Nuclear Reactions

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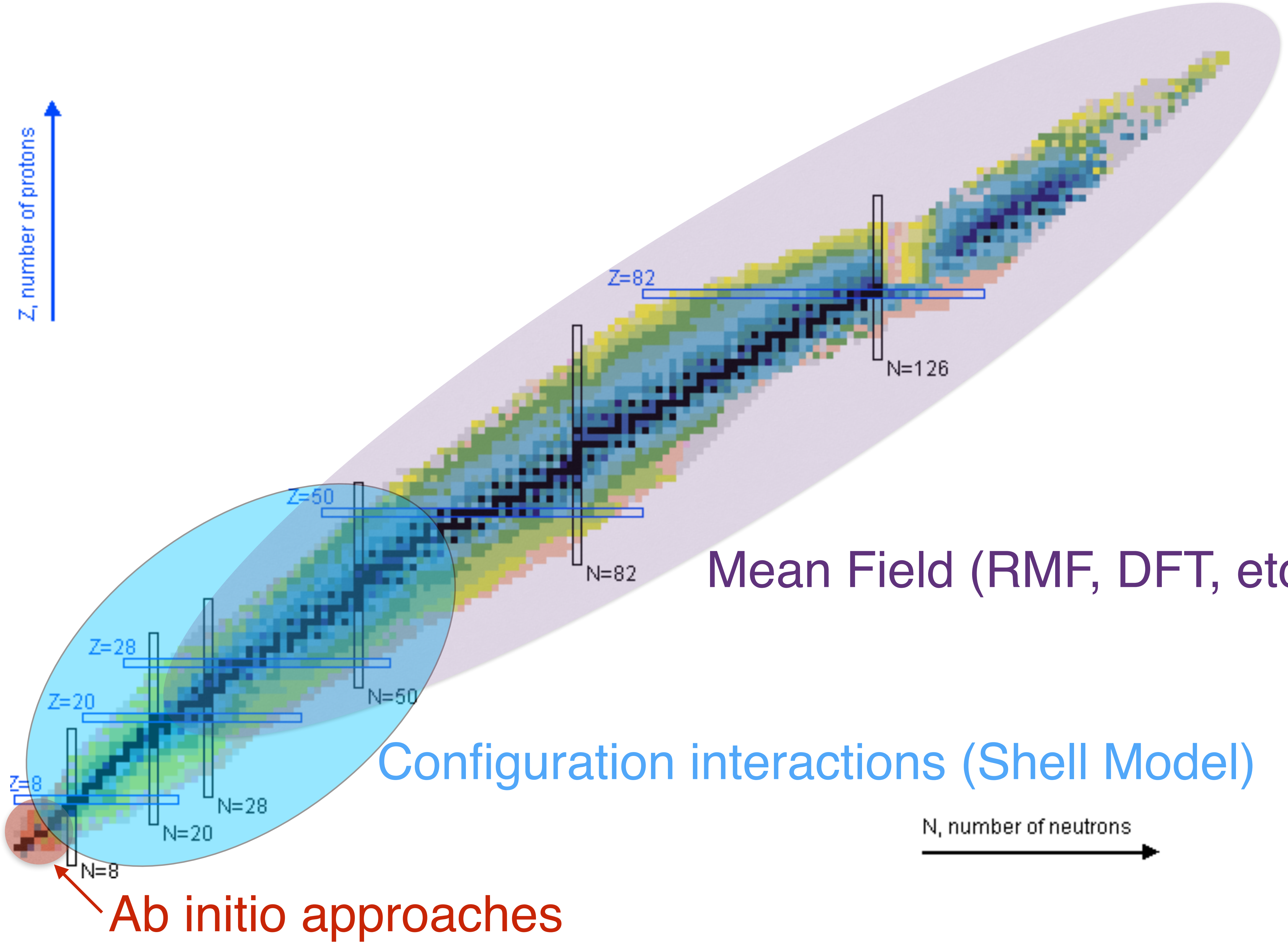


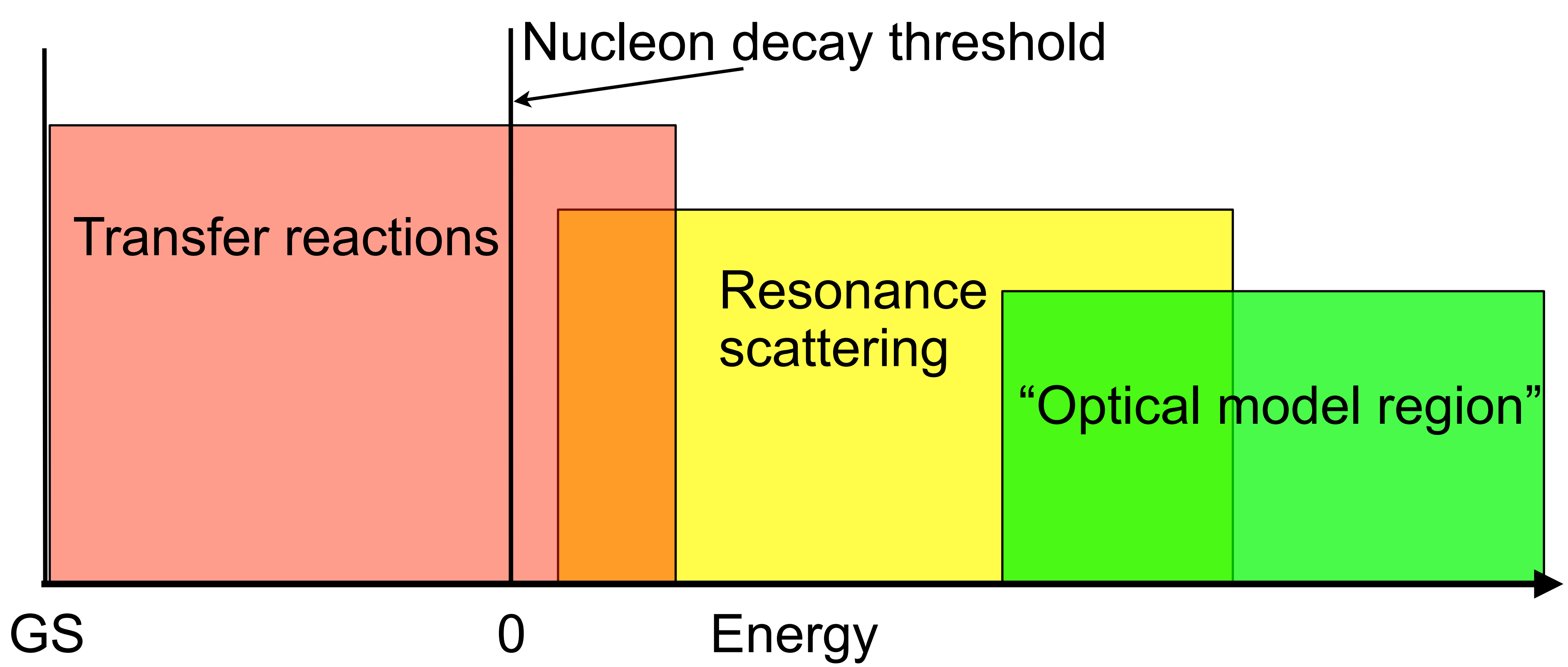
Z, number of protons
Z=82
N=126
Z=50
N=82
Mean Field (RMF, DFT, etc)
Z=28
Z=20
N=50
Z=8
N=28
N=20
N=8
Configuration interactions (Shell Model)
N, number of neutrons
Ab initio approaches

Ab initio approaches

- Structure of light nuclei can now be studied *ab initio*
  - *No Core Shell Model*
  - *Green’s Function Monte Carlo*
  - *Coupled Clusters*
  - *Effective Field Theory (EFT)*
  - *Lattice EFT*
- Various observables, such as excitation energies, ANCs, widths, **scattering phase shifts** can be calculated *ab initio* and verified experimentally
- Coupling to the continuum is a challenge

General outline

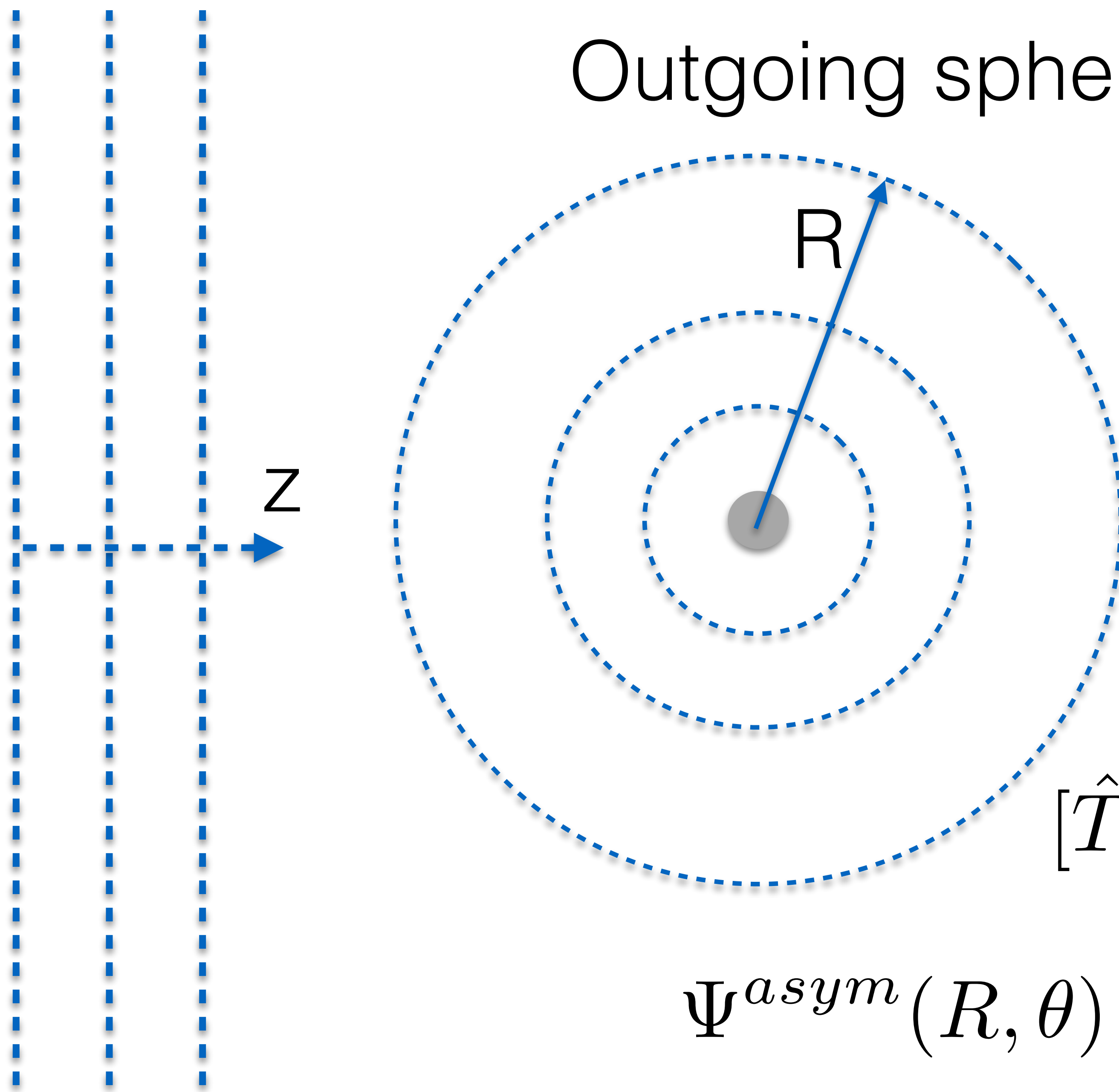
- Part 0. Introduction to scattering
- Part I. Elastic and scattering
- Part II. Transfer Reactions
- Part III. Resonance scattering



Part 0. Introduction to Scattering

Elements of scattering theory

Outgoing spherical wave ~ e^{ikR}/R

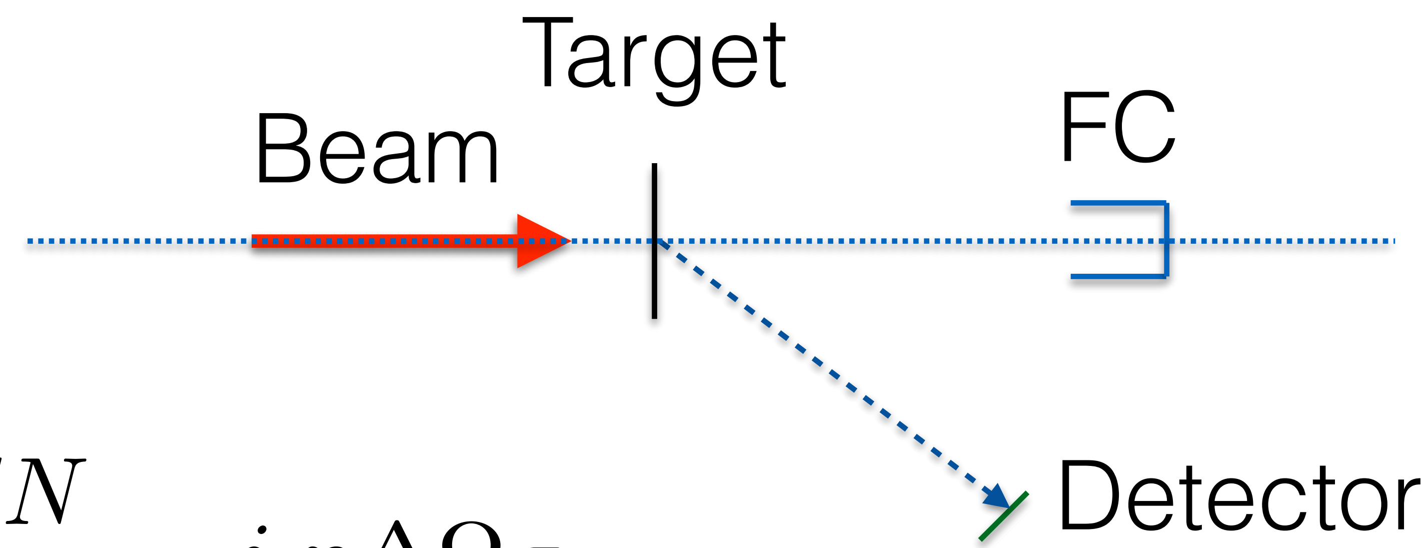


$$[\hat{T} + V - E]\Psi(R,\theta) = 0$$

$$\Psi^{asym}(R,\theta) = A(e^{ikz} + f(\theta)\frac{e^{ikR}}{R})$$

Incoming plane wave ~e^{ikz}

Cross section



$$\frac{dN}{dt} = j_i n \Delta\Omega \sigma$$

For n=1 $\sigma(\theta) = \frac{\hat{j}_f(\theta)}{j_i}$ where $\hat{j}_f(\theta) = \frac{dN}{dt\Delta\Omega}$

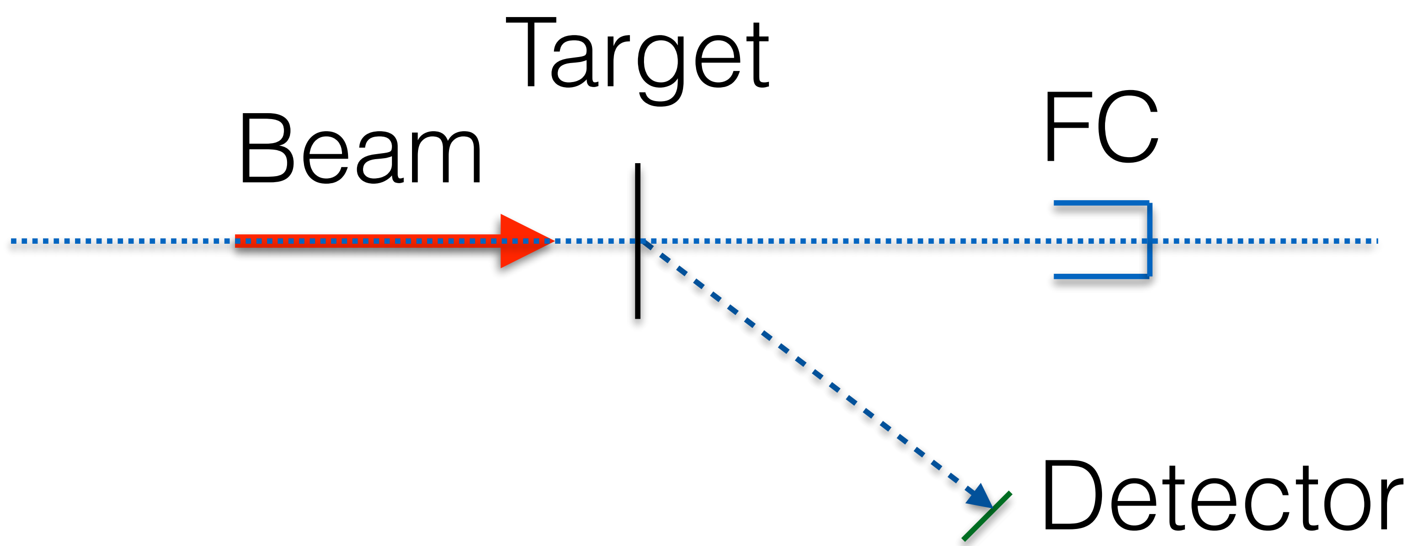
$$\vec{j} = \vec{v}|\Psi|^2$$

$j_i = v_i|A|^2$ - incident flux

$\hat{j}_f = R^2 j_f = v_f|A|^2|f(\theta)|^2$ - outgoing angular flux

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{v_f}{v_i}|f(\theta)|^2}$$

Cross section



Theory:

$$\frac{d\sigma}{d\Omega} = \frac{v_f}{v_i}|f(\theta)|^2$$

Experiment:

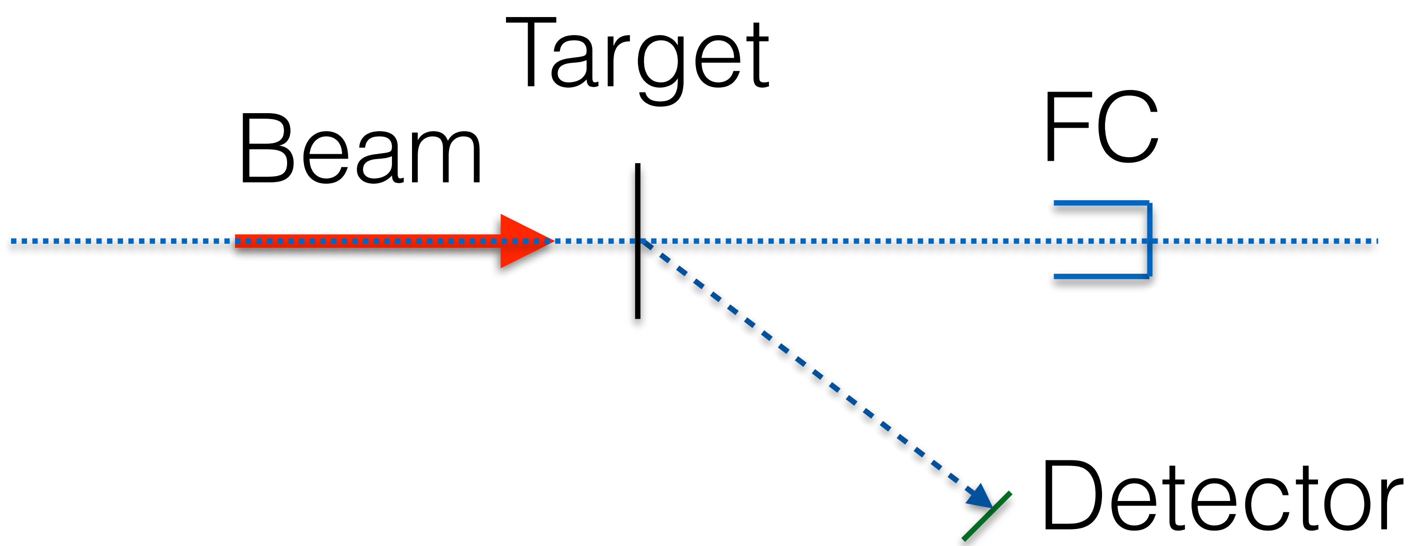
number of counts

$$\frac{d\sigma}{d\Omega}_{lab} = \frac{N}{I}\frac{1}{t\Delta\Omega}$$

number of beam ions

$$t = \frac{N_A \rho x}{M}$$ target thickness in cm^{-2}

Lab to cm



Theory:

$$\frac{d\sigma}{d\Omega} = \frac{v_f}{v_i}|f(\theta)|^2$$

$$A = \sqrt{\frac{M_1 M_3 E_{cm}}{M_2 M_4 (Q + E_{cm})}}$$

$$Q = \Delta M_1 + \Delta M_2 - \Delta M_3 - \Delta M_4 - E_3^* - E_4^*$$

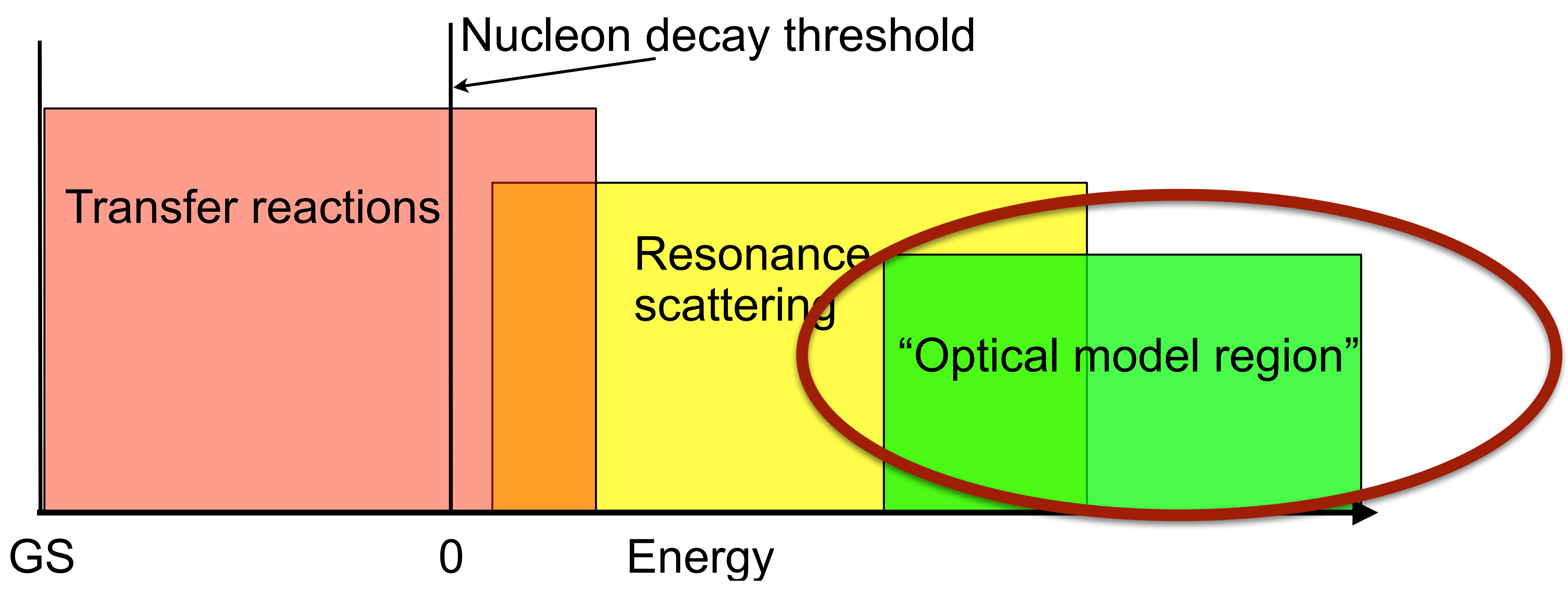
Experiment:

$$\frac{d\sigma}{d\Omega}_{lab} = \frac{(1 + A^2 + 2A\cos\theta)^{\frac{3}{2}}}{|1 + A\cos\theta|}\frac{d\sigma}{d\Omega}_{cm}$$

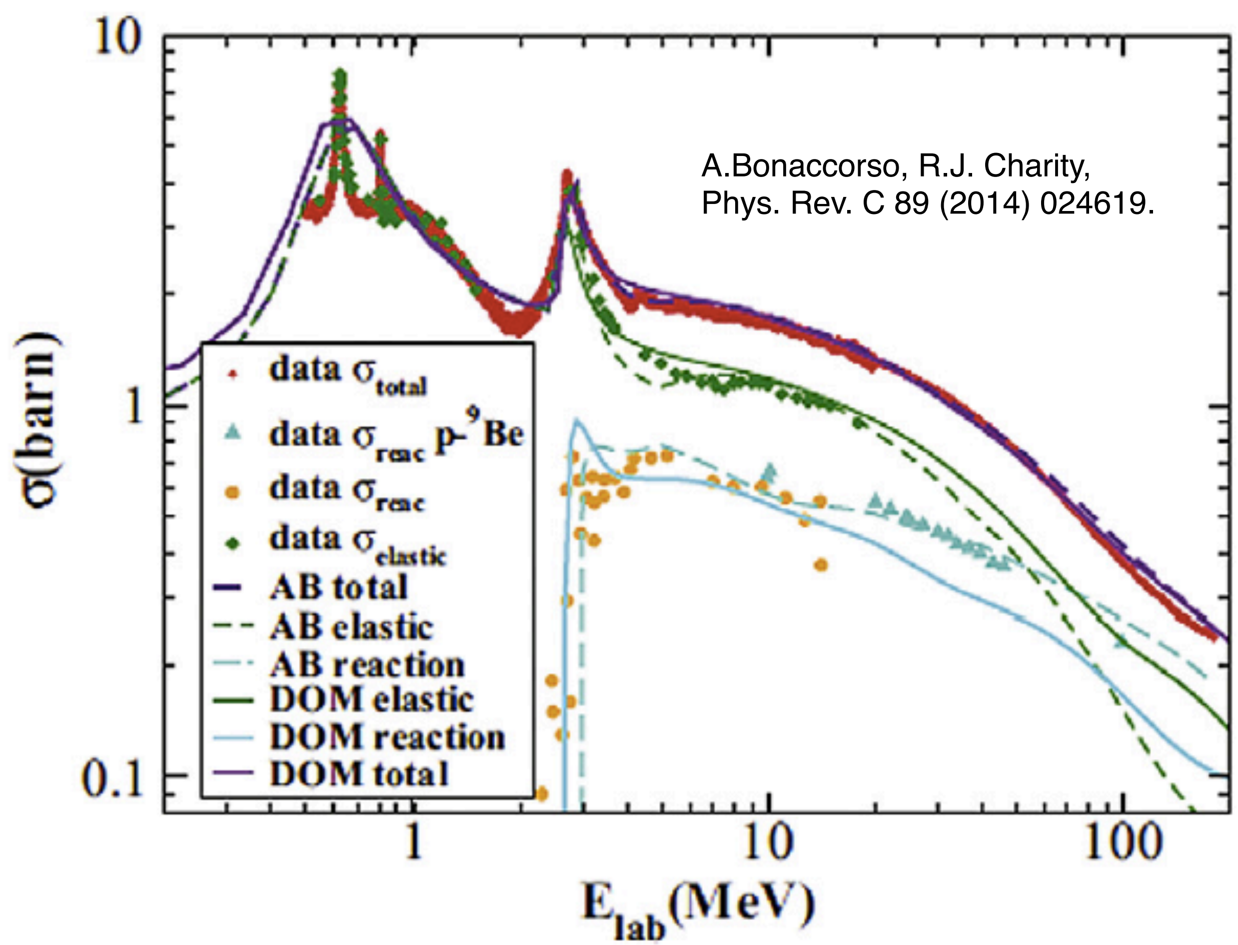
$$E_{cm} = \frac{M_2}{M_1 + M_2} E_1$$

$$\tan\theta_{lab} = \frac{\sin\theta}{A + \cos\theta}$$

Part I. Elastic scattering



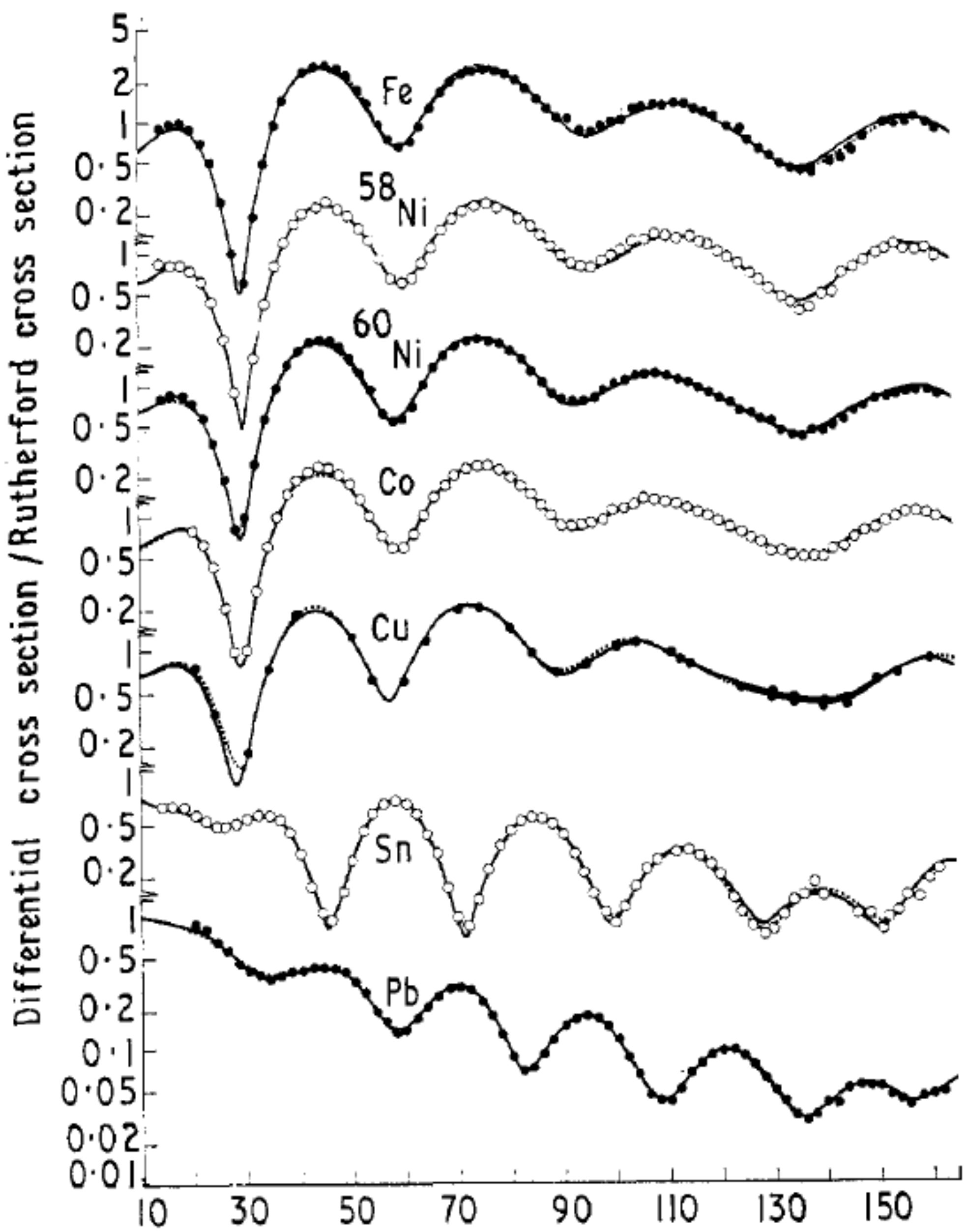
Nucleon decay threshold
Transfer reactions
Resonance scattering
“Optical model region”
GS
0
Energy



A.Bonaccorso, R.J. Charity,
Phys. Rev. C 89 (2014) 024619.
data σ total
data σ reac p-9Be
data σ reac
data σ elastic
AB total
AB elastic
AB reaction
DOM elastic
DOM reaction
DOM total
σ(barn)
E lab (MeV)

Optical model

Differential cross sections for proton elastic scattering



$$\nabla^2\Psi + \frac{2\mu}{\hbar^2}(E - V)\Psi = 0$$

$$\mu = \frac{mM}{m+M}$$

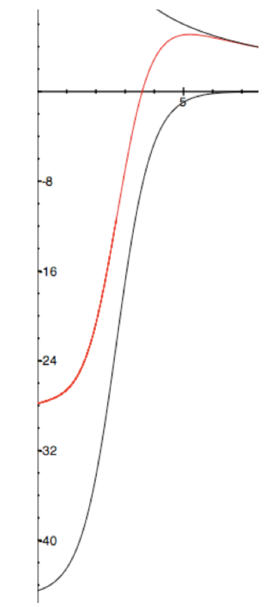
$$\Psi = \sum_{\ell} \frac{u(\ell)}{r} P_\ell(\cos\theta)$$

$$\frac{d^2 u_\ell}{dr^2} + \left\{\frac{2\mu}{\hbar^2}(E - V(r)) - \frac{\ell(\ell+1)}{r^2}\right\} u_\ell = 0$$

$$\begin{aligned}
\mathcal{U}(r,E,A,N,Z,\mathcal{P},MN) = & \\
& -\mathcal{V}_V(E,A,N,Z,\mathcal{P},MN) f_{WS}(r,\mathcal{R}_V,\mathcal{A}_V) \\
& -i\mathcal{W}_V(E,A,N,Z,\mathcal{P},MN) f_{WS}(r,\mathcal{R}_V,\mathcal{A}_V) \\
& +4\mathcal{A}_S\mathcal{V}_D(E,A)\frac{d}{dr} f_{WS}(r,\mathcal{R}_S,\mathcal{A}_S) \\
& +i4\mathcal{A}_S\mathcal{W}_D(E,A,N,Z,\mathcal{P})\frac{d}{dr} f_{WS}(r,\mathcal{R}_S,\mathcal{A}_S) \\
& +\frac{2}{r}\mathcal{V}_{SO}(E,A,N,Z,\mathcal{P})\frac{d}{dr} f_{WS}(r,\mathcal{R}_{SO},\mathcal{A}_{SO})(\mathbf{l}\cdot\sigma) \\
& +i\frac{2}{r}\mathcal{W}_{SO}(E,A,N,Z,\mathcal{P})\frac{d}{dr} f_{WS}(r,\mathcal{R}_{SO},\mathcal{A}_{SO})(\mathbf{l}\cdot\sigma) \\
& +f_{coul}(r,\mathcal{R}_C,A,N,Z,\mathcal{P}),
\end{aligned}$$

$$f_{WS}(r,\mathcal{R}_i,\mathcal{A}_i) = \left(1 + \exp\left((r - \mathcal{R}_i A^{1/3})/\mathcal{A}_i\right)\right)^{-1},$$

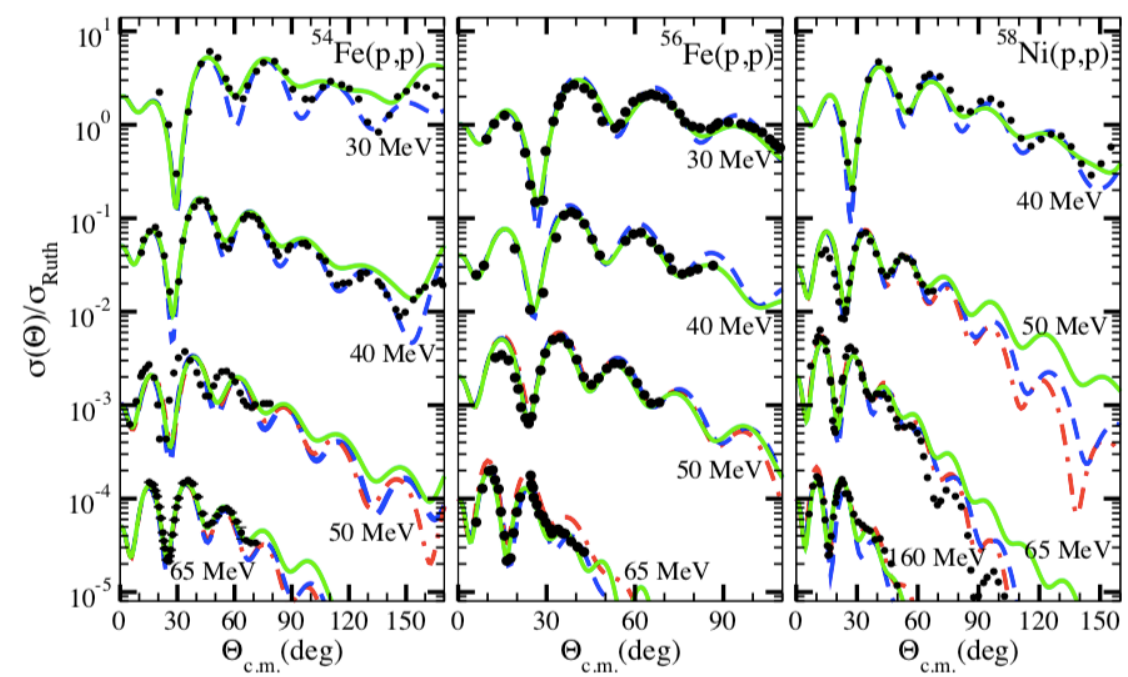
Phenomenological OM



$$\frac{d^2 u_\ell}{dr^2} + \left\{ \frac{2\mu}{\hbar^2}(E - V(r)) - \frac{\ell(\ell+1)}{r^2} \right\} u_\ell = 0$$

$$f_{WS}(r, \mathcal{R}_i, \mathcal{A}_i) = \left(1 + \exp\left((r - \mathcal{R}_i A^{1/3})/\mathcal{A}_i\right)\right)^{-1},$$

$$\begin{aligned}
\mathcal{U}(r, E, A, N, Z, \mathcal{P}, MN) = \\
& -\mathcal{V}_V(E, A, N, Z, \mathcal{P}, MN) f_{WS}(r, \mathcal{R}_V, \mathcal{A}_V) \\
& -i\mathcal{W}_V(E, A, N, Z, \mathcal{P}, MN) f_{WS}(r, \mathcal{R}_V, \mathcal{A}_V) \\
& +4\mathcal{A}_S \mathcal{V}_D(E, A) \frac{d}{dr} f_{WS}(r, \mathcal{R}_S, \mathcal{A}_S) \\
& +i4\mathcal{A}_S \mathcal{W}_D(E, A, N, Z, \mathcal{P}) \frac{d}{dr} f_{WS}(r, \mathcal{R}_S, \mathcal{A}_S) \\
& +\frac{2}{r} \mathcal{V}_{SO}(E, A, N, Z, \mathcal{P}) \frac{d}{dr} f_{WS}(r, \mathcal{R}_{SO}, \mathcal{A}_{SO})(1 \cdot \sigma) \\
& +i\frac{2}{r} \mathcal{W}_{SO}(E, A, N, Z, \mathcal{P}) \frac{d}{dr} f_{WS}(r, \mathcal{R}_{SO}, \mathcal{A}_{SO})(1 \cdot \sigma) \\
& +f_{coul}(r, \mathcal{R}_C, A, N, Z, \mathcal{P}),
\end{aligned}$$



$$\begin{aligned}
\mathcal{V}_V &= V_{V_0} + V_{V_1}A + V_{V_2}A^2 + V_{V_3}A^3 + V_{V_5}E + V_{V_6}E^2 + V_{V_7}E^3 \\
&+ \mathcal{P}(N - Z)\left(V_{V_{i0}} + V_{V_{i1}}A + V_{V_{i2}}A^2 + V_{V_{i3}}A^3 + V_{V_{i4}}A^4 + V_{V_{i5}}E + V_{V_{i6}}E^2\right) \\
&+ MN\left(V_{V_{m0}} + V_{V_{m1}}A + V_{V_{m2}}A^2 + V_{V_{m3}}A^3 + V_{V_{m5}}E + V_{V_{m6}}E^2\right),
\end{aligned}$$

$$\begin{aligned}
\mathcal{W}_V &= W_{V_0} + W_{V_1}A + W_{V_2}A^2 + W_{V_3}A^3 + W_{V_5}E + W_{V_6}E^2 + W_{V_7}E^3 \\
&+ \mathcal{P}(N - Z)\left(W_{V_{i0}} + W_{V_{i1}}A + W_{V_{i2}}A^2 + W_{V_{i3}}A^3 + W_{V_{i4}}A^4 + W_{V_{i5}}E + W_{V_{i6}}E^2\right) \\
&+ MN\left(W_{V_{m0}} + W_{V_{m1}}A + W_{V_{m2}}A^2 + W_{V_{m3}}A^3 + W_{V_{m5}}E + W_{V_{m6}}E^2\right).
\end{aligned}$$

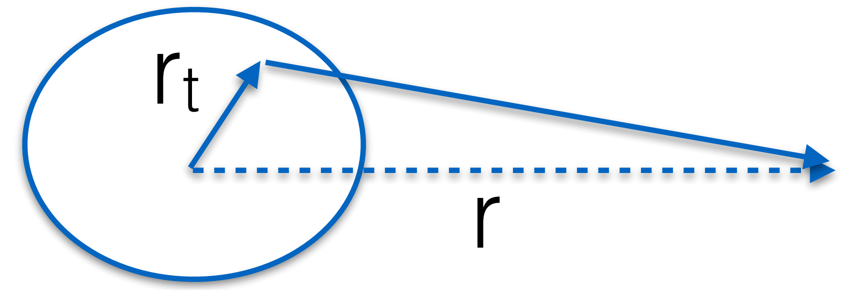
- A.J. Koning and J.D. Delaroche, Nucl. Phys. A 713 (2003) [Z=12-83, A=27-209, E<200 MeV]
- F.D. Becchetti, and G.W. Greenlees, Phys. Rev. 182 (1969) 1190. [Z=20-92, E=10-50 MeV]

S.P. Weppner, et al., arXiv 0906.3758v3 (2014)

OM from first principles

Single folded potential:

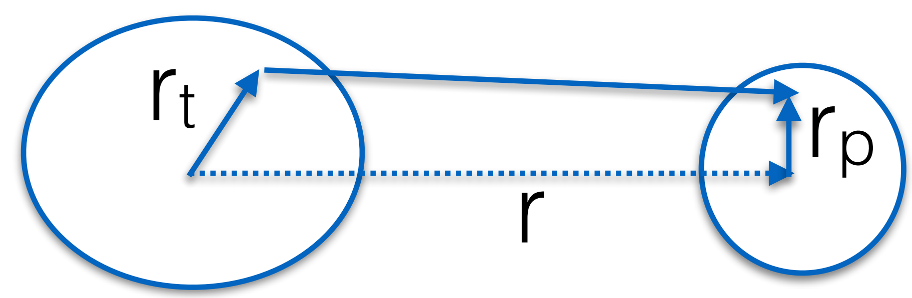
$$U_{pt}(\vec{r}) = \int d\vec{r}_t V_{NN}(\vec{r} - \vec{r}_t)\rho_t(\vec{r}_t)$$



Double folded potential:

$$U_{pt}(\vec{r}) = \int d\vec{r}_t \int d\vec{r}_p V_{NN}(\vec{r} + \vec{r}_p - \vec{r}_t)\rho_p(\vec{r}_p)\rho_t(\vec{r}_t)$$

$$V_{M3Y}(r) = 7999\frac{e^{-4r}}{4r} - 2135\frac{e^{-2.5r}}{2.5r} - 252\delta(r)$$

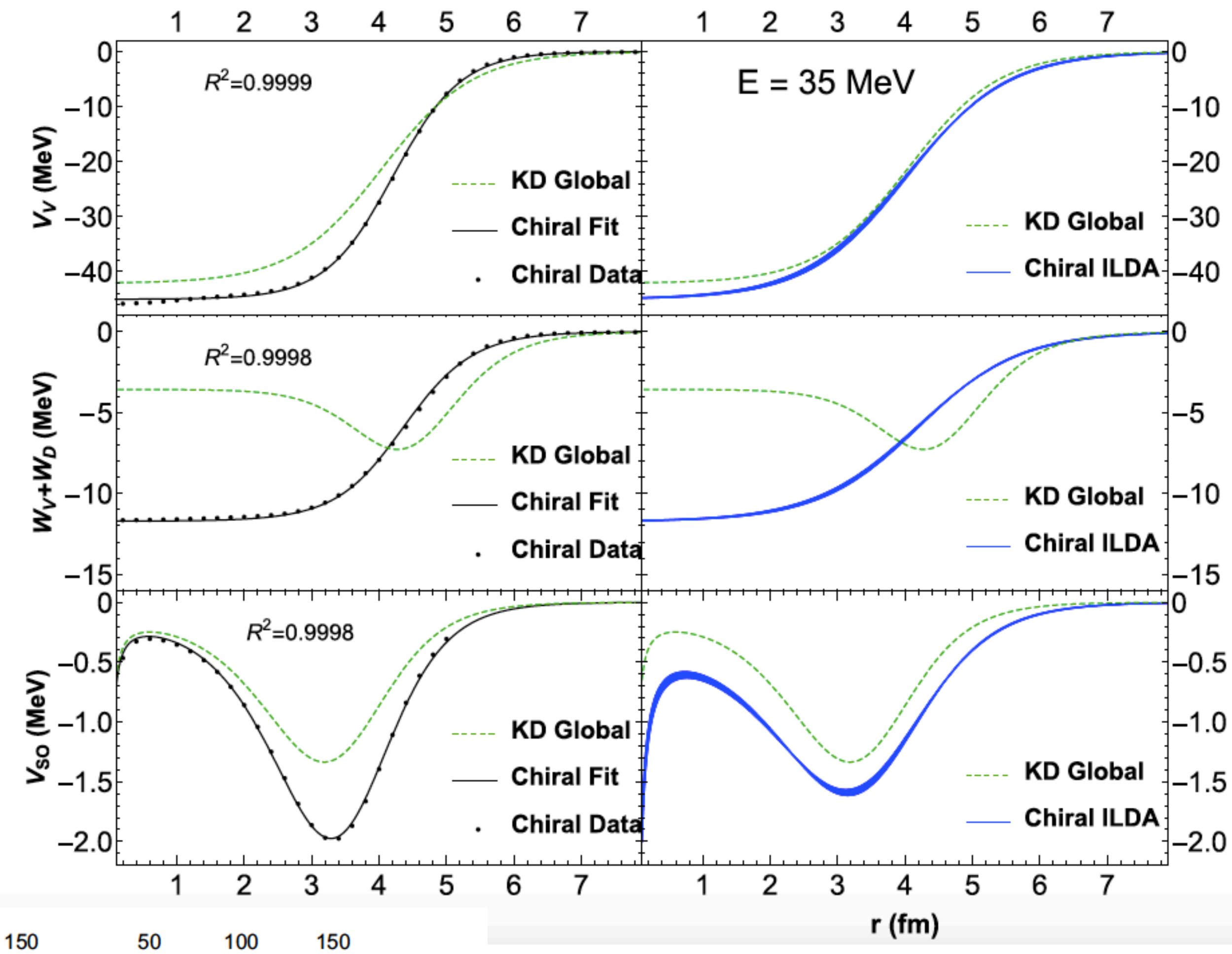


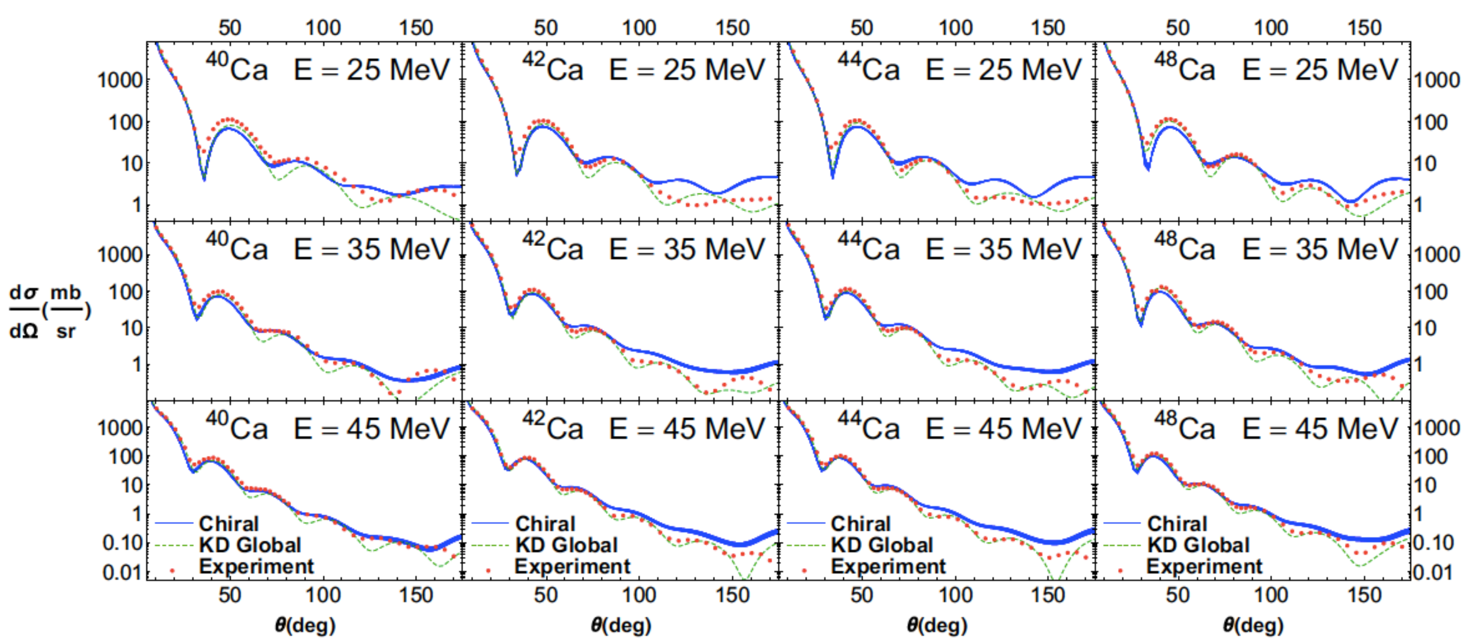
JLM - contains real and imaginary components, energy and density dependent
[J.P. Jeukenne, et al., Phys. Rev. C 16 (1977) 80]

OM from first principles

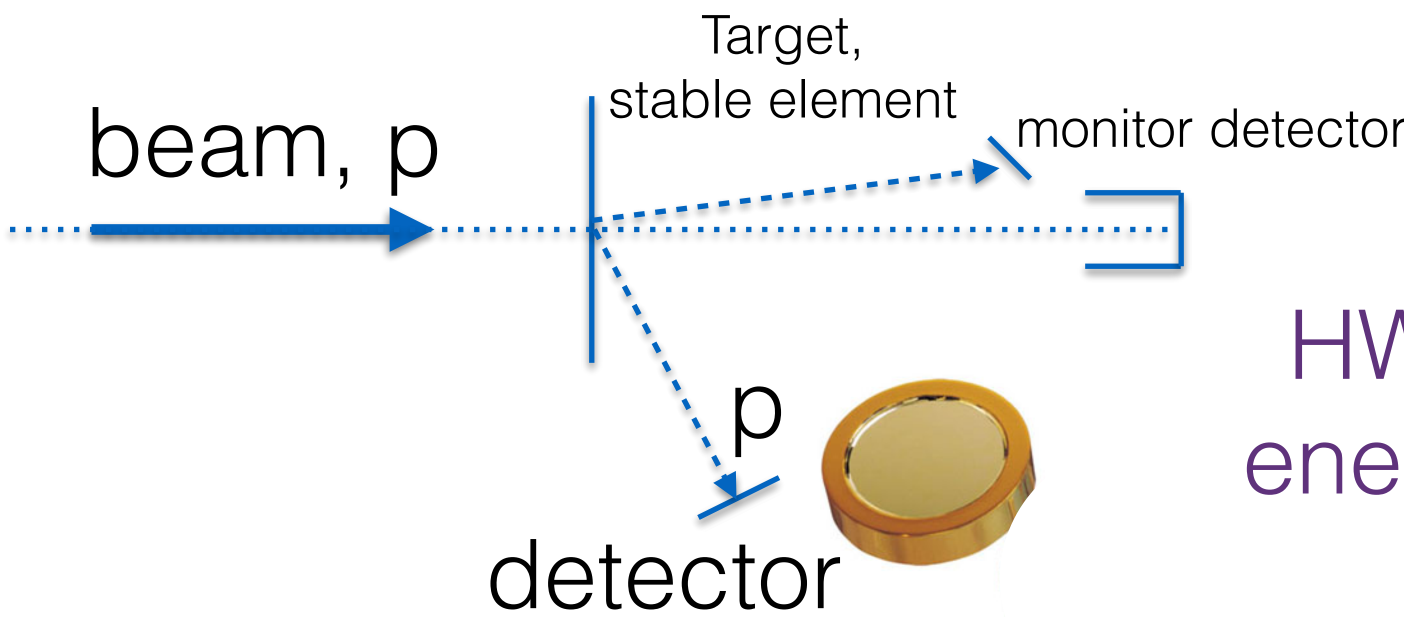
Effective Field Theory
chiral interactions

T.R. Whitehead, et al.,
Phys. Rev. C 100, 014601 (2019)



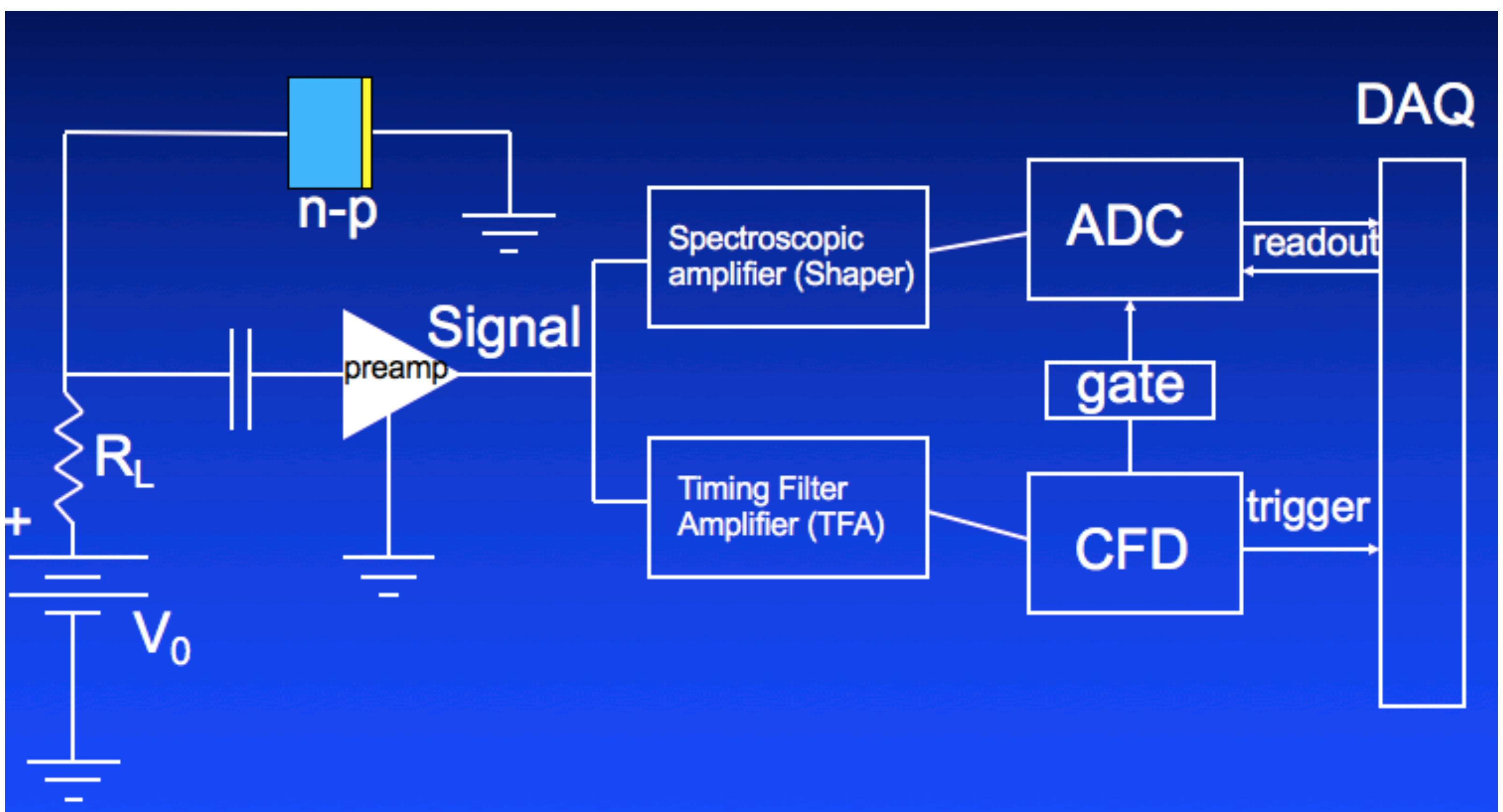


Measuring elastic scattering

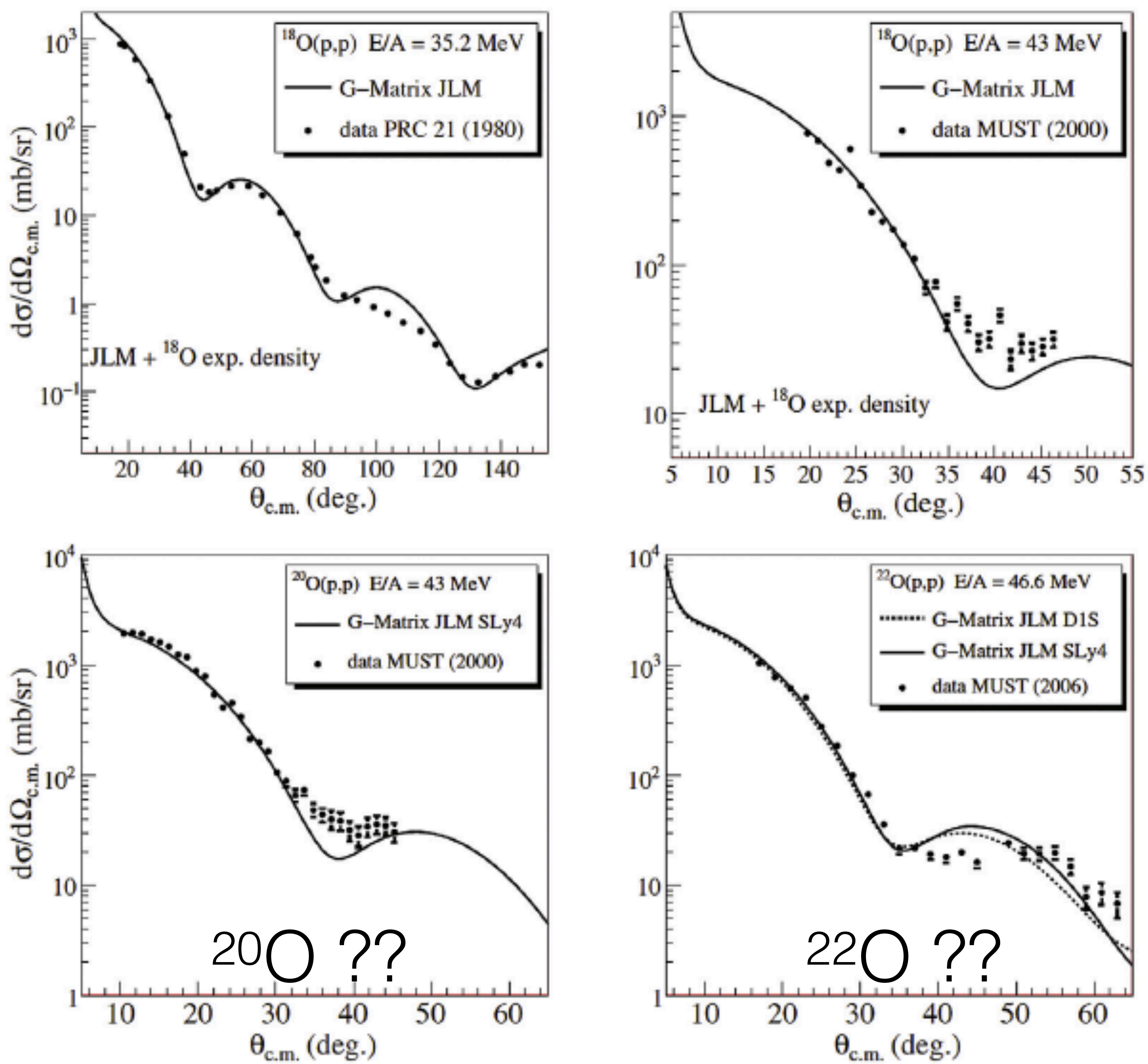


The good old days!

HW1: How important is energy resolution in such measurements?



Measuring elastic scattering



HW2: How long does it take to measure ^{18}O(p,p) elastic scattering at 130^{o} with a 5 mm (radius) Si detector, beam current 1nA and target thickness of

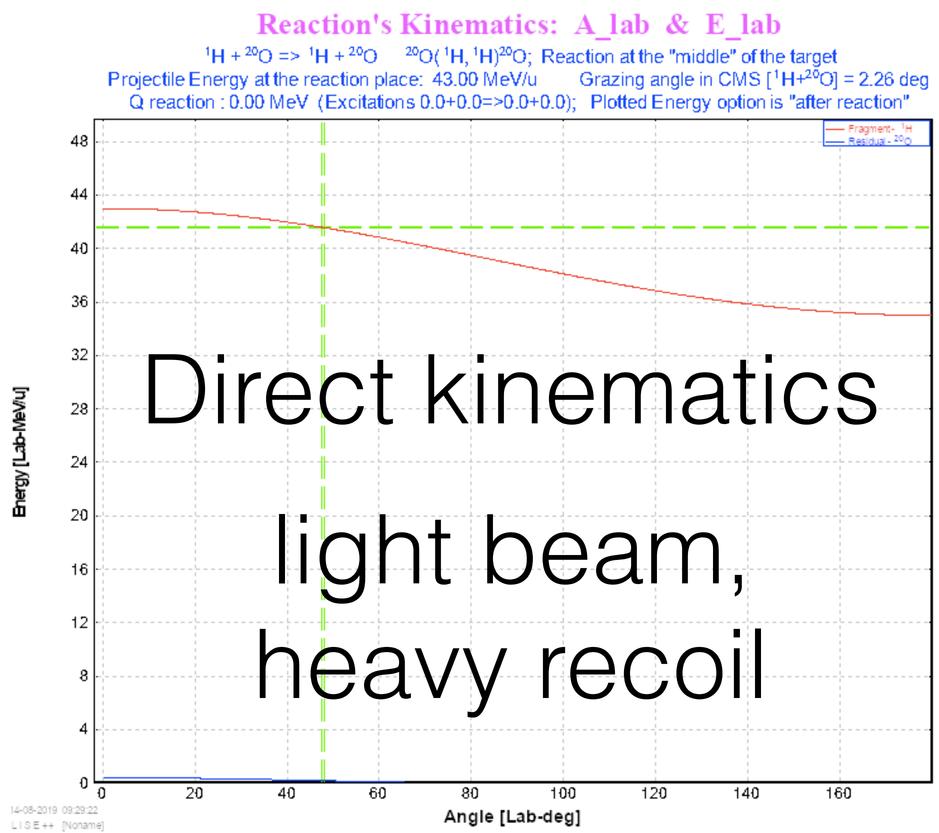
Radioactive beam experiment

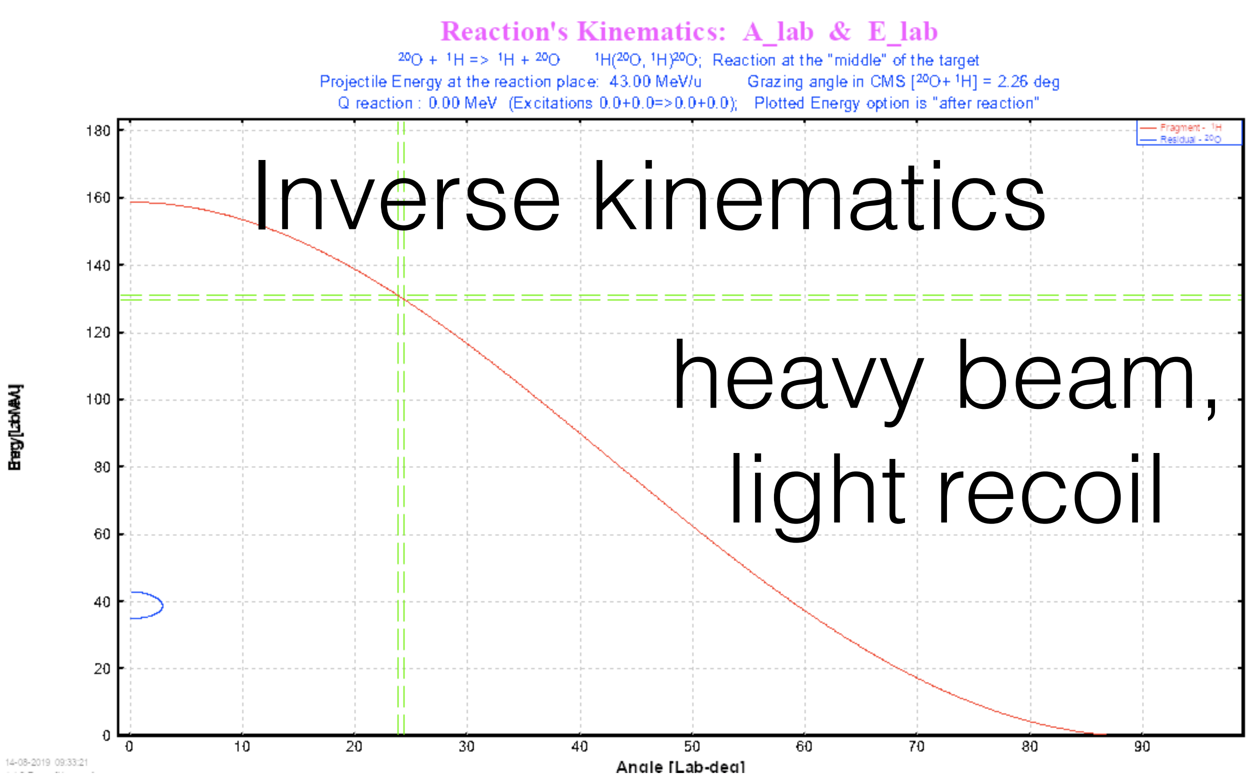
E. Khan, et al., Phys. Lett. B 490 (1) (2000) 45–52

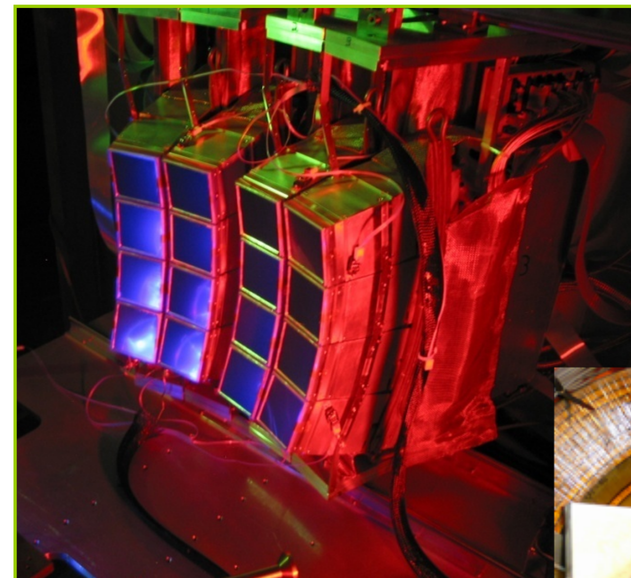
Measuring elastic scattering

HW2: How long does it take to measure $^{18}O(p,p)$ elastic scattering at 130^{o} with statistics 1000 counts, using a 2 mm (radius) Si detector at a distance of 30 cm away from the target. Proton beam current is 1 nA and target thickness is 1 micron of Al_2O_3.

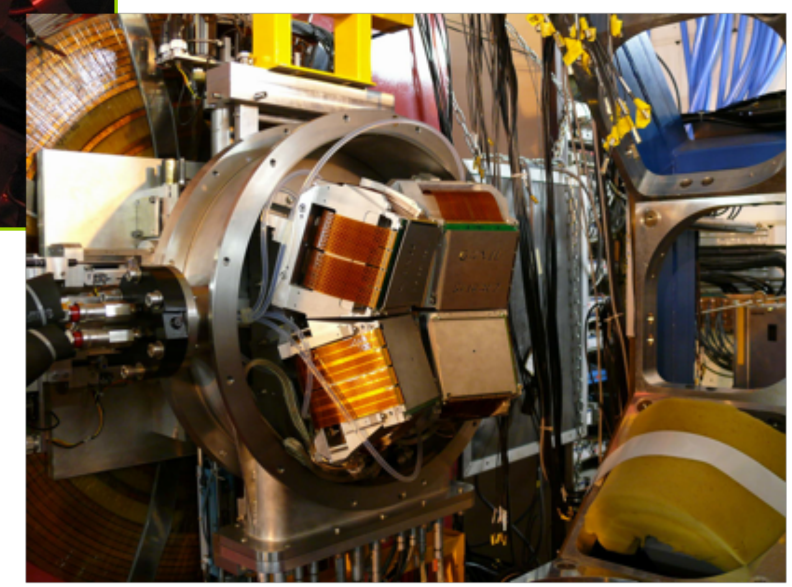
Measuring elastic scattering







HiRA



MUST2

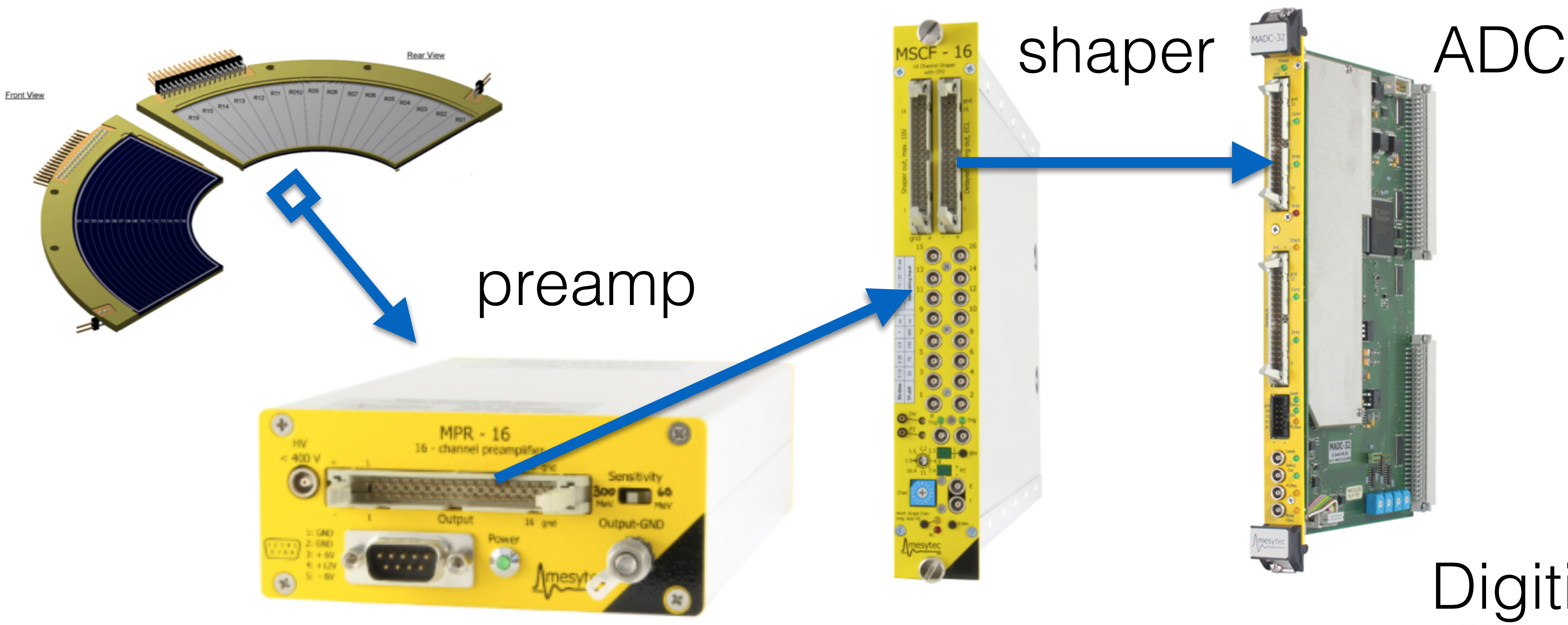
New approach: highly segmented, large solid angle, Si-CsI(TI) arrays

Measuring elastic scattering

HW3: Estimate the minimum ^{22}O beam intensity required to measure the $^{22}O+p$ elastic scattering at 60^o

Modern electronics for nuclear experiments

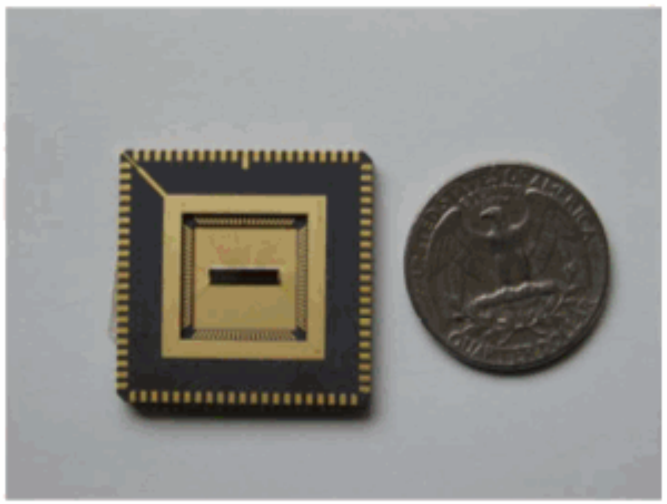
“Standard”, commercially available modules:

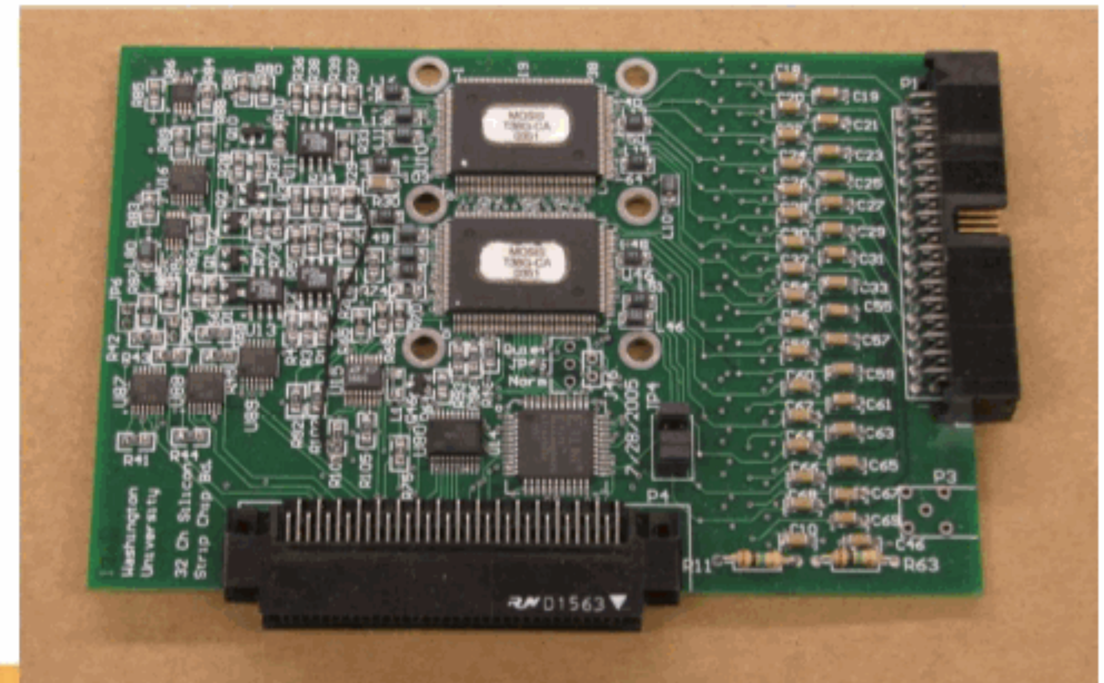


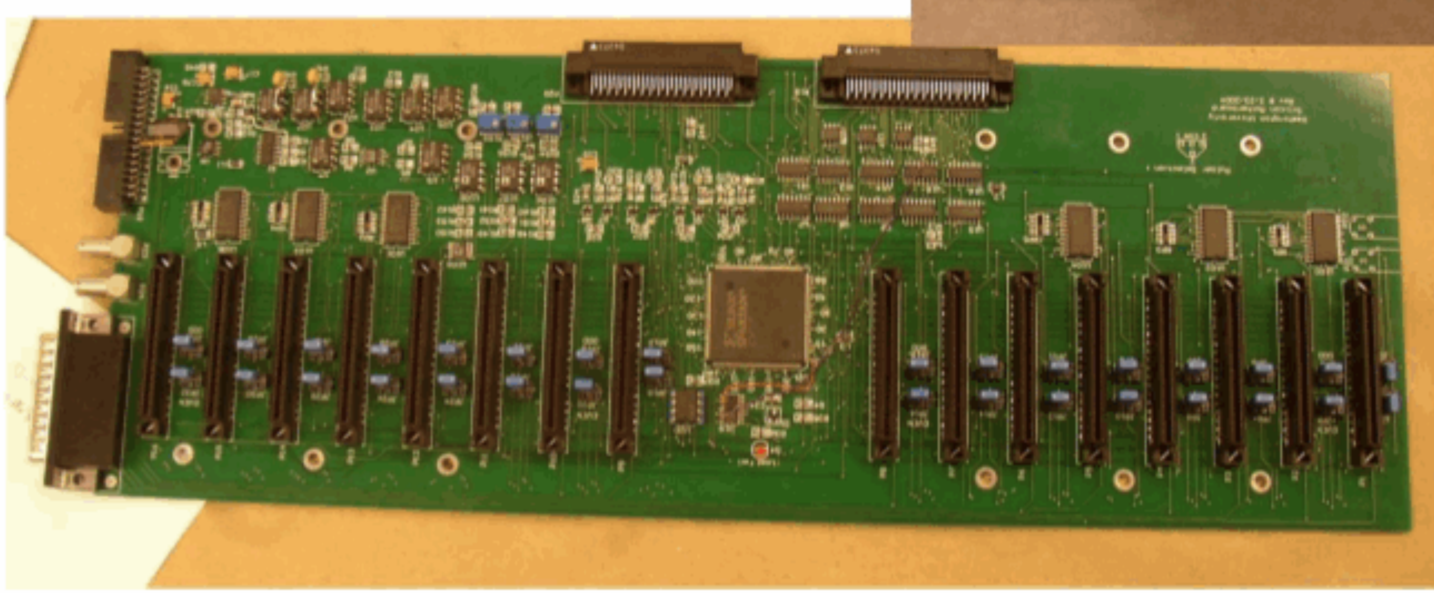
Waveform digitizers

Modern electronics for nuclear experiments

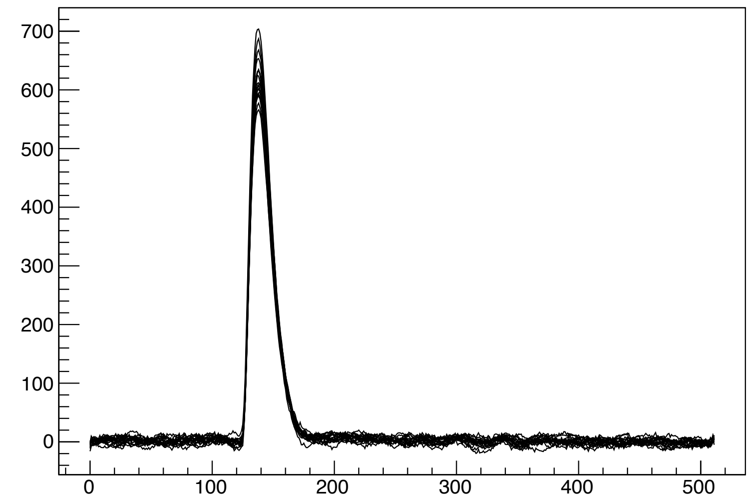
ASIC Chip -> Chip board -> Motherboard







GET - General Electronics for TPCs



More on scattering theory

$$[\hat{T} + V - E]\Psi(R,\theta) = 0$$

$$\Psi^{asym}(R,\theta) = A(e^{ikz} + f(\theta)\frac{e^{ikR}}{R})$$

Partial wave expansion

$$\Psi(R,\theta) = \sum_{L=0}^{\infty}(2L+1)i^L P_L(\cos\theta)\frac{1}{kR}\chi_L(R)$$

$$\left[-\frac{\hbar}{2\mu}\left(\frac{d^2}{dR^2} - \frac{L(L+1)}{R^2}\right) + V(R) - E\right]\chi_L(R) = 0$$

$$f(\theta) = \frac{1}{2ik}\sum_{L=0}^{\infty}(2L+1)P_L(\cos\theta)(S_L - 1)$$

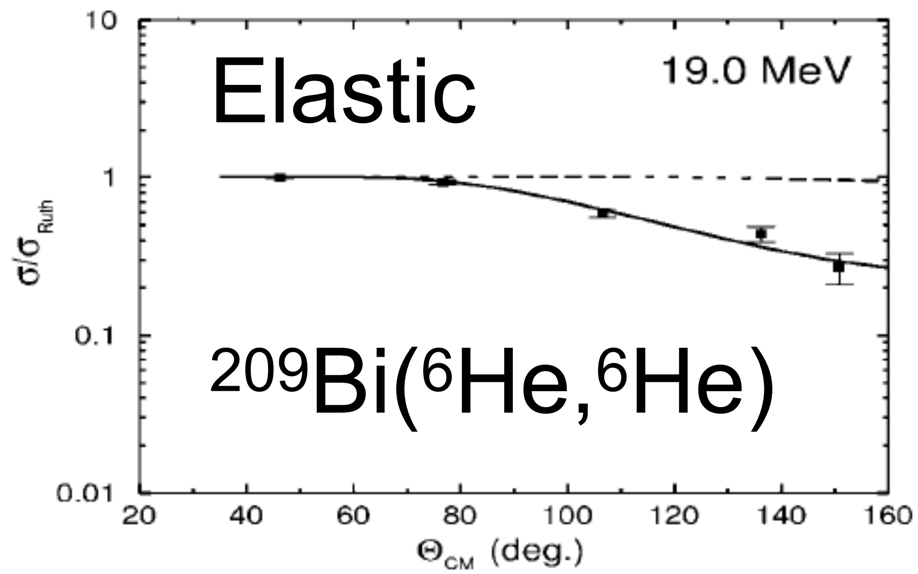
$$S_L = e^{2i\delta_L}$$

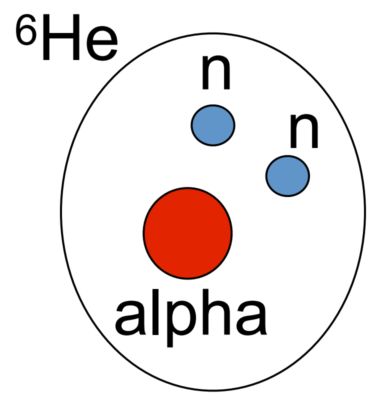
For real potentials $|S_L|=1$

For complex potentials $|S_L|<1$

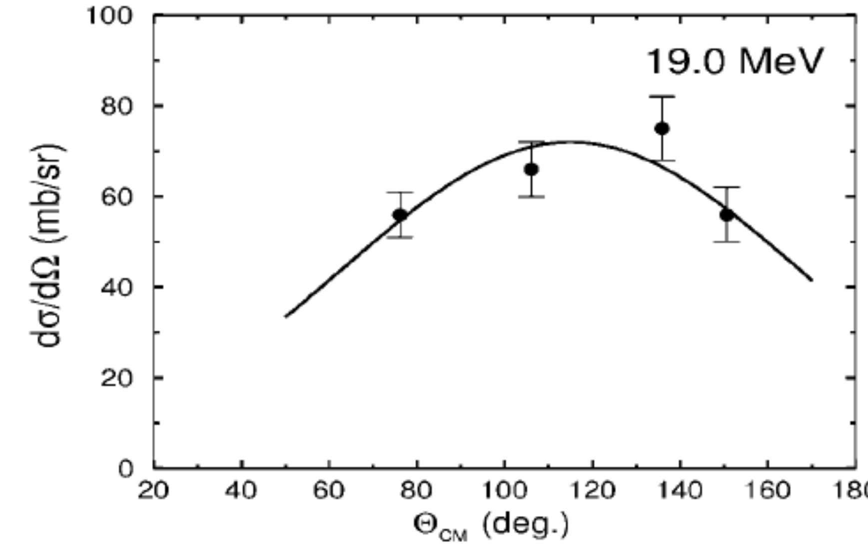
$$\sigma_R = \frac{\pi}{k^2}\sum_L (2L+1)(1-|S_L|^2)$$

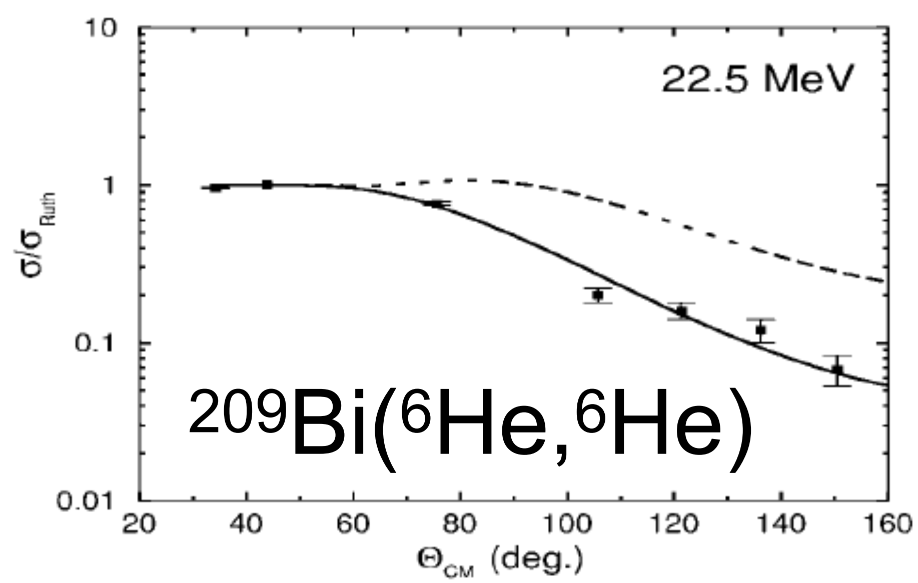
Elastic scattering. $^{4}He(^{6}He,^{6}He)$



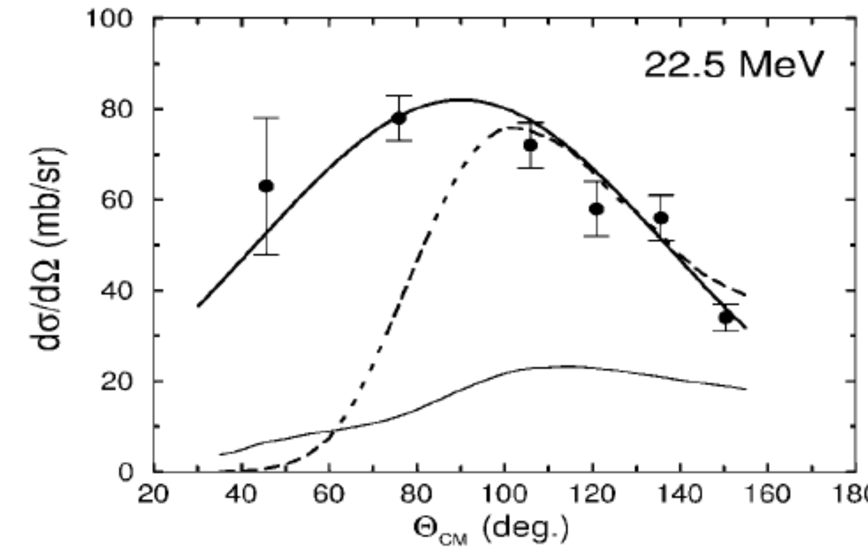


di-neutron configuration





It was found that reaction cross section is strongly enhance, indicating that neutron wave function is radially extended

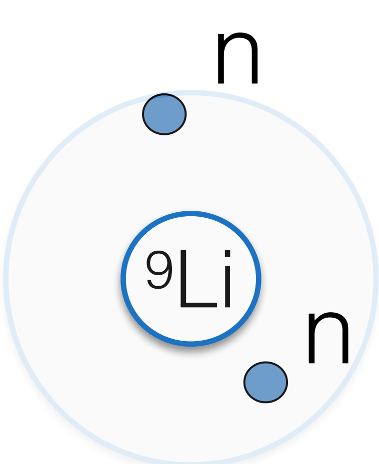


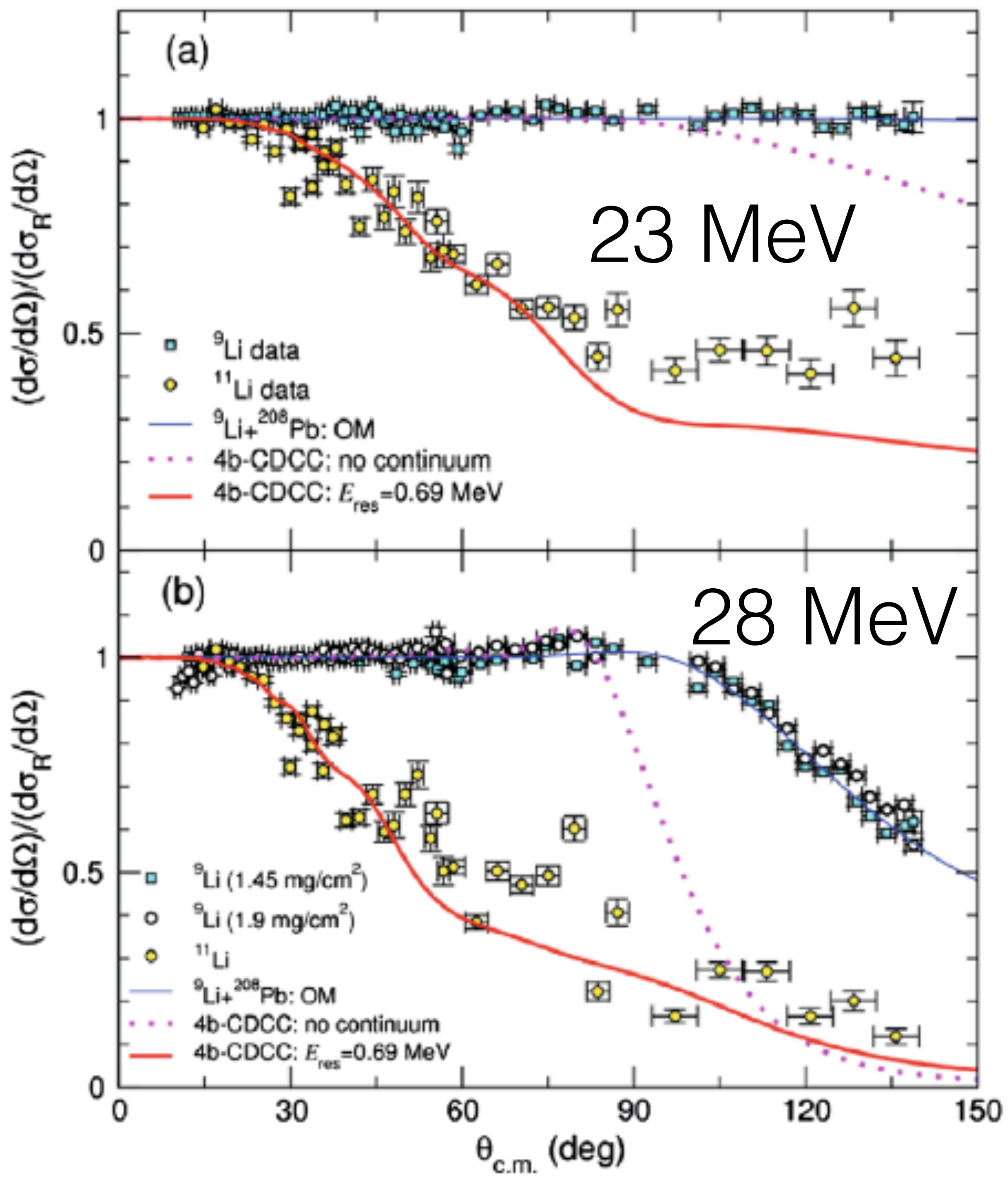
$^{209}Bi(^{6}He,alpha)$

E.F. Aguilera, et al., Phys. Rev. Lett. 84 (2000)

$$\sigma_{el} \sim (1 - S)^2$$
$$\sigma_r \sim (1 - |S|^2)$$

Elastic scattering. $^{208}Pb(^{11,9}Li,^{11,9}Li)$





M. Cubero et al., Phys. Rev. Lett. 109, 262701 (2012).

J. Kolata, et al., Eur. Phys. J. A (2016) 52: 123