Production of doubly heavy hadrons at the Z pole

Xu-Chang Zheng（郑绪昌）

Department of Physics, Chongqing University

In collaboration with Chao-Hsi Chang and Xing-Gang Wu

CEPC WS 2019@IHEP

Outline

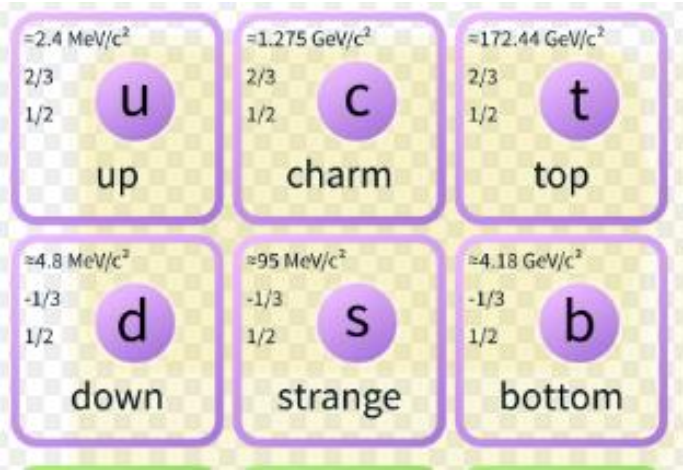
1. Introduction

2. Production of doubly heavy hadrons at e^+e^- colliders

3. NLO fragmentation functions for doubly heavy mesons

4. Conclusions

1. Introduction



Doubly heavy hadrons:

- Heavy quarkonia: $c\bar{c}$, $b\bar{b}$ (J/Ψ, $\Upsilon\cdots$)
- Doubly heavy-flavored mesons: $c\bar{b}$ (B_c, $B_c^*\cdots$)
- Doubly heavy baryons: Ξ_{cc}, Ξ_{bc}, Ξ_{bb} $\cdots$

➢ Production: perturbative, non-perturbative QCD

➢ Decay: weak interaction

The study of doubly heavy hadrons can test the NRQCD.

Advantages of the production at e^+e^- colliders:

- **The center-of-mass system of the process is known**

  Angle distributions and forward-backward asymmetry of doubly heavy hadrons have proper meaning in understanding the production.

- **There are less backgrounds at an e^+e^- collider**

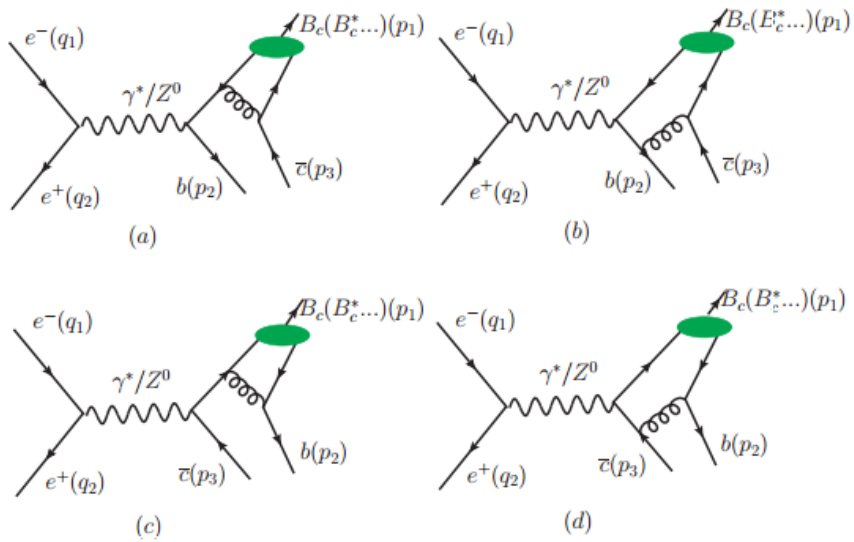
  An e^+e^- collider is a good platform for precision measurements.

Running at the Z pole:

- **Z-resonance effect**
- **CEPC**

2. Production of doubly heavy hadrons

- **LO calculation**



$$d\sigma(e^+ + e^- \to Bc + b + \bar{c})$$

$$= \sum_n d\hat{\sigma}(e^+ + e^- \to c\bar{b}[n] + b + \bar{c}) \langle O^{Bc}(n) \rangle$$

NRQCD factorization

Short-distance coefficients

Long-distance matrix elements

Numerical results

Phys. Rev. D 93, 034019, (2016), X.-C. Zheng, C.-H. Chang et al.

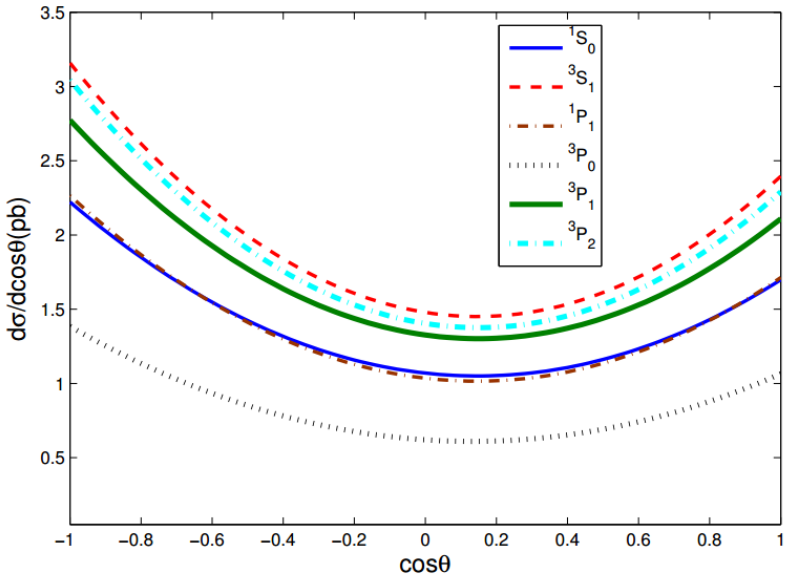
States	σ(**pb**)	Events/year
$B_c\ (\,^1S_0\,)$	2.73	2.7×10^6
$B_c^*(\,^3S_1\,)$	3.82	3.8×10^6
$B_c^{**}(\,^1P_1\,)$	0.27	2.7×10^5
$B_c^{**}(\,^3P_1\,)$	0.16	1.6×10^5
$B_c^{**}(\,^3P_2\,)$	0.34	3.4×10^5
$B_c^{**}(\,^3P_2\,)$	0.37	3.7×10^5

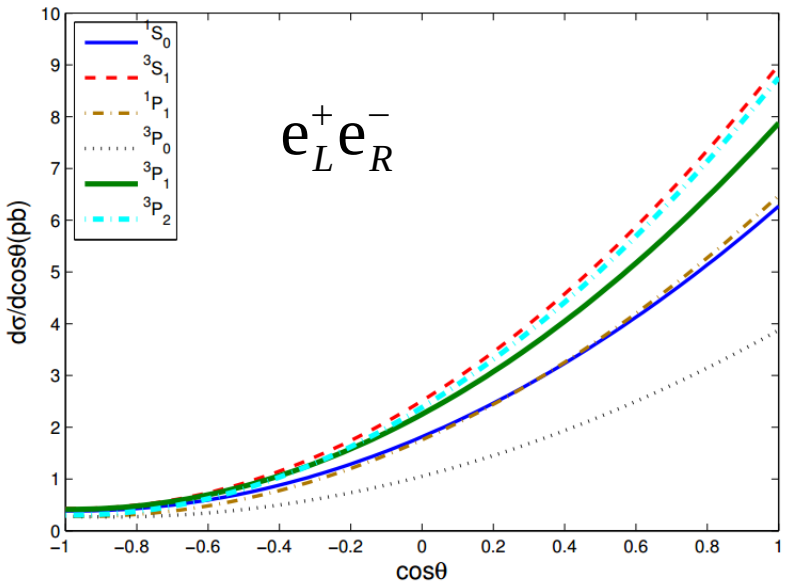
Cross sections at **the Z pole** with $L=10^{35}cm^{-2}s^{-1}$

States	σ(**fb**)	Events/year
$B_c\ (\,^1S_0\,)$	0.47	4.7×10^2
$B_c^*(\,^3S_1\,)$	0.72	7.2×10^2
$B_c^{**}(\,^1P_1\,)$	0.05	50
$B_c^{**}(\,^3P_1\,)$	0.03	30
$B_c^{**}(\,^3P_2\,)$	0.07	70
$B_c^{**}(\,^3P_2\,)$	0.07	70

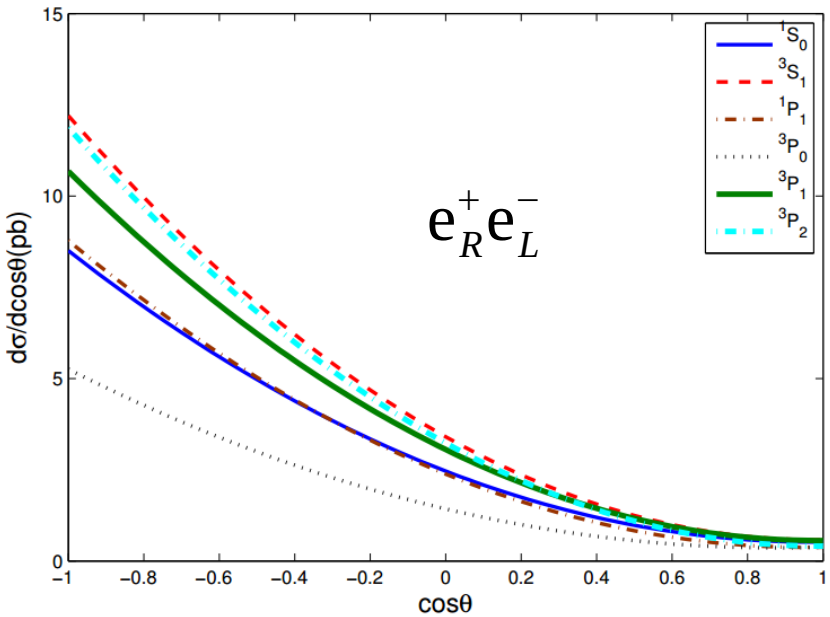
Cross sections at $\sqrt{s}=250GeV$ with $L=10^{35}cm^{-2}s^{-1}$

- **The Z-resonance effect is important for studying Bc and its excited states**
- **The luminosity of the e^+e^- collider should be $10^{35-36}cm^{-2}s^{-1}$**

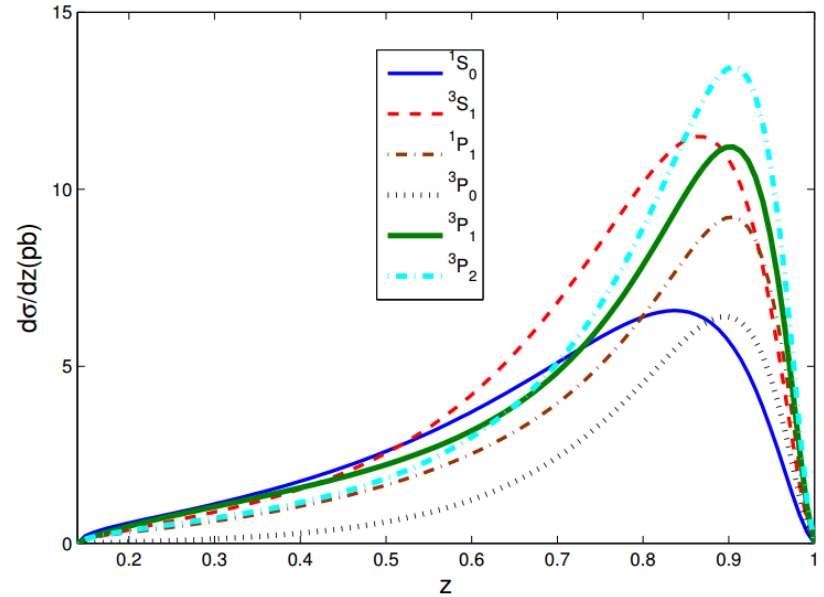


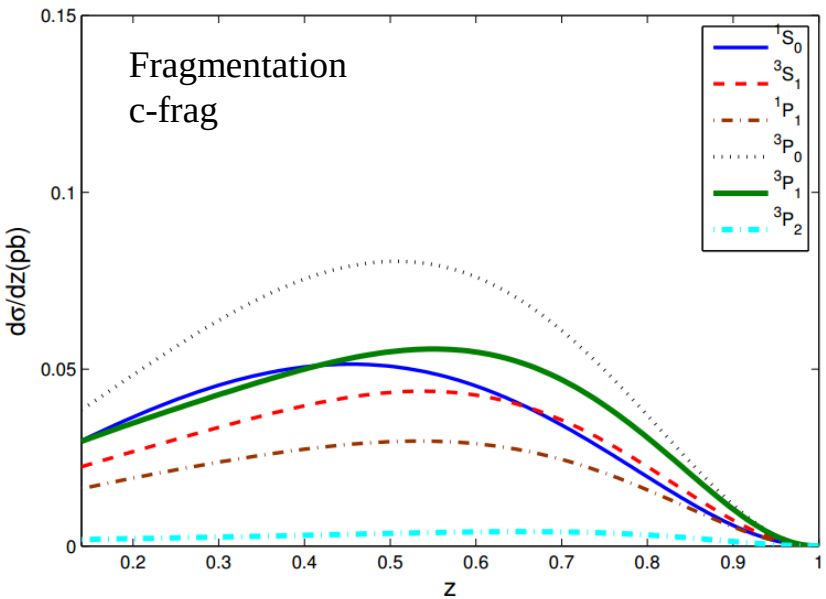


Phys. Rev. D 93, 034019, (2016),
X.-C. Zheng, C.-H. Chang et al.

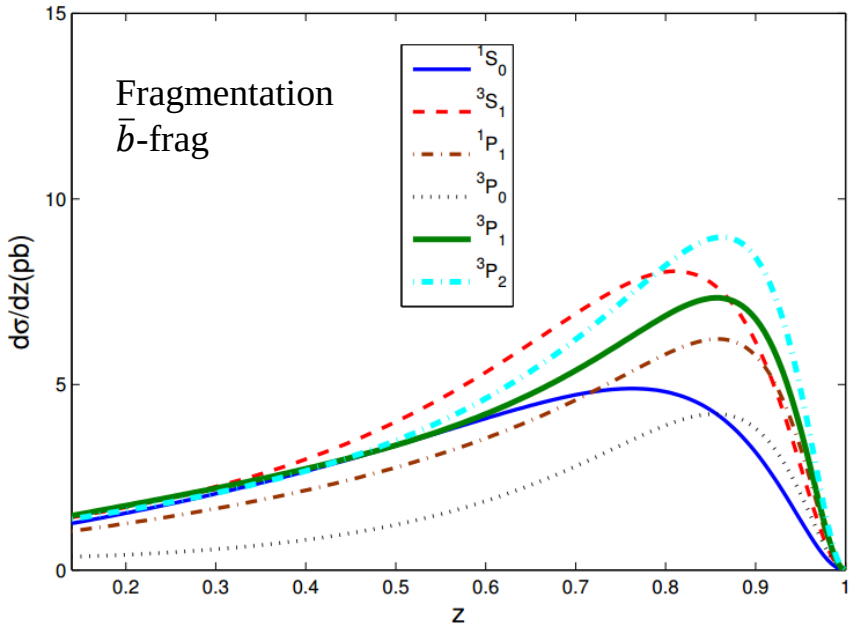


The angle distributions are forward-backward asymmetric.



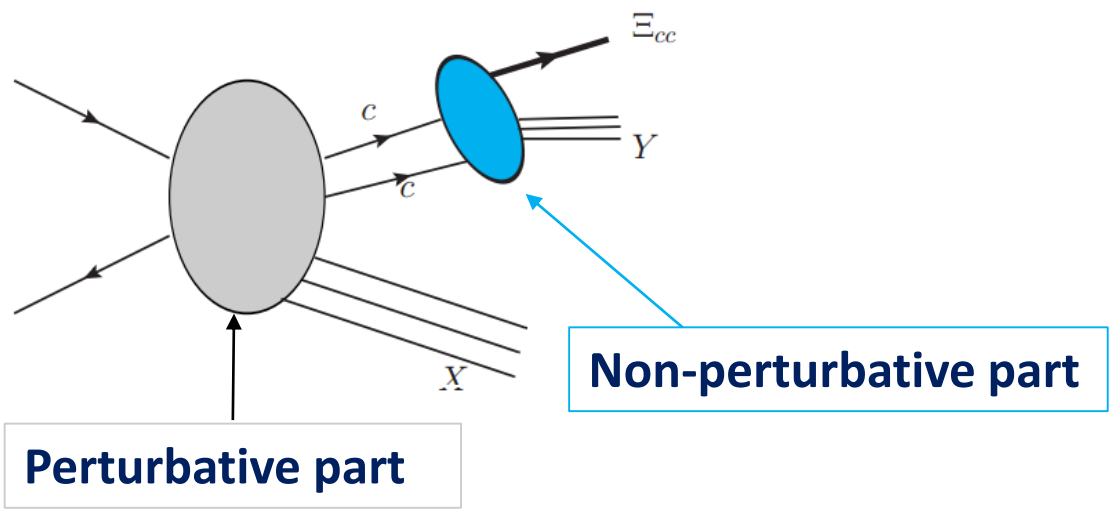


Phys. Rev. D 93, 034019, (2016),
X.-C. Zheng, C.-H. Chang et al.



The energy distributions.

Production of doubly heavy baryons



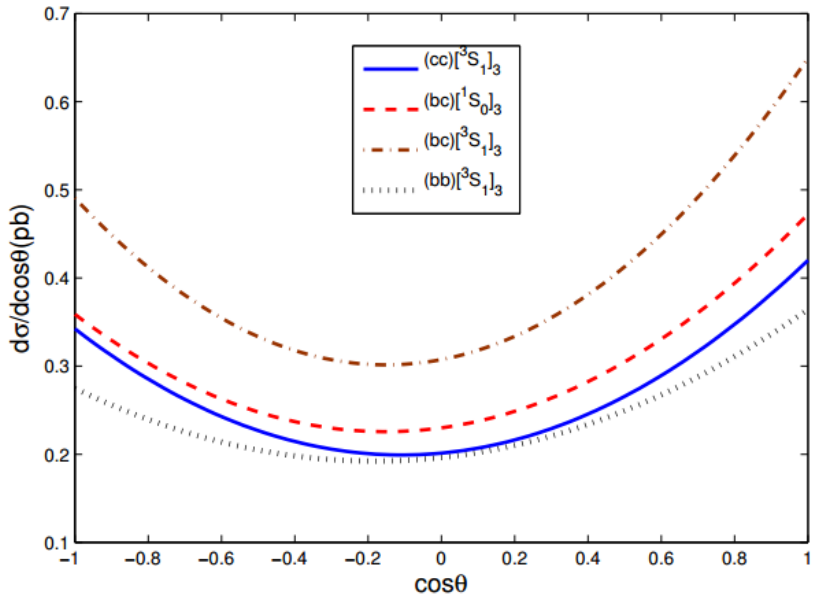
1) Production of diquark

2) The diquark fragments into the doubly heavy baryon

Phys. Rev. D 93, 034019, (2016),
X.-C. Zheng, C.-H. Chang et al.

States	σ(**pb**)	Events/year
Ξ_{cc}	0.52	5.2×10^5
Ξ_{bc}	1.37	1.4×10^6
Ξ_{bb}	0.05	5.0×10^4

Cross sections at **the Z pole**
with $L=10^{35} cm^{-2} s^{-1}$



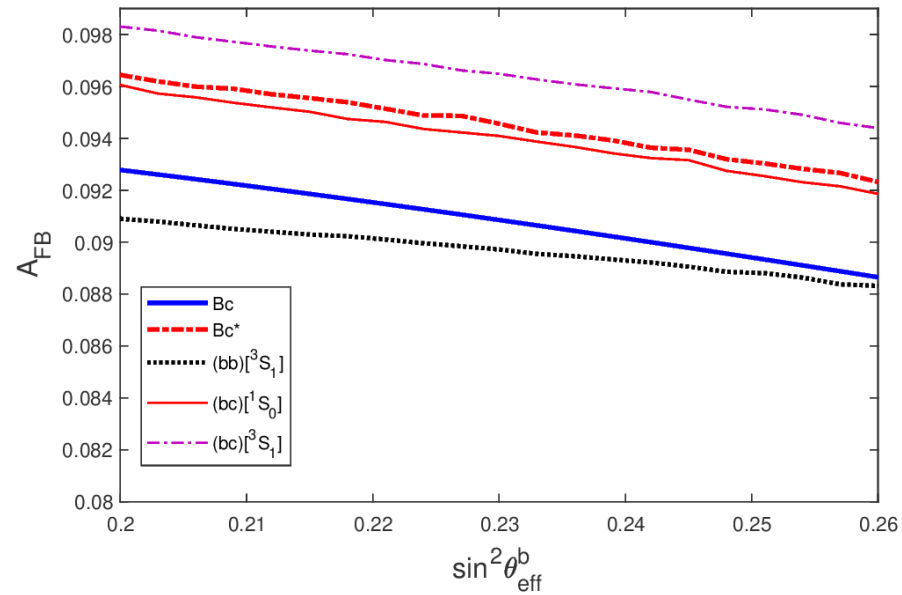
Differential angle distributions

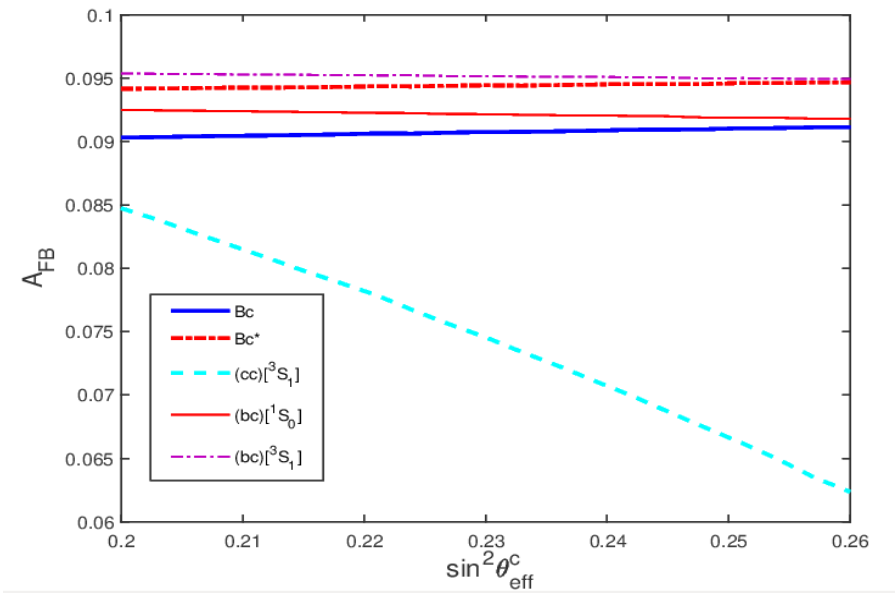
The angle distributions are also forward-backward asymmetric.

Forward-backward asymmetry:

X.-C. Zheng, C.-H. Chang et al, 2019.

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}.$$



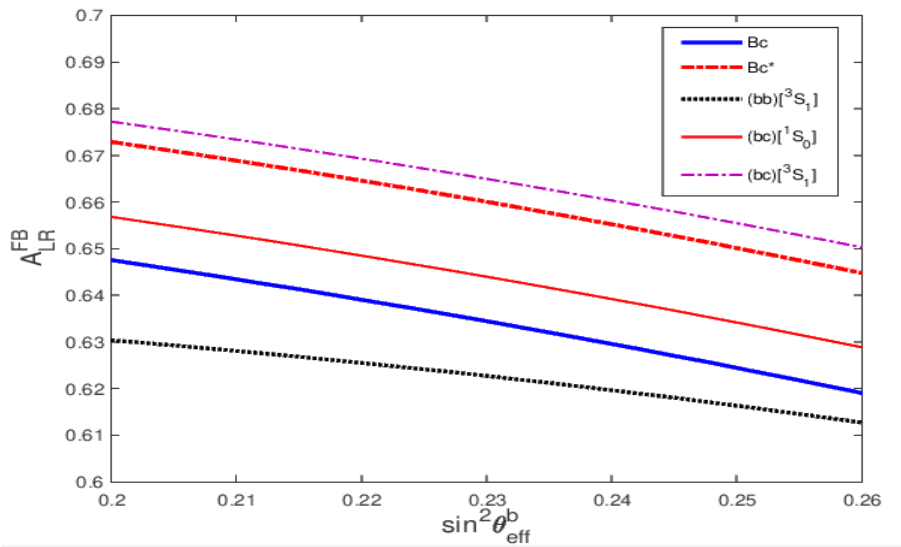


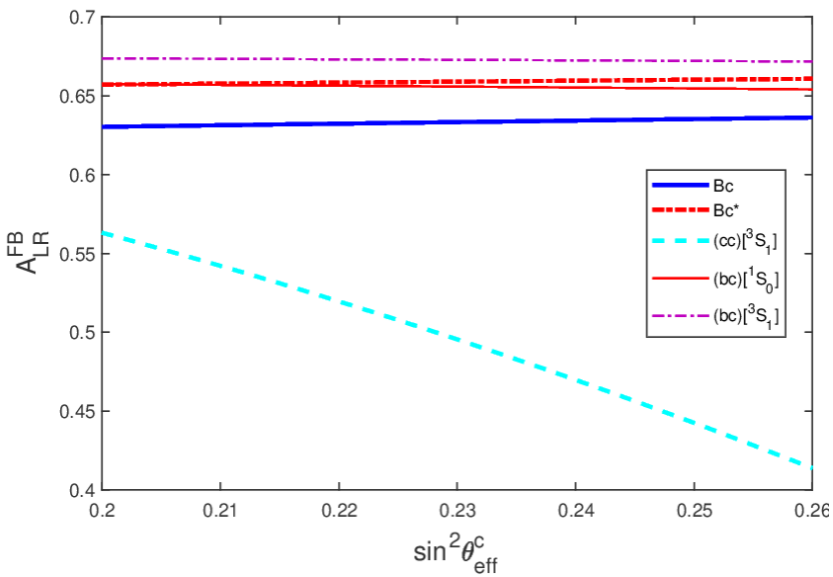
$\sin^2\theta^{f}_{eff}$ can be determined through measuring the forward-backward asymmetry of the doubly heavy-flavored hadrons.

Left-right forward-backward asymmetry:

X.-C. Zheng, C.-H. Chang et al, 2019.

$$A_{LR}^{FB} = \frac{\sigma_{LF} - \sigma_{LB} - \sigma_{RF} + \sigma_{RB}}{\sigma_{LF} + \sigma_{LB} + \sigma_{RF} + \sigma_{RB}}.$$

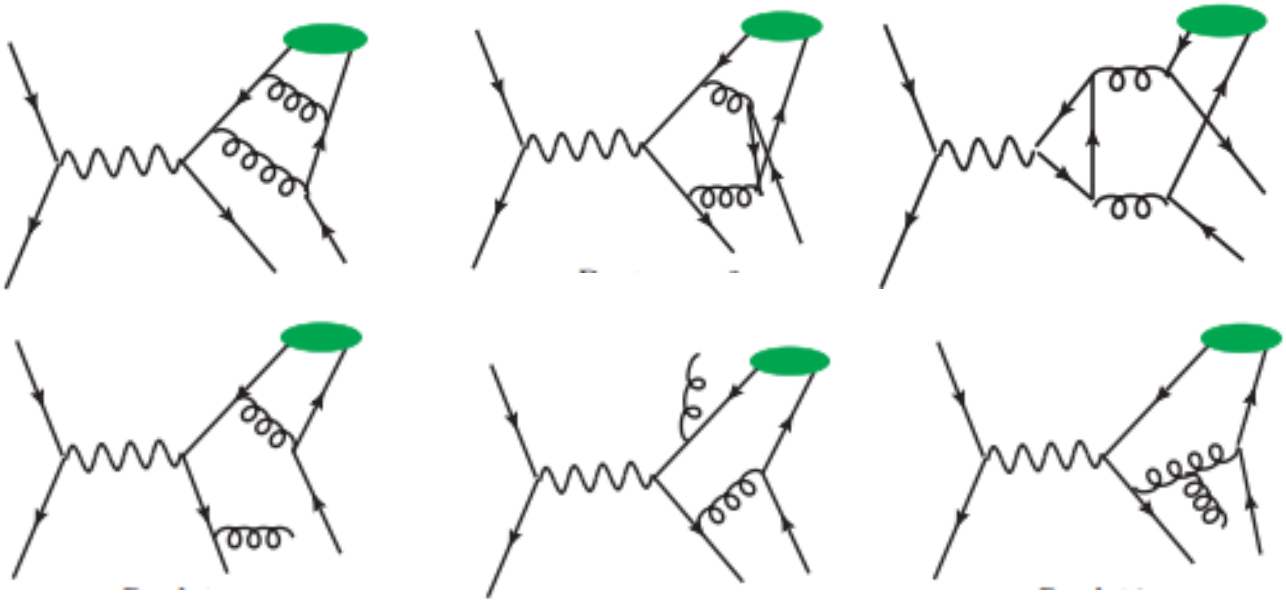




$\sin^2\theta_{eff}^f$ can be determined through measuring the left-right-forward-backward asymmetry of the doubly heavy-flavored hadrons.

NLO calculations for Bc and Bc*

- To see the changes of the physical observables from the LO calculations to the NLO calculations.
- To see how the dependence on the renormalization scale changes after including the NLO QCD corrections.



Numerical results

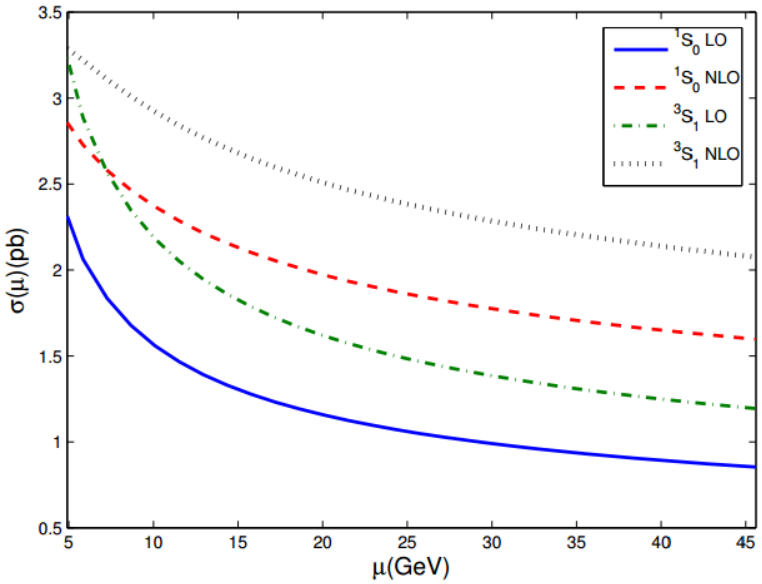
Sci. China-Phys.Mech. Astron. 61, 031012,(2018), X.-C. Zheng, C.-H. Chang et al.

μ	$\alpha_s(\mu)$	$\sigma_{\rm LO}$(pb)	$\sigma_{\rm NLO}$(pb)	$\sigma_{\rm NLO}/\sigma_{\rm LO}$
$2m_b$	0.180	1.58	2.38	1.51
$m_Z/2$	0.132	0.85	1.58	1.86

Cross section of Bc

μ	$\sigma_{\rm LO}$(pb)	$\sigma_{\rm NLO}$(pb)	$\sigma_{\rm NLO}/\sigma_{\rm LO}$
$2m_b$	2.20	2.93	1.33
$m_Z/2$	1.18	2.06	1.74

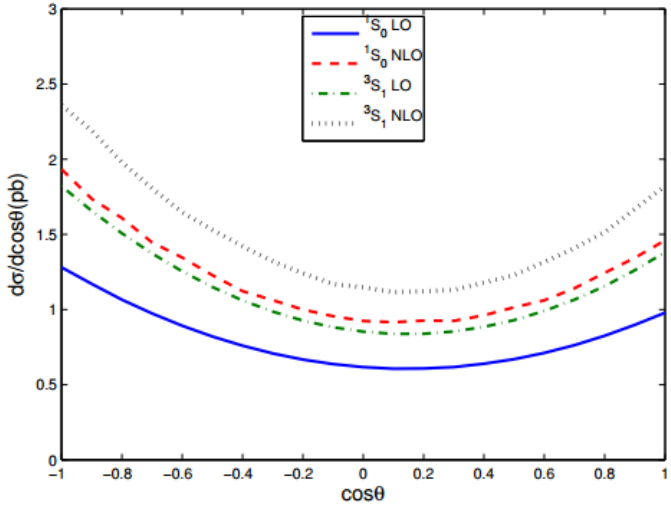
Cross section of Bc*



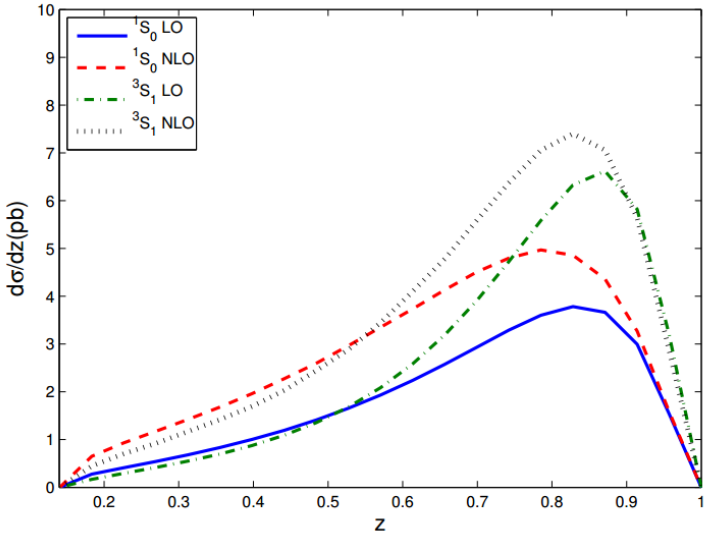
The dependence on μ is weaken significantly due to NLO corrections.

The NLO corrections are significant!

Sci. China-Phys.Mech. Astron. 61, 031012,(2018),
X.-C. Zheng, C.-H. Chang et al.



Differential angle distributions



Differential energy distributions

- The K-factor changes very little with different θ;
- The NLO corrections change the energy distributions significantly.

3. NLO fragmentation functions for doubly heavy mesons

Fragmentation approach

- NRQCD factorization

Energy scales: $\sqrt{s}, m_Q$

$$d\sigma(e^+ + e^- \to Bc + b + \bar{c})$$
$$= \sum_n d\sigma(e^+ + e^- \to (c\bar{b})[n] + b + \bar{c}) \langle O^{Bc}(n) \rangle$$

Log-terms appear in short-distance coefficients:

$$\alpha_s^m \sum_{n=0}^{\infty} \alpha_s^n \ln^n (s / m_Q^2)$$

Collinear gluon emission

Spoil or weak the convergence of the series

$\ln(p_t^2 / m_Q^2)$ appearing in the production at a hadron collider

- Perturbative QCD factorization

Involving $\ln(s/\mu_F^2)$

$\mu_F = O(\sqrt{s})$

$$d\sigma(e^+ + e^- \to Bc(p) + b + \bar{c})$$
$$= \sum_i d\hat{\sigma}(e^+ + e^- \to i + X)(p/z, \mu_F) \otimes D_{i\to Bc}(z, \mu_F) + O(m_Q^2/s)$$

Fragmentation function

NRQCD factorization:

$$D_{i\to Bc}(z, \mu_{F0}) = \sum_n d_{i\to c\bar{b}[n]}(z, \mu_{F0}) \langle O^{Bc}(n) \rangle$$

$\mu_{F0} = O(m_Q)$

Involving $\ln(\mu_{F0}^2/m_Q^2)$

Evolution of fragmentation functions

$$\frac{d}{d\ln\mu_F^2} D_{i\to Bc}(z, \mu_F) = \sum_j P_{ij}(z/y, \ \alpha_s(\mu_F)) \otimes D_{i\to Bc}(y, \mu_F)$$

$$P_{ij}(z, \ \alpha_s(\mu_F)) = P_{ij}^{(0)}(z)\frac{\alpha_s(\mu_F)}{2\pi} + P_{ij}^{(1)}(z)\left(\frac{\alpha_s(\mu_F)}{2\pi}\right)^2 + O(\alpha_s^3)$$

Collinear log-terms have been resumed through the DGLAP evolution.

- LO fragmentation functions for the Bc production
  - Extracting from the LO calculation of process $Z^0 \to Bc + b + \bar{c}$

    C.-H. Chang, Y.-Q. Chen, Phys. Rev. D 46, 3845, (1992);

  - Calculating from the definition:

    J.-P. Ma, Phys. Lett. B 332, 398, (1994);

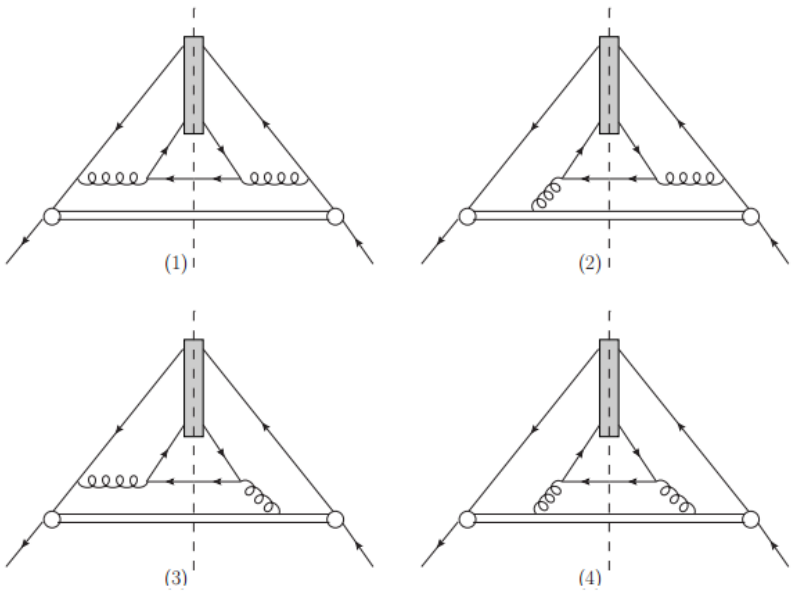
- ◆ There were no NLO results for $D_{i\to Bc}(z,\mu_F)$ before our calculation.
- ◆ In order to obtain the theoretical predictions under the fragmentation approach up to NLO QCD accuracy, the NLO results for $D_{i\to Bc}(z,\mu_F)$ are needed.

Fragmentation function calculation

Based on the definition of FFs by Collins and Soper.

Nucl. Phys. B 194, 445, (1982).

LO cut diagrams:



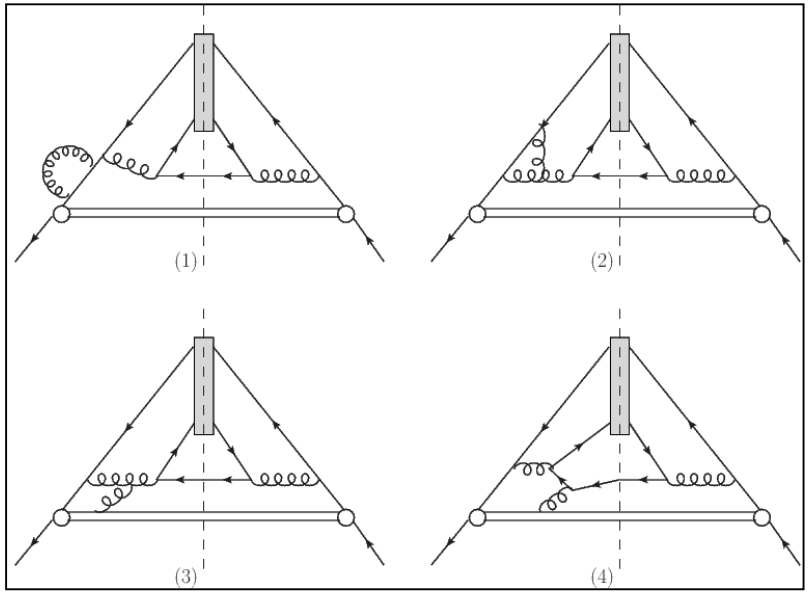
LO fragmentation functions:

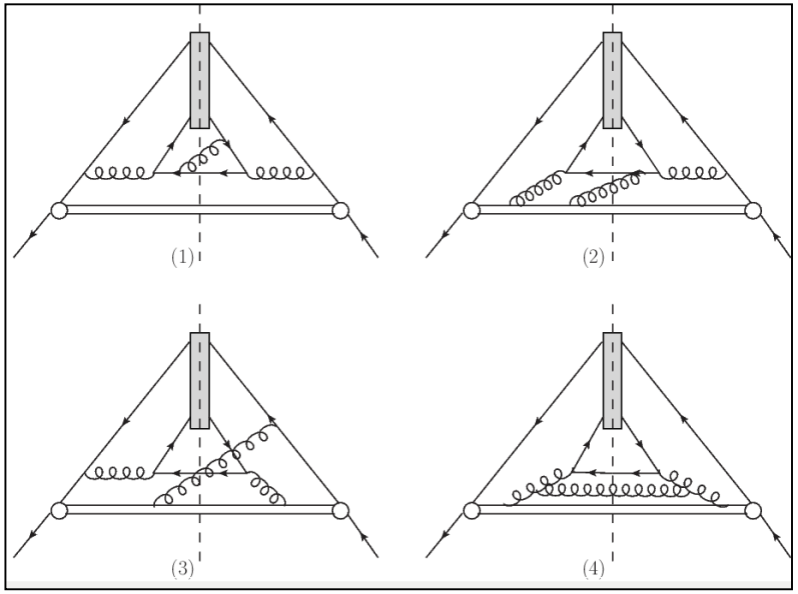
$$D^{\rm LO}_{\bar{b}\to B_c}(z) = \frac{2\alpha_s^2 z(1-z)^2|R_S(0)|^2}{81\pi r_c^2(1-r_b z)^6 M^3}[6-18(1-2r_c)z+(21-74r_c+68r_c^2)z^2 -2r_b(6-19r_c+18r_c^2)z^3+3r_b^2(1-2r_c+2r_c^2)z^4],$$

$$D^{\rm LO}_{\bar{b}\to B_c^*}(z) = \frac{2\alpha_s^2 z(1-z)^2|R_S(0)|^2}{27\pi r_c^2(1-r_b z)^6 M^3}[2-2(3-2r_c)z+3(3-2r_c+4r_c^2)z^2 -2r_b(4-r_c+2r_c^2)z^3+r_b^2(3-2r_c+2r_c^2)z^4].$$

NLO corrections

Typical NLO cut diagrams





54 virtual cut diagrams， 72 real cut diagrams.

➢ Virtual corrections

Tensor reduction, IBP reduction

Many integrals containing an eikonal line, e.g,

$$\int \frac{d^D l}{[(l-p_1)^2-m_1^2+i\varepsilon][(l-p_2)^2-m_2^2+i\varepsilon][(l-p_3)^2-m_3^2+i\varepsilon](l\cdot n+i\varepsilon)}$$

➢ Real corrections

UV and IR divergences!

$$\mathrm{D}^{real}_{\bar{b}\to Bc}(z)=\int N_{CS}\,d\phi_{real}(A_{real}-A_S)+\int N_{CS}\,d\phi_{real}A_S$$

Calculated in 4 dimensions

Calculated in d dimensions

Various types of subtraction terms need to be integrated!

➢ Renormalization

Renormalization of QCD:

$$\delta Z_2^{OS} = -C_F \frac{\alpha_s(\mu_R)}{4\pi}\left[\frac{1}{\epsilon_{UV}} + \frac{2}{\epsilon_{IR}} - 3\,\gamma_E + 3\,\ln\frac{4\pi\mu_R^2}{m^2} + 4\right],$$

$$\delta Z_m^{OS} = -3\,C_F \frac{\alpha_s(\mu_R)}{4\pi}\left[\frac{1}{\epsilon_{UV}} - \gamma_E + \ln\frac{4\pi\mu_R^2}{m^2} + \frac{4}{3}\right],$$

$$\begin{aligned}\delta Z_3^{OS} = \frac{\alpha_s(\mu_R)}{4\pi}\Bigg[&(\beta_0' - 2C_A)\left(\frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}}\right)\\ &-\frac{4}{3}T_F\left(\frac{1}{\epsilon_{UV}} - \gamma_E + \ln\frac{4\pi\mu_R^2}{m_c^2}\right)\\ &-\frac{4}{3}T_F\left(\frac{1}{\epsilon_{UV}} - \gamma_E + \ln\frac{4\pi\mu_R^2}{m_b^2}\right)\Bigg],\end{aligned}$$

$$\delta Z_g^{\overline{MS}} = -\frac{\beta_0}{2}\frac{\alpha_s(\mu_R)}{4\pi}\left[\frac{1}{\epsilon_{UV}} - \gamma_E + \ln\,(4\pi)\right], \qquad (71)$$

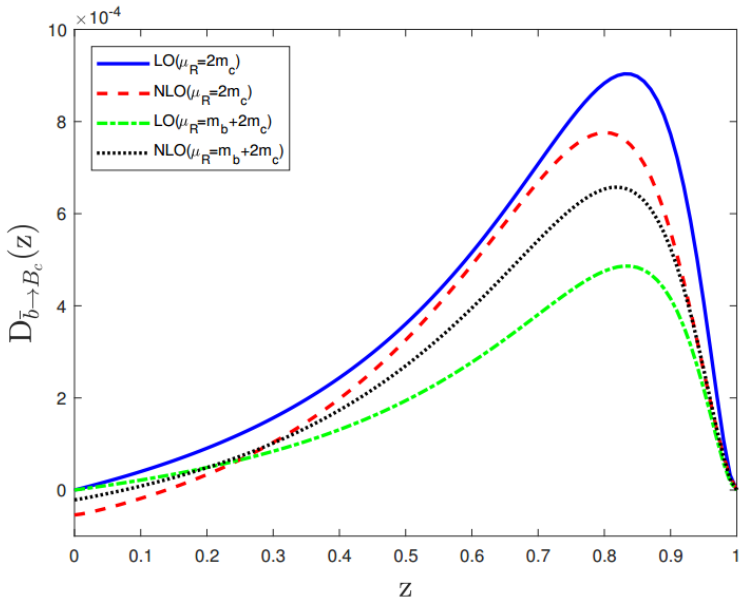
Renormalization of the operator:

$$\begin{aligned}&D^{\text{operator}}_{\bar{b}\to c\bar{b}[n]}(z)\\ &= -\frac{\alpha_s(\mu_R)}{2\pi}\left[\frac{1}{\epsilon_{UV}} - \gamma_E + \ln\,(4\pi) + \ln\frac{\mu_R^2}{\mu_F^2}\right]\\ &\times\int_z^1 \frac{dy}{y} P_{\bar{b}\bar{b}}(y) D^{\text{LO}}_{\bar{b}\to c\bar{b}[n]}(z/y),\end{aligned}$$

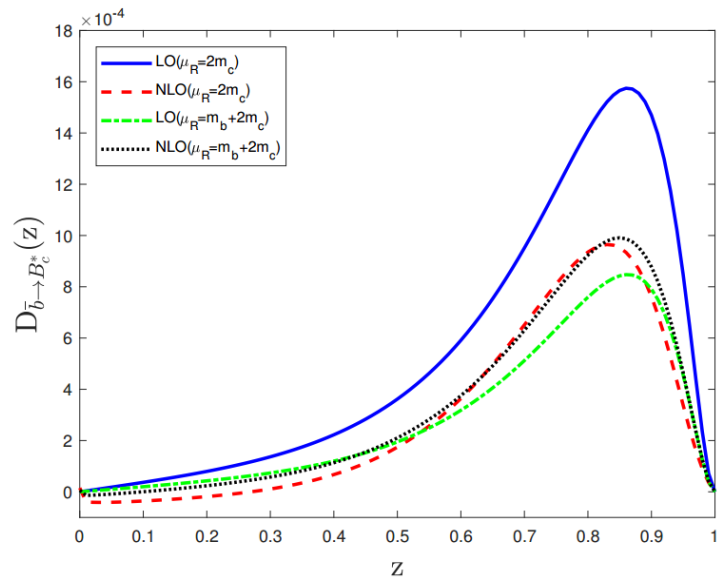
NLO results

Phys. Rev. D 100, 034004, (2019),
X.-C. Zheng, C.-H. Chang, X.-G. Wu.

NLO fragmentation functions



$$D_{\bar{b}\to Bc}(z,\mu_{F0}=m_b+2m_c)$$

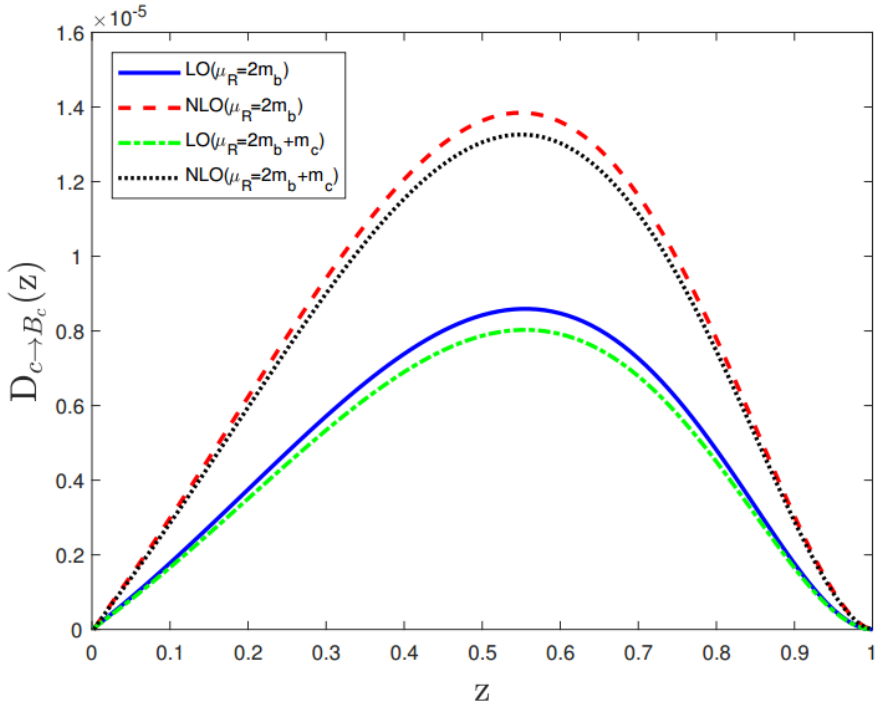


$$D_{\bar{b}\to B_c^*}(z,\mu_{F0}=m_b+2m_c)$$

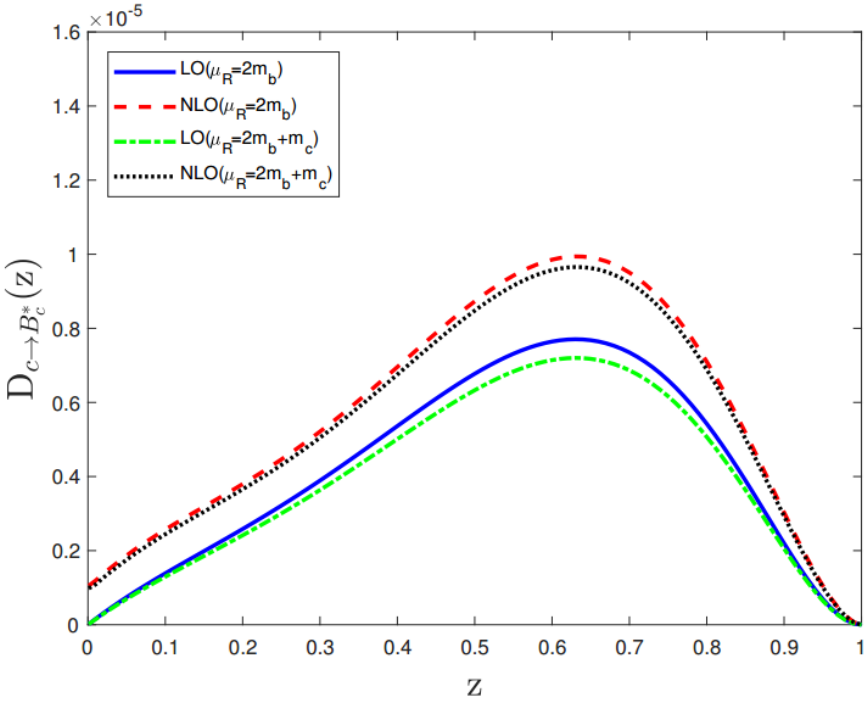
These fragmentation functions can be studied at the CEPC.

NLO results

Initial fragmentation functions



$$D_{c\to Bc}(z,\mu_{F0}=m_b+2m_c)$$



$$D_{c\to B_c^*}(z,\mu_{F0}=m_b+2m_c)$$

NLO results

Fragmentation probability and average value of z

$$P = \int_0^1 dz\, D(z), \qquad \langle z \rangle = \frac{\int_0^1 dz\, z\, D(z)}{\int_0^1 dz\, D(z)},$$

$\bar{\mathrm{b}} \to Bc$

μ_R	$\mathrm{P} \times 10^4$(LO)	$\mathrm{P} \times 10^4$(NLO)	$\langle z \rangle$(LO)	$\langle z \rangle$(NLO)
$2m_c$	3.82	3.14	0.68	0.70
$m_b + 2m_c$	2.05	2.73	0.68	0.69

$\bar{\mathrm{b}} \to B_c^*$

μ_R	$\mathrm{P} \times 10^4$(LO)	$\mathrm{P} \times 10^4$(NLO)	$\langle z \rangle$(LO)	$\langle z \rangle$(NLO)
$2m_c$	5.36	2.91	0.73	0.77
$m_b + 2m_c$	2.89	3.25	0.73	0.74

$c \to Bc$

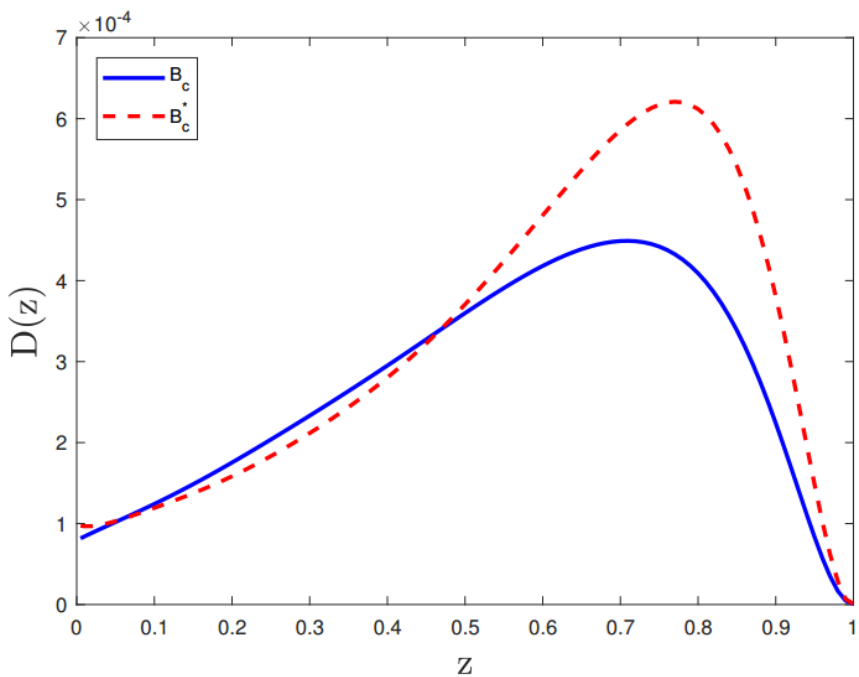
μ_R	$\mathrm{P} \times 10^6$(LO)	$\mathrm{P} \times 10^6$(NLO)	$\langle z \rangle$(LO)	$\langle z \rangle$(NLO)
$2m_b$	4.95	8.07	0.51	0.51
$2m_b + m_c$	4.63	7.72	0.51	0.51

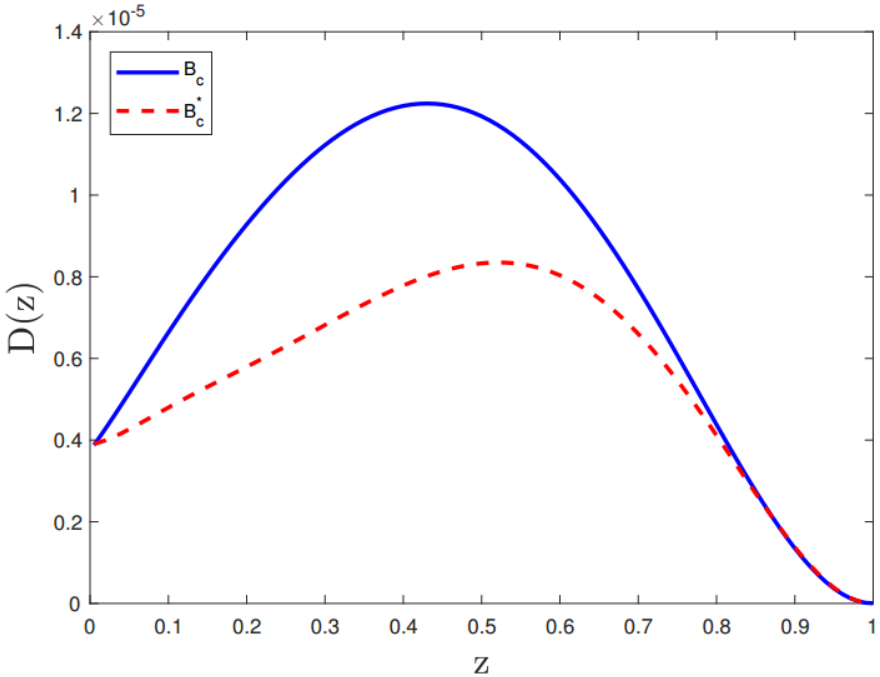
$c \to B_c^*$

μ_R	$\mathrm{P} \times 10^6$(LO)	$\mathrm{P} \times 10^6$(NLO)	$\langle z \rangle$(LO)	$\langle z \rangle$(NLO)
$2m_b$	4.28	5.75	0.55	0.54
$2m_b + m_c$	4.00	5.57	0.55	0.54

NLO results

The fragmentation functions at the scale of m_Z





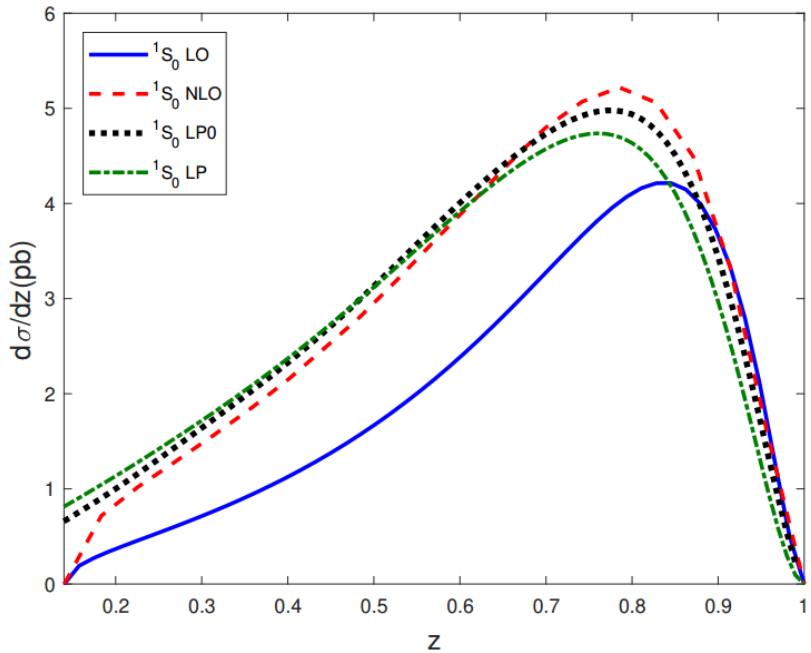
$$D_{\bar{b}\to Bc(Bc^*)}(z,\mu_F=m_Z)$$

$$D_{c\to(Bc)B_c^*}(z,\mu_F=m_Z)$$

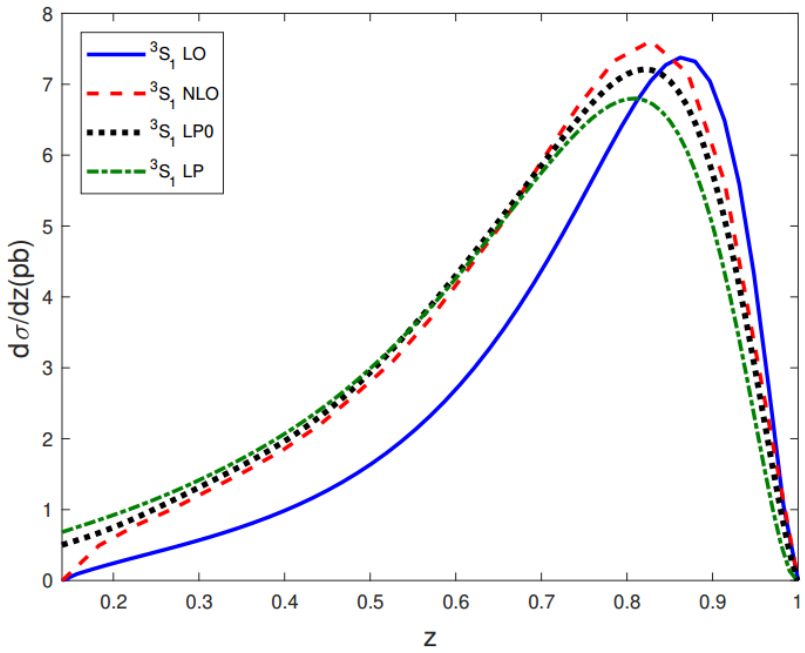
- Obtained through the DGLAP evolution from the fragmentation functions at the initial factorization scale.

Bc and Bc* production at the Z pole

Phys. Rev. D 100, 034004, (2019),
X.-C. Zheng, C.-H. Chang, X.-G. Wu.



$d\sigma / dz(Bc)$



$d\sigma / dz(Bc^*)$

LO,NLO: fixed-order approach

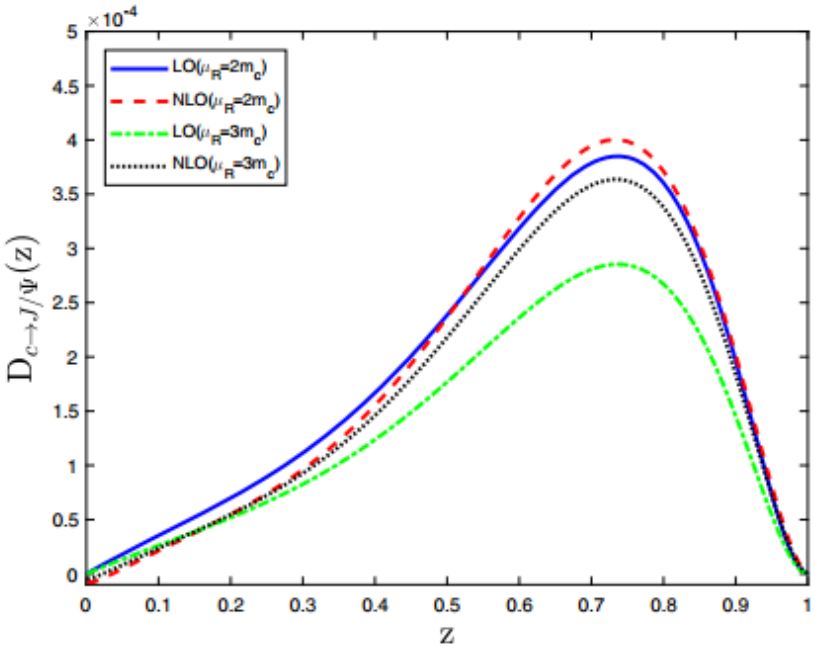
LP0： fragmentation approach, no DGLAP evolution

LP： fragmentation approach, evolved with DGLAP equation

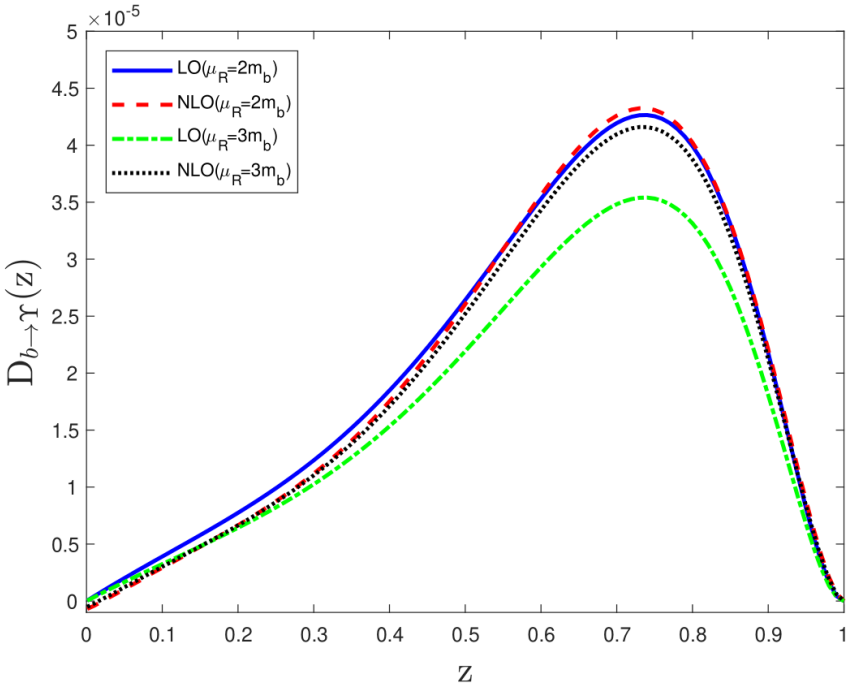
NLO results

Phys. Rev. D 100, 014005, (2019),
X.-C. Zheng, C.-H. Chang, X.-G. Wu.

NLO fragmentation functions for $c \to J/\psi$ and $b \to \Upsilon$



$D_{c\to J/\psi}(z, \mu_{F0} = 3m_c)$



$D_{b\to\Upsilon}(z, \mu_F = 3m_b)$

Conclusions

- The production of **doubly heavy hadrons** at an e^+e^- collider running at the **Z pole** is studied;
- $sin^2\theta^f_{eff}$ can be determined through measuring the asymmetries of the production of the doubly heavy hadrons at the Z pole;
- The **fragmentation functions** for a heavy quark to a doubly heavy hadron can be studied at the CEPC running at the Z pole.

Thank you！