I. Cluster DOF in nucleus

II. How to form?

III. How to probe?

IV. Some perspectives
Themes and challenges of Modern Science

• Complexity out of simplicity

How the world, with all its apparent complexity and diversity can be constructed out of a few elementary building blocks and their interactions

• Simplicity out of complexity

How the world of complex systems can display such remarkable regularity and simplicity

Copenhagen spirit:
Complementary and exclusive.
Exotic structures in unstable nuclei

shell evolution, halo, SDR, cluster, molecular......
Clustering happens more easily in an expanding & low-density environment

Fig. 1. Schematic figure for rich phenomena in nuclear systems.
Examples at high excitation

Members of the $K = 0^{+}_2$, $1^{-}_2$, and $0^{+}_4$ bands of $^{20}$O (with tentative assignments) are marked by downward hatched (blue), filled (cyan), and upward hatched (red) areas, respectively.
Our latest measurement for $^{18}$O:

- **Ex:** 7-19 MeV; 29 states;
- New states $Ex > 14$ MeV

High resolution:
- $11.15/1.47/111.72$ MeV,
- $12.38/12.58/12.94$ MeV
- Well resolved.

Clustering in the universe

Clustering in hadrons

QCD: There are many other possible color singlets.

dibaryon

pentaquark

glueball

diquark + di-antiquark
dimeson molecule

$q \bar{q} g$ hybrid
Impact on the nuclear-astrophysics

The famous Hoyle state
Furthermore:

- **Multi-nucleon/cluster transfer**;
- **Neutron-pair BEC state at the surface (crust) of neutron stars**
- ......
Clustering is featured by:

- non-linearity;
- self-stabilization;
- irregularity.

Exotic!
I. Cluster DOF in nucleus

II. How to form?

III. How to probe?

IV. Some perspectives
The Alpha-Particle Model of the Nucleus

L. R. HAFSTAD
Department of Terrestrial Magnetism, Carnegie Institution of Washington, Washington, D. C.

AND

E. TELLER
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(Received August 26, 1938)

Fig. 5. Left: The binding energy systematics leading to the \( \alpha \)-cluster model used by Hafstad and Teller [123]. The plot shows the change in binding energy versus the number of bonds in an alpha-particle model. For example, \(^{8}\text{Be}\) has 1 bond, \(^{12}\text{C}\)–3, \(^{16}\text{O}\)–6, \(^{20}\text{Ne}\)–9, etc., see text. Right: the \( \alpha \)–\( \alpha \) interaction potential used in Ref. [123], see also Fig. 12.
treme tightness. In this note, by applying the pion-theoretical potentials recently verified in two-nucleon problems, interactions between \( \alpha \)-particles are investigated from the viewpoint of the cluster model, without taking account of the polarization effects of \( \alpha \)-particles.

Fig. 1. The dashed lines represent the phenomenologically determined \( \alpha - \alpha \) interactions for \( S \)- and \( D \)-states.\(^{3)} \) The upper curves are obtained from the potential with an attractive TPEP only in the singlet even state and the lower curves from the potential with an additional attractive TPEP in the triplet even and triplet odd states.
The Systematic Structure-Change into the Molecule-like Structures in the Self-Conjugate $4n$ Nuclei

Kiyomi Ikeda, Noboru Takigawa and Hisashi Horiuchi

Department of Physics, University of Tokyo, Tokyo

The rotational bands with the diatomic-molecule-like structure in the self-conjugate $4n$ light nuclei, such as, $\alpha$-$\alpha$, $\alpha$-$^{12}$C and $\alpha$-$^{16}$O, appear systematically at near the threshold energy for the decay into the relevant subunit nuclei. The relations between the structure change into the molecule-like structure and the threshold energy for the decay into the subunit nuclei are discussed. According to this discussions, the diagram for the systematic structure changes into the molecule-like structures through the alpha particle release is presented as the function of the mass number and the energy. Upon this diagram the rotational bands with $K=0^\pm$ in light $4n$ nuclei can be summarized. The order of the degree of the polarization toward the separation of the subunit nuclei for the diatomic molecule-like structure case is discussed qualitatively.
Threshold rule

Fig. 1. Threshold energy for each decay mode. In the figure, the threshold energy for each decay mode is given in MeV. The systematics suggests the possible molecular nature around each energy. Some of the molecular states are already found and are represented in Fig. 2.
the rule\textsuperscript{1)} of the constancy of the binding energy per bond. The assumption of the relative tightness of the alpha particles in whole system was, however, considered to be doubtful\textsuperscript{2,3)} since alpha particles could easily dissolve into their constituents in the nucleus. The nucleus of \textsuperscript{8}Be which should be basic

Recognitions of the two different kinds of the structures in the ground states of the \(4n\) light nuclei lead us to an understanding that alpha particles lose its identity in the compact nucleus, where there remain the correlated characters like in \(^{16}\text{O}\), and, reversely, the correlated four particles in the whole system are able to tend to become alpha particles if the compactness of the whole nuclear system is released, such as the nucleus of \textsuperscript{8}Be. When we accord to this understanding the changes of the structures can be expected to arise in the excited states of the \(4n\) light nuclei, since the release of the compactness depends on not only the mass number but also the energy of the nuclear system.
Experimental Determination of In-Medium Cluster Binding Energies and Mott Points in Nuclear Matter


Cyclotron Institute, Texas A&M University, College Station, Texas 77843, USA

Typel et al. [1] have developed a quantum statistical approach which includes cluster correlations in the medium and interpolates between the exact low-density limit and the very successful relativistic mean field (RMF) approaches appropriate near the saturation density. The generalized RMF model developed attributes the decrease of the cluster fractions at high densities to a reduction of the cluster binding energies due to the Pauli blocking. This leads to the Mott effect of vanishing binding [2]. Well-defined clusters appear only for densities below approximately 1/10 of the saturation density and get dissolved at higher densities. The maximum cluster density is reached
FIG. 2. In-medium binding energies derived from the experiments as a function of density. $T$ and $\rho$ are changing in a correlated fashion. (See text.)
Theoretical descriptions

** Theory: ** AMD, FMD, THSR, GCM(RGM), MO, GTCM, FMD, TCSM, TCHO(DHO), ...

**Progress of Theoretical Physics Supplement No. 192, 2012**

**Recent Developments in Nuclear Cluster Physics**

Hisashi Horiuchi,¹,² Kiyomi Ikeda³ and Kiyoshi Kato⁴

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**Unified studies of chemical bonding structures and resonant scattering in light neutron-excess systems, **¹⁰⁷,¹²⁷Be

**Progress in Particle and Nuclear Physics 82 (2015) 78–132**

**Review**

Cluster models from RGM to alpha condensation and beyond

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c International Institute for Advanced Studies, Kizugawa 519-0225, Japan

**REVIEWS OF MODERN PHYSICS, VOLUME 90, JULY–SEPTEMBER 2018**

**Microscopic clustering in light nuclei**
It is not enough just to use the density distribution to define a cluster structure!

s, p and d orbital densities for single states.
Production and detection considerations

1) Large Q values

In favor of the selection of the reaction mechanism

\[
\begin{align*}
    d + ^9\text{Be} & \rightarrow ^{10}\text{Be} + p + 4.6 \quad \text{MeV}; \\
    ^6\text{Li} + ^9\text{Be} & \rightarrow ^{13}\text{C} + d + 9.2 \quad \text{MeV}; \\
    & \rightarrow ^{14}\text{C} + p + 15.1 \quad \text{MeV}; \\
    ^7\text{Li} + ^9\text{Be} & \rightarrow ^{13}\text{C} + t + 8.2 \quad \text{MeV}; \\
    & \rightarrow ^{14}\text{C} + d + 10.1 \quad \text{MeV}; \\
    & \rightarrow ^{15}\text{C} + p + 9.1 \quad \text{MeV}; \\
    ^8\text{Li} + ^9\text{Be} & \rightarrow ^{16}\text{C} + p + 11.3 \quad \text{MeV}; \\
    ^{11}\text{B} + ^7\text{Li} & \rightarrow ^{14}\text{C} + \alpha + 18.13 \quad \text{MeV} \\
    ^9\text{Be} + ^9\text{Be} & \rightarrow ^{14}\text{C} + \alpha + 17.3 \quad \text{MeV}; \\
    ^{10}\text{Be} + ^9\text{Be} & \rightarrow ^{15}\text{C} + \alpha + 11.7 \quad \text{MeV}; \\
    ^{11}\text{Be} + ^9\text{Be} & \rightarrow ^{16}\text{C} + \alpha + 15.4 \quad \text{MeV};
\end{align*}
\]
2) Optimum beam energy 20-30 MeV/u

Incident-energy dependence of the fragmentation mechanism reflecting the cluster structure of the $^{19}$B nucleus

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(Received 7 September 2000; published 21 February 2001)

The fraction of the dynamical component of the coincident cross sections between He and Li isotopes in $^{13}$B+$^{14}$N and $^{19}$B+$^{14}$N reactions.
Freer et al., PRL82(1999) 1383; PRC63 (2001)034301
Charity et al., PRC76(2007)064313

No small angle (low $E_{rel}$) detection

$^6\text{He}+^6\text{He}, \text{CH}_2$

$^4\text{He}+^8\text{He}, \text{with P}$

$^4\text{He}+^8\text{He}, \text{with C}$
I. Cluster DOF in nucleus

II. How to form?

III. How to probe?

IV. Some perspectives
1. Knockout reaction \((p,pC)\) for ground state clustering
QFS:

Differences to free NN:
- Fermi motion;
- Multiple collision & dissipation;
- Wave distortion;
- Medium effects;
- decay of the hole state

Observables:
- Separation energy
- Momentum correlation

NN scattering in nuclear field
(p,2p) reaction

Graphs showing data for different reactions with parameters:
- $T_o = 460$ MeV
- $\theta = 38.7^\circ$
- $S = 41.7$ MeV
- $S = 7.5$ MeV
• Being able to investigate the inner shell of nuclei

• Three arguments:
  i) mean free path: 
      \[ R = \frac{1}{\rho \sigma} \]
      must be large enough to avoid multi-scattering
  ii) wave length: small enough to avoid collective excitation (no problem for a proton)
  iii) high momentum transfer for a localized interaction
      requiring high energy (100-1000 MeV p) for probing inner orbits; but moderate energy is ok for probing surface nucleons (low binding)
Differing Sensitivity to the BSWF

Nonsudden Limits of Heavy-Ion Induced Knockout Reactions

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FIG. 1 (color online). Parallel-momentum distributions of projectile-like residues from inclusive one-nucleon knockout: (a) 15C and (b) 15B from 16C, (c) 13N and (d) 13O from 14O. Colors represent different settings for the S800 spectrometer. Calculated parallel-momentum distributions at mid-target energies convoluted with the momentum profile of the beam and shifted by a momentum (typically 100 MeV/c) corresponding to the residue’s energy loss in half of the target are shown as SE (dashed line) and TC (solid line) (see text). Theoretical distributions have been normalized to the data. Cutoff positions according to Eq. (1) are shown as vertical lines.

\[
P_\parallel = \sqrt{(T_p - S_n - \varepsilon_f)^2 + 2M_r(T_p - S_n - \varepsilon_f)},
\]
2. Direct reactions for excited cluster states

\[ \text{Dimers based on the } \alpha + \alpha \text{ potential and chain states of carbon isotopes} \]


W. von Oertzen

\[ ^9\text{Be} \rightarrow 2\alpha + n - 1.57 \text{ MeV} \]

\[ ^6\text{Li} \rightarrow \alpha + d - 1.47 \text{ MeV} \]

\[ ^7\text{Li} \rightarrow \alpha + t - 2.37 \text{ MeV} \]
Observation criteria

i) $E_x$ - spin systematics:
   high moment of inertia

ii) Large cluster decay width:
    large $\Gamma_{\text{Cluster}}/\Gamma$; $\gamma^2_{\text{Cluster}}$; $\theta^2_{\text{Cluster}}$

iii) Characteristic transition strength
    large $M(IS)$ !!

iv) Structural link in population and decay selective path

Cluster-decay: $E_x$ selectivity; AC-spin;
BR-SF; decay-path
Beam: $^{12}$Be, 29.0MeV/u, ~3000pps
Target: Carbon, 100 mg/cm$^2$
DSSD: 32 2mm-stip, 300μm, covering 0°-12° Lab.
CsI(Tl): 4 x 4, 2.5cm*2.5cm*3cm,
Detection focused on the most forward angles

$^{12}$Be: Molecular built on 2α cores
Observation of Enhanced Monopole Strength and Clustering in $^{12}\text{Be}$

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J. C. Pei (裴俊琛), R. Qiao (乔锐), H. B. You (游海波), H. Wang (王赫), Z. Y. Tian (田正阳), K. A. Li (李阔昂),
Y. L. Sun (孙世磊), H. N. Liu (刘红娜), J. Chen (陈洁), J. Wu (吴锦), J. Li (李晶), W. Jiang (蒋伟),
C. Wen (文超), B. Yang (杨彪), Y. Y. Yang (杨彦云), P. Ma (马朋), J. B. Ma (马军兵), S. L. Jin (金仕纶),
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(Received 10 December 2013; published 22 April 2014)

In a recent breakup-reaction experiment using a $^{12}\text{Be}$ beam at 29 MeV/nucleon, the $0^+$ band head of the
expected $^4\text{He} + ^8\text{He}$ molecular rotation was clearly identified at about 10.3 MeV, from which a large
monopole matrix element of $7.0 \pm 1.0 \text{ fm}^2$ and a large cluster-decay width were determined for the first
time. These findings support the picture of strong clustering in $^{12}\text{Be}$, which has been a subject of intense
investigations over the past decade. The results were obtained thanks to a specially arranged detection
system around zero degrees, which is essential in determining the newly emphasized monopole strengths to
signal the cluster formation in a nucleus.
State-of-the-art detection system at 0 degree; Uniform calibration of the Si strips; treatment of PID under intense direct beam:

i) Determination of the moment of inertia

For $E_x$:

- Q value in inelastic scattering, or reconstruction from binary decay:

$$E_{\text{rel}} = M^* - M_a - M_b = \sqrt{M^2 - M_a - M_b}$$

$$M^2 = M_a^2 + M_b^2 + 2(T_a + M_a)(T_b + M_b)$$

$$- 2\sqrt{(T_a^2 + 2T_aM_a)(T_b^2 + 2T_bM_b)} \cos \theta$$

$$E_x = E_{\text{rel}} + E_{\text{thre}}$$

For J (quite difficult):

- angular distribution of inelastic scattering, or angular correlation from binary decay.
Our exp: \( ^6\text{He}+^6\text{He} \) 11.7 13.3 MeV  
\( ^4\text{He}+^8\text{He} \) 10.3 12.1 13.6 MeV

Large and unusual 10.3 MeV state; pure 0+ !!
Angular correlation analysis for the 10.3 MeV state in $^{12}$Be decaying into $^4$He + $^8$He

$E_x$: 10.0 - 11.4 MeV

For small angle inelastic scattering leading to a resonant state with an angular momentum $J$, which subsequently breaks up into spin-0 fragments, the projected angular correlation spectrum is proportional to $|P_J(\cos(\Psi))|^2$, with $\Psi$ being the fragment c.m. angle relative to the beam direction.
ii) Determination of the cluster decay width and the cluster SF

\[ N(E) \propto \frac{\Gamma(E)}{[E - E_r - \Delta(E)]^2 + [\Gamma(E)/2]^2}. \]

\[ \Gamma = \Gamma_\gamma + \Gamma_n + \Gamma_p + \Gamma_\alpha + \cdots \]

the partial is of probability meaning, not energy meaning: \( \Gamma_i / \Gamma = \sigma_i / \sigma \)

\[ \Gamma_\alpha(E) = 2\gamma_\alpha^2 P_f(E), \quad P_f(E) = \frac{ka}{(F_l(ka))^2 + (G_l(ka))^2}, \]

\[ \theta_\alpha^2 = \frac{\gamma_\alpha^2}{\gamma_W^2}, \quad \gamma_W^2 = \frac{3\hbar^2}{2\mu a^2}. \]

where \( E \) is the decay energy (or relative energy) and \( a \) the channel radius. The latter is generally given by \( a = r_0(A_1^{1/3} + A_2^{1/3}) \) with \( r_0 \approx 1.4 \) fm. For \(^{12}\)Be decaying into \(^4\)He+\(^8\)He, the channel radius is about \( a = 5 \) fm. This value was also adopted in AMD calculations [32]. In eq. (2) \( F_l(ka) \) and \( G_l(ka) \) are regular and irregular Coulomb wave functions [9,31].
All possible decay channels

10.3 MeV(0+) state: $\Gamma = 1.5(2)$ MeV; $\Gamma = \Gamma_{\text{He}} + \Gamma_{\text{Be}}$

<table>
<thead>
<tr>
<th>反应道</th>
<th>阈值 [MeV]</th>
<th>(E_x=15MeV) 概率</th>
<th>(E_x=12MeV) 概率</th>
<th>文献结果 [33]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^4\text{He}+^6\text{He}$</td>
<td>9.6</td>
<td>39%</td>
<td>27.6%</td>
<td>36%</td>
</tr>
<tr>
<td>$^6\text{He}+^4\text{He}$</td>
<td>10.1</td>
<td>21%</td>
<td>13.1%</td>
<td>19.3%</td>
</tr>
<tr>
<td>$^4\text{He}+^5\text{He}+2n$</td>
<td>11.08</td>
<td>0.19%</td>
<td>1.1E-3%</td>
<td>0.027%</td>
</tr>
<tr>
<td>$^{11}\text{Be}+n$</td>
<td>3.17</td>
<td>44.3%</td>
<td>38.3%</td>
<td>40.9%</td>
</tr>
<tr>
<td>$^{10}\text{Be}+2n$</td>
<td>3.67</td>
<td>10.4%</td>
<td>5.6%</td>
<td>3.7%</td>
</tr>
<tr>
<td>$^9\text{Be}+3n$</td>
<td>10.48</td>
<td>0.098%</td>
<td>2.1E-3%</td>
<td>0.028%</td>
</tr>
<tr>
<td>$^8\text{Be}+4n$</td>
<td>12.06</td>
<td>6.9E-5%</td>
<td>---------------</td>
<td>3.4E-6%</td>
</tr>
</tbody>
</table>
Overall dominance of the $0^+$ state

**Figure:**

- Graph showing differential cross-section ($d\sigma/d\Omega$) as a function of center-of-mass angle ($\theta_{cm}$) for different $L$ values:
  - $L=0$
  - $L=2$, scaled by 0.1
  - $L=4$

- The graph highlights the comparison between Kosheninnilov's results and ours.

**Text:**

DWBA calculations for the excitation of $^{12}\text{Be}$ from its ground state to the 10.3 MeV excited state, when interacting with a C target.
10.3 MeV (0+) state: \( \Gamma = 1.5(2) \text{ MeV} \); \( \Gamma = \Gamma_{\text{He}} + \Gamma_{\text{Be}} \)

Kosheninnilov[12]: \( \Gamma_{\text{Be}} / \Gamma = 0.28 \pm 0.12 \)

\( \Gamma_{\text{He}} / \Gamma = 1 - \Gamma_{\text{Be}} / \Gamma = 0.72(12) \); \( \Gamma_{\text{He}} = 1.1(2) \text{ MeV} \)

\( \gamma_{\text{He}}^2 = 0.50(9) \); \( \theta_{\text{He}}^2 = 0.53(10) \) (comparable to \( ^8\text{Be} \))
iii) Determination of the monopole transition strength

T. Yamada et al., PRC85,034315(2012); PTP120,1139(2008)

Isoscaler monopole excitation means a jump of about 35 MeV in a simple single-particle picture. A strong $M(IS)$ for $E_x$ below 20 MeV is an indicator of cluster formation.
Determining the monopole strength
Fraction: 0.034(10) x 2.2.

EWSR: 6727.9 fm$^4$ MeV,

$M(IS)$: 7.0 $\pm$ 1.0 fm$^2$,

$M(IS)$ (cluster) $\sim$ 9.0 fm$^2$. 
$^{14}\text{C}$: Possible chain states based on $3\alpha$ cores

$^8\text{Be}(2\alpha)$

$^9\text{Be} \rightarrow 2\alpha + n - 1.57 \text{ MeV}$

$^6\text{Li} \rightarrow \alpha + d - 1.47 \text{ MeV}$

$^7\text{Li} \rightarrow \alpha + t - 2.37 \text{ MeV}$

W. Von Oerttzen et al.,
Major improvements:

- Gogny D1S force to better describe $E_x$;
- Projected single particle wave function for valence neutrons to distinguish the $\pi$-bond or $\sigma$-bond states;
- Core excitation included and the reduced decay-width deduced accordingly.

$$E''_{\pi} = \frac{\langle \Phi_{\pi}|H|\Phi_{\pi} \rangle}{\langle \Phi_{\pi}|\Phi_{\pi} \rangle} + v_\beta(\langle \beta \rangle - \beta_0)^2 + v_\gamma(\langle \gamma \rangle - \gamma_0)^2$$

$$\tilde{\phi}_s = \sum_{\alpha=1}^{A} f_{\alpha s} \tilde{\varphi}_\alpha.$$
Very high selectivity for $\sigma$-bond LCS
Structural link in decay scheme — exp.

$|^{10}\text{Be}_0^+(6.26 \text{ MeV}) > a_0(\sigma_{1/2} u)^2_0^+$

$|^{10}\text{Be}_2^+(5.958) > a_1(\pi_{3/2} - g)^2_{2^+}$

$|^{10}\text{Be}_1^-(5.960) > a_2[(\pi_{3/2} - g) \otimes (\sigma_{1/2} u)]_{1^-}$

$|^{10}\text{Be}_2^- (6.263) > a_3[(\pi_{3/2} - g) \otimes (\sigma_{1/2} u)]_{2^-}$

Q-value spectra for $^9\text{Be}(^9\text{Be}, ^{10}\text{Be} \alpha)$

Experimental probes
Basic considerations for experimentation

- Projectile and target in favor of cluster formation
- Large Q-value reaction in order to excite high-lying states in $^{14}$C and to have a good selection of the states in $^{10}$Be fragment;
Event sample I: Observed states

|             | This work | \(^{7}\text{Li}\left(^{9}\text{Be},\alpha^{10}\text{Be}\right)\alpha[11]| | \(^{14}\text{C}\left(^{14}\text{C},\alpha^{10}\text{Be}\right)^{14}\text{C}[12]| |
|-------------|-----------|-------------------------------------------------|-------------------------------------------------|
| \(^{10}\text{Be}_{gs}\)| 16.5(1)   | 16.4(1)                                         | 16.4(1)                                         |
| \(^{10}\text{Be}(2^+)\)| 17.9(1)   | 17.3(1)                                         | 17.3(1)                                         |
| \(^{10}\text{Be}(\sim 6 \text{ MeV})\)| 18.8(1)   | 18.6                                             | 18.6                                             |
| \(^{10}\text{Be}(2^+)\)| 19.8(1)   | 19.8(1)                                         | [19.0(2)]                                       |
| \(^{10}\text{Be}(\sim 6 \text{ MeV})\)| 20.8(1)   | 20.8                                             | 20.4(1)                                         |
| \(^{10}\text{Be}(2^+)\)| 21.4(1)   | 21.4(1)                                         | [21.6(2)]                                       |
| \(^{10}\text{Be}(\sim 6 \text{ MeV})\)| 22.0(1)   | 22.0(1)                                         | [21.9(1)]                                       |
| \(^{10}\text{Be}(2^+)\)| 22.5(1)   | 22.5(3)                                         | [22.5(1)]                                       |
| \(^{10}\text{Be}(\sim 6 \text{ MeV})\)| 23.5(1)   | 23.1(3)                                         | [23.1(2)]                                       |
| \(^{10}\text{Be}(2^+)\)| 24.0(1)   | 24.0(3)                                         | 24.0(3)                                         |
| \(^{10}\text{Be}(\sim 6 \text{ MeV})\)| 24.7(1)   | 24.7(3)                                         |                                                   |

\(^{14}\text{C}(^{14}\text{C},\alpha^{10}\text{Be})^{14}\text{C}[12]\)

---

These states were assigned by comparison with neighbouring decay path.
Relative decay strengths

\[ \sigma \text{-bond LCS: } \begin{array}{l} 22.5 \text{ MeV } \text{yes} \\ 23.1 \text{ MeV } \text{yes} \\ 24.7 \text{ MeV } \text{likely} \end{array} \]

\[ \text{Predictions: } \begin{array}{l} 22.16 \text{ MeV} \\ 22.93 \text{ MeV} \\ 24.30 \text{ MeV} \\ 26.45 \text{ MeV} \end{array} \]
### Properties of states with $E_x > 20$ MeV

<table>
<thead>
<tr>
<th>$E_x$ /MeV</th>
<th>Decay mostly to $^{10}\text{Be}(\sim 6$ MeV)</th>
<th>Decay to $^{10}\text{Be}(2_3^+)$</th>
<th>Decay to $^{8}\text{Be}+^{6}\text{He}$</th>
<th>Like $\sigma$-bond LCS</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 22</td>
<td>No</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>22.5</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes, lowest</td>
</tr>
<tr>
<td>23.1</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>24.7</td>
<td>likely</td>
<td>Not clear</td>
<td>Yes</td>
<td>Likely</td>
</tr>
<tr>
<td>25.7</td>
<td>Not clear</td>
<td>special</td>
<td>Yes</td>
<td>Different</td>
</tr>
</tbody>
</table>

Peak width $< 300$ keV
$^{14}$C: triangle, and $\pi$-bond or $\sigma$-bond linear-chain states
18O: Reflection asymmetric systems

- symmetry breaking under reflection operation
- parity inversion doublet bands: almost parallel, separated by \( \sim 5 \) MeV
  
  \[ \phi_r = |^{14}\text{C} \otimes \alpha\rangle \text{ (}\alpha\text{-cluster right)} \]
  
  \[ \phi_l = |\alpha \otimes ^{14}\text{C}\rangle \text{ (}\alpha\text{-cluster left)} \]
  
  \[ \Phi^\pm = N(\phi_r \pm \phi_l). \]

- observed in some even-even stable nuclei.

M. L. Avila, PHYSICAL REVIEW C 90, 024327 (2014)

$\alpha$-cluster structure of $^{18}O$

Resonant scattering; R-matrix analysis;

Finding:
- Previously classified molecular states are mostly with very small SF;
- The larger SFs are very much fragmented.

* Importance to determine the spins, cluster decay BRs (SFs) of the possible molecular states.
Reaction channel

\[ {^{13}\text{C}} + {^{9}\text{Be}} \rightarrow {^{18}\text{O}} + \alpha \]  \hspace{1cm} Q: 12.83 MeV

\[ {^{13}\text{C}} + {^{9}\text{Be}} \rightarrow {^{14}\text{C}} + \alpha \]  \hspace{1cm} Q: 6.60 MeV

- beam:  \( {^{13}\text{C}}; \)  65 MeV
- target:  \( {^{9}\text{Be}}; \)  1.4 \( \mu \text{m} \)
Detector setup

Combinations:

- L0+R0: $^{14}\text{C} + \alpha_{\text{dec.}}$
- L0+R0/R2: $\alpha_{\text{dec.}} + \alpha_{\text{rec.}}$
High resolution energy-spectra

\[ E_x(\text{IM}) : \]
7-19 MeV; 28 states;
New states \( E_x > 14 \) MeV

High resolution:
11.15/1.47/11.72 MeV, 
12.38/12.58/12.94 MeV
Well resolved.

Branching Ratio

\[ BR = \frac{N_{\text{IM}}/\epsilon_{\text{IM}}}{100 \times N_{\text{MM}}/\epsilon_{\text{MM}}} \]

Obtained for 14 states

<table>
<thead>
<tr>
<th>IM/MeV</th>
<th>MM/MeV</th>
<th>Branching Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.28</td>
<td>10.3</td>
<td>0.37±0.13</td>
</tr>
<tr>
<td>11.13</td>
<td>11.1</td>
<td>0.65±0.22</td>
</tr>
<tr>
<td>11.47</td>
<td>11.8</td>
<td>&gt; 0.23±0.08</td>
</tr>
<tr>
<td>11.72</td>
<td></td>
<td>&gt; 0.89±0.29</td>
</tr>
<tr>
<td>12.38</td>
<td>12.5</td>
<td>&gt; 0.41±0.14</td>
</tr>
<tr>
<td>12.58</td>
<td>12.5</td>
<td>&gt; 0.79±0.26</td>
</tr>
<tr>
<td>12.94</td>
<td>13.1</td>
<td>&gt; 0.94±0.31</td>
</tr>
<tr>
<td>13.64</td>
<td></td>
<td>&gt; 0.07±0.02</td>
</tr>
<tr>
<td>13.87</td>
<td>13.9</td>
<td>&gt; 0.32±0.11</td>
</tr>
<tr>
<td>14.18</td>
<td></td>
<td>&gt; 0.16±0.05</td>
</tr>
<tr>
<td>14.69</td>
<td>14.8</td>
<td>0.08±0.03</td>
</tr>
<tr>
<td>15.88</td>
<td></td>
<td>&gt; 0.57±0.19</td>
</tr>
<tr>
<td>16.06</td>
<td>15.9</td>
<td>&gt; 0.09±0.03</td>
</tr>
<tr>
<td>16.20</td>
<td></td>
<td>&gt; 0.09±0.03</td>
</tr>
</tbody>
</table>
Angular correlation and spin

\[ a(A, B^* \rightarrow c + C)b, \]

Spin 0 for c & C

\[ W(\theta^* = 0^\circ, \psi' = \psi - \alpha \theta^*) \]

\[ \alpha = \frac{l_i - J}{J}, \]

\[ l_i = r_0(A_p^{1/3} + A_t^{1/3}) \sqrt{2\mu E_{\text{c.m.}}} \]

\[ |P_J(\cos(\psi))|^2. \]

Fig. 3. (color online) Schematic diagram of the four symmetric reaction-decay processes in the chamber plane. (a) and (b) are parity-symmetric processes, while (c) and (d) are their axial-symmetric processes, respectively. All processes are identified by the angles \( \theta^* \) and \( \psi \) defined in various coordinate systems, as described in the text.
For the 10.3 MeV state

\( \alpha \text{ dec. (T0)+ } \alpha \text{ rec. (T2) events} \)

\( \theta_{\text{cm}}: \ 4^\circ \sim 15^\circ \)

\( \alpha \text{ rec. } \)

\( ^{14}\text{C} \) beam

4\(^+\) for 10.29 MeV
Positive-parity band confirmed

Prediction (2010):

- $0^+(3.63\text{MeV})$, bound;
- $2^+(5.24\text{MeV})$, bound
- $4^+(7.11\text{MeV})$, previously determined
- $6^+(11.69\text{MeV})$, now separated and large SF determined

Negative-parity band not-confirmed

Prediction (2010):
1- (9.6MeV),
3- (9.8MeV),
5- (13.1MeV)

SF fragmented?

Present data based on precise coincident measurement.
PRC99(2019)064315

Ref. PRC90(2014)024327
28 α-decay states in $^{18}$O were observed with high precision, including a few new states.

α-decay BR are extracted for 14 resonances, and their SF are deduced by using existing tentative spins.

Spin-parity of 4$^+$ is determined for the 10.3 MeV state, by using the AC method.

The positive parity band is confirmed for the 14C + α configuration in $^{18}$O, whereas the related negative-parity band is still questionable.
Outline

I.  Cluster DOF in nucleus
II. How to form ?
III. How to probe ?
IV. Some perspectives
A new experiment for $^{16}\text{C}$ (Apr. 2018)

60MeV/u 100-400enA $^{18}\text{O}^{8+}$

25MeV/u $^{16}\text{C}$ $10^5$pps

Primary Target: $^8\text{Be}$
Degradner Wedge

Poster by Y. Liu
A new experiment for $^{14}C$ (May 2019)

Poster by J. X. Han
Open problems for clustering in light nuclei

- $^{0}_{3}^{+}$ and $^{0}_{4}^{+}$ in $^{10}\text{Be}$;
- $^{12}\text{Be}$ systematics ($^{6}\text{He}+^{6}\text{He}?$);
- broad $^{0}_{3}^{+}$ state in $^{12}\text{C}$;
- chain states in $^{14}\text{C}$ and $^{16}\text{C}$;
- BEC condensation states...;
- molecular bands in $^{18-28}\text{O}$;
- cluster + GR;
- 2n, 4n correlations;
- ......
Close theory & experiment cooperation

2015.08. Hokkaido U & Osaka-RCNP
2016.11. Yokohama

2015.08. 2016.07. PKU

2017.11. Hokkaido U
2018.11. Sichuan U
Homework problems?

- Is cluster a new layer of nuclear structure?
- How cluster structure could affect the nuclear dynamics (nuclear-astrophysics; SHN synthesis; nuclear technique applications...)?
- How to create the clustering states?
- How to experimentally determine the cluster formation in nuclei?
- What would be the possible cluster-formation mechanism in heavy nuclei?
- Would you expect strong clustering in medium-mass nuclei?
Thank you for your attention!