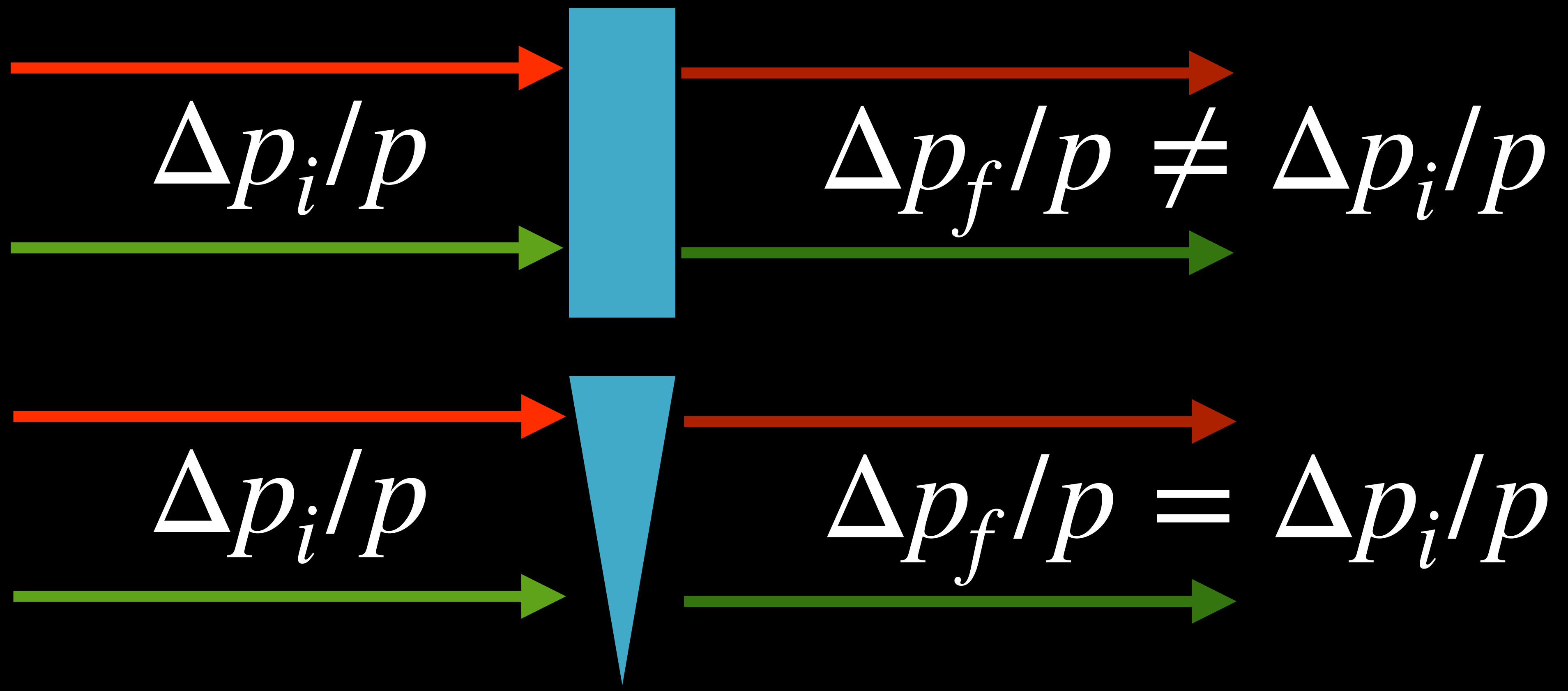
- Third generation fragment separator
- Second tagging stage used to identify (tag) isotopes contained in beam
- ZDS downstream of reaction target located at F8



WEDGE SELECTION

- First stage selection: magnetic rigidity $B\rho$
- Second stage selection: energy loss in material
- Energy loss varies as function of energy: uniform thickness changes dispersion!
- Angle of wedge-shaped material calculated to preserve dispersion therefore achromatism





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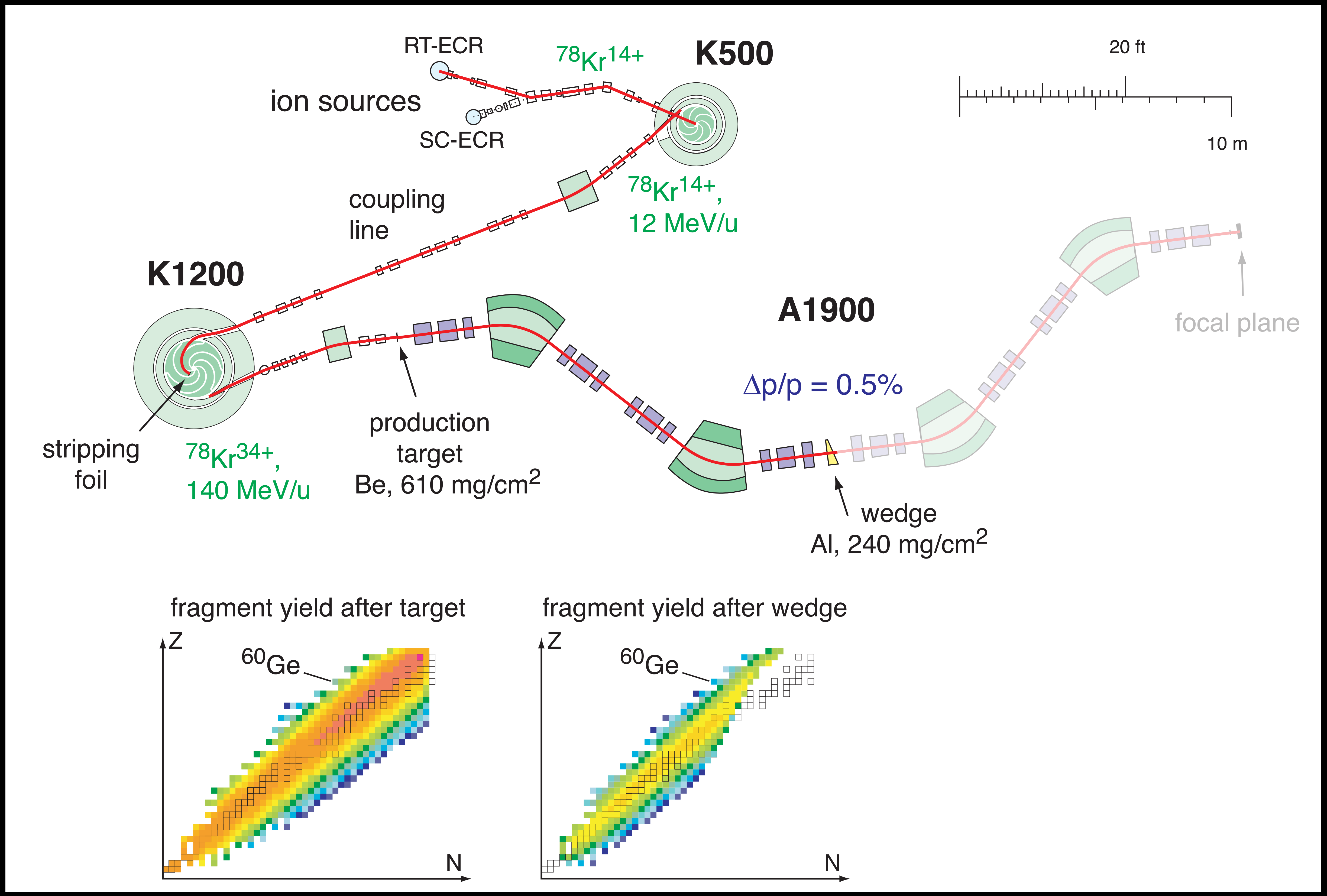

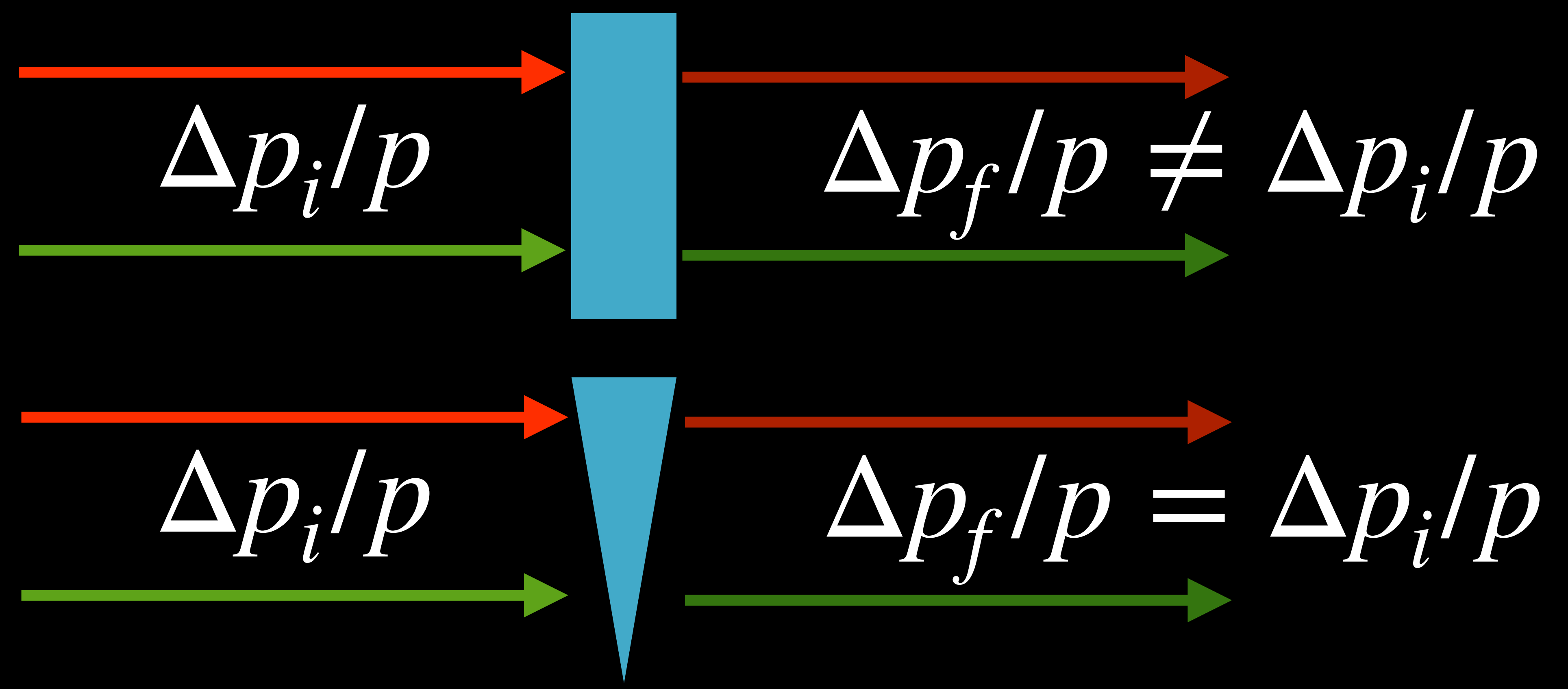

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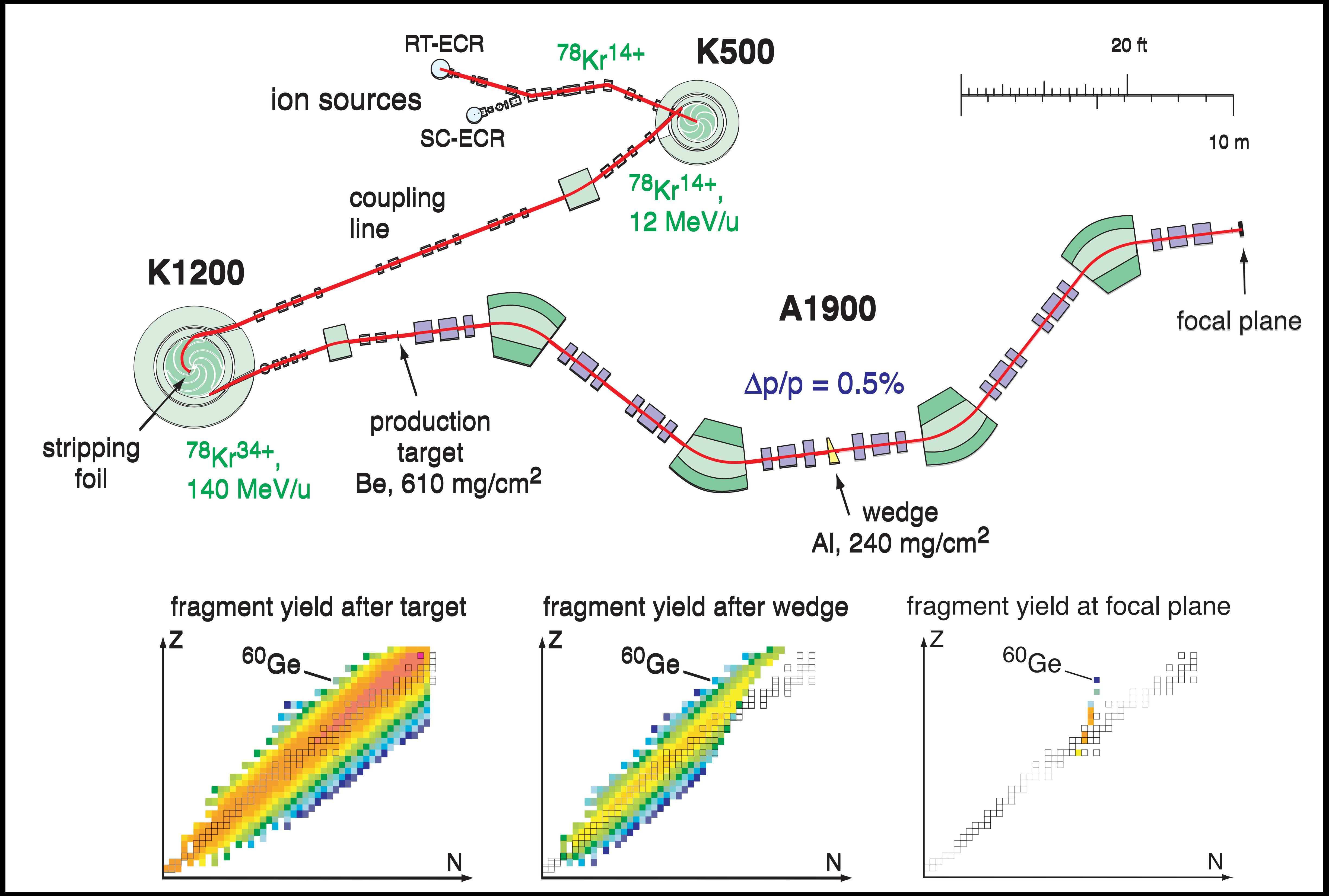
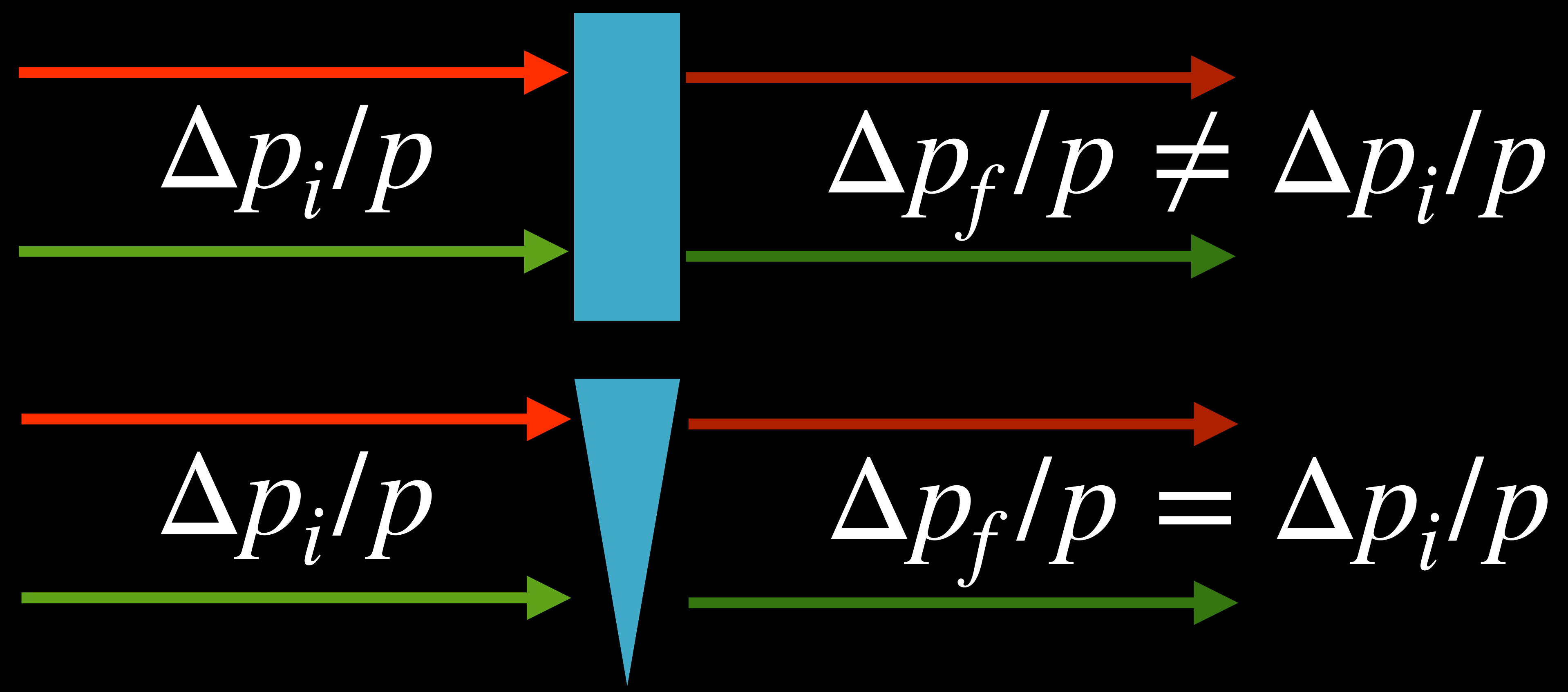
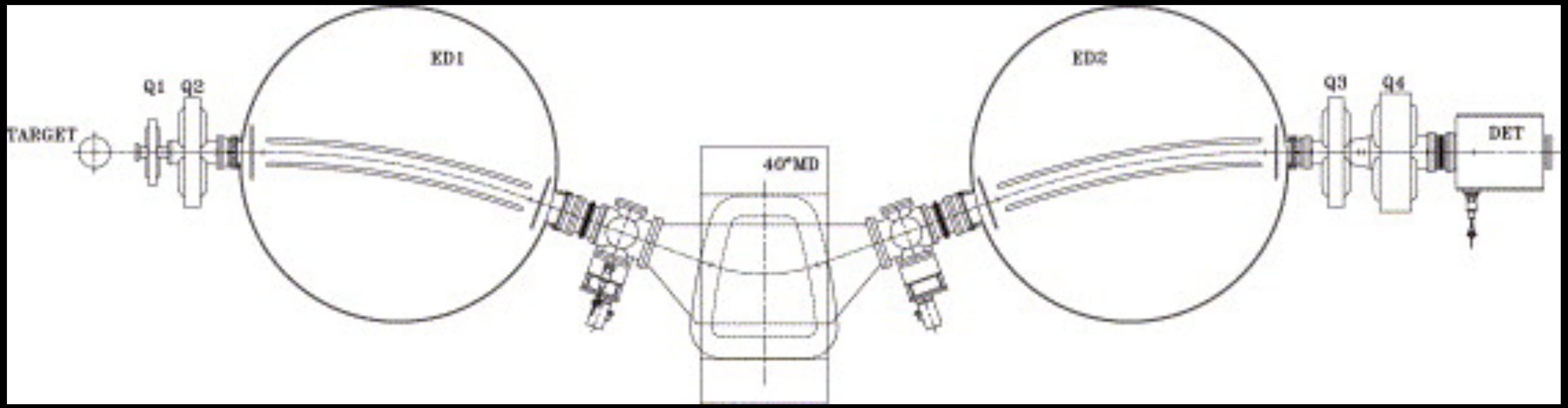


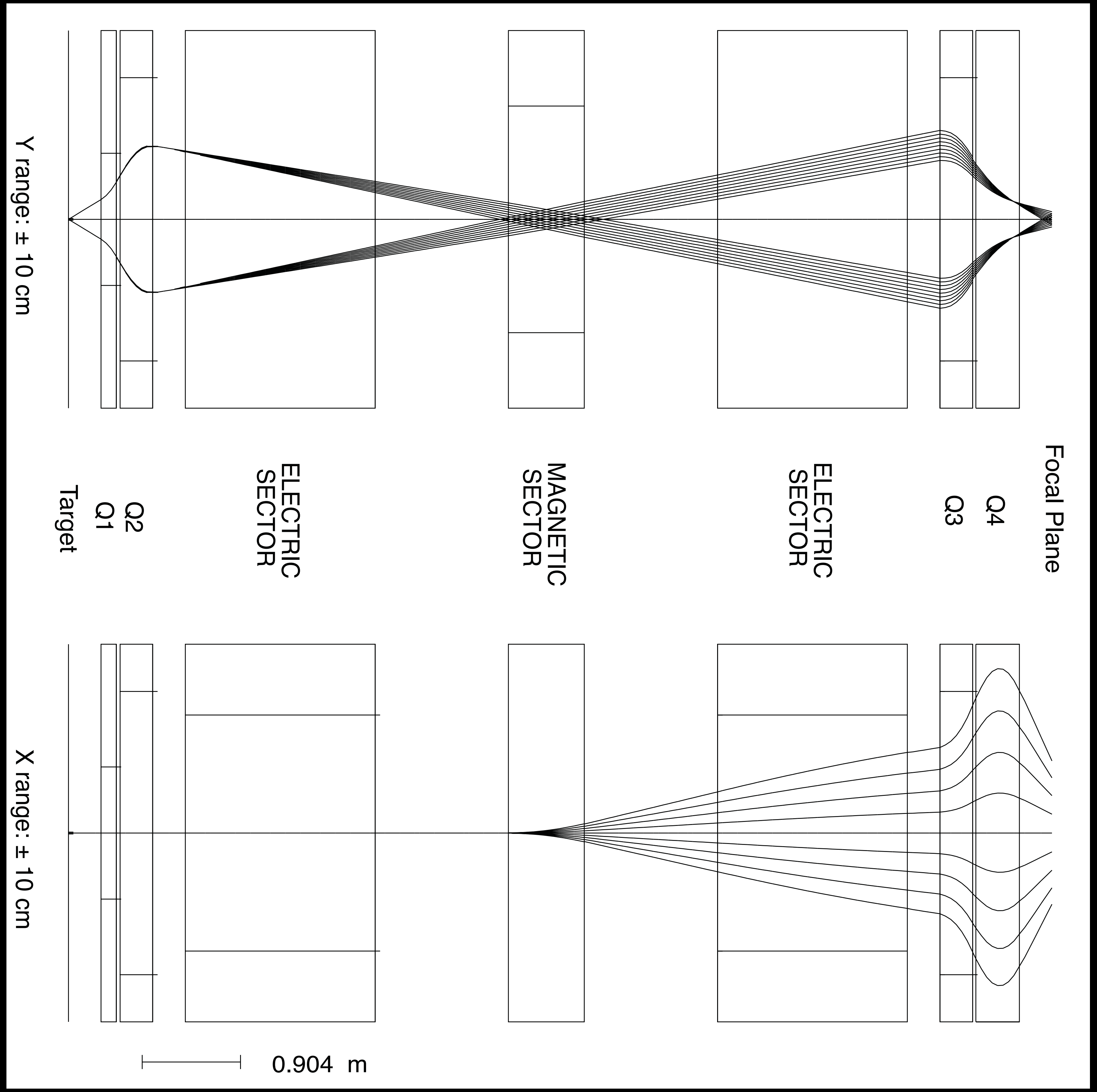


MASS SEPARATORS

- Examples: FMA (ANL) and EMMA (TRIUMF)
  - Use of both electric and magnetic dipoles
  - Gives possibility to achieve M/Q focussing and separation (hence the name "Mass separator")
  - Limited to energies around 10 MeV/u due to limits on electric field
  - Resolving power: 480 for 1mm beam spot, 10mm/% dispersion

EMMA separator @ TRIUMF

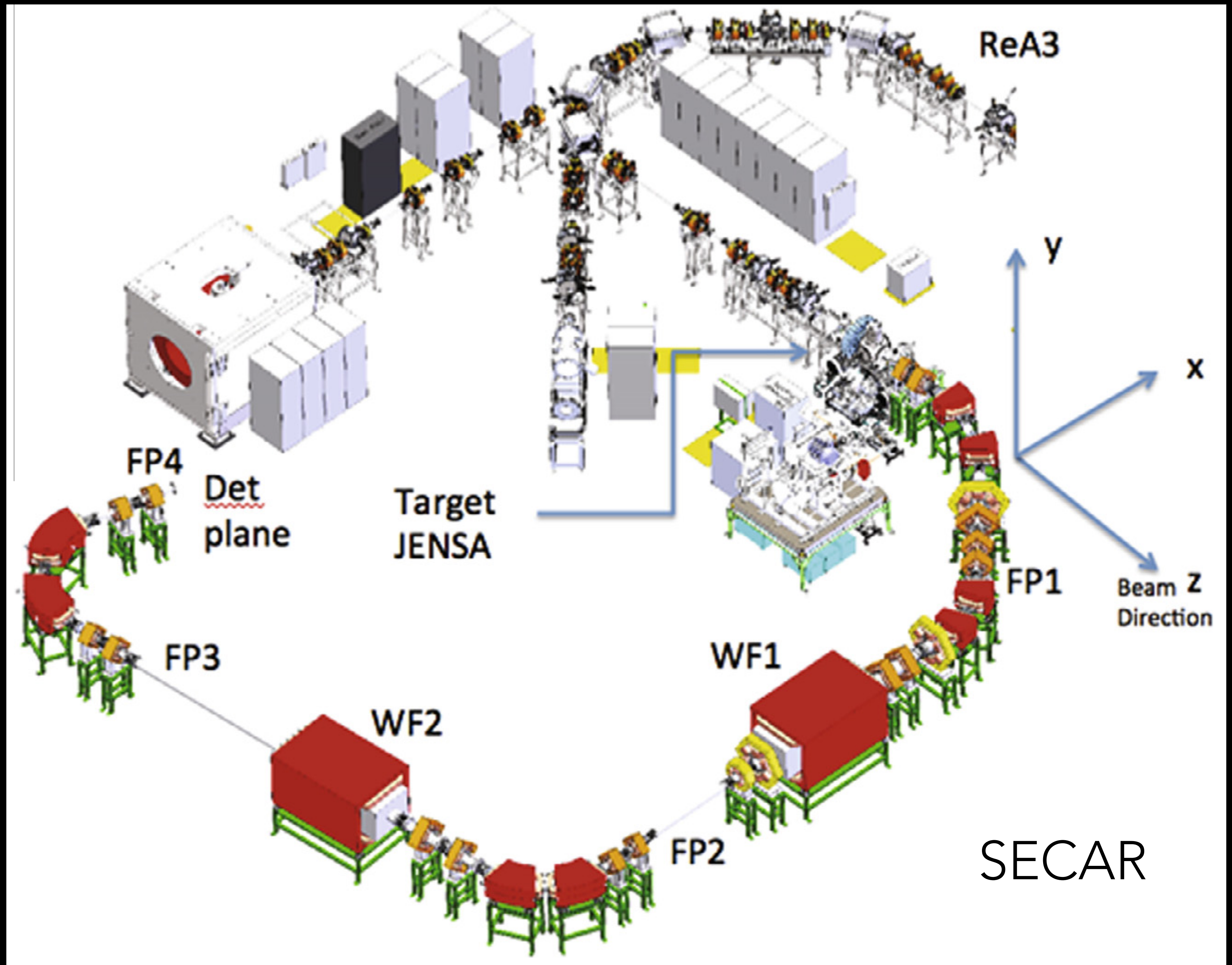




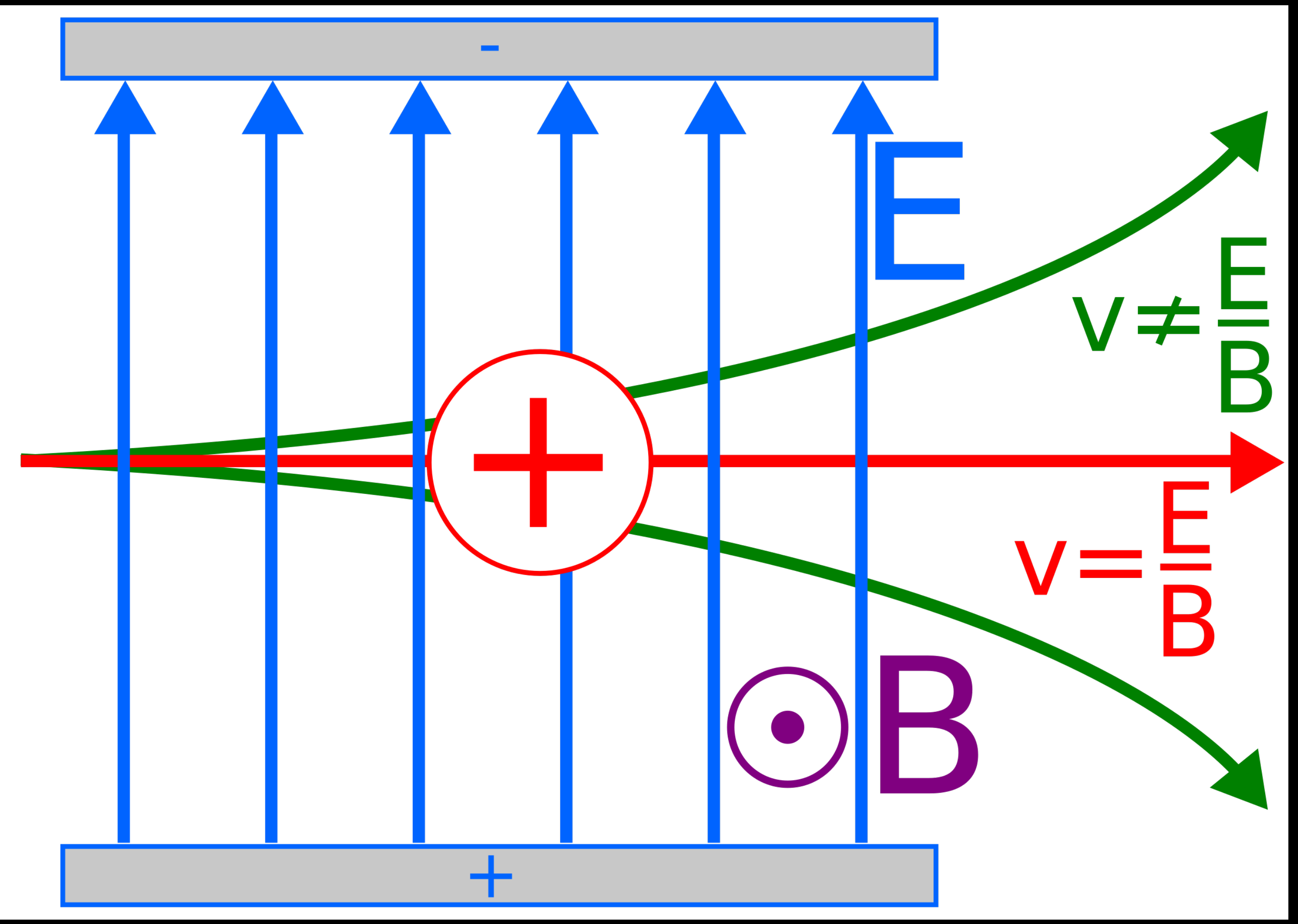
B. Davids and C. N. Davids, NIM A 544, 565 (2005)

RECOIL SEPARATORS

- Examples: DRAGON (TRIUMF) and SECAR (NSCL/FRIB)
  - Achieve very high resolving power to select recoils close to beam velocity, to measure very small cross sections
  - SECAR beam rejection is 10^{-13}
  - Aberrations need to be corrected via sextupoles and octupoles
  - Use of Wien velocity filters ($\vec{E}$ perpendicular to $\vec{B}$)
  - Exercise 6: show that the force is proportional to the velocity

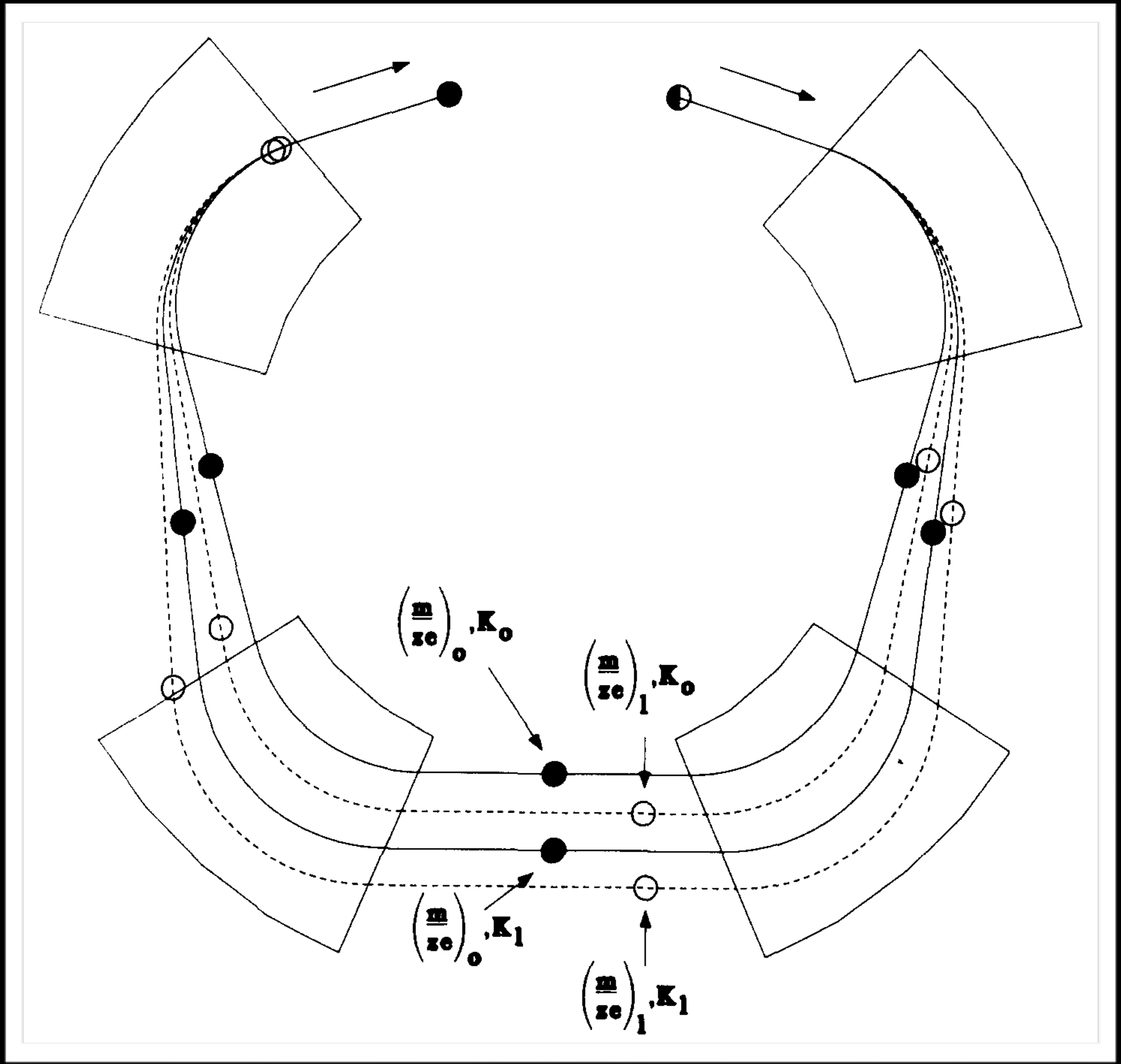


G.P.A. Berg et al., NIM B 376, 165 (2016)



ISOCHRONOUS SYSTEMS

- Isochronous = independent of time
  - Systems where the time-of-flight is independent of momentum and angles
  - Length of trajectories compensates for the differences in velocities
  - Isochronous systems are also usually achromatic
- Principal uses
  - Accelerator systems where beam is pulsed: conserve bunch structure
  - Mass measurements systems



TOFI mass spectrometer

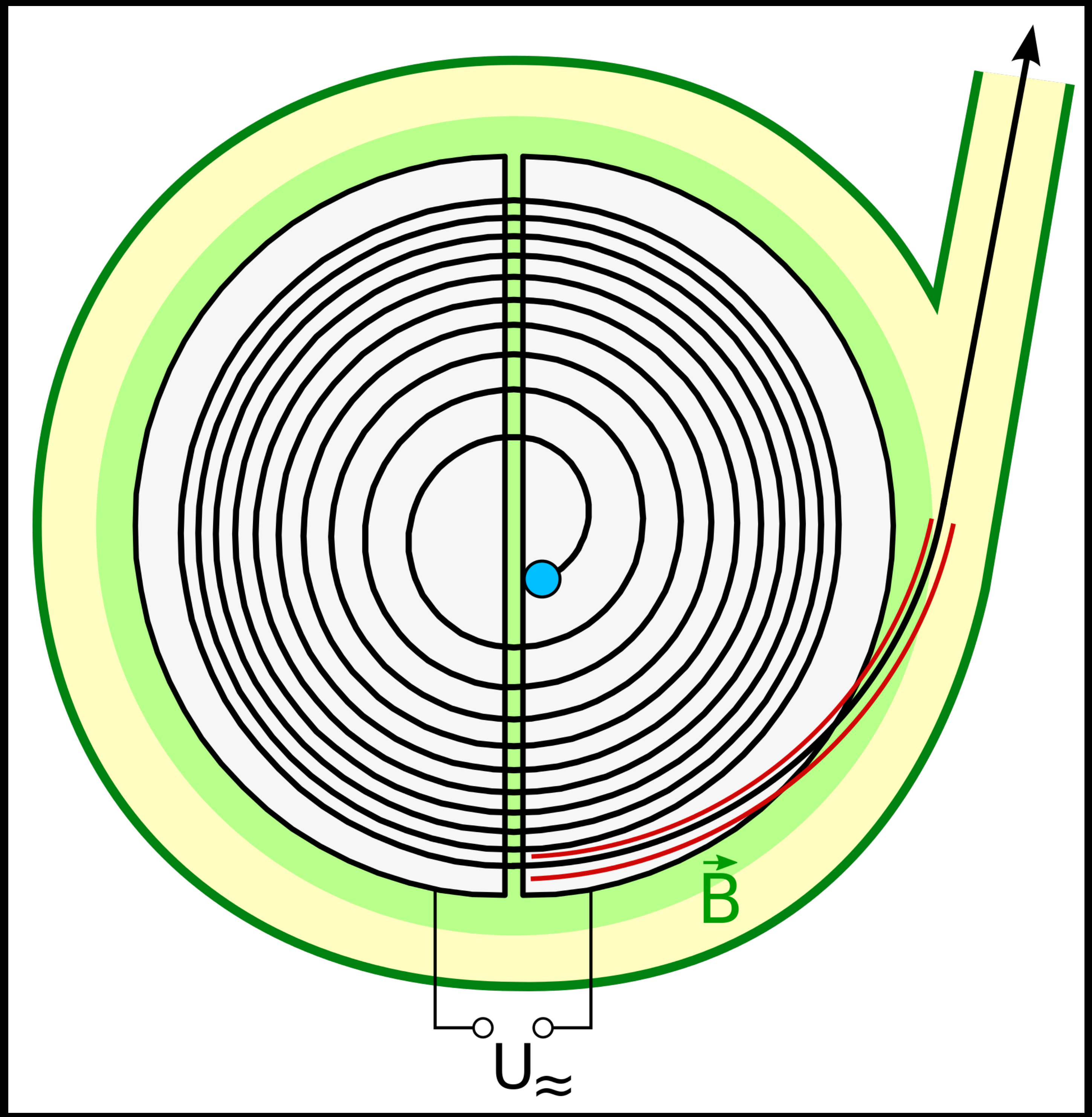
EXAMPLE1: CYCLOTRON

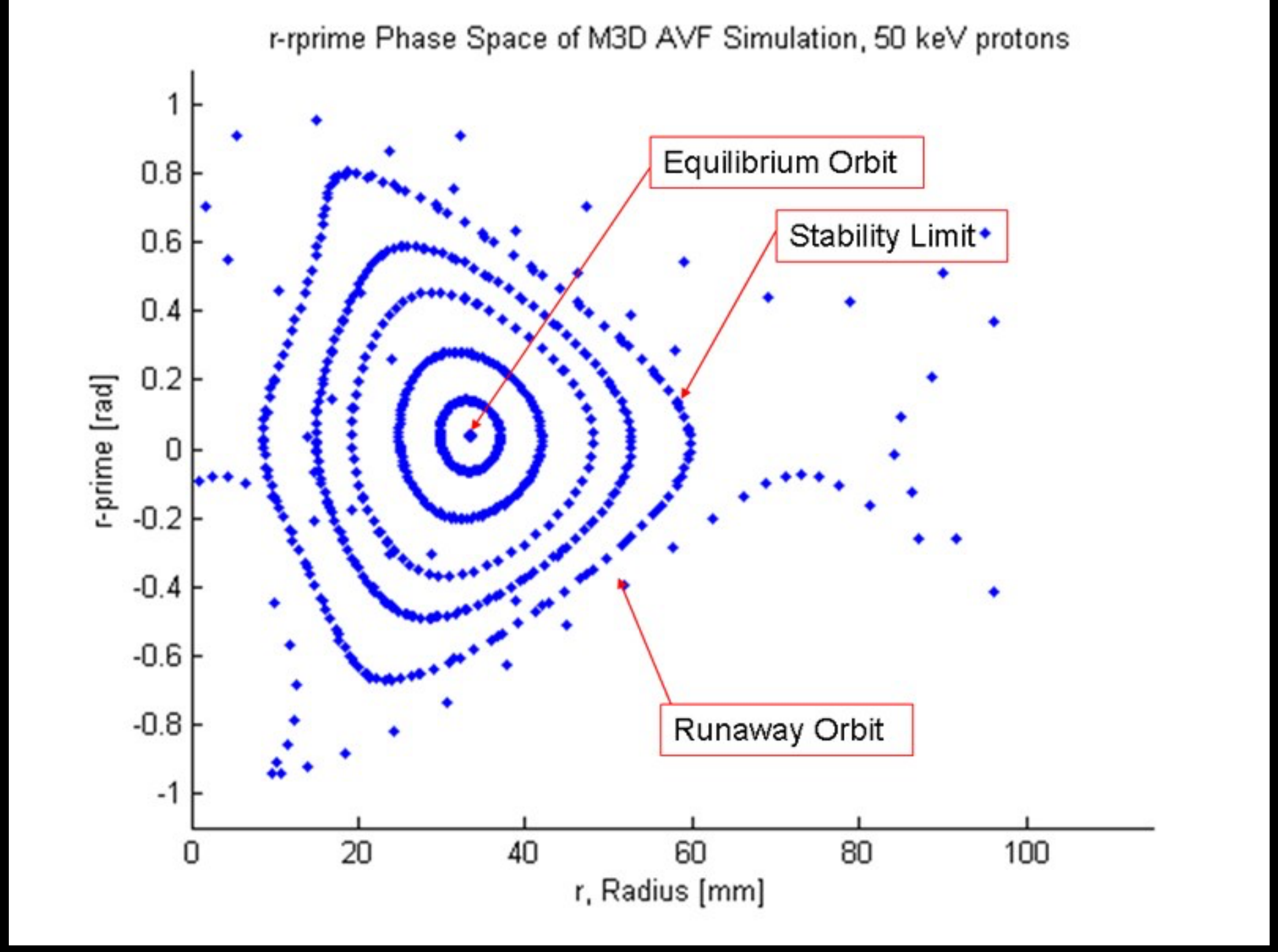
- Recall dipole first order Transfer matrix

- $$T_D = \begin{bmatrix} \cos\alpha & \rho\sin\alpha & 0 & 0 & 0 & \rho(1-\cos\alpha) \\ -\rho^{-1}\sin\alpha & \cos\alpha & 0 & 0 & 0 & \sin\alpha \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \sin\alpha & \rho(1-\cos\alpha) & 0 & 0 & 1 & \boxed{-L+\rho\sin\alpha} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

- For $\alpha = 360°$ we get dispersions $T_{16} = T_{26} = 0$ (achromatic) as well as $T_{56} = -L$ which is condition for isochronous system

- More sophisticated geometries require complex orbit calculations



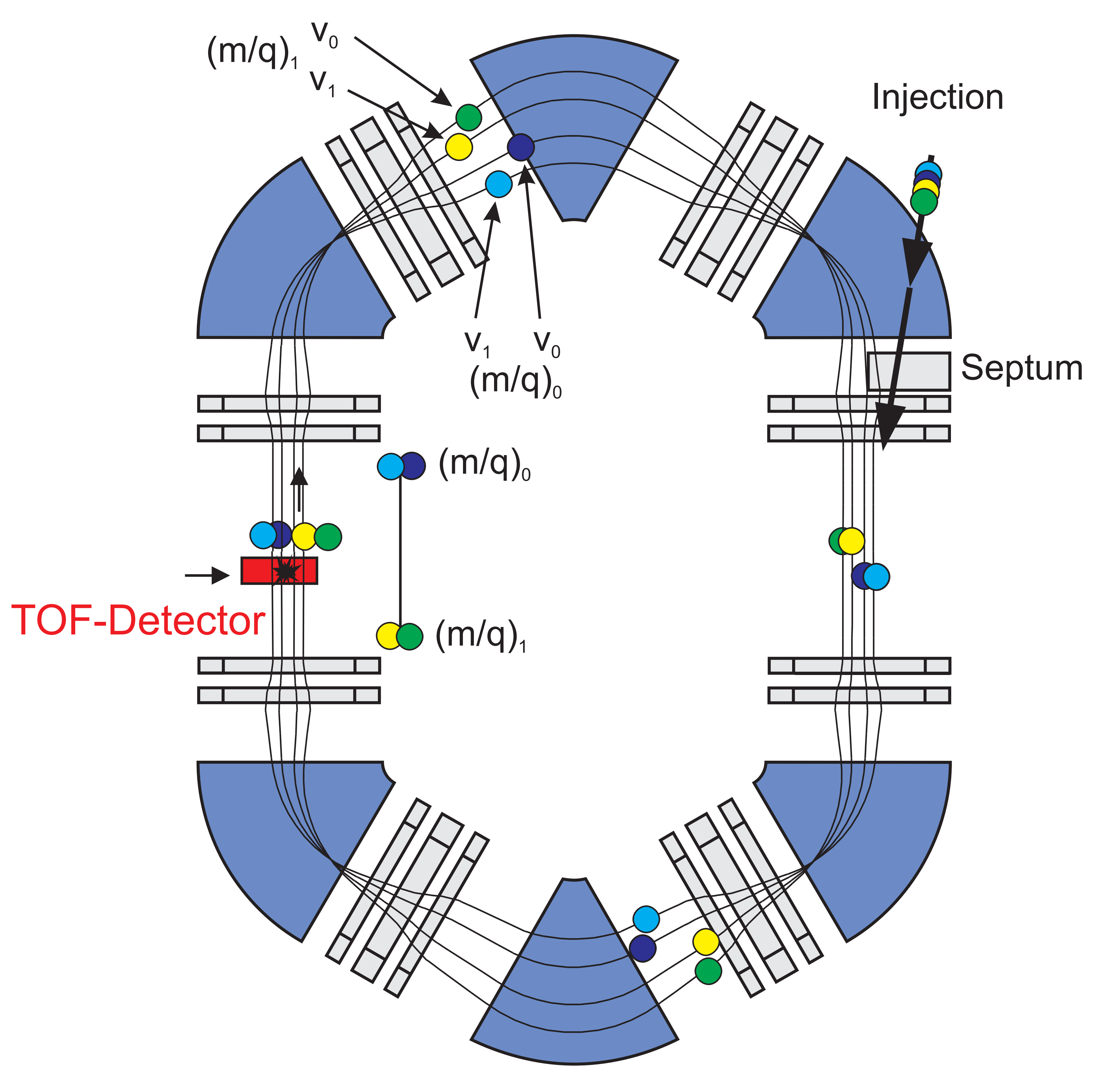


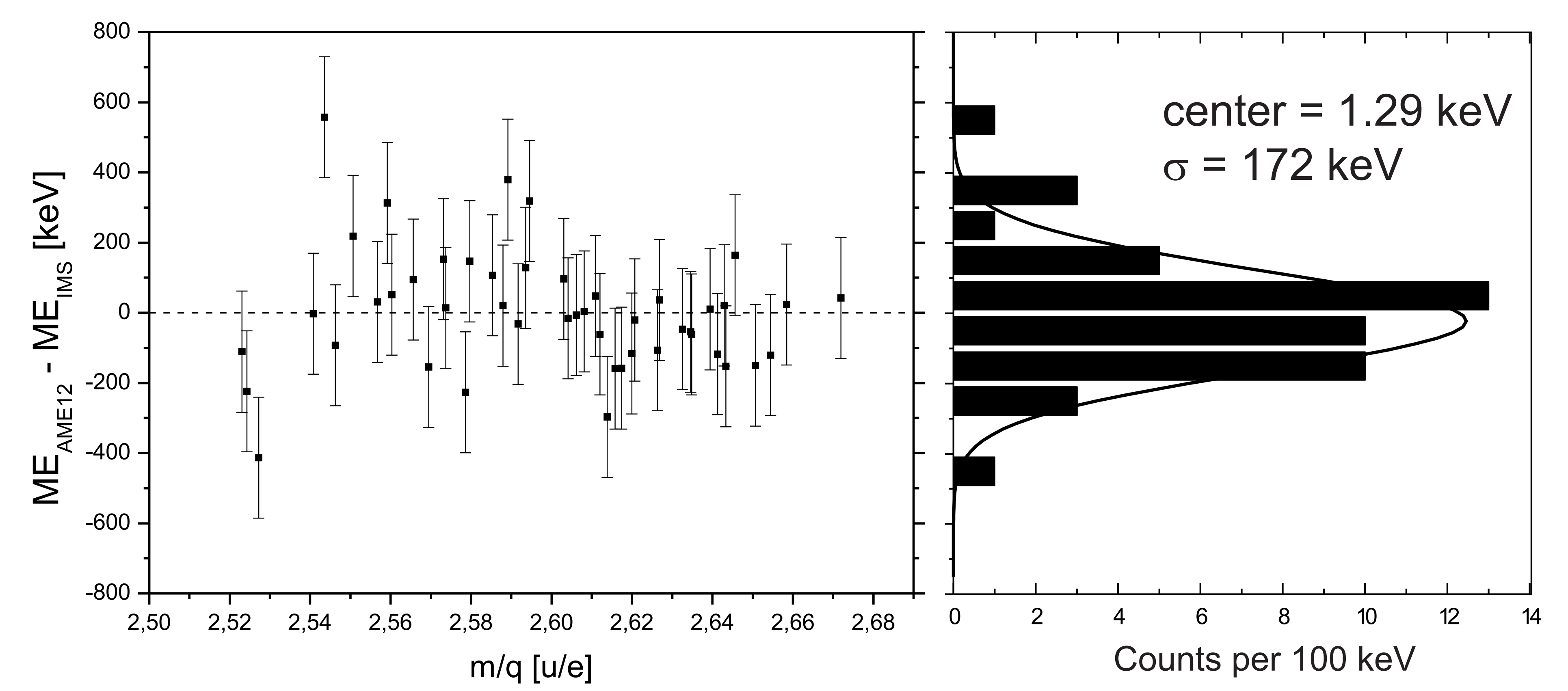
Koeth cyclotron

EXAMPLE2: STORAGE RINGS

- Can be tuned in isochronous mode
- Used to make high precision mass measurements as well as other studies (lifetime of isomers)
- Example shown on ESR @ GSI
- Resolution achieved: 172 keV (sigma)
- IMP Lanzhou has similar ring

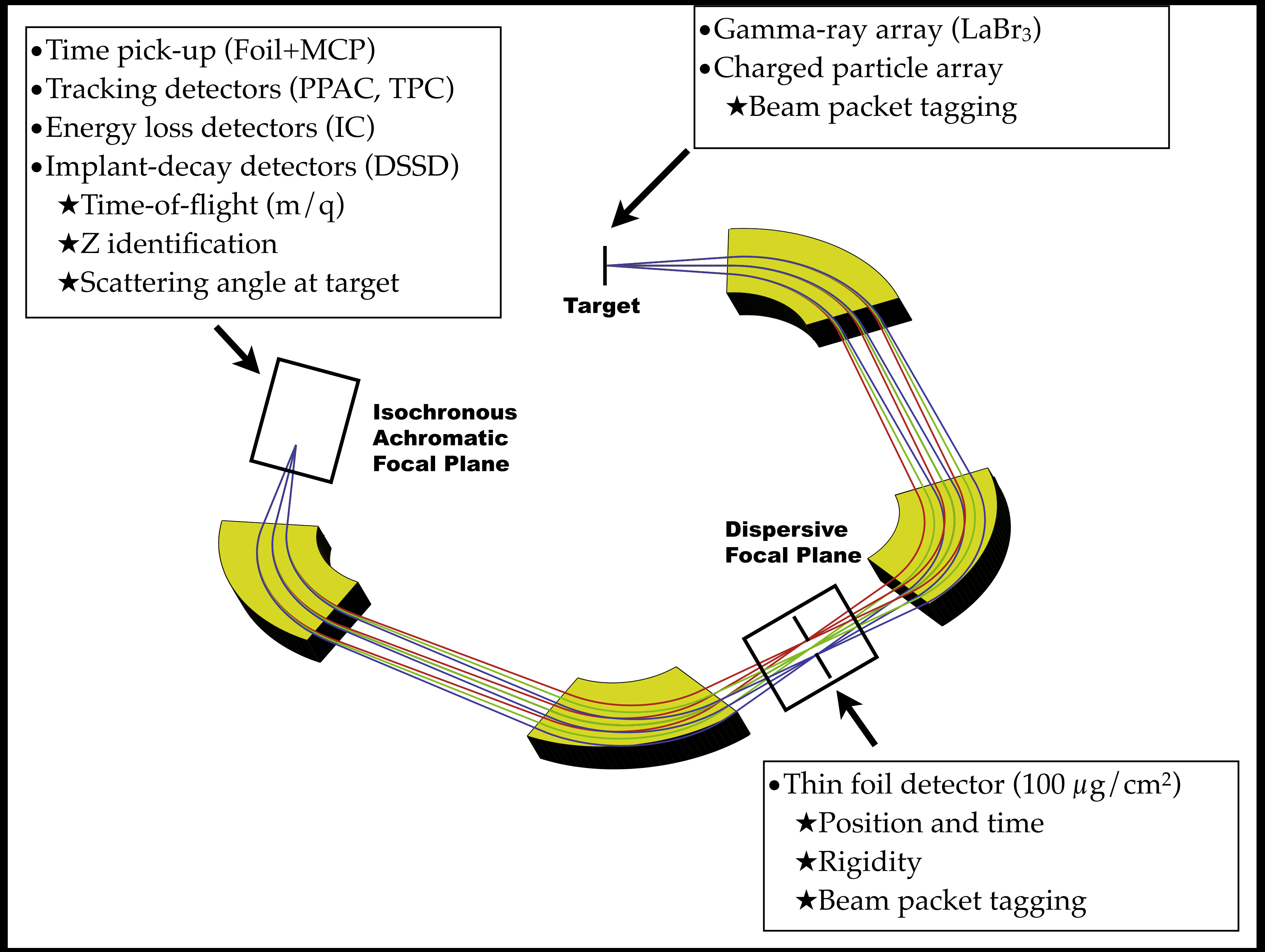
Marcel Diwisch's thesis

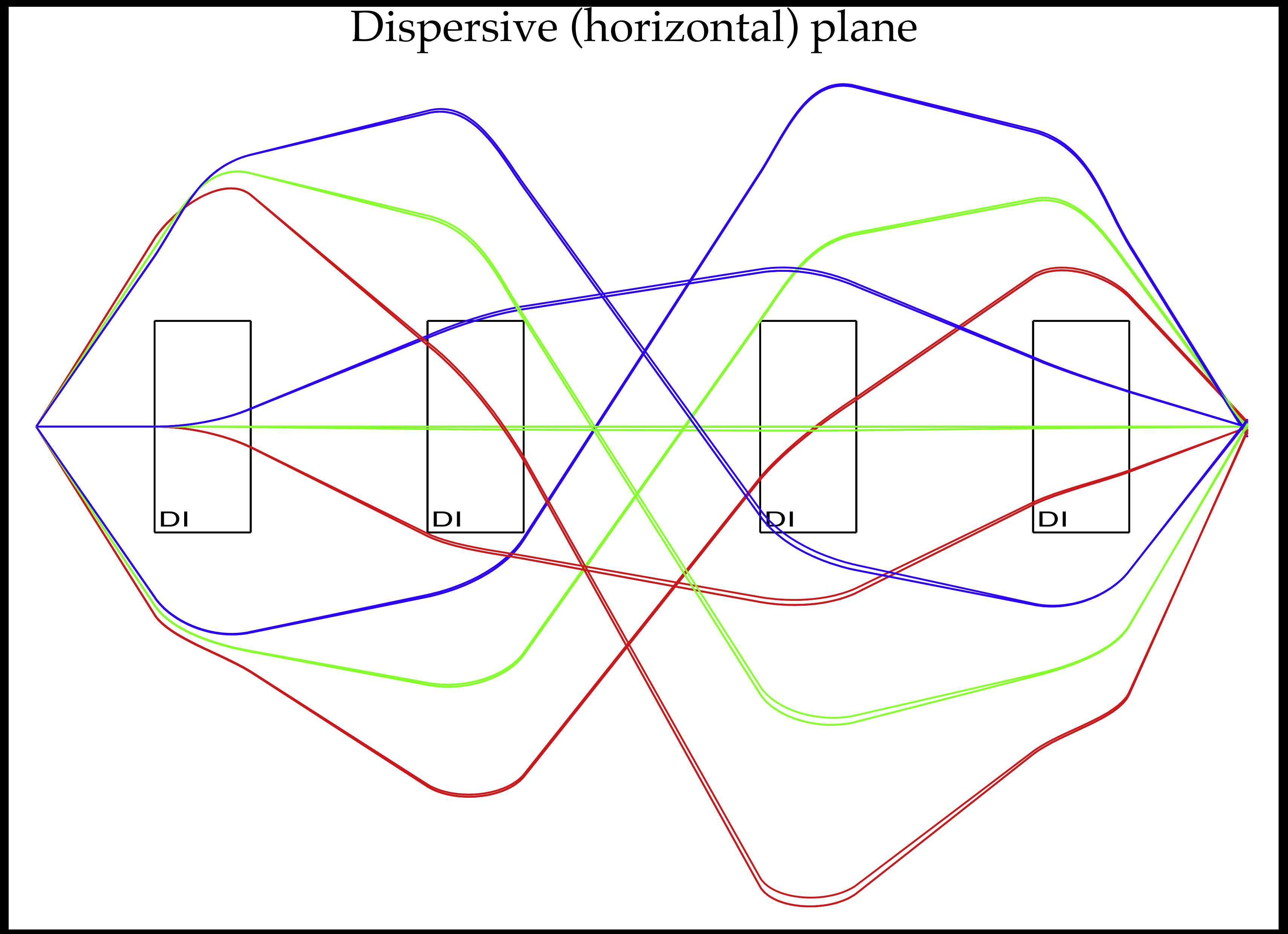




EXAMPLE: ISLA

- Isochronous Spectrometer with Large Acceptances
- Concept based on the TOFI spectrometer
- Acceptances: 20% momentum and 64 msr solid angle
- Focussing mainly done from edge angle on dipoles
- Symmetric configuration cancels most of the large aberrations





D. Bazin and W. Mittig, NIM B 317, 319 (2013)