Nuclear Reactions

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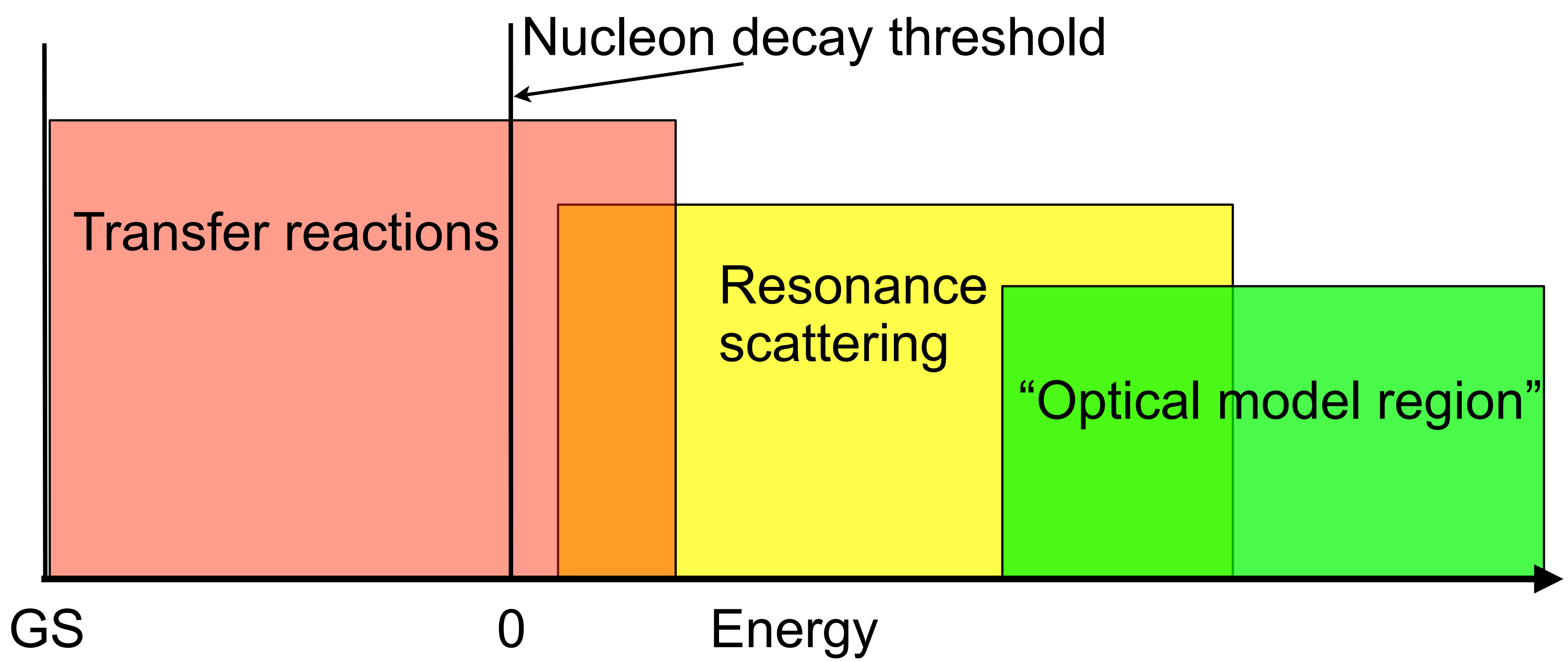
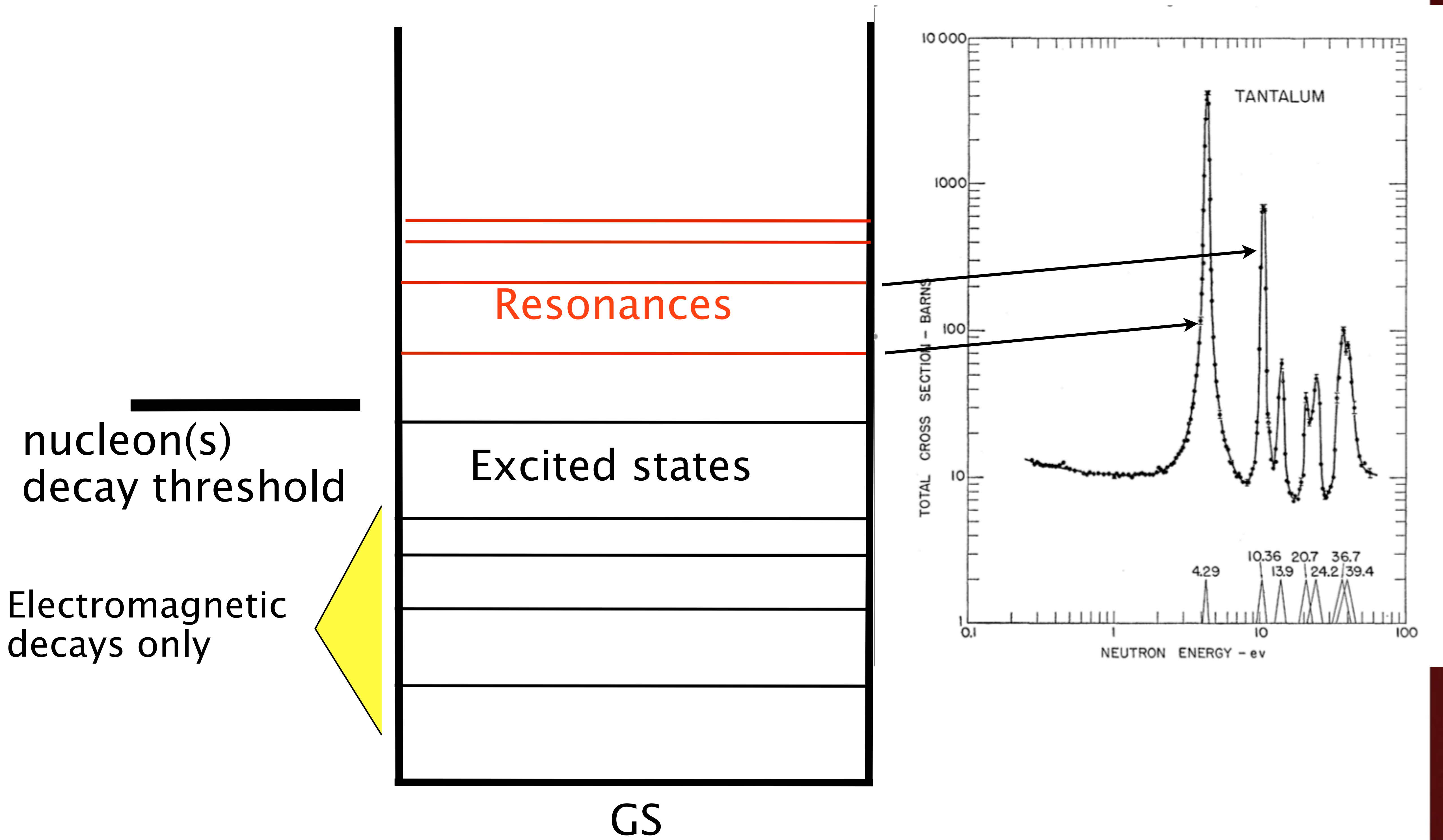
Part 3. Resonance reactions

General outline

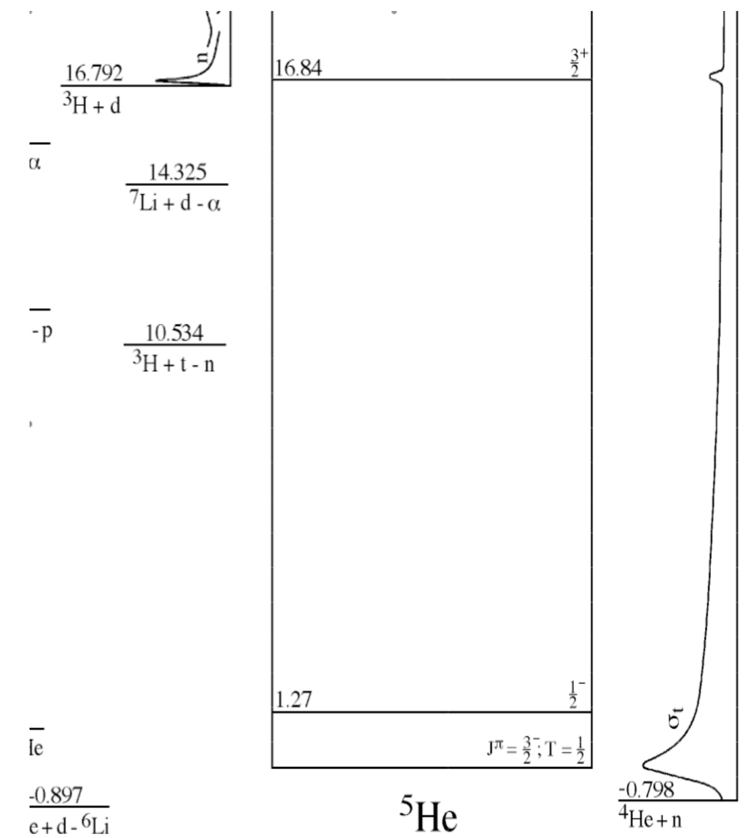
- Part 0. Introduction to scattering
- Part I. Elastic and scattering
- Part II. Transfer Reactions
- Part III. Resonance scattering





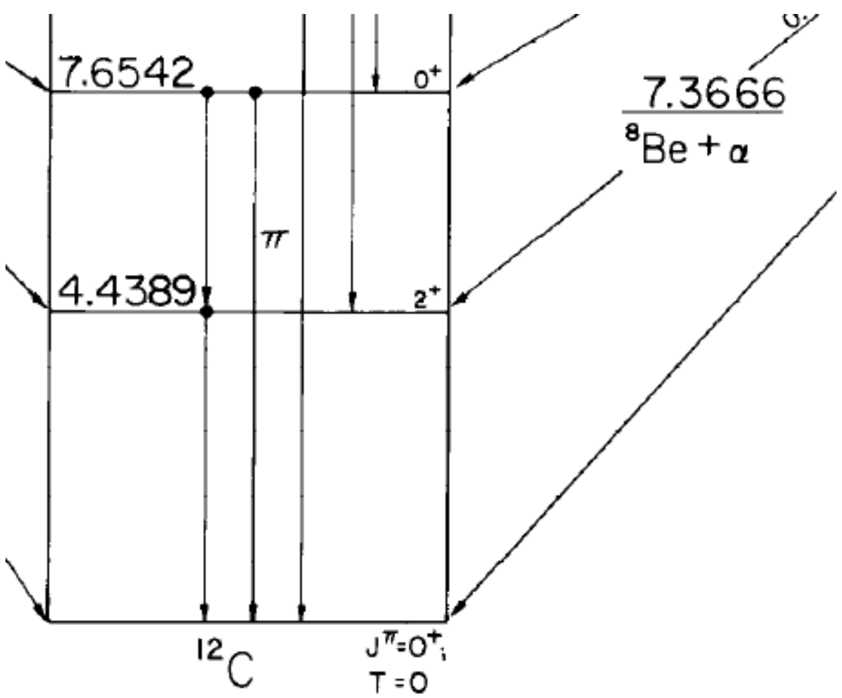
Resonances
nucleon(s)
decay threshold
Excited states
Electromagnetic
decays only
GS
TANTALUM
TOTAL CROSS SECTION - BARNS
NEUTRON ENERGY - ev
10 000
1000
100
10
1
0.1
1
10
100
4.29
10.36
13.9
20.7
24.2
36.7
39.4

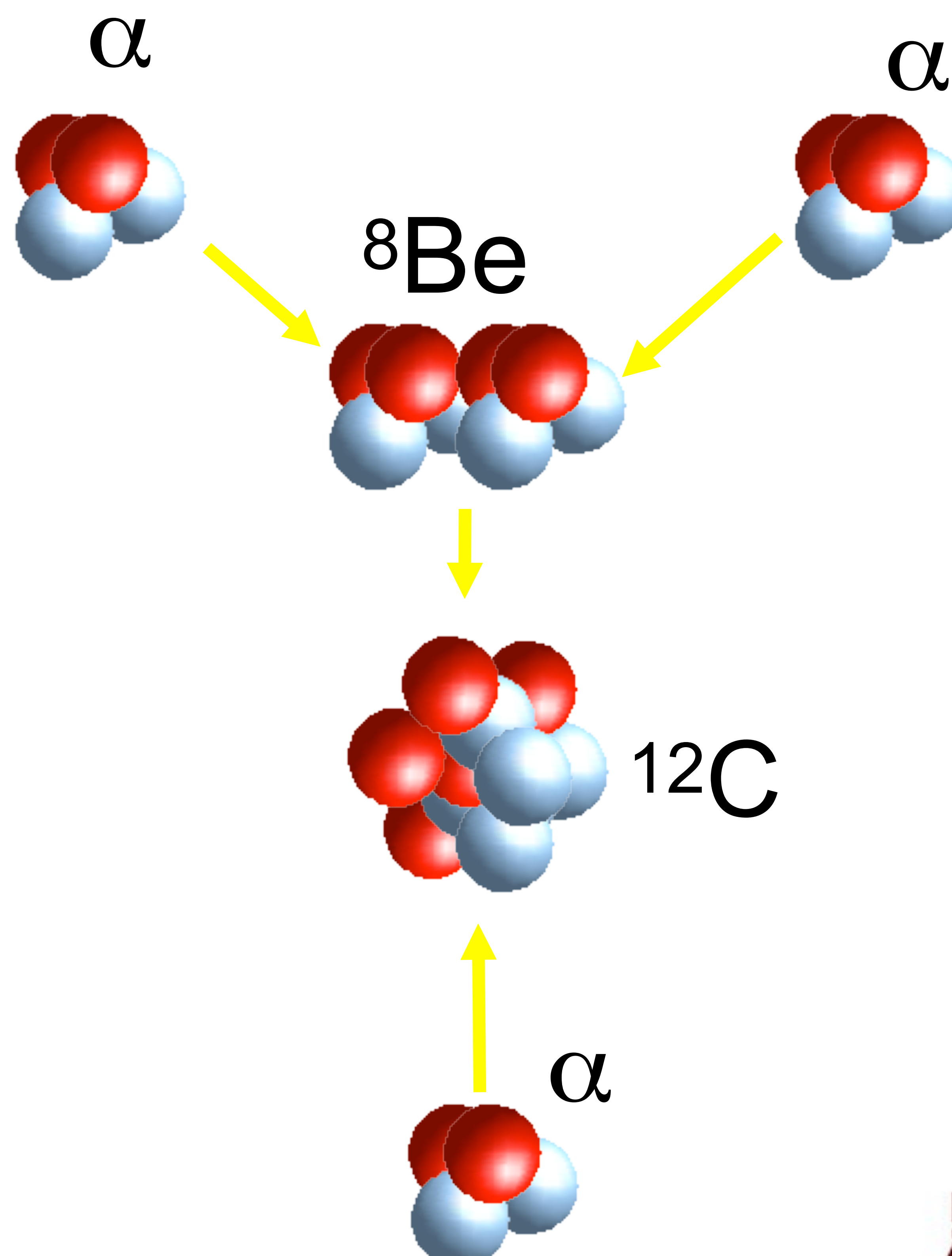
Practically unlimited yield of thermonuclear explosion is possible due to resonance in ^{5}He!



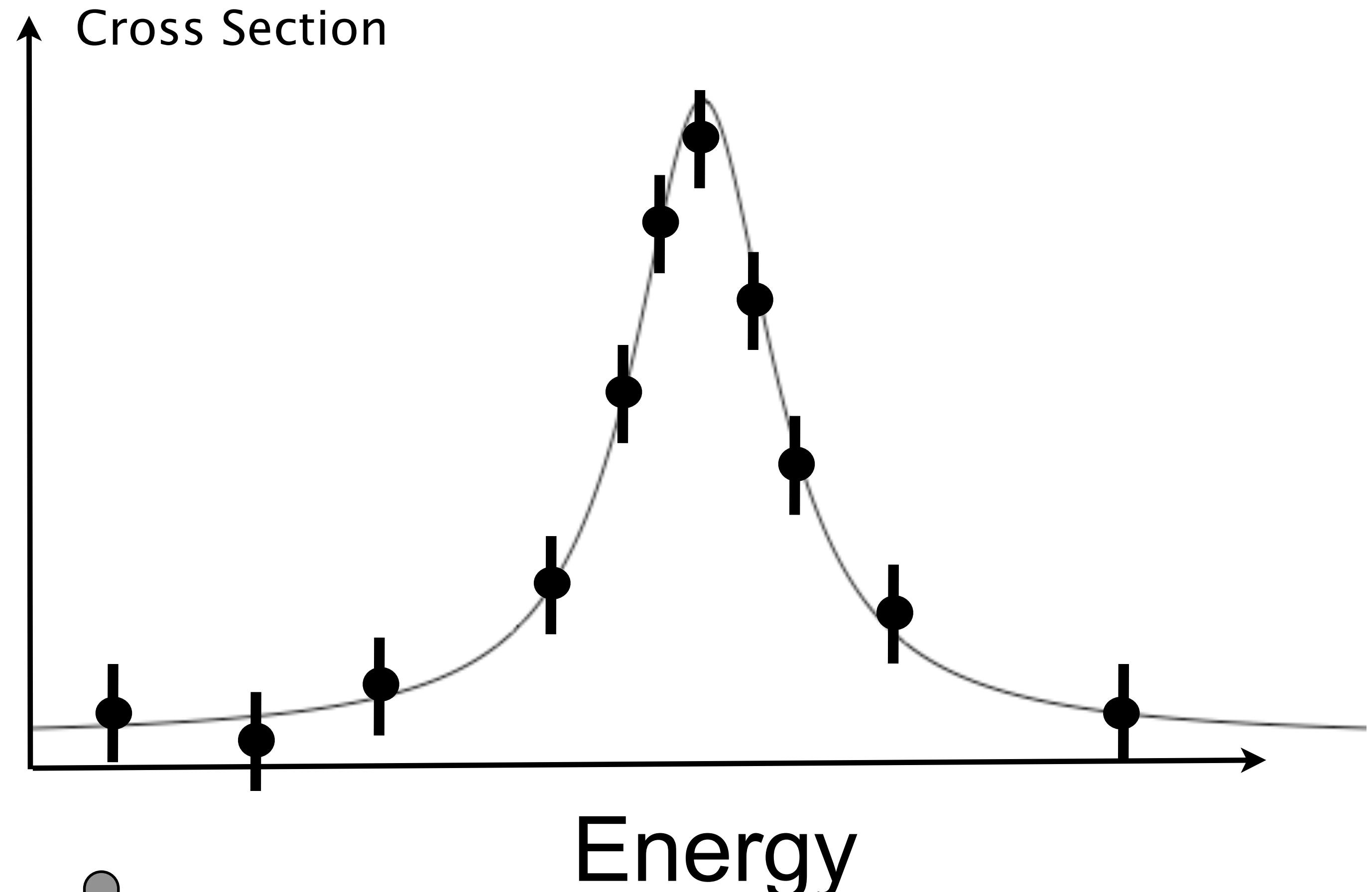
$^{3}H + ^{2}H \rightarrow ^{4}He + n + 17.8$ MeV

Hoyle state in ^{12}C at 7.65 MeV is responsible for production of ^{12}C in red giants and ultimately for our existence





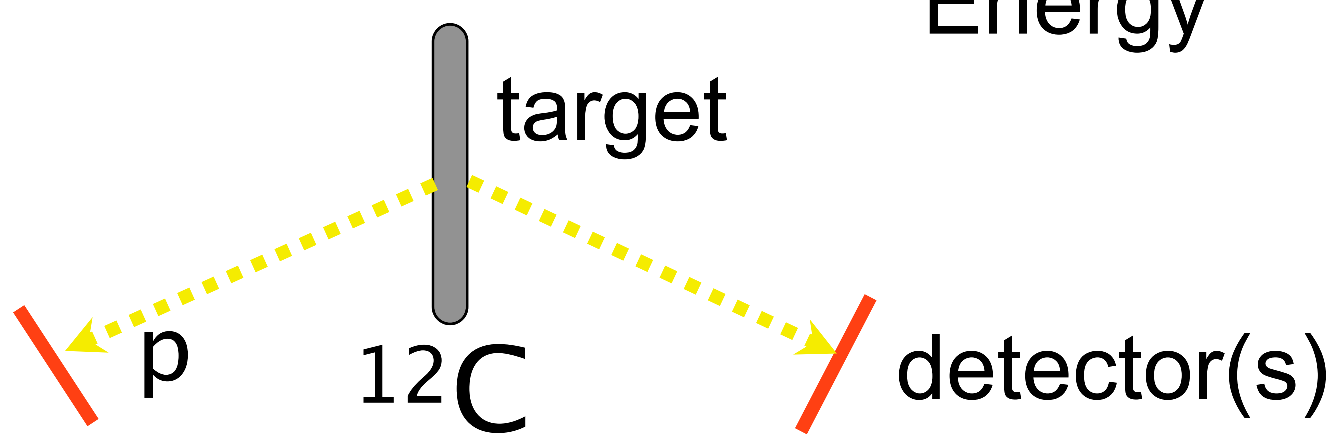
Observation of a resonance in an elastic scattering



E=3.1 MeV
E=3.0 MeV
E=2.9 MeV
E=2.8 MeV
E=2.7 MeV
E=2.6 MeV
E=2.5 MeV
E=2.4 MeV
E=2.2 MeV
E=2.1 MeV
E=2.0 MeV

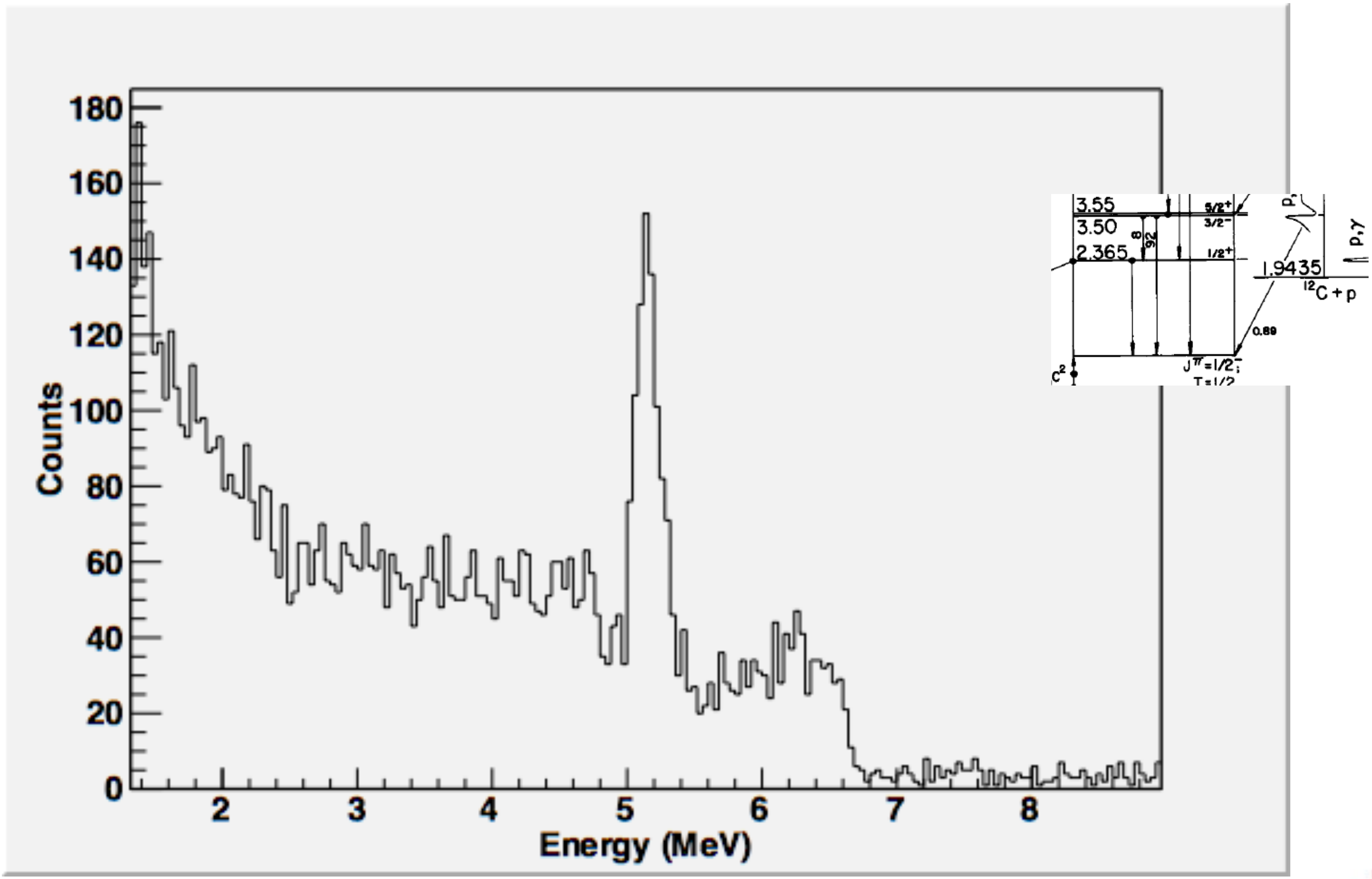
p

I~10^{12} pps

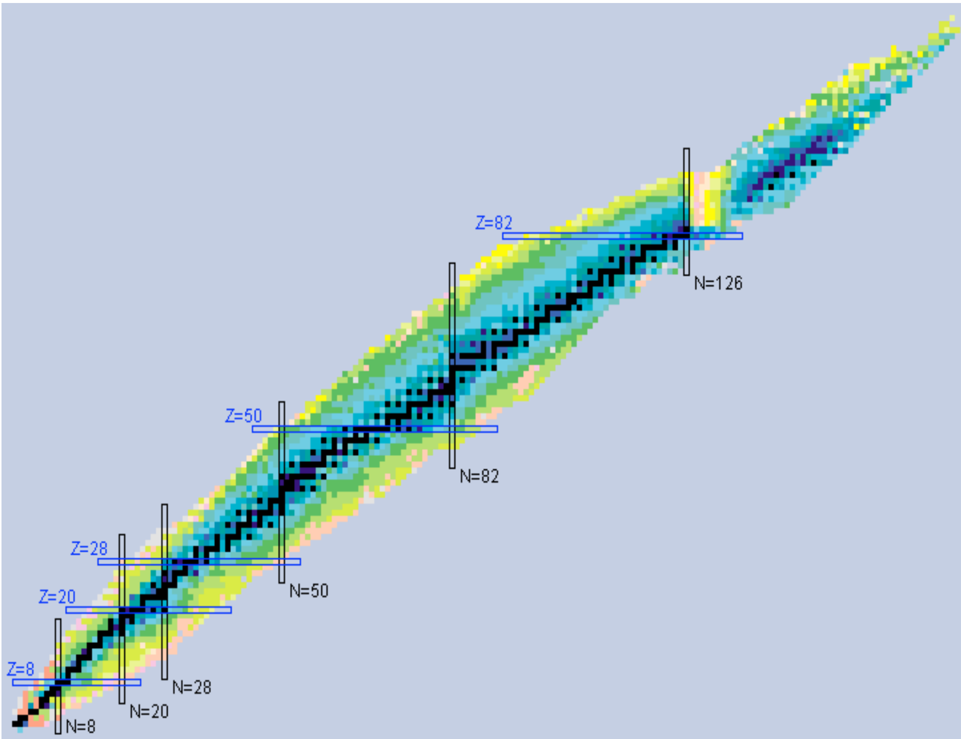


Excitation function for ^{12}C+p elastic scattering

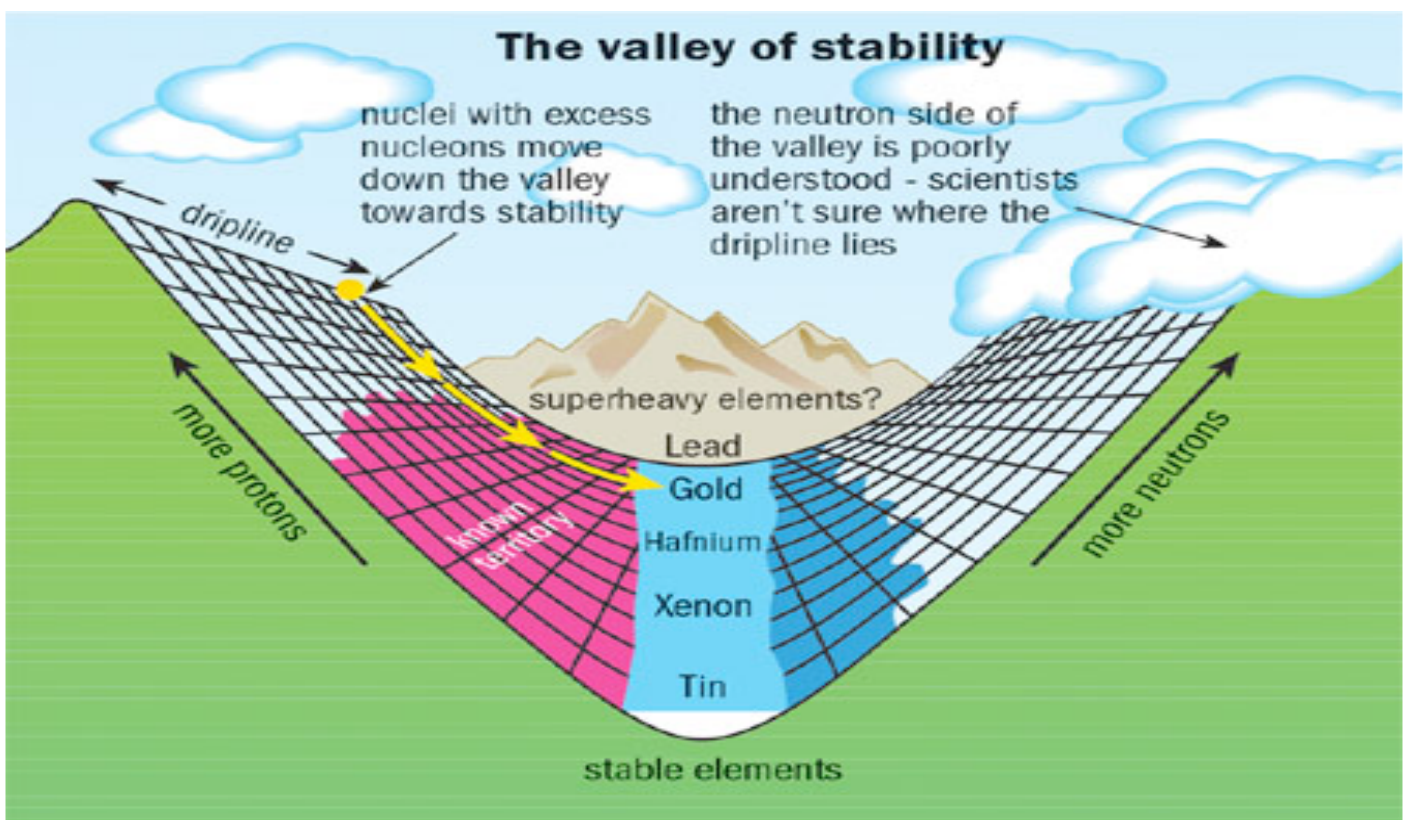
gs



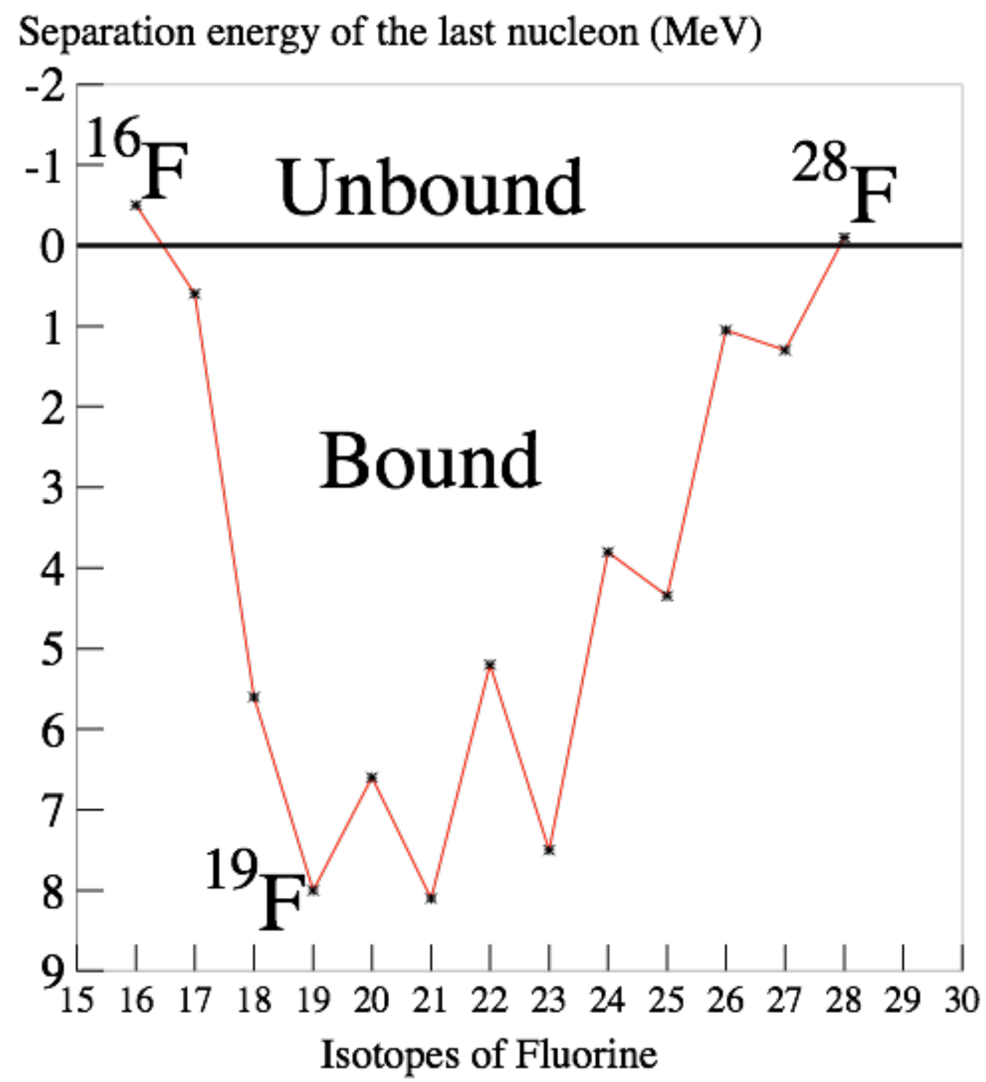
gs



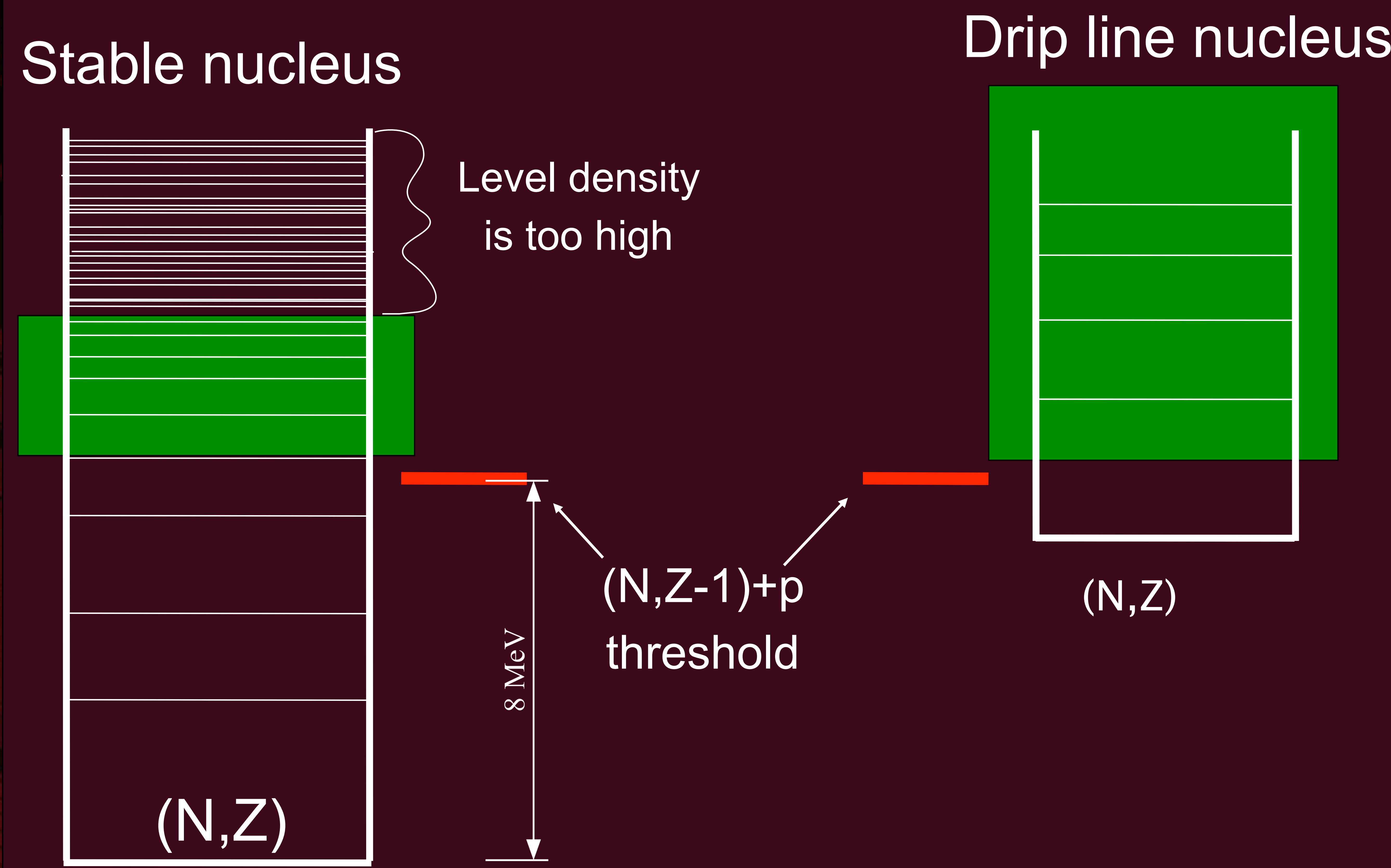
Z=82
N=126
Z=50
N=82
Z=28
N=50
Z=20
Z=8
N=28
N=20
N=8

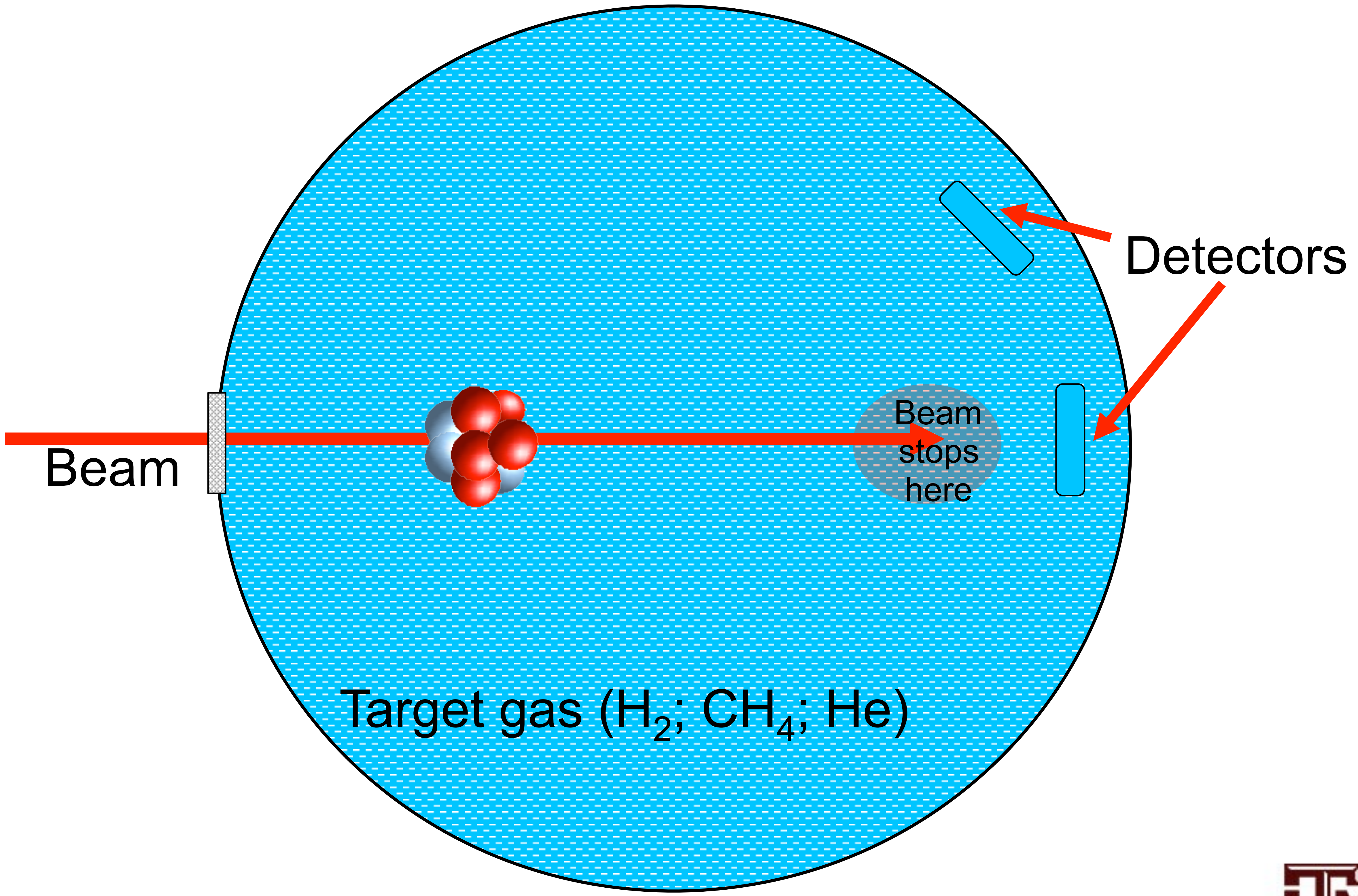


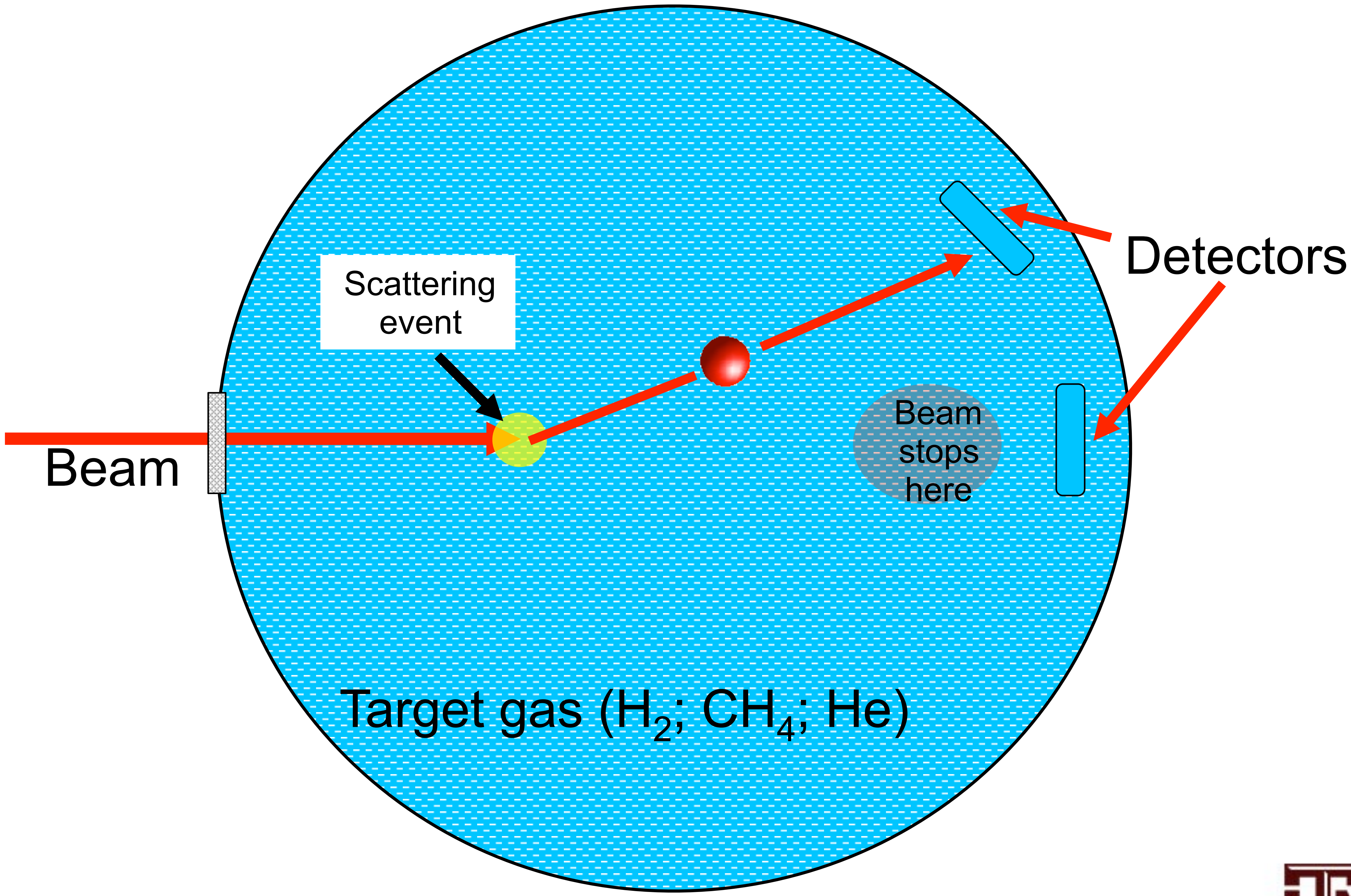
The valley of stability
nuclei with excess nucleons move down the valley towards stability
the neutron side of the valley is poorly understood - scientists aren't sure where the dripline lies
dripline
superheavy elements?
Lead
Gold
Hafnium
Xenon
Tin
known territory
more protons
more neutrons
stable elements



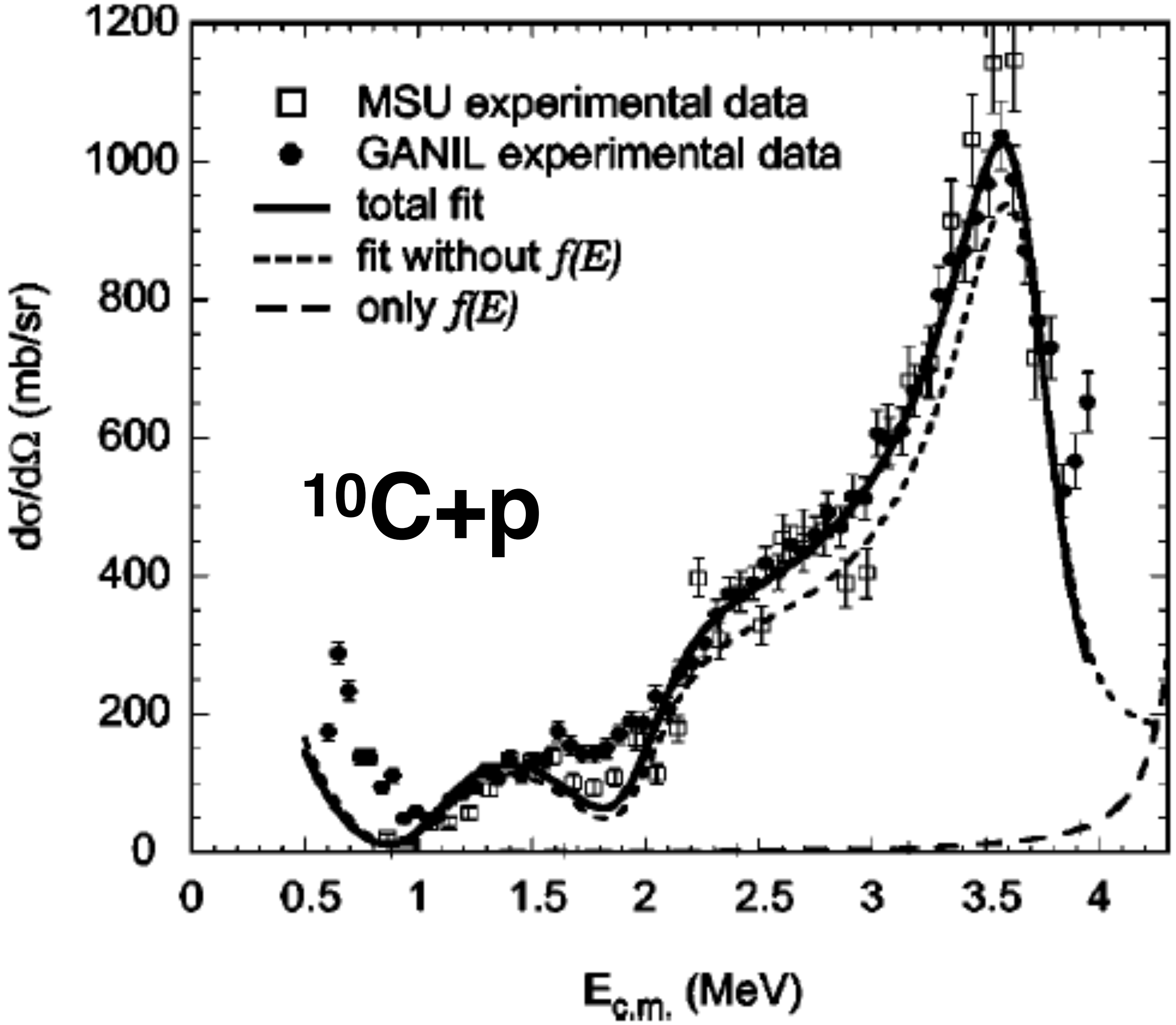
Separation energy of the last nucleon (MeV)
-2
-1
0
1
2
3
4
5
6
7
8
9
16F
28F
19F
Unbound
Bound
15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30
Isotopes of Fluorine

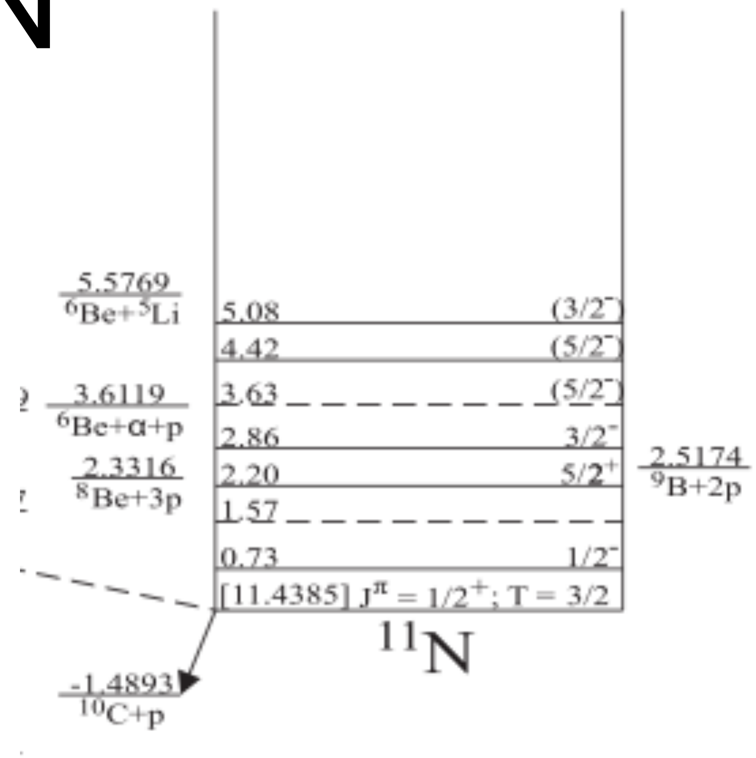

Stable nucleus
Drip line nucleus
Level density
is too high
(N,Z)
8 MeV
(N,Z-1)+p
threshold
(N,Z)

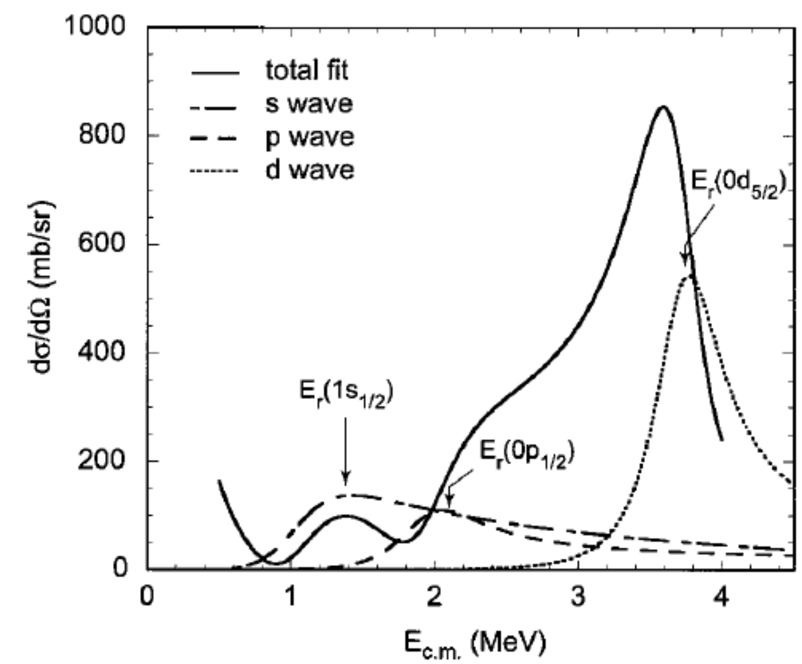

Detectors
Beam
Beam stops here
Target gas (H_2; CH_4; He)


Scattering
event
Detectors
Beam
Beam
stops
here
Target gas (H_2; CH_4; He)

Discovery of ^{11}N







L. Axelsson, et al., PRC 54, R1511 (1996)

K. Markenroth, et al., PRC 62, 034308 (2000)

R-matrix theory

$$u(r \to \infty) \sim e^{-ikr} - e^{2i\delta} e^{ikr} = I - UO$$

$$U = e^{2i\delta}$$

$$\Psi(r, \theta, \varphi) = A\,[e^{ikz} + (1/r)\, f(\theta,\varphi) e^{ikr}]$$

$$\sigma(\theta, E) = |f(\theta, E)|^2$$

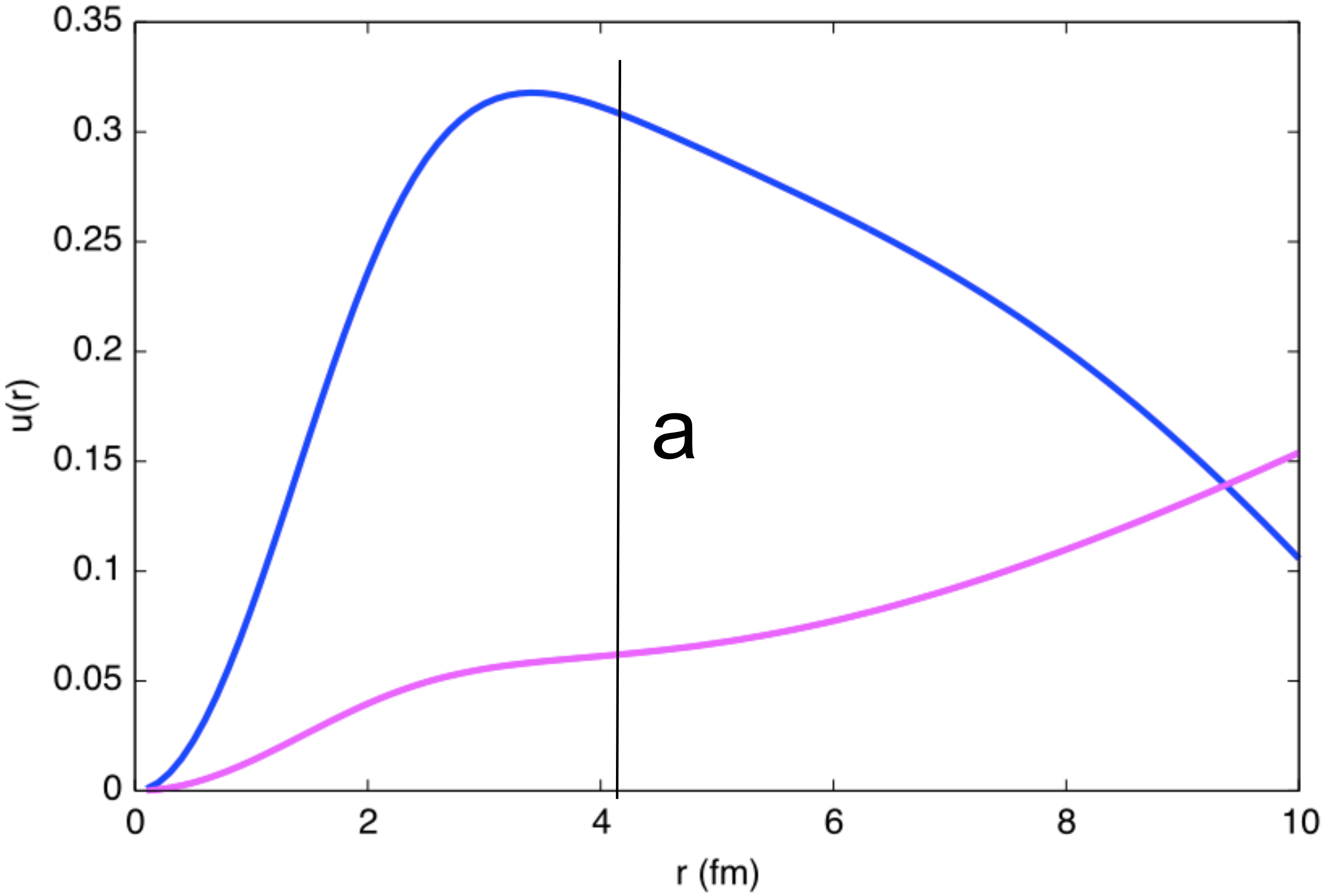
Straightforward manipulations can be used to show that:

$$f(\theta, E) = \frac{i}{2k} \sum_{\ell=0}^{\infty} (2\ell + 1)(1 - U_\ell) P_\ell(\cos(\theta))$$

Problem: How to relate the measured cross section to the properties of the wave function in the interior region.

R-matrix theory

On-resonance and off-resonance behavior of the interior wave function



$$R = \left(\frac{u_\ell}{\rho u_\ell'} \right)_{r=a}$$

$$\rho = kr$$

$$u = I - UO$$

$$R = \frac{I - UO}{\rho(I' - UO')}$$

R-matrix theory

gs

Applying Green's theorem to Schroedinger eq. leads to

$$R = \left(\frac{u_\ell}{\rho u_\ell'}\right)_{r=a} = \sum_\lambda \frac{\gamma_\lambda^2}{E_\lambda - E}$$

$$\gamma_\lambda = \sqrt{\frac{\hbar^2}{2\mu a}} u_\lambda(a)$$ reduced width amplitude

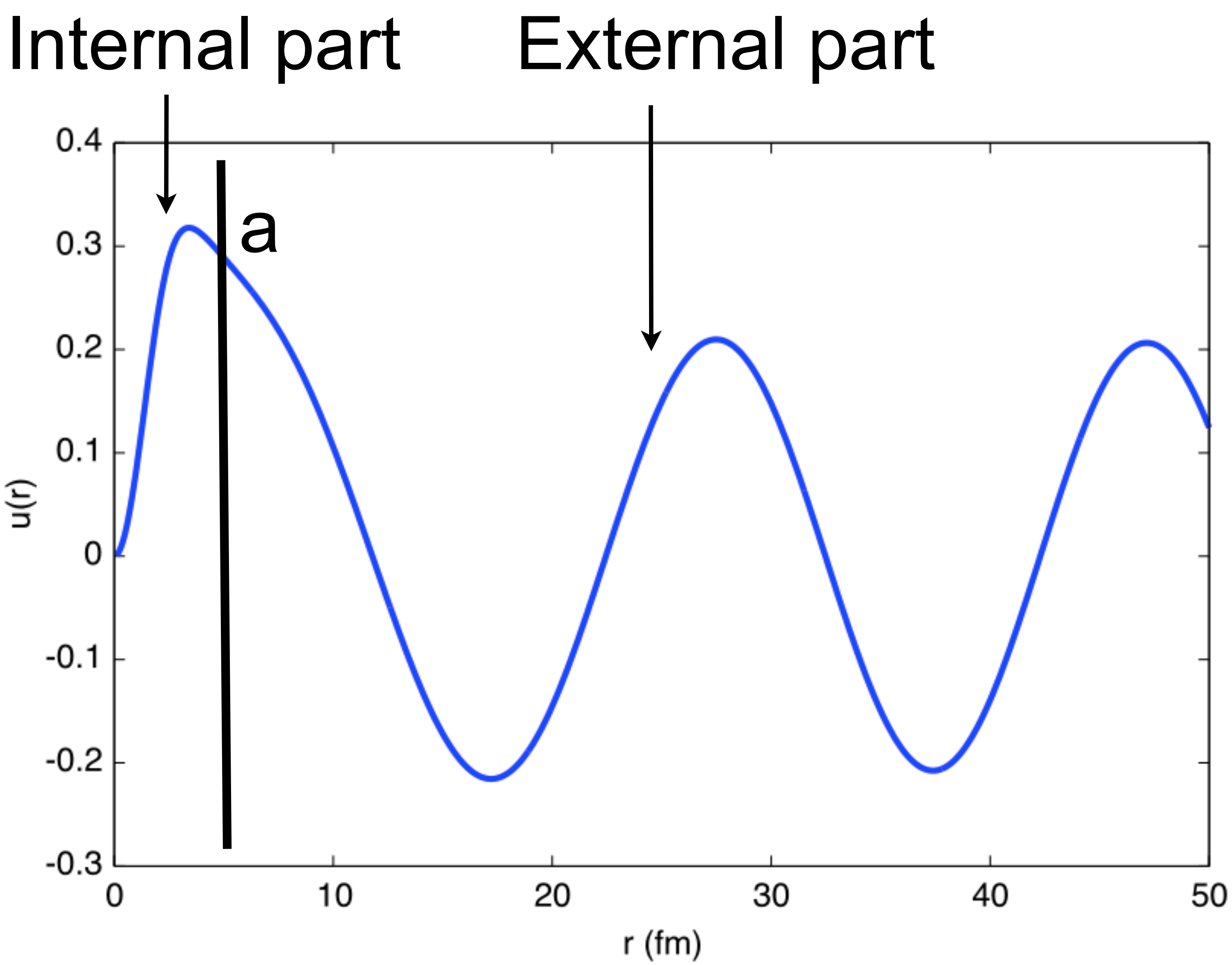
E_λ - Eigenvalues and

$u_\lambda(a)$ - eigenfunctions of Schroedinger eq. which satisfies

$$\frac{a}{u_\lambda(a)}\left(\frac{du_\lambda}{dr}\right)_{r=a} = B$$ boundary condition.

Interaction is unknown, so eigenvalues and values of eigenfunctions at **a** (channel radius) for EACH resonance are parameters of the theory. Other parameters are - channel radius "**a**" and boundary condition "**B**" (B is set independently for each partial wave).

R-matrix theory



In a nutshell:

1. The problem is split into two regions, internal and external.

2. Internal region, where interaction is important and unknown, is parametrized.

3. External part is described by asymptotic behavior of the wave functions under the assumption that there is no interaction (except for Coulomb!).

4. The phase shifts of the asymptotic wave functions are related to the R-function.

R-matrix theory

gs

If cross section is dominated by an isolated resonance:

$$R \approx \frac{\gamma_\lambda^2}{E_\lambda - E} \qquad \delta_\ell = \tan^{-1}\left(\frac{\gamma_\ell^2 P_\ell}{E_\lambda - E - \gamma_\ell^2 (S_\ell - B)}\right) - \phi_\ell + \omega_\ell$$

Since $\sigma \sim |1 - e^{2i\delta}|^2$ CS is maximum when δ_ℓ = 90°

and it is 1/2 of the maximum when δ_ℓ = 45°

$$E_r = E_\lambda - \gamma_\ell^2 (S_\ell(E_r) - B)$$ Observed resonance energy

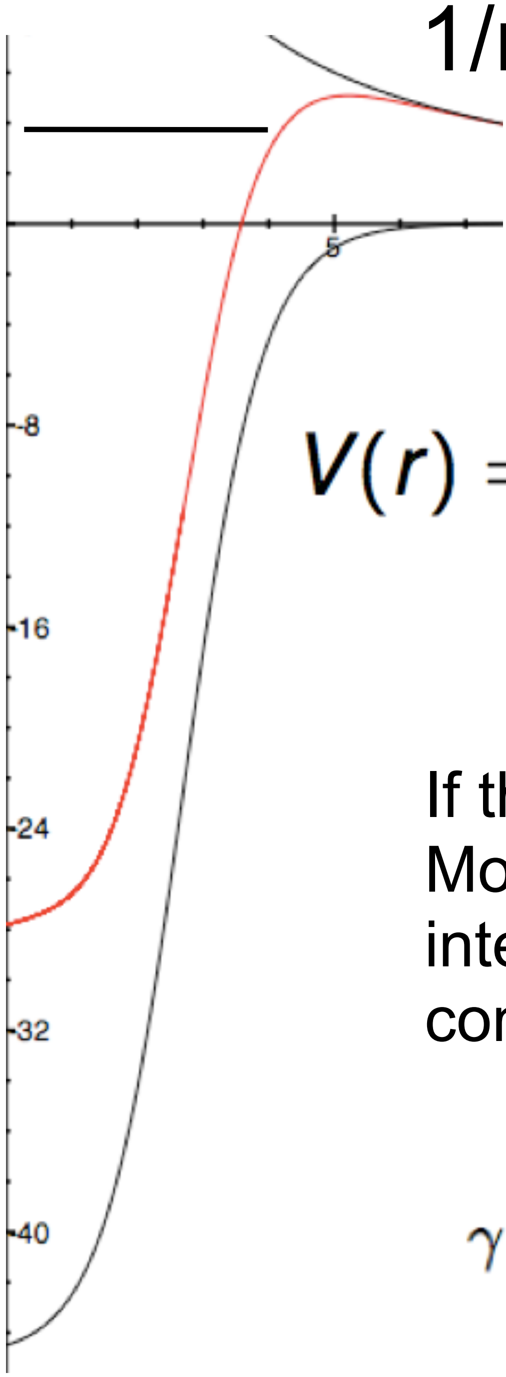
$$\Gamma = 2P_\ell(E_r)\gamma^2$$ Formal resonance width

$$\Gamma = \frac{2P_\ell(E_r)\gamma^2}{1 + \gamma^2 \frac{dS(E_r)}{dE}}$$ Observed resonance width

$$P_\ell = \frac{ka}{F_\ell^2 + G_\ell^2}$$ penetrability factor

$$S_\ell = \frac{FF' + GG'}{F^2 + G^2}\,ka$$ shift factor

R-matrix theory



$$V(r) = \frac{-V_o}{1+e^{\frac{r-R}{a}}}$$

$$\theta_\lambda^2 = \frac{\gamma_\lambda^2}{\gamma_{WS}^2}$$

More accurate dimensionless reduced width is determined using Woods-Saxon potential reduced width amplitude

If the wave function of the compound state is calculated (using Shell Model, for ex.) then the reduced width can be related to the overlap integral between the channel wave function and the wave function of the compound state calculated at the surface of radius **a**.

$$\gamma = \left(\frac{\hbar^2}{2\mu a}\right)^{1/2} \int [\psi(p) \times \psi(^{12}C)]\Psi(^{13}N)dS_c$$

R-matrix theory

Multi-level, multi channel problem for charged particles with non-zero spin.

$$A_{\alpha's'\nu',\alpha s\nu}(\Omega_{\alpha'}) = \frac{\pi^{\frac{1}{2}}}{k_\alpha}\Big[-C_{\alpha'}(\theta_{\alpha'})\delta_{\alpha's'\nu',\alpha s\nu} + i\sum_{JMll'm'}(2l+1)^{\frac{1}{2}}(sl\nu 0|JM)(s'l'\nu'm'|JM) \times T_{\alpha's'l',\alpha sl}{}^{J}Y_{m'}{}^{(l')}(\Omega_{\alpha'})\Big], \quad (2.3)$$

where

$$T_{\alpha's'l',\alpha sl}{}^{J} = e^{2i\omega_{\alpha'l'}}\delta_{\alpha's'l',\alpha sl} - U_{\alpha's'l',\alpha sl}{}^{J}.$$

In performing the absolute squaring operation, one introduces the two sets of summing integers

$$\{J_1M_1l_1l_1'm_1'\} \quad \text{and} \quad \{J_2M_2l_2l_2'm_2'\}$$

for the single set of (2.3), and thereby obtains for (2.1)

$$(2s+1)\frac{k_\alpha{}^2}{\pi}d\sigma_{\alpha s,\alpha's'}d\Omega_{\alpha'} = (2s+1)|C_{\alpha'}(\theta_{\alpha'})|^2\delta_{\alpha's',\alpha s}$$
$$+ \sum_{\substack{J_1J_2M_1M_2\\ l_1l_2l_1'l_2'\\ \nu\nu'm_1'm_2'}}(2l_1+1)^{\frac{1}{2}}(2l_2+1)^{\frac{1}{2}}(sl_1\nu 0|J_1M_1)$$
$$\times(sl_2\nu 0|J_2M_2)(s'l_1'\nu'm_1'|J_1M_1)(s'l_2'\nu'm_2'|J_2M_2)$$
$$\times(T_{\alpha's'l_1',\alpha sl_1}{}^{J_1}Y_{m_1'}{}^{(l_1')}(\Omega_{\alpha'}))$$
$$\times(T_{\alpha's'l_2',\alpha sl_2}{}^{J_2}Y_{m_2'}{}^{(l_2')}(\Omega_{\alpha'}))^*$$
$$\cancel{+}\,- \sum_{\substack{JMll'\\ m'\nu\nu'}}(2l+1)^{\frac{1}{2}}(sl\nu 0|JM)(s'l'\nu'm'|JM)$$
$$\times\delta_{\alpha's'\nu',\alpha s\nu}2\,\mathrm{Re}[iT_{\alpha's'l',\alpha sl}{}^{J}Y_{m'}{}^{(l')}(\Omega_{\alpha'})C_{\alpha'}(\theta_{\alpha'})]. \quad (2.4)$$

$$R \rightarrow R_{\alpha s\ell,\alpha's'\ell'} = \sum_\lambda \frac{\gamma_{\lambda c}\gamma_{\lambda c'}}{E_\lambda - E}$$

$$U \rightarrow U_{\alpha s\ell,\alpha's'\ell'}$$

$$\sigma_{\alpha\alpha} \sim C^2 + N^2 + C * N$$

$$\sigma_{\alpha\alpha'} \sim N^2$$

Available codes: SAMMY (Oak Ridge)
AZURE (Notre Dame)
MinRmatrix (FSU/TAMU)

A.M. Lane and R.G. Thomas, Rev. of Mod. Phys., 30 (1958) 257

R-matrix theory

R-matrix vs Exact solution of Schroedinger equation

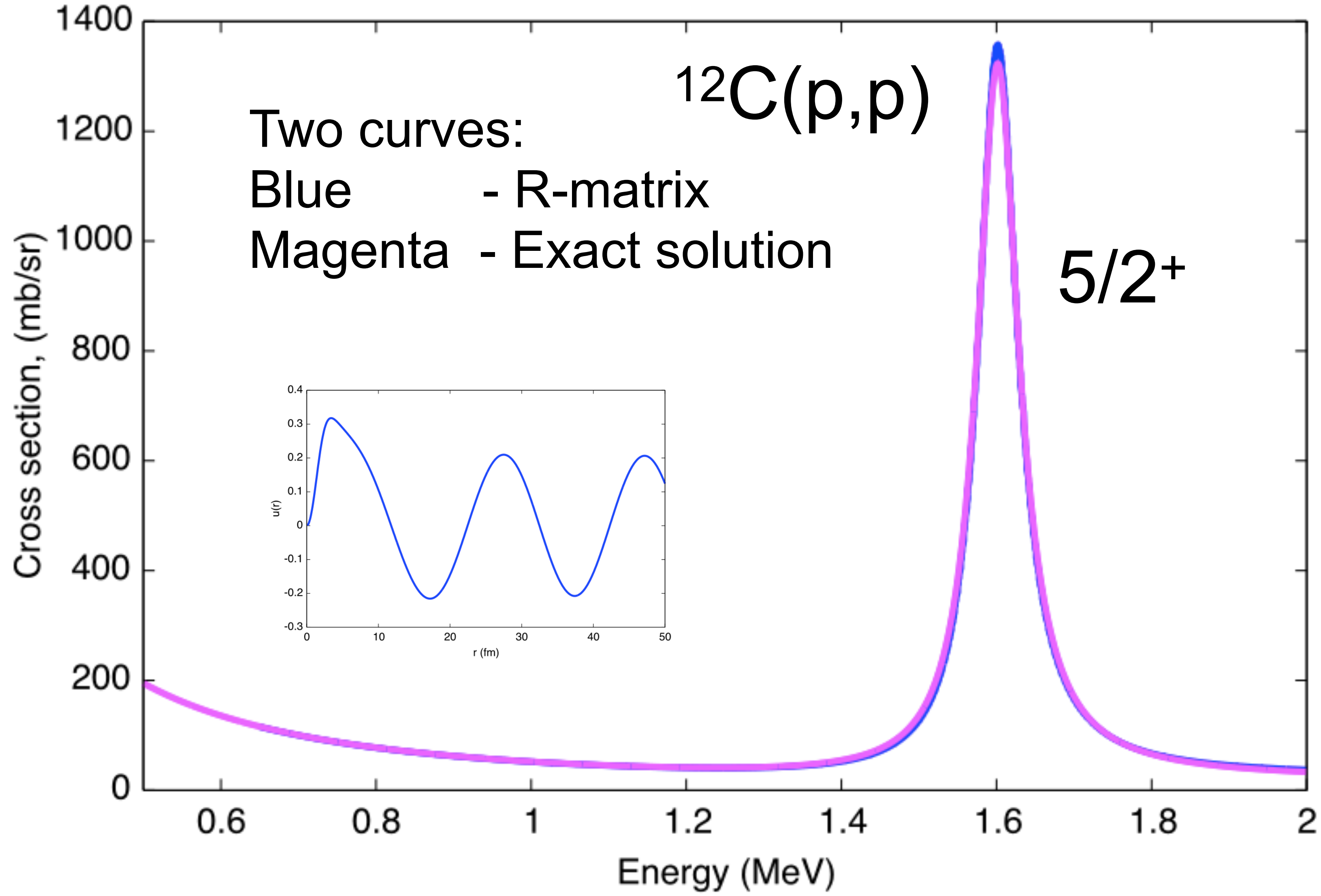
B = -2.0
a = 4.2 fm

$E_\lambda = 1.635$ MeV
$\gamma_\lambda = 1.4$ MeV$^{1/2}$

$E_{obs} = 1.603$ MeV
$\Gamma_{obs} = 64$ keV

W-S potential parameters:

V = -54.4 MeV
a = 0.662 fm
$r_o = 1.26$ fm
$V_{so} = 6.4$ MeV



R-matrix theory

Dependence on the channel radius and boundary condition

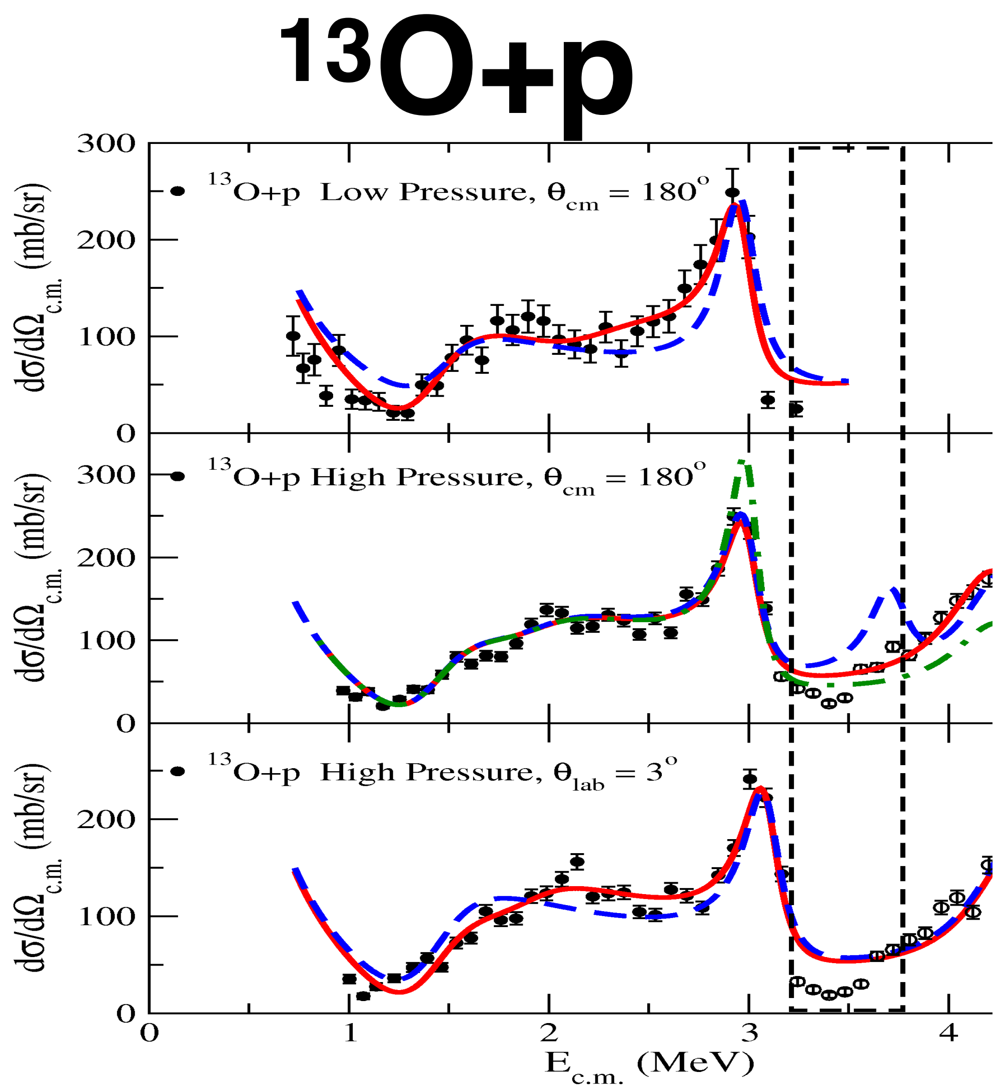
a fm	B	E_λ MeV	γ_λ $MeV^{1/2}$	E_{obs} MeV	Γ_{obs} keV	θ^2_{sw}
4.2	-2.0	1.635	1.4	1.603	64	0.76
4.2	0.0	-2.285	1.4	1.603	64	0.76
4.2	-1.0	-0.325	1.4	1.603	64	0.76
5.2	-2.0	1.685	0.75	1.603	64	0.33
6.2	-2.0	1.675	0.48	1.603	64	0.19
3.95	-2.008	1.603	1.7	1.603	64	1.0

$R_{^{12}C} + R_p = 2.61 + 0.84 = 3.45$ fm

Prescription that usually works well: **$a = 1.4 * A^{1/3} + 0.84$**

gs

Discovery of ^{14}F



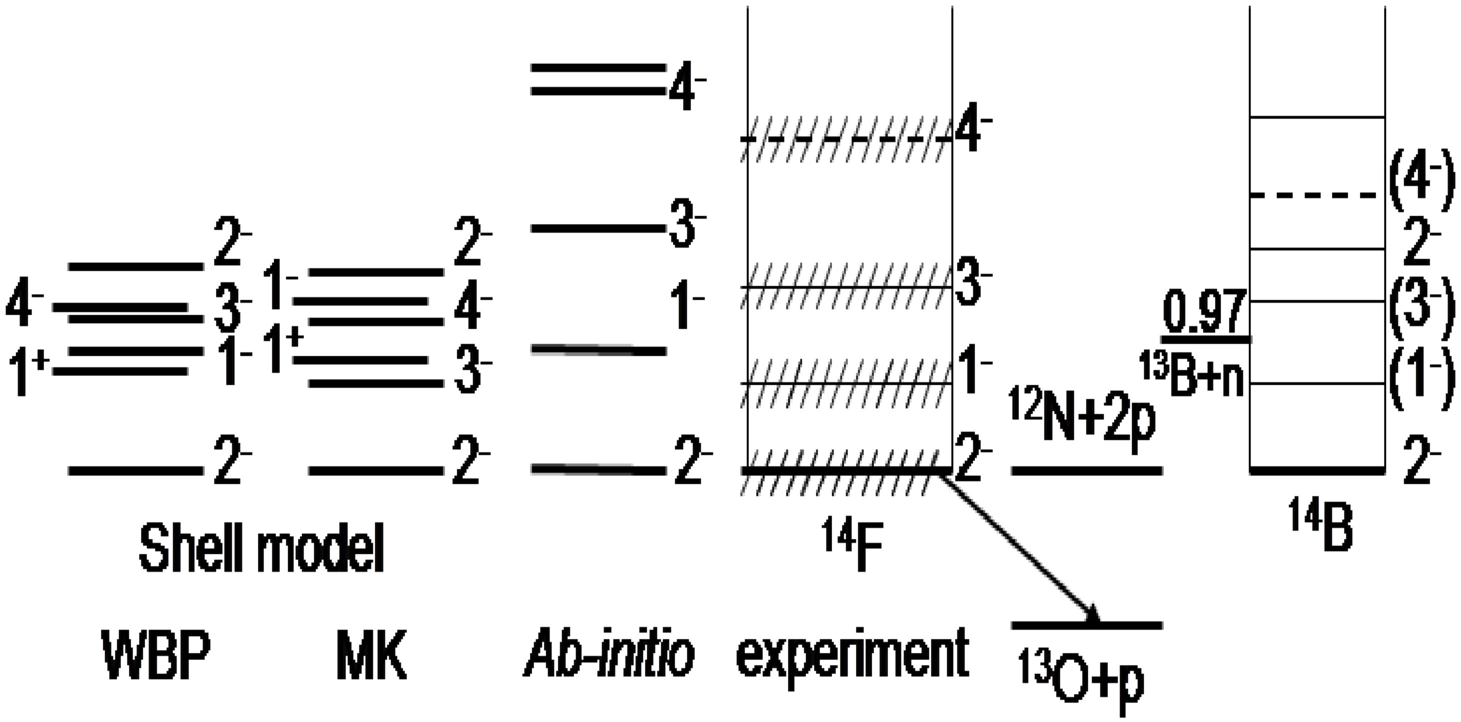


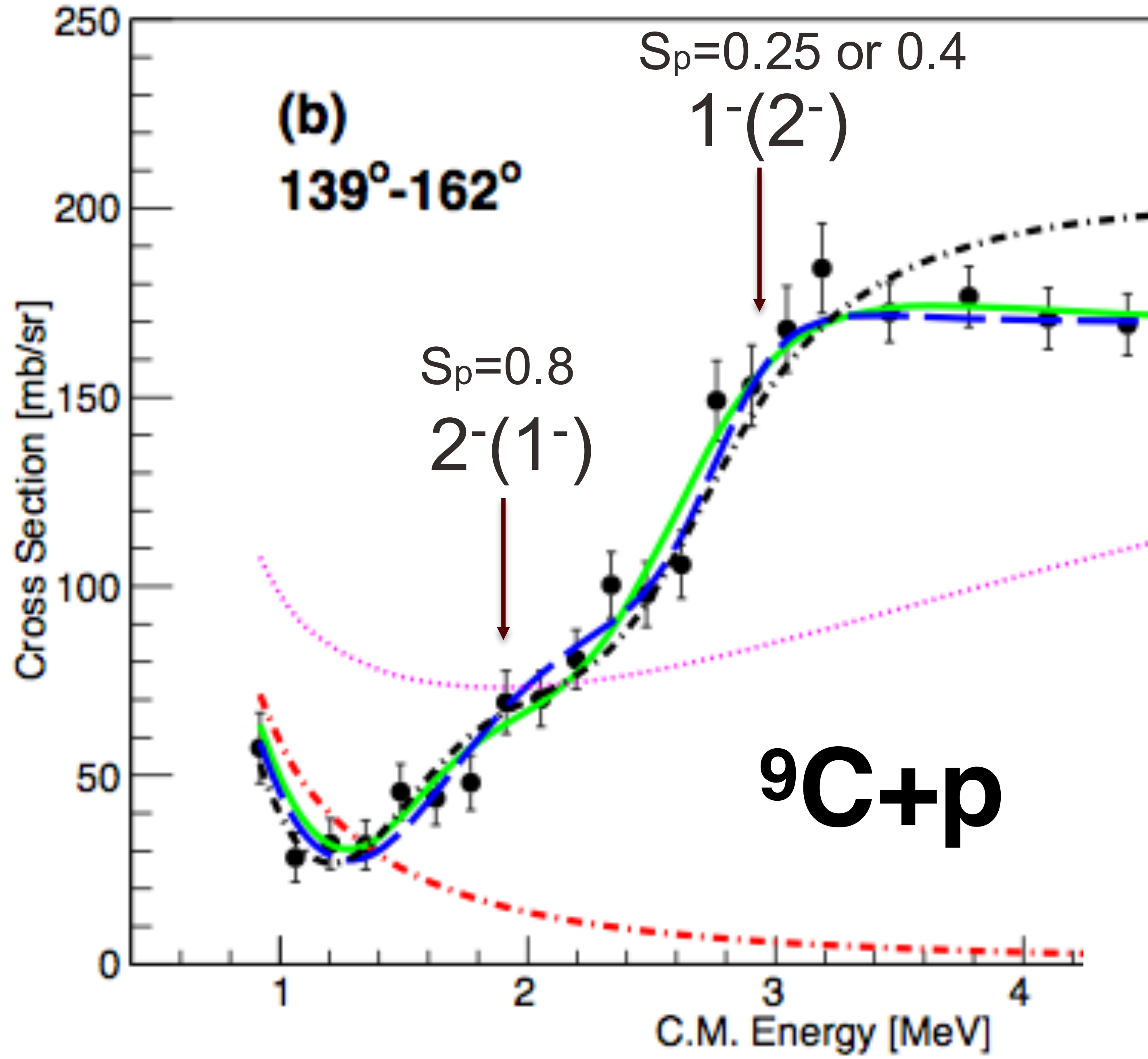
Table 1: Levels in ^{14}F

E_R(MeV)†	E_x^*	J^π	Γ (keV)	Γ/Γ_{sp}
1.56 ± 0.04	0.00	2^-	910 ± 100	0.85
2.1 ± 0.17	0.54	1^-	~ 1000	0.6
3.05 ± 0.060	1.49	3^-	210 ± 40	0.55
4.35 ± 0.10	2.79	4^-	550 ± 100	0.5

†Energy above ^{13}O+p decay threshold. *Excitation energy in ^{14}F.

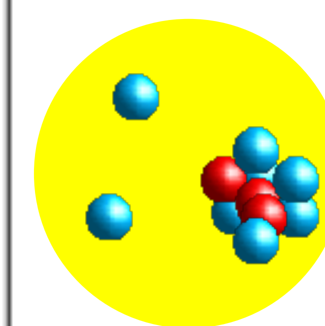
V.Z. Goldberg, et al., PRB 692 (2010) 307

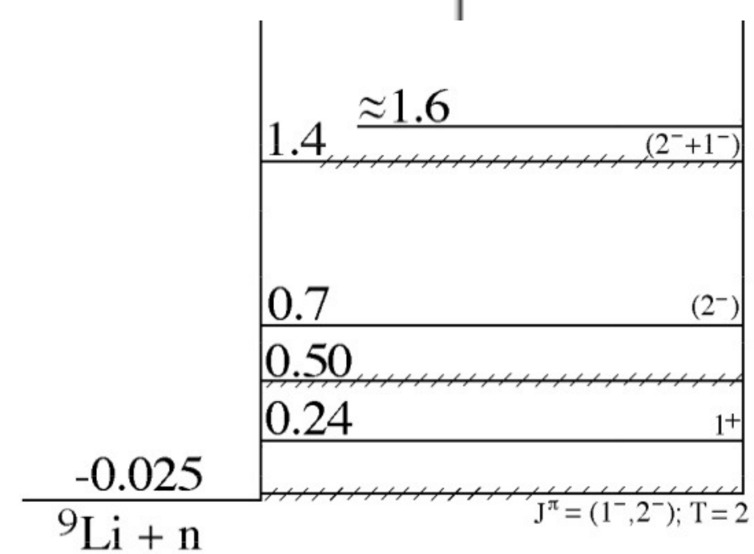
Discovery of ^{10}N



J. Hooker, GR, et al., PLB (2017)

^{11}Li

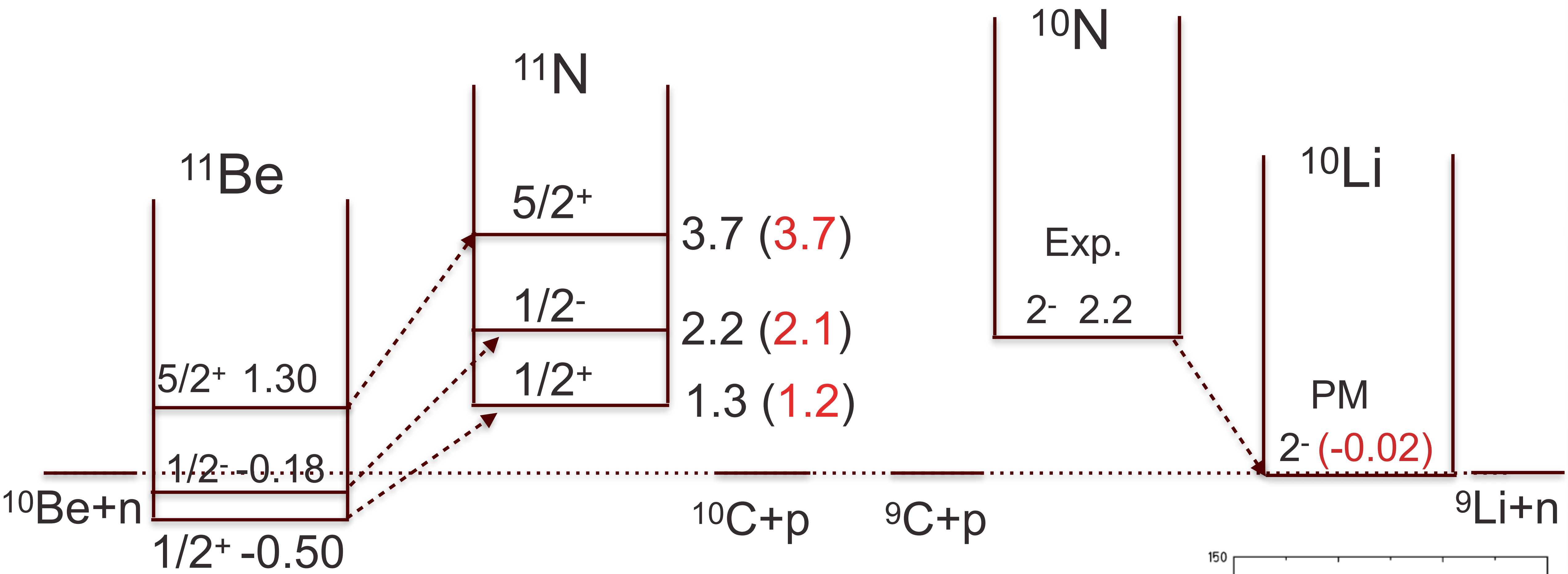




^{10}Li

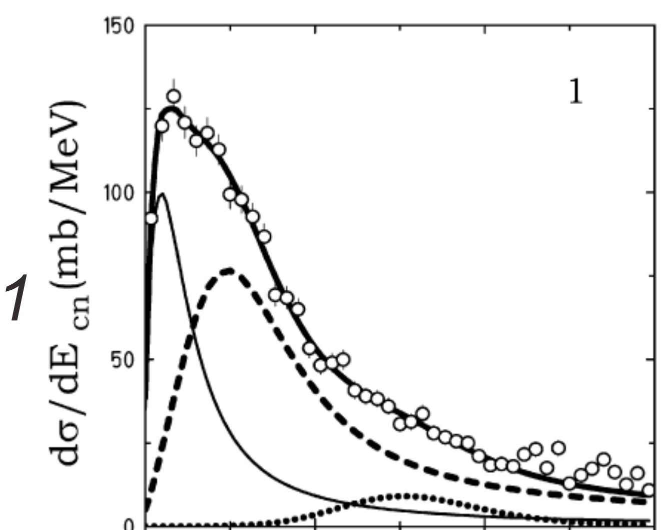
Potential model extrapolation

PM parameters: r_o = 1.25 fm, a=0.7 fm, r_c = 1.3 fm



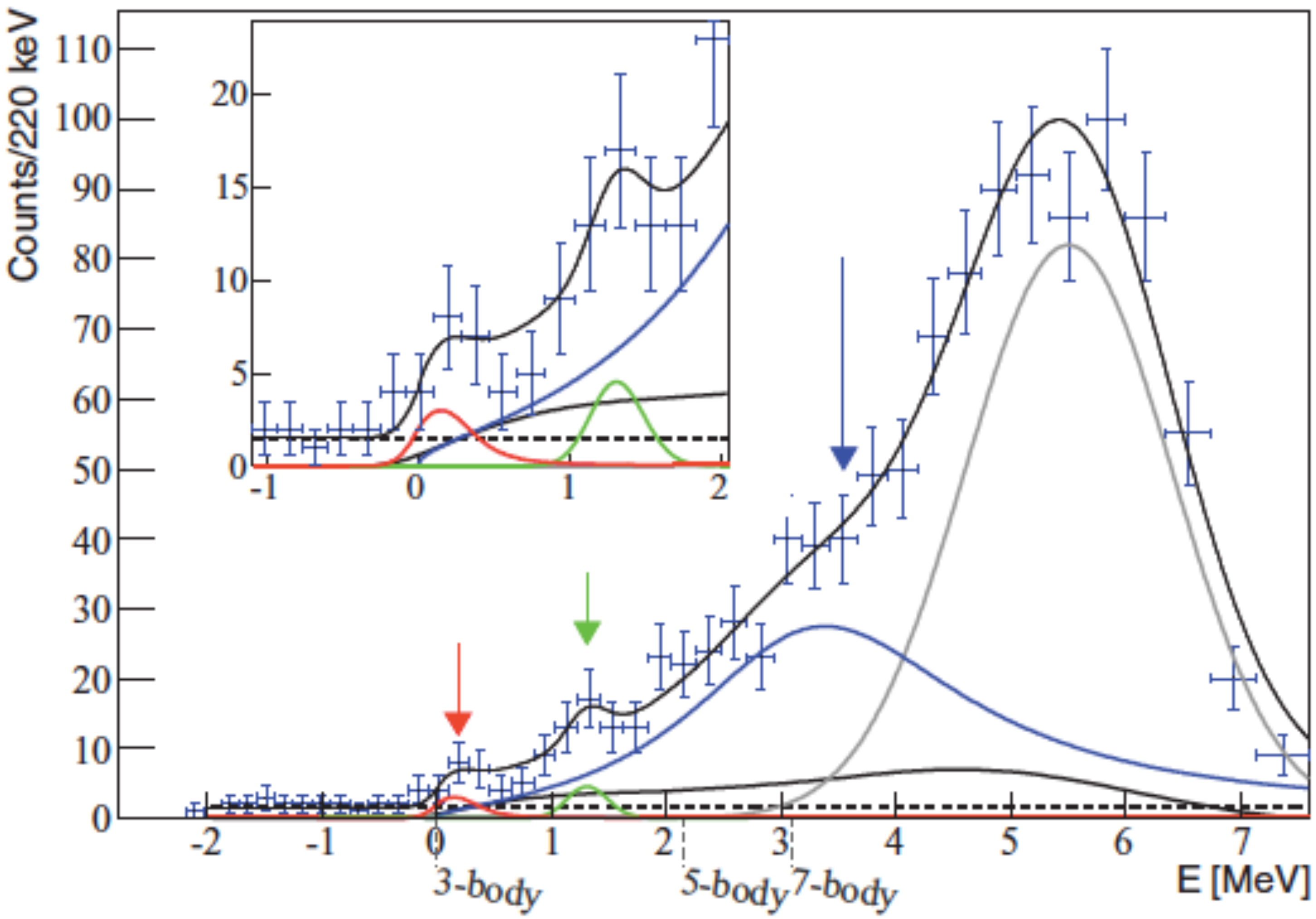
All values are in MeV. The experimental values for the known states are given. Potential model extrapolation are in parenthesis in red.

H. Simon, et al., Nucl. Phys. A 791 (2007) 267



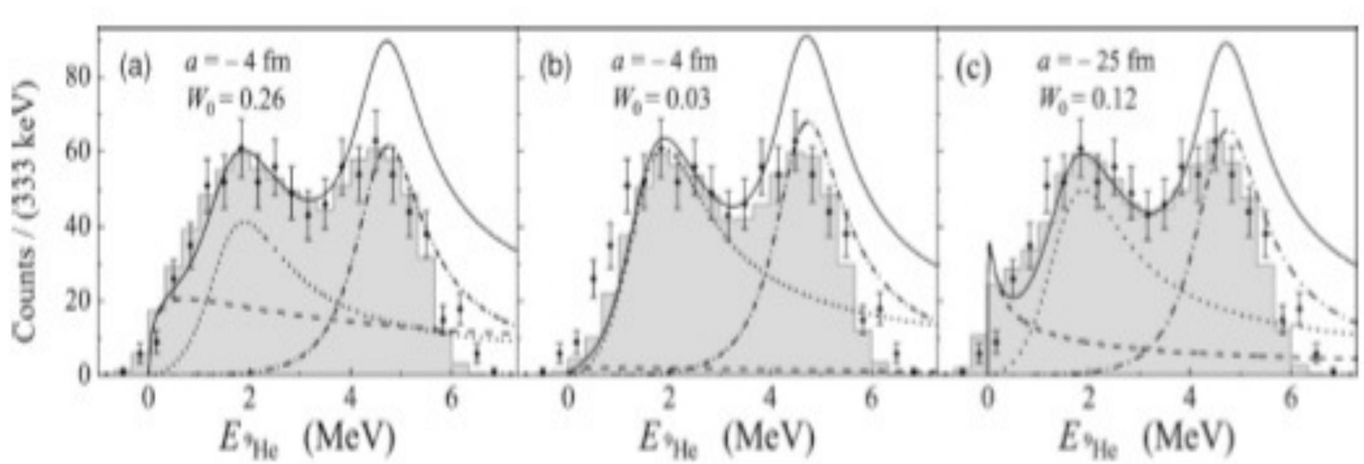
a = -30 fm - virtual s-state

Structure of ^{9}He



- Recent ^{8}He(d,p) measurements indicate low lying $1/2^{+}$ and $1/2^{-}$ states

  T.Al. Kalanee, et al., PRC 88 (2013) 034301

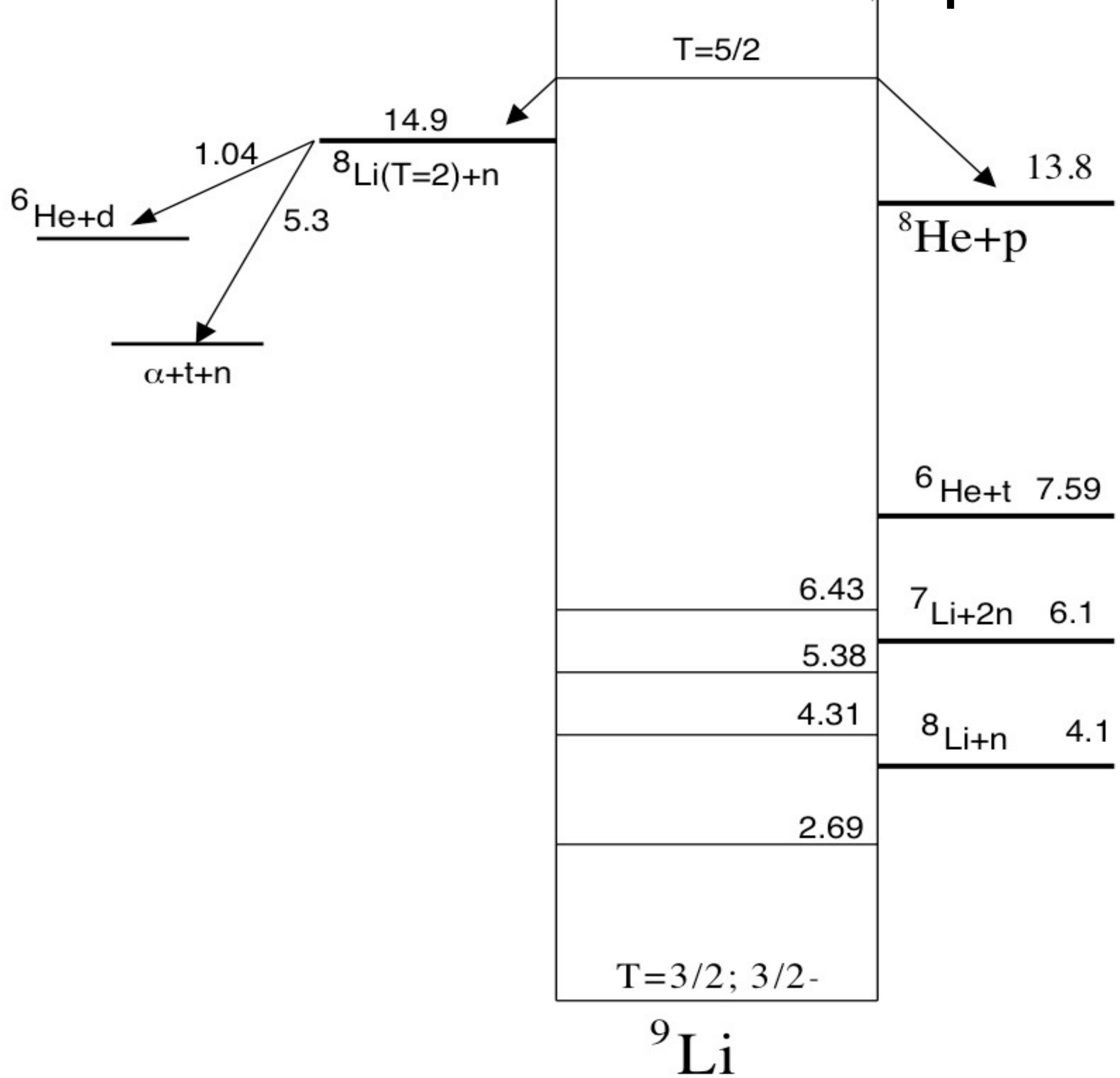


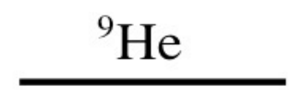
- This contradicts earlier ^{8}He(d,p) data

  M.S. Golovkov, et al., PRC 76 (2007) 021605

^{9}He through the T=5/2 IAR in ^{9}Li

$p + {}^{8}He \rightarrow {}^{9}Li(T=5/2) \rightarrow p + {}^{8}He$

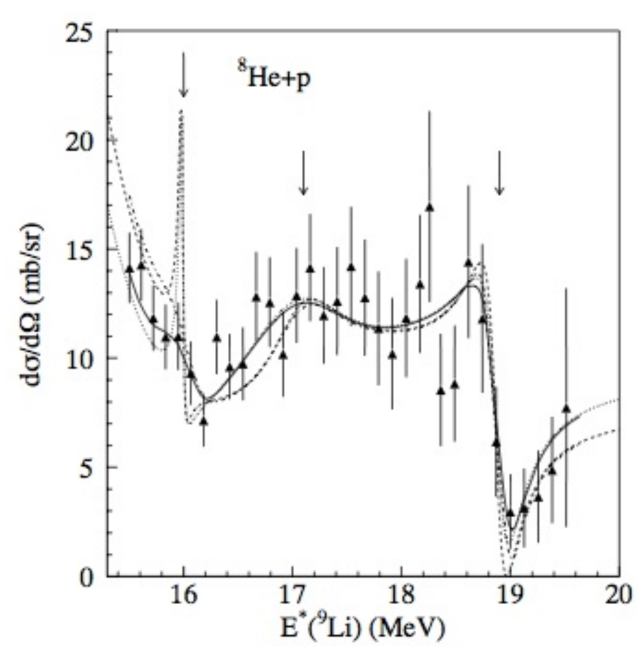




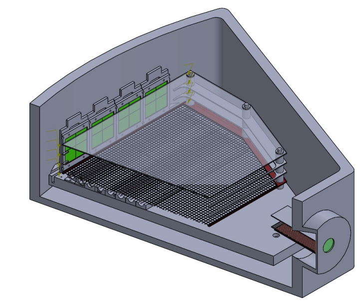
Decay of T=3/2 states back to elastic channel is suppressed due to the presence of the other channels.

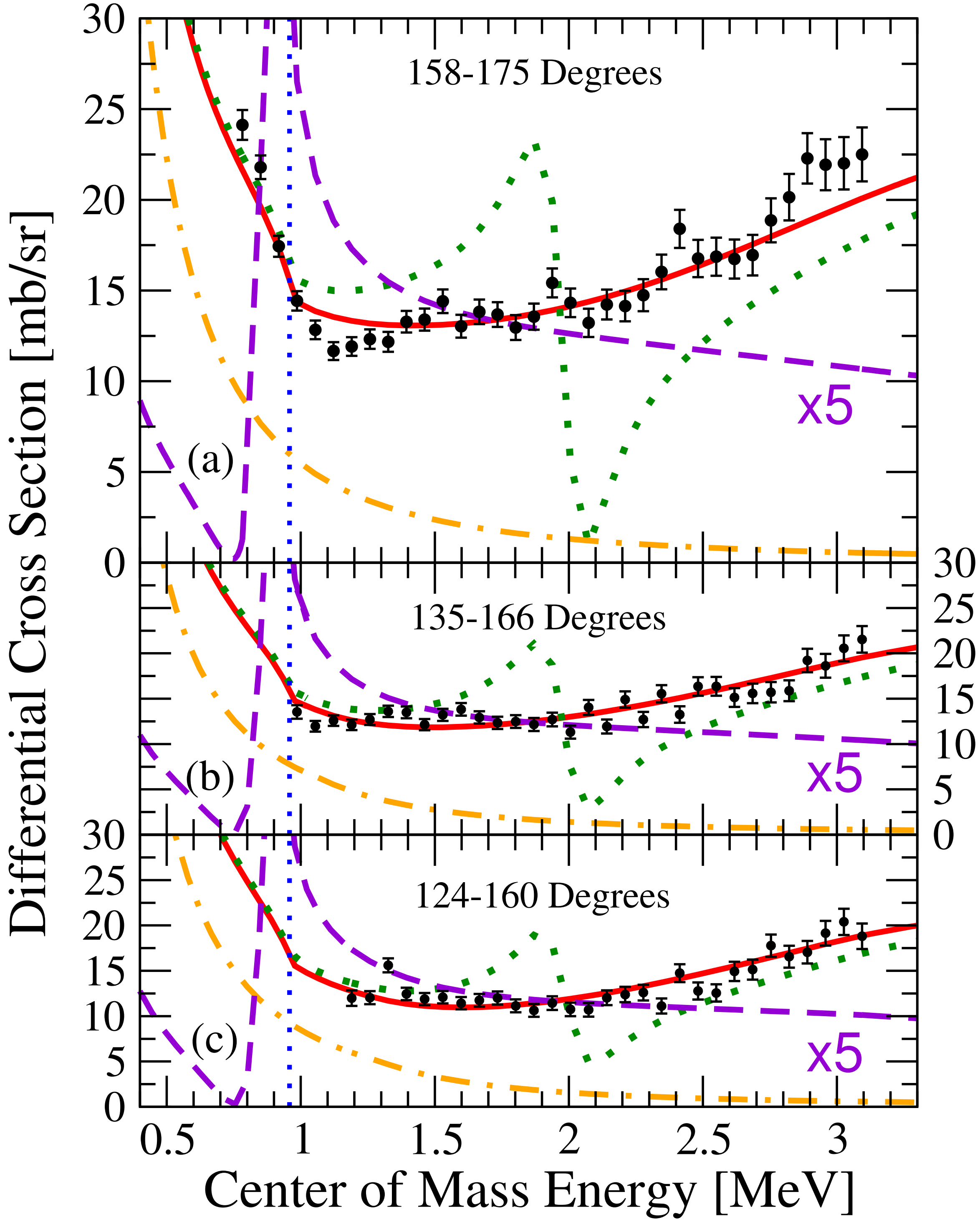
There are only two isospin allowed decay channels for T=5/2 states

GR, et al.,
PRC 67 (2003) 041603



Excitation function for ^{8}He(p,p) elastic scattering

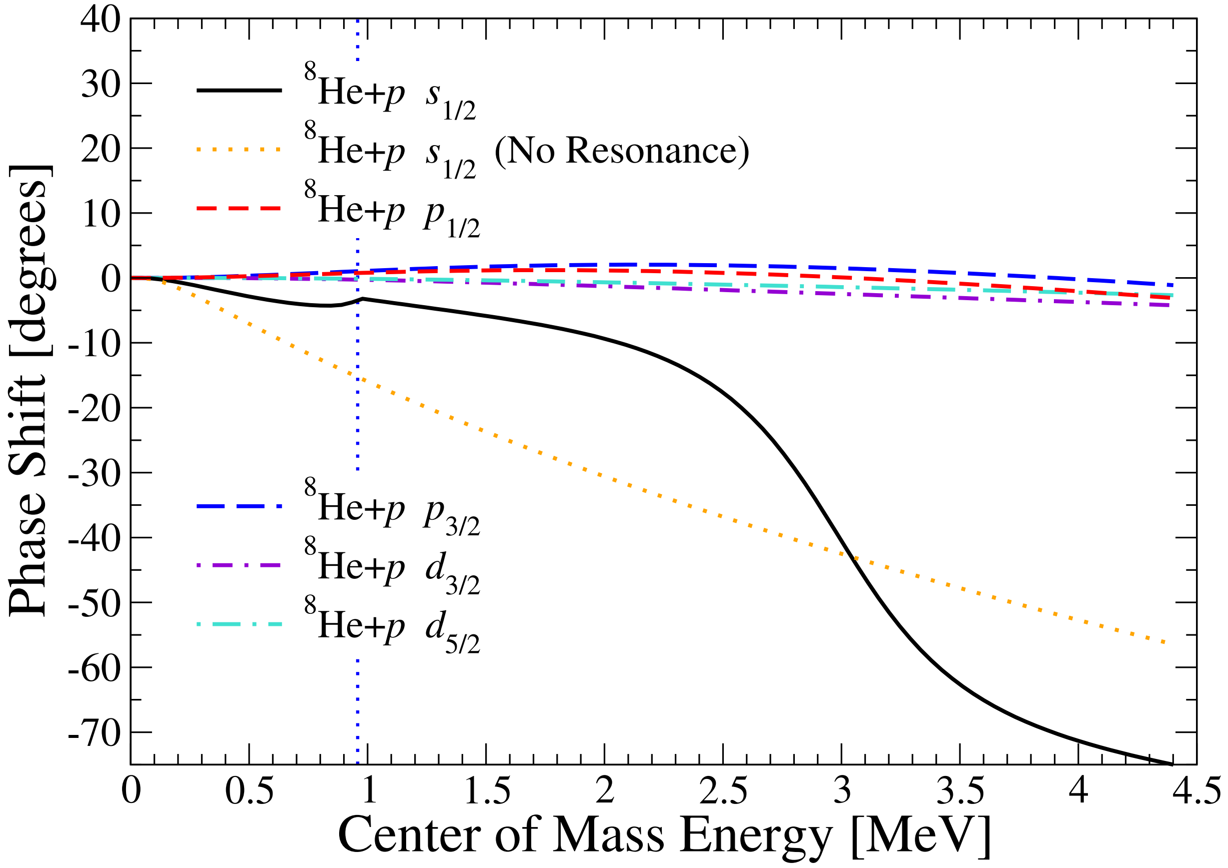


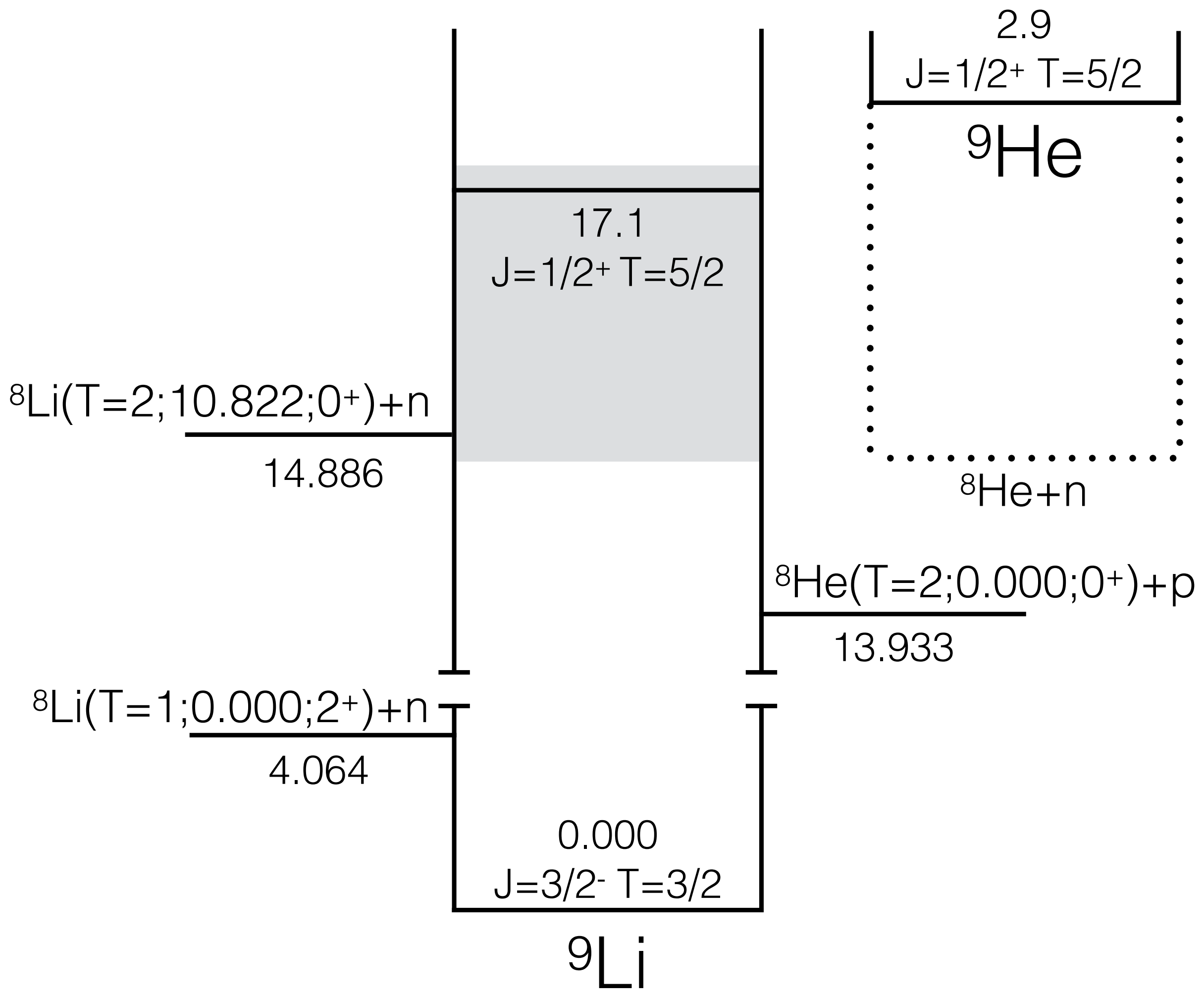


- T=5/2 states in ^{9}Li populated in ^{8}He+p resonance elastic scattering
- ^{8}He beam produced by ISAC facility at TRIUMF
- No narrow states were observed
- There is clear evidence for a very broad $1/2^{+}$ state at ~2.5 MeV above the proton threshold, this corresponds to a ground state of ^{9}He that is unbound by ~3 MeV

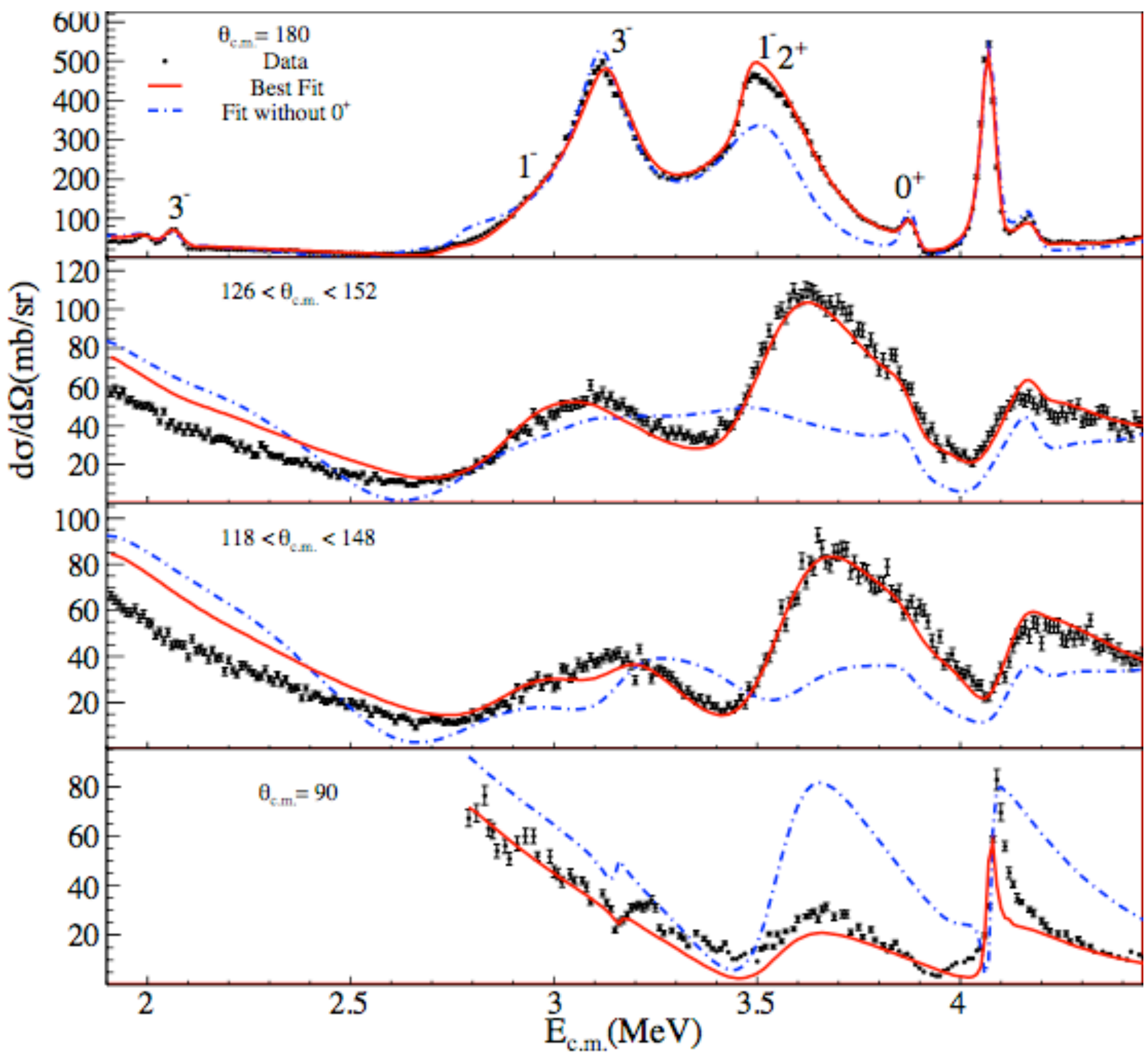
E. Uberseder, et al., Phys. Lett. B, 754, 323 (2016)

Level structure of ^{9}He inferred from the ^{8}He+p measurements and the phase shifts



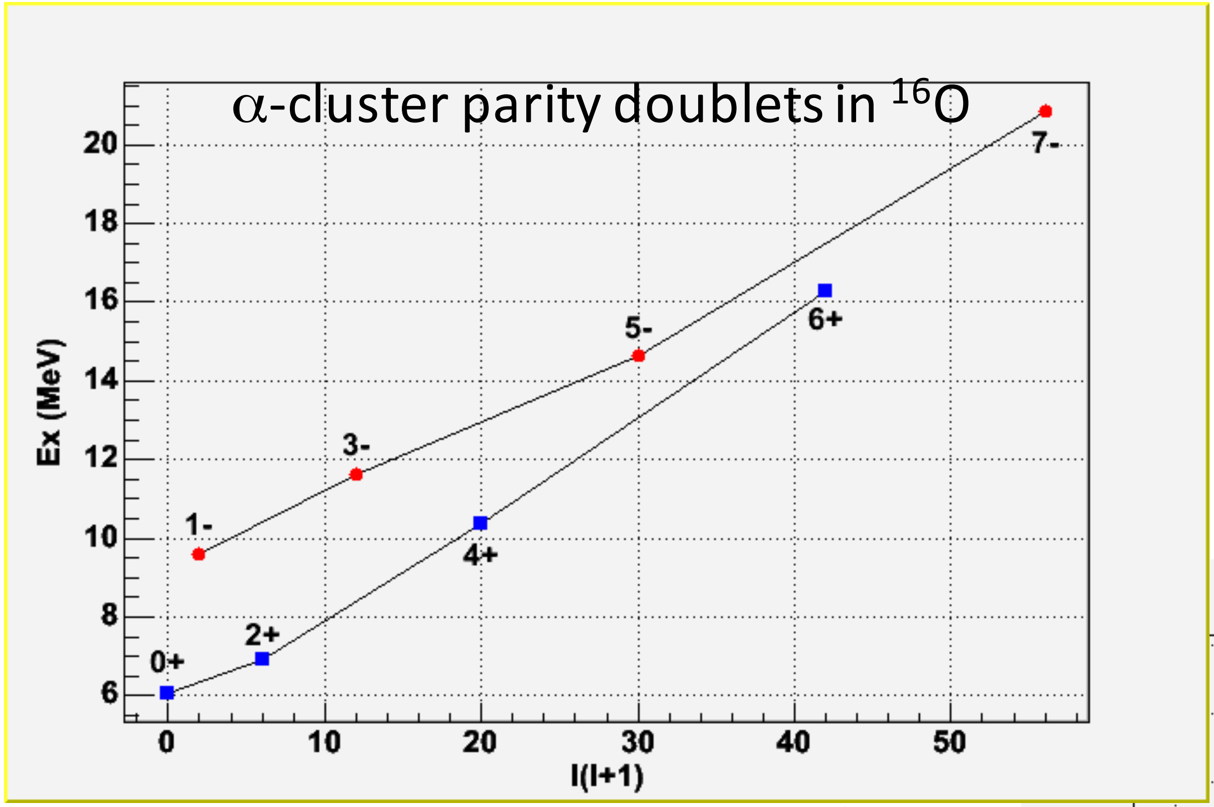


Cluster structure of ^{18}O

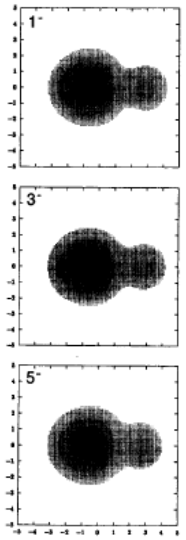


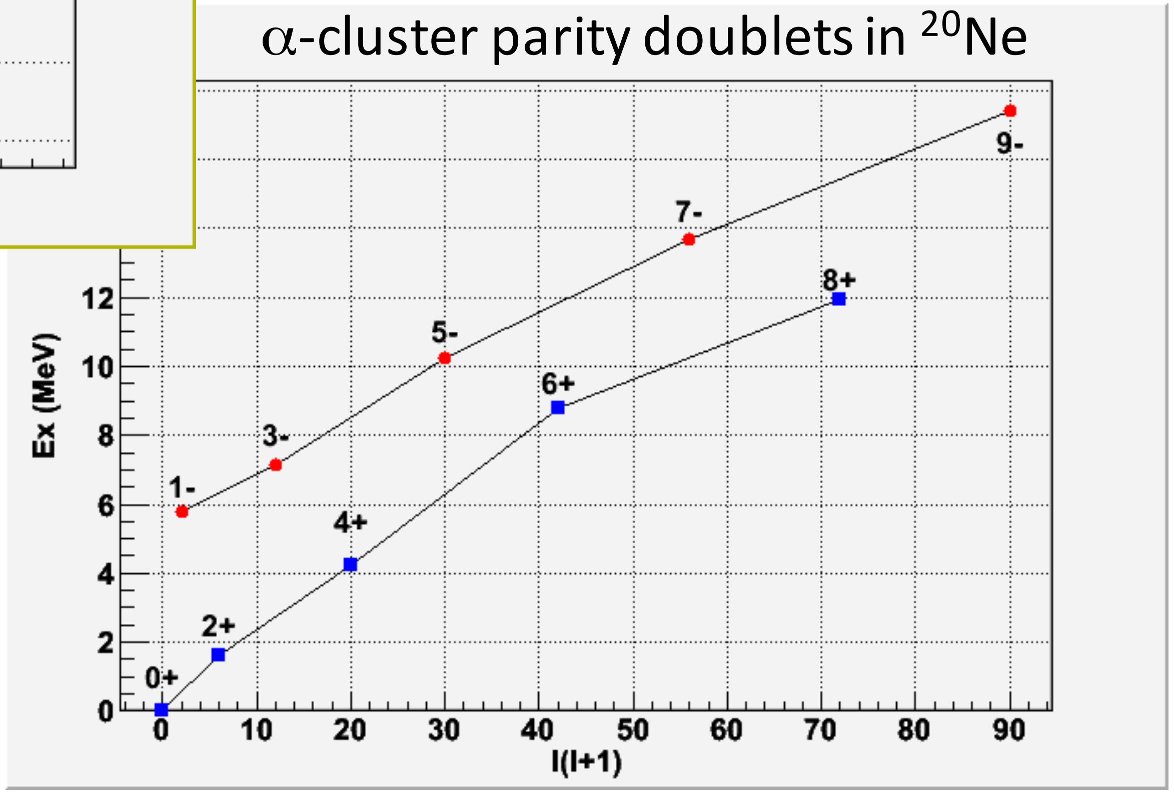
- ➢ $^{14}C(\alpha,\alpha)^{14}C$ excitation function was measured at John D. Fox Superconducting Linear accelerator facility at Florida State University.
- ➢ Method of Thick Target and Inverse Kinematics (TTIK) was used. [K.P. Artemov, et al., Sov.J.Nucl.Phys. (1990)]

[E.D. Johnson, GR, et al., EPJA, 42 135 (2009)]

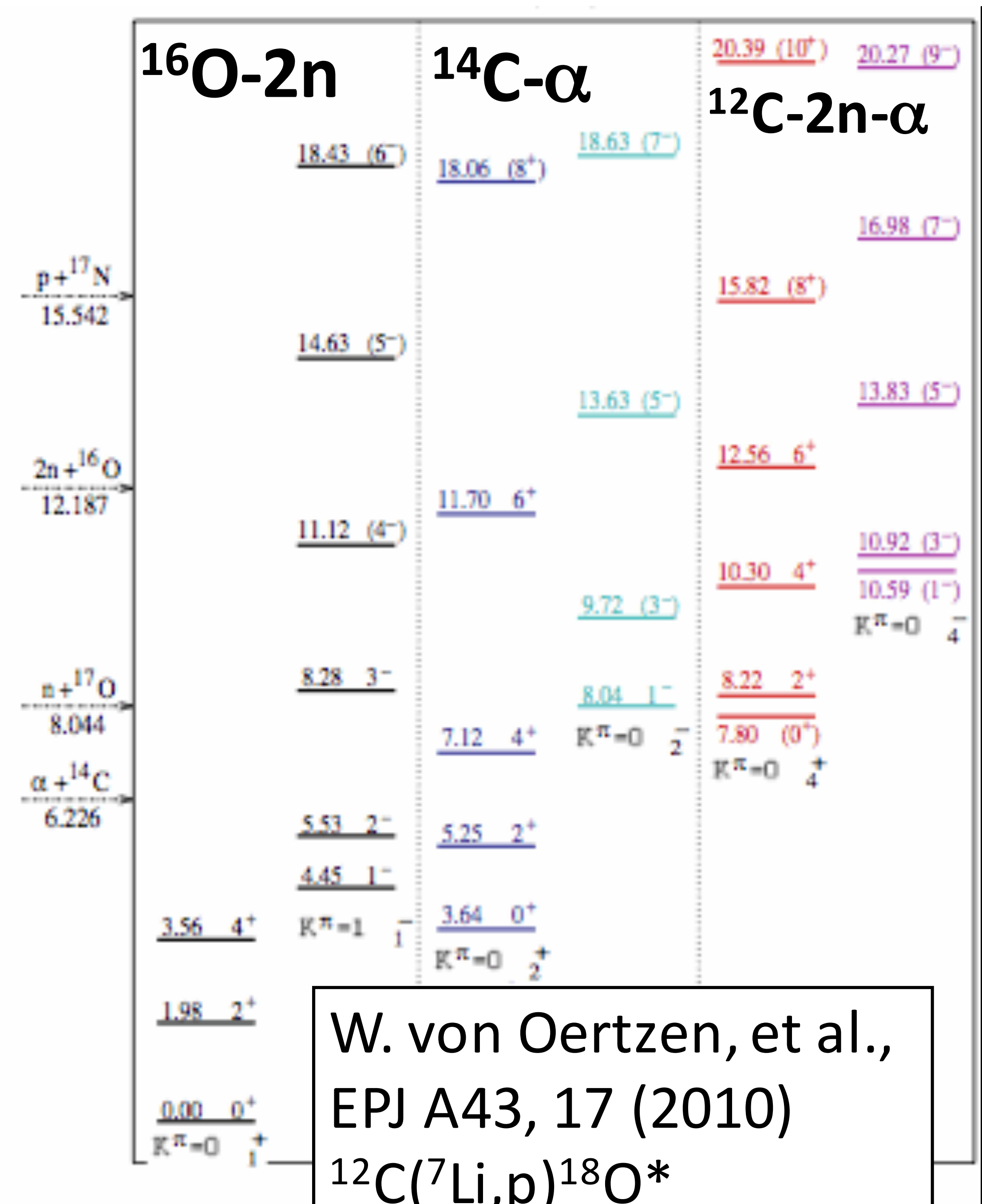


- Where is the α-cluster negative parity band in ^{18}O?





AMD calculations:
Y. Kanada-En'yo, H. Horiuchi
Prog.Th.Phys. 93 (1995), 115



W. von Oertzen, et al., EPJ A43, 17 (2010) $^{12}C(^{7}Li,p)^{18}O^*$

E* (MeV)	Jπ	Γtot(keV)	Γα(keV)	Γα/Γsp
8.04	1-	2	2	0.02
8.21	2+	1	1	<0.01
8.29	3-	8	2	0.09
8.78	2+	70	1	<0.01
8.98	2+	60	4	0.01
9.17	1-	240	205	0.24
9.36	2+	24	1	<0.01
9.39	3-	155	103	0.47
9.69	3-	56	0.1	<0.01
9.79	2+	263	167	0.20
9.76	1-	740	658	0.48
9.9	0+	2100	2100	1
10.1	3-	17	12	0.02
10.3	4+	23	16	0.08
10.34	2+	111	20	0.02
10.4	3-	48	17	0.02

Summary

- Nuclear reactions are powerful tools to probe nuclear structure
- Exciting recent theoretical developments open up a possibility to describe nuclear structure and reaction from first principles
- Advances in experimental techniques make it possible to probe structure of very exotic nuclei and challenge the theoretical predictions
- **We leave in truly exciting time for Nuclear Physics - you made the right choice !**