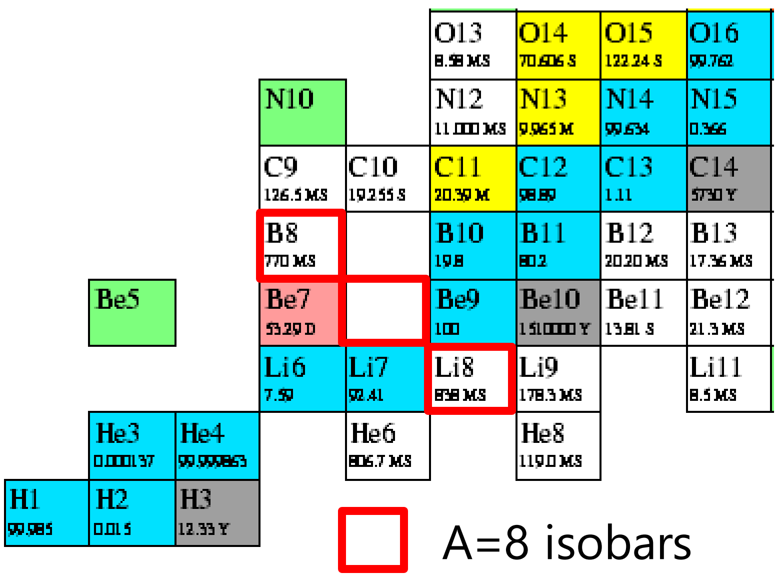
Nuclear Structure Theory for Cluster Physics

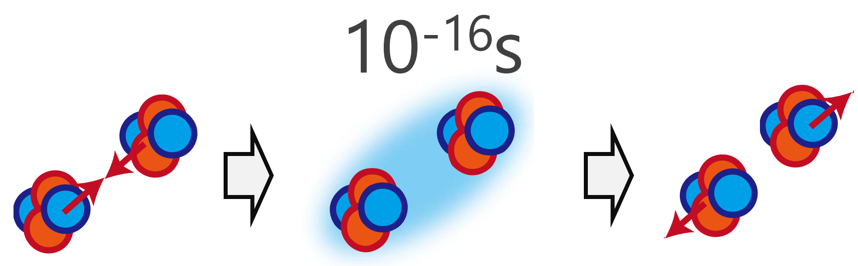
Masaaki Kimura (Hokkaido U.)

Part. 2　A story of the Hoyle state

Prediction and discovery of the Hoyle state

- Triple α reaction（Synthesis of ^{12}C）
- ◎ Elements are synthesized by the fusion reactions in stars
- ◎ There are no stable nucleus in A=8 isobars(^{8}B, ^{8}Be, ^{8}Li, ^{8}He)
- ◎ This means that the fusion reaction must stop at A=7 and no elements heavier than A=8 will be synthesized





Prediction and discovery of the Hoyle state

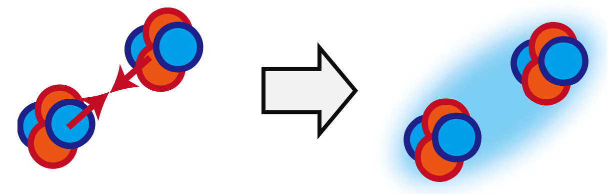
◎ Nevertheless, in our Universe, elements heavier than A=8 are abundant. (^{12}C, ^{16}O, ...)

◎ To resolve this puzzle, Porf. F. Hoyle proposed the following scenario in 1950's

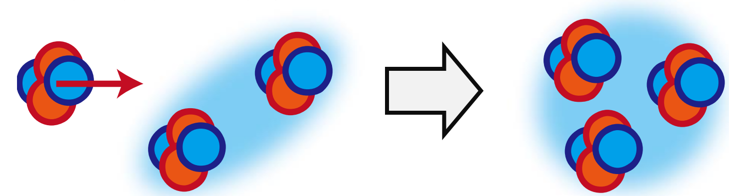
Prediction and discovery of the Hoyle state

- Triple α reaction （Synthesis of ^{12}C）

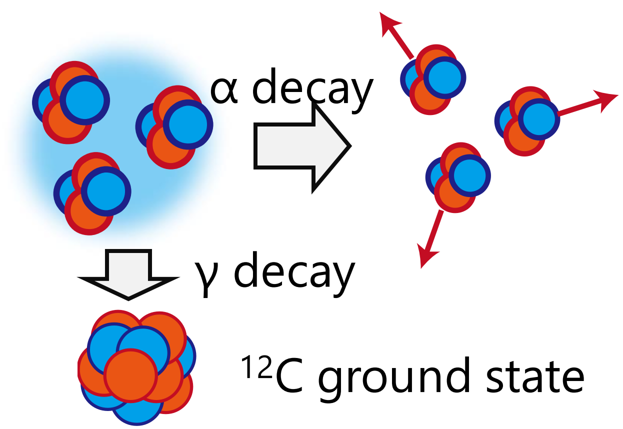
① Fusion of ^{4}He yields ^{8}Be (life 10^{-16} sec.)



② Before the decay of ^{8}Be, another ^{4}He comes.
They forms meta-stable state (resonance state) composed of three ^{4}He clusters

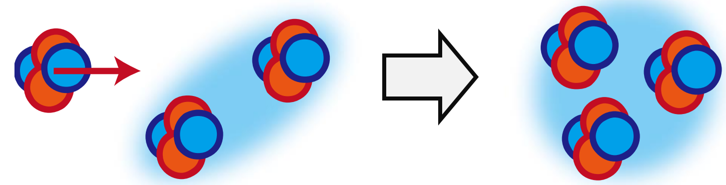


③ The resonance mainly decays into three ^{4}He nucleus, but occasionally decays by emitting γ-ray.
As a result, ^{12}C(g.s.) is yielded.



Prediction and discovery of the Hoyle state

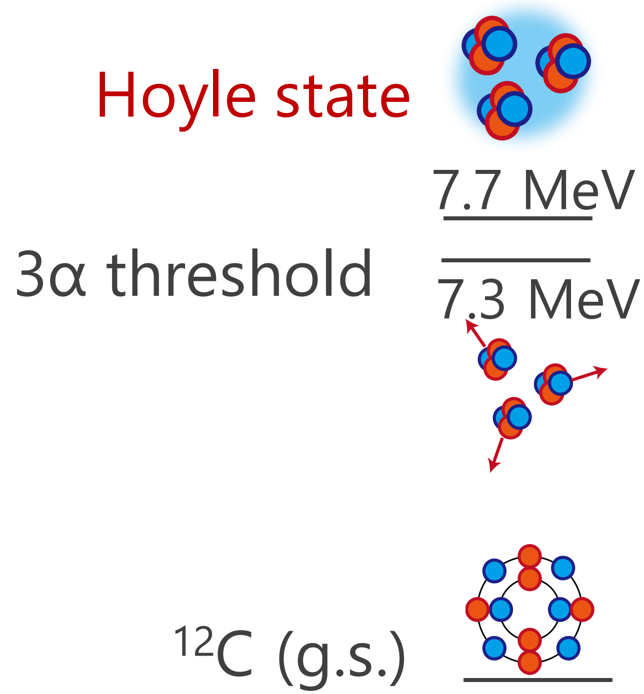
- Triple α reaction (Synthesis of ^{12}C)



Fred Hoyle predicted the existence of the resonance state in ^{12}C that is composed of three ^{4}He clusters just above the 3α threshold energy.

Otherwise, the reaction rate becomes small in order of magnitude and he cannot explain the origin of ^{12}C in the universe

Later, the experimentalists found the 0^{+} state at 7.7 MeV, which we call "Hoyle state" today.



Structure of the Hoyle state

- Many people have been interested in the structure of the Hoyle state

- Morinaga proposed "linear-chain" structure of the Hoyle state

Linear alignment of thee ^{4}He clusters

H. Morinaga, Phys. Rev. 101, 254 (1956).

- This idea attracted much interests
  - Extremely deformed state（1:3 deformation）
  - Quite different from the ordinary nuclear structure

- However, "linear-chain hypothesis" was denied, because it cannot explain short lifetime of α-decay

Structure of the Hoyle state

- Linear chain structure cannot explain the very short alpha decay lifetime of the Hoyle state

  Y. Suzuki, H. Horiuchi, and K. Ikeda, PTP47, 1517 (1972).

① Linear Chain state must have large orbital angular momentum of alpha particles

② If alpha particle has large orbital angular momentum, the alpha decay lifetime becomes much longer.

Exercise 2

Show that the wave function with angular momentum 0 is spherical

Hint: Rotation operator, for example about z axis, is given as

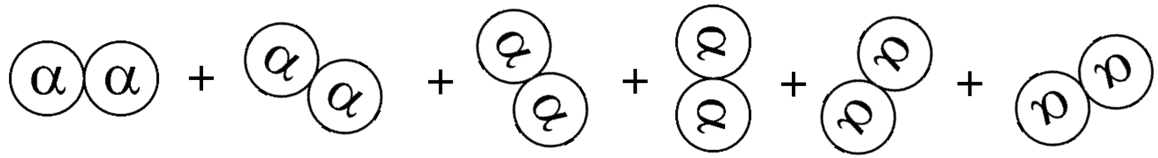
$$R_z(\theta) = e^{-iJ_z\theta}$$

Structure of the Hoyle state

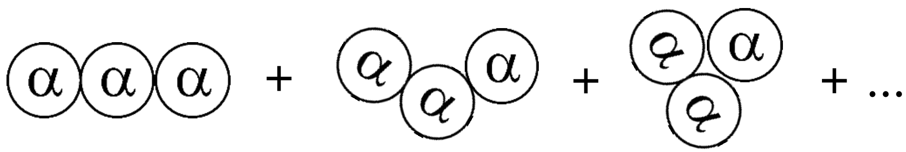
① Linear Chain state have large orbital angular momentum of α particle

Assume that the Linear chain has $^8Be(0^+)+\alpha$ structure, then

○ $^8Be(0^+)$ wave function is a superposition of various orientation

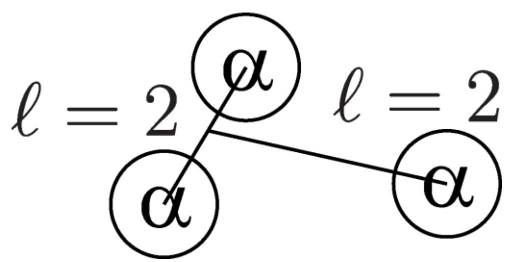
$\Psi_{^8\mathrm{Be}}(0^+) =$ 

○ $^8Be(0^+) + \alpha$ state is illustrated as

$\Psi_{^{12}\mathrm{C}}(0^+) =$ 

There are many non-linear-chain components

Thus, the linear chain must contain
$^8Be(2^+)$, $^8Be(4^+)$ components



Structure of the Hoyle state

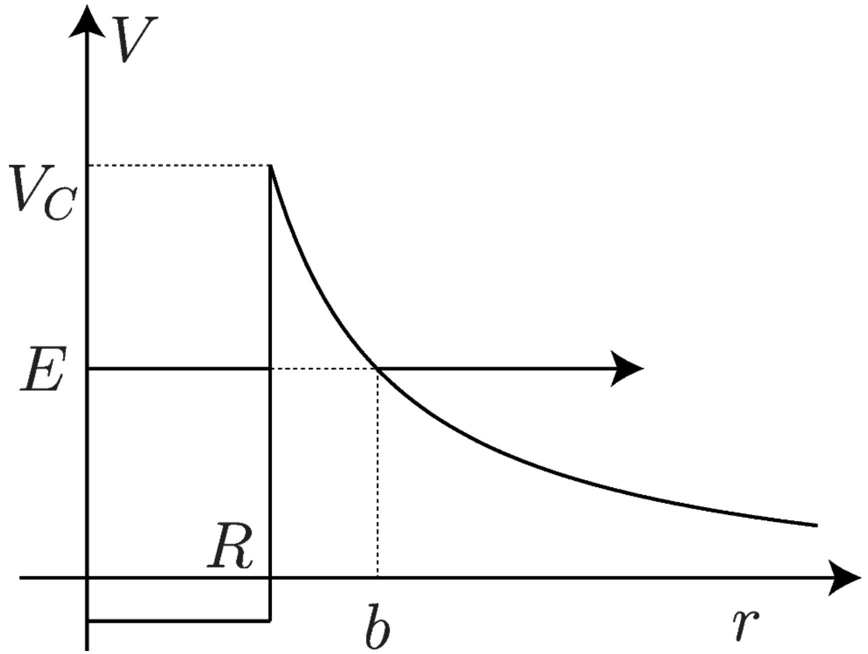
② If α cluster has large orbital angular momentum、
α decay lifetime becomes much longer

Exercise 2

○ Show that the log of the alpha decay lifetime λ is proportional to the $E^{-1/2}$ where E is the Q-value of the alpha decay (Geiger-Nuttall law, Gamow model for alpha decay)

$$\ln \lambda = -a_1 \frac{Z}{\sqrt{E}} + a_2$$

○ In the Gamow model, we assume that the orbital angular momentum of alpha particle is zero. Show that the lifetime becomes much longer if the alpha particle has orbital angular momentum



Structure of the Hoyle state

- Linear chain structure cannot explain the very short alpha decay lifetime of the Hoyle state

Y. Suzuki, H. Horiuchi, and K. Ikeda, PTP47, 1517 (1972).

① Linear Chain state must have large orbital angular momentum of alpha particles

② If alpha particle has large orbital angular momentum, the alpha decay lifetime becomes much longer.

Thus, it was shown that
○ Hoyle state is not a linear chain
○ If linear chain exist, it should be a mixture of the $^{8}Be(0^{+})+\alpha$, $^{8}Be(2^{+})+\alpha$, and $^{8}Be(4^{+})+\alpha$, and hence, it should decay to the $^{8}Be(2^{+})$ and $^{8}Be(4^{+})$ not only to $^{8}Be(0^{+})$

Structure of the Hoyle state

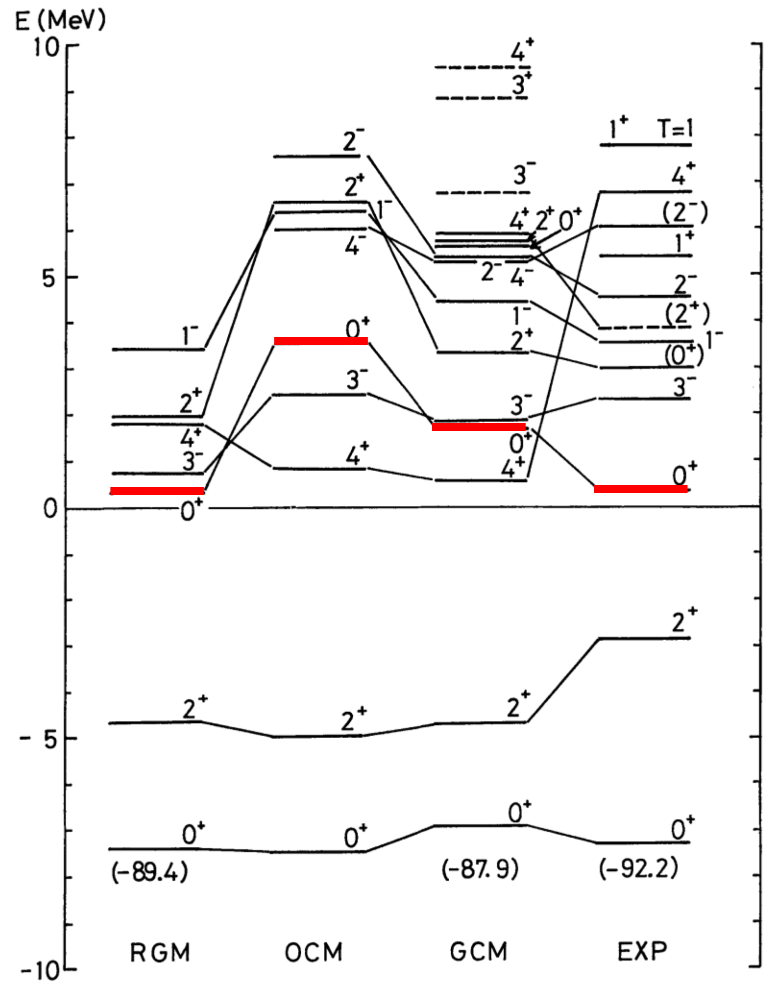
- So, how the Hoyle state looks like?

  3-body problem of alpha particles was numerically solved by using super computer

It was concluded that the Hoyle state is a "dilute bosonic gas state"

Y. Fujiwara et al., PTPS68, 29 (1980).

- ◯ It is composed of 3 α particles
- ◯ All α particles have orbital angular momentum 0
- ◯ Radius of the Hoyle state is large (bound state approx.)



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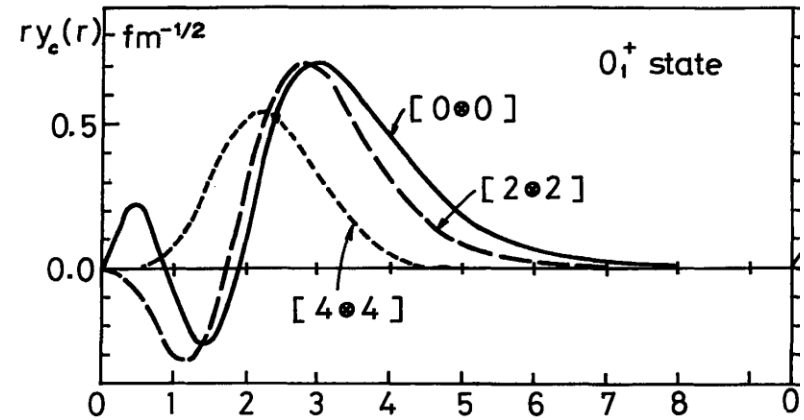
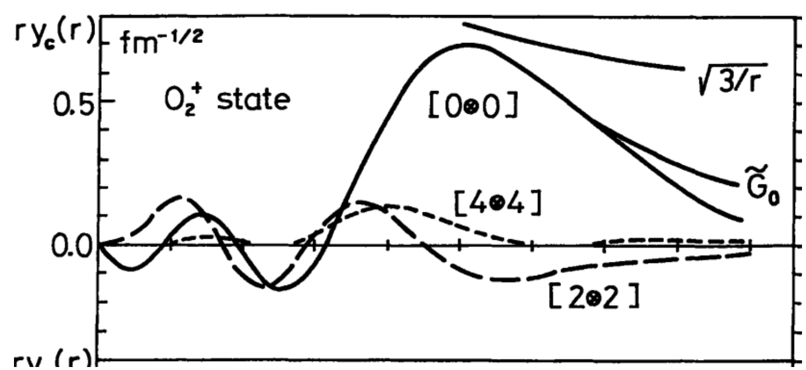




The Hoyle state can be considered as a finite system of alpha-boson gas.

Uegaki et al., PTP 57, 1262 (1977).

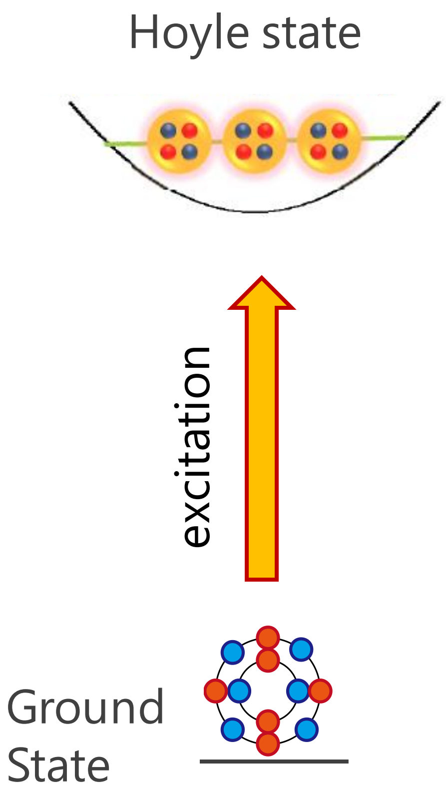
A new insight to the Hoyle state in this century

The Hoyle state is a Bose-Einstein condensate of alpha particles

A. Tohsaki, H. Horiuchi, P. Schuck, G. Röpke, PRL87, 192501 (2001)

THSR wave function $|\Phi_{n\alpha}\rangle = (C_\alpha^\dagger)^n |\mathrm{vac}\rangle$

$$C_\alpha^\dagger = \int d^3R\, e^{-\mathbf{R}^2/R_0^2} \int d^3r_1 \cdots d^3r_4 \times \varphi_{0s}(\mathbf{r}_1 - \mathbf{R}) a^\dagger_{\sigma_1\tau_1}(\mathbf{r}_1) \cdots \varphi_{0s}(\mathbf{r}_4 - \mathbf{R}) a^\dagger_{\sigma_4\tau_4}(\mathbf{r}_4)$$



○ All nucleons are confined in the alpha particles

○ All alpha particles occupies the same orbit (bosons)

（This is really simplified ansatz!）

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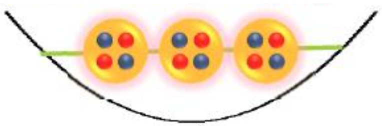
$$\times\, \varphi_{0s}(\mathbf{r}_1 - \mathbf{R}) a^\dagger_{\sigma_1\tau_1}(\mathbf{r}_1) \cdots \varphi_{0s}(\mathbf{r}_4 - \mathbf{R}) a^\dagger_{\sigma_4\tau_4}(\mathbf{r}_4)$$

Hoyle state



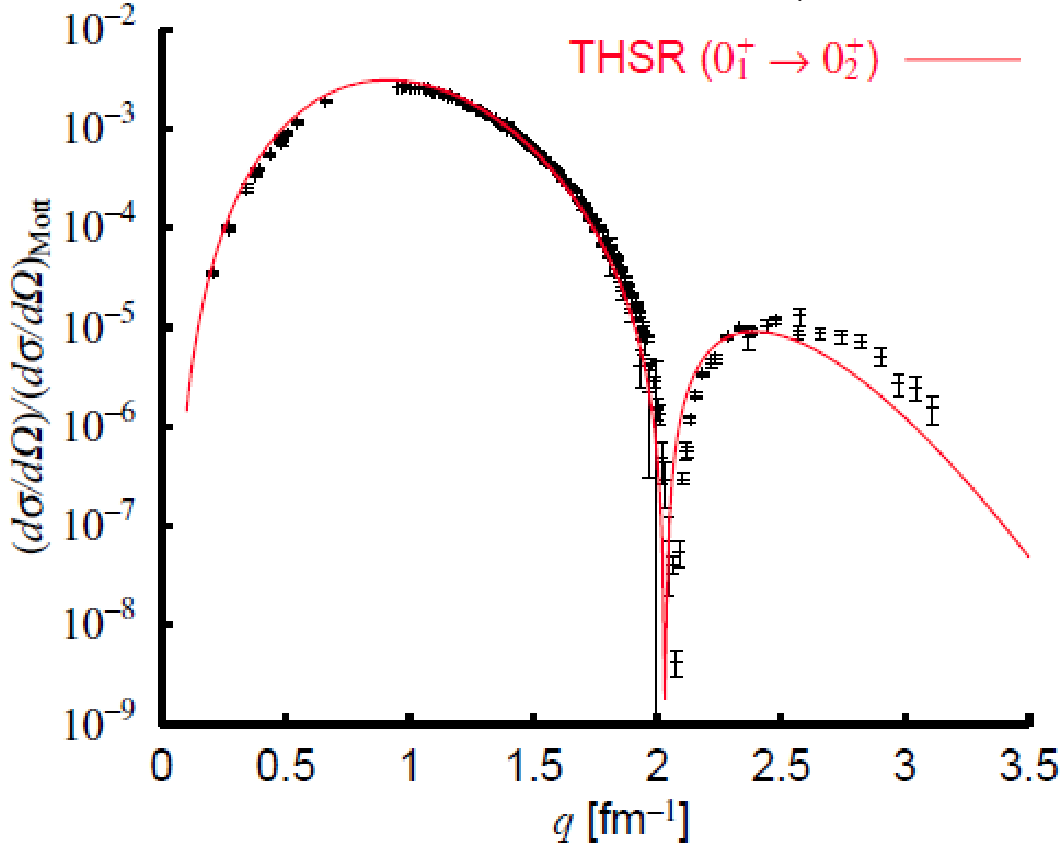
○ This simple ansatz works surprisingly well

E_k (MeV)	E_{exp} (MeV)	$E_k - E_{n\alpha}^{\mathrm{thr}}$ (MeV)	$(E - E_{n\alpha}^{\mathrm{thr}})_{\mathrm{exp}}$ (MeV)	$\sqrt{\langle r^2\rangle}$ (fm)	$\sqrt{\langle r^2\rangle_{\mathrm{exp}}}$ (fm)
−85.9	−92.16 (0_1^+)	−3.4	−7.27	2.97	2.65
−82.0	−84.51 (0_2^+)	+0.5	0.38	4.29	
−82.5	−84.89				

A new insight to the Hoyle state in this century

○ Inelastic form factor (information of wave func.)

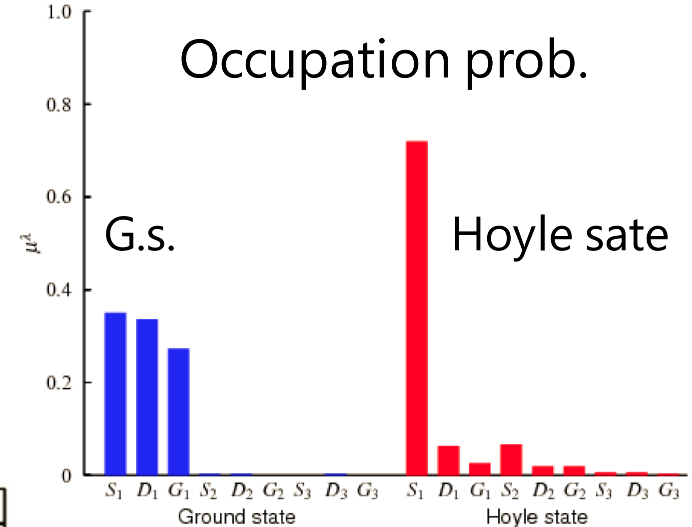
$$F(q) = \sqrt{4\pi} \int d\boldsymbol{x} e^{i\boldsymbol{q}\cdot\boldsymbol{x}} \langle \Phi_{\text{Hoyle}} | \sum_{i=1}^{Z} \delta(\boldsymbol{r}_i - \boldsymbol{x}) | \Phi_{\text{g.s.}} \rangle$$

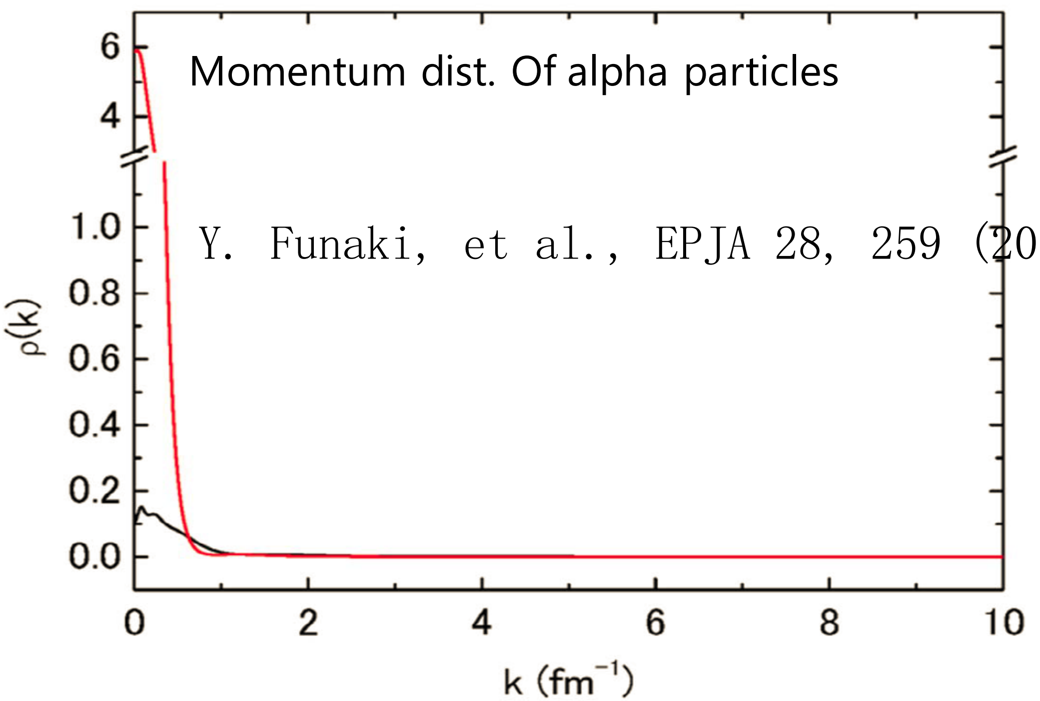


Y. Funaki, et al., Eur. Phys. J. 28, 259 (2006).

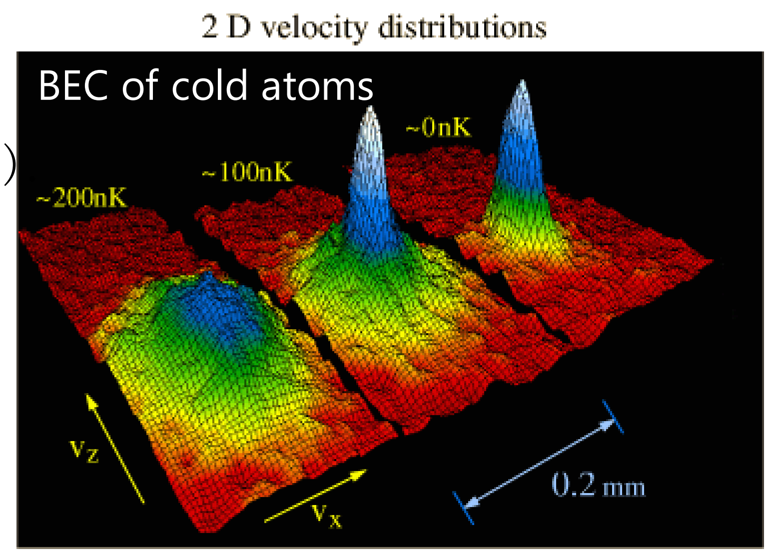
A new insight to the Hoyle state in this century

○ All alpha particles mostly occupy the 0S orbit
○ Momentum distribution of alpha particles are concentrated to 0. (Quite analogous to BEC of cold atoms)



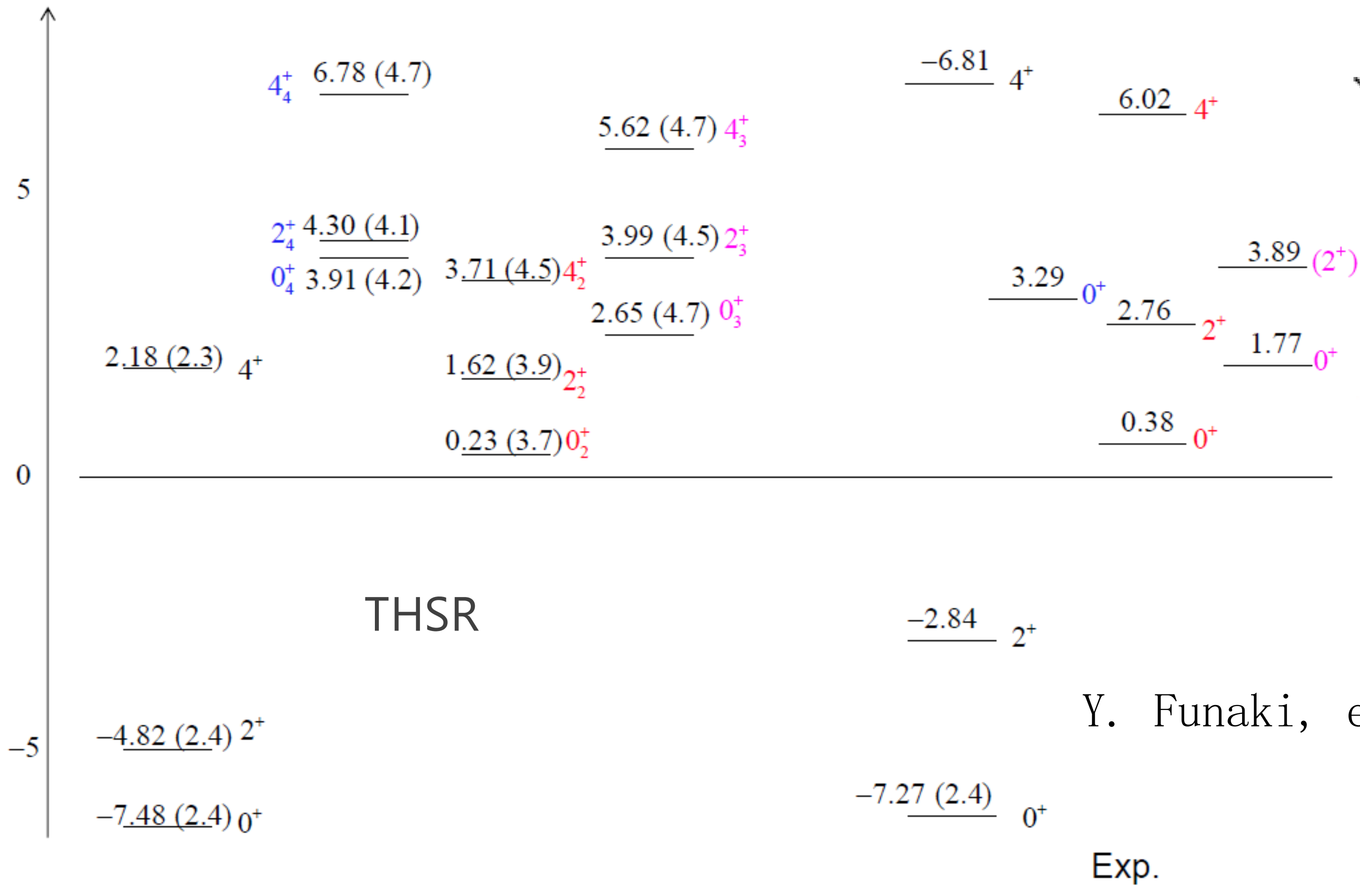


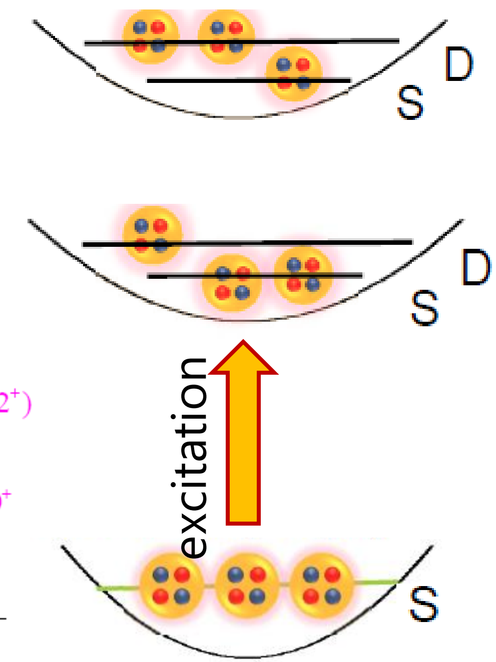
Y. Funaki, et al., EPJA 28, 259 (2006)



A new insight to the Hoyle state in this century

- If the Hoyle state can be regarded as a BEC, its excitation modes should be observed





Hoyle state

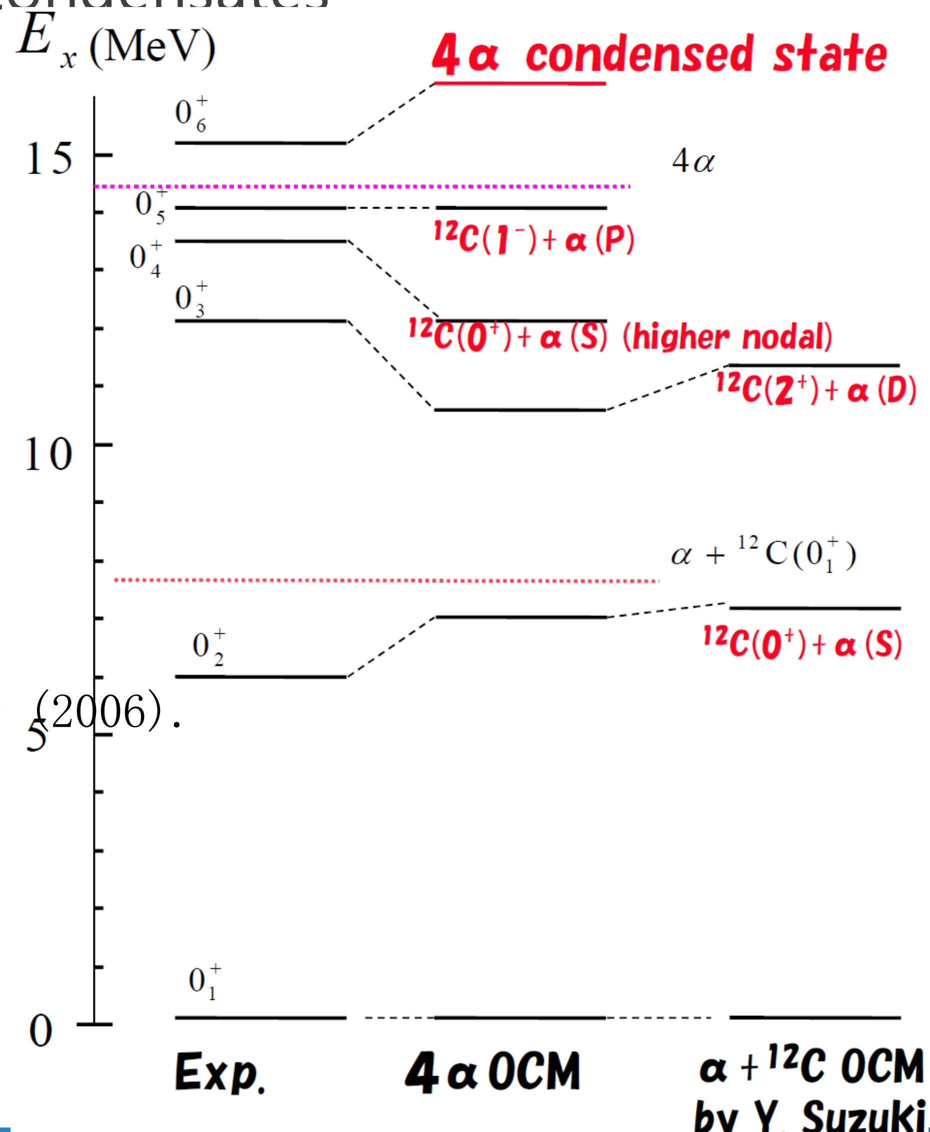
Y. Funaki, et al., EPJA 28, 259 (

Many Chinese (Nanjing U.) & Japanese researcher are working together

A new insight to the Hoyle state in this century

- There should be 4a, 5a condensates

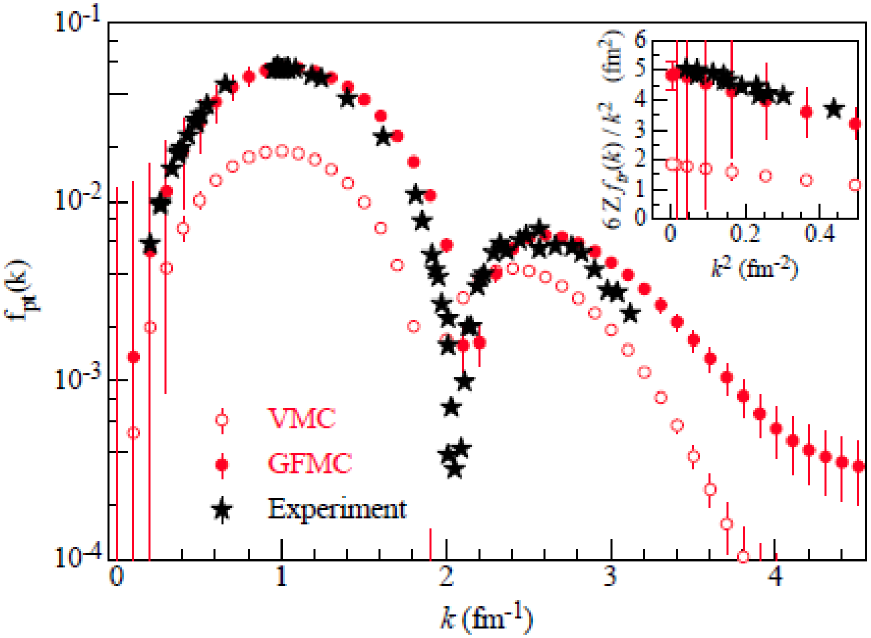
- A candidate for 4a condensate is theoretically predicted and experimentally suggested.



Y. Funaki, et al., EPJA 28, 259 (2006).

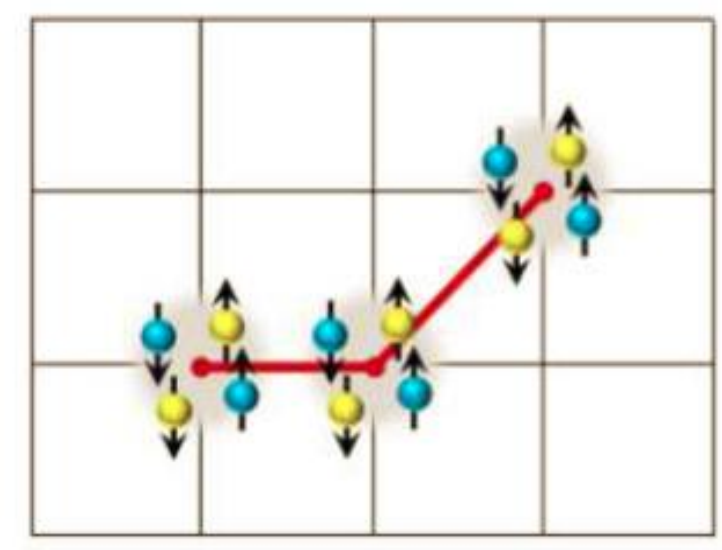
Ab-initio calculations for the Hoyle state

GFMC (AV18 + ILL)



J. Carlson, et al,
Rev. Mod. Phys. 87, 1067 (2015).

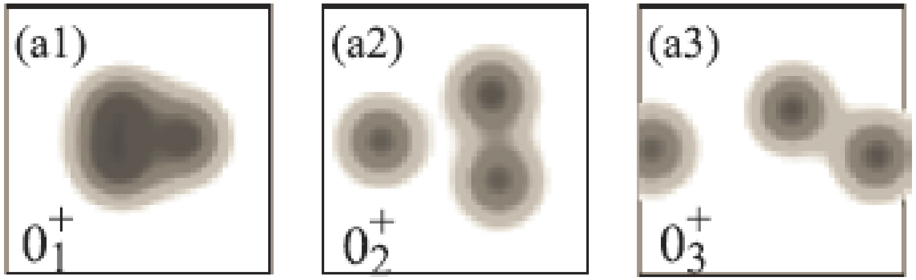
QMC (EFT)



E. Epelbaum, et al.,,
Phys. Rev. Lett. 109, 252501 (2012)

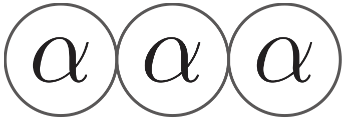
Summary of the Hoyle state

- The Hoyle state was predicted to explain 12C synthesis in the stars
- The linear-chain structure was proposed as the structure of the Hoyle state. But, it failed to explain the lifetime.
- Model calculations showed that the Hoyle state is a bosonic gas-like state
- It was found that the Hoyle state is the BEC of alpha particles
- Based on this idea, 4a, 5a condensates is being interested in.
- Linear chain is considered to be unstable in 12C



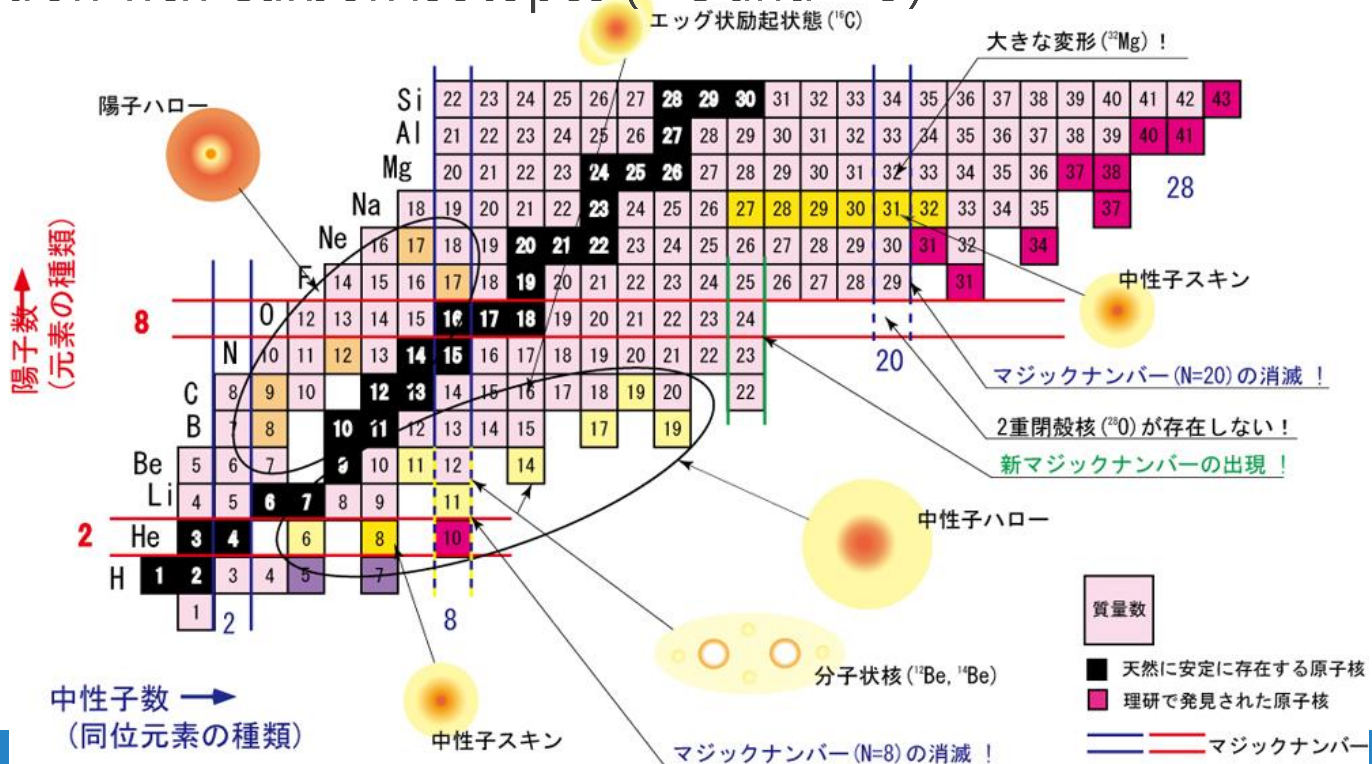
Recent story of the Linear Chain

- Does the Linear Chain state exist?



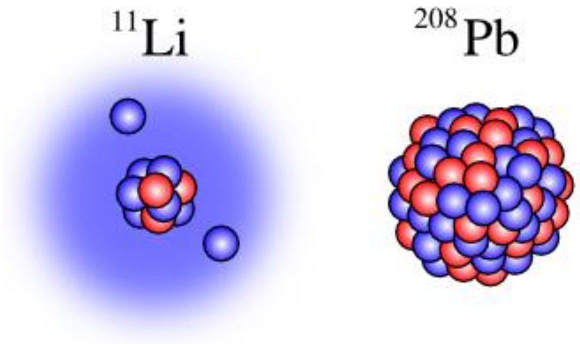
◎ Linear Chain proposed by Porf. Morinaga cannot be the structure of the Hoyle state (Lifetime)

◎ Recently, the Linear Chain story was revived in the study of the neutron-rich Carbon isotopes (^{14}C and ^{16}O)



Introduction: Clusters in extreme condition

○ Important ingredients of Ikeda diagram
  - Saturation of "energy density"
  - Saturation of "matter density"



○ Both of them breakdown in neutron-rich nuclei

○ Symmetry energy must be taken into account

$$E_B = a_V A - a_S A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - \underline{a_A \frac{(A-2Z)^2}{A}} \pm \delta(A,Z)$$

Introduction: Clusters in extreme condition

What will happen?

○ Energy/Matter densities should be kept constant

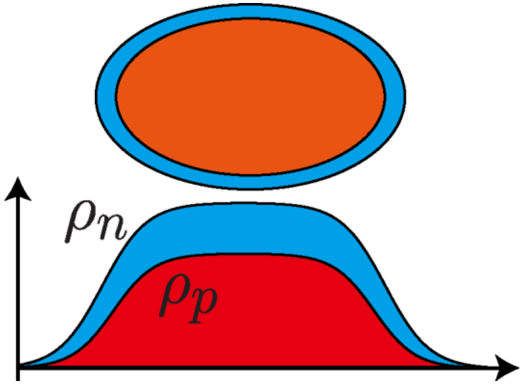
○ Proton and neutron overlap should be maximized to minimize symmetry energy

To possible ways to achieve them

1. Mean-field with neutron skin

○ Densities are **"globally"** kept almost constant

○ Symmetry energy is **"globally"** minimized



Introduction: Clusters in extreme condition

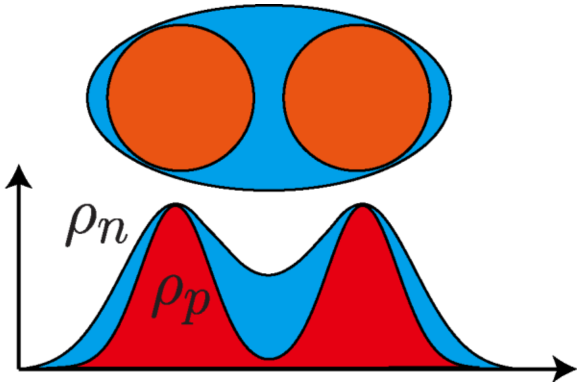
What will happen?

- ◯ Energy/Matter densities should be kept constant
- ◯ Proton and neutron overlap should be maximized to minimize symmetry energy

To possible ways to achieve them

2. Clustering

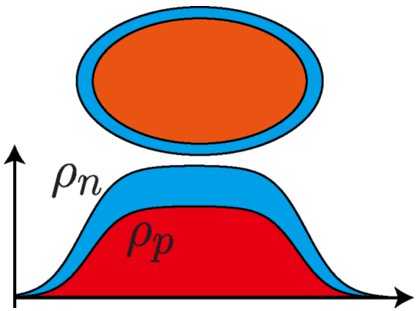
- ◯ Densities are **"locally"** kept constant
- ◯ Symmetry energy is **"locally"** minimized



Introduction: Clusters in extreme condition

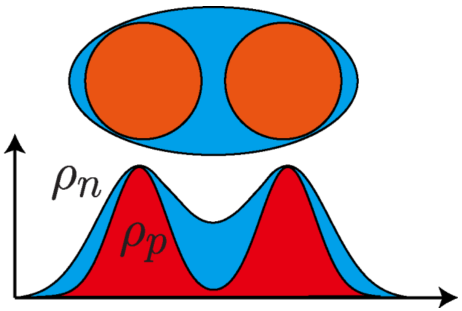
Which is better?

"Global" v.s. "Local"
"Neutron Skin" v.s. "Cluster"

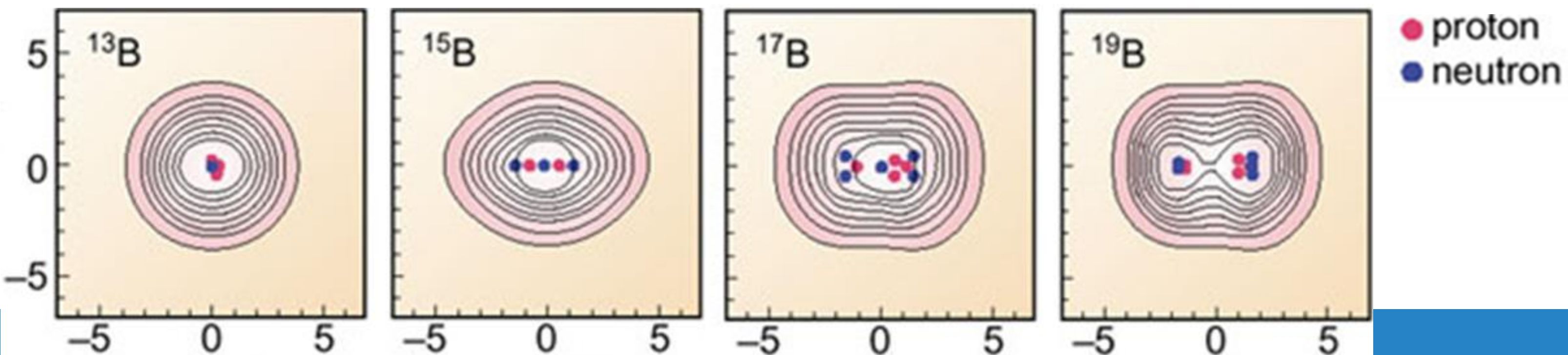


A possible theoretical answer

"neutron skin" ⇒ "clusters with skin"

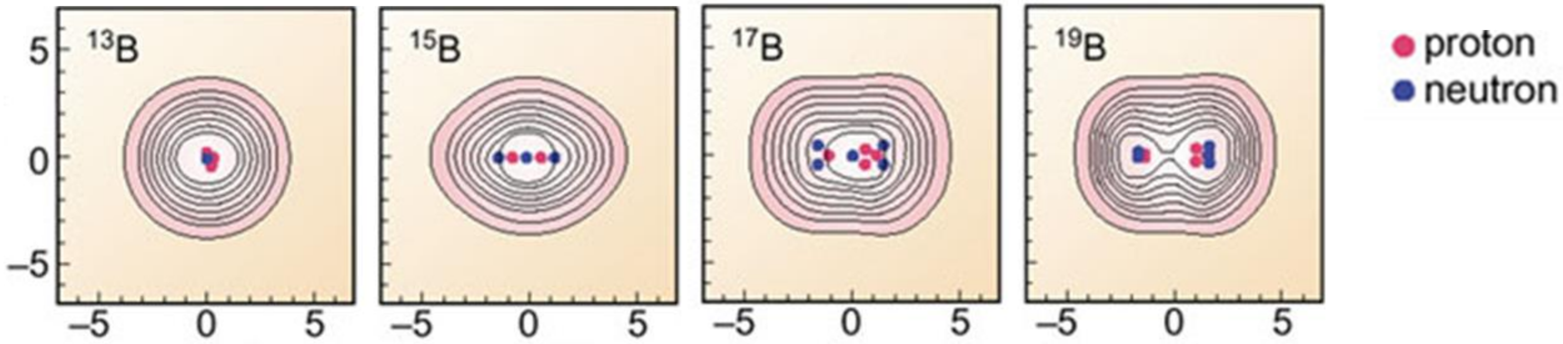


Y. Kanada-En'yo and H. Horiuchi, PRC52, 647(1995).



Introduction: Clusters in extreme condition

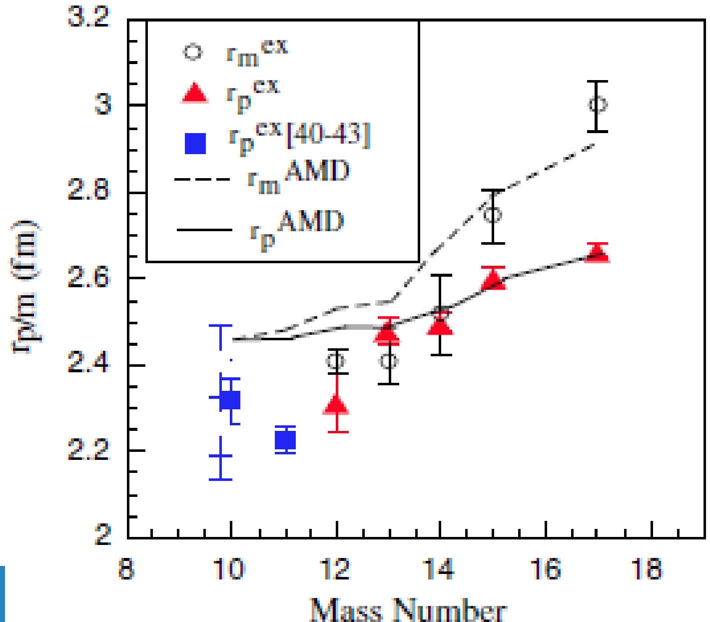
Experimental data



Observed increase of charge radius can be attributed to clustering

Charge radius of B isotopes:

A. Estrade, et al. PRL113, 132501 (2014).

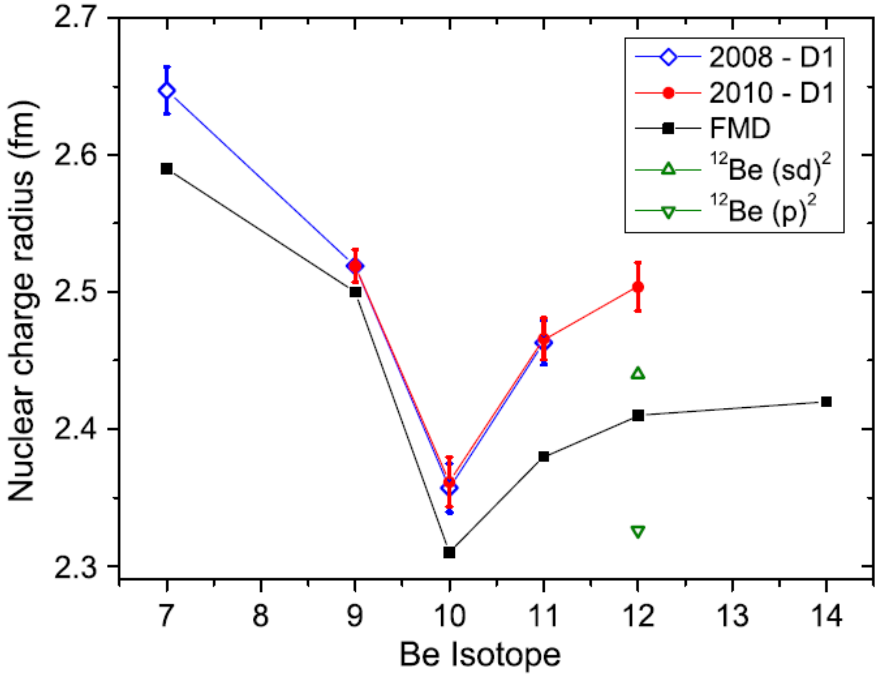


Introduction: Clusters in extreme condition

Experimental evidence for clustering

Charge radius (proton distribution radius) of Be isotopes

A similar behavior of charge radius was also reported in Be isotopes



A. Krieger, et al., PRL108, 142501 (2012)

What is behind the clustering in neutron-rich nuclei?

Molecular Orbits

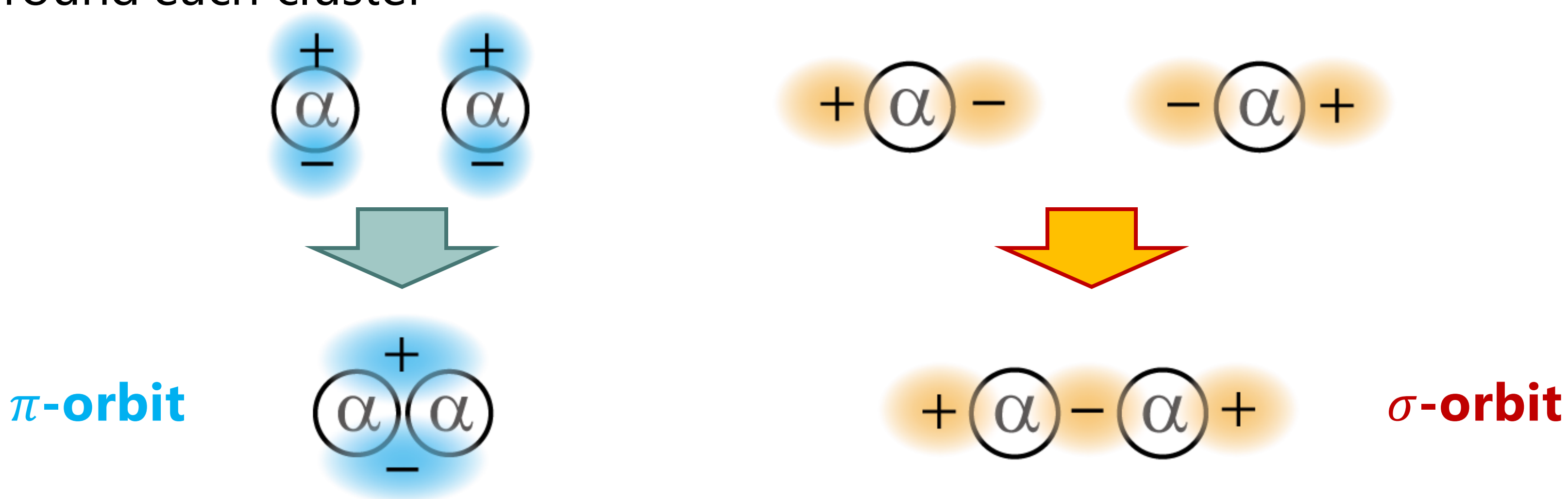
Underlying quantum shell effect; “molecular-orbits”

Herzberg G “molecular spectra and Molecular structure”, van Nostrand, Princeton (1950)

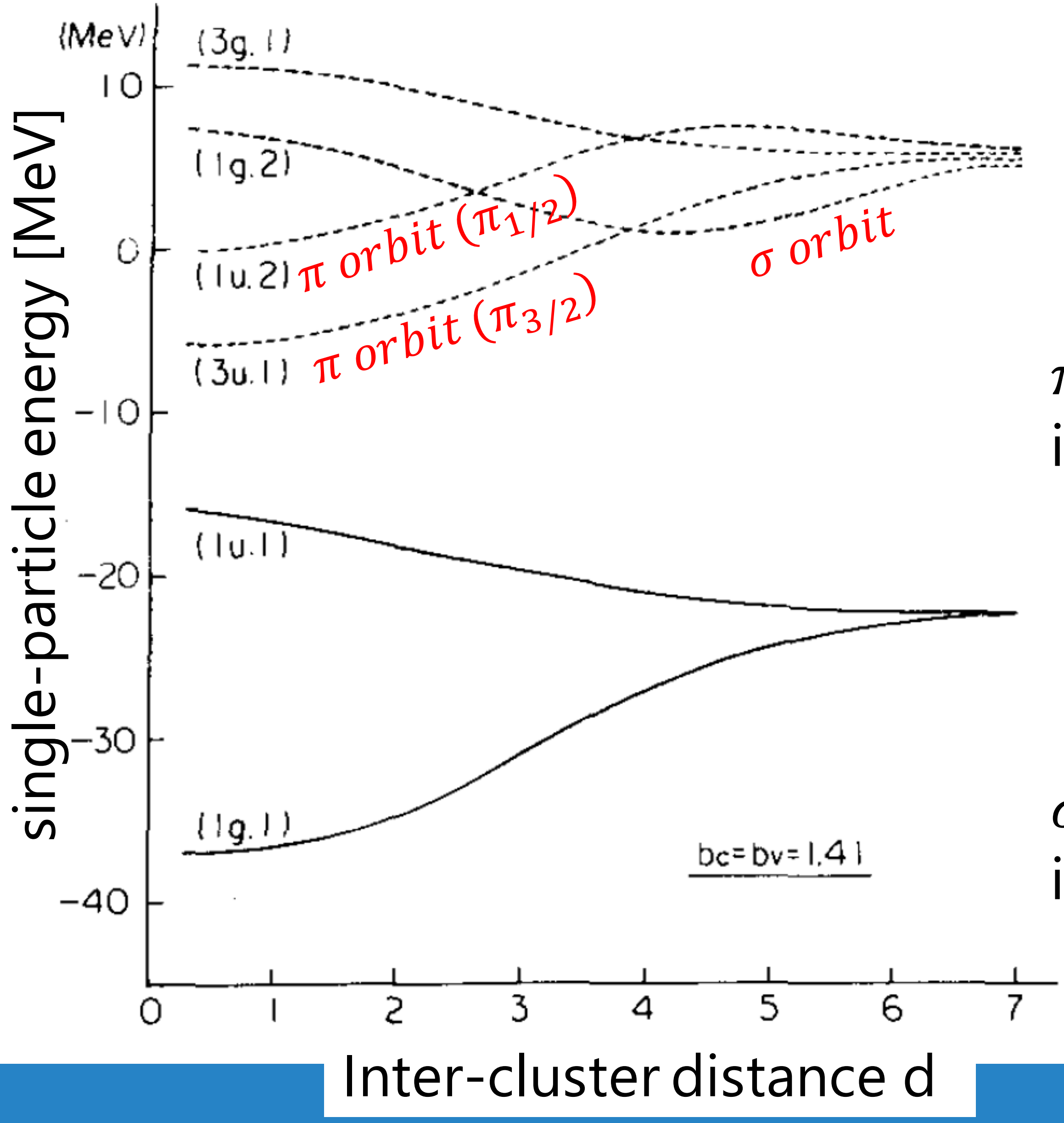
M. Seya, et al., PTP65, 204 (1981) W. von Oertzen et al., Phys. Rep. 432, 43 (2006).

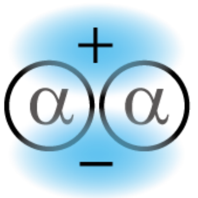
A special class of valence neutron orbits (molecular-orbits; MO) are formed around the clustered core

MO are described by the Linear Combination of Atomic Orbits (LCAO) around each cluster

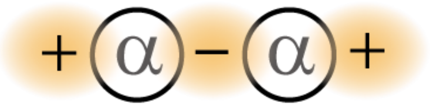


Molecular Orbits





π-orbit prefers shorter inter-cluster distance

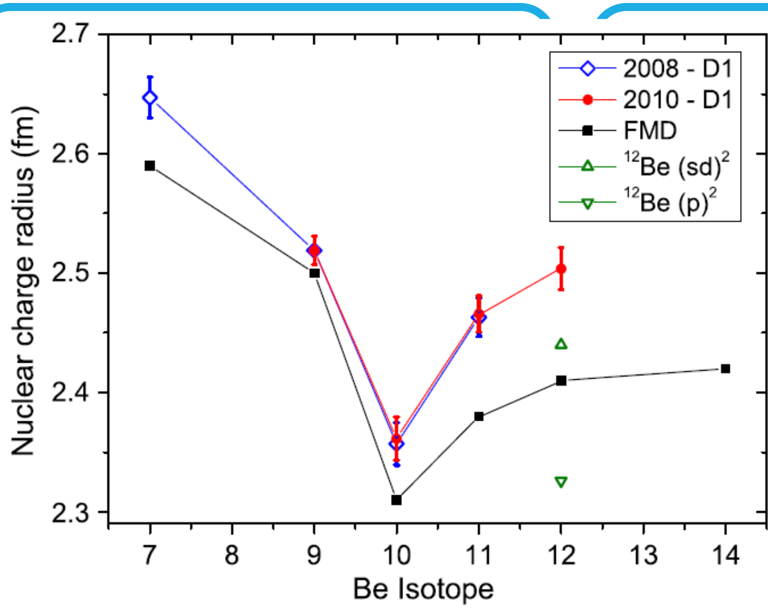


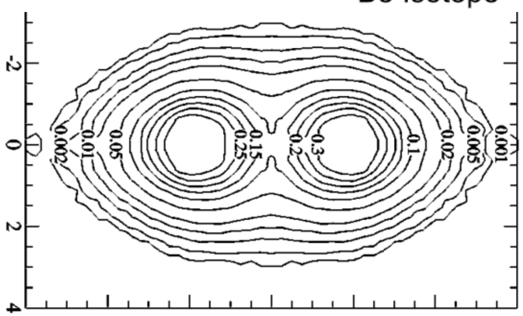
σ-orbit prefers longer inter-cluster distance

Molecular Orbits

It has been shown that the combination of π and σ orbits reasonably explains low-lying states of Be

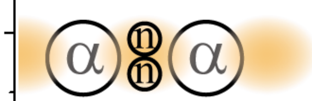
N. Itagaki, et al., PRC62 034301 (2000)





^{10}Be(^{8}Be+2n)

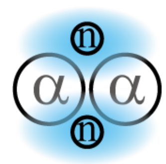
$(\sigma_{1/2})^2$



unced cluster

$(\pi_{3/2})^2$

0_1^+

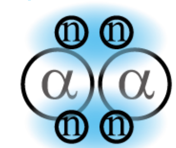


reduced cluster

^{12}Be(^{8}Be+4n)

$(\pi_{3/2})^2(\pi_{1/2})^2$

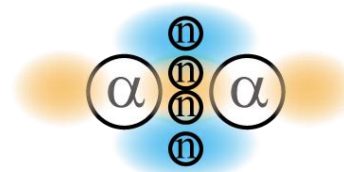
0_2^+



reduced cluster

$(\pi_{3/2})^2(\sigma_{1/2})^2$

0_1^+



pronounced cluster

Molecular Orbits

- Charge radii of Be isotope is reasonably explained by the molecular orbit
  - ◎ 2 α cluster core surrounded by the valence neutrons
  - ◎ Valence neutrons stabilizes 2α cluster core
  - ◎ π-orbit reduces the clustering, while σ-orbit enhances

- Molecular orbit states do not follow Ikeda diagram

- Molecular orbit is a novel type of the clustering and it stabilizes the cluster structure

How about the linear-chain state?

Linear Chains with MO

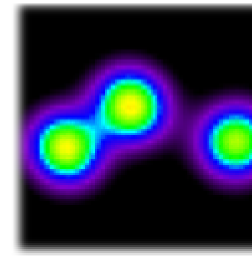
An interesting extension of MO is linear-chain of 3α

H. Morinaga, Phys. Rev. 101, 254 (1956).

○ Linear chain of 3α does not exist in ^{12}C

Y. Kanada-En'yo, PRL81,   T. Neff et al, PRL105

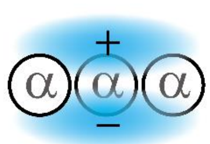
0^+ state above the Hoyle state is bent-armed 3α (not linear!)



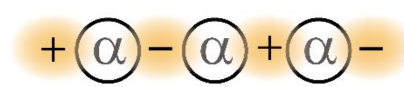
○ MO will stabilize the chain in 3α + neutrons system

A pilot study by N. Itagaki et al., PRC64, 014301 (2001)

π-orbit



σ-orbit

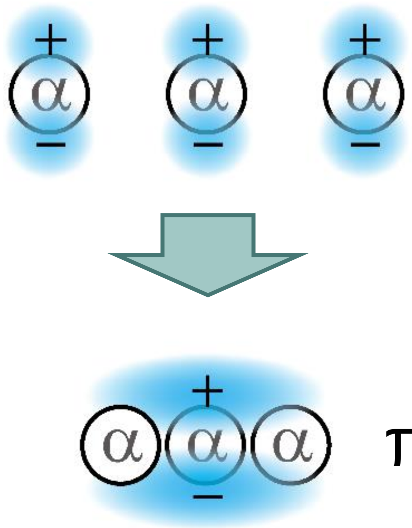


○ A series of recent experiments and MD calculations showed that the linear-chain formation in ^{14}C is convincing

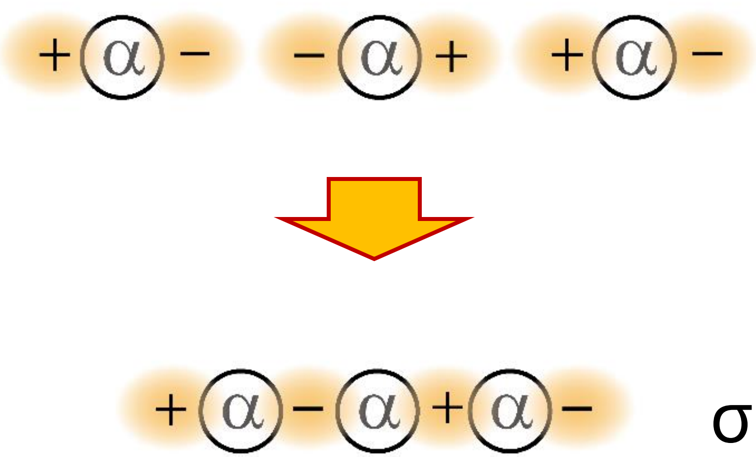
Linear Chain in neutron-rich nuclei

- Valence neutrons stabilizes the linear chain?

Molecular Orbit around the linear chain



π-orbit

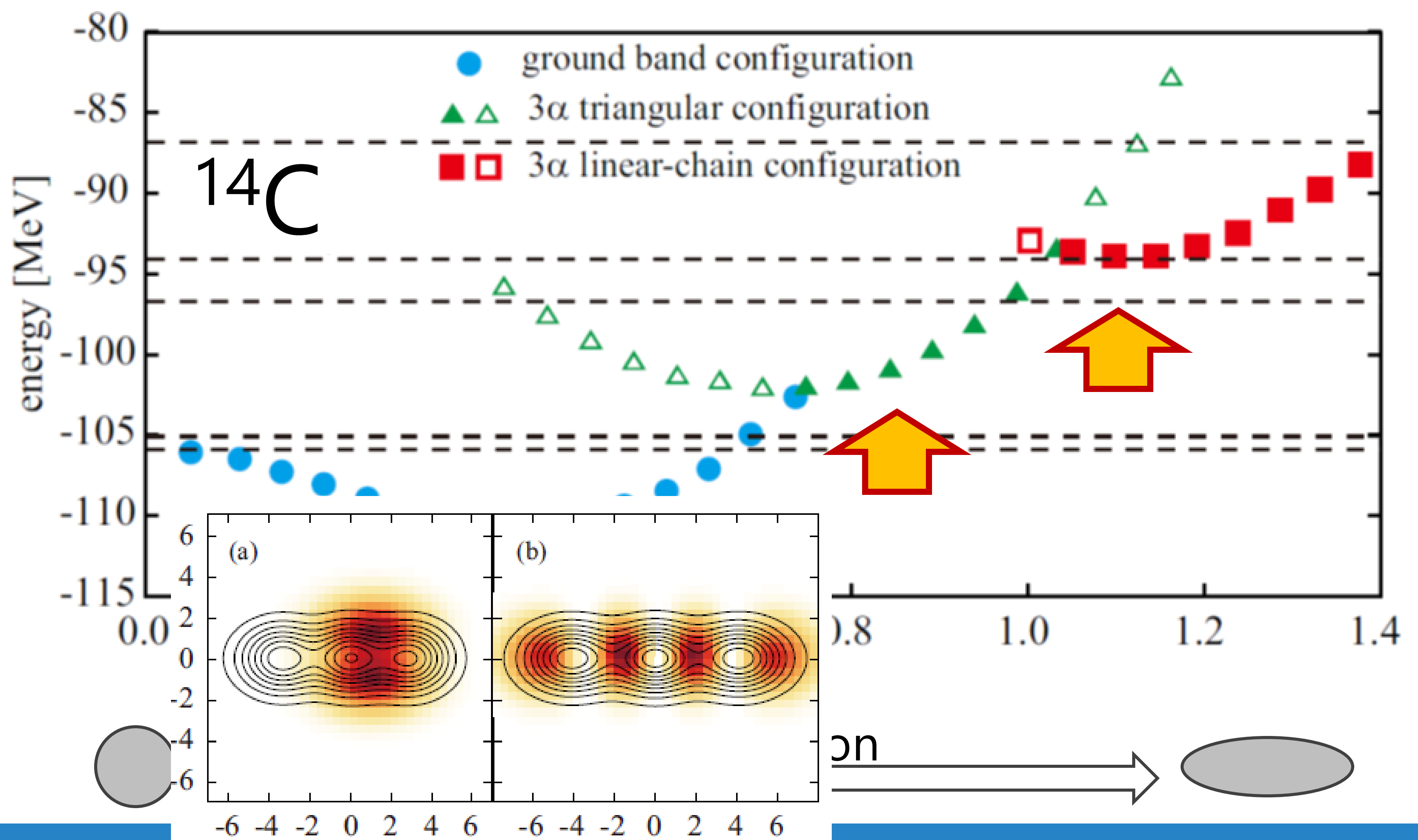


σ-orbit

N. Itagaki et al., PRC64, 014301

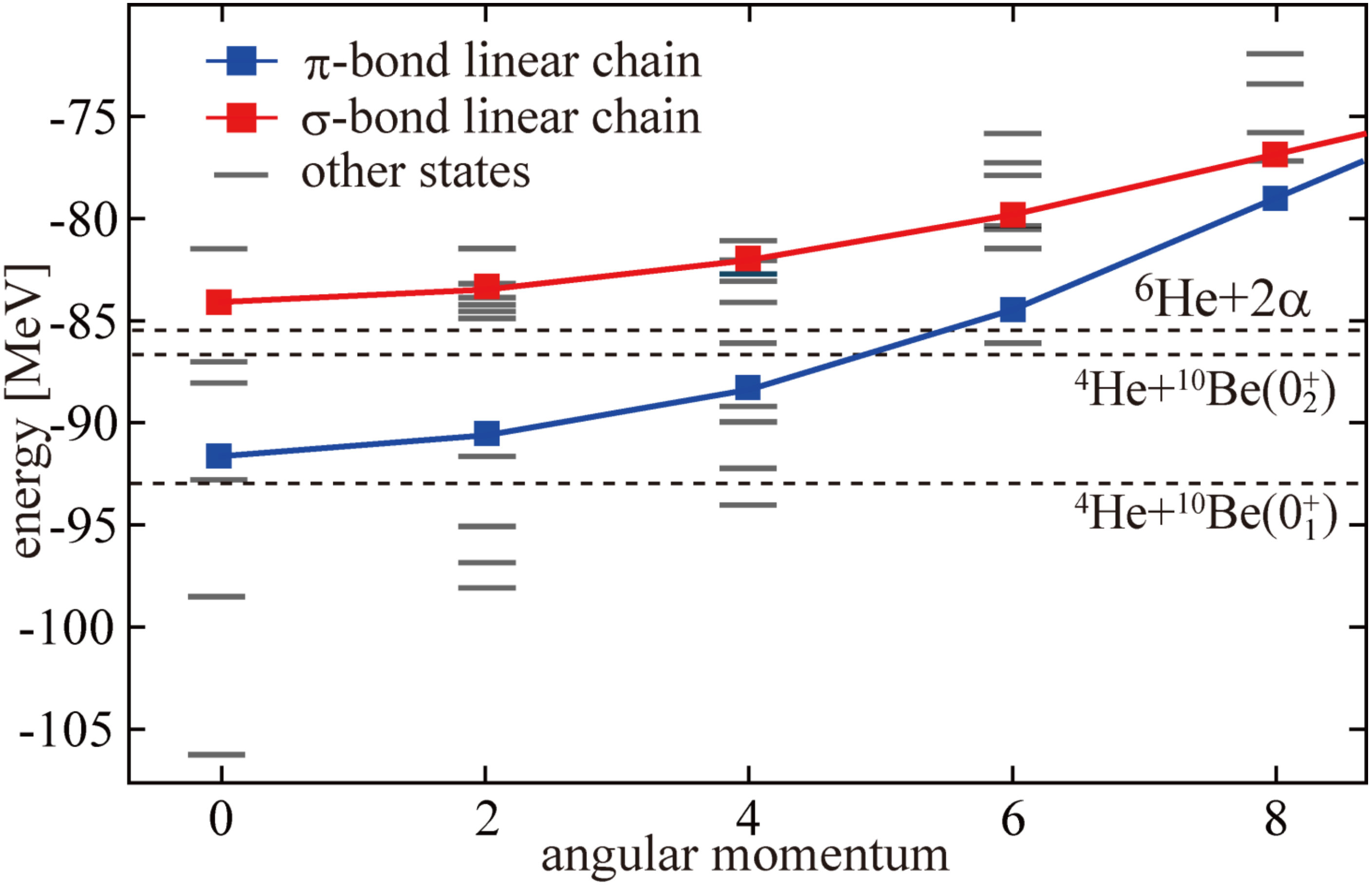
Search for the linear chain

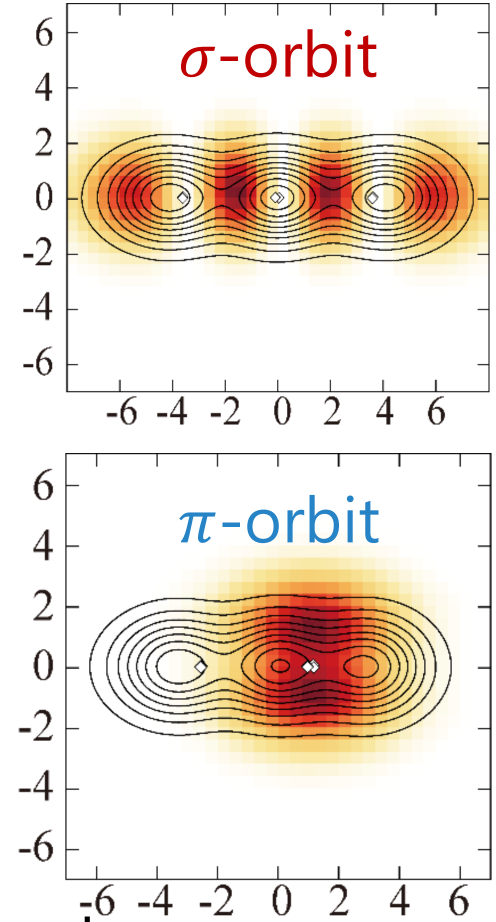
- Local energy minimum at extremely deformed region



Linear chains in ^{14}C (Theory)

T. Baba et al., PRC94, 044303 (2016) , PRC95, 064318 (2017)



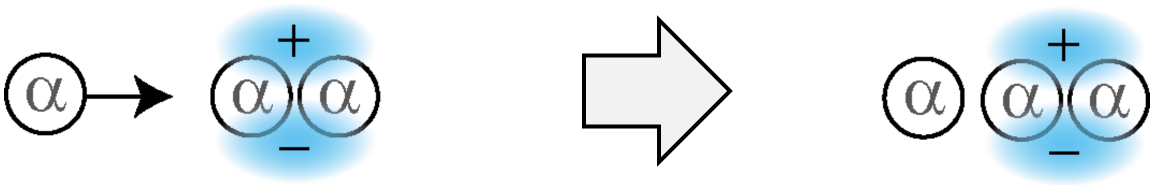


○ AMD calculation predicts two linear-chain bands

○ Valence neutron orbits are interpreted as π- and σ-orbits

Evidence for Linear-chain

① Linear chain should be observed as the resonances of $^{10}Be+^{4}He$



^{10}Be has two neutrons in π orbit

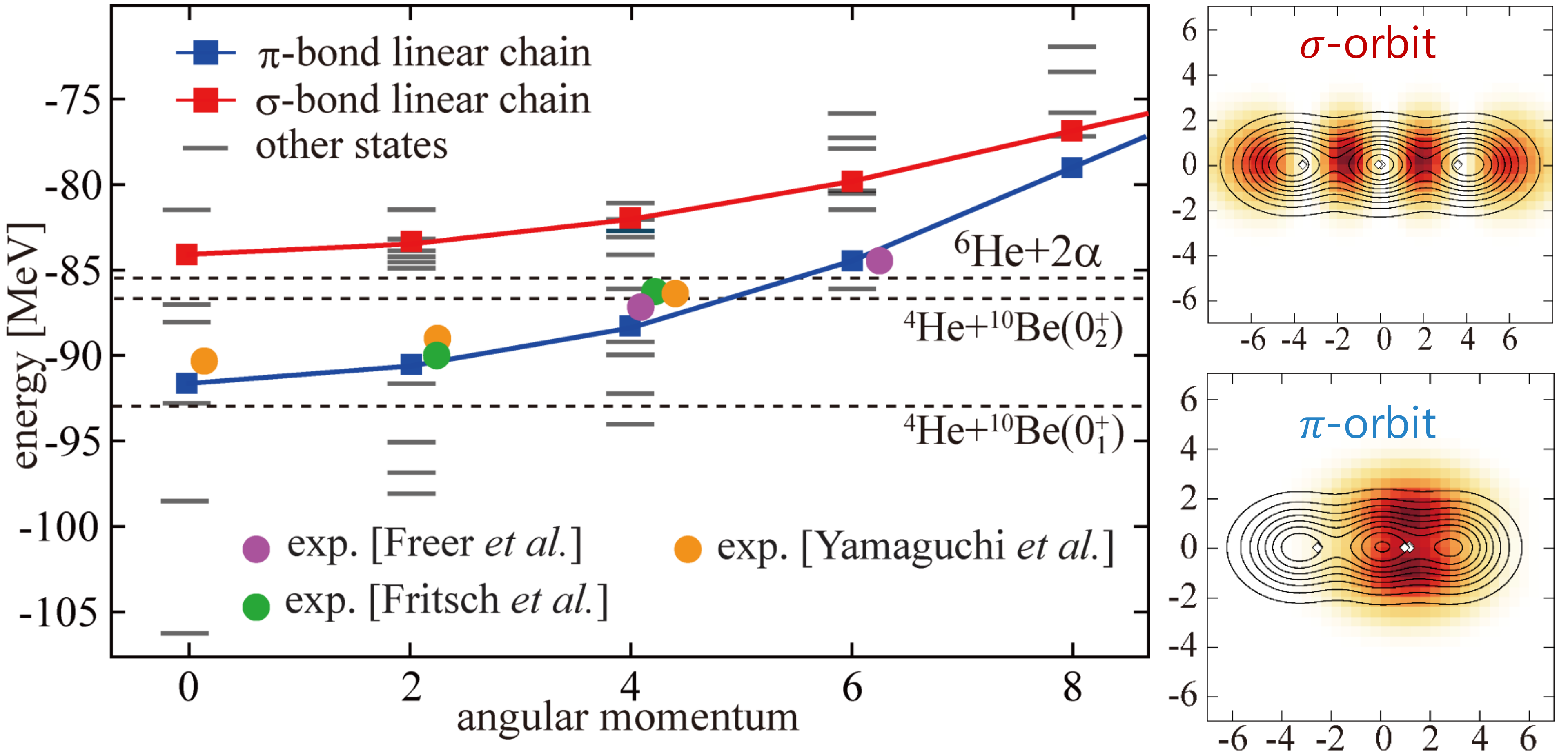
$^{10}Be+^{4}He$ resonance scattering should populate pi-bond linear chain

Experiments were performed

- M. Freer et al., PRC90, 054324 (2014)
- A. Fritsch et al., PRC93, 014321 (2016)
- H. Yamaguchi et al., PLB766, 11 (2017)

Linear chains in ^{14}C (Theory)

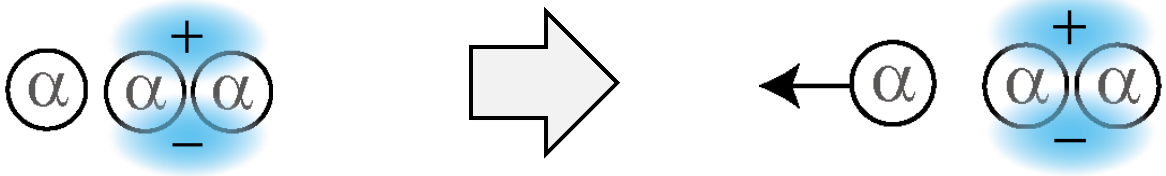
M. Freer et al., PRC90 (2014), Fritsch et al., PRC93 (2016), H. Yamaguchi et al., PLB766 (2017)



○ Observed resonances (α+^{10}Be resonant scattering) agree with π-bond linear chain ⇒ Huge moment-of-intertia !

Evidence for Linear-chain

② The linear chain should decay by the alpha particle emission



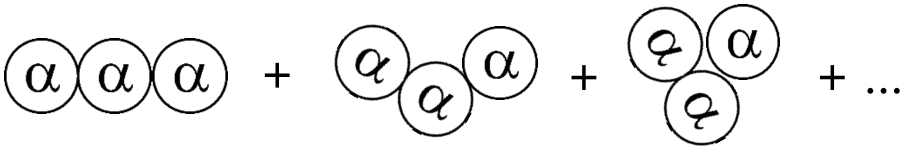
pi-bond linear chain should emit alpha

	π-bond linear chain			Expt. [36]		Expt. [37]		Expt. [39]	
J^π	E_x	Γ_α(5.2 fm)	Γ_α(6.0 fm)	E_x	Γ_α	E_x	Γ_α	E_x	Γ_α
0^+	14.64	250	179					15.07	760
2^+	15.73	214	188	(17.95)	(760)	15.0	290	16.22	190
4^+	17.98	149	147	18.22	200	19.0	340	18.87	45
6^+	21.80	123	151	20.80	300				
8^+	27.25	77	120						

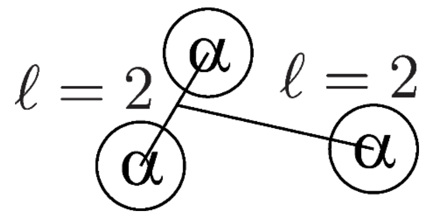
Evidence for Linear-chain

③ The linear chain should also decay to $^{10}Be(2^+)$ state

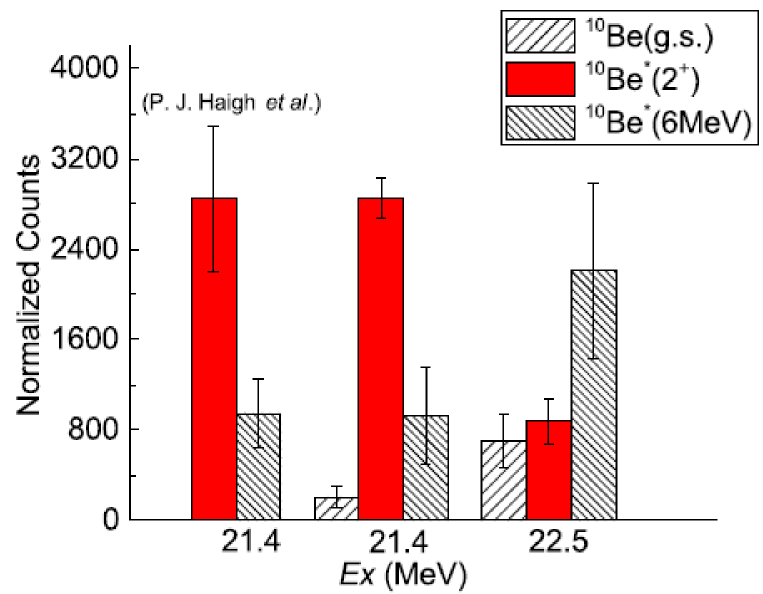
○ $^{8}Be(0^+)$ + α particle should be like this. It's is not a linear chain.

$\Psi_{^{12}\mathrm{C}}(0^+) =$ 

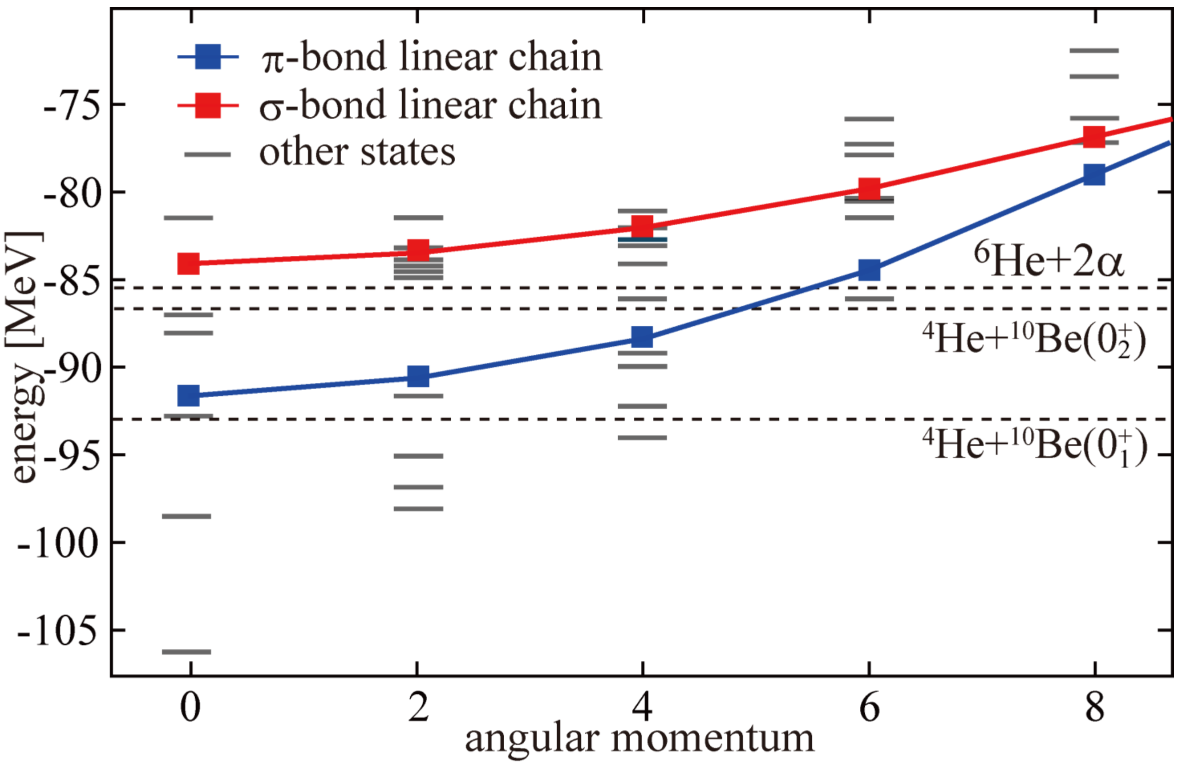
Therefore, the linear-chain is a mixture of $^{8}Be(0^+)$ $^{8}Be(2^+)$ $^{8}Be(4^+)$

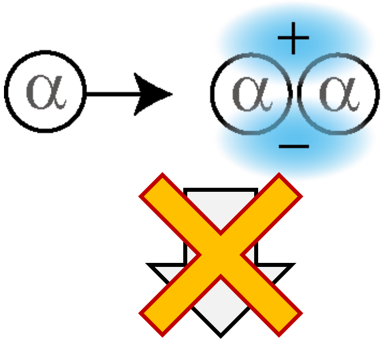


Experiment by Pekin U. group
(Prof. Yanlin Ye)
Li et al., PRC95, 021303(R), (2017)



Evidence for σ-bond Linear-chain





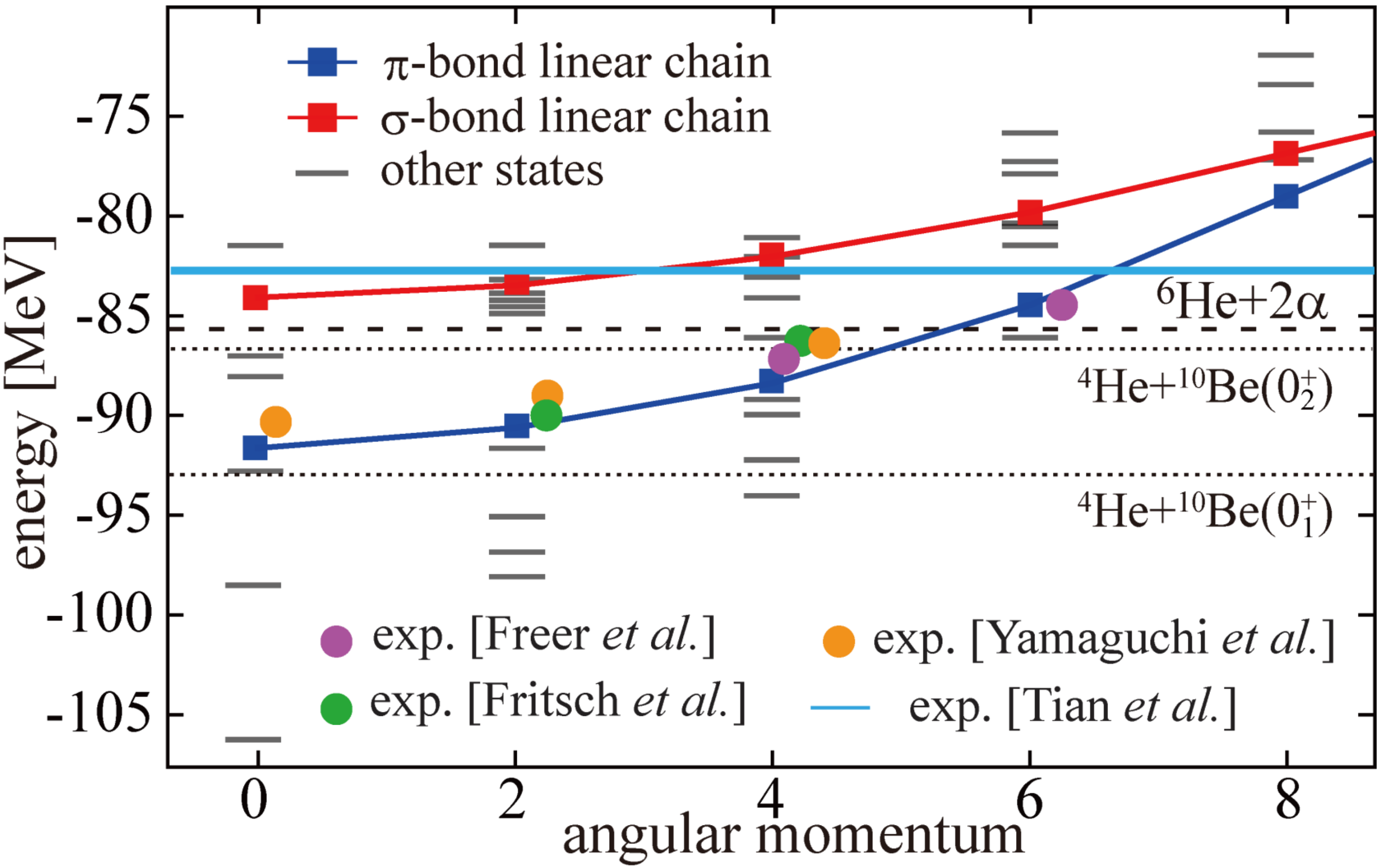
σ-bond linear chain is not easy to populate

Experiment by Pekin U. group, (Prof. Yanlin Ye)
Li et al., PRC95, 021303(R), (2017)

Breakup reaction ^{9}Be(^{9}Be,^{14}C*)^{4}He

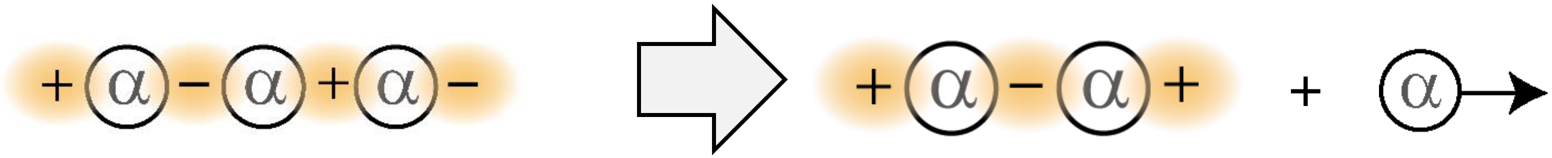
Evidence for σ-bond Linear-chain

- Candidate was reported but no J assignment



Evidence for σ-bond Linear-chain

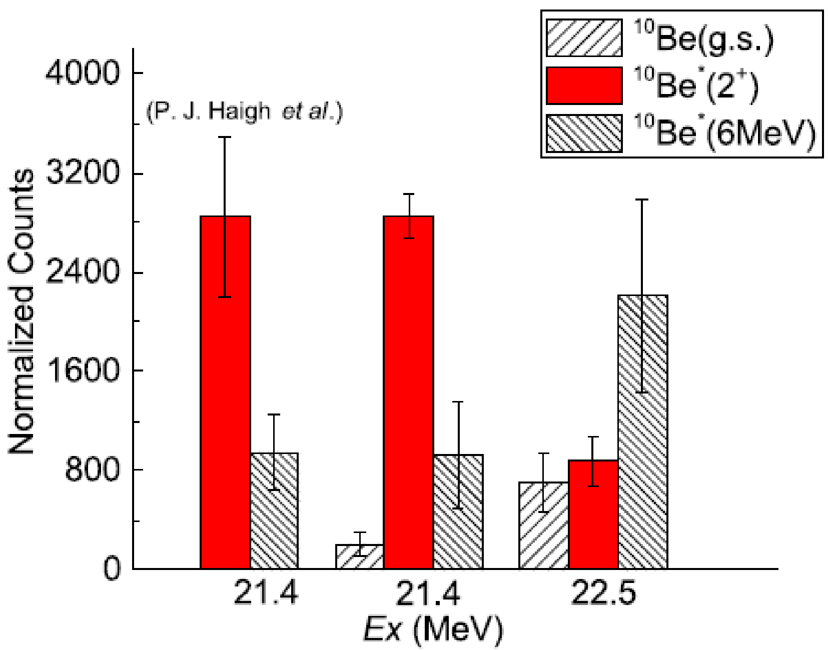
④ σ-bond Linear chain should decay to ^{10}Be*

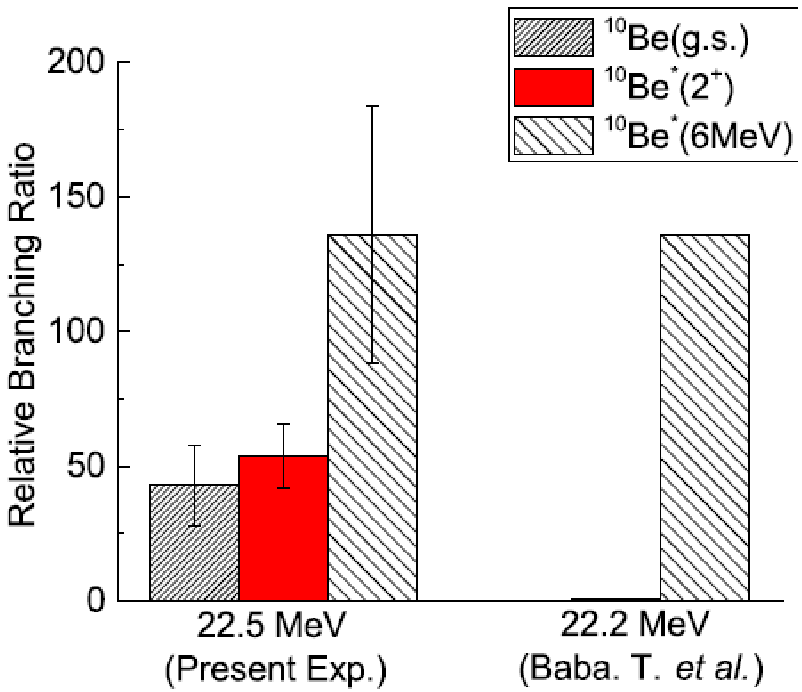


σ-bond linear chain

^{10}Be* with σ-bond

Li et al., PRC95, 021303(R), (2017)





Summary for the linear chain

- Valence neutrons in molecular orbit play crucial role to explain the clustering in neutron-rich nuclei

- Valence neutrons stabilize the clusters by their glue-like role

- Linear chains can be stabilized by the assist of the valence neutrons

- For ^{14}C, theories and experiments look being consistent for the linear-chain formation

- For ^{16}C, theories predicts most stable linear-chain